Projectile Motion

DISCLAIMER

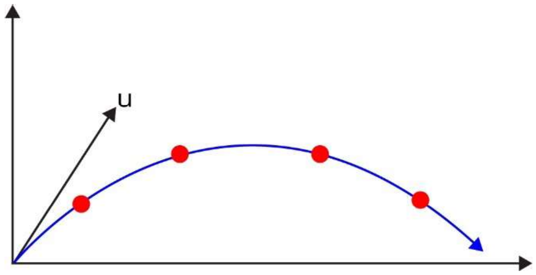
Projectile Motion

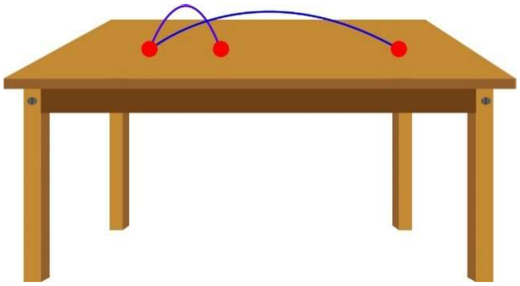
When a body is projected in effect of gravity in such a manner ‘that its path is parabolic, is called projectile motion. Projectile motion is an example of 2-D motion.

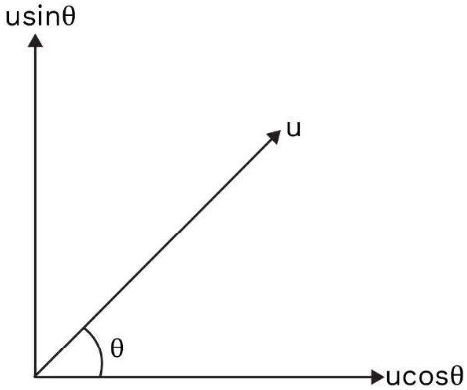
There are two types of projectile motion:-

1. Oblique or angular projectile motion.
2. Horizontal projectile motion.

- **OBLIQUE OR ANGULAR PROJECTILE MOTION:-**
When a body is thrown at an angle from horizontal then motion under gravity is known as oblique projectile motion







Definitions

When a body is projected in effect of gravity in such a manner, that its path is parabolic, is called projectile motion.

Definitions

When a body is thrown at an angle from horizontal then motion under gravity is known as oblique projectile motion

Concept Reminder

The shape of the trajectory of motion of an object is not determined by acceleration alone but also on initial conditions (like initial velocity).

For example, the trajectory of motion under gravity can be straight line or parabolic.

For x-axis	For y-axis
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$\vec{a}_x = 0$	$\vec{a}_y = -\vec{g}$
$\vec{F}_x = 0$	$\vec{F}_y = -m\vec{g}$
$\vec{v}_x = \vec{u}_x + \vec{a}_x t$	$\vec{v}_y = \vec{u}_y + \vec{a}_y t$
$\vec{S}_x = \vec{u}_x t + \frac{1}{2}\vec{a}_x t^2$	$\vec{S}_y = \vec{u}_y t + \frac{1}{2}\vec{a}_y t^2$
$\vec{v}_x^2 = \vec{u}_x^2 + 2\vec{a}_x S_x$	$\vec{v}_y^2 = \vec{u}_y^2 + 2\vec{a}_y S_y$

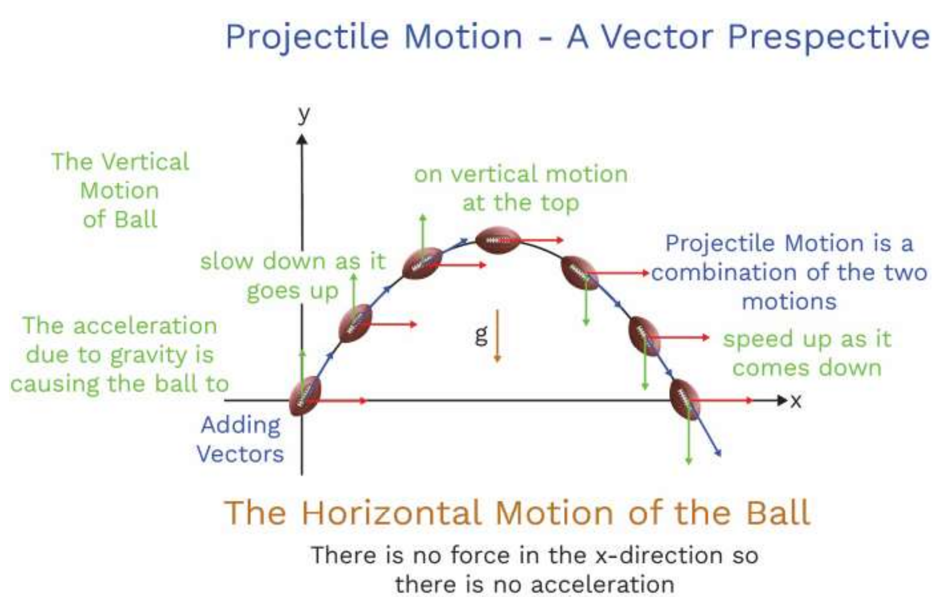
Keywords

- Oblique or angular projectile motion
- Horizontal projectile motion

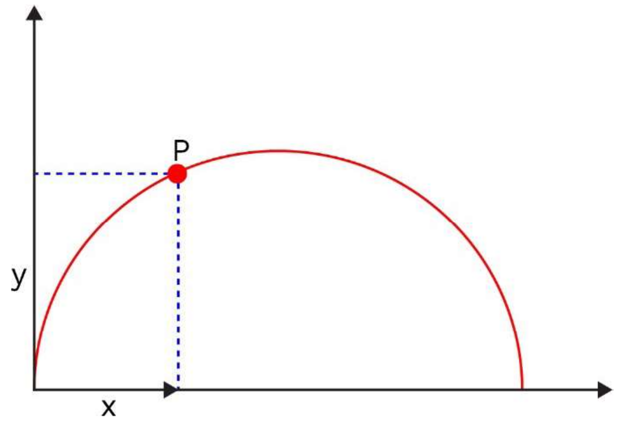
Concept Reminder

A projectile is any body which once thrown or projected into space with some initial velocity moves thereafter under the influence of gravity and air resistance alone.

Projectile Motion



- **EQUATION OF PATH:-**



At time t particle is at point P.

For x-axis:

$$x = u_x t + \frac{1}{2} a_x t^2 \qquad \left[\because a_x = 0\right]$$

$$x = u\cos\theta t \Rightarrow t = \frac{x}{u\cos\theta} \qquad \text{...(i)}$$

For y-axis:

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$y = u\sin\theta t - \frac{1}{2} g t^2 \qquad \text{...(ii)}$$

From equation (i) and (ii)

$$y = u\sin\theta \times \frac{x}{u\cos\theta} - \frac{1}{2} g \left(\frac{x}{u\cos\theta}\right)^2$$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$\Rightarrow$ Equation of trajectory.

Let $\tan\theta = A$ and $\frac{g}{2u^2\cos^2\theta} = B$

So, $y = Ax - Bx^2$

Concept Reminder:

Equation of Path:

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$y = x\tan\theta\left[1 - \frac{x}{R}\right]$$

KEY POINTS

*Equation of trajectory
* Point of projectile

Rack your Brain

The equation of projectile is $y = 16x - \frac{5x^2}{4}$.

