Ray Optics

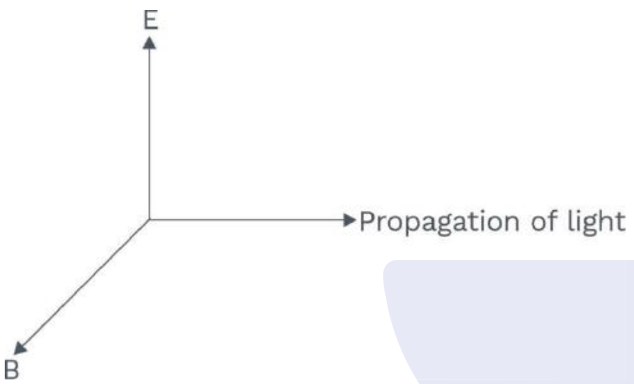
DISCLAIMER

Ray Optics

Properties Of Light

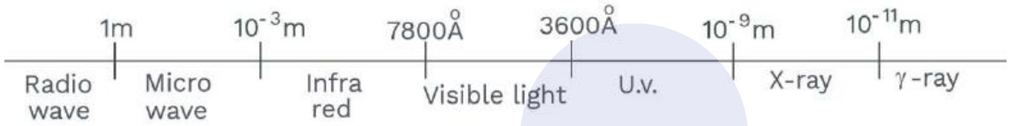
(i) Speed of light in vacuum is equal to 3×10^8 m/s approximately.

(ii) Light is electromagnetic wave. It consists of varying electric field and magnetic field.



(iii) Light carries energy and momentum.

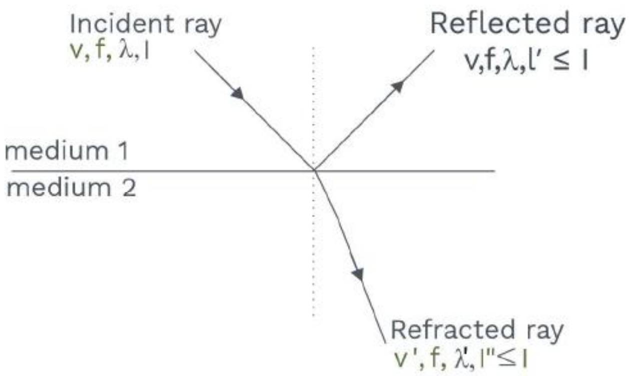
(iv) The formula = λ is applicable to light.



Electromagnetic spectrum

(v) When light gets reflected in same medium, it suffers no change in frequency, wavelength and speed.

(vi) Frequency of light remains unaffected when it gets reflected or refracted.



KEY POINTS

- Lights
- Electromagnetic wave
- Electromagnetic spectrum
- Ray Optics

Concept Reminder

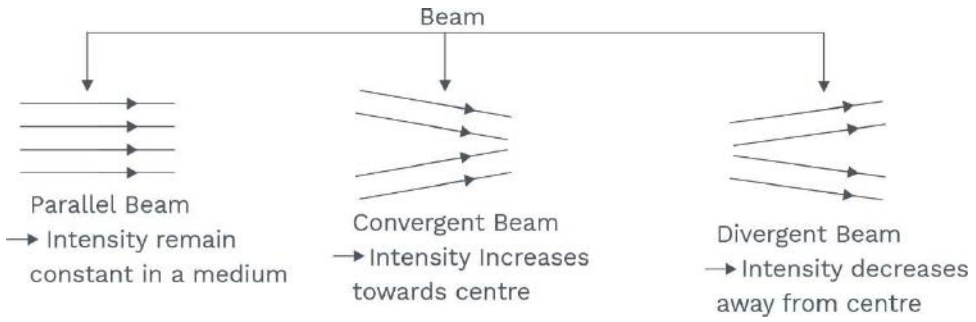
Electromagnetic radiation belonging to this region of the spectrum (wavelength of about 400 nm to 750 nm) is called light. It is mainly through light and the sense of vision that we know and interpret the world around us.

Concept Reminder

Frequency of light remains unchanged when it gets reflected or refracted.

Terms Used In Ray Optics :-

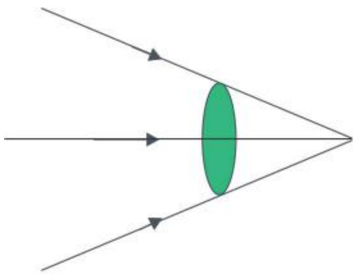
(a) Beam : Collection of bunch of Rays



Ray : The straight line path along which the light travels in a homogeneous medium is called a ray. It is symbolized by an arrow head on a straight line, the arrow head represents the direction of propagation of light.

Beam : A bundle or bunch of rays is called a beam. It is of following three types :

(a) Convergent beam : In this case diameter of beam decreases in the direction of ray.



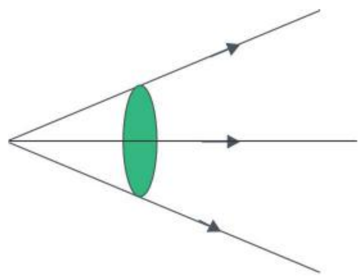
(b) Divergent beam : It is a beam in which all the rays meet at a point when produced backward and the diameter of beam goes on increasing as the rays proceed forward.

KEY POINTS

- Ray
- Beam
- Convergent beam
- Divergent beam
- Parallel beam

Concept Reminder

A light wave can be considered to travel from one point to another, along a straight line joining them. The path is called a ray of light, and a bundle of such rays constitutes a beam of light.



Concept Reminder

There are three types of beam-
(a) Convergent beam
(b) Divergent beam
(c) Parallel beam

(c) Parallel beam : It is a beam in which all the rays constituting the beam move parallel to each other and diameter of beam remains same.

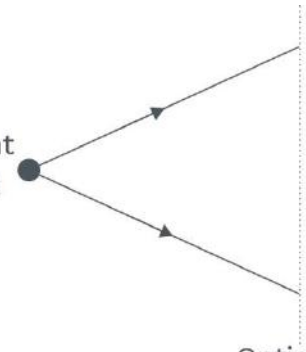
Concept Reminder

There are two things that we can intuitively mention about light from common experience. First, that it travels with enormous speed and second, that it travels in a straight line

Note : A very narrow beam is called pencil of light.

Object : It is decided by Incident Ray.

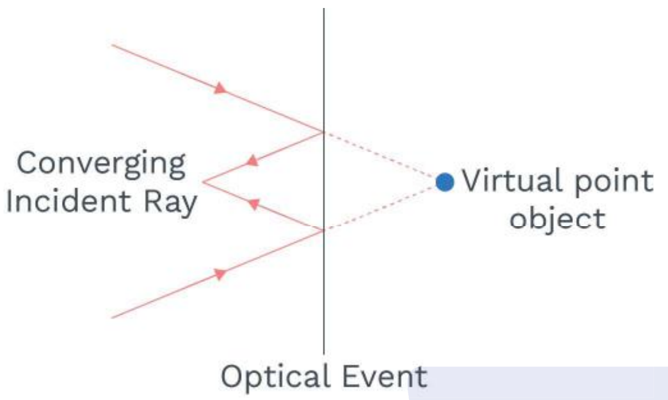
(i) Real Point Object



KEY POINTS

- Real object
- Virtual object
- Real image
- Virtual image

(ii) Virtual Point Object:



Concept Reminder

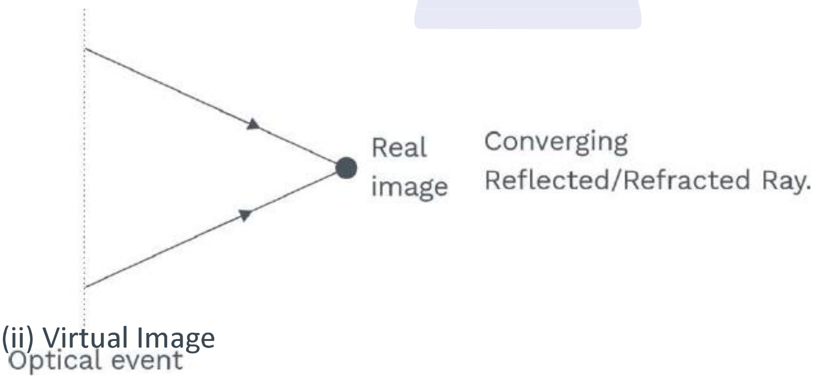
Object and image both can be of two types –
Real and Virtual

Concept Reminder

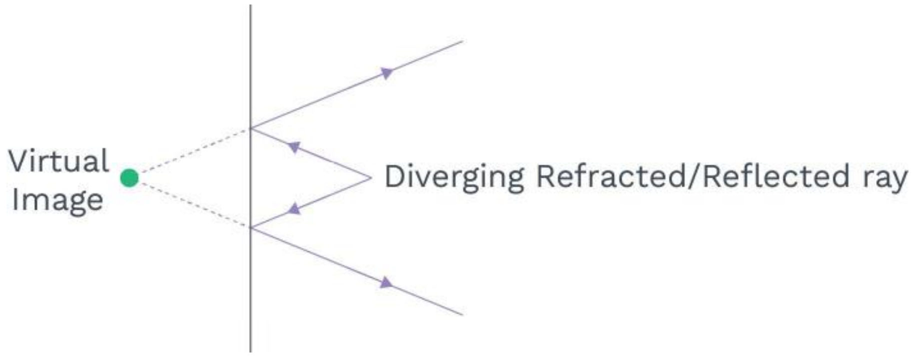
In physics, the difference between a real image vs virtual image is that a real image is formed when light converges at a point – like when you look at an apple on your desk – whereas a virtual image is formed from two divergent rays of light that never meet.

Image : It is decided by Reflected/Refracted Ray.

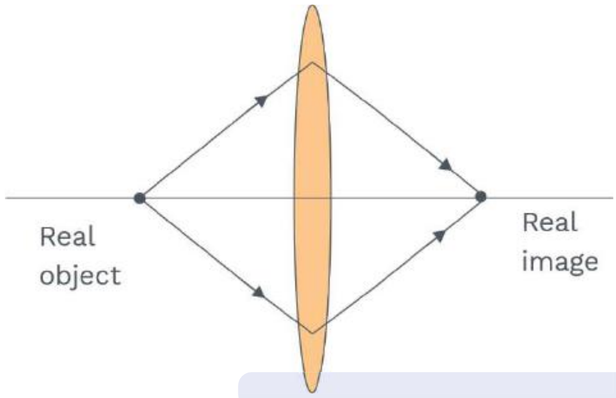
(i) Real Image



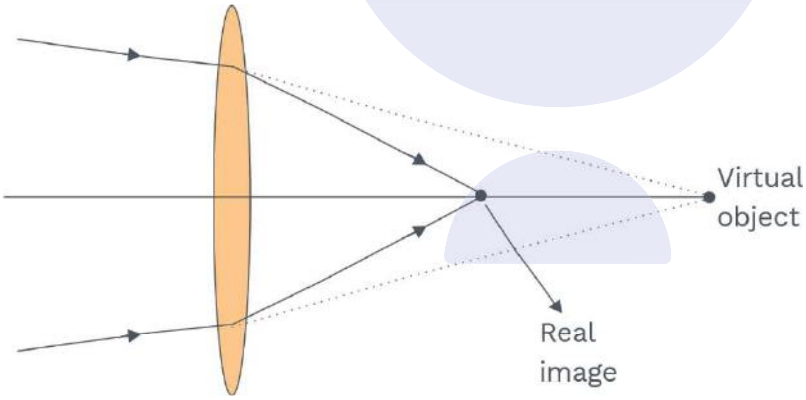
(ii) Virtual Image



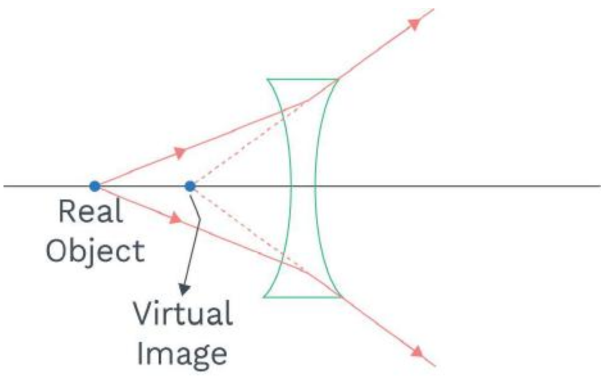
Example :-
(i)



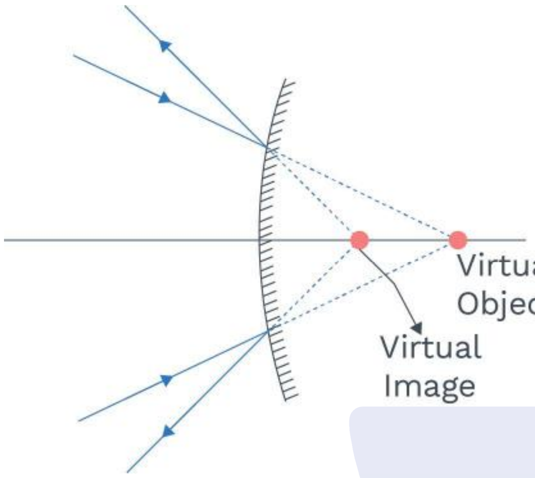
(ii)



(iii)



(iv)

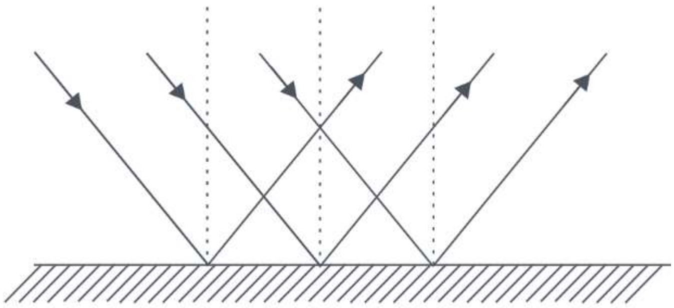


Reflection Of Light:-

When light rays strike the border of two media such as air and glass, a part of light is turned back into the same medium. This is called Reflection of Light.

(a) Regular Reflection:

When the reflection from a perfect plane surface it is known as Regular Reflection. In this case the reflected light has larger intensity in one direction and insignificantly small intensity in other directions.



(b) Diffused Reflection

When the surface is tough, we do not get a regular behaviour of light. While at each point light ray

Definitions

When light rays strike the boundary of two media such as air and glass, a part of light is turned back into the same medium. This is called Reflection of Light.

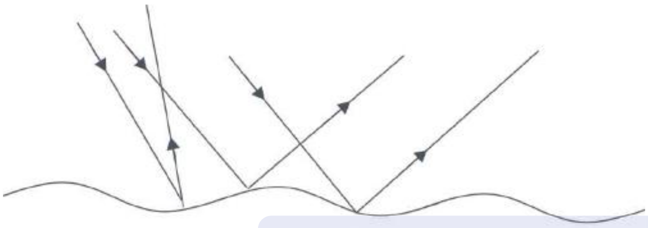
Concept Reminder

There are two types of reflections :-

(i) Regular reflection

(ii) Diffused reflection

gets reflected irrespective of the overall nature of surface, difference is observed because even in a narrow beam of light there are several rays which are reflected from different points of surface and it is fairly possible that these rays may transfer in different directions due to irregularity of the surface plane. This process allows us to see an object from any position. Such a reflection is known as diffused reflection.

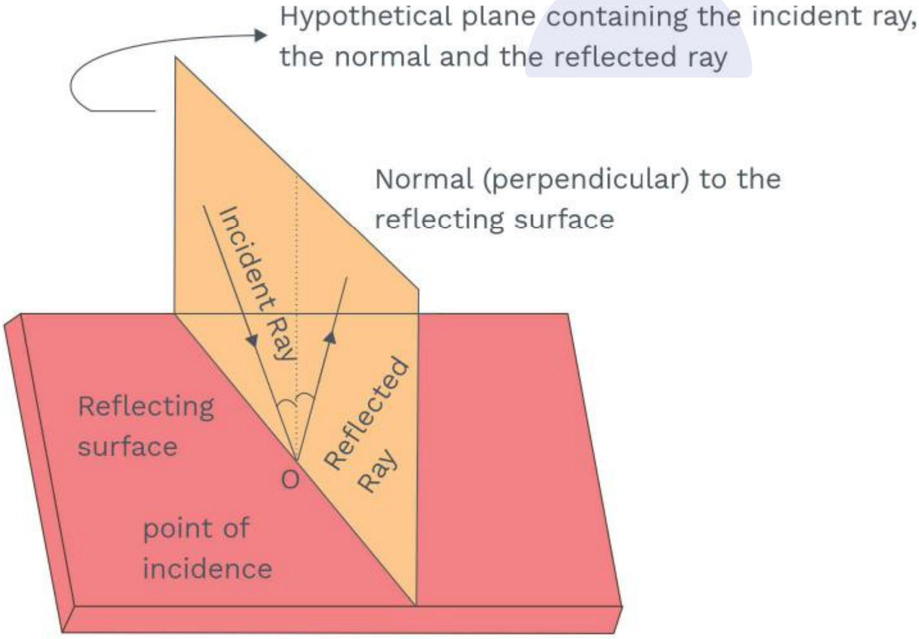


Diffused Reflection

For example: reflection from a news paper, from a wall etc. This is why we can not see our face in news paper and in the wall.

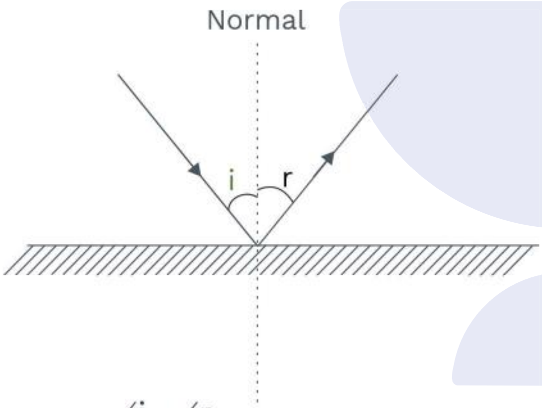
Laws of Reflection:-

(a) The reflected ray, the incident ray and the normal at the point of incidence lie in the same plane.



This plane is known as the plane of incidence (or plane of reflection). This condition can be expressed mathematically as $\vec{|}$ $\vec{m}\times\vec{4}$ $=\vec{4}$ $\vec{m}\times\vec{|}$ $=\vec{m}$ $\vec{4}\times\vec{|}$ $=$ where $\vec{4}$, $\vec{n}$ and $\vec{|}$ ≠ are vectors of any magnitude along the normal, incident ray and reflected ray respectively.

(b) The incident angle (the angle between normal and the incident ray) and the angle of reflection (the angle between the reflected ray and the normal) are equal, i.e.

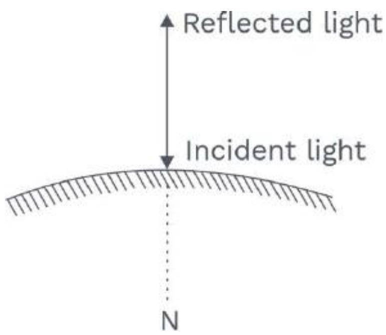


$\angle i = \angle r$

Special Cases :

Normal Incidence: In case light is incident normally,

$$i = r = 0$$



$\delta =$ °

Concept Reminder

According to law of reflection :-

(i) Incident ray, reflected ray and normal all lie in same plane.

(ii) $\angle$ = $\angle$

Rack your Brain

A watch shows time as 3 : 25 when see through a mirror, time appeared will be:

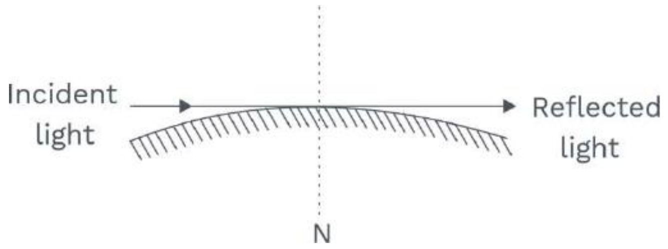
(1) 8 : 35
(2) 9 : 35
(3) 7 : 35
(4) 8 : 25

Note : We say that the ray has retraced its path.

Grazing Incidence: In case light strikes the reflecting surface tangentially,

$$i = r = 90°$$

$$\delta = \quad ° \qquad °$$



Note : In case of reflection speed (magnitude of velocity) of light remains unchanged and Grazing incidence velocity remains unchanged.

Ex. For the given figure show that for a light ray incident at an angle 'i' on getting reflected the angle of deviation is $\delta = \pi - \qquad +$

Sol.

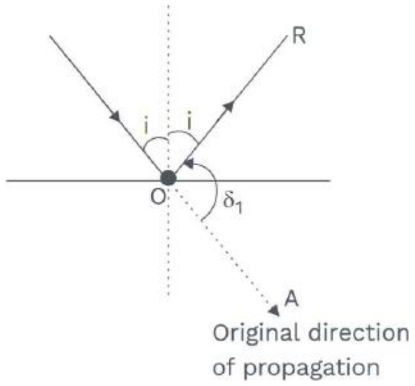


Figure (a)

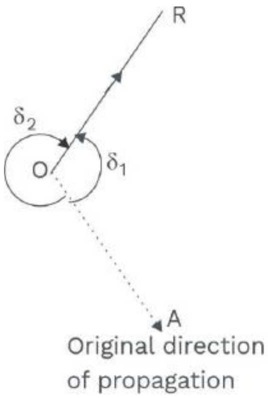


Figure (b)

Concept Reminder

Deviation produced by plane mirror on incident ray is (180°-2i).

From figure (b) it is clear that light ray bends either by δ anticlockwise

or by $\delta \;=\; \pi - \delta$ clockwise.

From figure (a) $\delta = \pi -$.

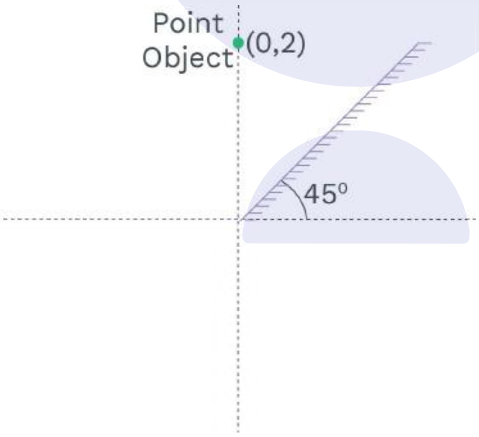
$\therefore \quad \delta = \pi +$

Plane Mirror

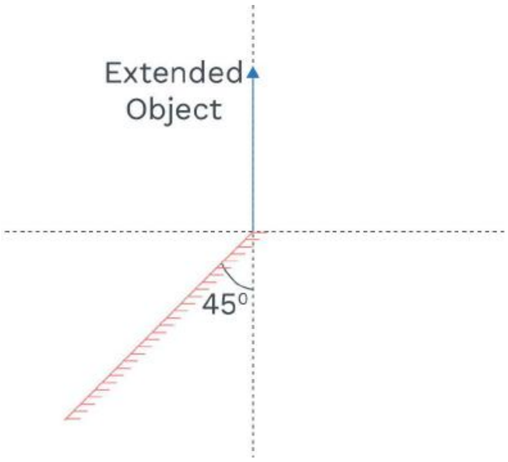
- A plane mirror has perfectly plane reflecting surface
- The image created by a plane mirror suffers lateral-inversion.
- In plane mirror image is just opposite to the object having same perpendicular distance from mirror as that of object.
- Lateral Magnification of a plane mirror is +1
- A plane mirror forms real image of virtual object or virtual image of real object.

Ex. Locate the image of following object?

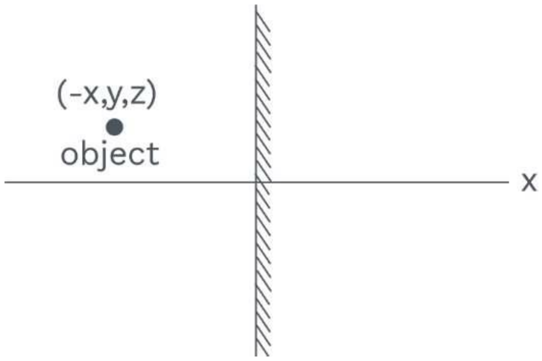
(i)



(ii)

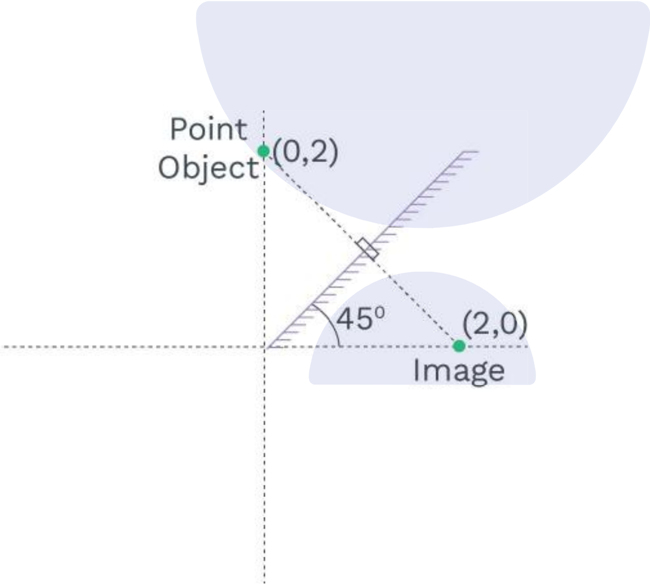


(iii)

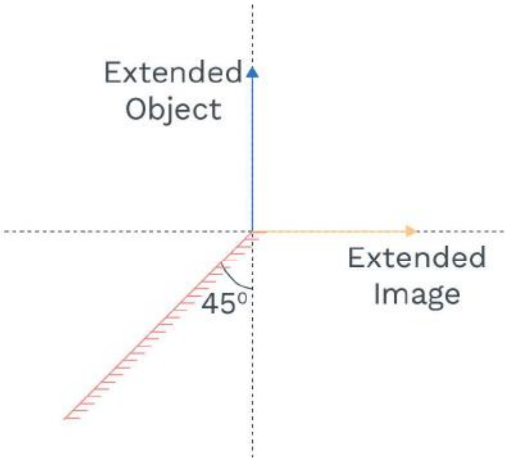

(-x,y,z)
object
x

Sol.

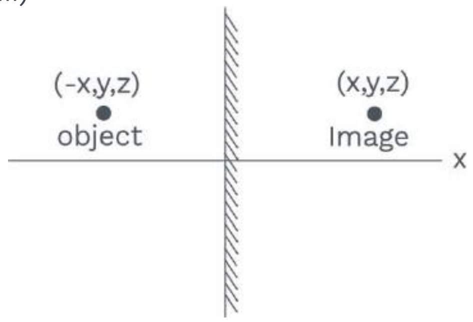
(i)


Point
Object
(0,2)
45°
(2,0)
Image

(ii)


Extended
Object
Extended
Image
45°

(iii)



Ex. A plane mirror is inclined at an angle of 45° with the horizontal and mirror starts from the origin, an object is kept at x = – 2 cm. Locate its image

Sol.

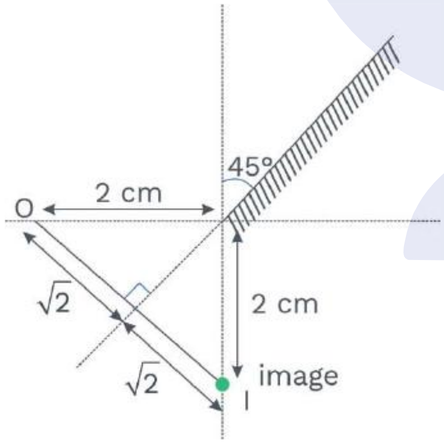
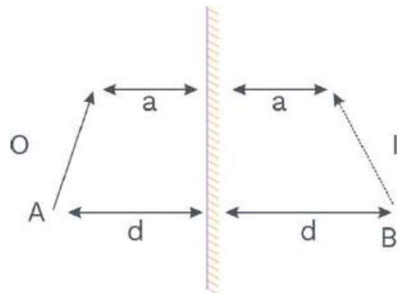


Image of an extended linear object:-

Draw the images of the extreme points and merged them with a straight line



Concept Reminder

- The image is erect, if the extended object is placed parallel to the mirror.
- The image is inverted if the extended object lies perpendicular to the plane mirror.

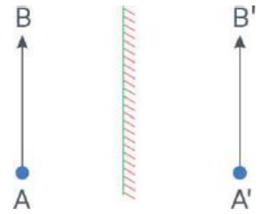
Concept Reminder

Image formation needs regular reflection / refraction. In principle, all rays from a given point should reach the same image point. This is why you do not see your image by an irregular reflecting object, say the page of a book.

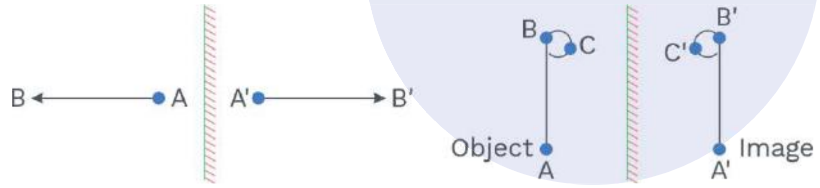
Characteristic of image of an extended object, formed by a plane mirror :

(1) Size of expanded object = size of expanded image.

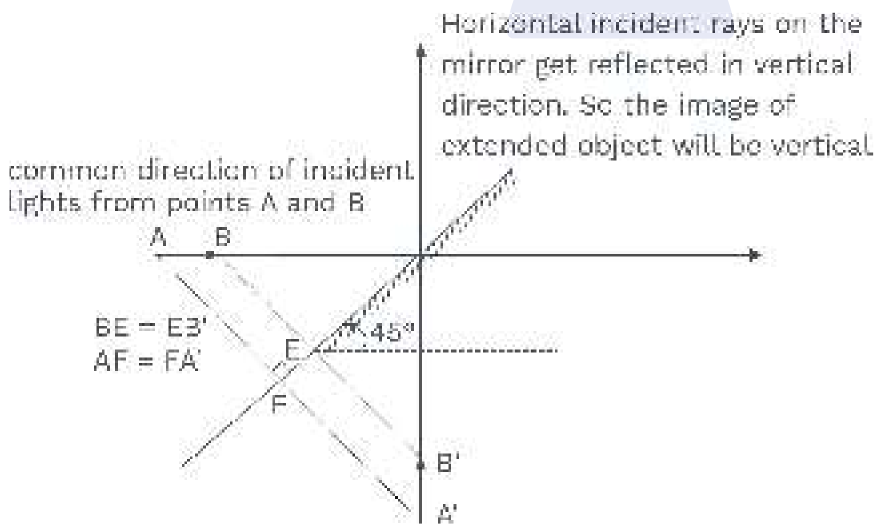
(2) The image is erect, if the expanded object is placed parallel to the mirror.

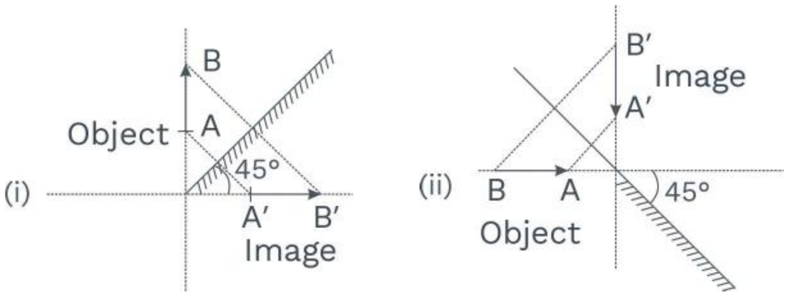


(3) The image is inverted if the expanded object lies perpendicular to the plane mirror.



(4) If an expanded horizontal object is placed in front of a mirror inclined at 45° with the horizontal, the image formed will be vertical. See figure.





Ex. Find the time observed by person in the image of clock formed by plane mirror?

(i) Actual time : 4 : 00

(ii) Actual time : 8 : 30

(iii) Actual time : 9 : 45 : 44

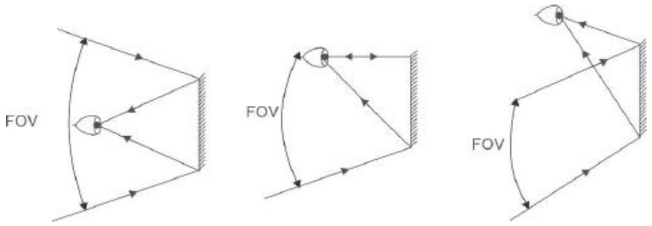
Sol.

(i) observed time : $\frac{-}{\quad}$

(ii) observed time $\frac{-}{\quad}$

(iii) observed time $\frac{-}{\quad}$

Field of View :
The extent of the scene that is reflected to the observer.



Rack your Brain

Two mirrors at an angle θ produce 5 images of a point. the number of images produced when θ is decreased to 30° is:

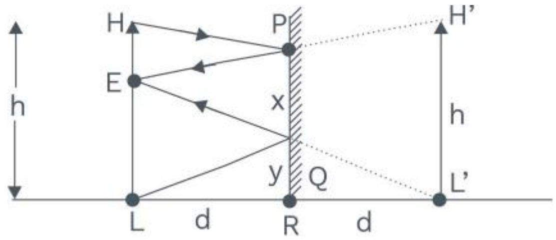
(1) 9
(2) 10
(3) 11
(4) 12

Concept Reminder

- The minimum size of mirror required by a person to view image of full back wall of height h, distance between wall and mirror is d and distance between person and mirror is x.

$\Rightarrow \quad = \frac{}{\quad + \quad}$

Minimum size of a mirror required by a person to observe his complete image.



(a) ΔY(~   ΔYl x

— = —— ⇒   = — (Minimum length of mirror required)

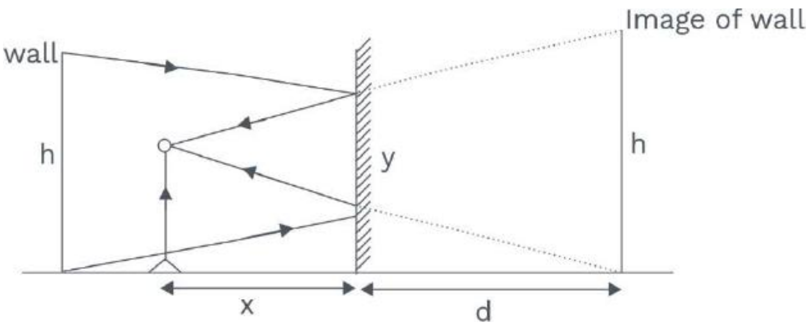
(b) Δ~ x | ΔYx x

$\frac{}{Yx}$ = —— ⇒ = $\frac{Yx}{}$ (Mirror should be placed half of the eye level above the ground)

Note: This Result is independent of the distance of person from the mirror

Ex. Find the minimum size of mirror required by a person to view image of full back wall of height h, distance between wall and mirror is d and distance between person and mirror is x.

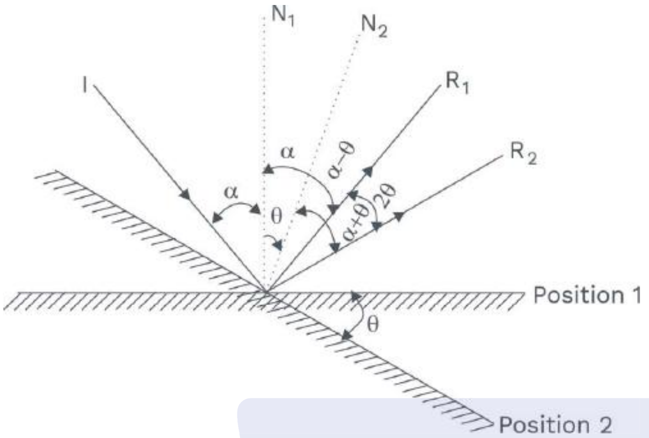
Sol.



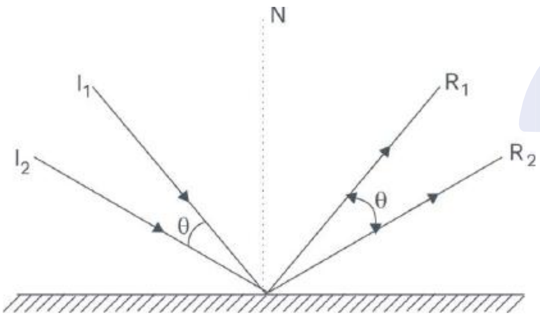
— = $\frac{}{+}$

⇒ = $\frac{}{+}$

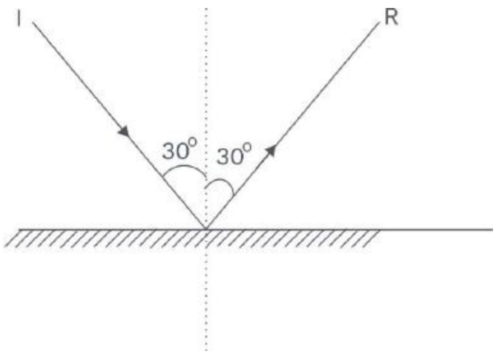
Rotation of plane mirror



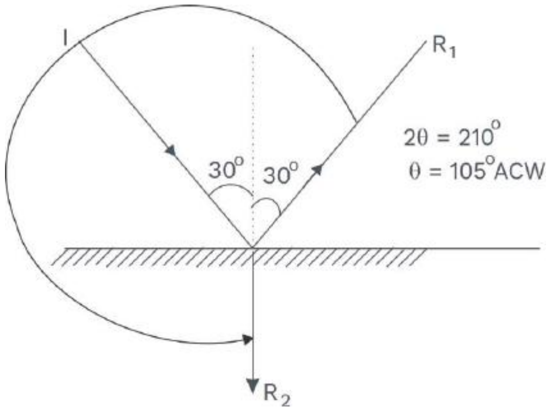
- Keeping the incident ray fixed. If the mirror is rotated by some angle θ, then the reflected ray rotates by double the angle θ in the same sense.
- If mirror is kept fixed & Incident ray is rotated by some angle then reflected ray rotated by the same angle but in opposite sense.



Ex. By what angle mirror should be rotated to obtain Reflected ray along negative y-axis?



Sol.

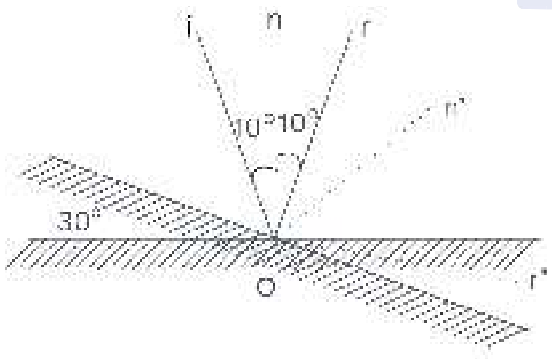


Concept Reminder

If mirror is kept fixed & Incident ray is rotated by some angle then reflected ray rotated by the same angle but in opposite sense.

Ex. Light falls on a plane mirror at an angle of incidence of 10°. The ray reflected from the mirror is r. If the mirror is now rotated through an angle 30°, the new reflected ray is r'. What is the angle between r & r'?

Sol. Let the normal of the mirror in the new position be n' then ∠ = ° Hence ∠ = °.



hence

∠ = °

According to laws of reflection ∠ = ∠

Here, in the figure :

∠ = °

Hence

∠ = ∠ + ∠ = °

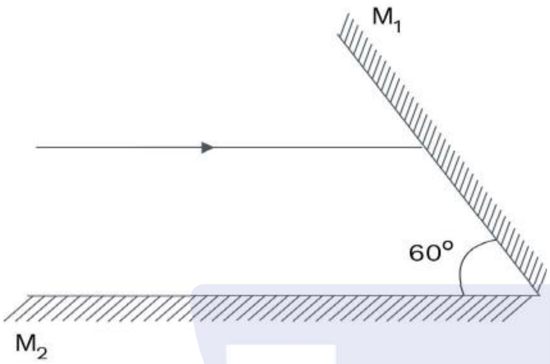
Rack your Brain

A beam of light from a source L is incident normally on a plane mirror fixed at a certain distance x from the source. The beam is reflected back as a spot on a scale placed just above the source I. When the mirror is rotated through a small angle θ, the spot of the light is found to move through a distance y on the scale. The angle θ is given by:

(1) — (2) ——

(3) — (4) —

Ex. As shown in figure two plane mirrors are inclined at an angle of 60° as shown in figure. A light ray incident on one mirror is parallel to the other. What will be the angle between the first incident ray and the last reflected ray?

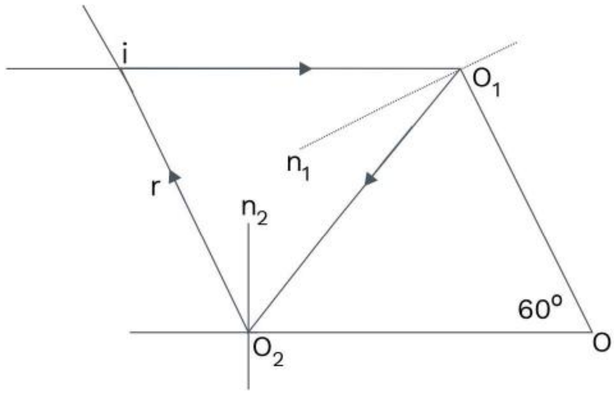


Sol. Given ∠8 88 = °

Hence ∠ = °

According to laws of reflection ∠ = ∠

So ∠ = °



According to geometry ∠ = °

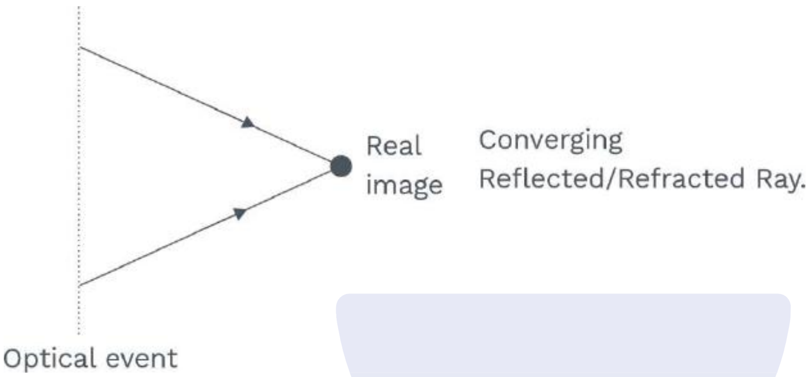
According to law of reflection ∠ = ∠

Hence ∠ = °

Hence angle between incident and reflected ray = 120°

Ex. A light incident normally on a plane mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away when mirror is rotated by 3.5°?

Sol.



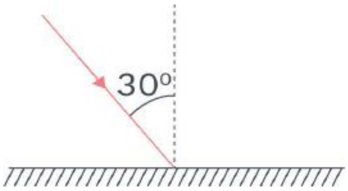
$\theta = \quad \times \frac{\pi}{}$

$= \theta$

$= \frac{\pi}{} \times \quad = \frac{\pi}{}$

= 18.31 cm

Ex. Find out the angle by which the mirror must be rotated such that the reflected ray becomes vertical.



Sol. The diagram shows the four ways in which the reflected ray can become vertical.

For case 1 :

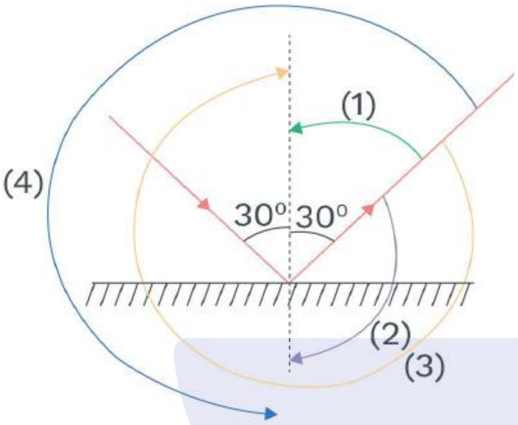
Angle by which Reflected ray rotated = 30°

Angle by which the mirror rotates $= \frac{^{\circ}}{} = \quad ^{\circ}$

(Anticlockwise)

For case 2 :

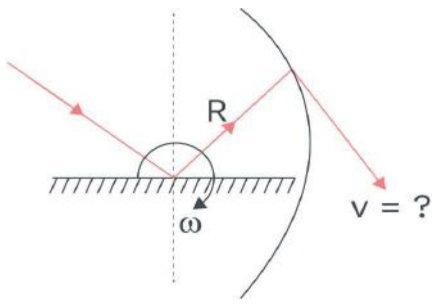
Angle by which the Reflected ray rotated = 150°
Angle by which the mirror rotates = 75°
(clockwise)



For case 3 :
Angle by which the Reflected ray rotated = 300°
Angle by which the mirror rotates = 150°
(clockwise)

For case 4 :
Angle by which the Reflected ray rotated = 210°
Angle by which the mirror rotates = 105°
(Anticlockwise)
But case (2) & case (3) are not possible as the incident ray falls on the polished part of mirror. after rotation of mirror.
∴ Answer is 15° (Anticlockwise) and 105°
(Anticlockwise)

Ex. Mirror is located at the centre of a sphere and it is rotating with an angular speed ω . Incident light strike on the mirror at the centre of the sphere. Find the linear speed of the light spot on the sphere?



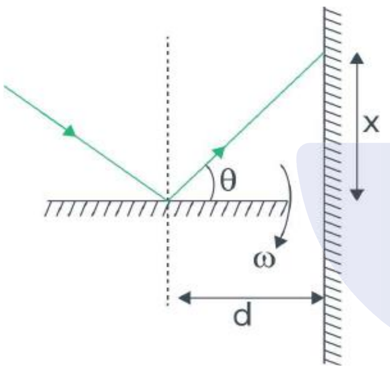
Sol. Angular speed of mirror = ω

Angular speed of Reflected Ray = ω

Speed of light spot on the mirror : ω |

Ex. In the previous example instead of spherical wall there is a vertical wall at a perpendicular distance d from the point where the light is incident.

Sol.



$$\theta = — \Rightarrow \quad = \qquad \theta$$

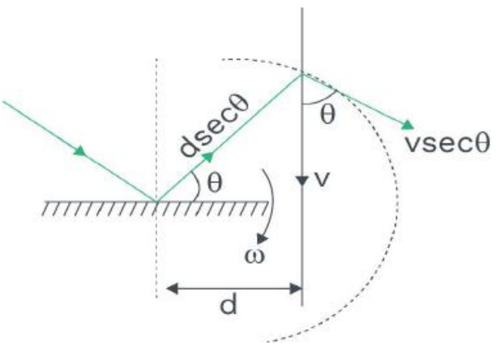
$$— = \qquad \theta \frac{\theta}{}$$

$$= \quad \omega \qquad \theta$$

$$\left(\because \frac{\theta}{} = \quad \omega \right)$$

OR

Considering an instantaneous circle of radius θ .



Concept Reminder

A plane mirror is a flat mirror that reflects light and produces a virtual image without the interference of an inward or outward curve.

Rack your Brain

An object is at a distance of 0.5 m in front of a plane mirror. Distance between the object and image is:

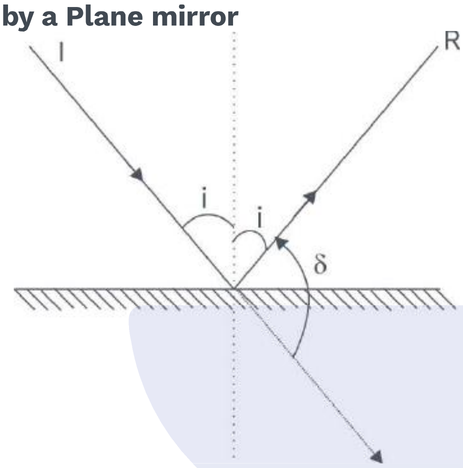
(1) 0.5 m (2) 1 m

(3) 0.25 m (4) 1.5 m

$= \omega \quad \theta$ ($\omega \quad \theta$ is a component of v.)

$$\Rightarrow \quad = \frac{\omega \quad \theta}{\theta} = \omega \quad \theta$$

Deviation produced by a Plane mirror



$\delta = \quad ^\circ -$

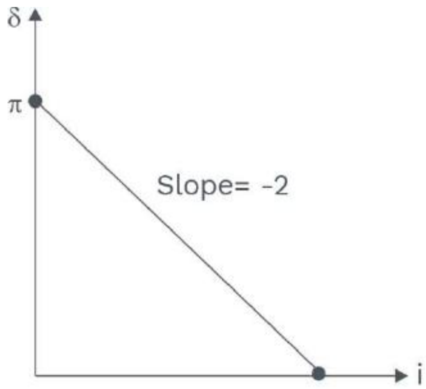
Case 1 When i = 0

$\delta = \quad ^\circ$

Case 2 When i = 90°

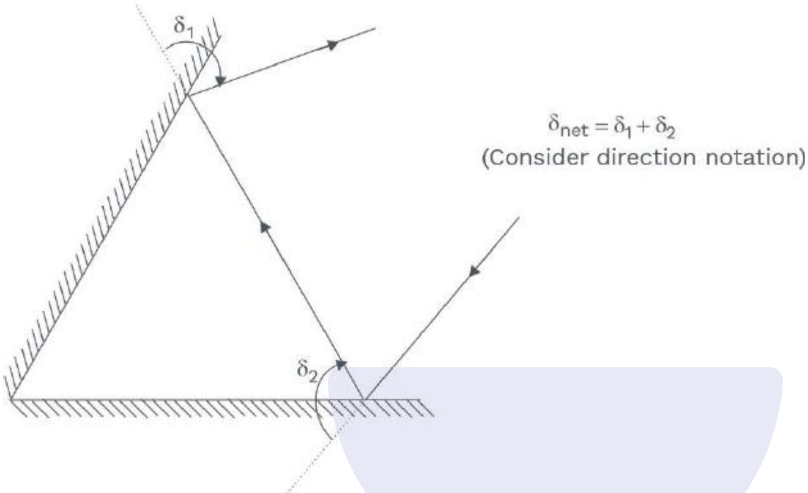
$\delta = \quad ^\circ$

$\delta = -2i + 180^\circ$



Multiple Deviation

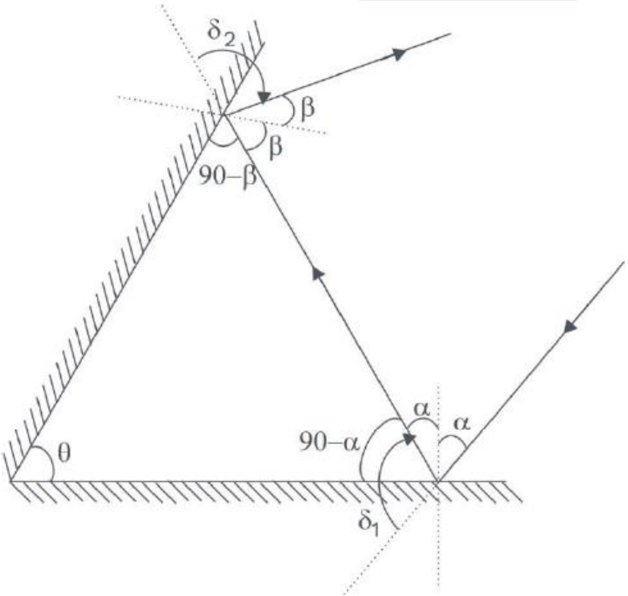
(a)



$\delta \quad = \delta \; + \delta$ (Consider direction notation)

Note :

(1) Deviation produced by two mirror after two successive reflection is "360- θ", where "θ" is the angle between



$\delta \; = \quad - \; \alpha \, ; \; \delta \; = \quad - \; \beta$

$\delta \quad = \delta \; + \delta \; = \qquad - \quad \alpha + \beta \; = \qquad - \;\; \theta$ Where $\theta = \alpha + \beta$

(2) Light retrace its path if it strike any mirror perpendicularly or normally.