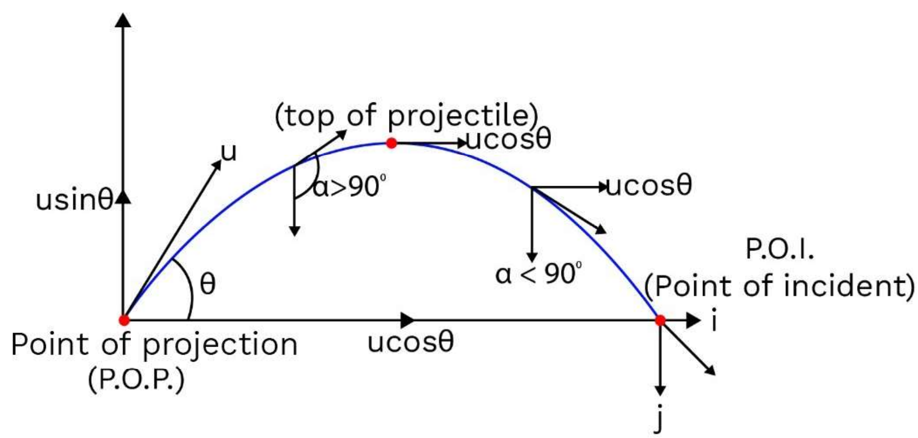
Find horizontal range ?

This is an equation of parabola so path of projectile is parabolic.

$$\because \quad y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$y = x\tan\theta\left[1 - \frac{x}{R}\right]$$

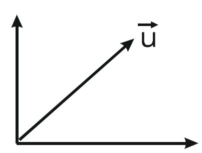
Where (R = horizontal range)



Concept Reminder

Vertical component of velocity is zero when particle moves horizontally i.e., at the highest point of trajectory.

(i) At P.O.P (Point of Projection)



$$\vec{u}_{P.O.P.} = u\cos\theta\hat{i} + u\sin\theta\hat{j}$$

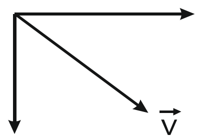
(ii) At T.O.P. (Top of Projectile)

$$\vec{v}_{T.O.P.} = u\cos\theta\hat{i} + 0\hat{j}$$

Rack your Brain

Two seconds after projection a projectile is travelling in a direction inclined at 30° to the horizontal after one more sec, it is travelling horizontally, what are the magnitude and direction of its velocity ?

(iii) At P.O.I. (Point of Impact)



$$\vec{v}_{P.O.I.} = u\cos\theta\hat{i} - u\sin\theta\hat{j}$$

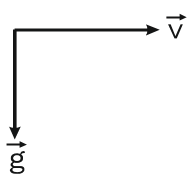
(iv) At any point for y-axis

$$\vec{v}_y = \vec{u}_y + \vec{a}_y t$$

Throughout the motion $F_x = 0, a_x = 0$,

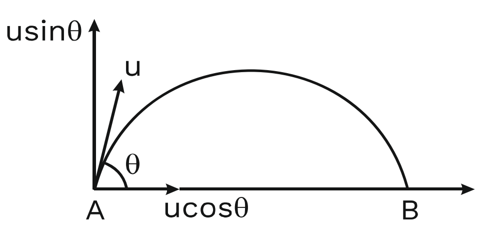
so horizontal component of velocity $(u\cos\theta = \text{constant})$

At highest point acceleration and velocity are mutually perpendicular to reach other.



Time of Flight:

The time taken by projectile between point of projection and point of impact is known as time of flight.



When projectile reach at point of impact then its height becomes zero.

Concept Reminder:

At highest point of projectile
$a \perp v$

Rack your Brain

The velocity of a projectile at point of projection is $2i + 3\hat{j}$. Find its velocity of point of impact.

Definitions

The time taken by projectile between point of projection and point of impact is known as time of flight.

If time taken is T during motion between points A to B

So, $\vec{S}_y = \vec{u}_y T + \frac{1}{2}\vec{a}_y T^2$

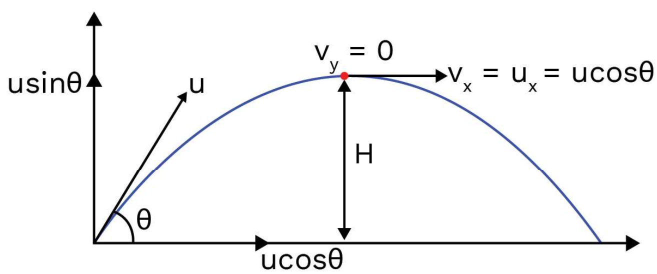
$$0 = \vec{u}_y T - \frac{\vec{g}}{2} T^2$$

$$T = \frac{2u_y}{g}$$

$$T = \frac{2u\sin\theta}{g}$$

Maximum Height :-

During projectile motion maximum height attained by object on vertical axis is known as maximum height.



At maximum height

$$v_y = 0$$

so, $v_y^2 = u_y^2 + 2a_y S_y$

$$0 = u_y^2 - 2gH$$

$$H = \frac{u_y^2}{2g}$$

$$H = \frac{u^2 \sin^2\theta}{2g}$$

at $\theta = 90°$

H becomes maximum, so greatest height

Definitions

During projectile motion maximum height attained by object on vertical axis is known as maximum height.

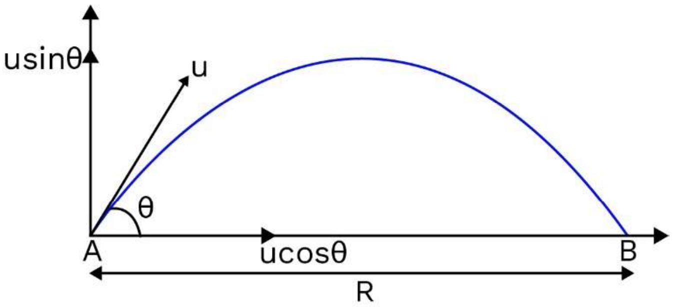
Rack your Brain

For a projectile, what is the ratio of maximum height reached to the square of flight time?

$$H_{max} = \frac{u^2 \sin^2 90°}{2g}$$

$$H_{max} = \frac{u^2}{2g}$$

Horizontal Range:- During projectile motion, distance travelled on x-axis is known as horizontal range.



Time taken to reach at point from point A is T (time of flight).

For x-axis

$$S_x = u_x T + \frac{1}{2} a_x T^2$$

$$R = u_x T + 0$$

where $a_x = 0$

$$R = u_x T$$

$$R = u_x \cdot \frac{2u_y}{g}$$

$$R = \frac{2u_x u_y}{g}$$

$$R = \frac{2u\cos\theta \cdot u\sin\theta}{g} = \frac{u^2 \cdot 2\sin\theta\cos\theta}{g}$$

$$R = \frac{u^2 \sin 2\theta}{g}$$

Concept Reminder

For oblique projectile

* $T = \frac{2u_y}{g}$
* $H = \frac{u_y^2}{2g}$
* $R = U \times T = \frac{2u \times u_y}{g}$

Keywords

- Time of flight
- Maximum height
- Horizontal range

Rack your Brain

The speed of a projectile at its maximum height is half of its initial speed. Find the angle of projection.

$$R = \frac{u^2 \sin 2\theta}{g}$$

at θ = 45° range becomes maximum so,

$$R_{max} = \frac{u^2 \sin 2 \times 45^\circ}{g} = \frac{u^2}{g} \Rightarrow \boxed{R_{max} = \frac{u^2}{g}}$$

Value of height when range is maximum,

$$h = \frac{u^2 \sin^2 45^\circ}{2g} = \frac{u^2}{4g}$$

Relation between R and H:

$$R = \frac{u^2 \sin 2\theta}{g} \qquad \text{...(i)}$$

$$H = \frac{u^2 \sin^2 \theta}{2g} \qquad \text{...(ii)}$$

From (i) and (ii)

$$\frac{R}{H} = \frac{\frac{u^2 \sin 2\theta}{g}}{\frac{u^2 \sin^2 \theta}{2g}}$$

$$\frac{R}{H} = \frac{4}{\tan\theta}$$

$$R \tan\theta = 4H$$

Concept Reminder

It was Gallileo who first stated the principle of physical independence of the horizontal and vertical motion of a projectile in his book “Dialogue on the great world system” in 1632