(3) If two mirrors are perpendicular then incident ray & emergent ray are always anti-parallel to each other, irrespective of any angle of incidence

$\delta \quad = \qquad ^\circ$

Ex. Two mirrors (plane) are inclined at an angle 'θ' with each-other. A light ray strikes one of them. Find out its deviation after it has been reflected twice-one from each mirror.

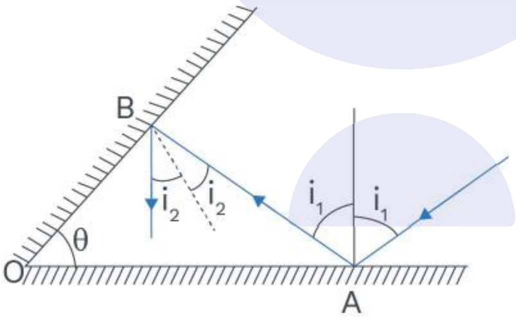
Sol. **Case I :**

δ = clockwise deviation at

A = 180° – $2i_1$

δ = anticlockwise deviation at

B = 180° – $2i_2$



Now, from $\Delta\, 8\, AN$, we have

$\angle$ BOA + $\angle$ OAB + $\angle$ ABO = 180°

θ + (90° – i_1) + (90° + i_2) = 180°

$\Rightarrow \qquad - \quad = \theta$

As $\quad > \quad \delta \; < \delta$

Hence, the net angle (anticlockwise) deviation $\qquad ^\circ - \qquad - \qquad ^\circ -$

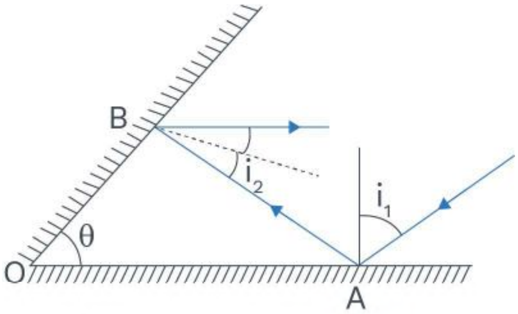
$= \qquad - \qquad = \theta$

δ = Clockwise deviation at A = 180° – $2i_1$

δ = Anticlockwise deviation at B = 180° – $2i_2$

Now, from Δ OAB, we have that

θ + (90° – i_1) + (90° – i_2) = 180°



$\Rightarrow$ + = θ

Hence, net clockwise deviation = δ + δ

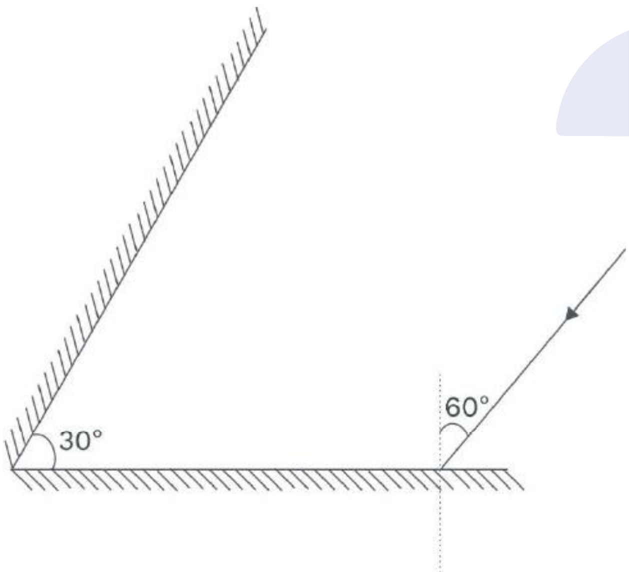
= (180° – $2i_2$) + (180° – $2i_1$)
= 360° – $2(i_1 + i_2)$
= 360° – θ

$\Rightarrow$ Net anticlockwise deviation = 360° – (360° – θ) = θ

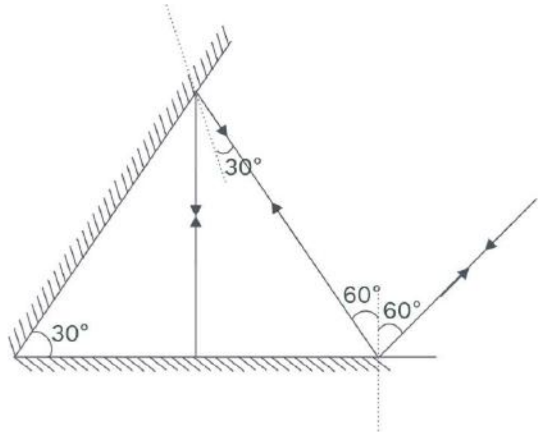
Concept Reminder

Deviation produced by two mirror after two successive reflection is "360- 2θ ", where "θ" is the angle between mirrors

Ex. Find net deviation



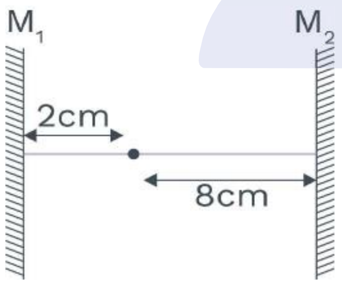
Sol. δ_{net} = 180° (Light retrace its path after 3 reflection)



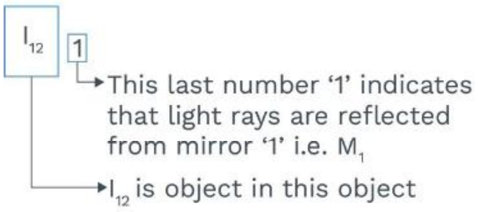
Images due to parallel plane mirrors:-

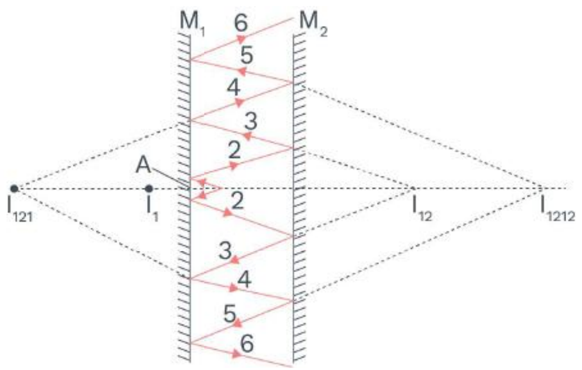
If rays after gets reflected from one mirror falls on second mirror, the image formed by first mirror will function as an object for second mirror, and this process will continue for every successive reflection.

Ex. Figure shows a point object located between two parallel mirrors. Object distance from M_1 is 2 cm and that from M_2 is 8 cm. Find out the distance of images from the two mirrors assuming reflection on mirror M_1 first.



Sol. You will need to know what symbols like I_{121} stands. According to the following diagram.

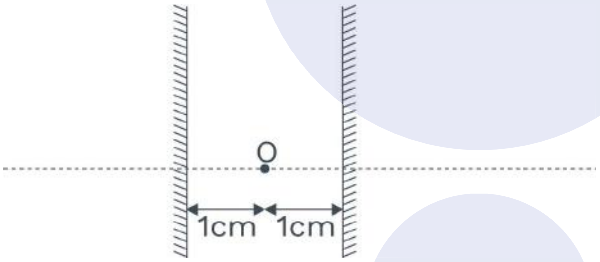




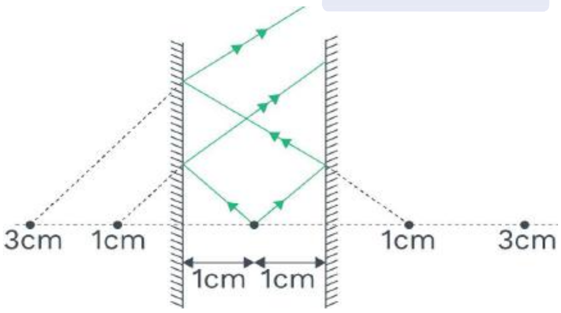
Incident Rays	Ref. by	Ref. rays	Object	Image	Object Distance	Image Distance
Rays 1	M_1	Rays 2	O	I_1	AO = 2 cm	AI_1 = 2 cm
Rays 2	M_2	Rays 3	I_1	I_{12}	BI_{12} = 12 cm	BI_{12} = 12 cm
Rays 3	M_1	Rays 4	I_{12}	I_{121}	AI_{121} = 22 cm	AI_{121} = 22 cm
Rays 4	M_2	Rays 5	I_{121}	I_{1212}	BI_{1212} = 32 cm	BI_{1212} = 32 cm

Similarly, images will be formed by the rays striking mirror M_2 first. Total number of images = ∞.

Ex. Two plane mirrors are put parallel to each other at a distance of 2 cm. A point object is kept at the midpoint of the line joining them. Represent the position of the images by drawing appropriate Ray diagram.



Sol.



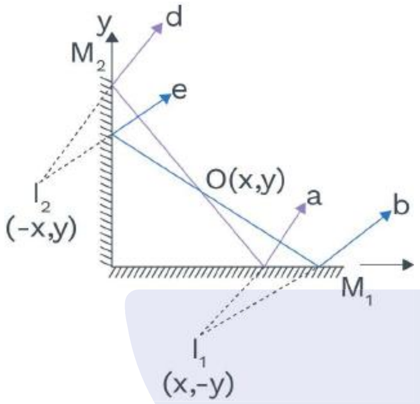
Thus, it forms an A.P.

Image due to perpendicular plane mirror:

Ex. Assume two perpendicular mirrors M_1 and M_2 and a point object O. Considering origin at the point of intersection of the mirrors and the coordinate of object as (x, y), find out the position and number of images.

Sol. Rays 'a' and 'b' strike mirror M_1 only and these rays will form image I_1 at (x, −y), such that O and I_1 are at equal distance from mirror M_1.

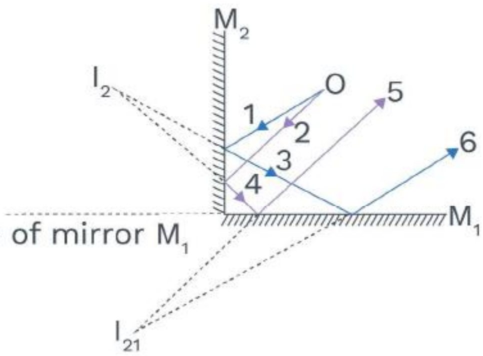
These rays do not form more image because they do not falls on any mirror again. Similarly rays ‘d’ and ‘e’ strike mirror M_2 only and these rays will form image I_2 at (–x, y), such that O and I_2 are equidistant from mirror M_2.



Now assume those rays which strike mirror M_2 first and then the mirror M_1.

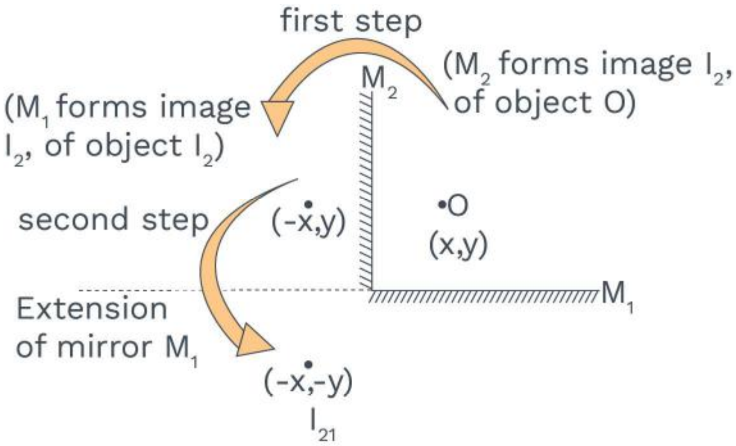
For incident light ray 1, 2 object is O, and reflected rays 3, 4 from image I_2.

Now light rays 3, 4 incident on M_1 (object is I_2) which reflect as rays 5, 6 and form image I_{21}. Light rays 5, 6 do not falls on any mirror, so image formation stops.



I_2 and I_{21}, are at same distance from M_1. To summarize see the following diagram. Now rays 3,4 incident on mirror M_1 (object is I_2) which reflect as light rays 5, 6 and form image I_{21}. Rays 5, 6 do not falls on any mirror, so image formation stops.

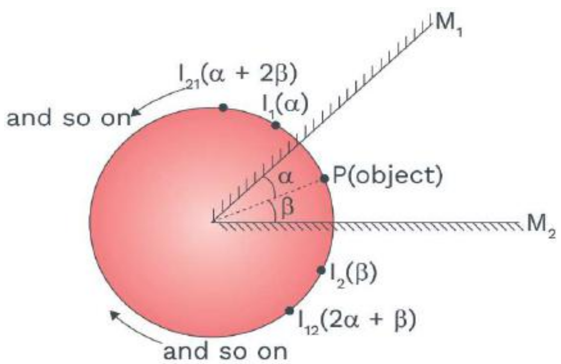
Ray Optics



For rays reflecting first from mirror M_1 and from mirror M_2, first image I_1 at (x, −y)) will be formed and this will function as object for mirror M_2 and then its image I_{12} (at (−x, −y)) will be formed. I_{12} and I_{21} coincide. So, three images are formed

Locating all the Images formed by two Plane Mirrors :

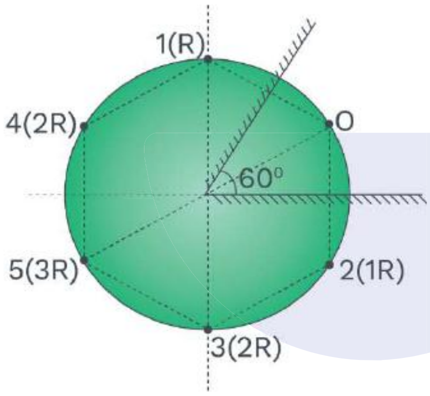
Assume two plane mirrors M_1 and M_2 inclined at an angle $\theta = \alpha + \beta$ as shown in diagram. Point 'P' is an object kept such that it makes angle 'α' with mirror M_1 and angle 'β' with mirror M_2. Image of object P made by M_1, denoted by I_1, will be inclined by angle 'α' on the other side of mirror M_1. This angle is written in bracket in the diagram besides I_1. Likewise image of object P formed by M_2, denoted by I_2, will be inclined by angle 'β' on the other side of mirror M_2. This angle is written in bracket in the diagram besides I_2.



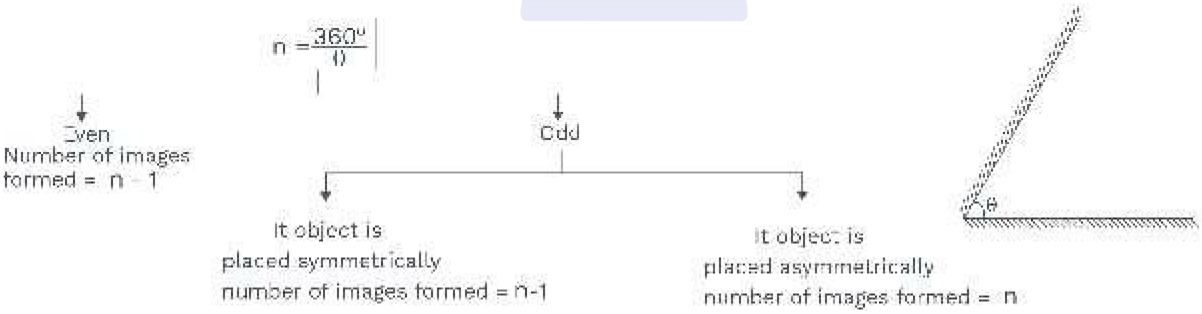
Now image I_2 will act as an object for mirror M_1 which is at an angle of

$\alpha + \beta$ on the opposite site of mirror M_1. This image will be define by I_{21}, and so on. Think when this process will stop [Hint : The virtual image formed by a plane mirror must note be in front of the mirror of its extension.]

Circle concept:
All the images created will lie on a circle whose centre is the intersection point of the mirror and radius equal to distance of object from the intersection point



Number of images formed by two inclined mirrors inclined at an angle θ



(i) If $\frac{{}^{\circ}}{\theta}$ = even number;

number of image = $\frac{{}^{\circ}}{\theta} -$

(ii) If $\frac{{}^{\circ}}{\theta}$ = odd number;

number of image = $\frac{{}^{\circ}}{\theta} -$

if the object is located on the angle bisector.

Concept Reminder

Number of images formed by combination of plane mirror inclined at an angle θ is $\frac{{}^{\circ}}{\theta} -$

if $\frac{{}^{\circ}}{\theta}$ = even number

(iii) If $\frac{\circ}{\theta}$ = odd number;

number of image = $\frac{\circ}{\theta}$,

if the object is not located on the angle bisector.

(iv) If $\frac{\circ}{\theta} \neq$ integer.

Ex. Two plane mirrors are inclined by an angle 30°. An object is kept making an angle of 10° with the mirror M_1. Find out the positions of first two images formed by each mirror and also the total number of images using (i) direct formula and (ii) counting the images.

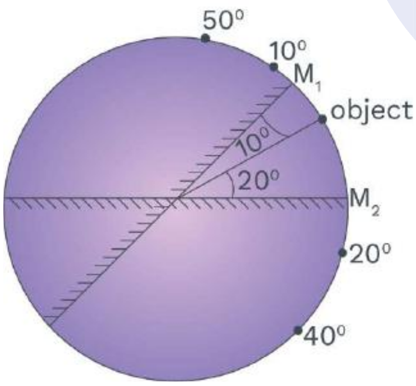
Concept Reminder

Number of images formed by combination of plane mirror inclined at an angle is.

(a) $\frac{\circ}{\theta}$ if = odd number and if object is not placed on the angle bisector.

(b) $\frac{\circ}{\theta} -$ if $\frac{\circ}{\theta}$ = odd number and if object is placed at an angle bisector.

Sol.



Number of images-

(i) Using direct formula : $\frac{\circ}{\circ}$ = (even number)

$\therefore$ number of images = 12 – 1 = 11

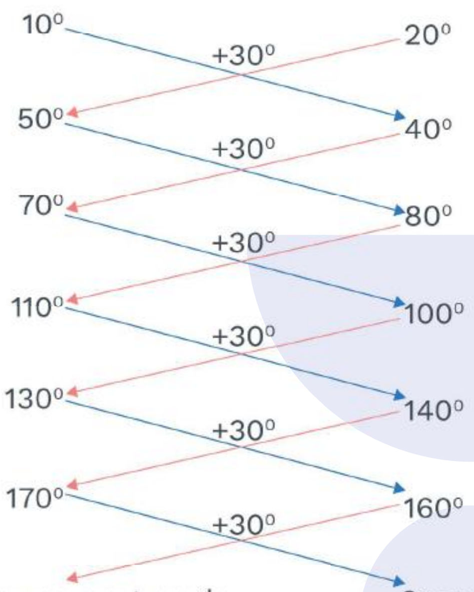
(ii) By counting: Observe the following table

To check that whether the final images formed by the both mirrors coincide or not : add the last angles and the angle between the plane mirrors. If it comes out to be exactly 360°, it implies that the final images created by the two plane mirrors coincide. At this time last angles made by the mirrors + the angle between the mirrors = 160° + 170° + 30° = 360°. Therefore in this case the last images coincide.

Hence the number of images = number of images formed by mirror M_1 + number of images formed by mirror M_2 – 1 (as the last images coincide) = 6 + 6 – 1 = 11.

Image formed by mirror M_1 (angles are measured from the mirror M_1)

Image formed by Mirror M_2 (angles are measured from the mirror M_2)

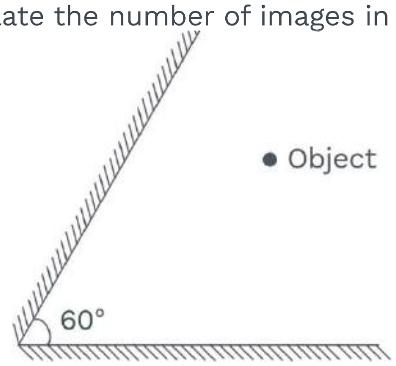


Stop because next angle will be more than 180°

Stop because next angle will be more than 180°

Ex. Calculate the number of images in the following cases.

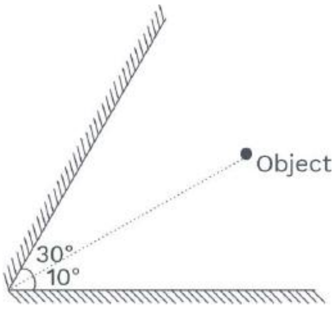
(i)



n = —— =

No. of image = 6 – 1 = 5

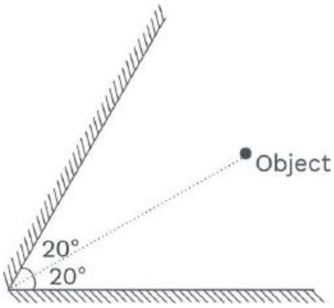
(ii)



= —— = A

No. of images = 9

(iii)

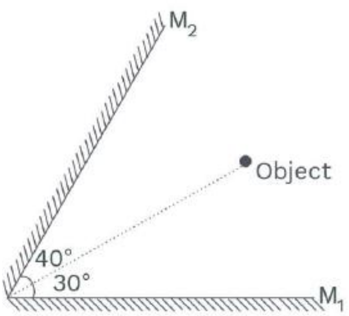


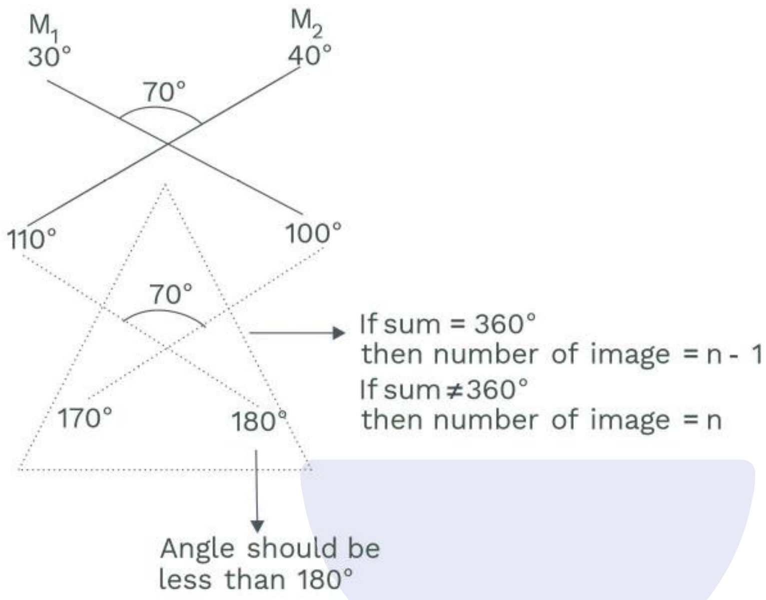
= —— =

No. of image = 9 – 1 = 8.

(b) If “n” is in fraction

eg:



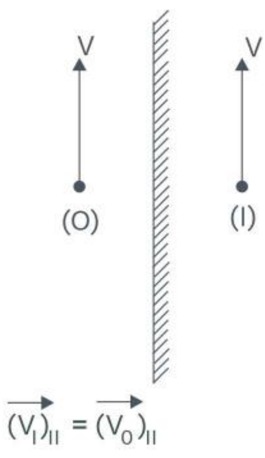


Note : All images lie on a circle.

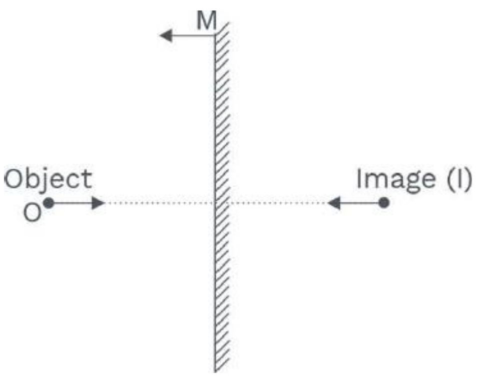
Motion of an object in front of a plane mirror:-

- When a plane mirror moves parallel with its plane then, there would not be any effect of its motion on motion of image
- When plane mirror moves perpendicular to the plane then motion of image depends on it.

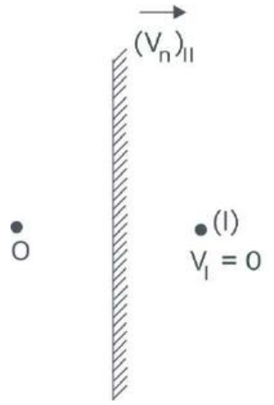
Case-1



Case-2



Case-3



83 = − m3

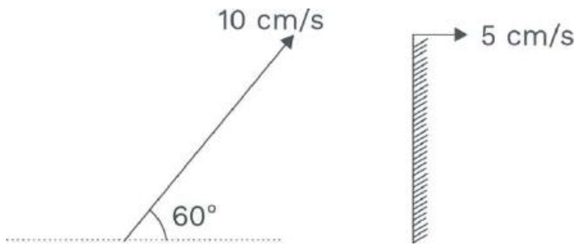
Differentiate wrt time

$\vec{8}3 = -\vec{m}3$

$\vec{}_{\perp} = \vec{}_{\perp} - \vec{}_{\perp}$

V_I, V_O, V_M are velocity of Image, object & mirror respectively, perpendicular to plane of mirror.

Ex. Find velocity of image in each case?



Sol. $\vec{8}_{\perp} =$ ° = +

$\vec{8} =$ ° = $\sqrt{\ }$

$\vec{}_{\perp} = +$

$\vec{}_{m\perp} = - =$

Rack your Brain

A man runs towards a plane mirror at a speed of 15 m/s. What is the speed of his image:

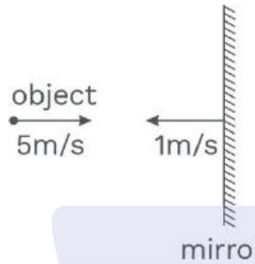
(1) 7.5 m/s (2) 15 m/s

(3) 30 m/s (4) 45 m/s

$_m = \,_8 = \sqrt{\ }$

$\vec{\ }_m = + \sqrt{\ }$

Ex. An object moves with 5 m/s towards right while the mirror moves with 1 m/s towards the left as shown. Find out the velocity of image.



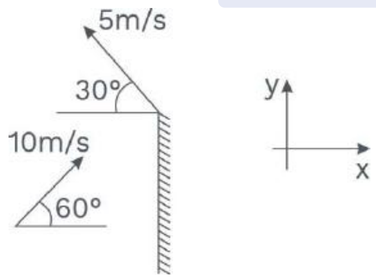
Sol. Take $\rightarrow$ as + direction. $v_i - v_m = v_m - v_0$

$\Rightarrow \quad - \; - \; = - \; -$

$\therefore \quad = -$

$\Rightarrow$ 7 m/s and direction towards left.

Ex. In the situation shown in figure, find out the velocity of image.



Sol. Along x-axis direction, applying $v_i - v_m = -(v_0 - v_m)$

$v_i - (-5 \cos 30°) = -(10 \cos 60° - (-5 \cos 30°))$

$\therefore \quad = - \quad + \sqrt{\ }$

Along y-axis direction $v_0 = v_i$

$\therefore \quad = \quad ° = \sqrt{\ }$

$\therefore \quad - \quad + \sqrt{\ } \quad + \sqrt{\ }$

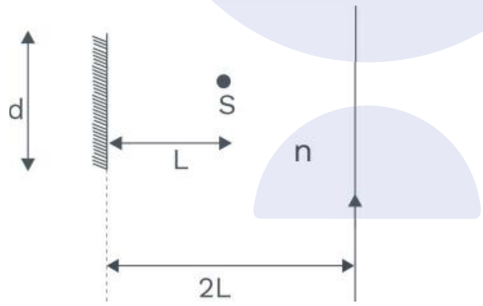
Ex. A plane mirror is lying along x – z plane & moving with velocity $\vec{\ } \ \vec{\ } = - \ + \ +$. An object is moving in front of it with velocity $\vec{\ } = \ - \ +$. Find velocity of image.

Sol. $\vec{\ }_m = \ - \ -$ (only velocity perpendicular to mirror will change)

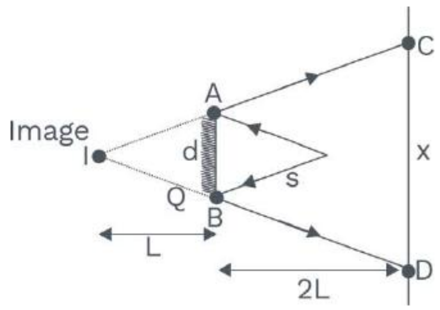
$=$

$\Rightarrow \vec{\ } = \ + \ +$

Ex. A source of light "S" is placed at a distance 'L' in front of the centre of a mirror of width 'd' and hung vertically on a wall. A person walks in front of the mirror along a line parallel to the mirror at a distance 2L from it as shown.
Find the greatest distance over which he can see the image of the light source in the mirror?

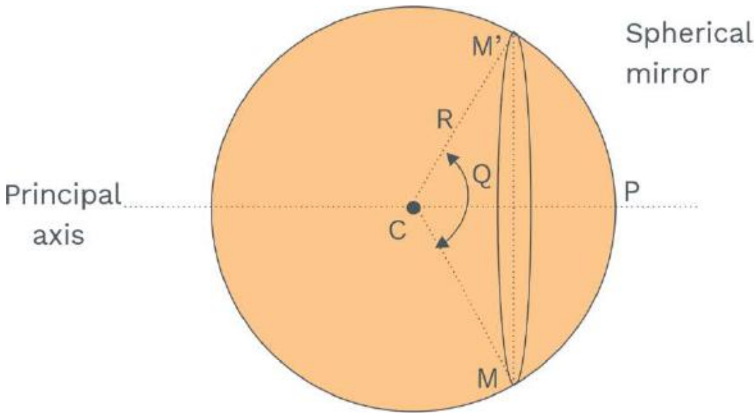


Sol.



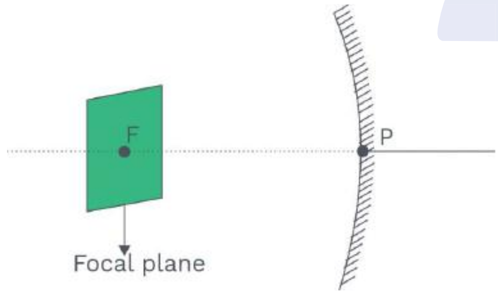
$\Delta\, AN \quad \Delta\, OS \Rightarrow - = \frac{x}{x} \Rightarrow \quad =$

Spherical Mirror



Aperture: It is the effective diameter of the light reflecting area of the mirror
R = Radius of curvature, C = Centre of curvature,
P = Pole

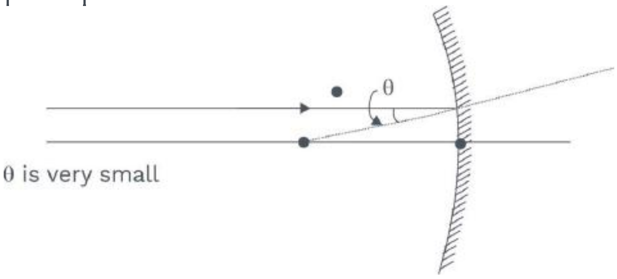
Focal plane :
It is the plane passing through focus & perpendicular to the principal axis



Definitions

Focal plane is defined as the plane passing through focus & perpendicular to the principal axis

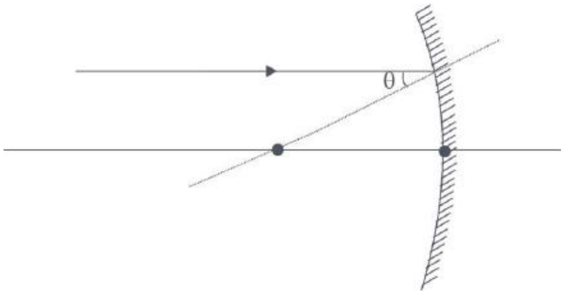
Paraxial Rays :
Those rays which make very small angle with normal at point of incidence & hence are close to principal axis



Definitions

Paraxial rays are those rays which make very small angle with normal at point of incidence & hence are close to principal axis

Marginal Rays:



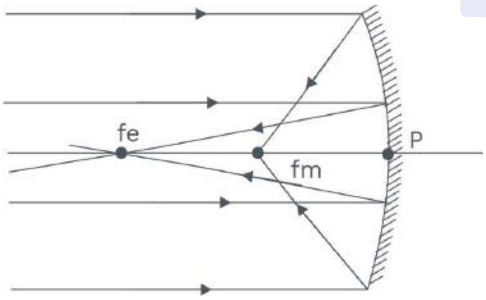
These rays have large angle of incidence

Spherical Aberration

When all rays are incident on a spherical mirror in a direction parallel to the axis, the marginal rays (i.e. the rays incident on the mirror just close to the edge) & the Paraxial rays (rays near to the Principal axis) intersect at different point on principal axis ∴ the image of the distant object is not formed at one point but is separated along principal axis, this defect is called spherical aberration

Definitions

The image of the distant object is not formed at one point but is separated along principal axis, this defect is called spherical aberration.

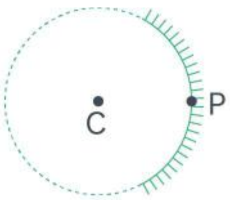


Concept Reminder

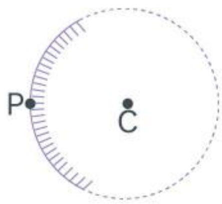
The word 'AMBULANCE' is opposite because when the driver of the vehicle ahead of the ambulance looks in her/ his mirror, the person can read it as 'AMBULANCE' and give way to it.

Some Important Definitions.

(i) Spherical Mirrors



Concave mirror

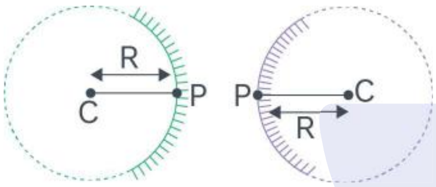


Convex mirror

(ii) Pole or Vertex (P) : It is a point on the mirror from where it is easy to measure object and image distance.

(iii) Centre of curvature : The centre C of the sphere of which the spherical mirror is a part, is the centre of curvature of the mirror.

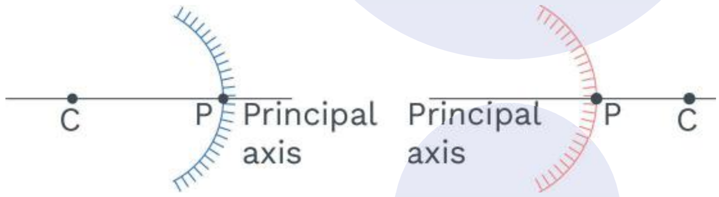
(iv) Radius of curvature (R) : Radius of curvature is the radius R of the sphere of which the mirror forms a part.



KEY POINTS

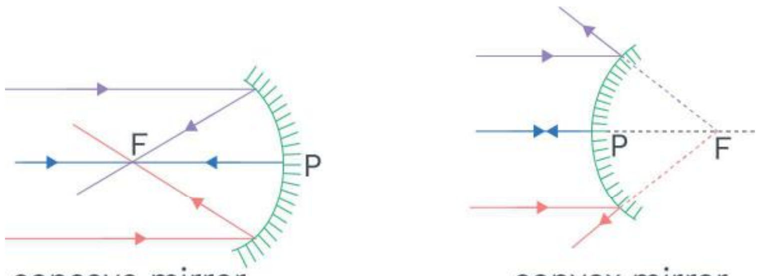
- Spherical mirror
- Centre of curvature
- Radius of curvature
- Principal axis
- Focus
- Focal length
- Aperture

(v) Principal axis :



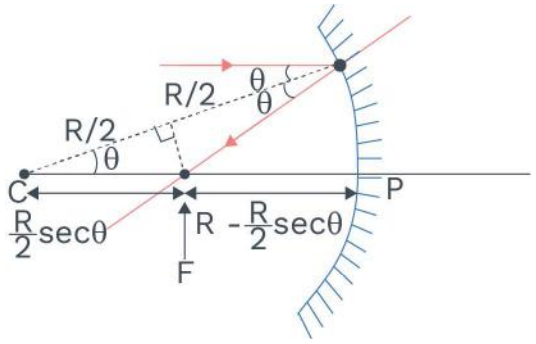
Line joining centre of curvature and pole of the mirror is known as principle axis or optical axis.

(vi) Focus (F) : If the rays are parallel to principal axis and paraxial then the point at which they appear to converge is defined as focus.



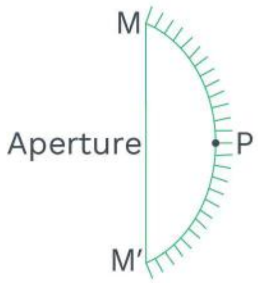
concave mirror　　　convex mirror

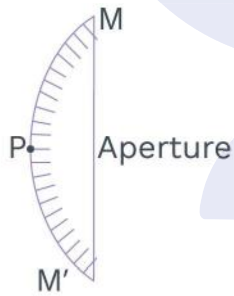
(vii) Focal Length (f) : Focal length is the distance PF between the pole P and focus F along the principal axis.

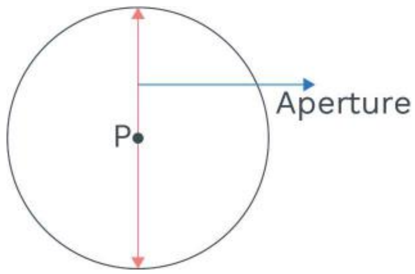


If θ is very small : $| \; - \frac{|}{\;} \qquad \theta \simeq \frac{|}{\;}$

(viii) Aperture : The line joining the end points of a spherical mirror is called the aperture or linear aperture. Aperature gives idea about size of mirror.







Ex. Find out distance on focal plane where parallel and paraxial rays which are not parallel to optic axis, meet after reflection.

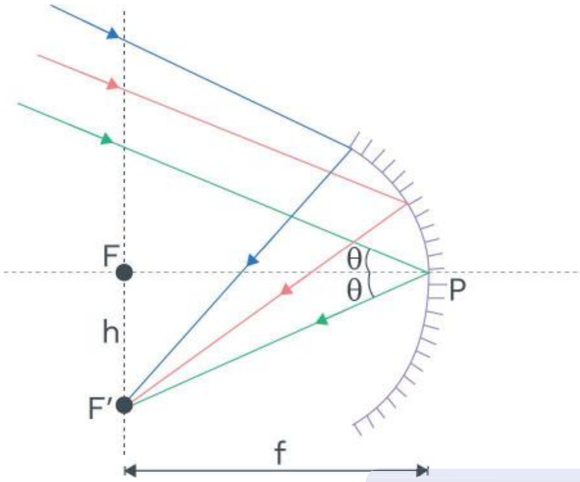
Definitions

Radius of curvature (R): Radius of curvature is the radius R of the sphere of which the mirror forms a part.

Concept Reminder

There are two types of spherical mirror–
(i) Concave mirror
(ii) Convex mirror

In principle, we can take any two rays emanating from a point on an object, trace their paths, find their point of intersection and thus, obtain the image of the point due to reflection at a spherical mirror.

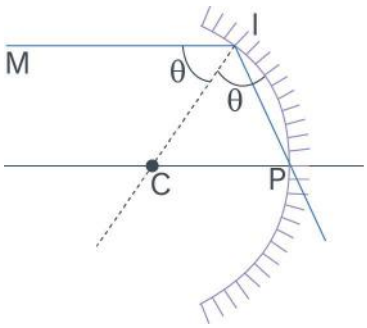


Sol. In Δi i ($\theta = -$

= θ (θ is small)

* If the rays are paraxial and parallel but not parallel to optic axis then they will meet at focal plane.

Ex. Find out the angle of incidence of ray for which it passes through the pole, given that MI || CP.



Sol. $\angle$ MIC = $\angle$ CIP = θ

MI || CP $\angle$ MIC = $\angle$ ICP = θ

CI = CP = R

$\angle$ CIP = $\angle$CPI = θ

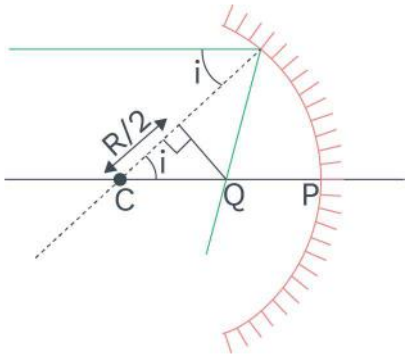
$\therefore$ In ΔCIP all angle are equal

$\theta =$ ° $\Rightarrow$ $\theta =$ °

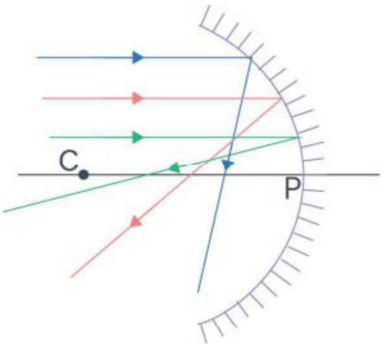
Concept Reminder

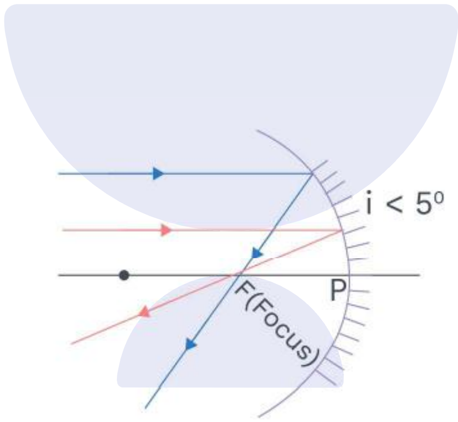
If the lower half of the concave mirror's reflecting surface is covered with an opaque (non-reflective) material, the image will be that of the whole object. However, as the area of the reflecting surface has been reduced, the intensity of the image will be low (in this case, half).

Ex. Find out the distance CQ if incident light ray parallel to principal axis is incident at an angle i. Also find the distance CQ if i → 0.

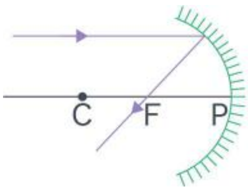


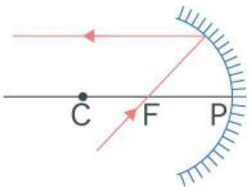
Sol.

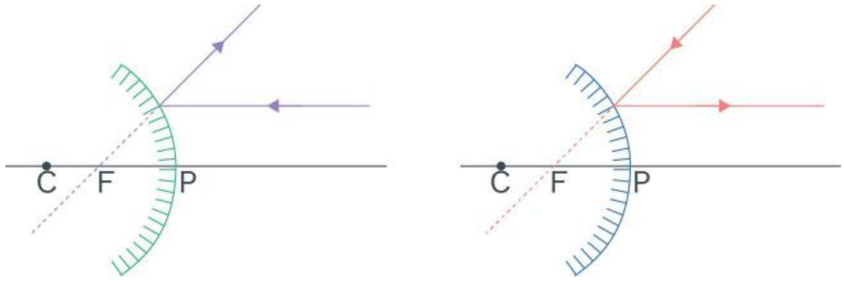




As i increases cos i decreases.
Therefore CQ increases
Hence, paraxial rays meet at a distance equal to R/2 from centre of curvature, which is defined as focus, Principal focus (F) is the point of meeting all the reflected rays for which the incident rays falls on the mirror (with small aperture) parallel to the principal axis. In convex mirror it is virtual and in the concave mirror it is real. The distance from pole to focus is known as focal length. Aperture (related to the size of spherical mirror) is the diameter of the spherical mirror.



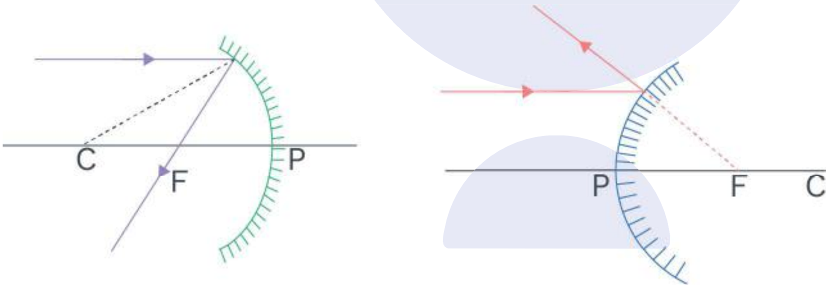




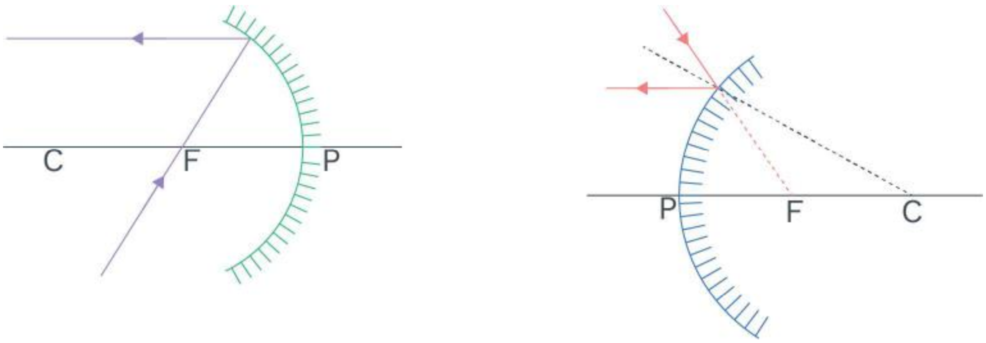
Rules For Image Formation (∠ = ∠):

The reflection of rays and formation of images are represented with the help of ray diagrams. Some specific incident rays and the corresponding reflected rays are shown below.

(i) A light ray passing parallel to the principal axis, after reflection from the spherical mirror passes or appears to pass across its focus (by the definition of focus)

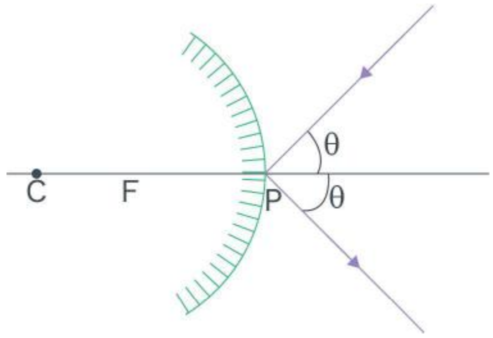


(ii) A ray passing through or directed towards focus, after reflection from the spherical mirror becomes parallel to principal axis (by the principal of reversibility of light).

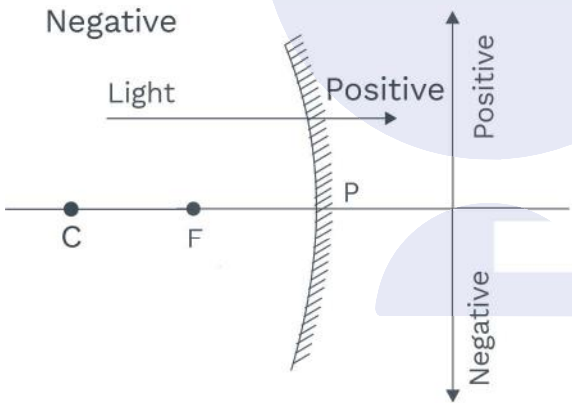


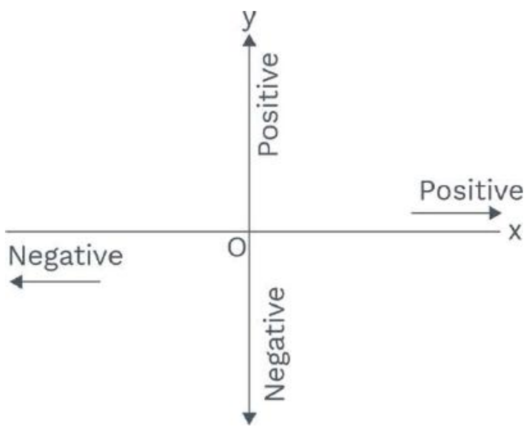
(iii) A ray passing through or directed in the direction on the centre of curvature, after reflection from the spherical mirror, retraces its path. (as for it $\angle i = 0$ and so $\angle r = 0$)

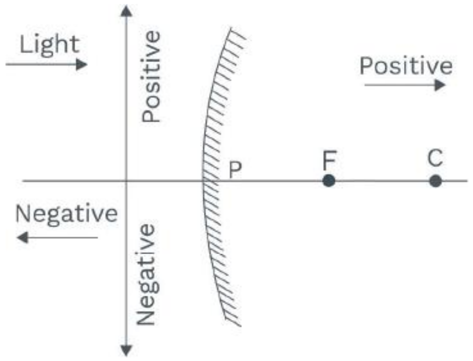
(iv) It is simple to make the ray tracing of a ray incident at the pole as shown in below.