Concept Reminder:

Relation between R and H:

$R \tan\theta = 4H$

Q1 **At what angle horizontal range becomes n times of maximum height.**

A1 According to question

$$R = nH$$

$$\frac{u^2 \sin 2\theta}{g} = \frac{n \times u^2 \sin^2 \theta}{2g}$$

$$\tan\theta = \frac{4}{n}$$

$$\theta = \tan^{-1}\left(\frac{4}{n}\right) \text{ (Angle of projection)}$$

if R = 4H

Then angle of projection is $\theta = \tan^{-1}\left(\frac{4}{n}\right) = 1$

$$\tan\theta = 1$$

$$\theta = 45°$$

If R = 2H

$$\tan\theta = \frac{4}{2}$$

$$\theta = \tan^{-1}(2)$$

$$\cos\theta = \frac{2}{\sqrt{5}},\ \sin\theta = \frac{1}{\sqrt{5}}$$

Q2 **A body of mass m is thrown at angle θ with horizontal from ground. The equation of trajectory is $y = \sqrt{3}x - \frac{gx^2}{2}$. Find-**

1. Angle of projection
2. Initial velocity
3. Time of flight
4. Maximum height
5. Range
6. Horizontal and vertical component of initial velocity.

A2 Equation of trajectory

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$y = \sqrt{3}x - \frac{gx^2}{2}$$

1. $\tan\theta = \sqrt{3}$

$$\theta = 60^\circ$$

2. $u^2\cos^2\theta = 1$

$$u^2 \times \cos^2 60^\circ = 1$$

$$u^2 \times \frac{1}{4} = 1$$

$$u = 2\,m/s$$

3. Time of flight $T = \frac{2u\sin\theta}{g}$

$$T = \frac{2\times 2\times \sin 60^\circ}{10} = 0.2\sqrt{3}\ \text{sec}.$$

4. $H = \frac{u^2\sin^2\theta}{2g} = \frac{4\times 3}{4\times 2\times 10} = 0.15\ \text{m}$

5. $y = \sqrt{3}x - \frac{gx^2}{2} = \sqrt{3}\left[1 - \frac{\frac{x}{2\sqrt{3}}}{g}\right]$

$$R = \frac{2\sqrt{3}}{g} = 0.2\sqrt{3}\,\text{m}$$

6. Horizontal component of initial velocity

$$u_x = u\cos\theta = 2\times\frac{1}{2} = 1\ m/s$$

Vertical Component of initial velocity

$$u_y = u\sin\theta = 2\times\frac{\sqrt{3}}{2} = \sqrt{3}\ m/s$$

Q3 **During a projectile motion initial velocity is $6\hat{i}+8\hat{j}$. Find -**

(i) Angle of projection

(ii) Time of flight

(iii) Range

(iv) Initial speed

(v) Maximum height

A3 Initial velocity

$$\vec{u} = u_x\hat{i} + u_y\hat{j} = 6\hat{i} + 8\hat{j}$$

$$u\cos\theta = 6$$

$$u\sin\theta = 8$$

1. $$\tan\theta = \frac{u_y}{u_x} = \frac{8}{6} = \frac{4}{3}$$

$$\tan\theta = \frac{4}{3}$$

$$\theta = 53^\circ$$

2. Time of flight

$$T = \frac{2u_y}{g} = \frac{2\times 8}{10} = 1.6\ \text{sec}.$$

3. Horizontal range

$$R = \frac{2u_x u_y}{g} = \frac{2\times 8\times 6}{10} = 9.6\ m$$

4. Initial speed

$$|\vec{u}| = \sqrt{6^2+8^2} = 10\ m/s$$

5. $$H_{max} = \frac{u_y^2}{2g} = \frac{8^2}{2\times 10} = 3.2\ m$$

Q4 A body of mass m is thrown at angle θ with horizontal from ground with initial velocity v. Find the angle of projection at a point where -

1. Velocity is half of initial velocity.

2. Velocity is $\frac{\sqrt{3}}{2}$ times of initial velocity

A4 Velocity at highest point = $u\cos\theta$

1. $u\cos\theta = \frac{u}{2}$

$\Rightarrow$ $\theta = 60°$

2. $u\cos\theta = \frac{\sqrt{3}u}{2}$

$\Rightarrow$ $\theta = 30°$

Q5 A body is thrown at angle θ from horizontal with initial velocity u. Find -

1. Time when direction of velocity become perpendicular to its initial velocity.

2. Also find velocity at that time.

A5

1. Let at point P, velocity becomes perpendicular to initial velocity

So, $\vec{u}\cdot\vec{v}_p = 0$

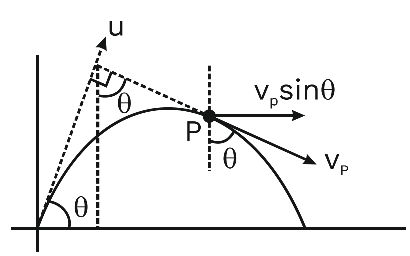
$$\left(u\cos\theta\hat{i} + u\sin\theta\hat{j}\right)\cdot\left[u\cos\theta\hat{i} + \left(u\sin\theta - gt\right)\hat{j}\right] = 0$$

$$u^2\cos^2\theta + u^2\sin^2\theta - u\sin\theta gt = 0$$

$$u^2\left(\cos^2\theta + \sin^2\theta\right) - u\sin\theta gt = 0$$

$$t = \frac{u}{g\sin\theta}$$

$$t = \frac{u}{g}\operatorname{cosec}\theta$$



2. $v_p\sin\theta = u\cos\theta$

$$v_p = u\cot\theta$$

Important Concept

Comparisons of Two Projectiles of Equal Range:

When two projectile are thrown with equal speeds at angle θ and $90-\theta$ then their ranges are equal but maximum height attained are different and time of flight are also different.

Range of projection angle θ

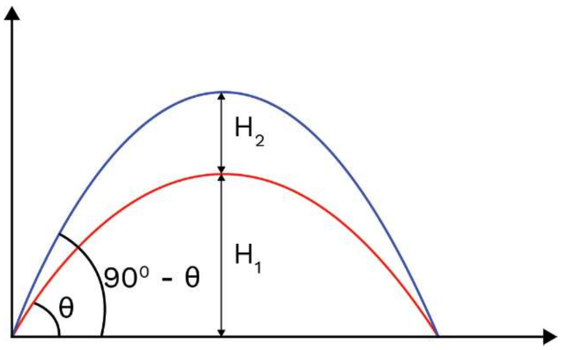
$$R_1 = \frac{u^2 \sin 2\theta}{g}$$

Range for projection angle $(90-\theta)$

$$R_2 = \frac{u^2 \sin 2(90-\theta)}{g} = \frac{u^2 \sin 2\theta}{g}$$

So, $R_1 = R_2$

Also equal for angle $(45° - \alpha)$ and $(45° + \alpha)$.



θ	$90 - \theta$	R_1 and R_2
30°	60°	$R_1 = R_2$
60°	30°	$R_1 = R_2$
16°	74°	$R_1 = R_2$
74°	16°	$R_1 = R_2$

Definitions

According to **Galileo**, in his book **"Two New Sciences"** stated that for elevations by which exceed or fall short of 45° by equal amounts, the ranges are equal.