Sing-Convention:

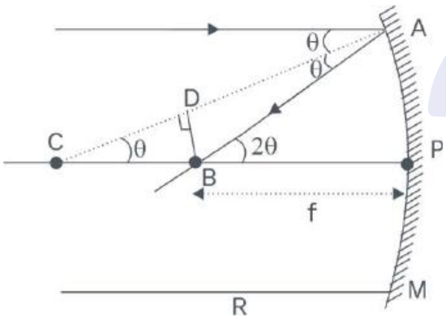






- Along the principal axis, distances are measured from the pole (pole is taken as the origin).
- Distances in the direction of incident light are taken positive while those along opposite direction negative.
- Whenever and wherever possible incident light is taken to travel from left to right.

Relation between f and R for a spherical mirror:



$$N(\; = \; = | - \frac{|}{\theta}$$

2. For paraxial rays

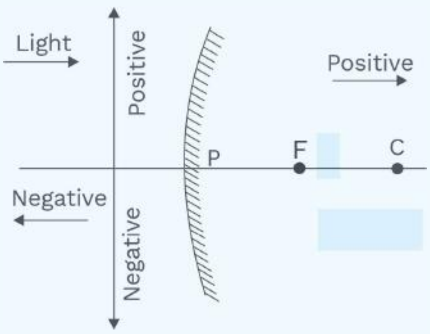
$$\theta \qquad \therefore \qquad \theta \approx$$

Hence $NO = \frac{|}{}$ and $N(\; = \frac{|}{}$. Thus, point B is the midpoint of PC and is defined as focus so $N(\; = \; = \frac{|}{}$ (only valid for paraxial rays).

Focal length (f) of mirror does not depend on the medium in which mirror is placed.

Concept Reminder

Sign convention in reflection by spherical mirror



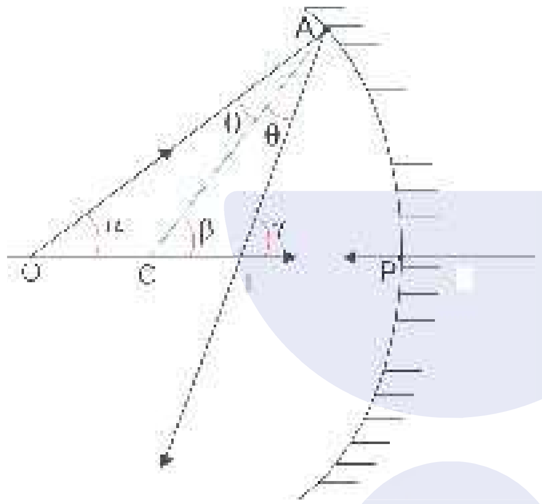
Concept Reminder

Relation between f and R for spherical mirror is

$$= | - \frac{|}{\theta}$$

Relation Between 'u', 'v' And 'R' For Spherical Mirrors:

Consider the situations shown in diagram. A point object is kept at the point 'O' of the principal axis of a concave mirror. A ray 'OA' is incident on the mirror at 'A'. It is reflected in the direction 'AI'. Another ray 'OP' travels along the principal axis. As 'PO' is normal to the mirror at 'P', the ray is reflected back along 'PO'. The reflected rays 'PO' and 'AI' intersect at I, where the image is produced.



Assume 'C' be the centre of curvature. The line CA is the normal at 'A'. Thus, by the laws of reflection, ∠OAC = ∠CAI. Let α β γ and θ represent the angles AOP, ACP, AIP and OAC respectively. As the exterior angle in a triangle is equal to the sum of the two opposite interior angles, we have,

from triangle OAC $\beta = \alpha + \theta$ ....(i)

and from triangle OAI $\gamma = \alpha + \ \theta$. ...(ii)

Eliminating θ from (i) and (ii),

$$\beta = \alpha + \gamma \qquad \text{...(iii)}$$

If the point 'A' is close to 'P', the angles α β and γ are small and we can write

$$\alpha = \frac{A(}{(8} \quad = \frac{A(}{(O} \text{ and } \gamma \approx \frac{A(}{(n}$$

or $\frac{}{(8} + \frac{}{(m} = \frac{}{(O}$ ...(iv)

The pole 'P' is taken as the origin and the principal

Concept Reminder

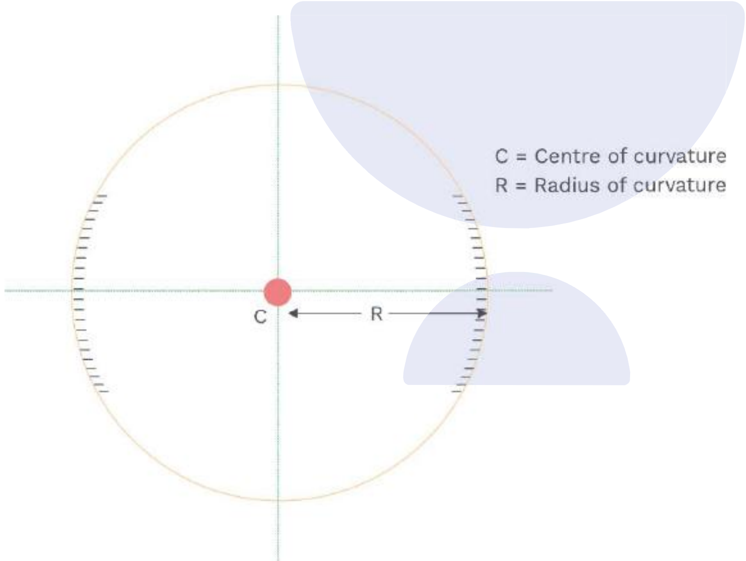
Mirror Equation

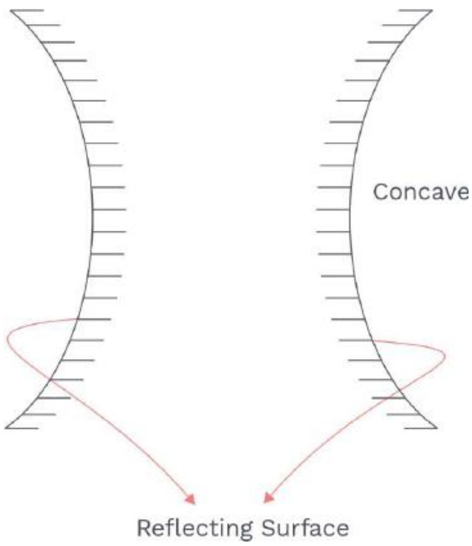
- $- + - = -$ I
- $- + - = -$

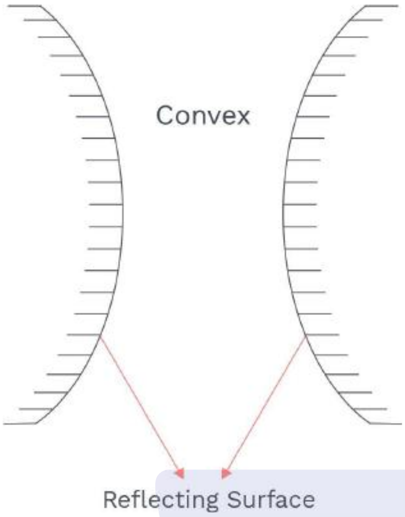
axis as the X-axis direction. The rays are incident from left to right. We consider the direction from left to right as the positive X-direction. The points ‘O’, ‘I’ and ‘C’ are situated to the left of the origin ‘P’ in the diagram. The quantities ‘u’, ‘v’ and ‘R’ are negative. As the distances PO, PI and PC are positives, PO = – u, PI = – v and PC = – R. Putting in equation (iv),

$$\frac{}{-} + \frac{}{-} = \frac{}{-|} \text{ or } \frac{}{} + \frac{}{} = \frac{}{|} \qquad ...(vii)$$

Even though equation (vii) is derived for a special situation shown in diagram, it is also valid in all other conditions with a spherical mirror. This is because we have chosen proper care of the signs of ‘u’, ‘v’ and ‘R’ appearing in figure shown.



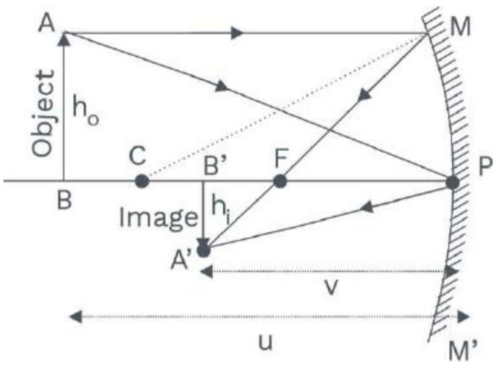




As per sign convention for object/image for spherical mirrors-

Real object ⇒ u negative
Real image ⇒ v negative
Virtual object ⇒ u positive
Virtual image ⇒ v positive

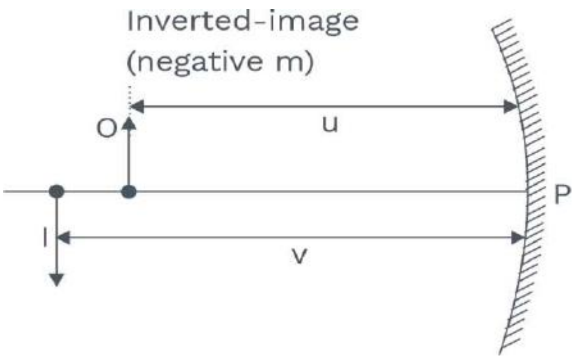
Magnification
Transverse or lateral or linear magnification

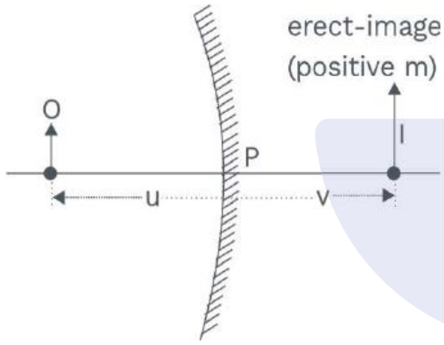


Linear magnification = —————— = ——

Δ ABP and Δ A'B'P are similar so —— = ——

⇒ —— = – ——





Concept Reminder

Magnification	Image
$|m| > 1$	enlarged
$|m| < 1$	diminished
$m < 0$	inverted
$m > 0$	erect

Concept Reminder

Longitudinal magnification :

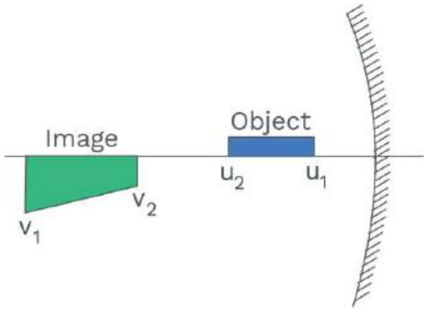
$$_{x} = - —— = \left[— \right] =$$

If one dimensional object is located perpendicular to the principal axis then linear magnification is called transverse or lateral magnification.

$$= —— = - —$$

Magnification	Image
>	enlarged
<	diminished
<	inverted
>	erect

- **Longitudinal magnification**
  If a rod is placed along the principal axis then linear magnification is called longitudinal or axial magnification.



Longitudinal magnification:

$$_{x} = \frac{\qquad}{\qquad} = \frac{\left| \; - \; \right|}{\left| \; - \; \right|}$$

For small objects only : $_{x} = - \frac{}{}$

differentiation of $\frac{}{} + \frac{1}{} = \frac{}{}$ yields $- \frac{}{} - \frac{}{} =$

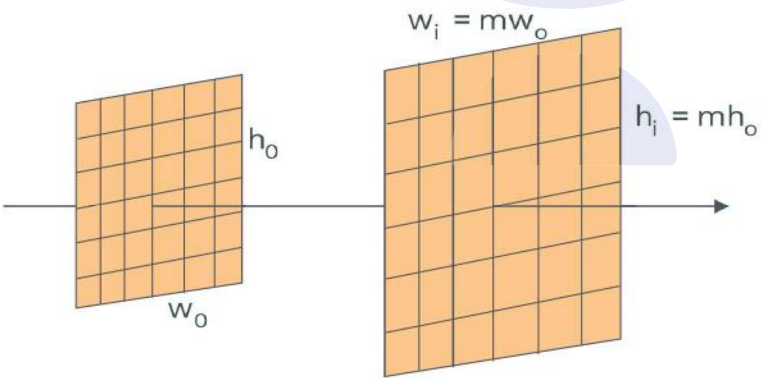
$\Rightarrow \frac{}{} = \left[\frac{}{} \right]$ so $_{x} = - \frac{}{} = \left[\frac{}{} \right] =$

KEY POINTS

- Longitudinal magnification
- Superficial magnification

Superficial magnification

If two dimensional object is placed with its plane perpendicular to the principal axis then its magnification is known as superficial magnification



Linear magnification $= \frac{}{} = \frac{}{}$

$=$ $=$ Also A $=$ $\times$

A $= \times = \times =$ A

Superficial magnification

$$= \frac{\qquad}{\qquad} = \frac{A}{A} =$$

Definitions

If two dimensional object is placed with its plane perpendicular to the principal axis then its magnification is known as superficial magnification

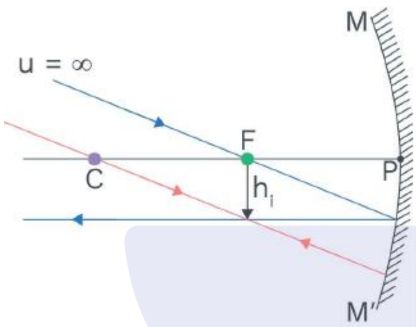
Image Formation By Spherical Mirrors

Concave Mirror

(i) **Object :** Placed at infinity

Image : real, inverted, highly diminished, at F

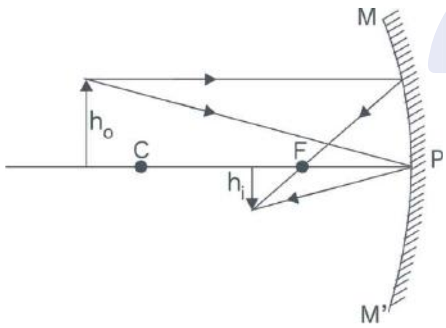
<< <



(ii) **Object :** Placed in between infinity and C

Image: real, inverted, diminished, in between C and F

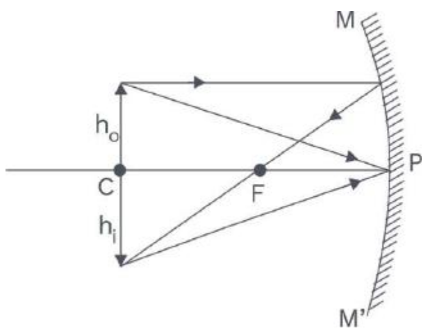
< <



(iii) **Object** : Placed at C

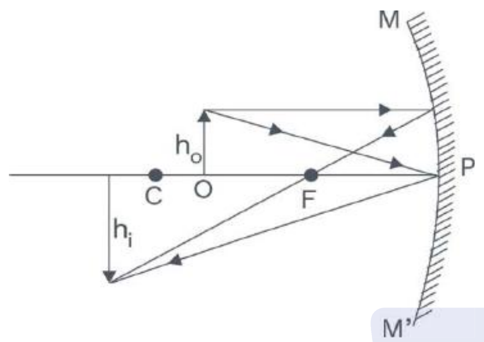
Image : real, inverted, equal, at C

(m = –1)

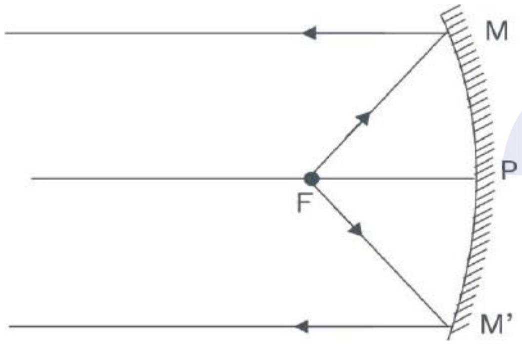


(iv) **Object :** Placed in between F and C
Image : real, inverted, enlarged beyond C

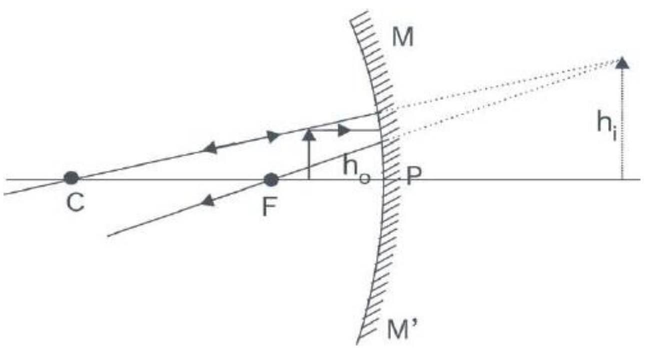
> <



(v) **Object :** Placed at F
Image : real, inverted, very large (assumed) at infinity (m<<–1)



(vi) **Object :** Placed between F and P
Image : virtual, erect, enlarged and behind the mirror (m > + 1)



Rack your Brain

An object is placed at a distance of 40 cm from a concave mirror of focal length 15 cm. If the object is displaced through a distance of 20 cm towards the mirror, the displacement of the image will be:

(1) 30 cm towards the mirror
(2) 36 cm away from the mirror
(3) 30 cm away from the mirror
(4) 36 cm towards the mirror

Concept Reminder

When a beam of light encounters another transparent medium, a part of light gets reflected back into the first medium while the rest enters the other.

(vii) For virtual object :

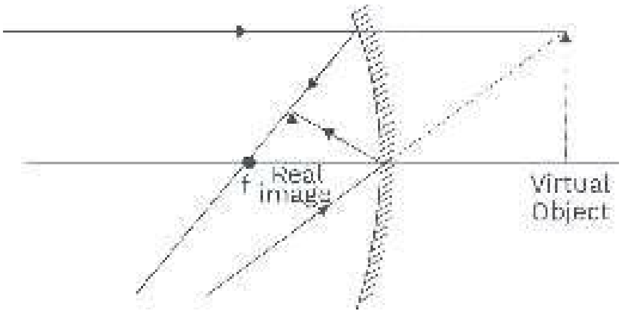
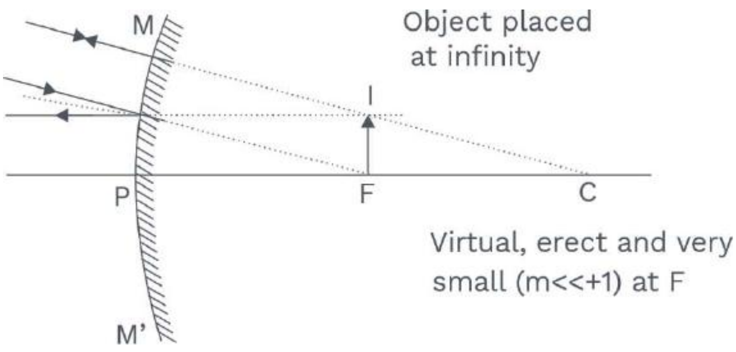


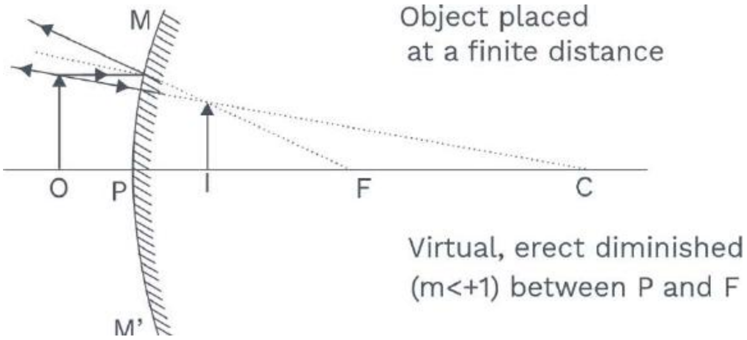
Image : Real, Erect and diminished

For concave mirror

Position of object	Position of image	Magnification
- ∞	F	|m|<<1 & m<0
- ∞ - C	C - F	|m|<1 & m<0
C	C	m = - 1
C - F	- ∞ - C	|m| > 1 & m < 0
Between C and F, near F	- ∞	m<< - 1
Between F and P, near F	+ ∞	m>> 1

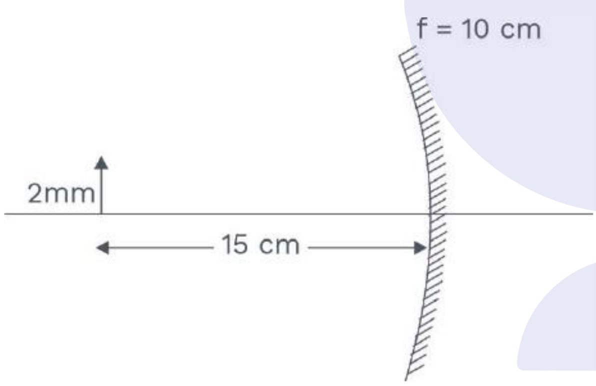
Convex mirror:
Image is always erect and virtual, for the position of the real object and 'm' is positive.





Ex. Find out position, nature & size of image in the following question.

(i)

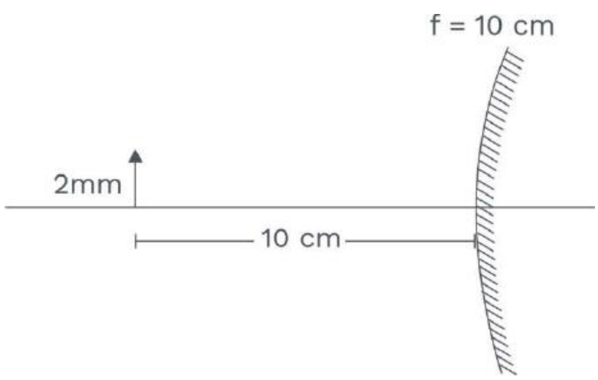


Concept Reminder

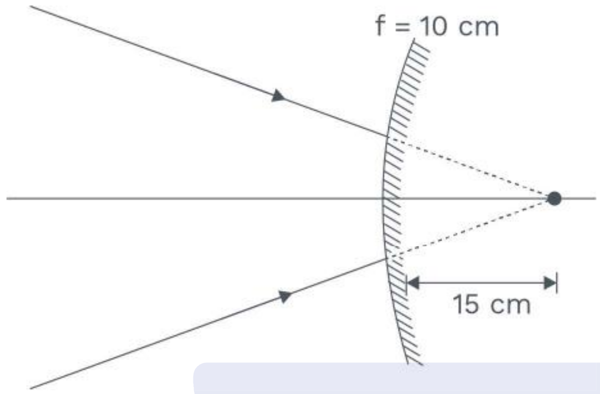
In convex mirror

Image is always virtual and erect, whatever be the position of the real object and m is positive.

(ii)



(iii)



Sol. (i) $= \frac{\quad}{-} = -$ (f = –10 cm & u = –15 cm)

$= \frac{-}{\quad} = -$

$- \;= \text{—} \Rightarrow \;= -$

Image formed is real, inverted and enlarged.

(ii) $= \frac{\quad}{-} = +$ (f = + 10 cm & u = –10 cm)

$= \frac{-}{\quad} = +$

$h_i = mh_o = 1$ mm

Image formed is erect, diminished & virtual

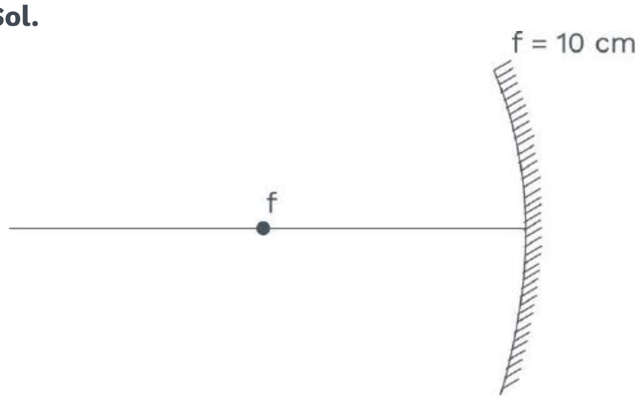
(iii) $= \frac{\quad}{-} =$ (f = 10 cm & u = 15 cm)

$= \frac{-}{\quad} = -$

Image formed is real, inverted & magnified.

Ex. Find out position of object placed in front of concave mirror of focal length 10 cm so that 3 times magnified image is formed.

Sol.



(a) if real and inverted image is formed

= –

——— = –
–

= –
———

(b) If virtual & erect image is formed

m = +3

——— =
–

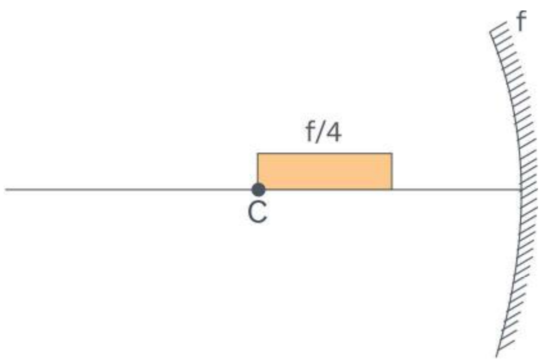
= –
———

Rack your Brain

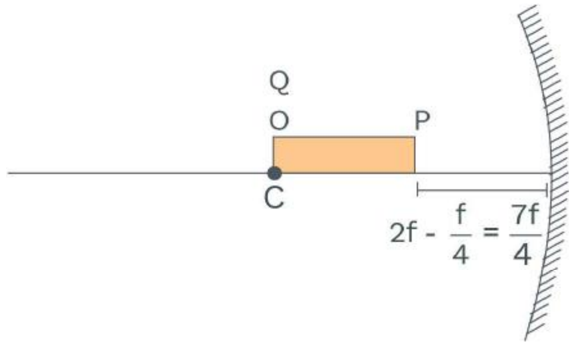
A concave mirror of focal length 15 cm forms an image having twice the linear dimensions of the object. the position of the object when the image is virtual will be:

(1) 22.5 cm
(2) 7.5 cm
(3) 30 cm
(4) 45 cm

Ex. Find the length of image & longitudinal magnification.



Sol.

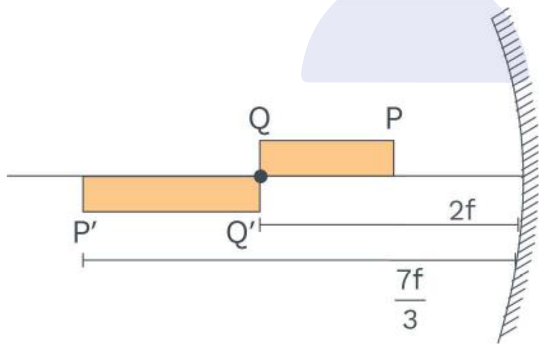


(a) Image of object at point Q will form at Q only. Also image will be real, same size and inverted.

(b) Image of object at point P is formed at

$$= \frac{\quad}{-} = \frac{-}{\quad} \left(= - \frac{\quad}{\quad} \right)$$

Length of image = —— – = —

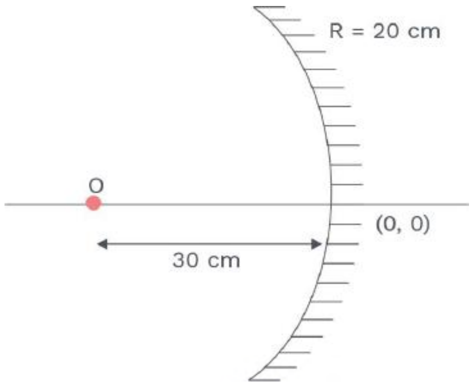


(c) Longitudinal magnification $(m_2) = \frac{x}{x} = \frac{-}{\quad} = \frac{-}{\quad}$ (negative sign for inverted image)

Ex. A square of side 4 mm is placed at a distance of 30 cm from a concave mirror which is of focal length of 10 cm. The centre point of the square is at the axis of the mirror and the plane is normal to the axis. Find the area enclosed by the image?

Sol. $= \frac{\quad}{-} = - - ; A = \quad A = - \quad =$

Ex. Find the position and type of image formed.

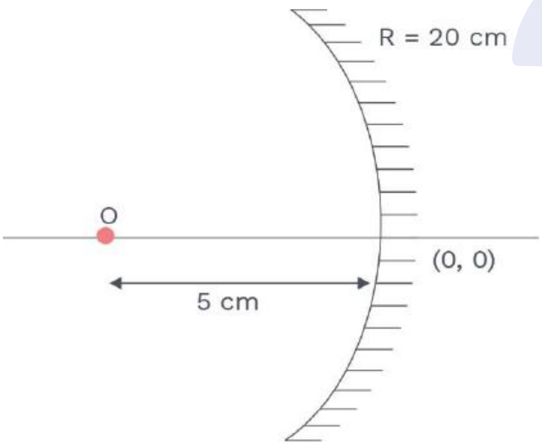


Sol. — = — + — ⇒ —— = —— + —

⇒ — = —— + —— = ——— = —— = ——

v = – 15 cm (Real image)

Ex. Find the position and type of image formed.



Sol. — = — + — ⇒ —— = —— + —

⇒ — = — – —— = ——— = ——

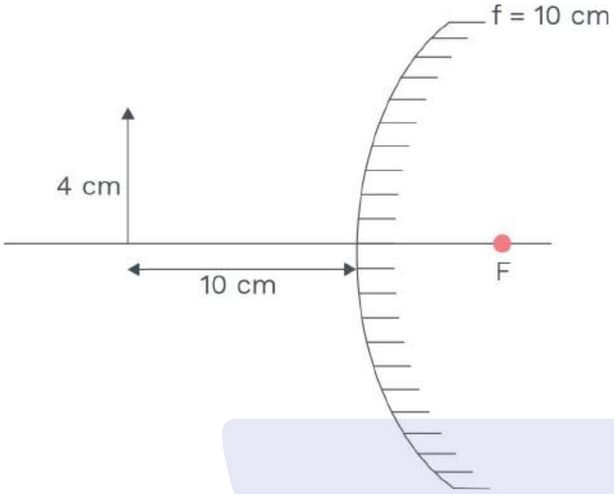
∴ v = + 10 (Virtual image)

Rack your Brain

A person wants a real image of his own, 3 times enlarged. Where should be stand infront of a concave mirror of radius of curvature 30 cm?

(1) 30 cm
(2) 20 cm
(3) 10 cm
(4) 90 cm

Ex. Find the position, height and type of image.



Sol. — = — + — ⇒ + — = — – —

⇒ — = + — + — ⇒ = +

— = — ⇒ = +

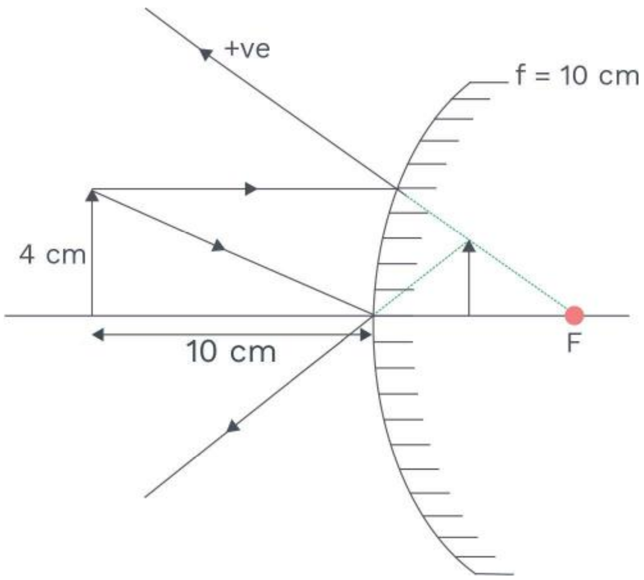


Image is virtual, erect and diminished.

Ex. An object is placed at a distance of 15 cm from a concave mirror of focal length 10 cm. If the object is moved by 0.1 cm towards the mirror, find by what distance image will get shift?

Sol. = –

= –

(= ——— = –)

= –0.4 cm

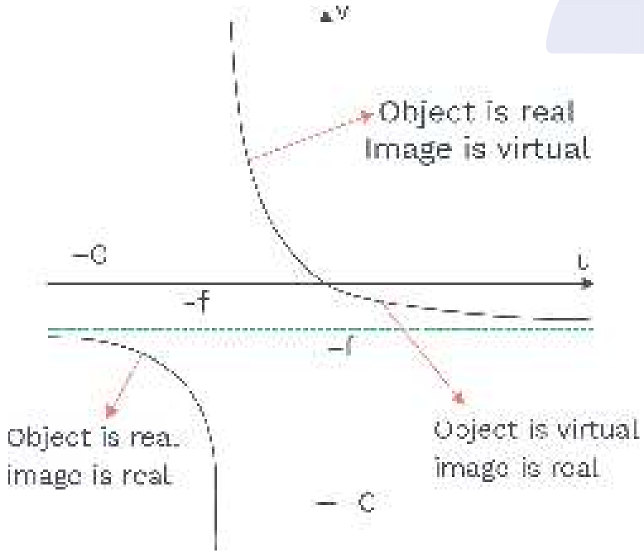
Image will get shift by 0.4 cm away from the mirror.

Power Of A Mirror

The power of a mirror in air is defined as

(= – ——— = – ———

Curve Between v and u

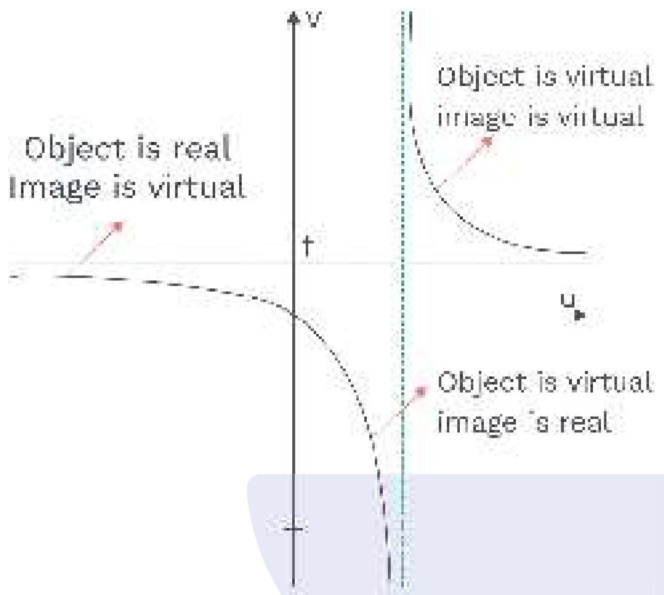


FOR CONCAVE MIRROR

Concept Reminder

The power of a mirror in air is defined as

(= – ——— = – ———



FOR CONVEX MIRROR

Velocity in Spherical Mirror :

Velocity of image

(a) Object moving along principal axis :

On differentiating the mirror formula w.r.t. time we get —— = – —— ——

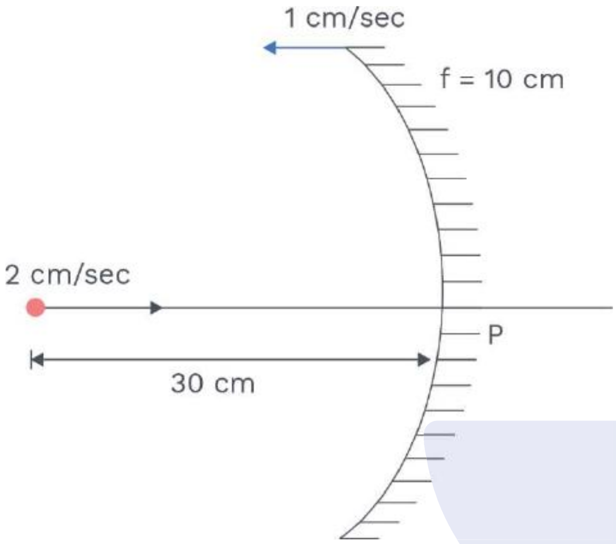
where —— is the velocity of image beside Principal axis and —— is the velocity of the object along Principal axis.

–ve sign implies that the image, in case of mirror, all the time moves in the direction opposite to that of object direction. This discussion is for the velocity with respect to mirror and along the x axis.

Therefore above equation can be written as

ra = – —— 83

Ex. Find out the velocity of image in the given figure.



Sol. Here u = – 30 c m

f = – 10 cm

From using mirror formula — + — = —

v = – 15 cm

a = – —— 83

m – – = – (—) – –

⇒ m = – —

(b) Object moving perpendicular to principal axis :

From the magnification formula we have,

—— = – —— or = – ——

If a point object goes perpendicular to the principal axis, x-coordinate of both the object and the image become constant. On differentiating the above equation with respect to time, we get,

—— = – – ——

Rack your Brain

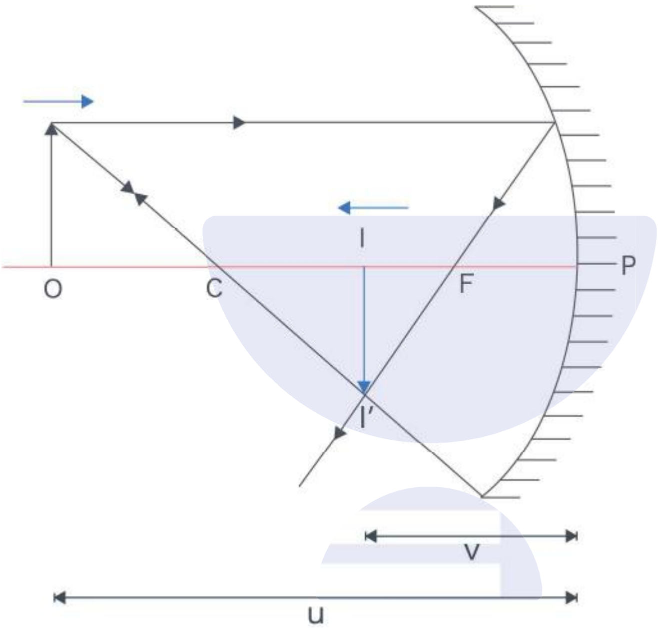
Under which of the following conditions will a convex mirror of focal length f produce an image that is erect, diminished and virtual:

(1) Only when 2f > u > f
(2) Only when u = f
(3) Only when u < f
(4) Always

Here, —— = signifies object velocity perpendicular to the principal axis

and —— = denotes image velocity perpendicular to the principal axis.

(c) Object moving parallel to Principal axis :



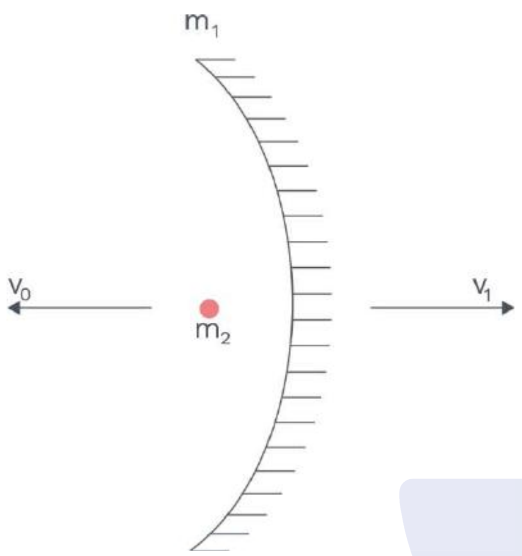
= —— = —— ——

On differentiating equation = – —

= —— = – [—— – – —— ——]

Ex. A gun of mass 'm_1' fires a bullet of mass 'm_2' with a horizontal speed 'v_0'. The gun is attached with a concave mirror of focal length 'f' facing towards a receding bullet. Find out the speed of separations of the bullet and the image just after the bullet was fired.

Sol. Assume v_1 be the speed of gun (or mirror) just after the firing of bullet. From conservation of linear momentum.



=

or = —— ...(i)

Now, —— is the rate at which distance between mirror and bullet is increasing

—— $= v_1 + v_0$ ...(ii)

∴ —— = (——) —— here —— = =

(as at the firing time bullet is at pole)

∵ —— = —— = + ... (iii)

Here —— is the rate at which distance between image (bullet) and mirror is increasing. So if 'v_2' is the absolute velocity of image (towards right) then,

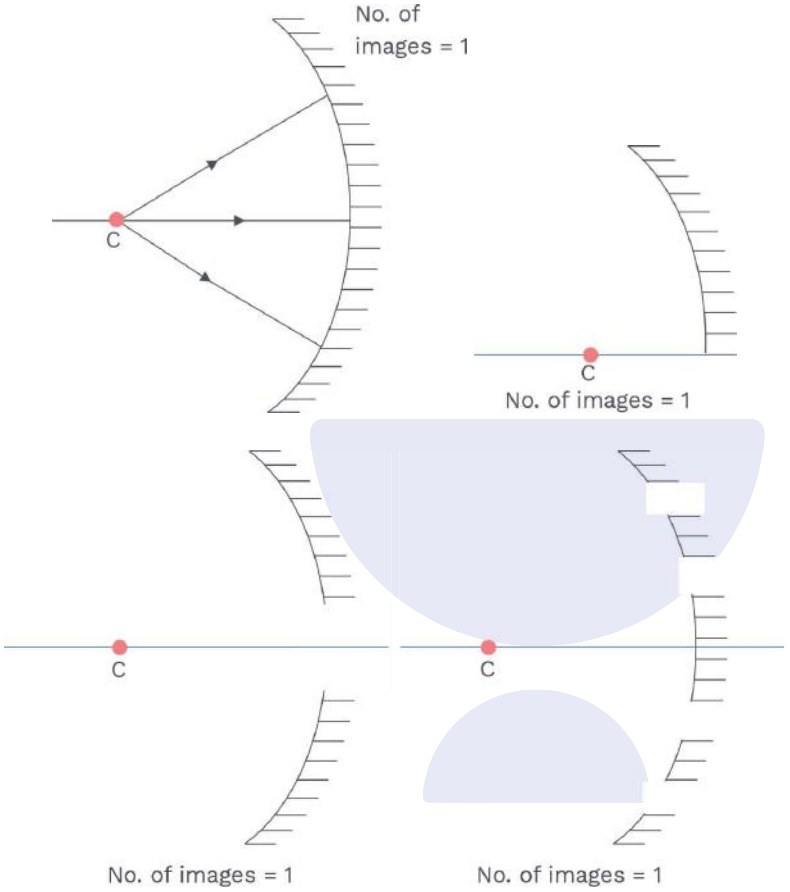
– = —— = + or = + ...(iv)

Hence, speed of separation of image and bullet will be,

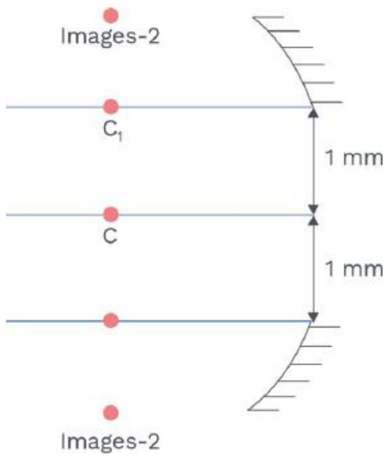
$v_r = v_2 + v_0 = 2v_1 + v_0 + v_0$ or $v_r = 2(v_1 + v_0)$

Replacing the value of v_1 from equation (i) we have, = (+ ——)

Cutting Of Mirrors

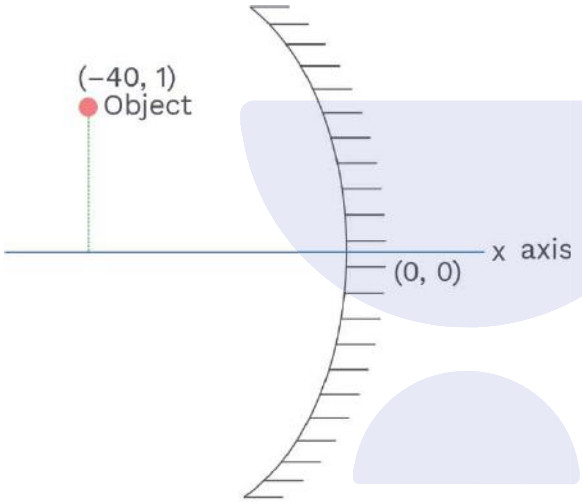


According to above figure both the part of mirror have similar hollow sphere so its radius of curvature is same therefore number of images found is 1.



If we cut the mirror and shift it the centre of curvature changes for example in the diagram shown above. A concave mirror is cut and each and every part is shifted by 1 mm. Then centre of curvature of each part shift by 1 mm and each part behaves as 2 independent concave mirror with its centre of curvature at the new position. Therefore two images are found.

Ex. As shown below a spherical concave mirror with its pole at (0, 0) and principal axis along x-axis. There is an object at (–40 cm, 1 cm), find out the position of image.



R.O.C = 10 cm

Sol. According to sign convention,
$u = -40$ cm, $h_1 = +1$ cm, $f = -5$ cm

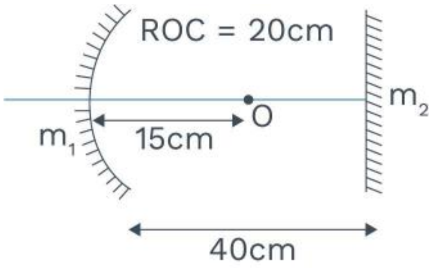
$$- + - = - \Rightarrow - + \frac{}{-} = \frac{}{-} \;; \quad = \frac{-}{} \quad ; - = \frac{-}{}$$

$$\Rightarrow \quad = - - \times \quad = \frac{-\left(- -\right)\times}{-} = - -$$

$\therefore$ The position of image is $\left(\frac{-}{} \quad - - \right)$

Combination Of Mirrors.

Ex. Find out the position of final image after three successive reflections taking first reflection on m_1.



Sol. **I reflection:**

Focus of mirror = – 10 cm and u = – 15 cm

Applying mirror formula :

— + — = —

⇒ = –

For II reflection on plane mirror :

u = –10 cm ∴ v = 10 cm

For III reflection on curved mirror again :

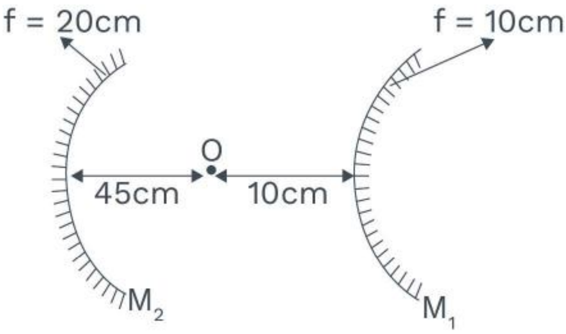
u = – 50 cm, f = – 10 cm

Applying mirror formula :

— + — = —

⇒ = –

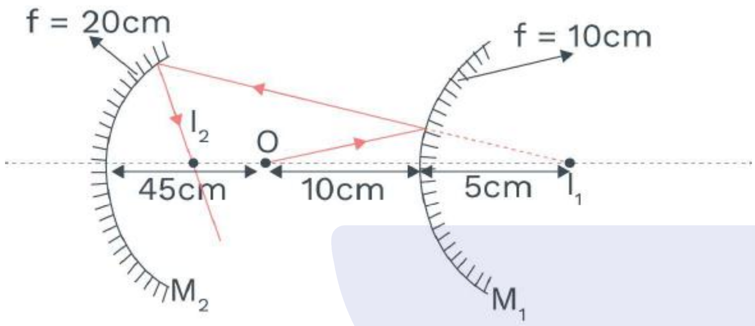
Ex. Find out the location of the final image formed by two reflections. Take the first reflection from M_1.



Sol. For M_1

$$— = — + —$$

$$\Rightarrow —— = — – —— \quad \Rightarrow — = —— \Rightarrow \quad = +$$



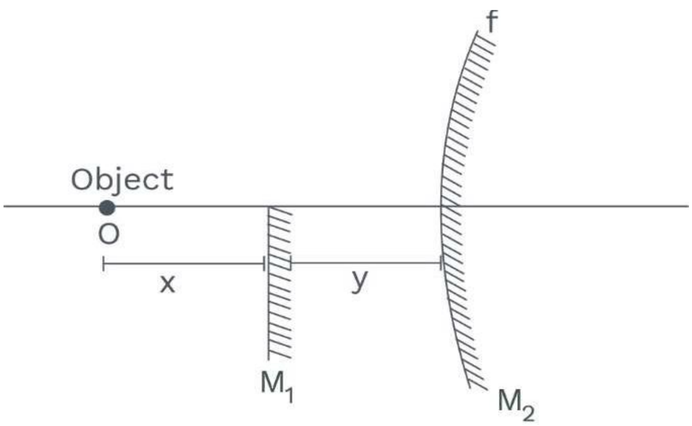
For M_2: u = –60, f = – 20

$$— = — + —$$

$$\Rightarrow –—— = — – —— \Rightarrow — = –\left(—— – ——\right)$$

$$\Rightarrow \quad = –$$

Ex. An object 'O' is placed in front of a small plane mirror 'M_1' and a large convex mirror 'M_2' of focal length 'f'. The distance between 'O' and 'M_1' is 'x', and the distance between 'M_1' and 'M_2' is 'y'. If the images of 'O' formed by 'M_1' and 'M_2' coincides, find focal length of the mirror



Sol. for convex mirror

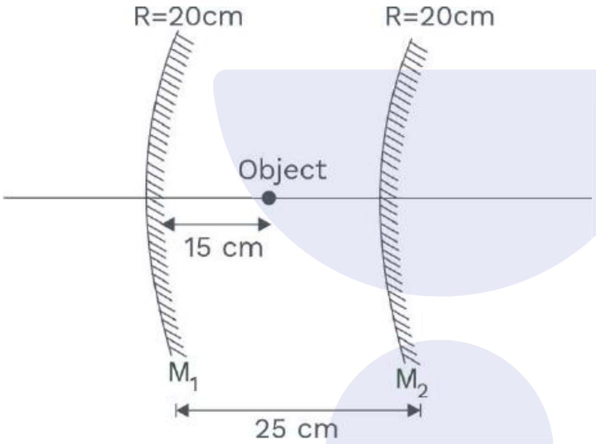
$u = -(x + y)$

$v = +(x - y)$

− = ——— − ——— = ———
 − + −

⇒ = ———
 −

Ex. Find out the position of the final image after two successive reflection, taking the first reflection on M_1.