Maximum Heights of Projectile at (θ) and ($90-\theta$):

$$H_1 = \frac{u^2 \sin^2 \theta}{2g}$$

$$H_2 = \frac{u^2 \sin^2 (90° - \theta)}{2g} = \frac{u^2 \cos^2 \theta}{2g}$$

$$\frac{H_1}{H_2} = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$$

$$\Rightarrow \frac{H_1}{H_2} = \tan^2\theta$$

$$H_1 \times H_2 = \frac{u^4 \sin^2\theta \cos^2\theta}{4g^2}$$

$$= \left(\frac{2u^2 \sin\theta \cdot \cos\theta}{g}\right)^2 \times \frac{1}{4} \times \frac{1}{4} = \frac{R^2}{16}$$

$$\Rightarrow H_1H_2 = \frac{R^2}{16}$$

$$R = 4\sqrt{H_1H_2}$$

$$H_1 + H_2 = \frac{u^2 \sin^2\theta}{2g} + \frac{u^2 \cos^2\theta}{2g} = \frac{u^2}{2g}$$

$$H_1 + H_2 = \frac{u^2}{2g}$$

(Maximum height when thrown vertically upward from ground)

Time of Flight of Projectiles at (θ) and (90−θ):

$$T_1 = \frac{2u\sin\theta}{g}, \quad T_2 = \frac{2u\sin(90-\theta)}{g} = \frac{2u\cos\theta}{g}$$

$$\frac{T_1}{T_2} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\Rightarrow \frac{T_1}{T_2} = \tan\theta$$

Concept Reminder:

At angle θ and 90 – θ:

- $R_1 = R_2$
- $\frac{H_1}{H_2} = \tan^2\theta$
- $H_1H_2 = \frac{R^2}{16}$
- $H_1 + H_2 = \frac{u^2}{2g}$
- $\frac{T_1}{T_2} = \tan\theta$
- $T_1 \times T_2 \propto R$

Rack your Brain

Two stones are projected with the same magnitude of velocity, but making different angles with horizontal. The angle of projection of one is $\pi/3$ and its maximum height is Y, what maximum height attained by the other stone with at $\pi/6$ angle of projection ?

$$T_1T_2 = \frac{4u^2 \sin\theta \cos\theta}{g^2} = \frac{2R}{g}$$

$$T_1 \times T_2 \propto R$$

If angle of projection is 45° then range would be maximum.

$$R_{max} = \frac{u^2 \sin 2 \times 45^\circ}{g} = \frac{u^2}{g}$$

$$\Rightarrow R_{max} = \frac{u^2}{g}$$

Concept Reminder

All projectile will go upto the maximum horizontal distance, if it is projected at 45° with the horizontal.

Q6 **When a projectile is thrown for maximum range then calculate maximum height attained by projectile.**

A6 Range is maximum at $\theta = 45°$

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \sin^2 45°}{2g} = \frac{u^2}{4g} = \frac{R_{max}}{4}$$

Q7 **A person can throw a body to attains maximum range. He can throw same body to attain maximum height with same velocity. Then what is relation between maximum height and maximum range.**

A7 For R_{max},

$\theta = 45°$,

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$R_{max} = \frac{u^2}{g} \qquad \text{...(i)}$$

For H_{max}

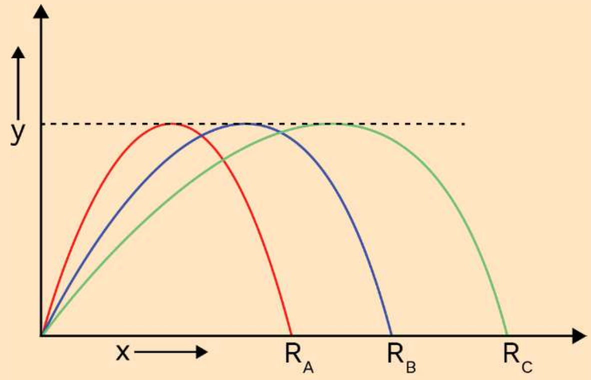
$\theta = 90°$

$$H_{max} = \frac{u^2}{2g} \qquad \text{...(ii)}$$

From equation (i) and (ii),

$$H_{max} = \frac{R_{max}}{2}$$

Q8 **Find the relation between -**



(1) R_A, R_B and R_C

(2) T_A, T_B and T_C

(3) H_A, H_B and H_C

(4) u_A, u_B and u_C

A8

1. $H_A = H_B = H_C \quad (u_y)_A = (u_y)_B = (u_y)_C$

2. $H = \dfrac{u_y^2}{2g}$

3. $T_A = T_B = T_C$

$T = \dfrac{2u_y}{g}$

4. $R_C > R_B > R_A$

$R = \dfrac{2u_x u_y}{g}$

$(u_x)_C > (u_x)_B > (u_x)_A$

Q9 **Body of mass m is thrown at an angle θ with horizonal from ground horizontal range is 200 m and time of flight is 10 sec. Find-**
1. Maximum height **2. Initial speed**

A9 Time of flight =

$$T = \frac{2u\sin\theta}{g} = 10$$

$$u\sin\theta = \frac{10\times10}{2} = 50 \text{ m/s}$$

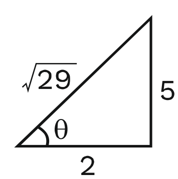
$$\text{Range}(R) = 200 = \frac{u^2\sin2\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g}$$

$$200 = \frac{2(u\sin\theta)^2\times\cos\theta}{g\sin\theta}$$

$$\tan\theta = \frac{2\times(50)^2}{10\times200} = \frac{5000}{2000} = \frac{5}{2}$$

$$H = \frac{u^2\sin^2\theta}{2g} = \frac{(50)^2}{2\times10} = 125\text{m}$$

$$\sin\theta = \frac{5}{\sqrt{29}}$$

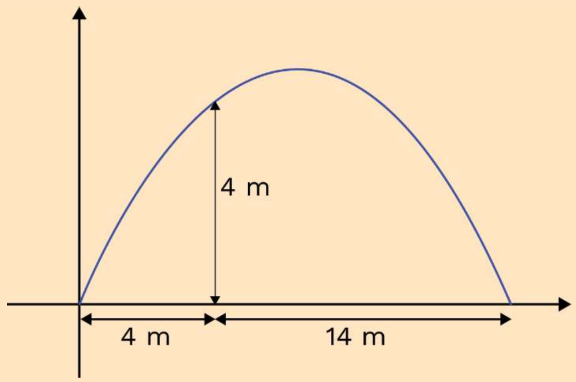


$$T = \frac{2u\sin\theta}{g}$$

$$u = \frac{g\cdot T}{2\sin\theta}$$

$$u = \frac{10\times10}{2\times5}\sqrt{29} = 10\sqrt{29}$$

Q10 **If projectile is thrown at an angle θ with horizontal with ground towards a wall 4m away from point of projection and height is 4m this projectile just touches and thrown on other side at 14m horizontal distance from foot of wall. Find, angle of projection and initial speed.**



A10

1.

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

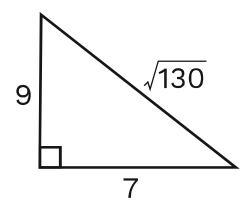
$$y = x\tan\theta\left(1 - \frac{x}{R}\right)$$

$$4 = 4\tan\theta\left(1 - \frac{4}{18}\right)$$

$$1 = \tan\theta\left(\frac{14}{18}\right)$$

$$\tan\theta = \frac{18}{14} = \frac{9}{7}$$

$$\theta = \tan^{-1}\left(\frac{9}{7}\right)$$



2.

$$R = \frac{2u^2\sin\theta\cos\theta}{g}$$

$$18 = 2\times u^2 \times \frac{9}{\sqrt{130}} \times \frac{7}{\sqrt{130}} \times \frac{1}{g}$$

$$18 = \frac{2u^2\times 9\times 7}{130\times g}; \qquad u^2 = \frac{18\times 130\times g}{9\times 7\times 2}$$

$$u^2 = \frac{1300}{7}$$

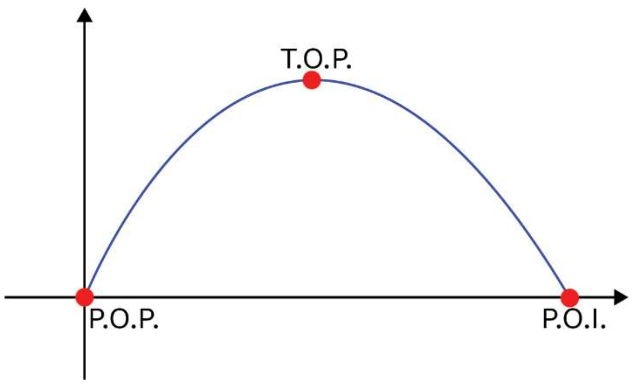
$$u = \sqrt{\frac{1300}{7}} \Rightarrow u = 10\sqrt{\frac{13}{7}}$$

Change in Linear Momentum:

We know that change in momentum

$$\Delta\vec{P} = m\left(\overrightarrow{v_F} - \overrightarrow{v_I}\right)$$

Now we discuss about change in momentum between different points on a projectile path.



1. $\vec{V}_{P.O.P} = u\cos\theta\hat{i} + u\sin\theta\hat{j}$
2. $\vec{V}_{T.O.P} = u\cos\theta\hat{i} + 0\hat{j}$
3. $\vec{V}_{P.O.I} = u\cos\theta\hat{i} - u\sin\theta\hat{j}$
4. $\vec{V}_{K.O.I} = u\cos\theta\hat{i} + \left(u\sin\theta - gt\right)\hat{j}$

1. Between P.O.P and T.O.P:

$$\overrightarrow{\Delta P} = -mu\sin\theta\hat{j}$$

2. Between P.O.P and P.O.I :

$$\overrightarrow{\Delta P} = -2mu\sin\theta\hat{j}$$

3. P.O.P and K.O.I :

$$\overrightarrow{\Delta P} = -mgt\,\hat{j}$$

$$\Delta\vec{P} = \text{weight} \times \text{time}\,\hat{j}$$

Rack your Brain

A particle of mass 100 g is fired with a velocity 20 m sec^{-1} making an angle of 30° with the horizontal. When it rises to the highest point of its path then find the change in its momentum.

Concept Reminder

Change in momentum of projectile during its half journey and full journey is

-Musin $\theta\hat{j}$ and -2 musin $\theta\hat{j}$ respectively.

$$t_{top} = \frac{u\sin\theta}{g} \Rightarrow \overrightarrow{\Delta P} = -mu\sin\theta\ \hat{j}$$

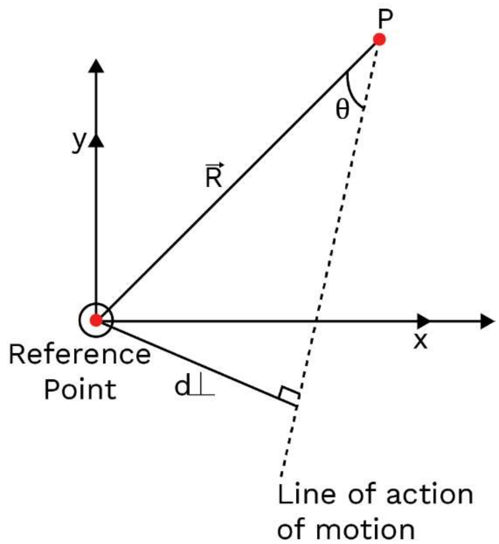
$$t = T = \frac{2u\sin\theta}{g} \Rightarrow \overrightarrow{\Delta P} = -2mu\sin\theta\ \hat{j}$$

Angular Momentum $\left(\vec{j}\right)$:-

It is moment of linear momentum.

OR

Or it is product of linear momentum and perpendicular distance between line of action of motion and reference point.



$$\frac{d\perp}{R} = \sin\theta$$

$$d\perp = R\sin\theta$$

$$J = P\cdot d\perp = PR\sin\theta$$

$$\vec{J} = \left(\vec{R}\times\vec{P}\right)$$

$$\vec{J}\perp\vec{R},\quad \vec{J}\cdot\vec{R} = 0$$

$$\vec{J}\perp\vec{P},\quad \vec{J}\cdot\vec{P} = 0$$

Concept Reminder

If in case of projectile motion range R is n times of maximum height H i.e., R = nH

Then angle of projection

$$\theta = \tan^{-1}\left(\frac{4}{n}\right)$$

Keywords

- Linear momentum
- Angular momentum

Rack your Brain

A particle is projected from ground with speed 80m/s at an angle 30 with horizontal from ground. Find magnitude of average in t = 25 s to t = 65 s.

Q11 **When a body of mass 'm' is thrown at an angle θ with height from ground with initial velocity (v). Find angular momentum about point of projection**

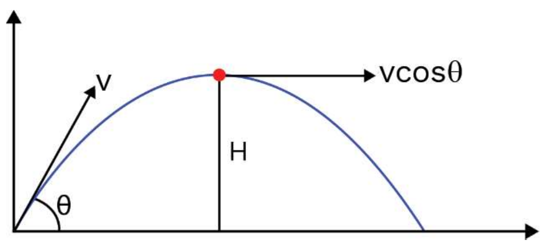
1. When it at highest point

2. In above is question if angle of projection are -

(a) 45° **(b) 60°**

3. In above question find angular momentum with respect to P.O.P when it is at point of impact.

A11



1. $J_0 = H \cdot mu\cos\theta$

$$J_0 = \frac{v^2 \sin^2\theta}{2g} \cdot mv\cos\theta = \frac{mv^3 \sin^2\theta\cos\theta}{2g}$$

2. (a) $J_0 = \dfrac{m \times v^3}{2 \times g} \times \dfrac{1}{2} \times \dfrac{1}{\sqrt{2}}$ $\left[\sin 45^\circ = \cos 45^\circ = \dfrac{1}{\sqrt{2}}\right]$

$$J_0 = \frac{mv^3}{4\sqrt{2}g}$$

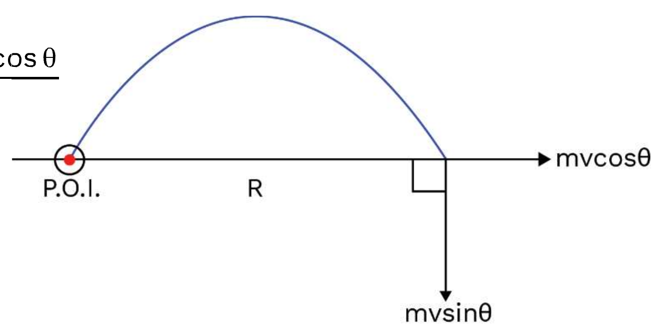
(b) $J_0 = \dfrac{mv^3}{2g} \times \dfrac{3}{4} \times \dfrac{1}{2}$ $\left[\sin 60^\circ = \dfrac{\sqrt{3}}{2},\ \cos 60^\circ = \dfrac{1}{2}\right]$

$$J_0 = \frac{3mv^3}{16g}$$

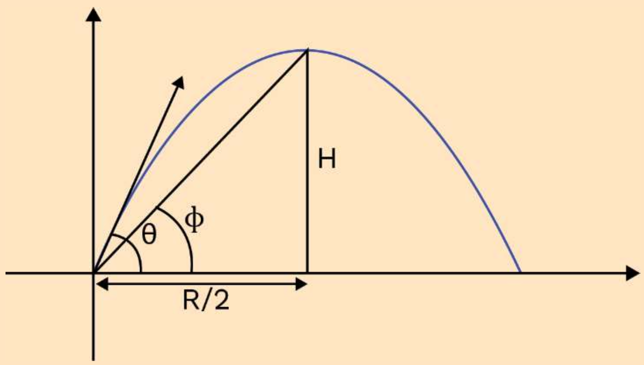
3. $J_{0_y} = R \cdot mv\sin\theta$

$$J_0 = mv\sin\theta \times \frac{2v^2 \sin\theta\cos\theta}{g}$$

$$J_0 = 2mv^3 \frac{\sin^2\theta}{g}\cos\theta$$



Q12 **Projectile of mass m is thrown with initial velocity u at an angle θ with horizontal from ground. Find angle made by straight line which join topmost point according P.O.P with horizontal.**