Sol. (i) Reflection on M_1 mirror.

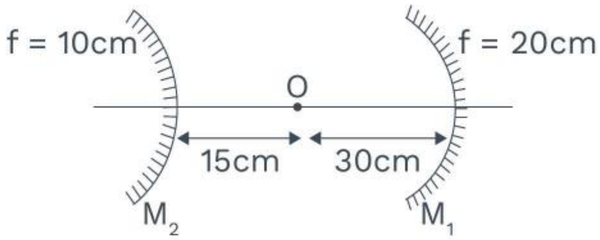
= ——— = −
 −

(ii) Reflection on M_2 mirror

= ——— = ——— = —— = −
 − − −

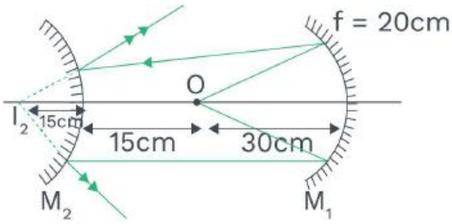
Final image is 10 cm left from pole of M_2 mirror.

Ex. Find out the position of the final image formed by two reflections. Take the first reflection from M_1.



Sol. **For M_1 :**

$u = -30$, $f = -20$



— = — + —

$\Rightarrow$ $v = -60$ cm

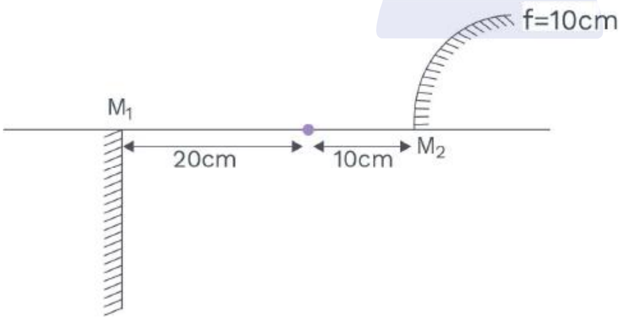
Similarly For M_2 :

$u = +15$, $f = +10$

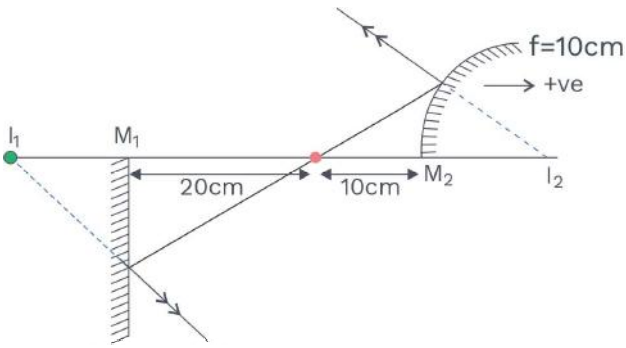
— = — + — ⇒ —— = —— + —

⇒ — = —— – —— = ——— ⇒ =

Ex. Find out the number and positions of all the images formed in the figure shown below.

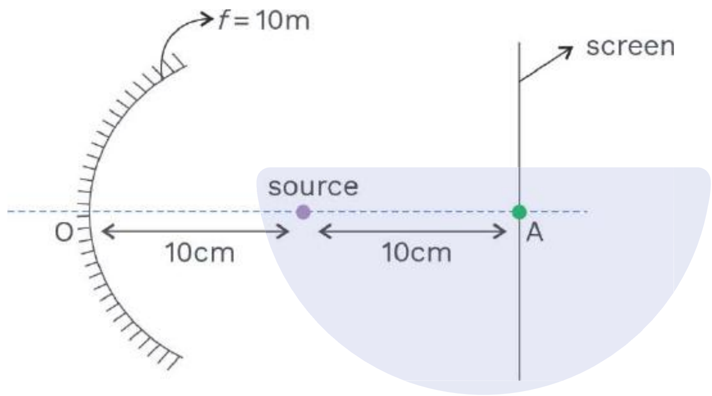


Sol.

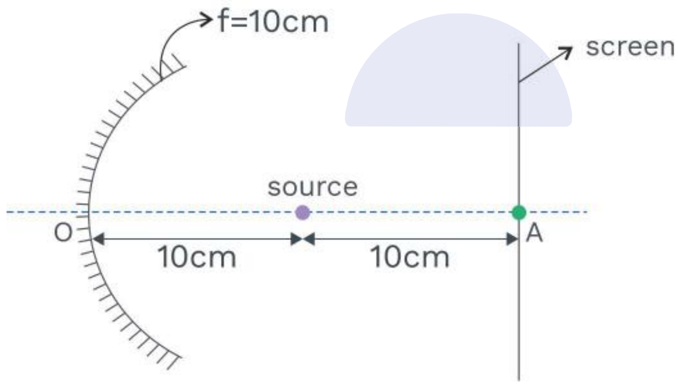


For M_2 :
$u = -10$ cm, $f = +10$ cm, $v = +5$ cm
Note : In the above case only one ray will go on the optic axis and the one ray is not responsible for image formation.
Intensity Of Light

Ex. Intensity at 'A' due to source is 'I'. Without concave mirror, then find the intensity of A after placing concave mirror.



Sol.



m= (/ π

— = — + —

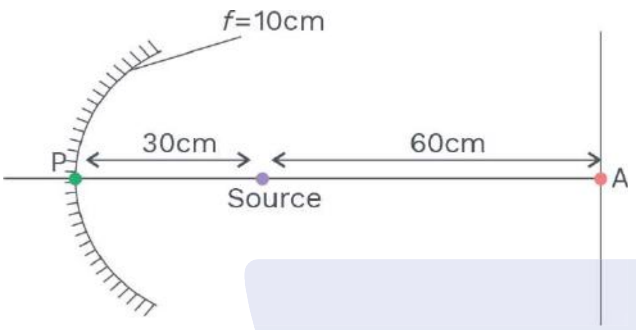
⇒ — = — — —

⇒ = ∞

Intensity at A due to reflection = I.
Total = I + I = 2I

Ex. Intensity at 'A' due to source is 'I' without concave mirror, then find the intensity of A after placing concave mirror in (i) and (ii) figure..

(I)

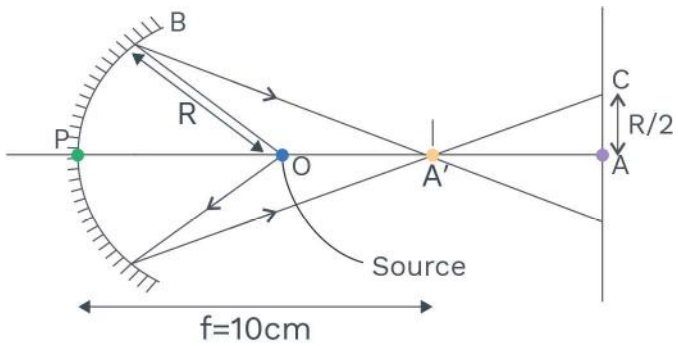


Sol. ($= m$ A $= m$ π

Intensity at point A $= \frac{(}{(\quad A} = \frac{\pi \quad m}{\pi} =$ n

Now $\Delta($ A′N $\quad \Delta$AOA′

$\Rightarrow \frac{(N}{(A'} = \frac{AO}{AA'} \qquad \Rightarrow \qquad \frac{|}{} = \frac{AO}{} \Rightarrow AO = \frac{}{} \times | = \frac{|}{}$



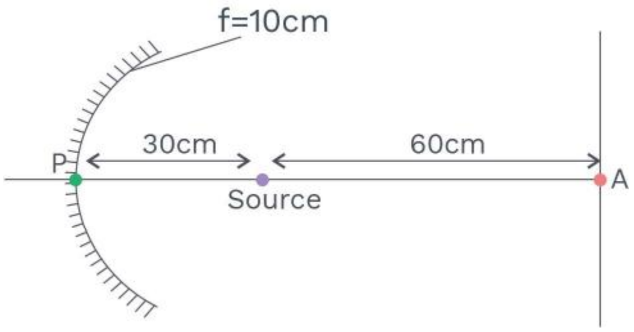
Energy at Area of R radius $= m \pi|$

Now the energy will fall on the screen but at an Area of radius $\frac{|}{}$

So intensity from mirror $= \frac{m \pi|}{\pi |} =$ n

Total Intensity = 16 I + I = 17 I

(II)



Sol. $(\ = m\!\!\!\times \ \ \pi$

Intensity at P = $\dfrac{(}{A} = \dfrac{(}{\pi} = \dfrac{n}{}$

Now $\dfrac{l}{} = \dfrac{}{}$

$= \dfrac{l}{}$

Power incident on mirror

$(_{(} = \dfrac{m}{} \times \ \pi l$

this power will be incident on π area

So Intensity from mirror = $\dfrac{m\!\!\!\times \ \pi l}{\pi \ l} = r$

So total I = I + I = 2I

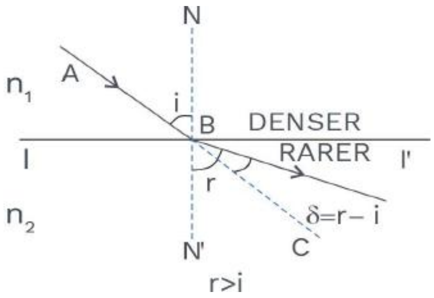
Refraction

Refraction is the phenomenon in which direction of propagation of light changes at the boundary when it passes from one medium to the other. During refraction frequency does not change.

Laws of Refraction :

(a) The incident ray, the refracted ray and the normal to any refracting surface at the point of incidence all lie in the same plane known as plane of incidence or plane of refraction.

(b) —— = Constant for any pair of medium and for light of a given wavelength.



This is called Snell's Law.

Also, —— = —— = —— = $\frac{\lambda}{\lambda}$

For applying in problems remember
$n_1 \sin i = n_2 \sin r$

— = is the refractive Index of the second medium w.r.t. the first medium.

c = speed of light in vacuum (or air) $= 3 \times 10^8$ m/s.

i & r should be taken from normal.

Absolute refractive index (μ)

It is defined as the ratio of speed of light in free space 'c' to that in a given medium v. Hence μ or

= —

Denser is the medium, lesser will be the speed of light and so greater will be the refractive index,

$\because$ < , $\therefore \mu_j > \mu$

Relative refractive index

When light passes from one medium to other, then refractive index of medium 2 relative to 1 is written as μ and is defined as

Definitions

Refraction is the phenomenon in which direction of propagation of light changes at the boundary when it passes from one medium to the other. During refraction frequency does not change.

Concept Reminder

Laws of Refraction :-

(a) Incident ray, refracted ray and normal all lie in same plane.

(b) —— = constant

Concept Reminder

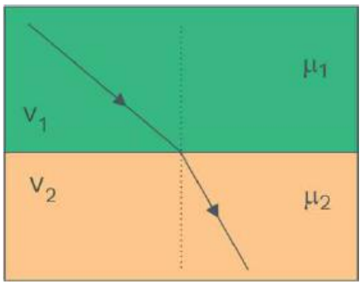
If light passes from rarer to denser medium then ∠ > ∠

Concept Reminder

If ray passes from denser medium to rarer medium then ∠ < ∠

KEY POINTS

- Refraction
- Refractive index
- Optical density
- Snell's law

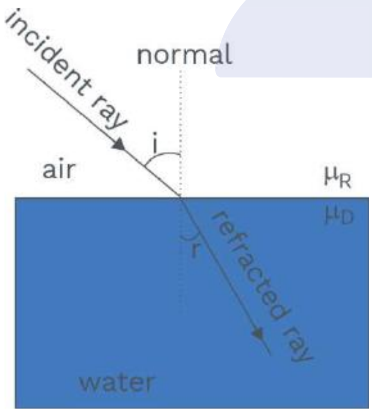


$$\mu \quad = \frac{\mu}{\mu} = \text{————} = \text{——}$$

- **Bending of light ray**

According to Snell's law, $\mu \qquad = \mu$

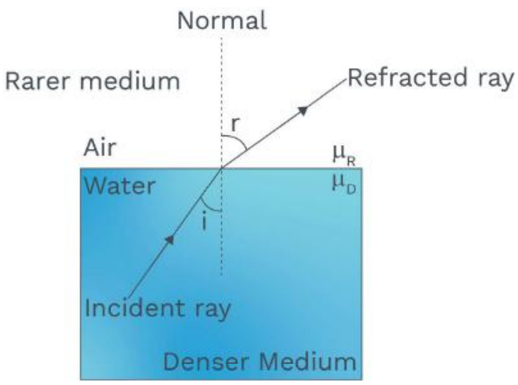
(i) If light passes from rarer to denser medium $\mu = \mu_l$ and $\mu = \mu_S$ so that $\text{——} = \frac{\mu_S}{\mu_l} > \quad \Rightarrow \angle > \angle$



In passing from rarer to denser medium, the ray bends towards the normal.

(ii) If light passes from denser (μ_S) to rarer (μ_l) medium

$$\text{——} = \frac{\mu_l}{\mu_S} < \quad \Rightarrow \angle < \angle$$



In passing from denser to rarer medium, the ray bends away from the normal.

Important points

(1) Refractive index is different for different wavelengths (Cauchy's theorem)

$$\mu = \quad \lambda \quad = A + \frac{N}{\lambda} + \frac{O}{\lambda} + ----$$

A, B & C are positive coefficients for a particular material.

(2) Absolute R.I. of any medium is more than one because speed of light is maximum in vacuum.

(3) Relative R.I. can be less than one.

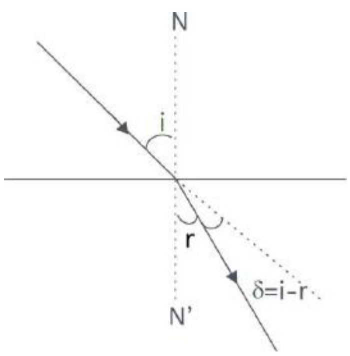
(4) Optical density & mass density has no relation.

(5) Optically rarer can be acoustically denser or vice versa.

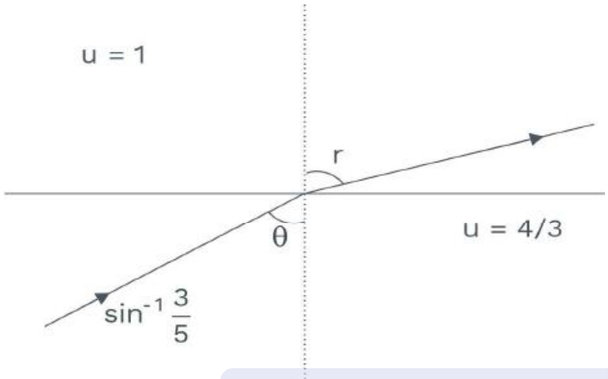
(6) Frequency of light does not change during refraction and colour of light is frequency dependent.

Deviation of a Ray Due to Refraction

Deviation δ of ray incident at $\angle$ and refracted at $\angle$ is given by $\delta = |i - r|$



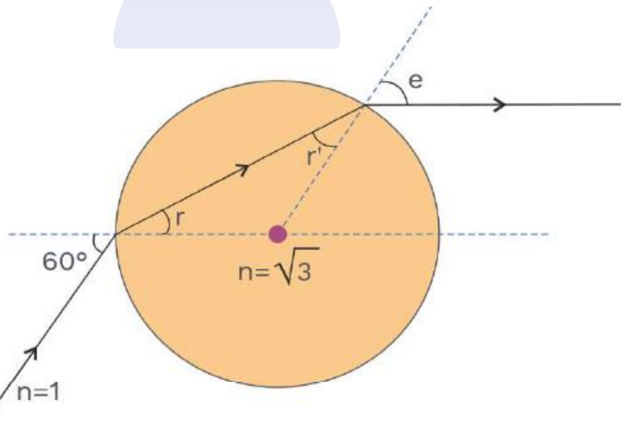
Ex. Find the angle of refraction when it gets refracted from water to air, as shown in figure.



Sol. — = ⇒ — =

sin r = 4/5 ⇒ ∠ = °

Ex. A light ray is incident on a glass sphere at an angle of incidence 60°as shown. Find the angles r, r', e and the total deviation after two refractions.



Sol. Applying Snell's law

1 sin 60° = $\sqrt{\ }$ ⇒ = °

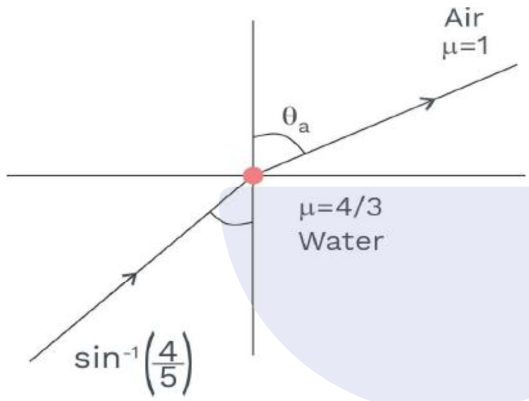
From symmetry r' = r = 30°

Again applying Snell's law at second surface

= $\sqrt{\ }$ ⇒ = °

Deviation at first surface = i – r = 60° – 30° = 30°
Deviation at second surface
= e – r′ = 60° – 30° = 30°
Therefore total deviation = 60°

Ex. Find the angle θ made by the light ray when it gets refracted from water to air, as shown in figure.



Sol. Snell's Law

$\mu \quad \theta = \mu \quad \theta$

$\Rightarrow \; - \times - = \quad \theta$

$\theta = -$

$\Rightarrow \theta = \quad ^{-} \; -$

Ex. Find the speed of light in medium 'a' if speed of light in medium 'b' is — where c = speed of light in vacuum and light refracts from medium 'a' to medium 'b' making 45° and 60° respectively with the normal.

Sol. Snell's Law

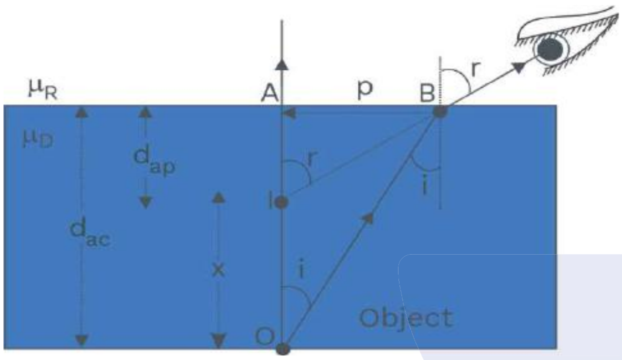
$\mu \quad \theta = \mu \quad \theta \Rightarrow — \quad \theta = — \quad \theta$

$— \quad ^{\circ} = —— \quad ^{\circ} \Rightarrow \quad = \frac{\sqrt{\;}}{\sqrt{\;}}$

Apparent Depth

(1) If a point object in denser medium is observed from rarer medium and boundary is plane, then from Snell's law we have

$\mu_S \quad = \mu_l$ ... (i)



If the rays OA and OB are close enough then p ≈ small

$\simeq \quad = \text{——} \quad$ and $\quad \simeq \quad = \text{——}$

here $\quad$ = actual depth, d_{ap} = apparent depth

So that equation (i) becomes

$$\mu_S = \text{——} = \mu_l \text{——} \Rightarrow \text{——} = \frac{\mu_S}{\mu_l} = \frac{\mu}{\mu}$$

(If $\mu_l = \mu_S = \mu$) then $\quad = \frac{}{\mu}$ ... (ii)

The distance between object and its image, is called normal shift (x)

$$= \quad - \quad \left[\because \quad = \frac{}{\mu}\right];$$

$$= \quad - \frac{}{\mu} = \left[\ - \frac{}{\mu}\right]$$ ... (iii)

If d_{ac} = d then $\boxed{= \left[\ - \frac{}{\mu}\right]}$

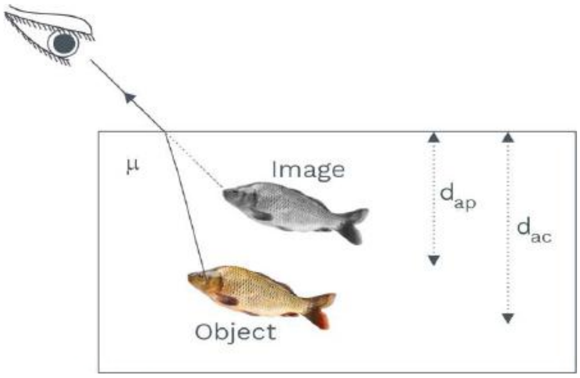
Concept Reminder

According to Snell's law

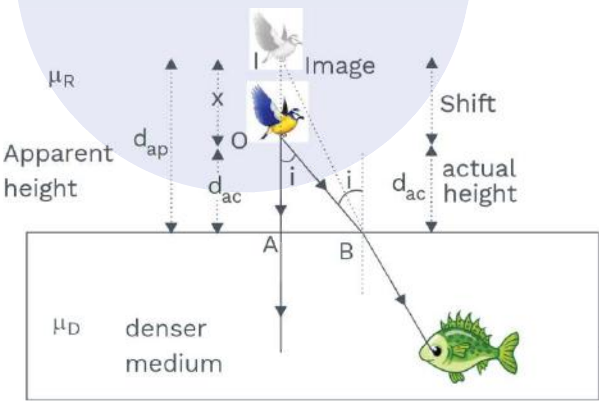
$$= \left[\ - \frac{}{\mu}\right]$$

Concept Reminder

Deviation of ray incident at ∠i and refracted at ∠r is given as |i–r|



(2) Object in a rarer medium as seen from a denser medium



$$\underline{\quad} = \frac{\mu}{\mu} = \frac{\mu_I}{\mu_S} = \frac{\quad}{\mu} <$$

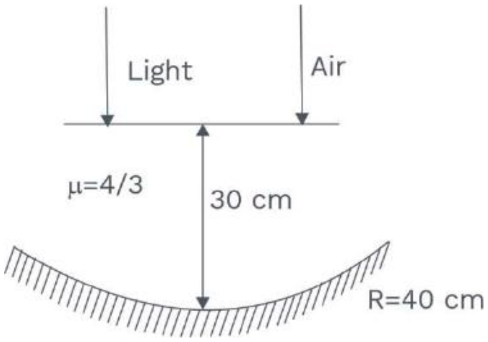
$$= \mu \qquad \text{i.e.,} \qquad >$$

A flying object appears to be higher than in reality.

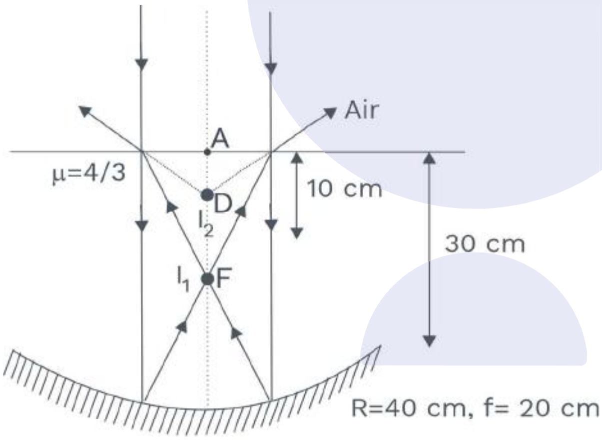
$$= \quad -$$

$$\Rightarrow \quad = \mu -$$

Ex. A concave mirror is placed inside water with its shining surface upwards & principal axis vertical as shown. Rays are incident parallel to the principal axis of concave mirror. Find the position of final image.



Sol.

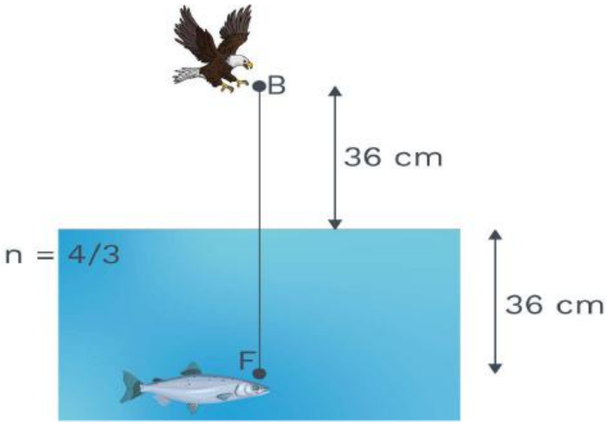


$AS = \frac{\quad}{\quad} =$

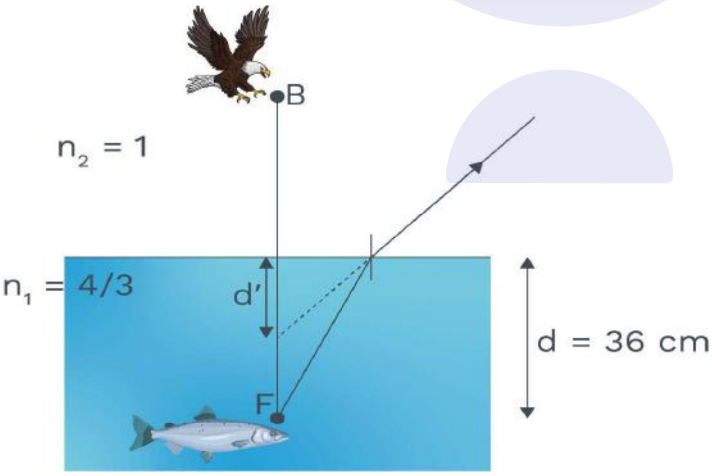
Ex. Find out the following in the figure shown below:

(a) The apparent distance of the fish from the surface as observed by the bird

(b) The apparent distance of the bird from the surface as observed by the fish

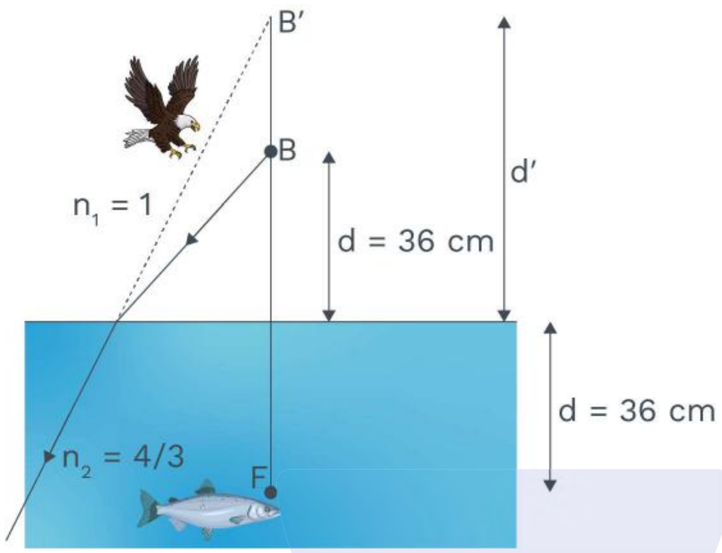


Sol. (a) = =



= ⇒ =

(b) = ⇒ =



Ex. A converging beam of light rays is incident on a concave spherical mirror whose radius of curvature is 0.8 m. Determine the position of the point on the optical axis of the mirror where the reflected rays intersect. If the extensions of the incident rays intersect the optical axis 40 cm from the mirror's pole.

Sol. u = + 40 cm

f = – 40 cm

$\Rightarrow$ —— = — + ——
–

$\Rightarrow$ — = – ——

$\Rightarrow$ v = –20 cm

So, required position is 0.2 m form the mirror's pole.

Ex. A point object is placed on the principal axis at 60 cm in front of a concave mirror of focal length 40 cm on the principal axis. If the object is moved with a velocity of 10 cm/s

(a) along the principal axis, find the velocity of image

(b) perpendicular to the principal axis, find the velocity of image at that moment

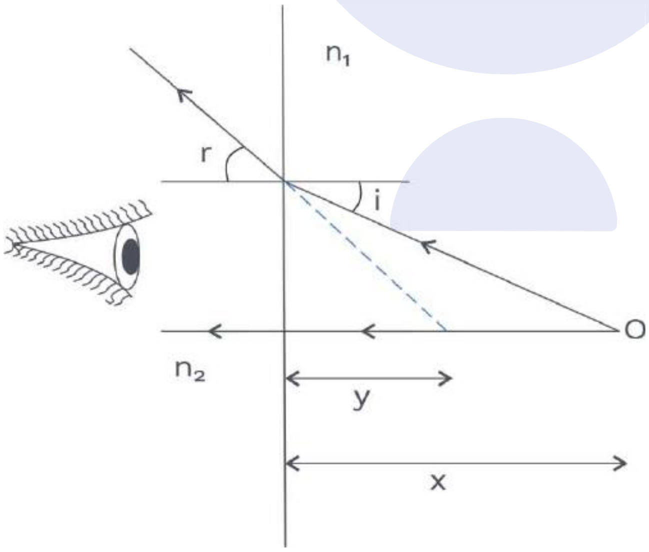
Sol. (a) $\Rightarrow - = - + - \Rightarrow \frac{\quad}{-} = - + \frac{\quad}{-}$

$\Rightarrow - = \frac{}{} + \frac{}{} = \frac{-}{\quad} \Rightarrow \quad = -$

$\frac{}{} = - \frac{}{}\frac{}{} = -\left(\frac{}{}\right) \times \quad = -$

(b) $\frac{}{} = \frac{-}{} \Rightarrow \frac{}{} = -\left(-\right)\left(\frac{}{}\right) = -\left(\frac{-}{-}\right) \times \quad = -$

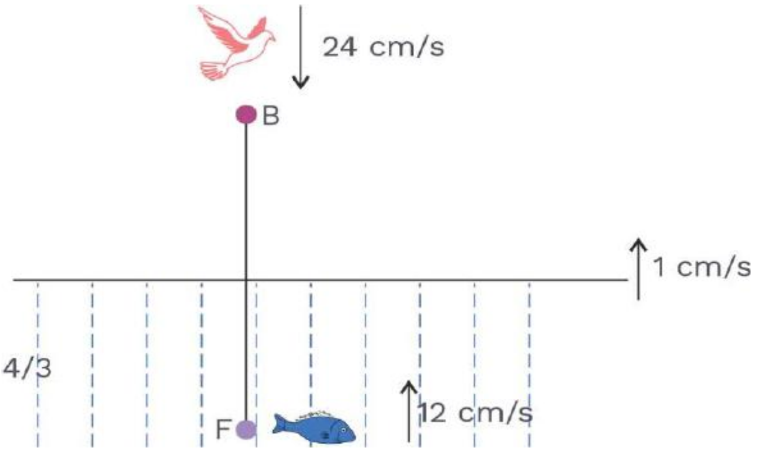
- **Velocity of the image in case of plane refraction:**



$\frac{}{} = - \Rightarrow \quad = \frac{}{}$

$\frac{}{} = \frac{}{}\frac{}{} \Rightarrow {}_{m} = \frac{}{} \quad 8$

Ex. Find out the following in the figure shown below:
(a) The apparent speed of the fish as observed by the bird
(b) The apparent speed of the bird as observed by the fish



Sol. (a) m = — 8 ⇒ m – = — –

m = — + = —

m = m – m = — + = —

(b) = =

m = — 8

= – = +

⇒ m = – = — – –

m = — + = —

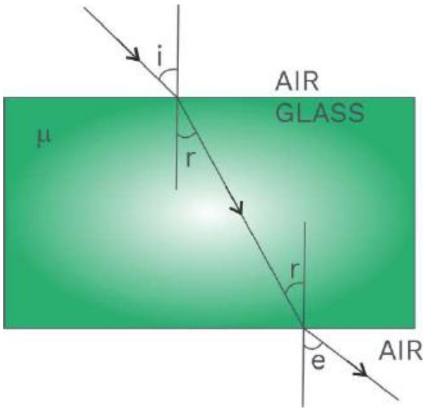
Refraction Through A Glass Slab:

When a light ray passes across a glass slab having parallel faces, it gets refracted twice times before finally emerging out of it.

First refraction gets place from air to glass.

So, μ = —— ... (i)

The second refraction takes place from glass to air.



Concept Reminder

When a light ray passes through a glass slab having parallel faces, it gets refracted twice before finally emerging out of it.

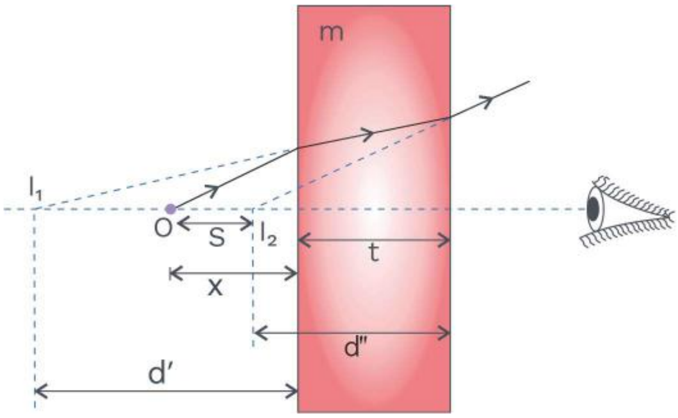
Therefore, $\frac{\;}{\mu} = \frac{\;}{\;}$ ... (ii)

From equations (i) and (ii), we find

$$\frac{\;}{\;} = \frac{\;}{\;} \Rightarrow \; =$$

Hence, the emergent ray is parallel to the incident ray.

- **Apparent shift due to slab when object is seen normally through the slab:**



I^st Refraction:

Because of the refraction at the first surface, the image of 'O' is formed at I_1. For this refraction, the real depth is 'x' and the apparent depth is d'. Similarly, the first medium is air and the second is the slab.

Thus,

$$\frac{\mu}{} = \frac{}{} \Rightarrow {}' = \mu$$

IInd Refraction:

The point I_1 shows as the object for the refraction at the second surface of slab. Due to this refraction, the image of I_1 is produced at I_2. Thus.

$$\frac{}{\mu} = \frac{}{\mu +} \Rightarrow {}'' = + \frac{}{\mu}$$

Shift $= + - - \frac{}{\mu}$

$$\boxed{= \left[- \frac{}{\mu} \right]}$$

If medium is not air out of the slab

$$\boxed{= \left[- \frac{\mu}{\mu} \right]}$$

Important points

1. Rays must be paraxial
2. Medium on both face of the slab should be same.
3. Shift comes out from the object
4. Shift does not depend of the distance of the object from the slab.
5. If shift comes out positive then shift is towards the direction of incident rays and vice versa.

Ex. Find out the shift produced by the slab having thickness 15 cm and refractive index 1.5 which is kept in air.

Sol. shift $= \left[- \frac{}{\mu} \right] = \left[- \frac{}{} \right] =$

Ex. See the figure. Find out the distance of final image formed by mirror

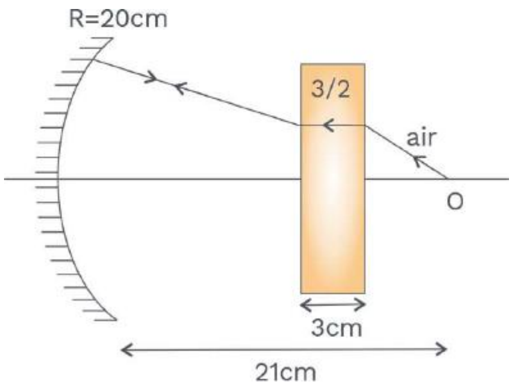
Concept Reminder

Shift is independent of the distance of the object from the slab

$$= \left[- \frac{}{\mu} \right]$$

Concept Reminder

Whether it is a wave or a particle or a human being, whenever two mediums and two velocities are involved, one must follow Snell's law if one wants to take the shortest time.



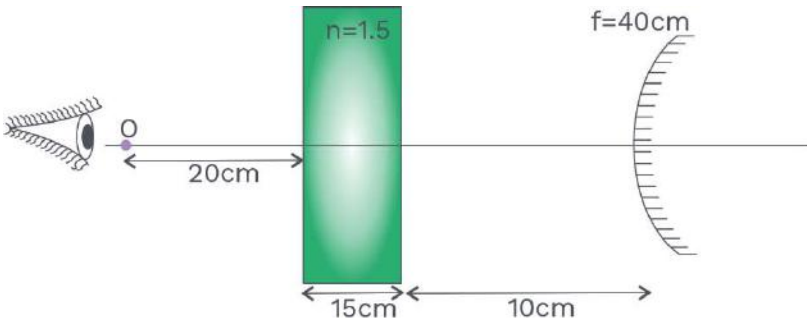
Sol. Shift $= \left(\ - \frac{}{} \right)$

For mirror object is at a distance

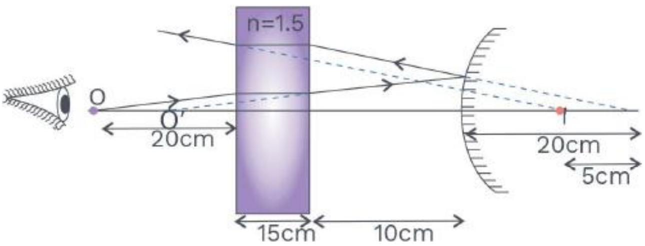
$$= \quad - \left(- \frac{}{} \right) =$$

$\therefore$ Object is at the centre of curvature of mirror. Hence the light ray will retrace and image will formed on the object itself.

Ex. Find out the distance between image and the mirror as observed by observer in the figure shown below



Sol.



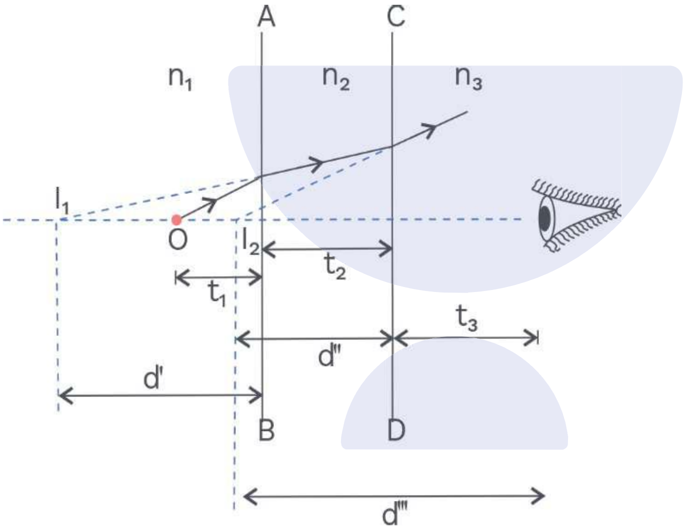
$$= \left(- \frac{-}{\mu} \right) = \left(- - \right) =$$

u = – 40, f = + 40

v = + 20 cm

the distance between mirror and the image as observed by observer = 20 – shift = 15 cm

- **Apparent distance between observer and object when both are in different medium:**



I[st] Refraction :

$$\frac{}{} = \frac{AN}{I \qquad AN} = \frac{}{}$$

$$= \frac{}{}$$

II[nd] Refraction :

$$\frac{}{} = \frac{m \qquad OS}{I \qquad m \qquad OS} = \frac{''}{\frac{}{} +}$$

$$= \frac{}{}\left[\frac{}{} + \right] = \left[\frac{}{} + \frac{}{} \right]$$

Final distance of image from observer = 3d″ +t_3

$$= \left[\frac{}{} + \frac{}{} + \frac{}{} \right]$$

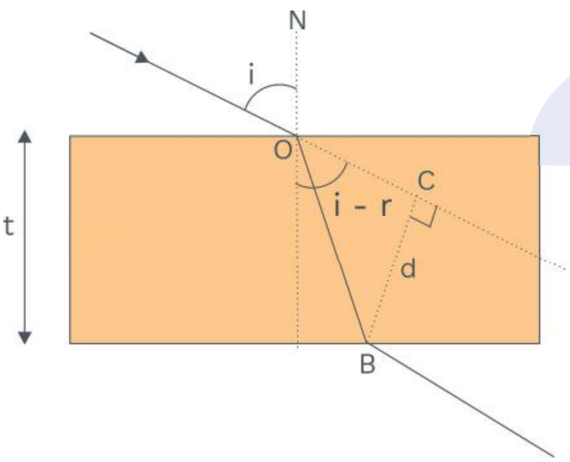
- If object and observer are in the same medium then shift formula should be used and if both are in different medium then the above formula of apparent distance should be used.

Concept Reminder

If object and observer are in same medium, then shift formula should be used and if both are in different medium then the above formula of apparent distance should be used.

Lateral Shift

The perpendicular distance between incident & emergent ray is known as lateral shift.



KEY POINTS

- Apparent depth
- Normal shift
- Lateral shift

Definitions

The perpendicular distance between incident & emergent ray is known as lateral shift.

Lateral shift (d) = BC & t = thickness of slab

In ΔN8 O :

$$- = \frac{NO}{8\,N} = \frac{}{8\,N}$$

$$= 8\,N \quad - \qquad \text{.... (i)}$$

In Δ8 NS

$$= \frac{8\,S}{8\,N} = \frac{}{8\,N}$$

8 N = ——— ... (ii)

from (i) & (ii) | = ——— − |

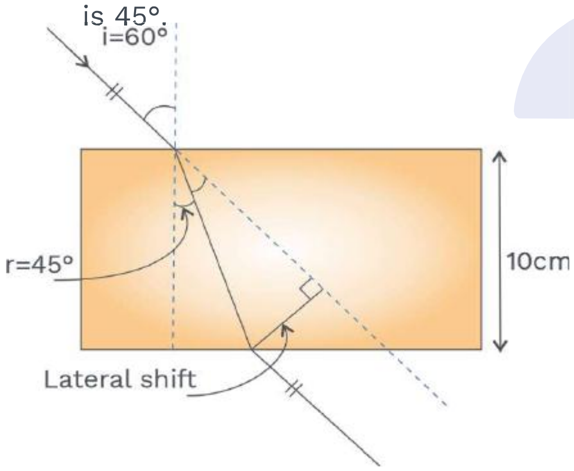
If angle is small

d = t (i − r)

$= \left(- - \right)$ (by Snell's law , sin i = μ sin r for small angle i = μ)

$= \left(- \frac{}{\mu} \right)$

Ex. Find out the lateral shift of light ray while is passes through a parallel glass slab of thickness 10 cm placed in air. The angle of incidence in air is 60° and the angle of refraction in glass is 45°.



Sol. = ——— = ——— ° ° °

= ——— = √

Concept Reminder

For small angles, lateral shift

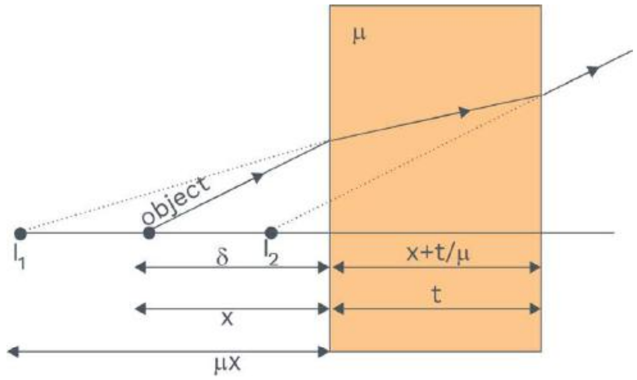
$= \left(- \frac{}{\mu} \right)$

Concept Reminder

Normal shift through a glass slab

$\delta = \left(- \frac{}{\mu} \right)$

Transparent Glass slab (Normal shift)



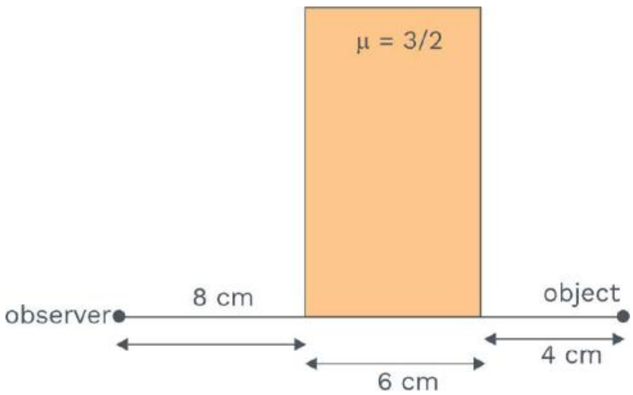
$$\delta = \quad + \quad - \quad + \quad \mu$$

$$\delta = \quad - \frac{\quad}{\mu} = \left(\quad - \frac{\quad}{\mu} \right)$$

$$\boxed{\delta = \left(\quad - \frac{\quad}{\mu} \right)}$$

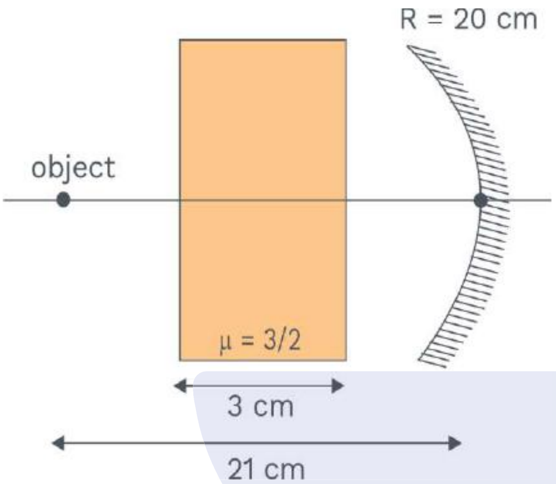
Note : - If medium outside slab is rarer then shift is in direction of propagation of light

Ex. Find out the location of object observed by observer.



Sol. $\delta = \left(\quad - \frac{\quad}{\quad} \right) = \left(- \right) =$ object appeared to be 16 cm from observer.

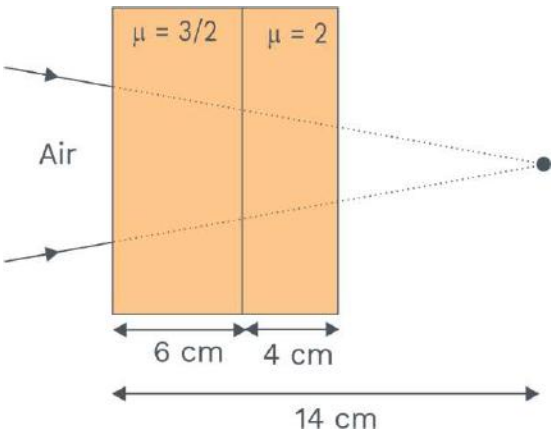
Ex. Find out the distance of final image formed by mirror in the given figure.



Sol. shift = 3 $\left(- \frac{}{} \right) =$

For mirror object is at a distance = 21–1 = 20 cm. Now object is at the centre of curvature of mirror. Hence the light rays will retrace & image will formed on the object itself.

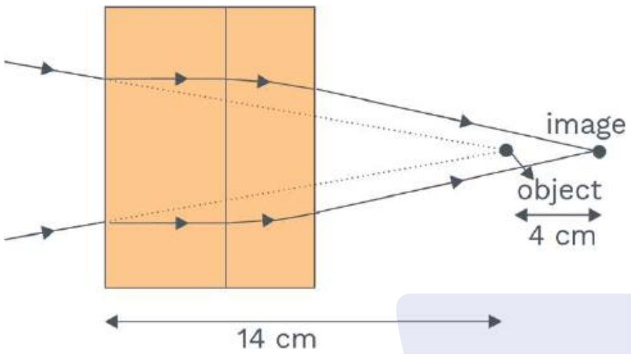
Ex. A convergent beam is incident on two slabs placed in contact with each other (as shown in figure) where will the rays finally converge?



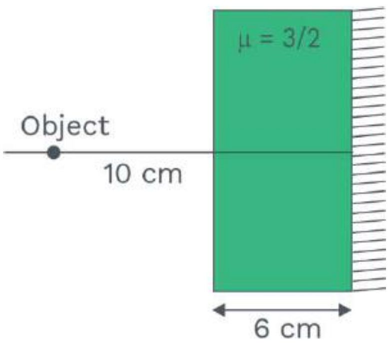
Sol. $\delta = \delta + \delta = \left(- \frac{}{\mu} \right) + \left(- \frac{}{\mu} \right)$

$$= \left(- - \right) + \left(- - \right)$$

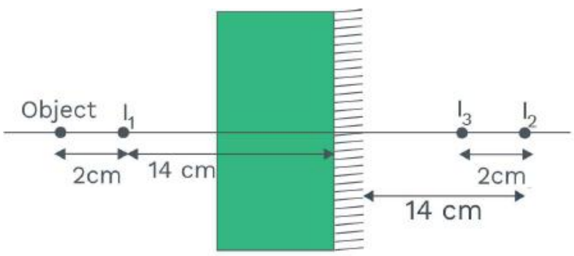
= 4 cm in the direction of propagation of ray.



Ex. Find out location of final image of object in given figure.

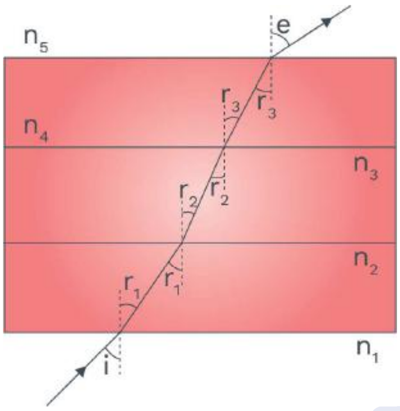


Sol. $\delta = \left(- \rule{2em}{0.4pt} \right) =$



Final image is I_3 which is 12 cm behind mirror.

Refraction Through Multiple Slab



=

=

=

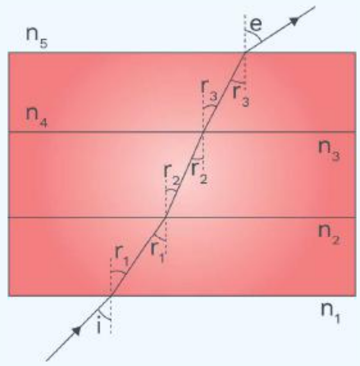
=

comparing all equation

=

if $n_1 = n_5$ then incident ray & emergent ray are parallel to each other.

Concept Reminder

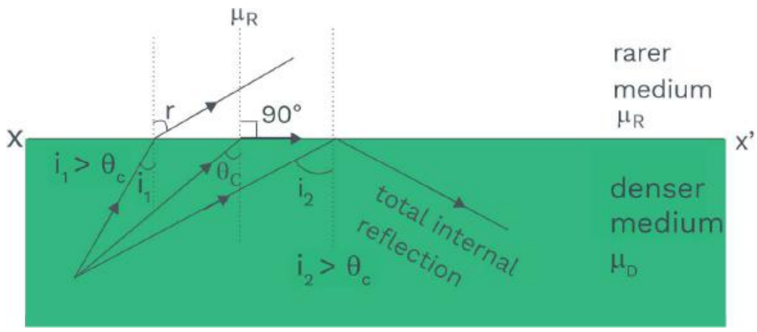


$n_1 \sin i = n_5 \sin e$

if $n_1 = n_5$ then $i = e$

Total Internal Reflection (TIR):

When light ray travels from denser to rarer medium, it bends away from the normal. If the incident angle is increased, the angle of refraction also increases. At a particular value of angle, the refracted ray subtends 90° angle with the normal, this angle of incidence is known as critical angle (θ_O). If angle of incidence increases further, the ray comes back to the same medium. This phenomenon is defined as total internal reflection.



Conditions

- Angle of incidence > critical angle

$$> \theta$$

- Light should travel from denser to rarer medium for example Glass to air, water to air, Glass to water

Applying Snell's Law at boundary xx' yields

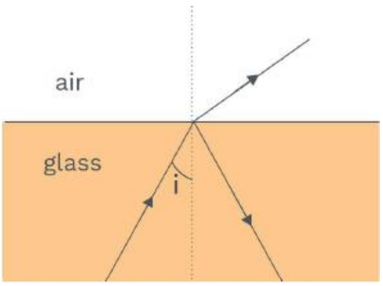
$\mu_S \quad \theta_O = \mu_l \quad ^\circ$

$$\Rightarrow \quad \theta_O = \frac{\mu_l}{\mu_S}$$

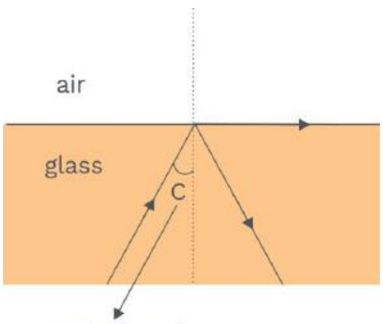
$$\mu_S \quad \theta_O = \mu_l \quad \frac{\pi}{}$$

$$\therefore \quad \theta_O = \quad - \left(\frac{}{{}_l\,\mu_S} \right)$$

(i) If i < C



(ii) If i = C



Definitions

The phenomenon in which ray travelling from denser to rarer medium reflect back to same medium is known as total internal reflection.

Definitions

At a particular value of angle, the refracted ray subtends 90° angle with the normal, this angle of incidence is known as critical angle (θ_c).

Rack your Brain

If the critical angle for total internal reflection from a medium to vacuum is 45°, then velocity of light in the medium is:

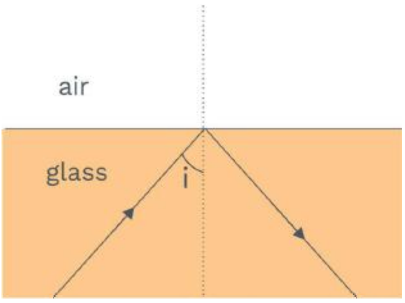
(1) 3×10^8 m/s

(2) 1.5×10^8 m/s

(3) $\frac{}{\sqrt{}} \times$

(4) $\sqrt{} \times$

(iii) If i > C



Some Facts

(i) In TIR, reflection is 100%.

(ii) In TIR, laws of reflection are valid.

(iii) In situation where there is continuous variation of refractive index, critical angle is approximately 90° and bending of ray takes place if angle of incidence approaches 90° while travelling successively from denser to rarer layers.

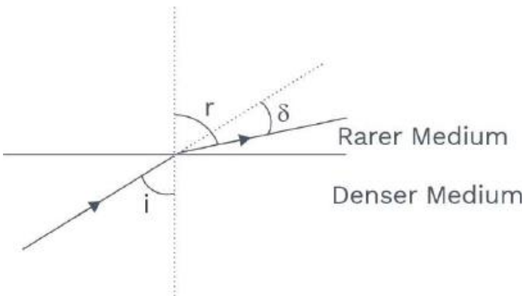
(iv) Critical angle increases with the increase in temperature of the medium

(v) Critical angle depends on nature of medium, temperature of medium and wavelength of light.

(vi) Air bubbles in glass appear silvery white due to total internal reflection.

Deviation versus angle of incidence graph for a ray travelling:

(i) from denser to rarer medium:



Concept Reminder

- Angle of incidence should be greater than critical angle.
- Light should travel from denser to rarer medium

Rack your Brain

In total internal reflection when the angle of incidence is equal to the critical angle for the pair of media in contact, what will be angle of refraction?

(1) 0°

(2) Equal to angle of incidence

(3) 90°

(4) 180°

Concept Reminder

The atmosphere is less dense as its height increase, and it is also known that the index of refraction decrease with a decrease in density.

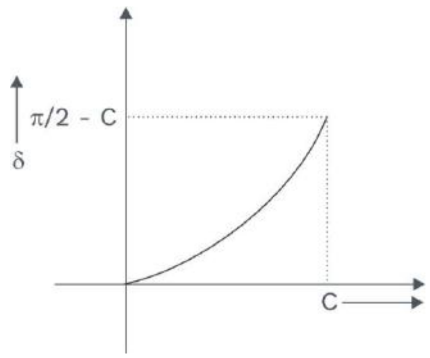
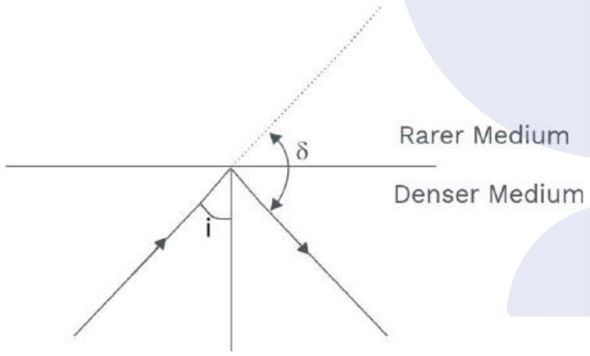


Fig. : Deviation versus angle of incidence graph when TIR is not taking place



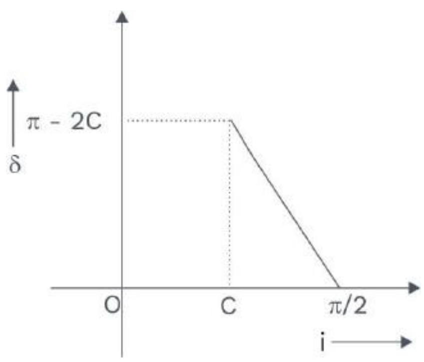
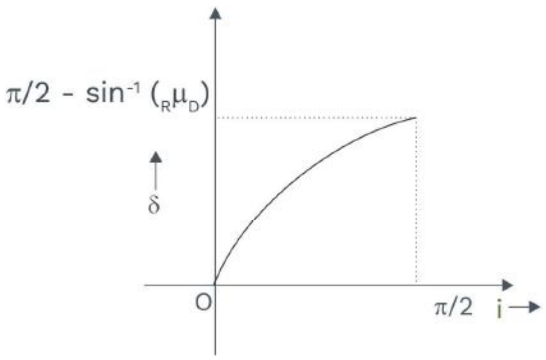


Fig.: Deviation versus angle of incidence graph when TIR is taking place

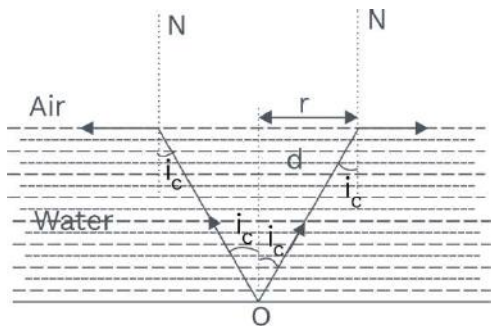
(ii) From rarer to denser medium



Ex. A tiny luminous body, at the bottom of a pool of water (μ = 4/3), d metres deep, emit rays upward in all directions. A circular area of light is formed at the surface of the water. Determine radius r of the circle of light.

Sol. The circular area is formed by the rays refracted into the air. For all angles greater than i_c the rays are totally internally reflected. Thus the circular lighted area of radius r at the surface should form a cone at the luminous object O, with angle $2i_c$ as the vertex angle.

Therefore, from the geometry of figure,



$— =$ or $=$

but $= \frac{—}{\mu}$,

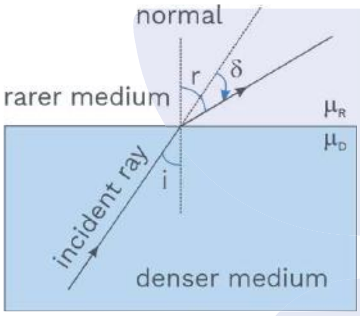
Therefore, $= \frac{}{\sqrt{\mu \; -}}$

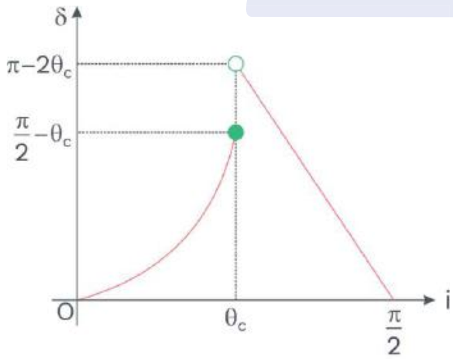
For water $\mu =$ $=$ $\sqrt{\ }$

or $r = 1.13\ d$

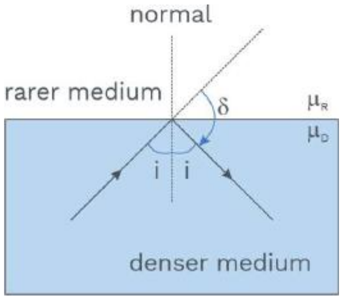
Graph between angle of incidence (i) and angle of deviation (δ) as ray goes from denser to rarer medium

- If $< \theta$ μ_S $= \mu_l$ $=$ $- \left(\frac{\mu_S}{\mu_l}\right)$
- so $\delta =$ $-$ $=$ $- \left(\frac{\mu_S}{\mu_l}\right) -$

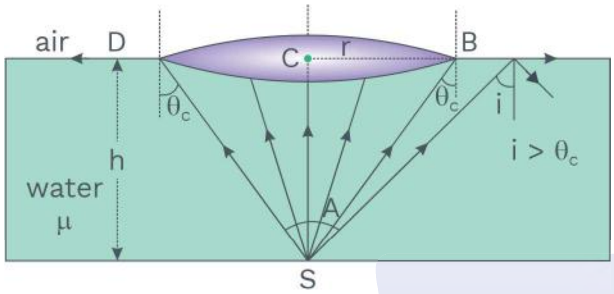




- If $> \theta$ $\delta = \pi -$



- A point source is situated at the bottom of a tank filled with a liquid of refractive index μ upto h height . It is found that light comes out of liquid surface through a circular portion above the source.

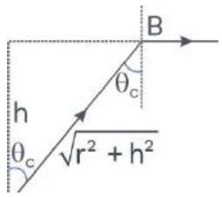


$$\theta_O = \frac{}{\sqrt{\quad + \quad}}$$

and

$$\theta_O = \frac{}{\mu} \Rightarrow \frac{}{\mu} = \frac{}{\sqrt{\quad + \quad}} \Rightarrow \frac{}{\mu} = \frac{}{\quad + \quad}$$

$$\Rightarrow \mu \quad = \quad + \quad \Rightarrow \mu \quad - \quad =$$



radius of circular portion $= \frac{}{\sqrt{\mu \quad -}}$ and

$= \pi$

Vertex angle $A = \quad \theta_O$

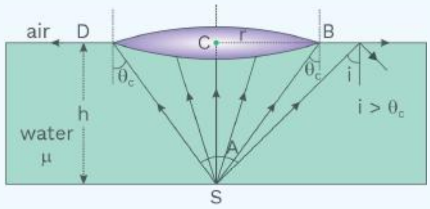
For pure water, A = 2 × 49° = 98°

Applications Of Total Internal Reflection:

- **Sparkling of diamond :** The twinkling of diamond is due to total internal reflection (TIR) inside it. As refractive index for diamond is 2.5 so $\theta \quad = \quad ^\circ$

Concept Reminder

Radius of circular portion



$= \frac{}{\sqrt{\mu \quad -}}$ and area = πr^2

Concept Reminder

- Application of TIR
- Sparkling of diamond
- Optical fibre
- Mirage and optical looming

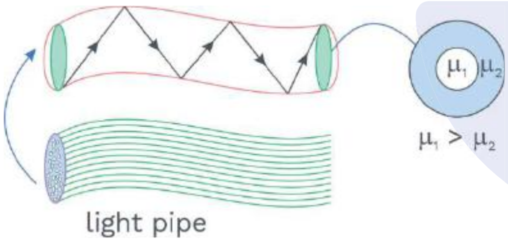
Concept Reminder

Diamond: Diamonds are known for their spectacular brilliance. Their brilliance is mainly due to the total internal reflection of light inside them. The critical angle for diamond-air interface (≅ 24.4°) is very small, therefore once light enters a diamond, it is very likely to undergo total internal reflection inside it.

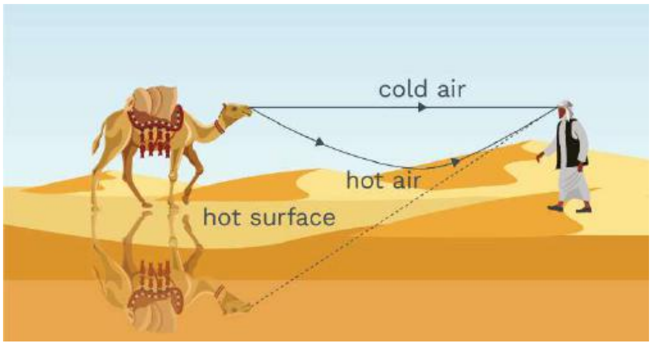
. Diamond is cut in such a manner that, once the light enters into it, when it tends come out then $> \theta_0$, So TIR will take place repeatedly inside it.

The light which beams out from a few places entering into the eyes of the observer makes it sparkle.

- **Optical Fiber :** In optical fiber light propagates through multiple total internal reflections along the axis of a glass fiber of few microns radius in which index of refraction of core is greater than that of surroundings (cladding).



- **Mirage and optical looming** : Mirage is caused due to total internal reflection in deserts and other hot regions where , refractive index of air near the surface of earth becomes lesser than that above it due to heating of the earth. Light from distant objects approach the surface of earth with successively increasing i, till $> \theta_0$ so that TIR takes place so that inverted images appear along with the objects as shown in figure.



Concept Reminder

Optical fibres are extensively used for transmitting and receiving electrical signals which are converted to light by suitable transducers. Obviously, optical fibres can also be used for transmission of optical signals.

Concept Reminder

Nowadays optical fibres are extensively used for transmitting audio and video signals through long distances. Optical fibres too make use of the phenomenon of total internal reflection.

Concept Reminder

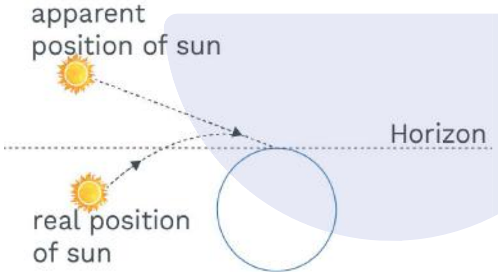
Optical density should not be confused with mass density, which is mass per unit volume. It is possible that mass density of an optically denser medium may be less than that of an optically rarer medium (optical density is the ratio of the speed of light in two media). For example, turpentine and water. Mass density of turpentine is less than that of water but its optical density is higher.

Similar to 'mirage' in deserts, 'optical looming' takes place in polar regions due to TIR. Here μ of different air layers decrease with height and so an inverted image of an object is formed in the sky which appears to be suspended in air.

Note :- In case of TIR, all incident light is reflected back into same medium. Therefore, image formed by TIR are much brighter than image formed by mirrors or lens.

Some Interesting facts about light :

(1) THE SUN RISES BEFORE IT ACTUALLY RISES AND SETS AFTER IT ACTUALLY SETS :



The atmosphere is less dense as its height increase, and it is also seen that the index of refraction reduces with a decrease in density. So, there is a decrement in the index of refraction with height. Due to this the light rays turned as they move in the earth's atmosphere

(2) THE SUN IS OVAL SHAPED AT THE TIME OF ITS RISE AND SET :

The rays diverging from the lower edge of the sun have to cover a greater thickness of air than the rays from the upper edge. Therefore the former are refracted more than the latter, and so the vertical diameter of the sun appears to be a little shorter than the horizontal diameter which remains unchanged.

(3) THE STARS TWINKLE BUT NOT THE PLANETS :

The refractive index of atmosphere varies by a small amount due to several reasons. This causes slight variation in bending of light due to which the apparent location of star also changes, producing the effect of twinkling.

(4) GLASS IS TRANSPARENT, BUT ITS POWDER IS WHITE :

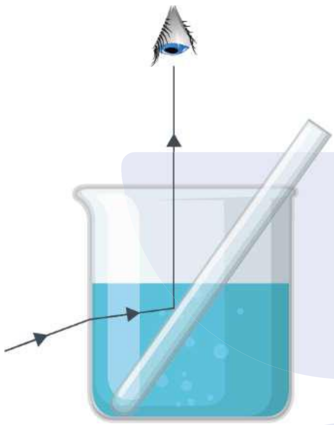
When powdered, light is reflected from the surface of innumerable small pieces of glass and so the powder appears white. Glass transmits most of the incident light and reflects very little hence it appears transparent.

(5) GREASED OR OILED PAPER IS TRANSPARENT, BUT PAPER IS WHITE :

The rough surface of paper diffusely reflects incident light and so it appears white. When oiled or greased, insufficient reflection takes place and most of the light is allowed to pass and therefore it appears transparent.

A TEST TUBE OR A SMOKED BALL SUBMERSED IN WATER APPEARS SILVERY WHITE WHEN VIEWED FROM THE TOP :

This is due to Total internal reflection



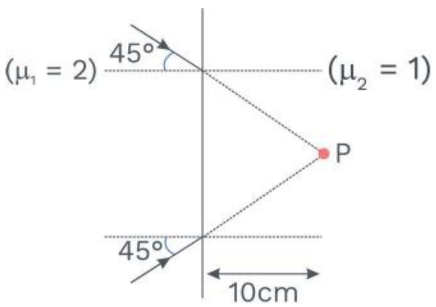
Concept Reminder

The main requirement in fabricating optical fibres is that there should be very little absorption of light as it travels for long distances inside them. This has been achieved by purification and special preparation of materials such as quartz.

(7) TREES HANG INVERTED UNDERGROUND IN DESERTS AND SHIPS HANG INVERTED IN THE AIR IN COLD COUNTRIES:

This is due to Total internal reflection

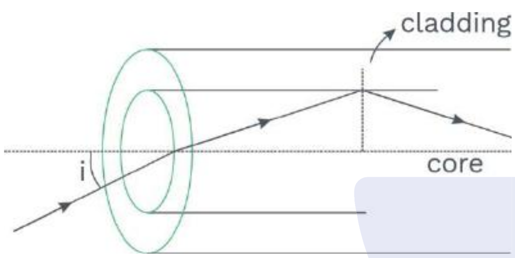
Ex. A plane surface separates two media of refractive indices, $\mu =$ & $\mu =$. A converging ray of light incident on the surface, whose converging point is 10 cm behind the surface as shown. Find the new converging point.



Sol. $\theta_O = - \Rightarrow \theta_O = \quad ^\circ$

Angle of incidence > critical angle therefore TIR will occur & light ray converge before the surface at a distance of 10 cm.

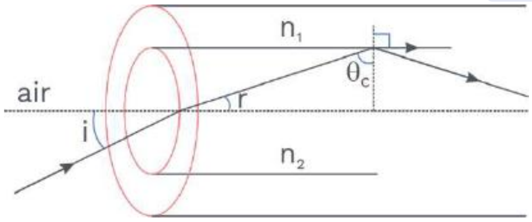
Maximum Acceptance Angle In Optical Fibre



It is defined as the maximum external angle of incidence at which the external light rays must strike the core-cladding intersection & enters the fibre core & propagate within it.

Derivation

Using Snell's law



$\rightarrow \quad = \quad$ ... (i)

$\rightarrow \quad \theta_O = \quad ^\circ \quad \theta_O = \quad -$

$= \quad$ ... (ii)

square (i) & (ii) equation & add them

$= \quad -$

$$= \sqrt{\quad - \quad}$$

Concept Reminder

If outside region of optical fibre is not air

$= \sqrt{\frac{\quad - \quad}{\quad}}$

Concept Reminder

If outside region of optical fibre is air then,

$= \sqrt{\quad - \quad}$

Definitions

The angle between refracting faces/ surface is called the refracting angle or angle of the prism (A) or prism angle (A).

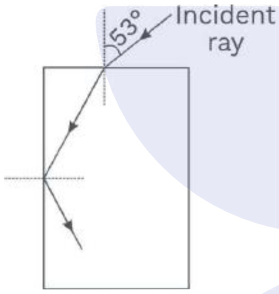
$\angle$ = maximum acceptance angle

If outside medium is not air, but having refractive index n_3 then

$$= \frac{\sqrt{\quad - \quad}}{\quad}$$

Angle of incidence for light ray propagation in optical fibre must be less than angle "i"

Ex. For the given incident ray as shown in figure, find the minimum refractive index of slab.



Sol. $- \quad =$

$$- \quad = (\qquad)$$

$$= \quad + \frac{}{} = \frac{}{} \Rightarrow \quad = \sqrt{\frac{}{}}$$

Ex. The wavelength of light in two different liquids are 3500 Å & 7000 Å, find the critical angle for the pair of liquid.

Sol. $\theta_O = \frac{\mu_l}{\mu_S} = \frac{\lambda_S}{\lambda_l}$

$$\theta_O = \frac{}{} = \frac{}{} \Rightarrow \theta_O = \quad ^\circ$$

Ex. A bulb is kept at a depth of $\sqrt{\quad}$ m in water and a floating opaque disc is kept over the bulb so that the bulb is not visible from the surface. Find out the minimum diameter of the disc? $\mu \quad =$

Sol. $= \frac{}{\sqrt{\mu \; -}} = \frac{\sqrt{}}{\sqrt{— \; -}} =$

Diameter = 12 m

Ex. A ray of light travelling in glass (μ = 3/2) is incident on a horizontal glass air surface at the critical angle θ . If a thin layer of water (μ = 4/3) is now poured on the glass air surface, at what angle will the ray of light emerge into the air at the water air surface?

Sol. Initially $_A\mu$ = so θ = −

when water is poured μ =

so new critical angle (i_c) = $\sin^{-1}$ (8/9) , i.e. θ < & hence now TIR will not take place at AB and light will be incident on water air boundary at and angle r such that

$$-\theta = \mu \qquad \Rightarrow \qquad = \frac{}{\mu}$$

if θ is the angle of emergence at water air boundary CD

$$\mu \qquad = \qquad \theta$$

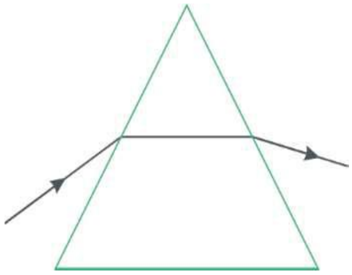
$$\theta =$$

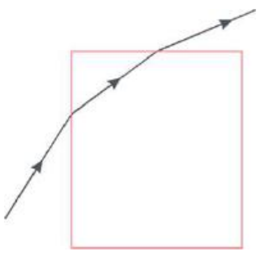
$$\theta = \quad ^{\circ}$$

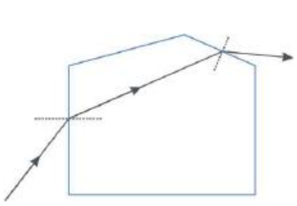
i.e. light ray will emerge parallel to water air surface with $\theta = \quad ^{\circ}$

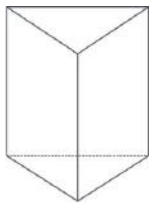
Prism

It is a transparent medium bounded by two refracting faces non parallel to each other.







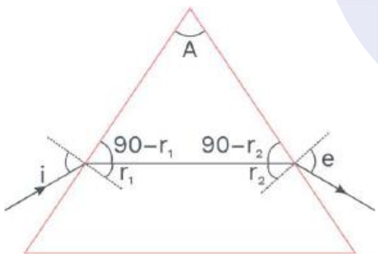


The angle between refracting faces/ surface is called the refracting angle or angle of the prism (A) or prism angle (A).

Relation between angle of prism (A) & refracted angle (r_1 & r_2).

$$A + 90 - r_1 + 90 - r_2 = 180$$

$$\boxed{A = \quad + \quad}$$



Concept Reminder

For one δ (except δ min) there are two values of angle of incidence. If i and e are inter-changed then we get the same value of δ because of reversibility principle of light.

Concept Reminder

- At angle of minimum deviation,

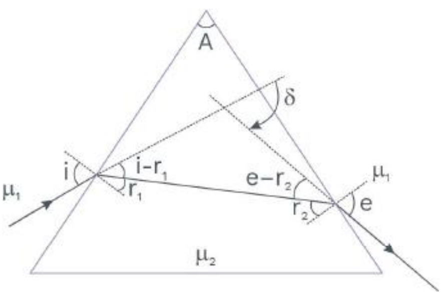
  $i = e$ and $r_1 = r_2 = r$

Angle of deviation

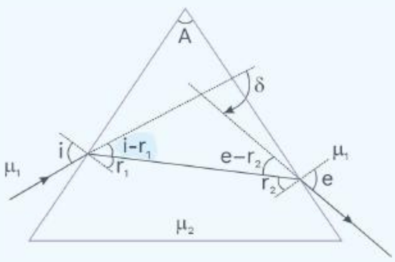
$$\delta = \quad - \quad + \quad -$$

$$\delta = \quad + \quad - \quad +$$

$$\boxed{\delta = \quad + \quad - A}$$



Concept Reminder



$$A = r_1 + r_2$$
$$\delta = i + e - A$$

Snell's law

$$\mu \quad = \mu$$

$$\mu \quad = \mu$$

Condition Of Minimum Deviation

For Minimum Deviation

In this condition $= \quad = \Rightarrow \quad = \quad =$ and

since $+ \quad = A$

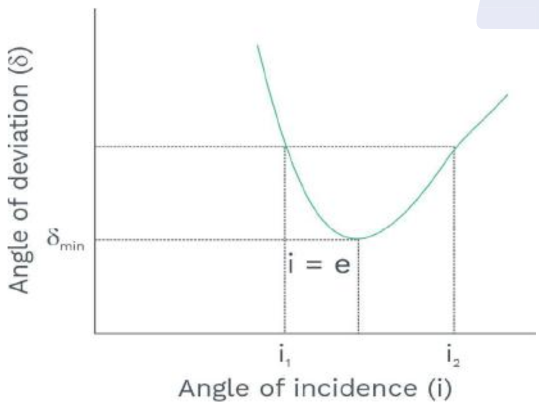
$$\therefore \quad + \quad = A \Rightarrow \quad = A \Rightarrow \quad = \frac{A}{}$$

Minimum deviation

$$\delta \quad = \quad - A \quad = \frac{A + \delta}{} \quad = \frac{A}{}$$

If prism is placed in air $\mu \; = \quad \times \quad = \mu$

$$\left[\frac{A + \delta}{}\right] = \mu \quad \frac{A}{} \Rightarrow \mu = \frac{\left[\frac{A + \delta}{}\right]}{\frac{A}{}}$$



if angle of prism is small A < 10° then $\theta \approx \theta$

$$\mu = \frac{(A + \delta \quad)}{} = \frac{A + \delta}{A} \Rightarrow A + \delta \quad = \mu A$$

$$\Rightarrow \delta \quad = \mu - \quad A$$

Concept Reminder

- As angle of incidence increases angle of deviation first decreases then increases.
- Refractive index of prism is given as,

$$\iota = \frac{\left(\frac{A + \delta}{}\right)}{\left(\frac{A}{}\right)}$$

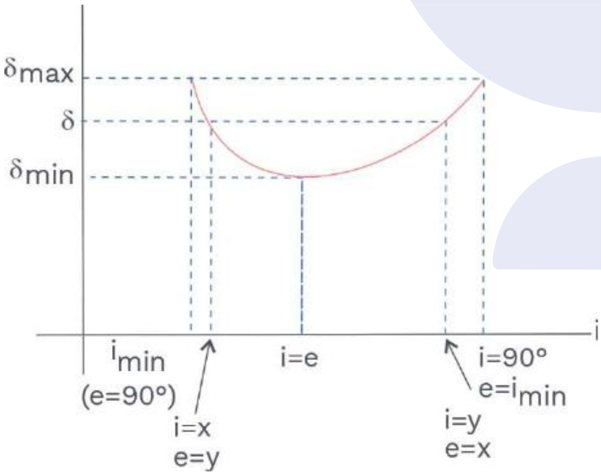
Ex. A beam of light is incident on one face of a prism $\mu =$ at an angle of 60°. The angle of refraction of the prism is also 60°. Find out the angle of emergence and the angle of deviation. Is there any other angle of incidence, which will generate the same deviation?

Sol. Angle of incidence = i = 60°

At point P, $\frac{\quad}{\quad}^{\circ} = \frac{\quad}{\quad} \Rightarrow \quad = \frac{\quad}{\sqrt{\quad}}$

or, $\approx \quad ^{\circ}$

Using $r_1 + r_2 = A$, we get
$r_2 = A - r_1 = 60° - 35°6' = 24°44'$



At point Q, $\frac{\quad}{\quad} = \frac{\quad}{\quad}$

$\Rightarrow$ sin e = 1.5 sin 24° 44' $\Rightarrow$ sin e = 0.63

$\Rightarrow$ e = 39°

$\therefore$ deviation = δ = (i + e) – A = 60°+39° – 60° = 39°

If e and i are interchanged, deviation remains the same. Hence same deviation is obtained for angles of incidence 60° and 39°.

Concept Reminder

- If i and e are interchanged, deviation remains the same. Hence same deviation is obtained for angles of incidence 60° and 30°.

Ex. A ray of light makes an angle of 60° on one of the front of a prism and suffers a total deviation of 30° on emergence from the other face. If the angle of the prism is 30°, show that the emergent ray is perpendicular to the other face. Also find out the refractive index of the material of the prism.

Sol. The angle of deviation $\delta = \quad + \quad - A$

Here, $\delta = \quad ^\circ \quad = \quad ^\circ \; A = \quad ^\circ$

Therefore 30° = 60° + i_2 – 30° = 30° + i_2

$\Rightarrow i_2 = 0$

The emergent angle is zero. This means that the emergent ray is perpendicular to the second face.

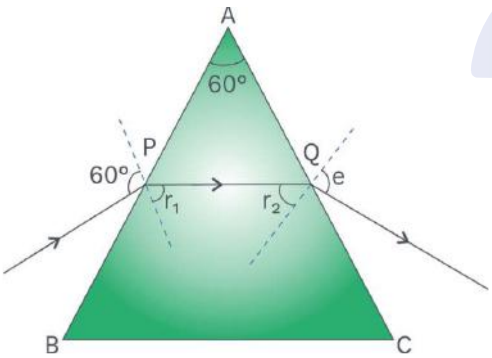
Since $i_2 = 0$, the angle of incidence at the second face is zero.

$\therefore \quad r_2 = 0$

Now, $r_1 + r_2 = A$ or, $r_1 = A = 30°$

We know, $\mu = \frac{}{} = \frac{\quad^\circ}{\quad^\circ} = \frac{\sqrt{\ }}{} = 1.732$

Graph Between $\angle\delta$ and $\angle$:



(1) Variation of δ versus i (shown in figure).
For one deviation angle δ (except δ min) there are two values of incident angle. If i and e are interchanged then we get the equal value of δ because of reversibility principle of light

(2) There is one and only one incident angle for which the angle of deviation is minimum.

(3) Right hand side part of the graph is more tilted then the left hand side.

Relation Between Refractive index and the angle of Minimum Deviation:

When $\delta = \delta$, we have

$e = i$ and $r_1 = r_2 = r$ (say)

We know

$A = r_1 + r_2 = r + r = 2r$ or, $= \frac{A}{\;}$

Also, $A + \delta = i + e$

or, $A + d_m = i + i$

or, $= \frac{A + \delta}{\;}$

The refractive index of the substance of the prism is given by

$\mu = \frac{\;}{\;}$ (Snell's law)

or, $\mu = \frac{\left(\frac{A+\delta}{\;}\right)}{\frac{A}{\;}}$ ... (2)

If surrounding medium has refractive index = n_s

then $\left| \frac{\;}{\;} = \frac{\left(\frac{A+\delta}{\;}\right)}{\frac{A}{\;}} \right|$

Ex. A ray of light incident at an angle of 49° on the face of an equilateral prism passes symmetrically Find out the refractive index of the substance of the prism.

Sol. As the prism is an equilateral triangle, A = 60°. As the ray of light passes symmetrically, the prism is in the position of minimum deviation.

So, $= \frac{A}{\;} = \frac{\;^\circ}{\;} = \;^\circ$

also, $i = 49°$

$\therefore \quad \mu = \frac{\;}{\;} = \frac{\;^\circ}{\;^\circ} = \frac{\;}{\;} =$

Ex. The refracting angle of the prism is 60° and the refractive index of the material of the prism is 1.632. Calculate the angle of minimum deviation.

Sol. Here, A = 60° ; μ = 1.632

Now, $\mu = \frac{\left(\frac{A+\delta}{\ }\right)}{\left(\frac{A}{\ }\right)}$

o

$= \frac{\left(\frac{\circ+\delta}{\ }\right)^{r}}{\frac{\circ}{\ }} = \frac{\left(\frac{\circ+\delta}{\ }\right)}{\circ}$,

or, $\left(\frac{\circ+\delta}{\ }\right)$ = 1.632 × sin 30° = 1.632 × 0.5

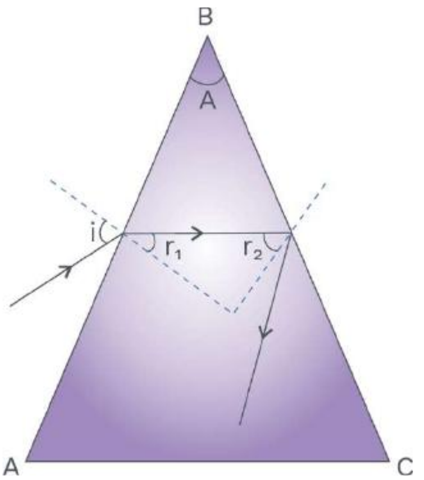
Or, $\left(\frac{\circ+\delta}{\ }\right) =$

Or, $\frac{\circ+\delta}{\ } = \ \circ$

$\delta = \ \circ$

Condition For Prism :

(a) Relation between prism angle A & critical angle C such that ray will always show T I R at BC :

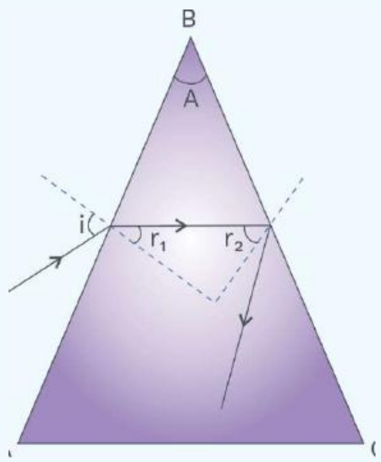


Rack your Brain

The refractive index of the material of a prism is $\sqrt{\ }$ and the angle of the prism is 30°. of the two refracting surfaces of the prism is made a mirror inwards, by silver coating. A beam of monochromatic light entering the prism from the other face will retrace its path (after reflection from the silvered surface) if its angle of incidence on the prism is:

(1) 30° (2) 45°
(3) 60° (4) Zero

Concept Reminder



(i) For A > 2C, all rays are reflected back from second surface.

(ii) For A ≤ C, no rays are reflected from second surface.

For this condition $(r_2)_{min} > C$ ...(i)

For $(r_2)_{min}$, r_1 must be maximum and

for $\Rightarrow$ = °

= O

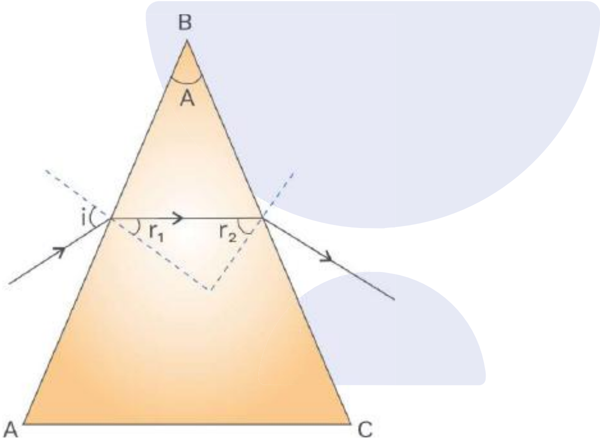
= A – O

Now from eq. (i) A – C > C

A > O

i.e. A > 2C, all rays are reflected back from the second surface.

(b) The relation between A and C such that ray will always cross surface BC.



For this $(r_2)_{max} < C$

A – < O

A – < O ... (2)

$(r_1)_{min} = 0$ when $i_{min} = 0$

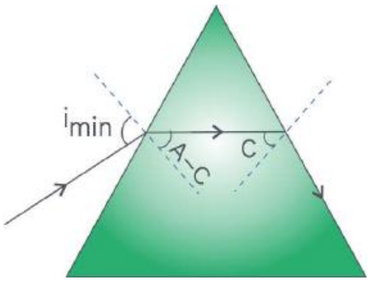
from eq. (2) A – 0 < C

A < O

i.e. If $A \leq O$, no rays are reflected back from the second surface i.e. all rays are refracted from second surface.

(c) If 2C $\geq$ A > C, some rays are reflected back from the second surface and some rays are refracted from second surface, depending on the angle of incidence.

δ is maximum for two values of i

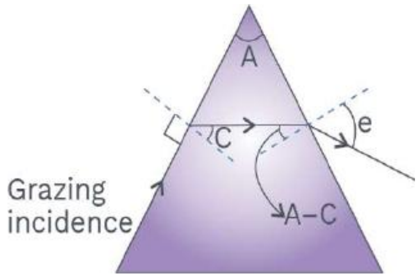


$\Rightarrow$ i_{min} (corresponding to e = 90°)
and i = 90°
(corresponding to e_{min}).

For i_{min} : $n_s \sin i_{min} = n_p \sin (A - C)$

If $i < i_{min}$ then T.I.R. takes place at second refracting surface PR.

Condition for δ_{max}:



$$i = 90° \text{ or } e = 90°$$

$$n \sin (A - C) = \sin e$$

$$e = \sin^{-1} [n\sin(A - C)]$$

$$\delta \quad = \quad + \quad - A$$

$$\delta \quad = \quad + \quad ^{-} \quad A - O \quad - A$$

Ex. Find the minimum and maximum angle of deviation for a prism with angle A = 60° and μ = 1.5

Sol. **Minimum deviation**

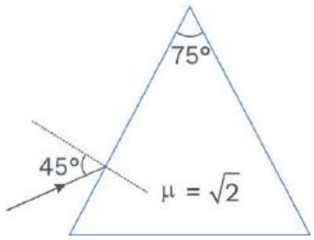
The angle of minimum deviation occurs when i = e and $r_1 = r_2$ and is given by

$$\mu = \frac{\left(\frac{A+\delta}{\quad}\right)}{\frac{A}{\quad}} \Rightarrow \delta \quad = \quad ^{-}\left(\mu \quad \frac{A}{\quad}\right) - A$$

Substituting μ = 1.5 and A = 60°, we get

$\delta \quad = 2 \sin^{-1} (0.75) - 60° = 37°$

Ex. Find r_1, r_2, e & δ from given diagram?



Sol. By Snell's law sin 45° = $\sqrt{\ }$

$\angle \ = \ ^\circ$

$\Rightarrow$ A = $\ +$

$= \ + \ = \Rightarrow \angle \ = \ ^\circ$

$\Rightarrow$ By Snell's law at emerging surface

$\sqrt{\ }$ $^\circ =$

$\angle \ = \ ^\circ$

$\Rightarrow \delta = \ + \ - A$

= 45° + 90 – 75 = 60°

Ex. If minimum, deviation is equal to angle of prism (A) then final refractive index of prism?

Sol. $= \dfrac{\left(\dfrac{\delta \ + A}{\ }\right)}{A} = \dfrac{A}{A}$

$= \dfrac{A \qquad A}{A} = \qquad A$

Ex. If μ = cot (A/2), find minimum deviation of prism?

Sol. $\mu = \dfrac{\left(\dfrac{\delta \ + A}{\ }\right)}{A}$

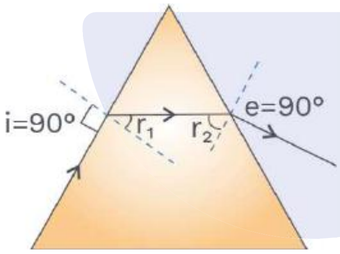
$\dfrac{A}{A} = \dfrac{\left(\dfrac{\delta \ + A}{\ }\right)}{A}$

$$- A \quad = \quad \left(\frac{\delta \quad + A}{\quad}\right)$$

$$\Rightarrow \quad - \frac{A}{\quad} = \frac{\delta \quad + A}{\quad}$$

$$\boxed{\delta \quad = \quad - \quad A}$$

Maximum deviation (Grazing incidence or Grazing emergence) :
The deviation is maximum when i = 90° or e = 90° that is at grazing incidence or grazing emergence.



Let i = 90°

$$\Rightarrow \quad = O = \quad - \left(\frac{\quad}{\mu}\right)$$

$$\Rightarrow \quad = \quad - \left(\frac{\quad}{\quad}\right) = \quad \circ$$

$$\Rightarrow \quad = A - \quad = \quad \circ - \quad \circ = \quad \circ$$

Using $\frac{\quad}{\quad} = \frac{\quad}{\mu}$, we have

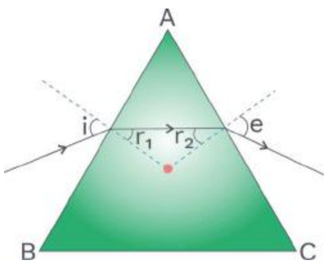
$$= \mu \quad = \quad \times \quad \circ$$

$$\Rightarrow \quad = \quad \Rightarrow \quad = \quad \circ$$

$\therefore$ Deviation $\delta \quad = \quad + \quad - A$

$$= 90° + 28° - 60° = 58°$$

Deviation Through a Prism of Small Angle:
If the angle of the prism A is small, r_1 and r_2 (as $r_1 + r_2 = A$) and i and e will be small.



For the refraction at the face AB, we have

$\mu =$ ——

or, $\mu =$ — (since i and r_1 are small angles, $\approx$

and $\approx$)

$\Rightarrow$ For refraction at the face AC, we have

$\mu =$ ——

or, $\mu =$ — ($\because$ e and r_2 are small angles, so

$\simeq$ and $\simeq$)

$\Rightarrow$ $= \mu$

Now, deviation produced by a prism

$\delta = \quad + \quad - A$

or, $\delta = \mu \quad + \mu \quad - A$ or, $\delta = \mu \quad + \quad - A$ or,

$\delta = \mu A - A \because \quad + \quad = A$

or, $\delta = \mu - \quad A$ ... (20)

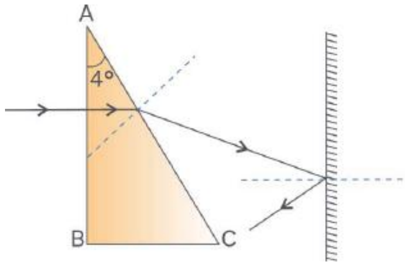
The above formula is valid for all locations of the prism provided the angle of the prism A is small (say $\leq 10°$).

Ex. A prism having a refracting angle 4° and refractive index 1.5 is kept in front of a vertical plane mirror as shown in the figure. A horizontal ray of light is incident on the prism. Find out the angle of incidence at the mirror?

Concept Reminder

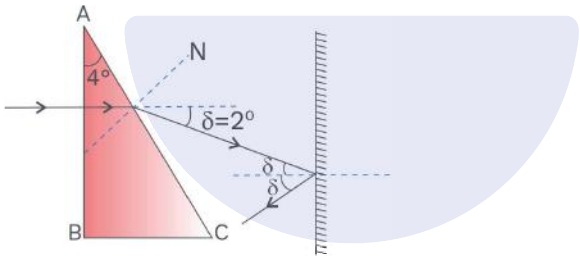
Deviation through a prism of small angle is,

$\delta = (\mu - 1)A$



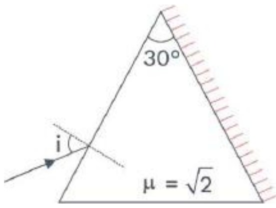
Sol. The deviation suffered by refraction through the small angled prism is given by

$$\delta = \mu - \quad A = \quad - \quad \times \quad ^\circ = \quad ^\circ$$

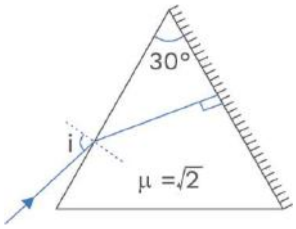


This gives the angle of incidence 2° at the mirror.

Ex. Find angle of incidence if light retrace its path after reflection from silvered surface.



Sol. To retrace the path, light must strike the silvered surface normally.



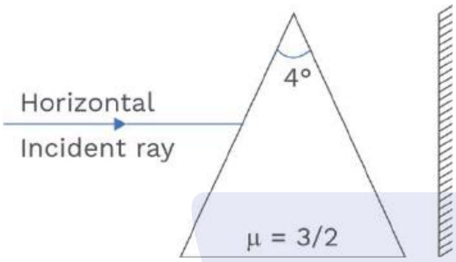
$$= \quad = \quad ^\circ \qquad = A -$$

By Snell's law $\quad = \sqrt{\ }$

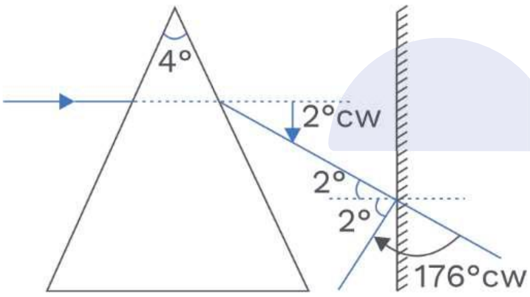
$= \sqrt{\ } \quad ° = \frac{}{\sqrt{\ }}$

$\Rightarrow \angle = \quad °$

Ex. Find net deviation of light ray after reflection from plane mirror.



(i) $\delta \ = \mu - \ A = \left(- - \right) ° = \ °$



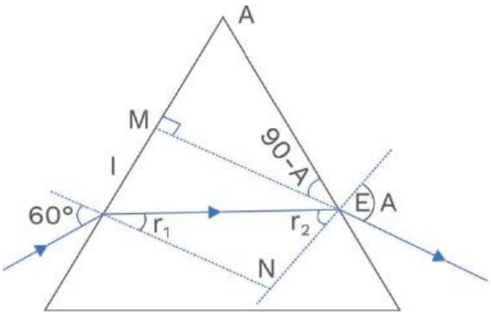
(ii) deviation due to plane mirror
= 180 – 2 i = 180 –2 (2) = 176° cw

(iii) net deviation $\delta \ + \delta \ = \quad °$

Ex. A monochromatic ray of light is incident at an angle 60° to a face of a transparent trihedral prism having a refractive index of '3'. Having undergone double refraction, the ray emerges in a direction normal to this face. Find the angle of prism.

Sol. For refraction at point I

$° = \sqrt{\ }$ ... (i)



For refraction at point E

$\sqrt{\ } \quad = \quad A$

$\sqrt{\ } \quad A - \quad ° = \quad A$

... (ii)

On solving we get A = 60°

Ex. Refracting angle of a prism is 60° and its refractive index is 3/2, what is the angle of incidence i to get minimum deviation. Also find out the minimum deviation. Consider the surrounding medium to be air (n = 1).

Sol. For minimum deviation,

$$= \quad = \frac{A}{} = \quad ^\circ$$

Applying Snell's law at I surface

$$\times \quad = - \quad ^\circ \Rightarrow \quad = \quad ^{-}\left(-\right)$$

$$\Rightarrow \delta \quad = \quad ^{-}\left(-\right) - \frac{\pi}{}$$

Dispersion Of Light

The angular splitting of a ray of white light into a number of components and spreading in different directions is called Dispersion of Light. [It is for

KEY POINTS

- Dispersion
- Cauchy's formula
- Angular disperion
- Mean deviation

Definitions

The angular splitting of a ray of white light into a number of components and spreading in different directions is called Dispersion of Light.

Concept Reminder

Dispersion takes place because the refractive index of medium for different frequencies (colours) is different. For example, the bending of red component of white light is least while it is most for the violet.

Concept Reminder

Cauchy's formula:

$$\lambda \ = \ + \frac{}{\lambda}$$

whole Electro Magnetic Wave in totality]. This phenomenon takes place because waves of different wavelength move with same speed in vacuum but with different speeds in a medium.

Therefore, the refractive index of a medium depends slightly on wavelength also. This variation of refractive index with wavelength is given by Cauchy's formula.