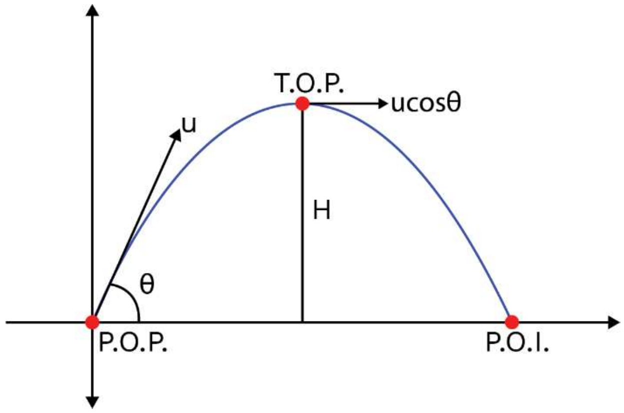
A12

$$\tan\phi = \frac{2H}{R}$$

$$\tan\phi = \frac{2 \times u^2 \sin^2\theta \cdot g}{2g \times 2u^2 \sin\theta \cos\theta}$$

$$\tan\phi = \frac{\tan\theta}{2}$$

Kinetic energy, gravitational, potential energy and total mechanical energy when projectile thrown from grown:-



(i) $\quad$ $G.P.E = mgh$

(ii) $\quad$ $KE = \frac{1}{2}mu^2$

(iii) Total $M.E = K.E + P.E$

A. P.O.P (H = 0)

1. $G.P.E = 0$
2. $KE = \frac{1}{2}mu^2 = E_0$
3. Total $M.E = E_0$

B. T.O.P (H = H)

1. $G.P.E = mgH = \frac{mgu^2 \sin^2\theta}{2g} = E_0 \sin^2\theta$
2. $K.E = \frac{1}{2}mv^2 = \frac{1}{2}m \times u^2 \cos^2\theta = E_0 \cos^2\theta$
3. Total $M.E = E_0 \cdot \left(\sin^2\theta + \cos^2\theta\right) = E_0$

$$\frac{G.P.E}{K.E} = \tan^2\theta$$

C. P.O.I (H = 0)

1. $G.P.E = 0$
2. $K.E = \frac{1}{2}mu^2 = E_0$
3. Total $M.E = E_0$

Total M.E. at every point (P.O.P, T.O.P and P.O.I) is same.

Q13 **When body is at top of projection and initial kinetic energy is 4J then find kinetic energy and gravitational potential energy at T.O.P when angle of projection is 45°.**

A13

$$\frac{1}{2}mv^2 = E_0 = 4J$$

$$\left(KE\right)_{TOP} = E_0 \cos^2\theta$$

$$= 4 \times \frac{1}{2} = 2J$$

$$\left(P.E\right)_{T.O.P} = E_0 \sin^2\theta$$

$$= 4 \times \frac{1}{2} = 2J$$

Q14 **Find angle of projection when kinetic of energy at topmost point is $\frac{3}{4}$ times of initial kinetic energy.**

A14

$$(K.E)_{T.O.P} = \frac{3}{4}(K.E)_{P.O.P}$$

$$\frac{1}{2} \times u^2 \cos^2\theta = \frac{3}{4} \times \frac{1}{2} mu^2$$

$$\cos^2\theta = \frac{3}{4}$$

$$\cos\theta = \sqrt{\frac{3}{4}}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

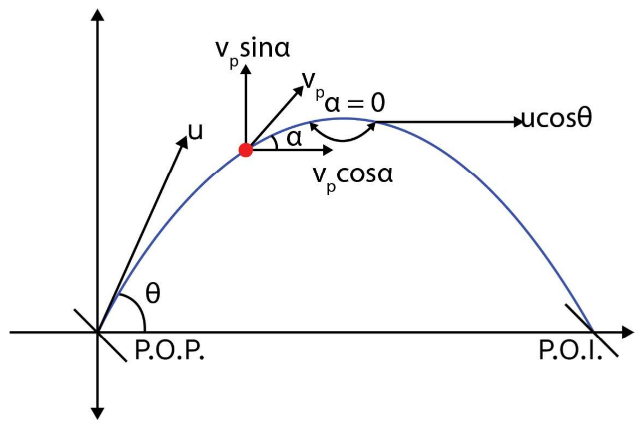
$$\theta = 30°$$

- **Direction of velocity of projectile when thrown by making angle θ with horizontal from ground:-**

$$\tan\alpha = \frac{(v_p)_y}{(v_p)_x} = \frac{u\sin\theta - gt}{u\cos\theta}$$

$$(v_p)_x = u\cos\theta = v_p\cos\alpha$$

$$(v_p)_y = u\sin\theta - gt$$



1. T.O.P $= t_{TOP} = \dfrac{u\sin\theta}{g}$

$$\tan\alpha = \frac{u\sin\theta - \frac{g\times u\sin\theta}{g}}{u\cos\theta}$$

$$\tan\alpha = 0°$$

$$\alpha = 0$$

2. P.O.I $t = T = \dfrac{2u\sin\theta}{g}$ = Time of flight

$$\alpha = \theta$$

In terms of H :-

$$\tan\alpha = \frac{(v_p)_y}{(v_p)_x} = \frac{u^2\sin^2\theta - 2gh}{u\cos\theta},$$

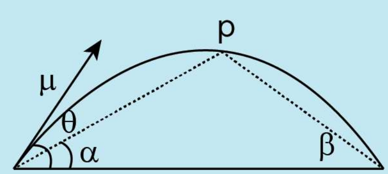
$$h = \frac{u^2\sin^2\theta}{2g}$$

At T.O.P

$v_y^2 = u_y^2 + 2a_y s_y$, $\qquad \tan\alpha = 0°$

$$\alpha = \theta$$

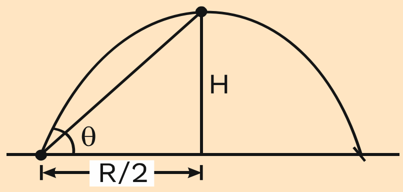
Concept Reminder



$$\tan\theta = \tan\alpha + \tan\beta$$

Q15 **If a projectile of mass m is thrown at an angle θ with horizontal from ground then average velocity in between**

1. P.O.P and T.O.P **2. P.O.P and P.O.I**



A15 **1.**

$$\Delta v = \frac{\text{Total displacement}}{\text{Total time}}$$

$$\Delta v = \frac{\sqrt{\left(\frac{R}{2}\right)^2 + H^2}}{T}$$

$$\Delta v = \frac{\sqrt{\left(\frac{2u^2 \sin\theta\cos\theta}{2\times g}\right)^2 + \left(\frac{u^2 \sin^2\theta}{2g}\right)^2}}{\frac{u\sin\theta}{g}}$$

$$\Delta v = \frac{\sqrt{\frac{u^4\sin^2\theta}{4g^2}\left[\left(4\cos^2\theta\right) + \left(\sin^2\theta\right)\right]}}{\frac{u\sin\theta}{g}}$$

$$\Delta v = \frac{\frac{u^2\sin\theta}{2g}}{\frac{u\sin\theta}{g}}\sqrt{3\cos^2\theta + 1}$$

$$\Delta v = \frac{u}{2}\sqrt{3\cos^2\theta + 1}$$

2.