Cauchy's formula $\lambda \;=\; \;+\frac{}{\lambda}$

where a and b are positive constants of a medium.

$$\xrightarrow{\text{rNj}\quad 8|}$$

$$\lambda \uparrow \qquad \downarrow$$

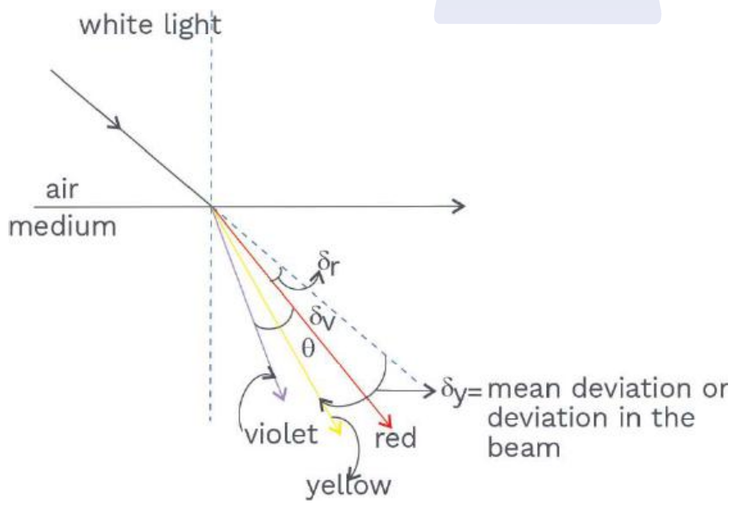
Note :-

- Such phenomenon is not exhibited by sound waves.

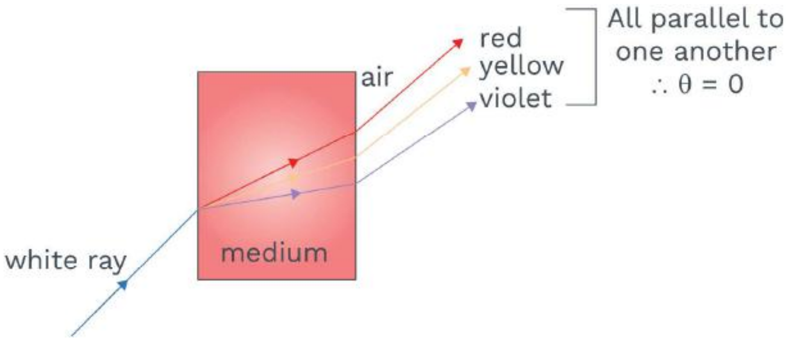
Angle between the rays of the extreme colour in the refracted (dispersed) light is called angle of dispersion

$$\theta - \delta \;\; - \delta$$

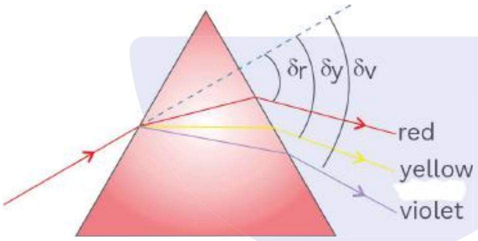
Fig (a) and (c) represents dispersion, whereas in fig. (b) there is no dispersion.



(a)



(b)



(c)

For prism of small 'A' and with small 'i'.

$\theta = \delta \;\; - \delta \;\; = \quad\; - \quad A$

As $\delta \quad = \mu - \quad A \because \lambda_l > \lambda$

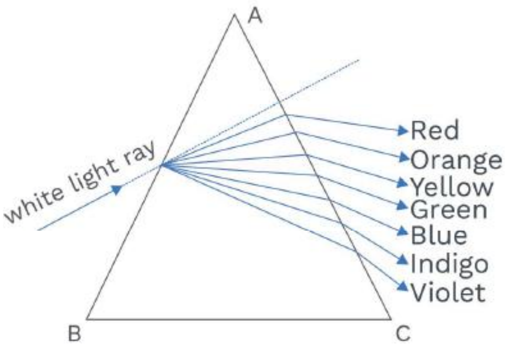
So $\mu \;\; > \mu_l \Rightarrow \delta \qquad > \delta$

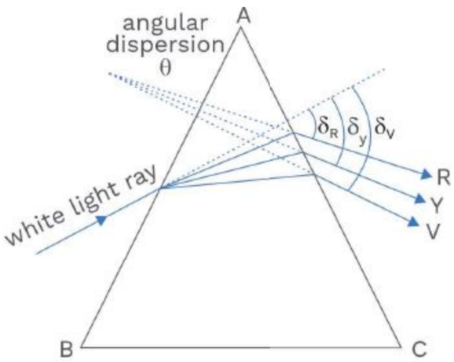
Angular Dispersion

It is the difference between the angles of deviation for violet colour and red colour Angular dispersion

$\theta = \delta \;\; - \delta_l \; = \mu \;\; - \;\; A - \mu_l \; - \;\; A = \mu \;\; - \mu_l \;\; A$

It depends on prism material and on the angle of prism $\theta = \mu \;\; - \mu_l \;\; A$





Ex. The refractive indices of flint glass for red and violet light are 1.613 and 1.632 respectively.
Find the angular dispersion produced by a thin prism of flint glass having refracting angle 5°.

Sol. Deviation of the red light is $\delta = \mu - \quad A$

and deviation of the violet light is $\delta = \mu - \quad A$.

The dispersion $\delta \ - \delta = \mu \ - \mu \ A$

$= (1.632 - 1.613) \times 5° = 0.095°$

Note: Deviation of beam (also called mean deviation)
Numerical data reveals that if the average value of μ is small deviation is also small and if the average value of μ is large $\mu \ - \mu$ is also large. Therefore, larger the mean deviation, greater will be the angular dispersion.

Concept Reminder

Angular dispersion

$\theta = \delta_v - \delta_R = (\mu_v - \mu R)A$

Concept Reminder

Dispersive power

$$\omega = \frac{-}{-} = \frac{\delta \ - \delta}{\delta} = \frac{\theta}{\delta}$$

$$= \frac{A}{S}$$

Dispersive power of the medium of the material of prism is given by :

$$\omega = \frac{\mu \ - \mu}{\mu \ -}$$

- ω is the property of a medium.

For small prism angle (A $\leq$ 10°) with light incident at small angle i :

$$\frac{\mu \ - \mu}{\mu \ -} = \frac{\delta \ - \delta}{\delta} = \frac{\theta}{\delta} = \underline{\qquad\qquad\qquad}$$

$[\ \mu \ = \frac{\mu \ + \mu}{\ }$ if μ_y is not given in the question]

- $\mu - 1$ = refractivity of the medium for the corresponding colour.

Ex. Refractive index of glass for the red and violet colours are 1.50 and 1.60 respectively. Find
(a) The refractive index for yellow colour, approximately
(b) Dispersive power of the medium.

Sol. (a) $\mu \simeq \frac{\mu \quad + \mu_{|}}{} = \frac{\quad + \quad}{} =$

(b) $\omega = \frac{\mu \quad + \mu_{|}}{\mu \quad -} = \frac{\quad + \quad}{\quad - \quad} =$

Dispersion without average deviation and average deviation without dispersion:

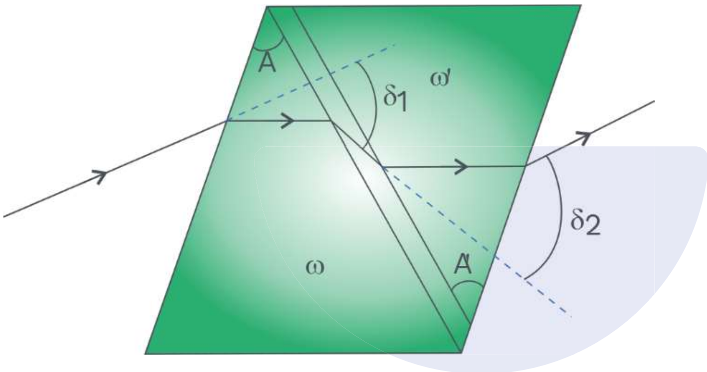


Figure shows two thin prism placed in the contact in such a way that the two refracting angles are reversed w.r.t. each other. Let, the refracting angles of the two prism are A and A' and their dispersive power are ω and ω' respectively.

Assume a ray of light for which the refractive indices of the substances of the two prisms are μ and μ . Assuming that the ray passes through the prism in symmetrical situation, the deviations produced by the two prism are

$$\delta \;=\; \mu - \quad A \text{ and } \delta \;=\; \mu - \quad A$$

As the two deviations are opposite to each other, the net deviation is

$$\delta = \delta \;\; - \delta$$

$$= \;\mu - \quad A - \;\mu - \quad A \qquad \text{... (1)}$$

If violet light passes through the combination, the net deviation of the violet ray is

$$\delta \;=\; \mu \;\; - \quad A - \;\mu \;\; - \quad A$$

and that of the red ray is

$$\delta \;=\; \mu \;\; - \quad A - \;\mu \;\; - \quad A$$

The angular dispersion produced by the combination is

$$\delta - \delta = \mu - \mu \ A - \mu - \mu \ A \qquad \text{... (2)}$$

The dispersive power are given by

$$\omega = \frac{\mu - \mu}{\mu -} \quad \text{and} \quad \omega = \frac{\mu - \mu}{\mu -}$$

Thus, by (2), the net angular dispersion is

$$\delta - \delta = \mu - \ \omega A - \mu - \ \omega \ A \qquad \text{... (3)}$$

The net deviation of the yellow ray i.e., the average deviation, is, by (1)

$$\delta = \mu - \ A - \mu - \ A \qquad \text{... (4)}$$

Combination Of Prisms

Deviation without dispersion $(\theta = 0°)$

Two or more thin prisms are combined in such a way that deviation occurs i.e. emergent light ray makes certain angle with incident light ray but dispersion does not occur i.e., white light does not split into different colours.

Total dispersion

$$= \theta = \theta + \theta = \mu - \mu_l \ A + \mu - \mu_l \ A$$

For no dispersion

$$\theta = \ \mu - \mu_l \ A + \mu - \mu_l \ A =$$

Therefore, $A = \left(\frac{\mu - \mu_l \ A}{\mu' - \mu_l} \right)$

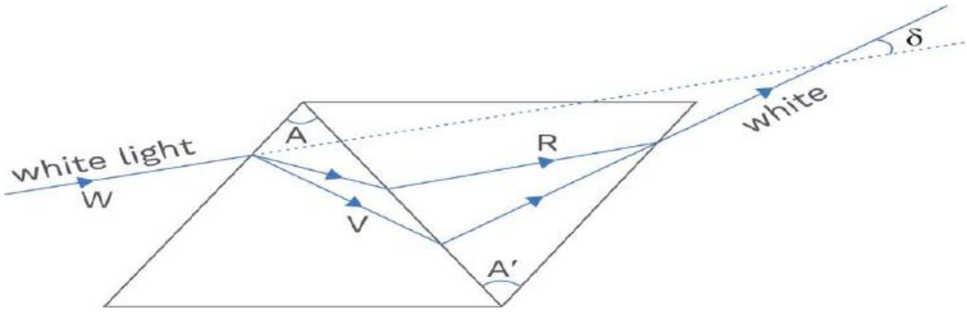
–ve sign indicates that prism angles are arranged in opposite manner.

Concept Reminder

Condition of dispersion without deviation

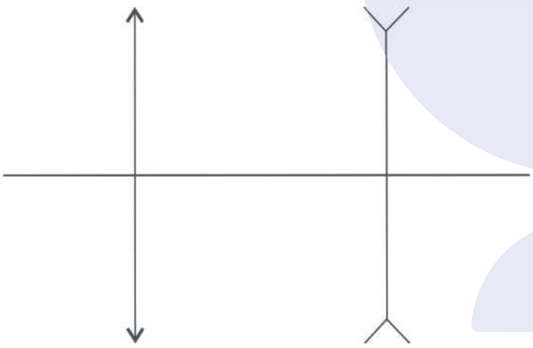
$$A' = -\frac{(\mu - \ )A}{\mu' -}$$

–ve sign indicates that prism angles are arranged in opposite manner.



Dispersion without deviation ($\delta = 0°$)

Two or more thin prism combine in such a way that dispersion occurs i.e., white light is splitted into different colours but deviation does not occur i.e., emergent light ray remains parallel to incident light ray.



Total deviation is $\delta = d_1 + d_2$

$$\Rightarrow \delta = \quad \mu - \quad A + \mu - \quad A = \qquad \Rightarrow A \quad = -\frac{\mu - \quad A}{\mu -}$$

–ve sign indicates that prism angles are arranged in opposite manner.

Ex. Find the angle of the flint glass prism which should be combined with a crown glass prism of 5° so as to give dispersion but no deviation.

For crown glass : $\mu \quad = 1.523; \ \mu_l \quad = 1.515$

For flint glass : $\mu \quad = 1.688 \ ; \ \mu_l \quad = 1.650$

Sol. For no deviation

$$\frac{A}{A} = \left(\frac{\mu -}{\mu -}\right) \text{ or, } A \quad = \left(\frac{\mu -}{\mu -}\right) A$$

Now, $\mu = \dfrac{\mu \quad + \mu_l}{} = \dfrac{+}{} =$

$$\mu = \frac{\mu \; + \mu_l}{} = \frac{+}{} =$$

$$\therefore \qquad A = \left(\frac{-}{-}\right)^{\circ} = \quad {}^{\circ}$$

Ex. White light is passed through a prism of angle 4°. If the refractive indices for red & blue colours are 1.63 & 1.65 respectively calculated the angle of dispersion between them and also find dispersion power.

Sol. (i) $\theta = \mu \;\; - \mu_l \;\; A = \quad - \quad {}^{\circ}$

$= \quad {}^{\circ} = \quad {}^{\circ}$

(ii)

$$\omega = \frac{\theta}{\delta} = \frac{}{\mu \;\; - \;\; A} = \frac{}{-} =$$

Ex. A light ray is incident on a thin prism of angle 15° having refractive index 1.5, it is combined with another prism of refractive index 1.75 to produce dispersion without deviation. Find the prism angle of another prism.

Sol. $\mu \; - \;\; A \; = \mu \; - \;\; A$

$- \quad {}^{\circ} = \quad - \;\; A$

$A \; = \quad {}^{\circ}$

Ex. Two thin prisms are combined to form an achromatic combinations. For prism –1 $\angle A = \; {}^{\circ} \; \mu_l \; = \quad \mu \; = \quad \mu \; =$ and for prism –2 $\mu_l \; = \quad \mu \; =$ and $\mu \; =$ find the prism angle of second prism and the net mean deviation?

Sol. (i) $\theta = \theta$

$$\mu \;\; - \mu_l \;\; A = \mu \;\; - \mu_l \;\; A$$

$$A \; = \frac{-}{-}{}^{\circ} = \quad {}^{\circ}$$

Rack your Brain

A thin prism having refracting angle 10° is made of glass of refractive index 1.42. This prism is combined with another thin prism of glass of refractive index 1.7. This combination produces dispersion without deviation. The refracting angle of second prism should be:

(1) 6° (2) 8°

(3) 10° (4) 4°

(ii) $\delta \quad = \delta \;- \delta$

$= \quad - \quad ^\circ \quad - \quad ^\circ = \quad ^\circ$

Ex. Find the angle of a prism of dispersive power 0.021 and refractive index 1.53 to form an achromatic combination with the prism of angle 4.2° and dispersive power 0.045 having refractive index 1.65. Also calculate the resultant deviation.

Sol. $\omega = \quad ; \mu = \quad ; \omega \; = \quad ; \mu \; =$

A' = 4.2°

For no dispersion

$\omega\delta + \omega \;\; \delta \;\; =$

or, $\omega \; \mu - \quad A + \omega \;\; \mu - \quad A \;\; =$

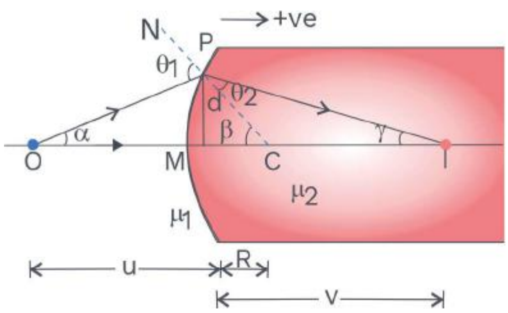
or, $A = -\left(\dfrac{\omega \;\; A \;\; \mu -}{\omega \; \mu -}\right) = \left(\dfrac{\quad \times \quad ^\circ \times \quad -}{\quad \times \quad -}\right) \Rightarrow A = - \quad ^\circ$

Net deviation $\delta + \delta \;\; = \;\; \mu - \quad A + \;\; \mu - \quad A$

= – 11.04° (1.53 – 1) + 4.2° (1.65 – 1) = – 3.12°

Refraction From A Spherical Surface

Consider two transparent media having indices of refraction m_1 and m_2, where the boundary between the two media is a spherical surface of radius R. We assume that $m_1 < m_2$. Let us consider a single ray leaving point O and focussing at point I. Snell's law applied to this refracted ray gives,



$\mu \quad \theta = \mu \quad \theta$

Because q_1 and q_2 are assumed to be small, we can use the angle approximation

$$\theta \approx \theta$$

(angles in radians) and say that

$$\mu \theta = \mu \theta \qquad \text{... (1)}$$

From the geometry shown in the figure.

$$\theta = \alpha + \beta \qquad \text{... (2)}$$

and $\beta = \theta + \gamma$ ... (3)

Eqs. (1) and (3) can be combined to express θ in terms of α and β. Substituting the resulting expression into Eq. (2) then yields.

$$\beta = \frac{\mu}{\mu} \alpha + \beta + \gamma$$

So $\mu \alpha + \mu \gamma = \mu - \mu \beta$ ... (4)

Since, the arc PM (of length S) subtends an angle β at the centre of curvature

$$\beta = \frac{}{|}$$

Also in the paraxial approximation

$$\alpha = — \qquad \gamma = —$$

Using these statements in Eq. (4) with proper signs, we are left with,

$$\frac{\mu}{-} + \frac{\mu}{} = \frac{\mu - \mu}{|}$$

or $\boxed{\frac{\mu}{} - \frac{\mu}{} = \frac{\mu - \mu}{|}}$ ... (5)

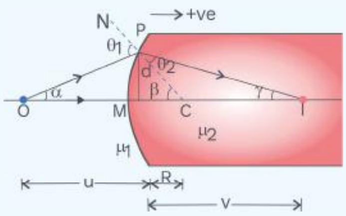
m_1 = refractive index of the medium in which the incident ray lies.

m_2 = refractive index of the medium in which refracted ray lies.

O = Object

P = pole

Concept Reminder



$$\frac{\mu}{} - \frac{\mu}{} = \frac{\mu - \mu}{|}$$

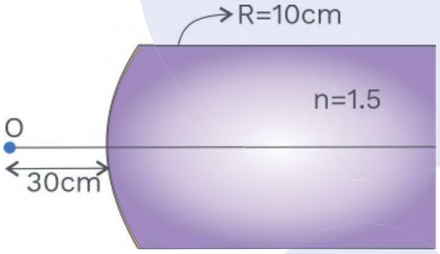
C = centre of curvature
R = PC = radius of curvature

Although the formula (5) is obtained for a particular situation, it is applicable for all other conditions of refraction at a single spherical surface.

Important point for above formula:-

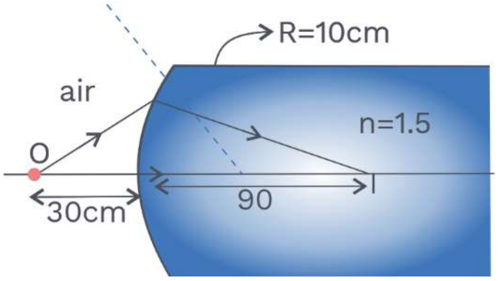
- Above formula is applicable only for paraxial ray.
- u,v,R must be use along with sign
- μ is r.i. of medium in which rays is going away and μ is the r.i. of medium from which rays are coming.

Ex. Find the position of the image formed and draw the appropriate ray diagram



u = – 30 cm
R = + 10 cm

Sol.



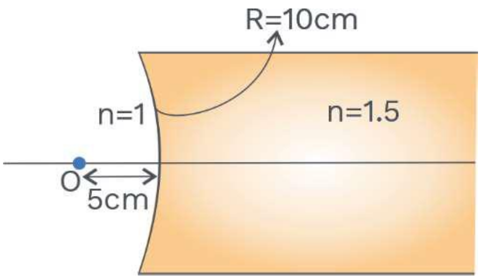
$\Rightarrow$ —— – —— = ——

$\Rightarrow$ —— + —— = ——

$\Rightarrow$ —— = —— – ——

$\Rightarrow$ —— = —— $\Rightarrow$ = —— = |

Ex. Find out the position of the image formed and draw the appropriate ray diagram



Sol. $n_2 = 1.5,\ n_1 = 1$

$u = -5$ cm $R = -10$ cm

$$— - — = \frac{-}{|}$$

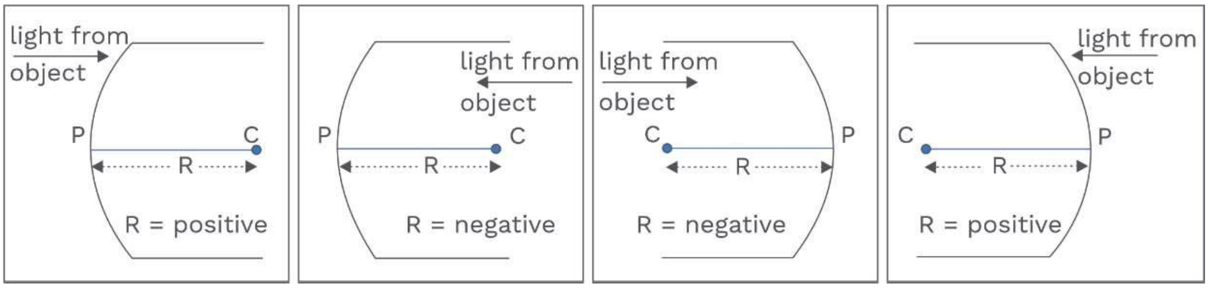
$$\Rightarrow\ — + — = - —$$

$$\Rightarrow\ — = \frac{-}{—} - —$$

$$\Rightarrow\ — = \frac{-}{—}$$

$$\Rightarrow\ \ = -$$

Sign Convention For Radius Of Curvature



These are valid for all types of refracting surfaces – convex, concave or plane. In case of plane refracting

surface $| \to \infty$,

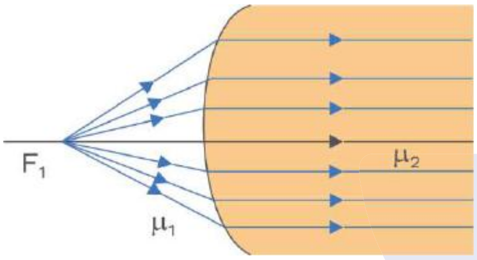
$$\frac{\mu}{} - \frac{\mu}{} = \frac{\mu - \mu}{|} \Rightarrow \frac{\mu}{} - \frac{\mu}{} =$$

or $\frac{}{} = \frac{\mu}{\mu}$ or $\frac{A}{A} = \frac{\mu}{\mu}$

Focal Length Of A Single Spherical Surface

A single spherical surface as two principal focal points which are as follows:

(i) **First focus:** The first principal focus is the point on the axis where an object should be placed so that the image is formed at infinity. That is when



$= \quad = \infty$ then from $-\frac{\mu}{} + \frac{\mu}{} = \left(\frac{\mu - \mu}{}\right)$

We get $-\frac{\mu}{} = \frac{\mu - \mu}{} \Rightarrow \quad = \frac{-\mu}{\mu - \mu}$

(ii) Second focus: Similarly, the second principal focus is the point where parallel rays get focussed. That is $= -\infty \quad =$, then

$$\frac{\mu}{} = \frac{\mu - \mu}{} \qquad = \frac{\mu}{\mu - \mu}$$

(iii) **Ratio of Focal lengths:** $\boxed{\frac{}{} = -\frac{\mu}{\mu}}$ or $\boxed{\frac{}{\mu} + \frac{}{\mu} =}$

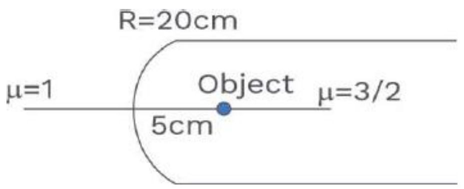
Ex. A convex surface of radius 30 cm separates glass from air. If a point object is placed in air at 15 cm from interface then find position & nature of image. (n_{glass} = 3/2)

Sol. $\frac{}{} - \frac{}{} = \frac{-}{}$

$\frac{}{} + \frac{}{} = \frac{}{}$

$= -\frac{}{} = -$ (virtual image)

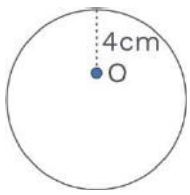
Ex. A concave refractive surface of diameter 40 cm separates air from glass, if a point object is placed in glass at a distance of 5 cm from interface then find nature & position of image. (m_g = 3/2)



Sol. — — ——— = — —— ⇒
×

— = — —— + —— = —— ⇒ = ——

Ex. An air bubble is placed inside a glass sphere of radius 10 cm as shown. Find the position of image as seen by observer from outside? (μ_g = 3/2)



Sol. —— — —— = ——— – ———
|

— + ——— = ——— ⇒ — = ——— — — = — ——
× –

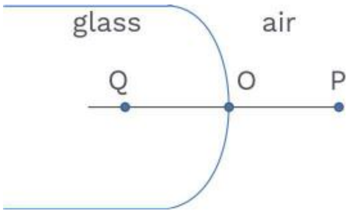
⇒ = — ——

Ex. A spherical surface of radius R separates air from glass such that centre of curvature

Concept Reminder

Thick lenses give coloured images due to dispersion. The variety in colour of objects we see around us is due to the constituent colours of the light incident on them. A monochromatic light may produce an entirely different perception about the colours on an object as seen in white light.

is in glass. A point object P is placed in air whose real image Q is formed in glass. The line PQ cuts the surface such that PO = OQ. Then find the value of PO :



Sol. $\frac{}{} - \frac{}{} = \frac{-}{|}$

$\frac{}{} - \frac{}{(-\ )} = \frac{}{|}$

$\frac{}{} = \frac{}{|}$

v = 5R = u

Ex. A parallel beam of light travelling in water (μ = 4/3) is refracted by a spherical air bubble of radius 2 mm situated in water. Assuming the light rays to be paraxial (a) find the position of the image due to refraction at the first surface and the position of final image. (b) draw a ray diagram showing the positions of both the images.

Sol. (a) In case of refraction from spherical surface

$\frac{\mu}{} - \frac{\mu}{} = \frac{\mu \ - \mu}{|}$

so, for refraction at first surface, i.e., P_1

μ = μ = | = and

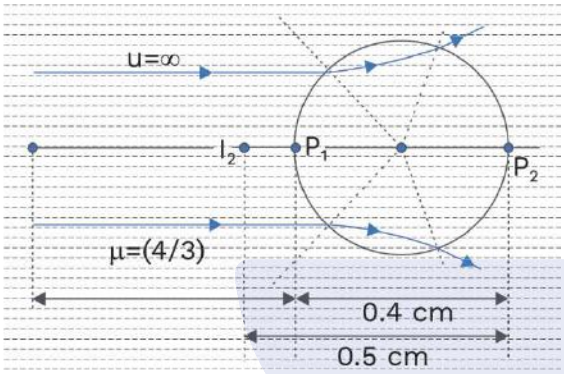
μ = $-\infty$

so, $\frac{}{} - \frac{}{-\infty} = \frac{-}{}$, i.e. v_1 = −0.6 cm

i.e., the first surface will form virtual image

at a distance 0.6 cm to the left to P_1 as shown in figure.
This image will act as an object for the second surface. So for second surface

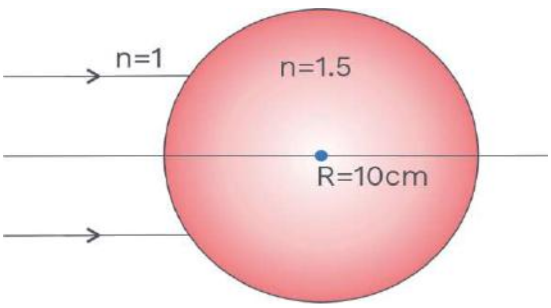
$$\mu = \quad \mu = \quad | = -$$



so, $\quad — - \dfrac{}{-\infty} = \dfrac{-}{} \quad$ i.e. $\quad = -$

i.e., the final image is at a distance of 0.5 cm to the left of second surface P_2. So the final image is at a distance of 0.5 – 0.4 = 0.1 cm to the left of first surface as shown in figure.

Ex. Find out the position where parallel rays will meet after coming out of the sphere and draw the appropriate ray diagram

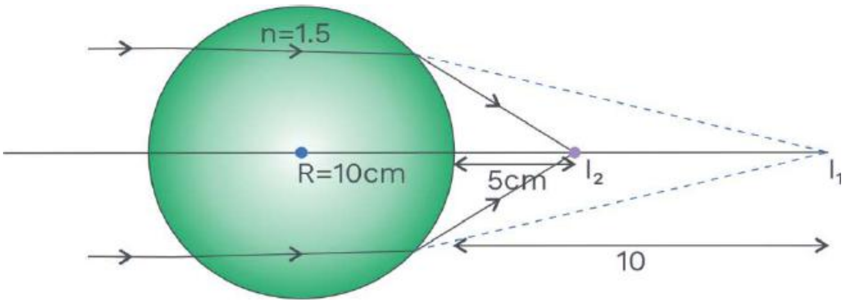


Sol. $\quad — - — = \dfrac{-}{|}$

$$\Rightarrow \frac{1}{} - \frac{}{\infty} = \frac{}{}$$

$$\Rightarrow \frac{1}{} = \frac{}{}$$

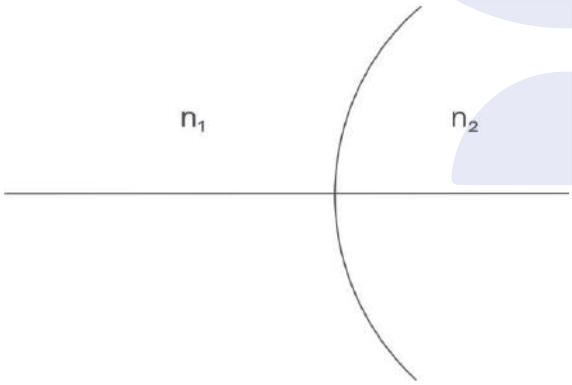
$\Rightarrow \quad = \frac{}{} =$



For IInd

$\frac{}{} - \frac{}{} = \frac{-}{-} \Rightarrow \quad = +$

Velocity Of Spherical Refraction



$\frac{}{} - \frac{}{} = \frac{-}{I}$

differentiate with respect to time

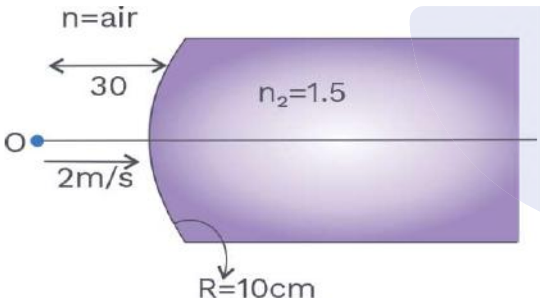
$-\frac{}{}\frac{}{} + \frac{}{}\frac{}{} =$

$\frac{}{} = +\left(\frac{}{}\right)\left(\frac{}{}\right)\frac{}{}$

m = —— 8

m – = — — 8 –

Ex. Find the velocity of image as shown below



Sol. — – — = –/|

⇒ — = –/| + —

⇒ — = (– + |)/|

⇒ = |/(– + |) =

— – — = –/|

By differentiating :

– — — + — — = μ

⇒ — = — — —

⇒ — = — — × = +

Concept Reminder

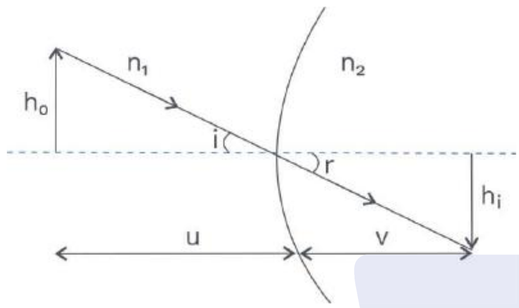
A magician during a show makes a glass lens with n = 1.47 disappear in a trough of liquid. The refractive index of the liquid must be equal to 1.47 in order to make the lens disappear. This means $n_1 = n_2$. This gives 1/f =0 or f → ∞. The lens in the liquid will act like a plane sheet of glass. No, the liquid is not water. It could be glycerine.

Transverse Magnification

If i and r are very small

$\approx \quad \approx$

$= \quad =$



$\Rightarrow \quad = — \Rightarrow \quad = —$ ... (1)

$= — \Rightarrow \quad \approx —$ ... (2)

Again, by applying Snell's law:

$n_1 \sin i = n_2 \sin r$

$\Rightarrow \quad \approx$ ... (3)

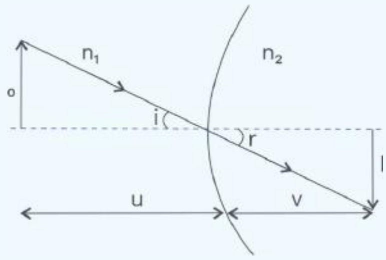
$\Rightarrow$ From (1), (2), (3)

$= — = —\left(—\right)$

Concept Reminder

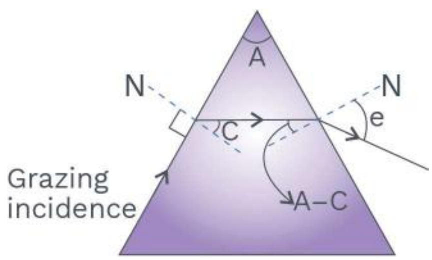
Transverse magnification in case of lens is $= — = —$.

Concept Reminder



$= — = —\left(—\right)$

Ex. Find the position of the image formed and represent the appropriate ray diagram.



R = + 10 cm, u = –30 cm

Sol. $— - — = —$

$$\Rightarrow \frac{}{} + \frac{}{} = \frac{}{} \qquad \Rightarrow$$

$$\frac{}{} + \frac{}{} - \frac{}{}$$

$$\Rightarrow \frac{}{} = \frac{}{}$$

$\Rightarrow$ v = + 90

Mirror will form the image of I_1 30 cm behind it as shown in the figure.

For the second refraction:

u = – 150 cm, R = – 10 cm,

n_1 = 1.5, n_2 = 1

$$\frac{}{} + \frac{}{} = \frac{}{-}$$

$$\Rightarrow \frac{}{} + \frac{}{} - \frac{}{}$$

$$\Rightarrow \frac{}{} = \frac{}{}$$

$\Rightarrow$ v = 25 cm (Real)

KEY POINTS

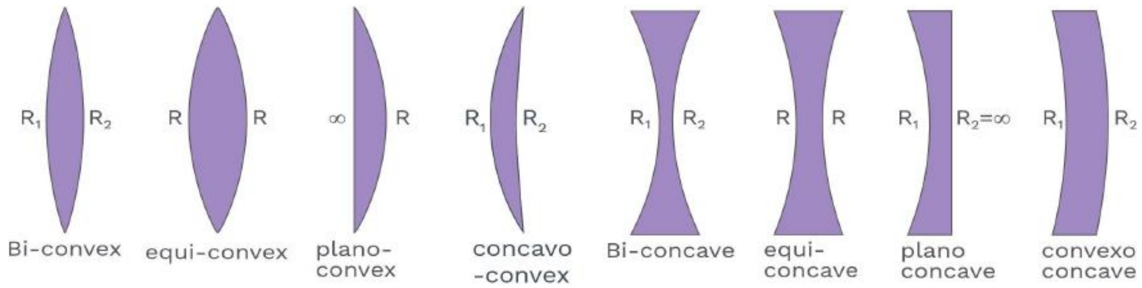
- Optical centre
- Principal axis
- Principal
- ocus
- Aperture

Lens

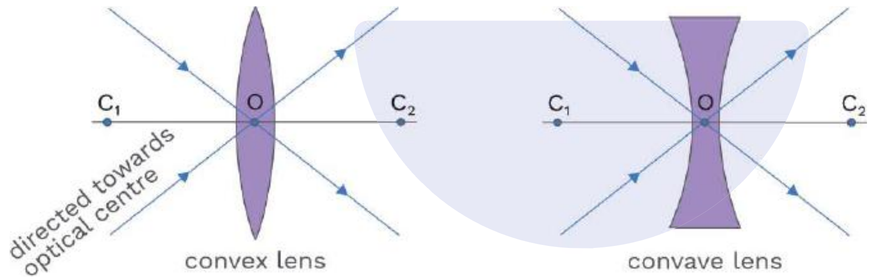
- A lens is a portion of a transparent material with two refracting surfaces such that at least one is curved with refractive index of its material being different from that of the surroundings.
- A thin spherical lens with refractive index greater than that of surroundings behaves as a convergent or convex lens, i.e., converges parallel rays if its central (i.e. paraxial) portion is thicker than marginal one.
- However if the central portion of a lens is thinner than marginal one, it diverges parallel rays passing through it and behaves as divergent or concave lens. This is how we classify convergent and divergent lenses.

Definitions

Point through which ray passing parallel to the principal axis after refraction through the lens passes or appear to pass is known as focus.

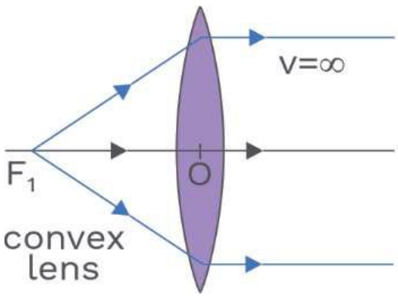


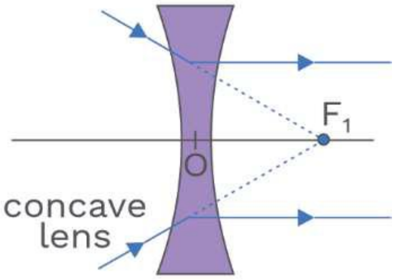
Optical Centre : It is a point O for a given thin lens through which any ray passes undeviated.



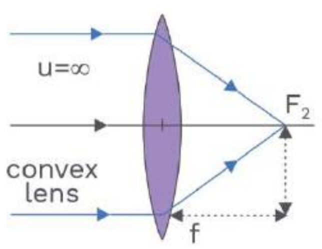
- **Principal Axis :** $C_1 C_2$ is a line passing through optical centre and perpendicular to the lens.

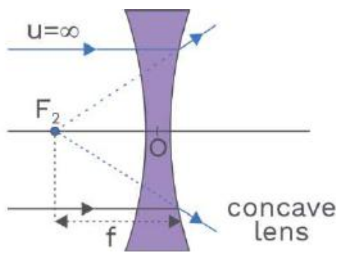
- **Principal Focus:-** A lens has two focal points. First focal point is an object point on the principal axis corresponding to which the image is formed at infinity.





Whereas second focal point is an image point on the principal axis corresponding to which object lies at infinity





- Focal Length f is defined as the distance between optical centre of a lens and the point where the parallel beam of light converges or appears to converge.

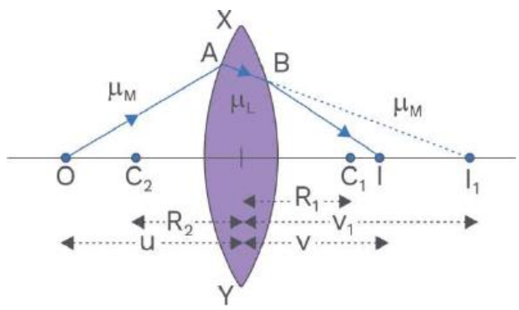
- **Aperture :-** In reference to a lens, aperture means the effective diameter of the circular area through which light enters the lens. Intensity of image produced by a lens depends on the light passing through the lens will equivalently depend on the square of aperture,
i.e.,

$$m \quad \propto \quad A$$

Lens Maker's Formula

In case of image creation by a lens

Image formed by first surface acts as object for the second surface.



So, from the formula of refraction at curved surface $\frac{\mu}{} - \frac{\mu}{} = \frac{\mu \ - \mu}{|}$

For first surface A, $\frac{\mu_x}{} - \frac{\mu_3}{} = \frac{\mu_x - \mu_3}{|}$ ... (i)

$$\because \mu \ = \mu_x \ \ \mu \ = \mu_3$$

For second surface B,

Concept Reminder

Intensity of image formed by a lens depends on the light passing through the lens will equivalently depend on the square of aperture, i.e.,

$$m \quad \propto \quad A$$

$$\frac{\mu_3}{} - \frac{\mu_x}{} = \frac{\mu_3 - \mu_x}{|} = -\frac{\mu_x - \mu_3}{|} \quad \text{... (ii)}$$

$$\because \mu \;= \mu_3 \;\; \mu \;= \mu_x \;\; \mu \;= \mu \quad \rightarrow$$

Adding (i) and (ii)

$$\mu_3 \left[- - - \right] = \mu_x - \mu_3 \left[\frac{}{|} - \frac{}{|} \right]$$

$$\Rightarrow \quad - - - = \frac{\mu_x - \mu_3}{\mu_3} \left[\frac{}{|} - \frac{}{|} \right]$$

$$= \mu - \left[\frac{}{|} - \frac{}{|} \right] \quad \text{... (iii)}$$

$$\left(\because \mu = \frac{\mu}{\mu_3} \right)$$

Now if object is at infinity, Image will be formed at the focus, i.e, $= -\infty$, $=$,

So $\boxed{- = \mu - \left[\frac{}{|} - \frac{}{|} \right]}$ ... (iv)

This is known as lens makers formula. By equating (iii) and (iv), $- - - = -$ this is known as lens formula or Gaussian form of lens equation.

Concept Reminder

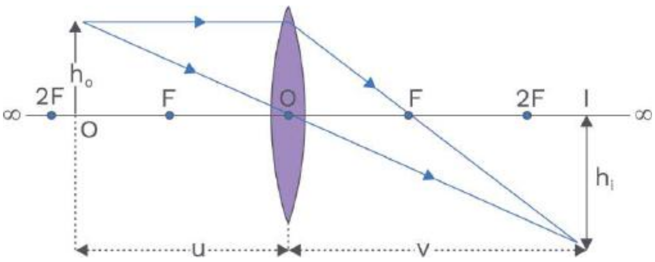
Lens-maker formula,

$$- = \mu - \left(\frac{}{|} - \frac{}{|} \right)$$

Thin lens formula,

$$- = - - -$$

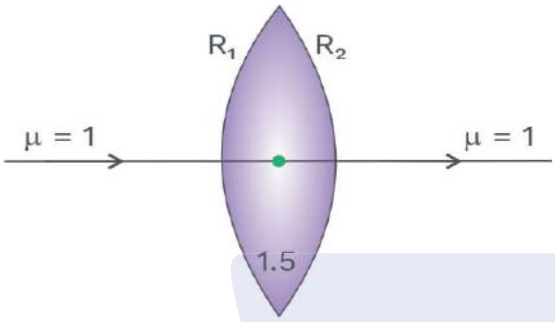
Magnification :



$$= \frac{}{} = \frac{}{} = - = \frac{}{+} = \frac{-}{}$$

Ex. Find out the focal length of a biconvex lens in air if the radius of its surfaces are 60 cm and 15 cm and refractive index of glass = 1.5

Sol. Regard as a light ray going through the lens as shown in figure. It strikes the convex side of '60' cm radii and concave side of 15 cm radii while coming out.



$R_1 = +60$ cm, $R_2 = -15$ cm

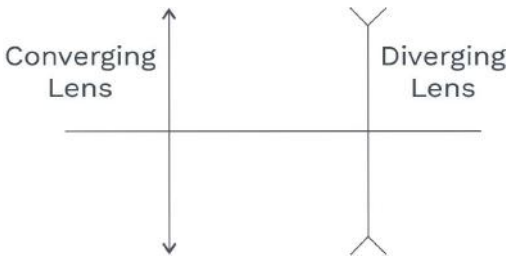
$$\therefore \quad \frac{}{} = \left[\frac{\mu}{\mu} - \right]\left[\frac{}{|} - \frac{}{|}\right]$$

$$\text{or,} \quad \frac{}{} = \left[\frac{}{} - \right]\left[\frac{}{} + \frac{}{}\right]$$

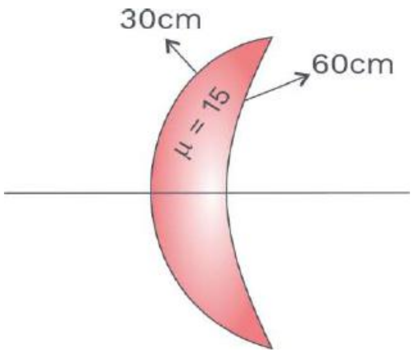
$\Rightarrow \quad f = +24$ cm

Note :

- Since for converging and diverging lenses
- Focal length of lens depends on surrounding medium.
- If f = + ve implies converging and if f = – ve implies diverging lens.



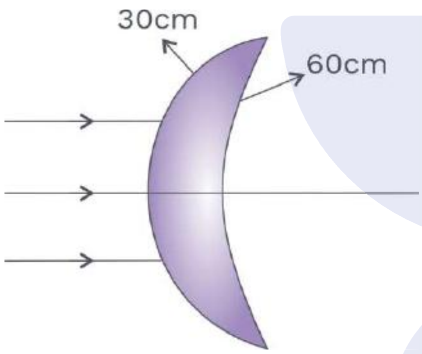
Ex. Calculate the focal length of the lens shown in the figure.



Concept Reminder

A ray passing through the optical centre of the lens proceeds undeviated through the lens.

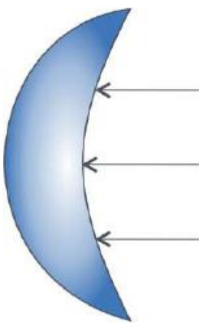
Sol.



$$-=\left(\frac{\mu}{\mu}-\right)\left(\frac{}{|}-\frac{}{|}\right)$$

$$\Rightarrow\ -=\quad -\left(+\frac{}{}-\left(+\frac{}{}\right)\right)\ \Rightarrow\ f = 120\text{ cm}$$

If in the above case direction of rays is reversed.



Concept Reminder

The Interplay of light with things around us gives rise to several beautiful phenomena. The spectacles of colours that we see around us all the time is possible due to sunlight.

$$\frac{}{} = \left(\frac{\mu}{\mu} - \right)\left(\frac{}{|} - \frac{}{|}\right)$$

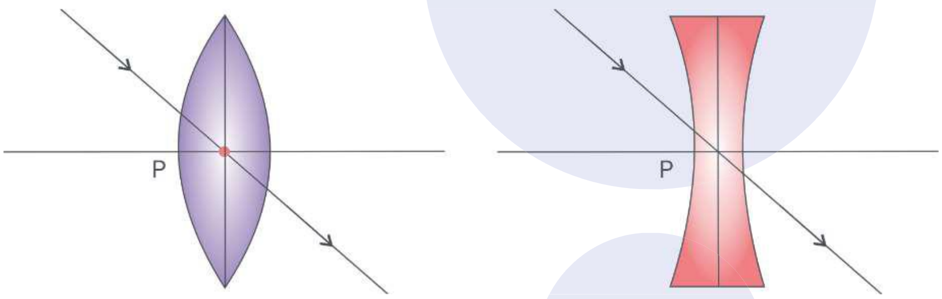
$$\Rightarrow \frac{}{} = \left(\quad - \right)\left(-\frac{}{} - \left(-\frac{}{}\right)\right)$$

f = + 120 cm

This Illustration shows that focal length does not depend on the incident ray direction.

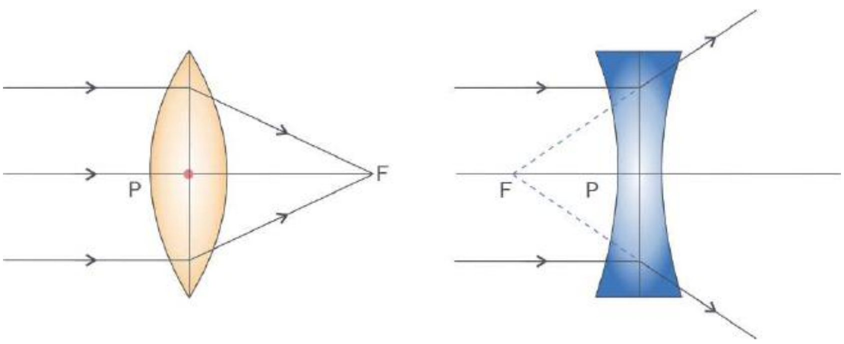
Rules For Image Formation:-

(i) A light ray passing through the optical centre (P) of the lens proceeds un-deviated across the lens. (By definition of optical centre)

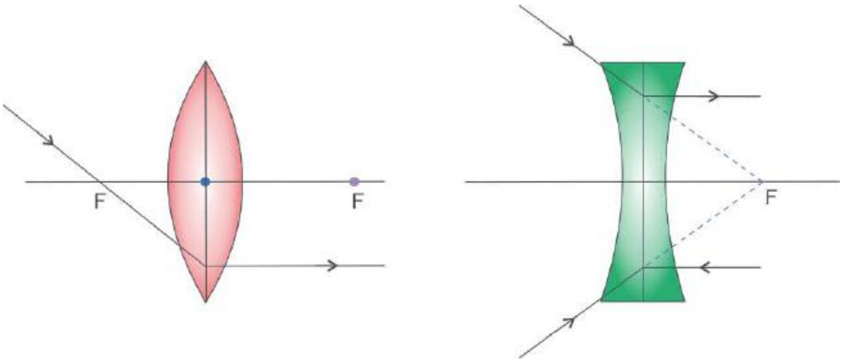


Pole is the junction of the ray which goes undeviated through the lens and the optical axis.

(ii) A ray passing parallel to the principal axis after refraction pass by the lens passes or appear to pass through the focus. (By the definition of the focus)



(iii) A ray passes through the focus or directed towards the focus, after refraction from the lens, becomes parallel to the principal axis. (Principle of reversibility of light)



Only two rays from the same point of an object are needed for image formation and the point where the rays after refraction through the lens intersect or appear to intersect, is the image of the object. If they actually intersect each other, the image is real and if they appear to intersect the image is said to be virtual.