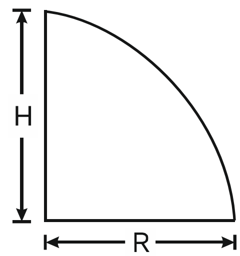
$$\Delta v = \frac{R}{T}$$

$$\Delta v = \frac{2u^2\sin\theta\cos\theta \times g}{g\times 2u\sin\theta}$$

$$\Delta v = u\cos\theta = \left(v_{avg}\right)$$

Horizontal Projectile:-

When a projectile thrown from certain height H horizontal direction then



$\overrightarrow{u_x} = u,\qquad \overrightarrow{a_x} = 0$

$\overrightarrow{u_y} = 0,\qquad \overrightarrow{a_y} = -g$

1. x-direction:

$$\overrightarrow{s_x} = \overrightarrow{u_x}t + \frac{1}{2}\overrightarrow{a_x}t^2$$

$$x = ut$$

$$t = \frac{x}{u}$$

2. $$\overrightarrow{s_y} = \overrightarrow{u_y}t + \frac{1}{2}\overrightarrow{a_y}t^2$$

$$\text{so, } \overrightarrow{s_v} = -\frac{1}{2}g\cdot\left(\frac{x^2}{v^2}\right)$$

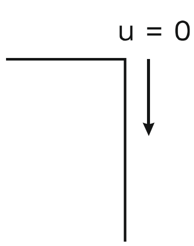
1. Time of flight [T]:

$$\overrightarrow{s_y} = \overrightarrow{u_y}t + \frac{1}{2}\overrightarrow{a_y}t^2$$

$$-H = -\frac{1}{2}gT^2$$

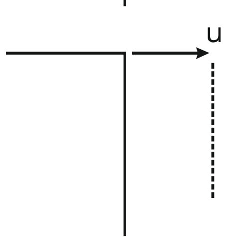
$$T = \sqrt{\frac{2H}{g}}$$

(a)



$T = \sqrt{\frac{2H}{g}}$

(b)



$T = \sqrt{\frac{2H}{g}}$

Concept Reminder

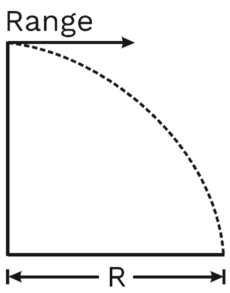
If an object is dropped or projected horizontally, in both cases Time of flight remains game i.e., $T = \sqrt{\frac{2H}{g}}$

KEY POINTS

* Horizontal projectile

When a projectile is thrown from height H horizontally and dropped from same height strike to the ground in same time.

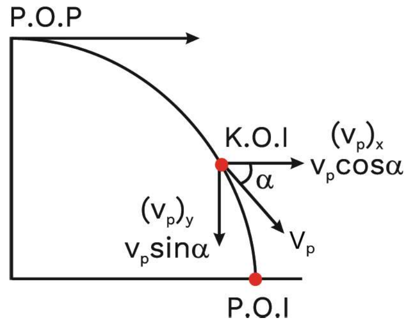
2.



$$\vec{s_x} = \vec{u_x}t + \frac{1}{2}\vec{a_x}t^2$$

$$R = uT = u\sqrt{\frac{2H}{g}}$$

3.



Direction: $\tan\alpha = \frac{(v_p)_y}{(v_p)_x} = -\frac{gt}{u}$

$$(v_p)_y = \vec{v_y} = \vec{u_y} + \vec{a_y}t$$

$$= -gt$$

$$\vec{v_p} = \vec{v_x}\hat{i} + \vec{v_y}\hat{j} = 4\hat{i} - gt\hat{j}$$

$$v_{speed} = \sqrt{u^2 + g^2t^2}$$

Concept Reminder

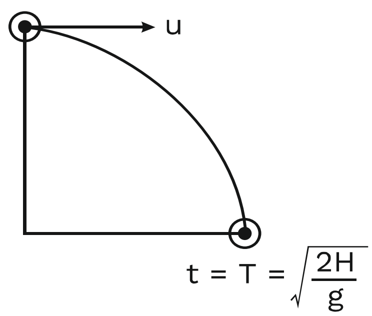
In Horizontal projectile

$$T = \sqrt{\frac{2H}{g}}$$

$$R = uT = u\sqrt{\frac{2H}{g}}$$

Rack your Brain

Two paper screens A and B are seperated by 100 m distance. A bullet pierces A and them B. The hole in B is 10cm below the hole in A. If the bullet is travelling horizontally at A calculate the velocity of bullet at A.

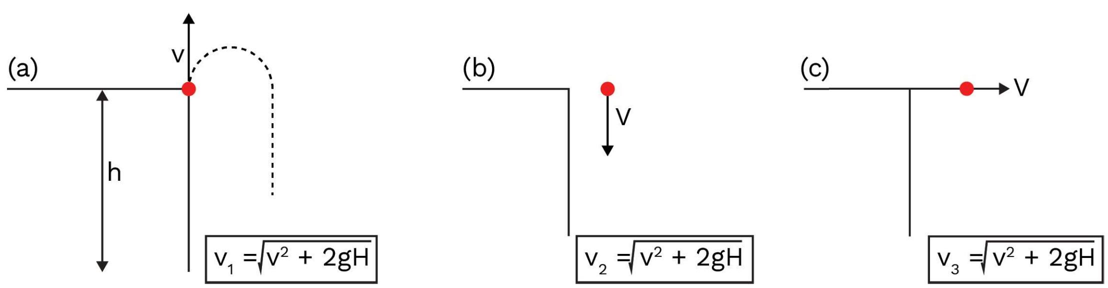


Concept Reminder

The path of a projectile as seen from another projectile is a straight line.

$$v = \sqrt{u^2 + g^2t^2}$$

$$v = \sqrt{u^2 + 2gH}$$



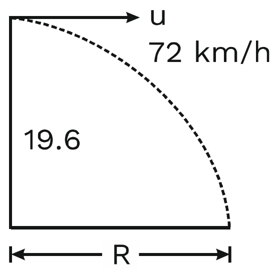
When a particle is thrown vertically upward and thrown vertically downward and horizontal project the its speed when it strike to the ground is same.