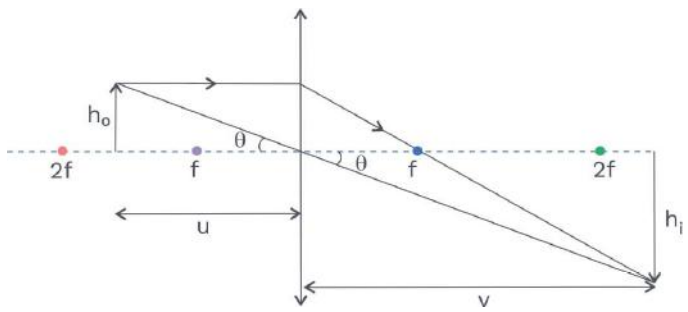
Ex. Find the position of the image formed.
$u = +30$ cm, $f = +10$ cm

Sol. — = — – —

⇒ — = — – — ⇒ — = — + —

⇒ = —— = 7.5 cm

Transverse Magnification

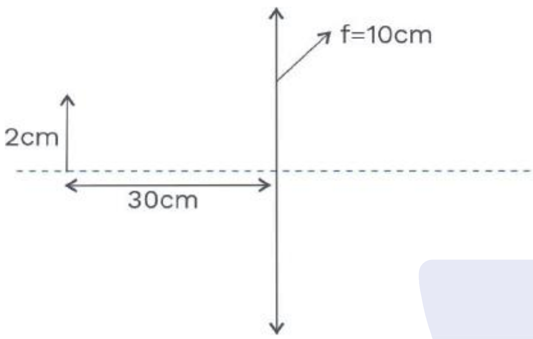


$\theta =$ —— ... (1),

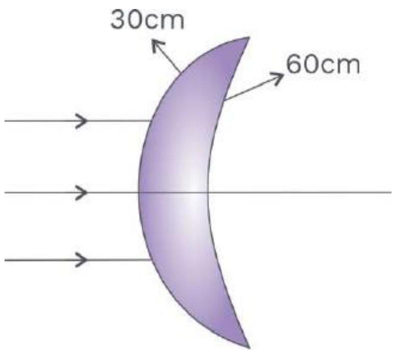
$\theta =$ — ... (2)

from eq. (2) / (1) = — = —

Ex. Find out the position, height and nature of the image formed.



Sol. u = – 30 cm, f = + 10 cm, h_0 = + 2 cm



— = — – —

⇒ — = — + — ⇒ — = —

⇒ = +

— = —
–

⇒ — = —
–

⇒ = –

∴ Real, inverted, diminished

Concept Reminder

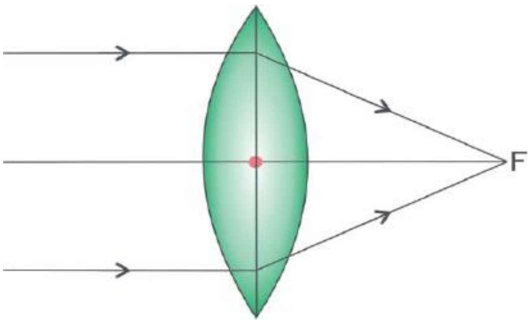
A lens of shorter focal length bends the incident light more, while converging it in case of a convex lens and diverging it in case of a concave lens.

Definitions

A lens is a portion of a transparent material with two refracting surfaces such that at least one is curved with refractive index of its material being different from that of the surroundings.

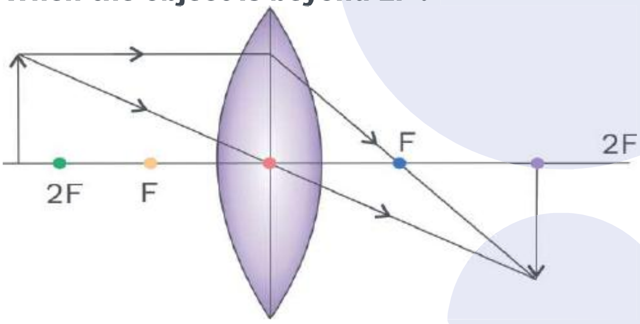
Image Formation By a Convex Lens Of The Linear Object:-

(i) When the object is at infinity:-



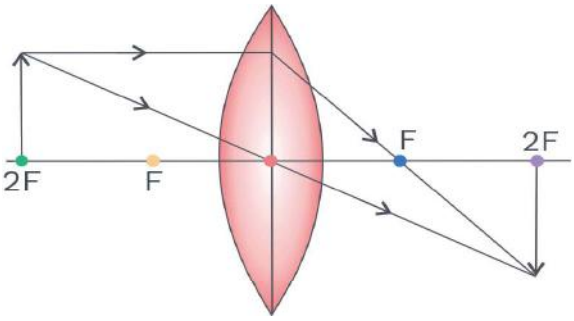
The image is formed at ‘F’. It is inverted, real and highly diminished.

(ii) When the object is beyond 2F :-



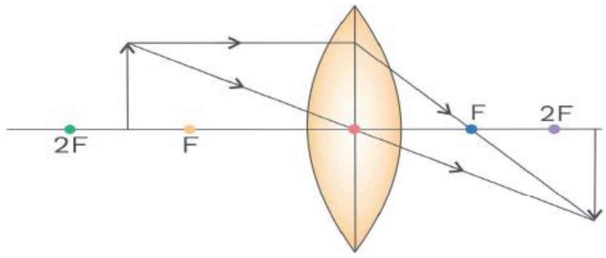
The image is produced between F and 2F. It is inverted, real and diminished

(iii) When the object is at 2F :



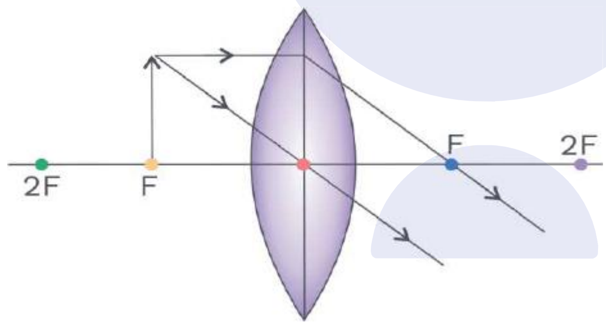
The image is formed at 2F. It is real, inverted and the same size as the object.

(iv) When the object is between F and 2F :



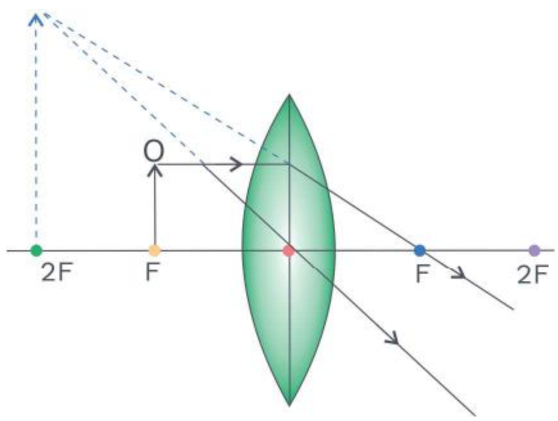
The image is formed beyond 2F (i.e., between 2F and ∞). It is real, inverted and enlarged.

(v) When the object is at F :



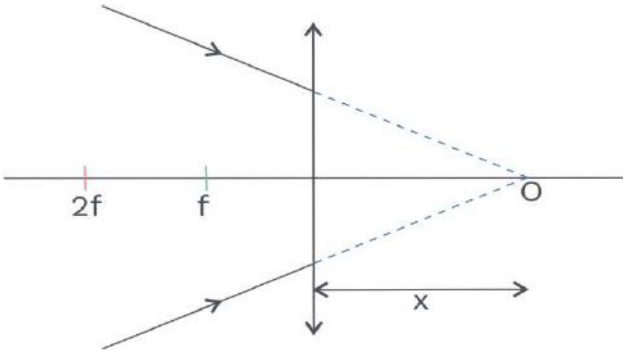
The image is formed at infinity. It is real, inverted and highly magnified.

(vi) When the object is between F and O :



The image is on the same side as the object is. It is virtual, erect and magnified.

(vii) Virtual object case for converging lens:-



u = + x

f = + f

from lens formula — — — = —

— = — + —

if = → ∞

if = ∞ →

Position of Object	Position of Image	Magnification
$-\infty$	F	$\|m\| << 1$ & $m<0$
$-\infty$ – 2F	F – 2F	$\|m\| < 1$ & $m<0$
2F	2F	$m = -1$
F – 2F	∞ – 2F	$m << -1$
Between C & F, near F	$+\infty$	$m << -1$
Between F & O, near F	$-\infty$	$m >> 1$
F – O	In front of lens	$m > 1$

Graphs for converging lens

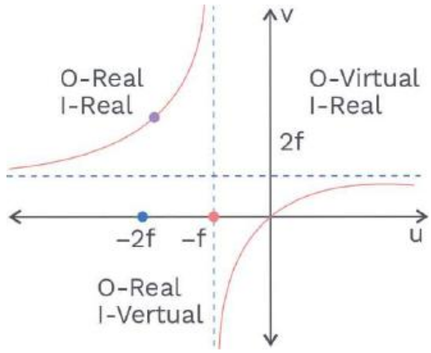
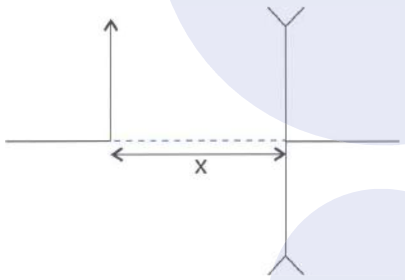


Image Formation By a Concave Lens Of A Linear Object:-

(a) Real object case



u = − x, f = − f

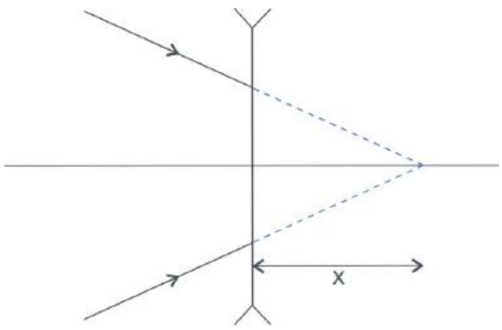
from lens formula — + — = — —

— = — — = —

If → ∞ → −

If → →

(b) Virtual object case:



u = +x, f = – f

from lens formula — = — – –

If x = 0 → v = 0

If x < f ; v = + u

If x is just smaller than f

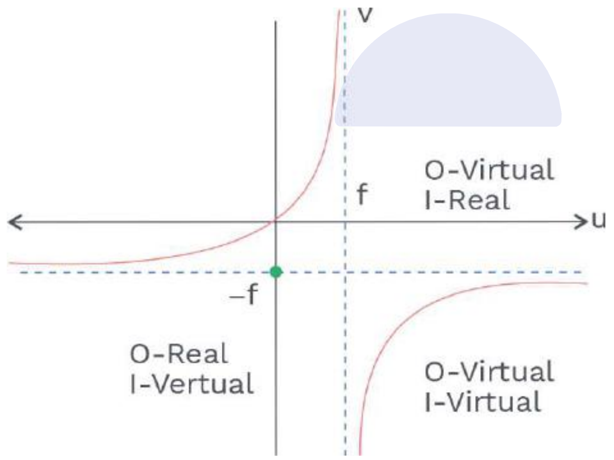
→ +∞

If x is just greater than f → –∞

If → ∞ , → –

Real object	u = –ve
Real image	v = +ve
Virtual object	u = +ve
Virtual image	v = –ve

Graphs for diverging lenses.



Ex. An object is positioned in phase of a converging lens of focal length 10 cm and image formed is double the size of object. Then find the position of object.

Sol. **Case I** : If the image formation is real –— =

⇒ — = –

⇒ = –

⇒ —— = – —— – –

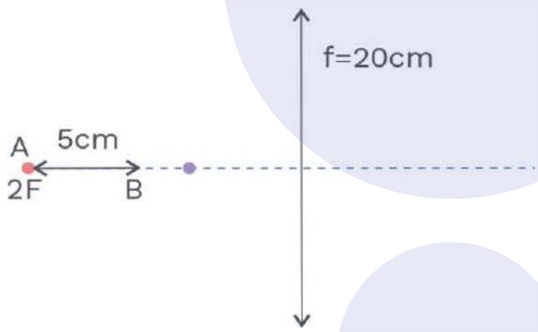
⇒ —— = – —— ⇒ = –

Case II : If the image formation is virtual

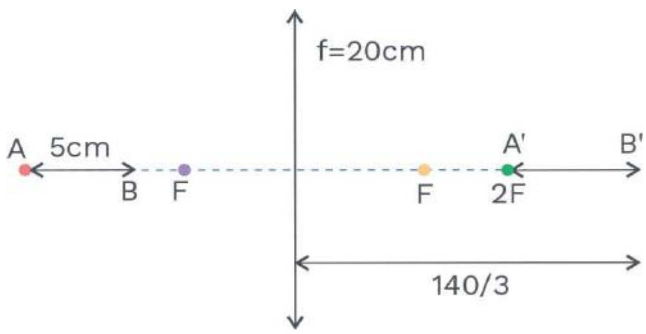
—— = ⇒ v = 2u

⇒ —— = —— – – ⇒ —— = – —— ⇒ = – cm

Ex. Find out the linear length of the image of the object AB shown in figure.



Sol. **For B** :



– = — – –

⇒ —— = — + ——

⇒ – = —— – ——

$\Rightarrow \quad = \frac{\quad}{\quad}$

$A \; N = \;_{N} - \;_{A} = \frac{\quad}{\quad} - \quad = \frac{\quad}{\quad}$

Ex. Find location & nature of image formed if object is placed at 30 cm from convex lens of focal length 10 cm?

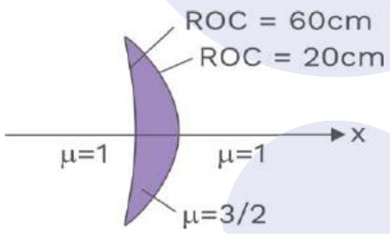
Sol. $- = - - -$

$\frac{\quad}{\quad} = - + \frac{\quad}{\quad}$

v = +15 (Real image)

$= - = \frac{\quad}{-} = - -$ (inverted & diminished)

Ex. Find out the focal length of the lens shown in figure. If



(a) the light is incident from left side.
(b) the light is incident from right side.

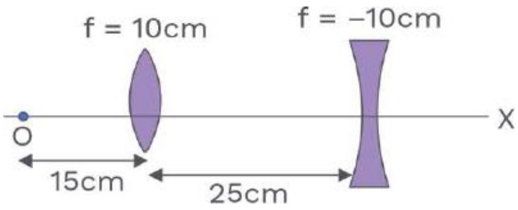
Sol. (a) $- = \quad - \left(\frac{\quad}{|} - \frac{\quad}{|}\right) = \left(- -\right)\left(\frac{\quad}{-} - \frac{\quad}{-}\right)$

f = 60 cm

(b) $- = \quad - \left(\frac{\quad}{|} - \frac{\quad}{|}\right) = \left(- -\right)\left(\frac{\quad}{\quad} - \frac{\quad}{\quad}\right)$

f = 60 cm

Ex. Find the position of final image formed as shown in figure.



Sol. For converging lens

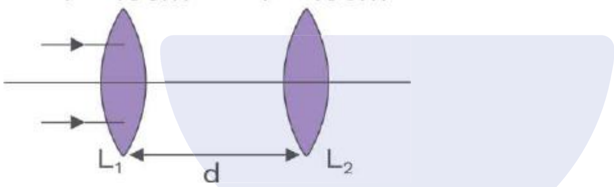
$u = -15$ cm, $f = 10$ cm $\quad = \frac{\quad}{+} =$

For diverging lens

$u = 5$ cm

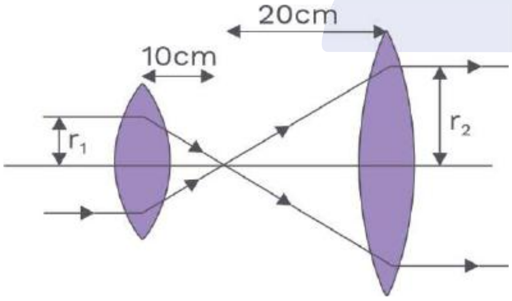
$f = -10$ cm $\quad = \frac{\quad}{+} =$

Ex. Figure displays two converging lenses. Incident rays are parallel to principal axis. What should be the supposed value of d so that final rays are also parallel.



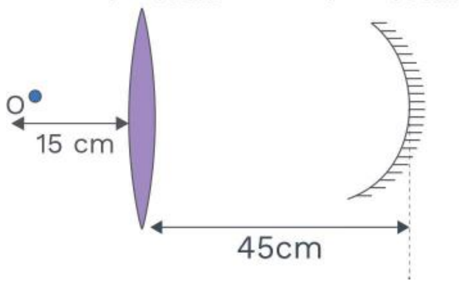
Sol. Final rays should be parallel. For this the II focus of L_1 must coincide with I focus of L_2 lens.

$d = 10 + 20 = 30$ cm



Here the diameter of ray beam becomes wider.

Ex. Find the position of final image formed.



Sol. For lens,

— — — = –

— — —— = ——
–

⇒ = +

Hence it is object for mirror
u = –15 cm

— + —— = ——
– –

⇒ = –

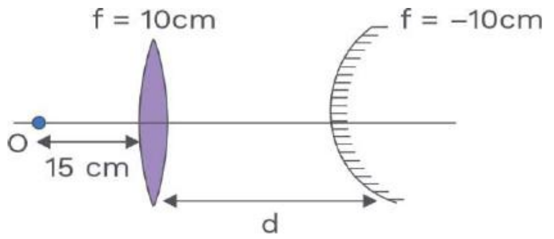
Now for second time it again passes through lens
u = –15 cm
v = ? ; f = 10 cm

— – —— = ——
–

⇒ = +

Therefore final image will form at a distance 30 cm from the lens towards left.

Ex. What should be the value of ‘d’ so that image is formed on the object itself.

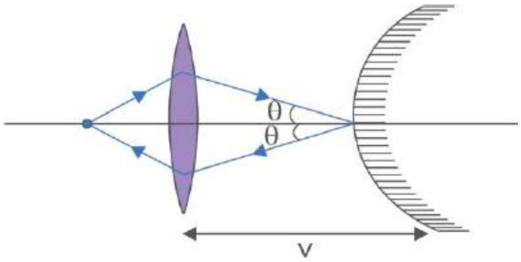


Sol. For lens :

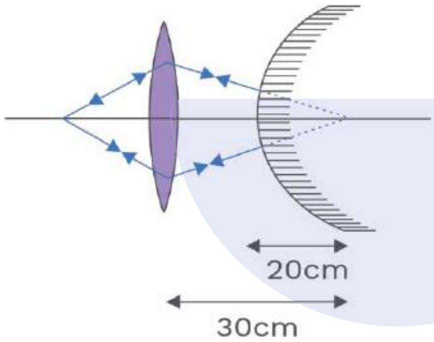
— – —— = ——
–

v = +30 cm

Case I : If ‘(d = 30)’, the object for mirror will be at pole and its image will be created there itself.

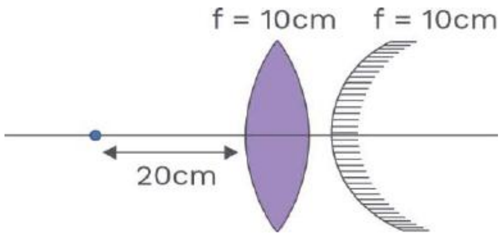


Case II : If the rays incident the mirror normally, they will retrace and the image will be formed on the object itself



∴ d = 30–20 = 10 cm

Ex. Find out the position of final image created.
(The gap shown in diagram is of negligible width)



Sol. For lens :

— — — = — ⇒ — — —— = —
–

— = —— – —— = —— ⇒ =

For mirror :

— + — = — ⇒ — + —— = ——

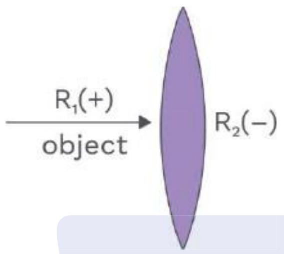
— = —— – —— = —— ⇒ =

Focal length of equiconvex lens placed in air:

refractive index of lens $\mu_x = \mu$, | = +|

refractive index of surrounding medium μ_3 =

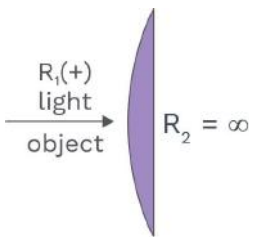
| = −|



— = μ — [— − (− —)]
| |

⇒ Focal length = $\frac{|}{\mu -}$

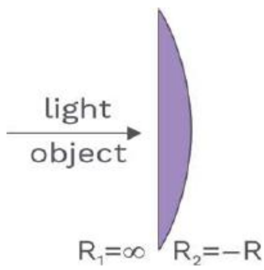
- Focal length of planoconvex lens placed in air:



— = μ — [— − —]
| ∞

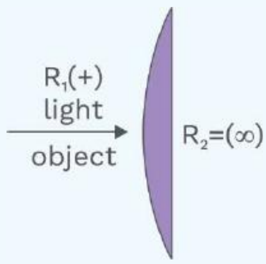
⇒ Focal length = $\frac{|}{\mu -}$

If the object is placed towards the plane surface, then



Concept Reminder

Focal length of planoconvex lens placed in air:



= $\frac{|}{\mu -}$

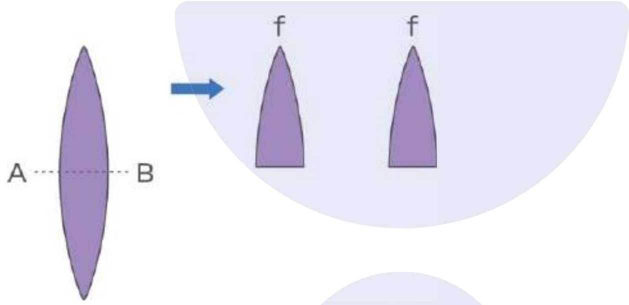
$- = \mu - \left[\frac{}{\infty} - \left(-\frac{}{|}\right)\right]$

$\Rightarrow$ Focal length $= \frac{|}{\mu -}$

- If an equiconvex lens of focal length f is cut into two identical parts by a horizontal plane AB then the focal length of each part will be equal to that of the initial lens; because μ, R_1 and R_2 will remain unchanged. Only intensity of image will be reduced.
  $\therefore$ intensity I $\propto$ $(\text{aperture})^2$

Concept Reminder

If an equiconvex lens of focal length f is cut into two identical parts by a horizontal plane AB then the focal length of each part will be equal to that of the initial lens.

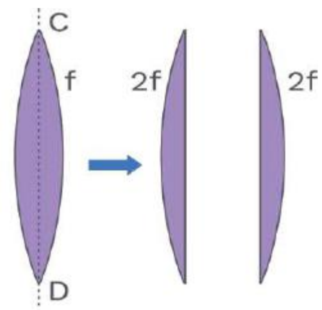


- If the same lens is cut into equal parts by a vertical plane CD, the focal length becomes double.
  For equiconvex lens $- = \frac{\mu -}{|}$ For plano convex lens $— = \frac{\mu -}{|}$

  So $- = - \Rightarrow \quad =$

Concept Reminder

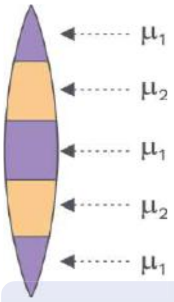
If the same lens is cut into equal parts by a vertical plane CD, the focal length becomes double.



focal length of each part becomes 2f

(f = focal length of original lens)

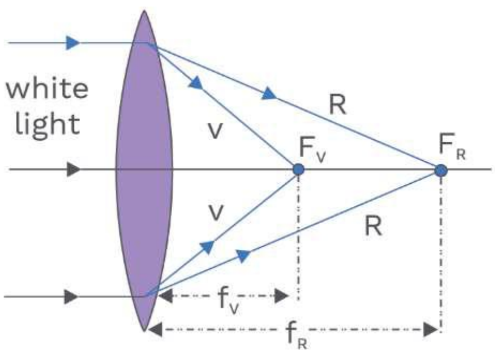
- If a lens is created of number of layers of different refractive indices for a given wavelength of light, the no. of images corresponding to an object is equal to types of refractive indices, as $- \propto \mu -$

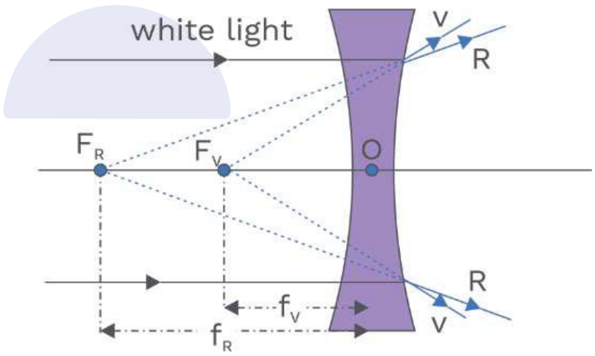


For the figure shown number of images = 2

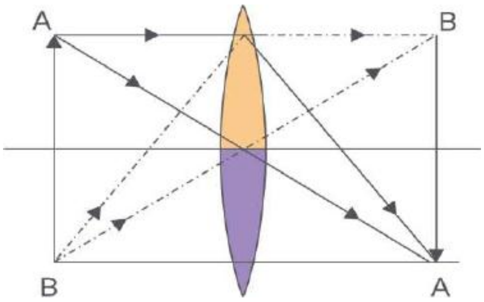
- Focal length of a lens varies on wavelength.

$\because - \propto \mu - \quad \propto \frac{}{\lambda} \Rightarrow \quad \propto \lambda \qquad \therefore \ | >$





- If half portion of a lens is covered by black paper then intensity of image will be reduced but complete image will be formed.



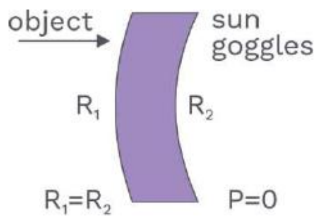
Concept Reminder

The number of images formed by lens corresponding to an object is equal to types of refractive index.

Concept Reminder

If half portion of a lens is blackened then image formed will be complete but intensity of image will be reduced.

Sun–glasses or goggles:
radii of curvature of two surfaces are equal with centres of curvatures on the same side of the lens

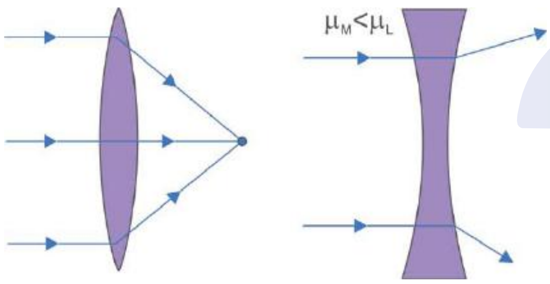


| = | = +| so — = μ – [— —]

⇒ — = ⇒ = ∞ and P = 0

⇒ sun glasses or goggles have no power.

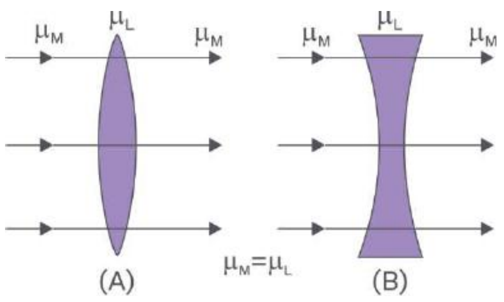
- If refractive index of medium < refractive index of lens



If $\mu_3 < \mu_x$ then $\frac{\mu_x}{\mu} >$ or $\left(\frac{\mu_x}{\mu} - \right) >$

Convex lens behave as convergent lens.
While concave lens behave as divergent lens.

- Refractive index of medium = Refractive index of lens ($\mu_M = \mu_L$)



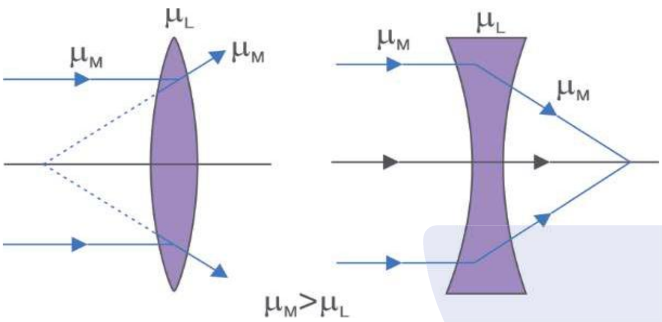
Concept Reminder

If refractive index of medium is equal to refractive index of lens then lens will behave as plane transparent plate.

$- = \left(\frac{\mu_x}{\mu_3} - \right)\left(\frac{}{|} - \frac{}{|}\right) - =$ $\Rightarrow f = \infty$ & $P = 0$

Lens will behave as plane transparent plate

- Refractive index of surrounding medium > Refractive index of lens



$\mu_M > \mu_L$

$\mu_3 > \mu_x \Rightarrow \left(\frac{\mu_x}{\mu} - \right) <$

convex lens will behave as divergent lens and concave lens will behave as convergent lens. An air bubble in water behaves as a concave lens.

Concept Reminder

If $\mu_M > \mu_L$ there convex lens will behaves as divergent lens and concave lens will behave as convergent lens.

Ex. A magnifying lens has a focal length of '10' cm. (a) Where should an object be placed if the image is to be 30 cm away from the lens? (b) What will be the magnification

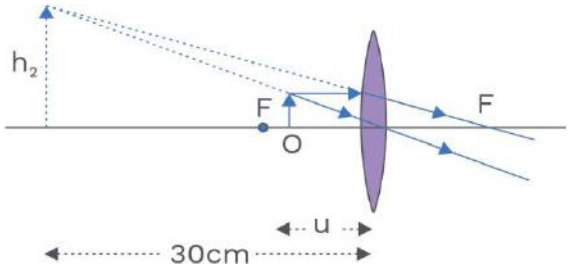
Sol. (a) In case of magnifying lens, it is converging in nature and the image is enlarged, erect, virtual between infinity and object and on the same side of the lens.

$f = 10$ cm and $v = -30$ cm

and hence from lens-formula,

$- - - = -$

we have $\frac{}{-} - - = -$ i.e., $u = -7.5$ cm



So the object must be put at a distance of 7.5 cm (which is < f) in front of the lens.

(b) $= \left[\frac{}{} \right] = \frac{}{} = \frac{-}{-} =$ i.e., image is erect, virtual and four times the size of object.

Ex. An object 25 cm high is located in front of a convex lens of focal length 30 cm. If the height of the image produced is 50 cm, find the distance between the object and the image?

Sol. As the object is in front of the lens, it is real. If the image is inverted and real then

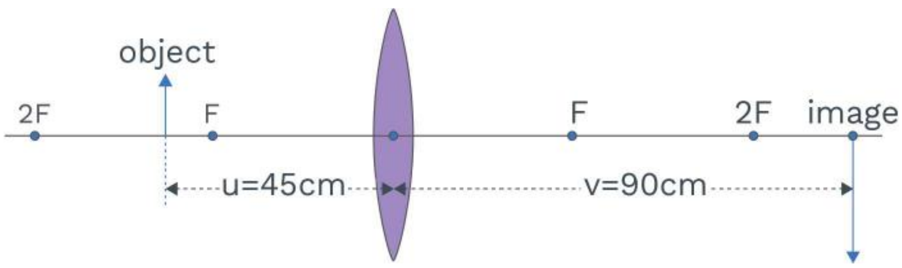
h_1 = 25 cm, f = 30 cm, h_2 = – 50 cm

$$= \frac{}{} = \frac{-}{} = - \quad \Rightarrow \quad = \frac{}{+} \Rightarrow - = \frac{}{+}$$

u = – 45 cm $\Rightarrow \quad = - \Rightarrow - = \frac{}{-} \Rightarrow$ v = 90 cm

As in this situation, the object and image are on the opposite sides of the lens, the distance between object and image is

d_1 = u + v = 45 + 90 = 135 cm



If the image is erect (i.e., virtual)

$$= \frac{}{+} \Rightarrow = \frac{}{+} \Rightarrow = - \quad \Rightarrow \quad = -$$

$$\Rightarrow \quad = \frac{\quad}{-}$$

$$\Rightarrow \quad = - \qquad .$$

As in this situation both image and object are in front of the lens, the distance between object and image is $d_2 = v - u = 30 - 15 = 15$ cm.

Velocity of the image formed by a lens:-

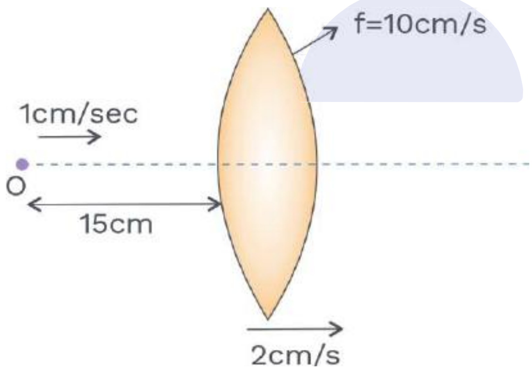
from — – — = –

differentiate the above eq.

$$- \text{——} + \text{——} = \quad \Rightarrow \text{—} = \text{——}$$

$$x = \text{—} \quad \alpha$$

Ex. Find out the velocity of the image of the object shown in the figure.



Sol. – = – – –

$$\Rightarrow \text{—} = - + \text{—} \Rightarrow - = \text{—} - \text{—}$$

$$\Rightarrow \quad = \text{——} =$$

$$- = \text{—} - -$$

By differentiating :

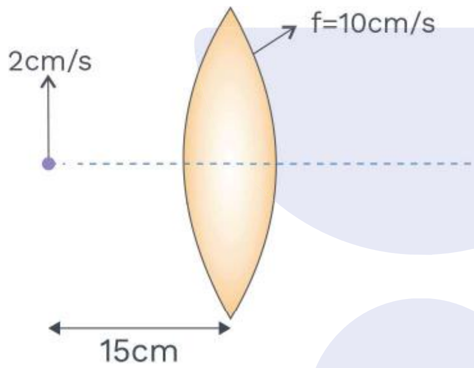
$$= \frac{}{} - \frac{}{} \Rightarrow \; = - \frac{}{} + \frac{}{} \Rightarrow \frac{}{} = \frac{}{}$$

$$\Rightarrow \; \frac{}{} = \frac{}{} \Rightarrow \; = \frac{}{} \times \; - \; = -$$

$$\Rightarrow \; m^{-} \; x \; = \frac{}{} \; - \; x$$

$$\Rightarrow \; m^{-} \; = \; \times - \; \Rightarrow \; m = -$$

Ex. Find out the velocity of the image of the object shown in the figure.



Sol. $- - \frac{}{-} = \frac{}{} \Rightarrow - = \frac{-}{} = \frac{}{}$

v = 30 cm

$$\frac{}{} = -$$

$$\Rightarrow \; \frac{}{} = - \qquad \Rightarrow \; \frac{}{} = - \frac{}{}$$

$$\Rightarrow \; \frac{}{-} \times \; = -$$ (downwards)

Combination Of Lenses And Mirrors

When several lenses or mirrors are used, the image formation is considered one after another in sequences of steps, The image formed by the lens facing the object serves as an object for the next lens or mirror, the image formed by the second lens acts as an object for the third, and so on, The total magnification in such situations will be given by

$$= \frac{m}{8} = \frac{m}{8} \times \frac{m}{m} \times \; \Rightarrow \; = \; \times \; \times$$

Power of Lens [in air] $(_x = \frac{}{x}$

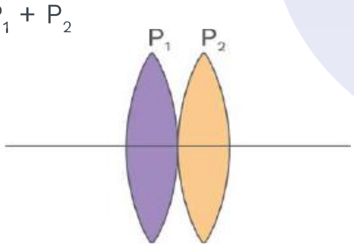
(f_L is always take in SI units)
Converging lens P_L = +ve
Diverging lens P_L = –ve
Power of lens (in medium) $(_x = \frac{\mu}{}$

Power of mirror $(\quad = -\frac{}{}$

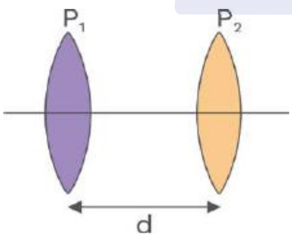
Convex mirror P_M = –ve
Concave mirror P_M = +ve

Power of lens in combination

(i) $P_{eq} = P_1 + P_2$



(ii) $P_{eq} = P_1 + P_2 - dP_1P_2$



Silvering Of Lens
The focal length of the equivalent mirror of a equiconvex lens silvered at one side.

$$(\; = (_x + (_3 + (_x = \; (_x + (_3$$

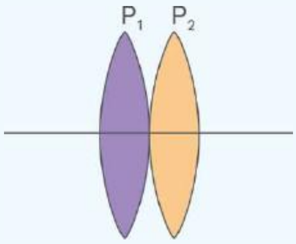
$$\frac{}{i} = \frac{}{x} + \frac{}{} + \frac{}{x} = \frac{2}{x} + \frac{}{} = \frac{\mu - \quad \times}{|} + \frac{|}{|}$$

$$= \frac{\mu - \quad +}{|} \Rightarrow i \; = \frac{|}{\mu -}$$

Ex. Radius of curved surface of a plano convex lens is 20 cm and refractive index of lens material is 1.5. Calculate equivalent focal length of lens if :–

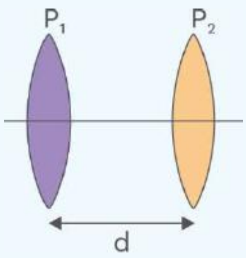
Concept Reminder

i.



$P_{eq} = P_1 + P_2$

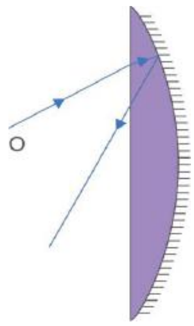
ii.



$P_{eq} = P_1 + P_2 - dP_1P_2$

(i) curved surface is silvered.
(ii) plane surface is silvered.

Sol. (i)



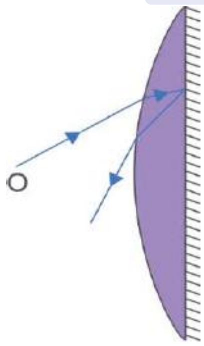
$$(= (_x + (_3$$

$$(= \frac{\mu -}{|} + \frac{}{|}$$

$$(= \frac{\mu}{|} \Rightarrow (= \frac{\times}{} = --$$

$$\Rightarrow \quad = - \frac{}{\times} = - \frac{}{}$$

(ii)



$$(= (_x + (_3$$

$$(= \frac{\mu -}{|} + \frac{}{\infty}$$

$$(= \frac{\mu -}{|} \Rightarrow \quad = - \frac{}{(}$$

Rack your Brain

A plano-convex lens of unknown material and unknown focal length is given. With the help of a spherometer we can measure the:

(1) Refractive index of the material
(2) Focal length of the lens
(3) Radius of curvature of the curved surface
(4) Aperture of the lens

$\Rightarrow \quad = -\frac{|}{\mu -} = \frac{-}{-} = -$

Ex. An object is kept 30 cm in front of a concave lens that is made of a glass of refractive index 1.5 and has equal radii of curvature of its two surface, each 30 cm. The surface of the lens beyond away from the object is silvered. Find out the nature and position of the final image.

Sol. Focal length of concave lens, using the lens-maker's formula, is

$\frac{}{x} = \quad - \left(- \frac{}{} - \frac{}{} \right)$

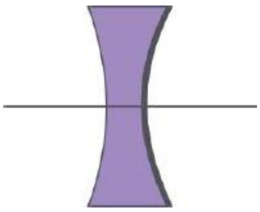
[R_1 = –30 cm, R_2 = 30 cm]

$\therefore f_L$ = –30 cm = – 0.3 m

In a silvered lens, light as it enters the lens suffers refraction, then it gets reflected at the silvered surface and gain undergoes refraction as it comes out in air.

In such a situation, power of the silvered lens will be

$P = P_L + P_M + P_L = 2P_L + P_M$



but $(_x = \frac{}{x} = \frac{}{}$ S

and $(_3 = -\frac{}{3} = \frac{}{}$ S

[Silvered surface behaves as a spherical convex mirror of radius of curvature 30 cm so that focal length will be 15 cm = 0.15 m]

$\therefore \quad (= (_x + (_3 = -\frac{}{} - \frac{}{} = -$ S

and focal length of the equivalent mirror

$i = -\frac{}{(} = -\frac{}{-} = \qquad =$

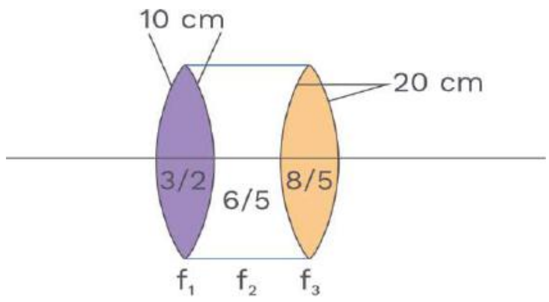
Rack your Brain

The power of a biconvex lens is 10 dioptre and the radius of curvature of each surface is 10 cm. Then the refractive index of the material of the lens is:

(1) – (2) –

(3) – (4) –

Ex. Find the focal length of equivalent system



Sol. — = (— –)(— + —) = – × — = —

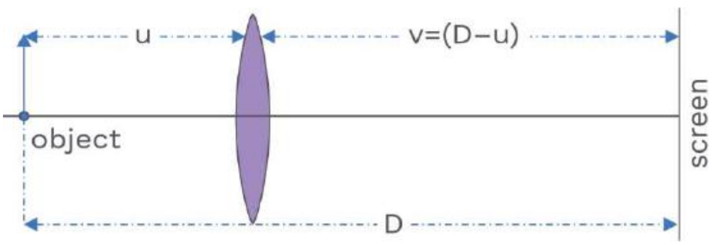
— = (— –)(— – —) = – × (— / ×) = —

— = (— –)(— + —) = —

– = — + — + — = — + — + — ⇒ = —

Displacement Method

It is used for calculation of focal length of convex lens in laboratory. A thin convex lens of focal length 'f' is arranged between an object and a screen fixed at a distance D apart. If D > 4f there are two positions of lens corresponding to which a sharp image of the object is formed on the screen.



By lens formula — – — = – ⇒ — – — = –

$$\Rightarrow \quad -S \quad +S \quad = \quad \Rightarrow \quad = \frac{S \pm \sqrt{S\ S -}}{}$$

there are three possibilities:

(i) for D < 4f, u will be imaginary hence physically no position of lens is possible

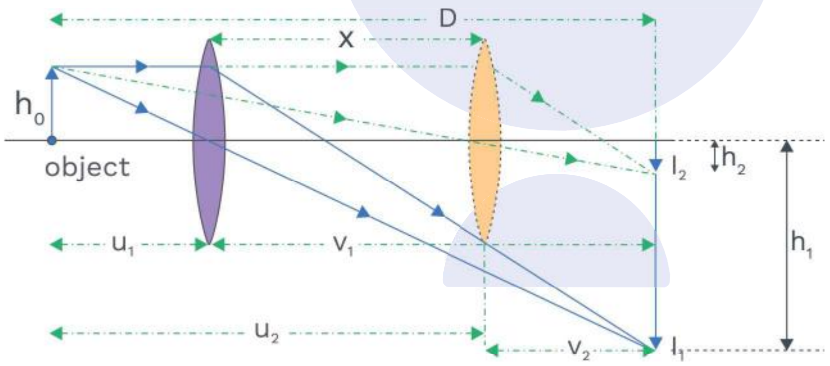
(ii) for D = 4f, $= \frac{S}{} =$ so only one position of lens is possible and

since

v = D – u = 4f – 2f, v = 2f

(ii) for D > 4f, $= \frac{S - \sqrt{S\ S -}}{}$ and $= \frac{S + \sqrt{S\ S -}}{}$

So there are two locations of lens for which image will be created on the screen. (for two distances 'u_1' and 'u_2' of the object from lens)



If the distance between two locations of lens is x then

$$= \quad - \quad = \frac{S + \sqrt{S\ S -}}{} - \frac{S - \sqrt{S\ S -}}{}$$

$$= \sqrt{S\ S -} \quad \Rightarrow \quad = S \quad - \quad S \quad \Rightarrow \quad = \frac{S \quad -}{S}$$

Distance of image corresponds to two locations of the lens:

$$= S - \quad = S - — S - \sqrt{S\ S -} \quad = — S + \sqrt{S\ S -} \quad = \quad \Rightarrow \quad =$$

$$= S - \quad = S - — S + \sqrt{S\ S -}$$

$= — S - \sqrt{S\ S -}\quad = \quad \Rightarrow \quad =$

Distances of image and object are interchangeable. for the two locations of the lens

Now $x = u_2 - u_1$ and $D = v_1 + u_1 = u_2 + v_2$ $[\because v_1 = u_2]$

so $= \ = \frac{S -}{}$ and $= \ = \frac{S +}{}$;

$= \frac{m}{8} = — = \frac{S +}{S -}$

and $= \frac{m}{8} = — = \frac{S -}{S +}$

Now $\times \ = \frac{S +}{S -} \times \frac{S -}{S +} \Rightarrow \frac{mm}{8} = \ \Rightarrow 8 = \sqrt{mm} \Rightarrow \ = \sqrt{\quad}$.

Ex. In the displacement method a convex lens is put in between an object and a screen. If the magnification in the two positions are m_1 and m_2 ($m_1 > m_2$), and the distance between the two locations of the lens is x, the focal length of the lens is :

Sol.

$= — \quad = — \quad = \quad + \quad = \quad +$

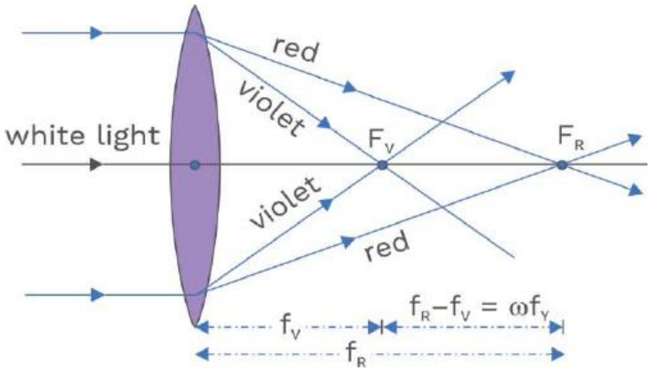
$= \ - \ = \ - \ \Rightarrow \ = \frac{}{-}$

Concept Reminder

Thick lenses could be assumed as made of many prism, therefore, thick lenses show chromatic aberration due to dispersion of light. When white light passes through thick lenses, red and blue colours focus at different points. This phenomenon is known as chromatic aberration.

Chromatic Aberration

The image of an object due to white light created by a lens is usually coloured and blurred. This type defect of image is known as chromatic aberration which arises due to the fact that the focal length of a lens is different for different colours. For a single lens $— = \ \mu - \left[\frac{—}{|} - \frac{—}{|} \right]$ and as μ of lens is maximum for violet while minimum for red so, violet gets focussed nearest to the lens while red is farthest from it.



Longitudinal or Axial Chromatic Aberration

When an object O situated on the axis of a lens is illuminated by white light, then images of different colours are formed at different points along the axis. The formation of images of different colours at different positions is called 'axial' or longitudinal chromatic aberration. The axial distance between the red and the violet images $I_R - I_V$ is a measure of longitudinal aberration. When white light is incident on lens, image is obtained at different point on the axis because focal length of lens depends on wavelength. $\propto \lambda \Rightarrow {}_{|} >$

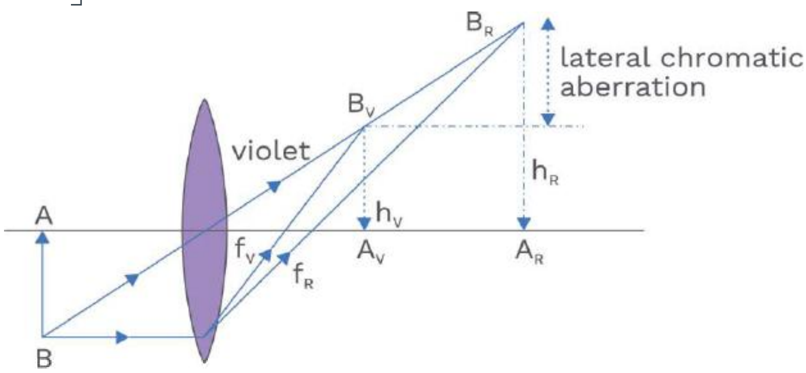
$f_R - f_V = \omega f_y \Rightarrow$ Axial or longitudinal chromatic aberration

If the object is at placed infinity, then the longitudinal chromatic aberration is equal to the difference in focal–lengths ($f_R - f_V$) for the red and the violet rays.

Lateral Chromatic Aberration

As the focal–length of the lens varies from colour to colour, the magnification

$= \left[\frac{\quad}{+}\right]$ produced by the lens also varies from colour to colour.



Therefore, for a finite–sized object AB, the images due to different colours formed by the lens are of different sizes.
The creation of images of different colours in different sizes is called lateral chromatic aberration. The difference in the heights of the red image B_R A_R and the violet image B_V A_V is a measure of as lateral chromatic aberration.

$$LCA = h_R - h_V$$

Achromatism
If two or more lenses are combined together in such a way that this combination produces images due to all colours at the same point then this combination is known as achromatic combination of lenses. Condition for achromatism, [when two lenses are in contact].

$$\frac{\omega}{\;} + \frac{\omega}{\;} = \;\Rightarrow \frac{\omega}{\omega} = -\frac{\;}{\;}$$

and equivalent focal length $\frac{\;}{i} = \frac{\;}{\;} + \frac{\;}{\;}$ (Apply sign convention while solving numerical)

Ex. Dispersive power of one lens is double of another. Find individuals focal length of lenses if net focal length is 60 cm.

Sol. $\omega = \omega$

$$\frac{\omega}{\omega} = -\frac{\;}{\;} = \;\Rightarrow\; = -$$

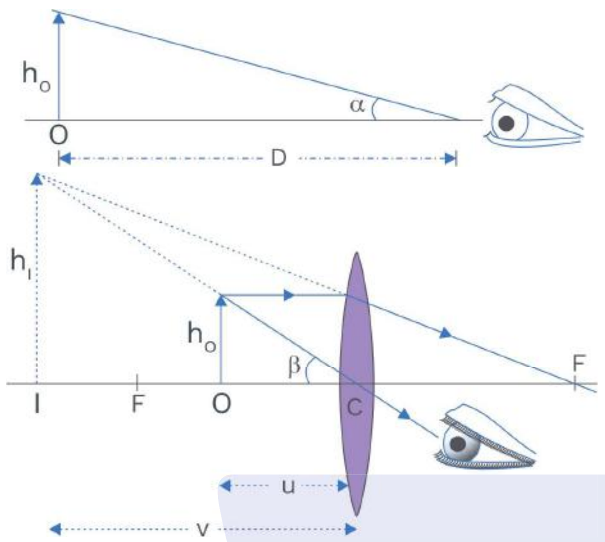
$$\frac{\;}{\;} + \frac{\;}{\;} = \frac{\;}{\;}$$

$$\frac{\;}{\;} - \frac{\;}{\;} = \frac{\;}{\;}$$

$f_1 = -60$ cm
$f_2 = 30$ cm

Optical Instruments:-

Simple microscope : It is a convergent lens.
When the object is positioned between the focus and the optical centre a virtual, magnified and erect image is formed.



Magnifying power (MP)

$$= \frac{\beta}{\alpha}$$

$$\Rightarrow 3\ (\ = \frac{\ }{-S} = \frac{S}{\ }$$

(i) When the image is produced at infinity:

From lens equation — — — = — ⇒ — $\frac{\ }{-\infty}$ — $\frac{\ }{-}$ = —

⇒ = So 3 ($= \frac{S}{\ } = \frac{S}{\ }$

(ii) If the image is at minimum distance of clear vision D:

$\frac{\ }{-S} - \frac{\ }{-} = - \Rightarrow - = \frac{\ }{S} + -$ [v = –D and u = –ve]

Multiplying both the sides by D

$$\frac{S}{\ } = \ + \frac{S}{\ } \Rightarrow 3\ (\ = \frac{S}{\ } = \ + \frac{S}{\ }$$

Ex. A man with normal nearby point 25 cm away reads a book with small print using a magnifying glass, which is a thin convex lens of focal length ‘5’ cm. What is the closest and furthest distance at which he can read the book when showing through the magnifying glass?

Sol. As for normal eye far and closed points are ∞ and 25 cm away respectively, so for magnifier $= -\infty$ and $= -$. However,

for a lens as $- - - = - \Rightarrow \; = \frac{\quad}{-}$

So, u will be minimum when, v = minimum = –25 cm
i.e.

$$= \frac{\quad}{-} = - \frac{\quad}{-} = -$$

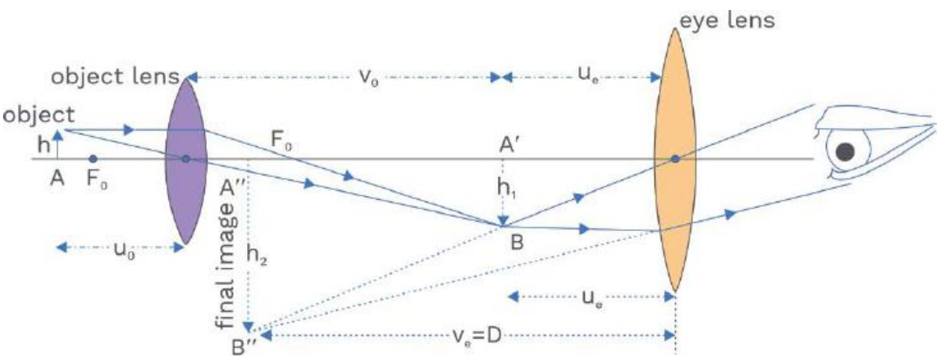
And u will be maximum when, v = maximum = ∞

i.e., $= \frac{\quad}{\left(\frac{\quad}{\infty} - \right)} = -$

So the nearest and farthest distance of the book from the magnifier (or eye) for clear viewing are 4.17 cm and 5 cm respectively.

Compound Microscope:-

Compound microscope is used to linked more magnified image as compared to a simple microscope. Object is positioned in front of the objective lens and the image is seen across the eye piece. The aperture of objective lens is less as comparison to eye piece because object is very near so collection of more light is not required. Normally object is placed between F – 2F due to this a inverted, real and magnified image is produced between 2F – ∞. It is recognized as intermediate image A'B'. The intermediate image behaves as an object for the eye piece. Now the separation between both the lens are adjusted in such a way that intermediate image drops between the optical centre of eye piece and its focus. In this condition, the final image is inverted, virtual and magnified.



Total magnifying power = Linear magnification of objective lens × angular magnification MP of eye lens = $= \frac{}{} \frac{S}{}$

(i) When final image is created at least distance of distinct vision.

$$3 (= \frac{}{}\left[+ \frac{S}{} \right] = \frac{}{+}\left[+ \frac{S}{} \right]$$

$$= \frac{-}{}\left[+ \frac{S}{} \right] = \frac{}{}\left(+ \frac{S}{} \right)$$

Length of the tube $L = v_0 + |u_e|$

(ii) When final image is created at infinity.

$$\frac{}{} - \frac{}{} = \frac{}{} \Rightarrow \frac{}{\infty} + \frac{}{} = \frac{}{} \Rightarrow \quad =$$

$$3 (= \frac{}{}\left[\frac{S}{} \right] = \frac{}{+}\left[\frac{S}{} \right] = \frac{-}{}\left[\frac{S}{} \right] = \frac{x}{} \frac{S}{}$$

Where L' is tube length (distance between second focus of objective and first focus of eye piece.Length of the tube $L = v_0 + f_e$

Sign convention for solving numerical

$u_0 = -v_e$, $v_0 = +ve$, $f_0 = +ve$,

$u_e = -ve$, $v_e = -ve$, $f_e = +ve$, $m_0 = -ve$, $m_e = +ve$, $M = -ve$

Ex. A compound microscope has a magnifying power '30'. The focal length of its eye-piece is '5' cm. If the final image to be at the least distance of distinct vision (25 cm), calculate the magnification produced by objective.

Sol. In case of compound microscope,

$MP = m \times m_0$ ...(i)

And in case of final image at minimum distance of distinct vision,

$$= \left[+ \frac{S}{} \right] \quad ...(ii)$$

So, from eq^n. (i) and (ii),

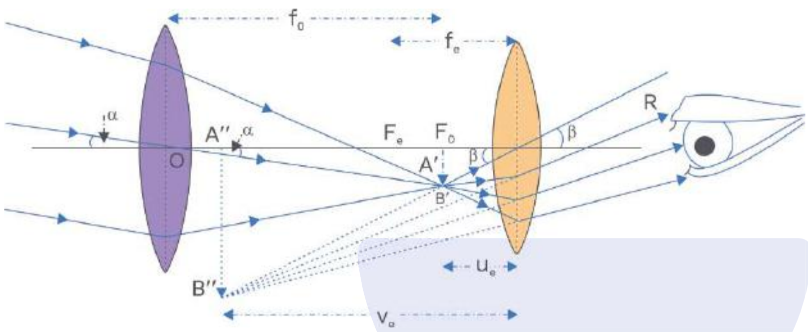
$$3 (= \left[+ \frac{S}{} \right]$$

Here, MP = –30; D = 25 cm and f_e = 5cm

So, – = [+ —] ⇒ = $\frac{–}{\quad}$ = –

Negative sign indicate that image formed by objective is inverted.

Astronomical Telescope:-



A telescope is used to see far-away objects. The objective forms the image A'B' at its focus. This image A'B' behaves as an object for eyepiece and it forms the final image A"B".

$$3\ (\;=\; \frac{\beta}{\alpha}$$

⇒ 3 (= —— = – —   A N =

(i) If the final image is formed at infinity then, $v_e = -\infty$, $u_e = -ve$

$\frac{\quad}{-\infty} - \frac{\quad}{-} = — ⇒ \quad =$ So 3 (= – — and length of the tube

$L = f_0 + f_e$

(ii) If the final image is at least distance of distinct vision then: $v_e = -D$, $u_e = -ve$

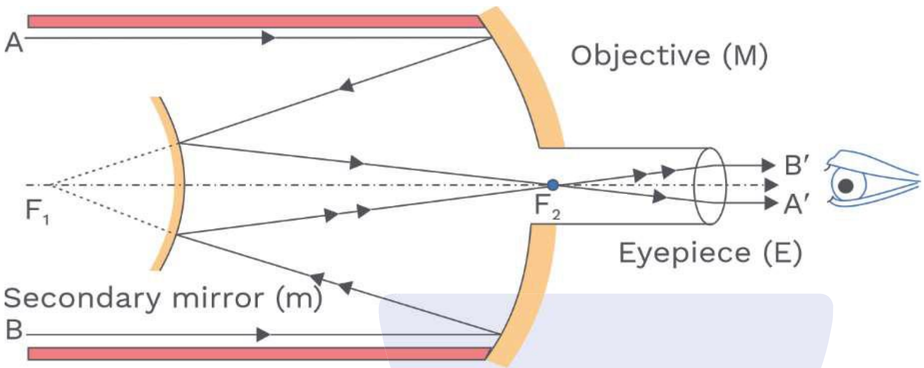
$\frac{\quad}{-S} - \frac{\quad}{-} = — ⇒ — = — + \frac{\quad}{S} = —\left[\; + \frac{\quad}{S}\right]$

So 3 ($= - — = - —\left[\; + \frac{\quad}{S}\right]$

Length of the tube is $L = f_0 + |u_e|$

Cassegrain's telescope:-

This telescope consists of a paraboloidal mirror 'M' as the objective, and a convex elliptical mirror 'm' called the secondary mirror. 'F_1' and 'F_2' are the two conjugate foci of the mirror 'm'.



It is easy to find that the angular magnification of the telescope, i.e.,

$$3 (= \frac{-}{\quad} \Rightarrow 3 = - -$$

Ex. A compound microscope contains of an objective of focal length 2.0 cm and an eye piece of focal length 6.25 cm, separated by a distance of 15 cm. How far should an object be placed from the objective in order to obtain the final image at (a) the least distance of distinct vision (25 cm).

Sol. Here, f_0 = 2.0 cm; f_e = 6.25 cm, u_0 = ?

(a) $= -$ $\because - - - = -$

$$\therefore - = - - - = \frac{\quad}{-} - - = \frac{- \ -}{\quad} = \frac{-}{\quad} \Rightarrow \quad = -$$

As distance between objective and eye piece = 15 cm; v_0 = 15 − 5 = 10 cm

$$\because - - - = - \therefore - = - - - = - - - = \frac{-}{\quad}$$

$$\Rightarrow \quad = \frac{-}{\quad} = -$$

$$\because \frac{}{} - \frac{}{} = \frac{}{} \Rightarrow \frac{}{} - \frac{}{} - \frac{}{} = \frac{}{} - \frac{}{} = \frac{-}{}$$

$$\Rightarrow \quad = \frac{-}{} = -$$

Ex. A small telescope has an objective of focal length '144' cm and an eyepiece of focal length '6' cm. What is the magnifying power of the telescope? What is the separation between the objective and the eyepiece? The final image is formed at infinity.

Sol. Here, f_0 = 144 cm; f_e = 6.0 cm, MP = ?, L = ?

3 ($= \frac{-}{} = \frac{-}{} = -$ and L = $f_0 + f_e$ = 144 + 6.0 = 150.0 cm.

Ex. Diameter of the moon is 3.5 × 10^3 km and its distance from earth is 3.8 × 10^5 km. It is seen by a telescope whose objective and eyepiece have focal lengths 4 m and 10 cm respectively. What will the angular diameter of the image of the moon?

Sol. 3 ($= -\frac{}{} = \frac{}{} = -$. Angle subtended by the moon at the objective $= \frac{\times}{\times} =$ radians.

Thus angular diameter of the image = MP × visual angle of moon = 40 × 0.009 = 0.36 radians $= \frac{\times}{} = \ ^{\circ}$

Ex. Magnifying power of Astronomical telescope is 5. If Focal length of eye piece is 4 cm. Find focal length of objective lens & length of tube when image is formed at infinity.

Sol. MP = 5

f_e = 4 cm

$$3 \ (= -\frac{}{} \Rightarrow - = -\frac{}{}$$

f_0 = 20 cm

L = $f_0 + f_e$ = 20 + 4 = 24 cm

Ex. An astronomical telescope involving of an objective of focal length 60 cm and eye-piece of focal length 3cm is focused on the moon so that the final image is formed at least distance of distinct vision, i.e., 25 cm from the eye-piece. If the angular diameter of Moon as (1/2)° at the objective, calculate (a) angular size and (b) linear size of image seen through the telescope.

Sol. As final image is at least distance of distinct vision,

$$3\,(\quad = \text{—} = \left[\; + \frac{}{S} \right] = \text{—} \left[\; + \text{—} \right] =$$

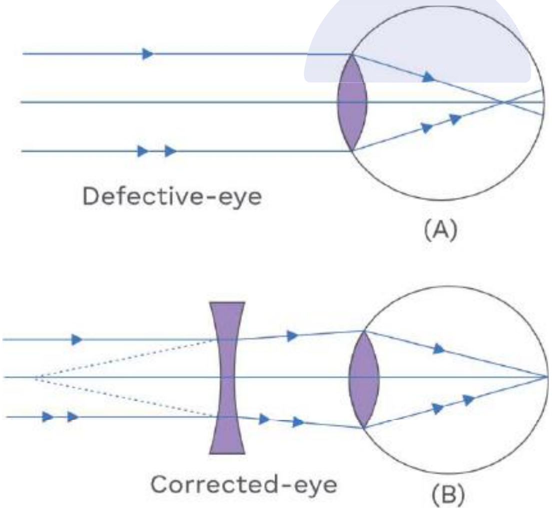
Now as by definition MP = (θ/θ_0), so the angular size of image

$$\theta = 3\,(\; \times \theta \quad = \quad \left[\text{—} \right]^{\circ}$$

And if I is the size of final image which is at least distance of distinct vision, θ = (I/25), i.e., I = 25 × θ = 25 × 0.2 = 5 cm

Defects Of Vision

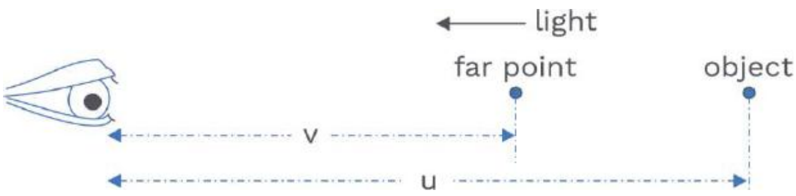
MYOPIA [or Short–sightedness or Near – sightedness]



(i) Distant objects are not obviously visible, but nearby objects are clearly noticeable because image is developed before the retina.

(ii) To resold the defect concave lens is used.

- The extreme distance which a person can see without the help of spectacles is recognized as far point of distinct vision.



- If the position of object is not given then it is taken as infinity.
- In this situation image of the object is produced at the far point of the person.

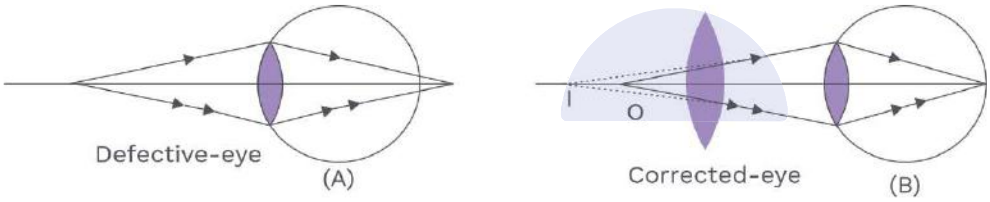
— — — = — = (⇒ ————————————

– ———————————— = – = (

———————————— – ———————————— = (

distance of far point = –ve, distance of object = –ve, P = –ve

Long-Sightedness Or Hypermetropia:-



(i) Nearby objects are not obviously visible.
(ii) The image of near objects is formed behind the retina.
(iii) To eliminate this defect convex lens is used.

Near Point:–

The minimum distance which a person can see without the help of spectacles.

- In this case the image of the object is formed at the near point.
- If reference of object is not given it is taken as 25 cm.

— — — = — = (⇒ ————————————

– ———————————— = — = (

distance of near point = –ve, distance of object = –ve, P = +ve

Presbyopia
In this case, both nearby and distant objects are not clearly visible. To remove this defect, two separate lenses one for myopia and other for hypermetropia are used or bifocal lenses are used.

Astigmatism
In this defect a person cannot see object in two orthogonal directions clearly. It can be removed by using cylindrical lens in a certain orientation.

Ex. A person cannot see clearly an object kept at a distance beyond of 100 cm. Find out the nature and the power of lens to be used for seeing clearly the object at infinity.

Sol. For lens u = – ∞ and u = – 100 cm

$\therefore$ — — — = — ⇒ — = — ⇒ f = v = –100 cm (concave)

$\therefore$ Power of lens (= — = — — = – S

Ex. A far sighted person has a near point 60 cm away. Find out the power of a lens he should use for eye glasses so that he can read a book at a distance of 25 cm?

Sol. Here v = – 60 cm, u = –25 cm

— = — — — = — — + —

⇒ = ——

$\therefore$ Power = ——— = ——— = + S

Ex. A person is able to see between 50 cm & 4 m clearly. Find power of lens required to view nearby & far objects clearly.

Sol. (i) u = –25 cm
v = – NP = –50 cm

— = — —— + ——

f = 50 cm

P = 2D

(ii) u = $-\infty$

v = −F.P. = −4 m

— = — — —

f = −4 m

(= — —S = — S

Ex. A far sighted person can see object beyond 94 cm clearly if separation between glasses and eye lens is 2 cm, then find focal length of glass?

Sol. — = — — — = — + ——

— = — + = —

Some Natural Phenomenon Due to Sunlight:-

- **Rainbow:-**

After a light sprinkle, an observer with the sun facing his back, realizes a number of concentric coloured arcs looming in the sky, with the general centre of these arcs resting on the line joining the sun and the observer. These arcs represent the primary rainbow.

The inner frame of the primary rainbow is violet and the outer edge is red. Likewise the primary rainbow, a bigger but a fainter rainbow is also seen. This is known as the secondary rainbow. The colours sequence in the secondary rainbow is the reverse of that in the primary rainbow, i.e., the outer edge is violet the inner edge is red. Both these rainbows are produced by :

(i) Dispersion and

(ii) Internal reflection (TIR) of the Sun's rays in the rain drops floating in the atmosphere.

Rack your Brain

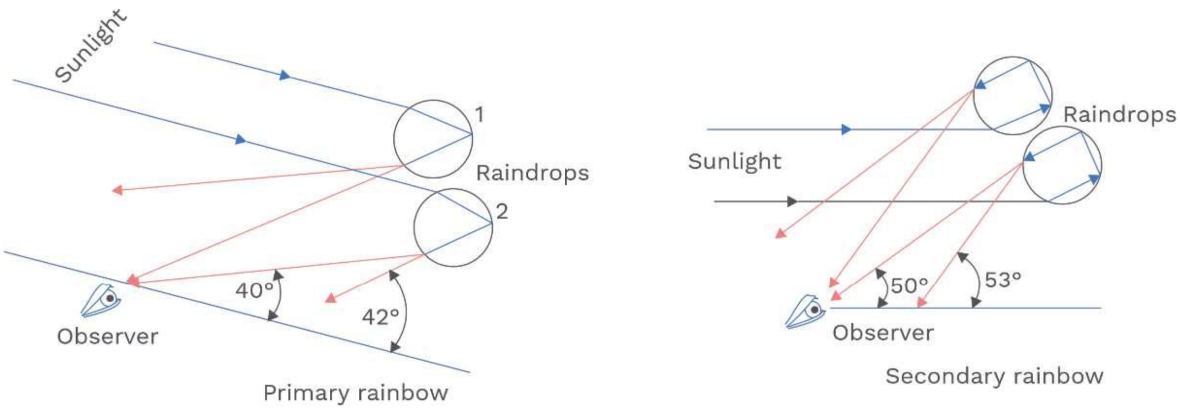
Pick the wrong answer in the context with rainbow.

(1) The order of colours is reversed in the secondary rainbow

(2) An observer can see a rainbow when his front is towards the sun

(3) Rainbow is a combined effect of dispersion refraction and reflection of sunlight

(4) When the light rays undergo two internal reflections in a water drop, a secondary rainbow is formed



- **Scattering of light:-**

  The deflection of light energy by fine fragments of solid, liquid or gaseous matter from the main direction of the beam is known as the scattering of light.

  The basic method involved in scattering is the absorption of light by the molecules followed by its re-radiation in different directions. The intensity of the scattered light varies on:

  (i) the wavelength (λ) of light

  (ii) the size of the particles causing scattering.

  Varying upon the size of the scatterers, the following two situations arise:

(a) If the scattering fragments (air molecules) are of size smaller than the wavelength of light, the intensity of the scattered light (I) fluctautes inversely as the fourth power of the wavelength of light,

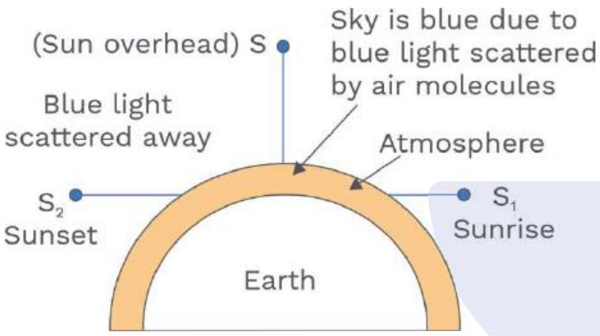
This declaration, which holds for elastic scattering, is known as the Rayleigh's law of scattering. While the wavelength of blue light is less than that of red light, blue light is scattered extremely while the red light least. Blue colour of sky, reddish look of Sun during sunrise and sunset are due to this incident as discussed below.

Further, it is due to this cause that red signals are managed to indicate danger. Such signals go to great distances without an significant loss due to scattering.

(b) If the scattering fragments are of sizes greater than the wavelength of light (e.g., dust fragments, water droplets), Rayleigh's law of scattering is not valid and all colours are scattered equally. It is due to this cause that clouds generally appear white.

Blue colour of the sky

If an observer (O) looks at the sky when the Sun is overhead at noon as shown by position S in figure it is the scattered light that is received by the observer. Since blue light is scattered more (almost ten times) than red, the sky appears blue to the observer.



- **Reddish appearance of the Sun during sunrise and sunset**

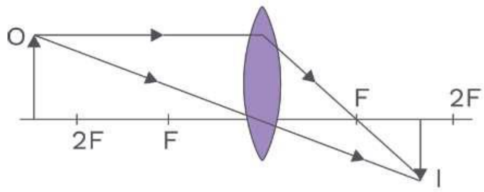
During sunrise (S_1) and sunset (S_2), the light coming from the sun has to travel a larger distance through the atmosphere (than it does at noon) before entering the observers eye. As a result of this, most of the blue light is scattered on its way to the observer. The transmitted light (sunlight minus the scattered light), which reaches the observer, is rich in red and orange colour and makes the Sun appear reddish orange.

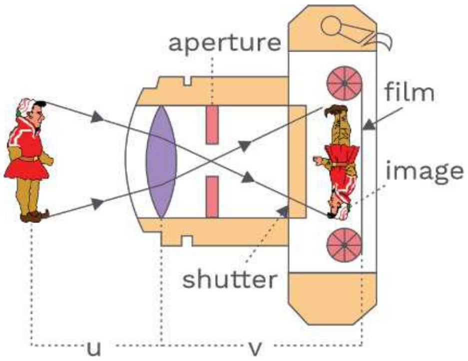
Camera

A camera has a convex lens whose aperture and distance from the film screen can be adjusted. Object is real and placed between ∞ and 2F, so the image is real, inverted, diminished and between F and 2F.

Concept Reminder

The blue of the sky, white clouds, the red-hue at sunrise and sunset, the rainbow, the brilliant colours of some pearls, shells, and wings of birds, are just a few of the natural wonders we are used to.





If I is the intensity of light, S is the light transmitting area of lens and t is the exposure time, then for proper exposure, I × S × t = constant
light transmitting area of a lens is proportional to the square of its aperture D ; I × D^2 × t = constant
If aperture is kept fixed, for proper exposure, I × t = constant, i.e., $I_1 t_1 = I_2 t_2$
If intensity is kept fixed, for proper exposure, D^2 × t = constant

Time of exposure ∝ ———— ... (i)

The ratio of focal length to aperture of lens is called the f–number of the camera,

– = ————

⇒ A ∝ ———— ... (ii)

From equation (i) and (ii)
⇒ Time of exposure ∝ (f–number)^2

Ex. With the diagraphm of a camera lens set at —, the exposure time is —— , then calculate the correct exposure time with diaphragm set at — .

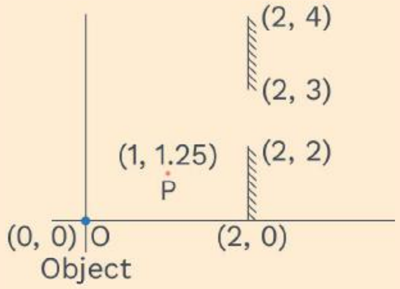
Sol. As exposure time ∝ ———— ⇒ ∝ ——— and ∝ ———
[ ] [ ]

here = —— then — = — =

⇒ = = ——

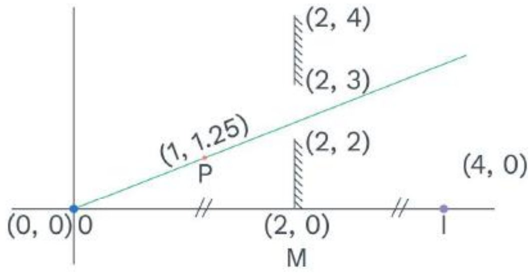
EXAMPLES

Q1 **Two plane mirrors are placed as shown in the diagram and a point object 'O' is placed at the origin**



(a) How many images will be formed.
(b) Find out the position(s) of image(s).
(c) Will the incident ray passing across a point 'P' (1, 1.25) take part in image formation.

Sol.



(a) While both mirrors are in same plane images produced by both mirror concide

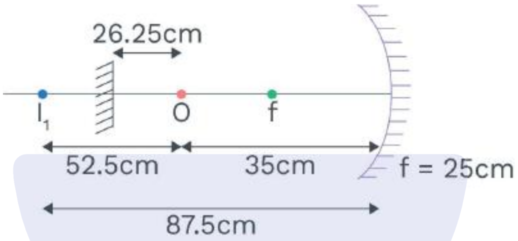
(b) MI = MO = 2 ∴ m≡

(c) Since ray passing across P is not falling on mirror.

Q2

A source is at a distance '35' cm on the optical axis from a spherical concave mirror having a focal length '25' cm. At what distance determined along the optical axis from the concave mirror should a plane mirror (perpendicular to principal axis) be placed for the image it forms (due to rays falling on it after reflection from the concave mirror) to coincide with the point source?

Sol.



— = — + — ⇒ —— = — + ——

= —— = —

$OI_1 = 87.5 - 35 = 52.5$ cm.

Distance between mirrors = 35 + 26.25 = 61.25 cm

Q3

A object is placed on the principal axis at '60' cm in front of a concave mirror of focal length '40' cm on the principal axis. If the object is travelled with a velocity of 10 cm/s (a) along the principal axis, find out the velocity of image (b) perpendicular to the principal axis, find the velocity of image at that moment.

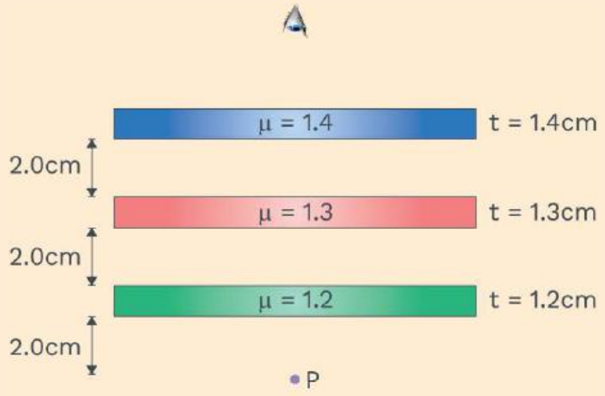
Sol. (a) — = — + — ⇒ —— = — + —— ⇒ — = —— − —— = ——

⇒ = −

—— = − —— —— = − (——) × = −

(b) $\frac{}{} = \frac{-}{} \Rightarrow \frac{}{} = -\left(\frac{}{-}\right)\left(\frac{}{}\right) = -\left(\frac{-}{-}\right) \times \qquad = -$

Q4 **Locate the image of the point P as seen by the eye in the figure.**

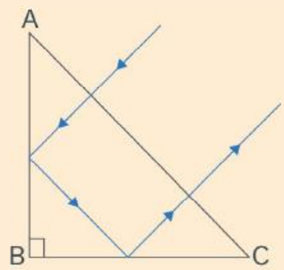


Sol. Apparent. Shift $= \left(- \frac{}{}\right) + \left(- -\right) + \left(- \frac{}{}\right)$

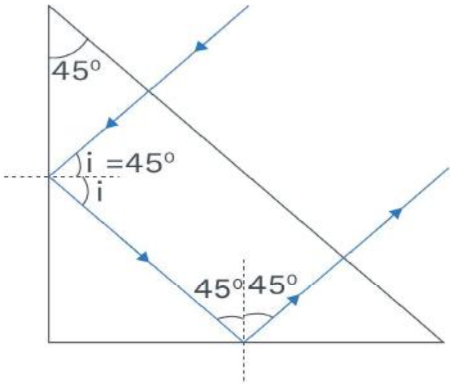
$+ \left(- -\right) + \left(- \frac{}{}\right) + \left(- -\right)$

= 0.4 + 0.3 + 0.2 = 0.9 cm towards the eye.
So image is established 0.9 cm above P.

Q5 **At what values of the refractive index of a rectangular prism can a ray travel as shown in diagram. The section of the prism is an isosceles triangle and the light ray is normally incident onto the side AC.**



Sol.



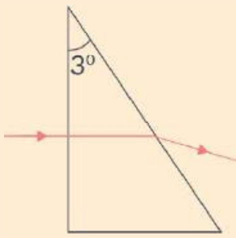
$$> \quad - \left(\frac{}{\mu}\right)$$

$$\Rightarrow \quad \frac{\pi}{} > \frac{}{\mu}$$

$$\Rightarrow \quad \mu > \frac{}{\pi}$$

$$\Rightarrow \quad \mu > \sqrt{\ }$$

Q6 **Find out the angle of deviation suffered by the light ray shown in figure for following two conditions. The refractive index for the prism material is** $\mu = 3 \quad 2$



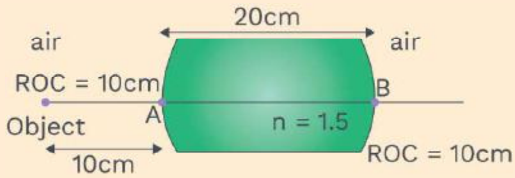
(i) When the prism is set in air $\mu = 1$

(ii) When the prism is set in water $\mu = 4 \quad 3$

Sol. (i) $\delta = A\,\mu - \quad = \quad {}^{\circ}\left(- - \right) = \quad {}^{\circ}$

(ii) $\delta = A\,\mu - \quad = \quad {}^{\circ}\left(\text{———} - \right) = {}^{\circ}\left(- - \right) = - \quad {}^{\circ}$

Q7 **A point object is placed 10 cm away from a glass piece (n = 1.5) of length 20 cm bounded by spherical surfaces of radius of curvature 10 cm. Find out the position of final image formed after two refractions at the spherical surfaces.**



Sol. For first refraction:

$$\frac{}{} - \frac{}{-} = \frac{-}{} \Rightarrow \quad = -$$

For second refractions

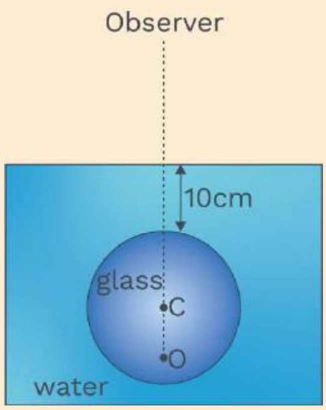
u = – (30 + 20) = – 50 cm

$$\therefore \quad \frac{}{} - \frac{}{-} = \frac{-}{-}$$

$$\Rightarrow \quad =$$

Therefore, final image is formed 50 cm right of B.

Q8 **There is a small air bubble inside a glass sphere $\mu = 1\;5$ of radii 5 cm. The bubble is at 'O' at 7.5 cm lower than the surface of the glass. The sphere is placed inside water $\left(\mu = \frac{4}{3}\right)$ such that the top surface of glass is 10 cm below the surface of water. The bubble is viewed normally from air. Find the apparent depth of the bubble.**



Sol. For first refraction, (at the glass-water interface)

$$\frac{}{} - \frac{}{-} = \frac{-}{-}$$

$$\frac{}{} + \frac{}{} = \frac{}{}$$

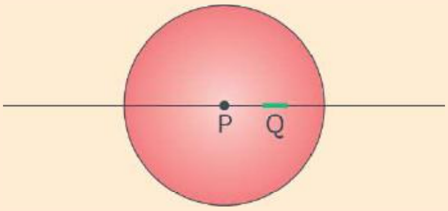
$$\frac{}{} = \frac{-}{} \Rightarrow \quad = -$$

For second refraction: (at air-water interface)

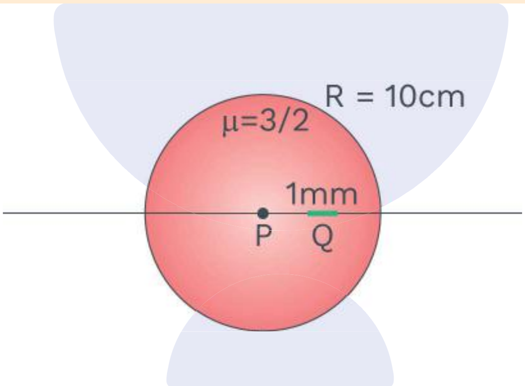
Apparent depth $= \frac{+}{}$

$$= \frac{\times}{} = \frac{}{} = \frac{}{}$$

Q9 **A small object 'Q' of length '1' mm lies along the principal axis of a spherical glass of radii R = 10 cm and refractive index is 3/2. The object is noticed from air beside the principal axis from left. The distance of small object from the centre P is '5' cm. Find out the size of the image. Is it real, inverted?**



Sol.



— — — = —

— — — = —

— + — = — ⇒ = —

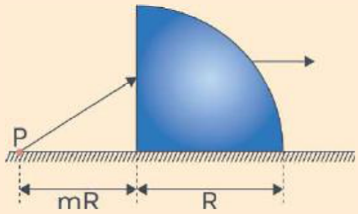
— — + — =

— = — ×

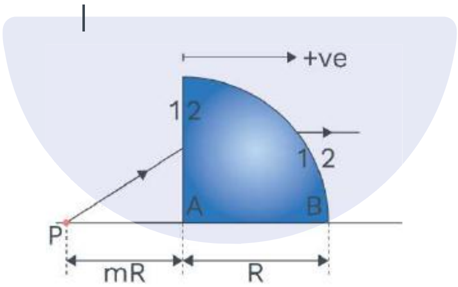
= — = —

+ve dv ⇒ no inversion

Q10 **A quarter cylinder of radii R and refractive index 1.5 is placed on a table. A point object P is stayed at a distance of mR from it. Find out the value of m for which a ray from P will emerge parallel to the table as shown in the figure.**



Sol. Applying $\frac{\mu}{} - \frac{\mu}{} = \frac{\mu - \mu}{}$



First on plane surface

$$\frac{}{} - \frac{}{- } = \frac{-}{\infty} \qquad = \infty$$

$$\therefore \qquad = -$$

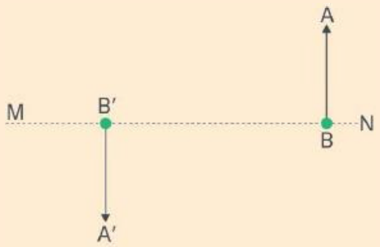
Then on curved surface

$\frac{}{\infty} - \frac{}{- \quad + } = \frac{-}{-}$ [$= \infty$ because final image is at infinity]

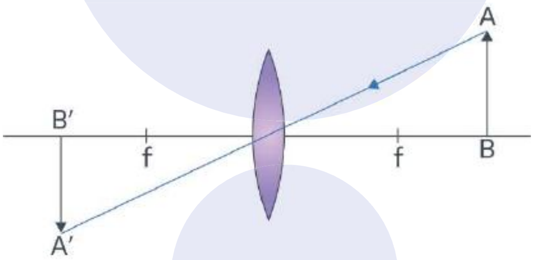
$\Rightarrow \frac{}{+} = \frac{}{} \Rightarrow \quad = \quad +$

$\Rightarrow \frac{}{} = \quad$ or $\quad =$

Q11 **Given an optical axis MN and the positions of a real object AB and its image A′B′, determine diagrammatically the position of the lens (its optical centre O) and its foci. Is it a converging or diverging lens? Is the image real or virtual?**



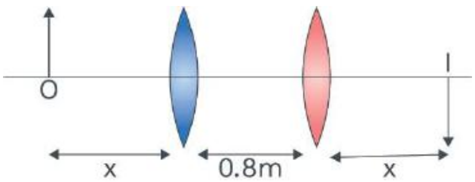
Sol.



It is a convergent lens. So, image is real.

Q12 **A lens placed between a candle and a stable screen forms a real triply magnified image of the candle on the screen. When the lens is shifted away from the candle by 0.8 m without changing the position of the candle, a real image one-third the size of the candle is formed on the screen. Determine the focal length of the lens.**

Sol.



for first position

= — = + / — = −

⇒ + = ⇒ =

− = — − − ⇒ − = — − — / −

⇒ − = — + — = + / — ⇒ =

Q13 **A 2.5 dioptre lens forms a virtual image which is 4 times the object set perpendicularly on the principal axis of the lens. Find out the required distance of the object from the lens.**

Sol. = — = — = = — = ⇒ =
(

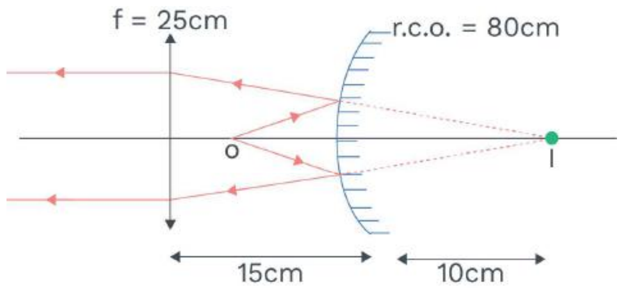
Using lens formula

— = — − − = — — = — ⇒ = −

So, required distance = 30 cm.

Q14 **A convex lens and a convex mirror are placed at a separation of '15' cm. The focal length of the lens is '25' cm and radius of curvature of the mirror is 80 cm. Where should a point source be positioned between the lens and the mirror so that the light, after getting reflected by the mirror and then getting refracted by the lens, comes out parallel to the principal axis?**

Sol.

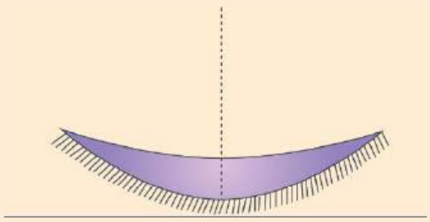


For final light ray to be parallel to the axis of the lens, the image formed by the mirror should be at the focus of the lens.

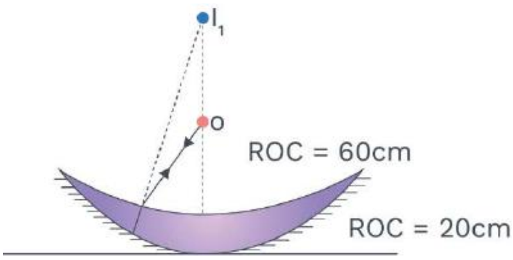
By mirror formula — + — = —   v = + 10 cm, f = 40 cm

on solving   = – —— cm so distance of object from lens =   – —— = — cm

Q15 **The convex surface of a thin concavo-convex lens of glass of refractive index '1.5' has a radius of curvature '20' cm. the concave surface has a radius of curvature '60' cm. The convex side is silvered and set on a horizontal surface as shown in diagram. (a) Where should a pin be put on the axis so that its image is developed at the same place? (b) If the concave side is filled with water $\mu = 4 \quad 3$, find the distance through which the pin should be moved so that the image of the pin again coincides with the pin.**



Sol. (a)

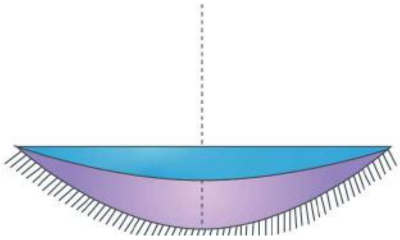


First image should form at centre of curvature of mirror after refraction by concave surface.

—— – — – = ——   — = —— – —— = ——   ⇒   = – —— = –

(b) After refraction by water lens image should form at 15 cm (Answer of part a)

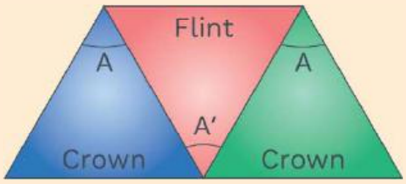
—— = —— – – = (– –)(— – ——)   ∞



$\Rightarrow$ u = – 13.86 cm

$\Rightarrow$ So, distance through which the pin should be moved = (15 – 13.86) = 1.14 cm towards lens.

Q16 **Three thin prisms are combined as shown in diagram. The refractive index of the crown glass for red, yellow and violet rays are μ μ and μ respectively and those for the flint glass are μ' μ' and μ' respectively. Find the ratio r ′ r for which (a) system produces deviation without dispersion(achromatic combination) and (b) system produces dispersion without deviation(direct vision arrangement).**



Sol. (a) For no net angular dispersion,

$$A(\mu - \mu) - A'(\mu' - \mu') =$$

$$\Rightarrow \frac{A'}{A} = \frac{(\mu - \mu)}{(\mu' - \mu')}$$

(b) For no net deviation

$$A\ (\mu_O - \ ) - A\ (\mu - \ ) =$$

$$A(\mu - \ ) - A'(\mu' - \ ) =$$

$$\frac{A'}{A} = \frac{(\mu - \ )}{(\mu' - \ )}$$

Q17 **A certain material has refractive indices 1.53, 1.60 and 1.68 for red, yellow and violet light respectively. (a) Find the dispersive power. (b) Find out the angular dispersion produced by a thin prism of angle 6° made of this material.**

Sol. (a) $\omega = \frac{\mu \ \ - \mu_r}{\mu \ \ -} = \frac{-}{-} = \frac{}{} = \frac{}{}$

(b) $\theta = A\left(\mu \ \ - \mu_r\right) = \ ^{\circ} \ \ - \ \ = \ \ ^{\circ}$

Q18 **A point object is to be found at a distance of 15 cm from a convex lens. The image is produced on the other side at a distance of 30 cm from the lens. When a concave lens is put in contact with the convex lens, the image shifts away further by 30 cm. Calculate the focal lengths of the two lenses.**

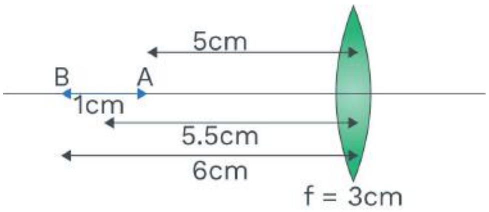
Sol. $\frac{}{} = \frac{}{} - \frac{}{-} = \frac{+}{} = \frac{}{} \Rightarrow \ \ =$

$\frac{}{} = \frac{}{} + \frac{}{} = \frac{}{} - \frac{}{-} = \frac{+}{}$

$\Rightarrow \frac{}{} + \frac{}{} = \frac{}{} \qquad \frac{}{} = \frac{}{} - \frac{}{} = \frac{-}{} \Rightarrow \ \ = -$

Q19 **A pin of length 1 cm lies along the principal axis of a converging lens, the centre being at a distance of 5.5 cm from the lens. The focal length of the lens is '3' cm. Find the size of the image.**

Sol.



For A

$$- = \frac{}{_A} - \frac{}{-} \Rightarrow \frac{}{_A} = - - - = - \Rightarrow {}_A =$$

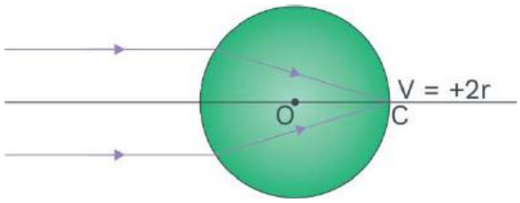
For B

$$- = \frac{}{_N} - \frac{}{-} \Rightarrow \frac{}{_N} = - - - = - \Rightarrow {}_N =$$

size of image $= {}_A - {}_N = 1.5$ cm.

Q20 **A narrow parallel beam of light is incident paraxially on a solid transparent sphere of radius r kept in air. Find out should be the refractive index if the beam is to be focused (a) at the farther surface of the sphere, (b) at the centre of the sphere.**

Sol. (a)



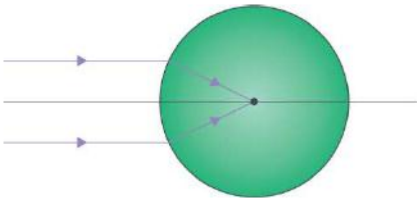
$$\frac{\mu}{} - \frac{\mu}{} = \frac{\mu - \mu}{|} \quad \text{or} \quad \frac{\mu}{} - \frac{}{-\infty} = \frac{\mu -}{}$$

$$\frac{\mu}{} = \frac{\mu -}{}$$

$\Rightarrow \mu = \mu -$

$\Rightarrow \mu =$

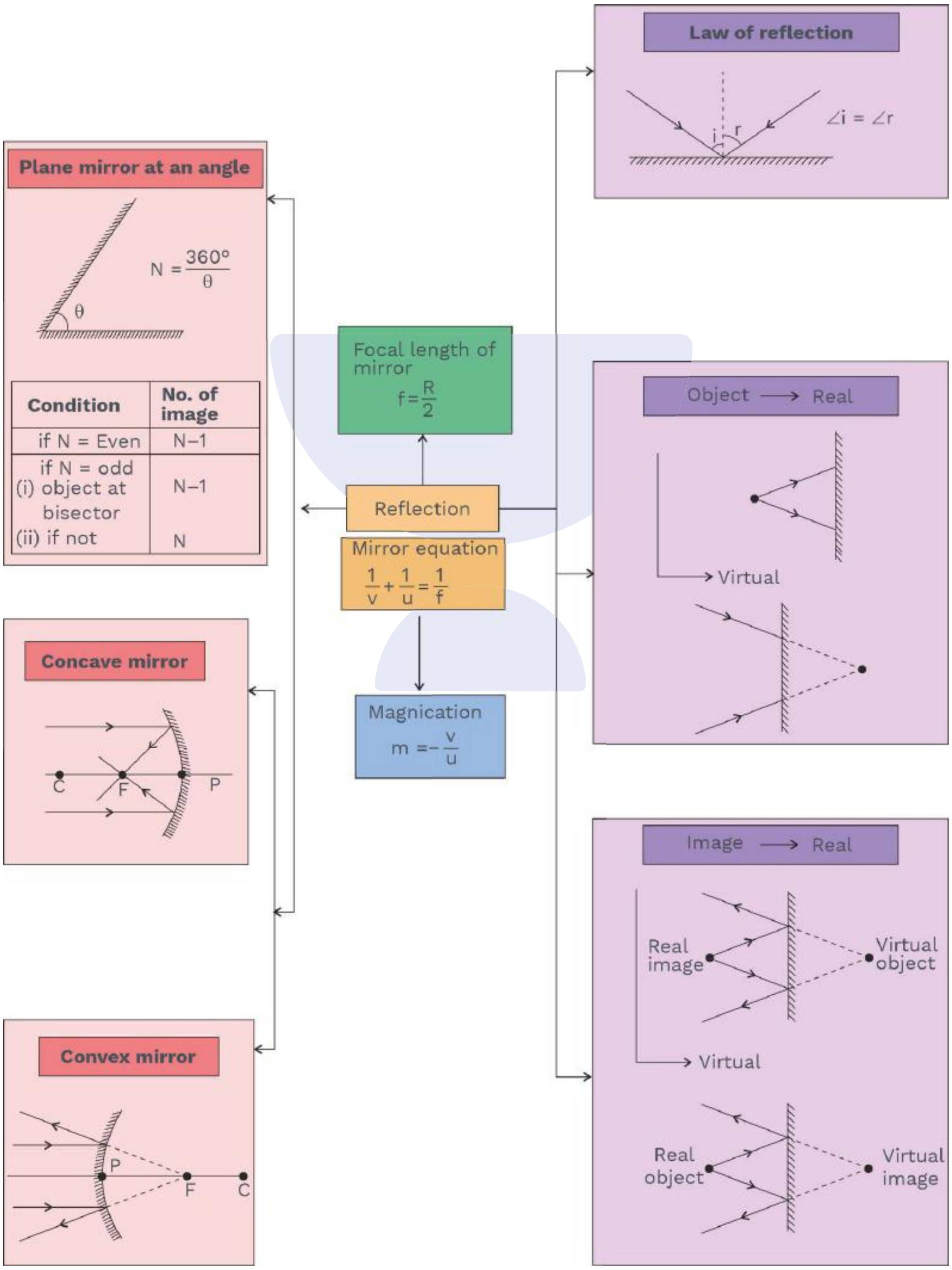
(b)



$\frac{\mu}{} - \frac{}{-\infty} = \frac{\mu -}{} \Rightarrow \frac{\mu}{} = \frac{\mu -}{} \mu = \mu -$

$\Rightarrow$ 0 = – 1 not possible

MIND MAP


Law of reflection
i
r
∠i = ∠r
Plane mirror at an angle
N = 360°/θ
θ
Condition
No. of image
if N = Even
N–1
if N = odd
(i) object at bisector
N–1
(ii) if not
N
Focal length of mirror
f = R/2
Reflection
Mirror equation
1/v + 1/u = 1/f
Magnication
m = –v/u
Object ⟶ Real
Virtual
Concave mirror
C
F
P
Image ⟶ Real
Real image
Virtual object
Virtual
Real object
Virtual image
Convex mirror
P
F
C