Q16 **Aeroplane is moving horizontal with 72 km/hr. when it is at height 19.6 m. standing on ground a bomb will strike to the person. If not then time to the strike to the ground of person when strike to the ground.**

A16

1. No

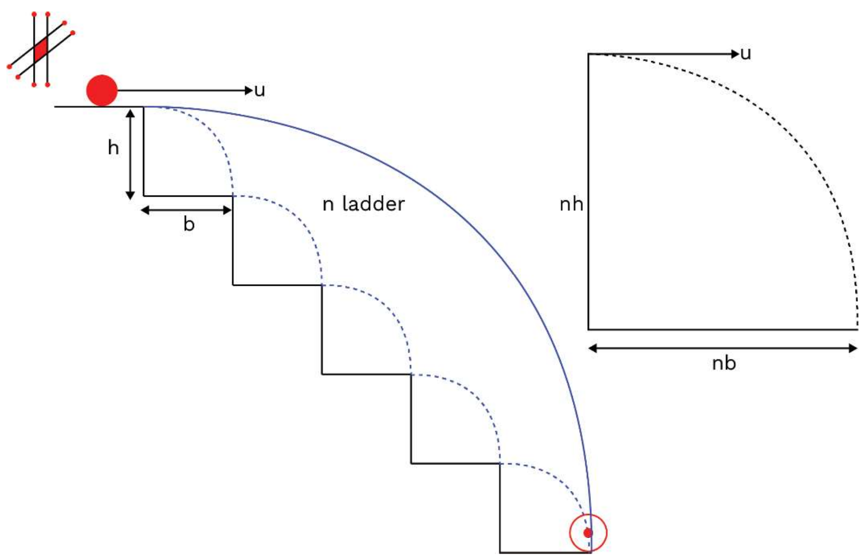
2. $T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 19.6}{9.8}} = 2 \text{ sec.}$



3. $R = u\sqrt{\frac{2H}{g}} = 72 \times \frac{5}{18} \times 2 = 4 \times 2 \times 5 = 40\text{m}$

IMPORTANT CONCEPT

- **When a ball is thrown from a stair in horizontal direction then-**



$$R = u\sqrt{\frac{2H}{g}}$$

$$nb = u\sqrt{\frac{2(nh)}{g}}$$

$$n^2b^2 = u^2 \cdot \frac{2nh}{g}$$

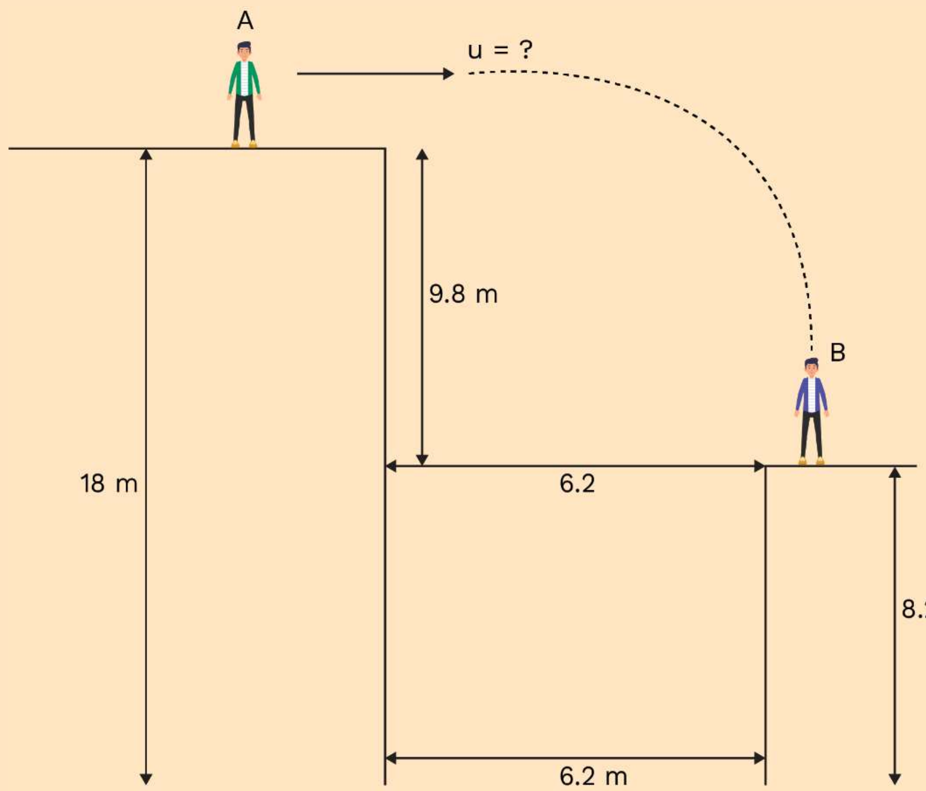
$$n = \frac{2hu^2}{gb^2}$$

Rack your Brain

A ball rolls off the top stair way with a horizontal velocity $10\sqrt{10}$ m/s. If the steps are 0.25m high and 5m wide then ball will hit the edge of the n^{th} step. Find value of n.

Projectile Motion

Q17 **A person is throwing a ball in horizontal direction from a 18m height building. The ball is caught by another person which is at another building of 8.2m height from ground. If horizontal range is shown in figure. Then calculate initial velocity of ball.**



A17 $H = 18 - 8.2$

$H = 9.8\ m$

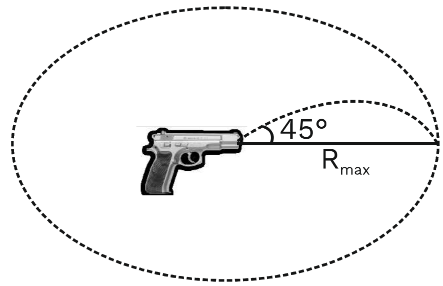
$R = 6.2m$

$$R = u\sqrt{\frac{2H}{g}}$$

$$6.2 = u\sqrt{\frac{2 \times 9.8}{9.8}} \quad \Rightarrow \quad u = \frac{6.2}{\sqrt{2}}$$

$$u = \frac{6.2}{1.41} = 4.4\ m/s$$

When bullets are fired with same velocity u from stair give in different direction then maximum area covered by any bullet.



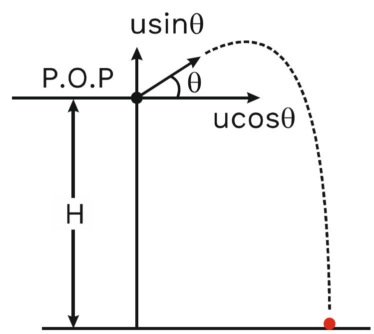
$$R_{max} = \frac{u^2}{g}$$

$$Area = \pi R^2$$

$$\pi R^2_{max}$$

$$Area = \pi \times \frac{u^4}{g^2}$$

When a projectile thrown at an angle θ with horizontal upward with at some height:-



(i) Time of flight (T):-

$$\vec{s_y} = \vec{u_y}t + \frac{1}{2}\vec{a_y}t^2$$

$$-H = u\sin\theta T - \frac{1}{2}gT^2$$

$$gT^2 - 2u\sin\theta T - 2H = 0$$

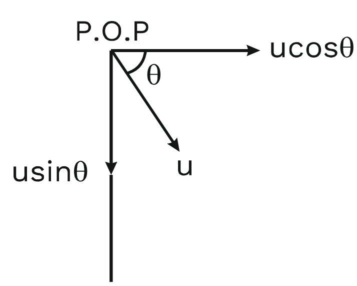
$$T = \frac{u\sin\theta + \sqrt{u^2\sin^2\theta + 2gH}}{g}$$

(ii) Range (R):-

$$\vec{s_x} = \vec{u_x}t + \frac{1}{2}\vec{a_x}t^2$$

$$R = u\cos\theta T$$

When a body is thrown at an angle θ with horizontal downward from some height:-



$$u_x = +u\cos\theta$$

$$a_x = 0$$

$$u_y = -u\sin\theta$$

$$a_y = -g$$

Time of flight (T):-

$$\vec{s_y} = \vec{u_y}t + \frac{1}{2}\vec{a_y}t^2$$

$$+H = +u\sin\theta T + \frac{1}{2}gT^2$$

$$2H = 2u\sin\theta T + gT^2$$

$$gT^2 + 2u\sin\theta T - 2H = 0$$

$$T = \frac{-u\sin\theta + \sqrt{u^2\sin^2\theta + 2gH}}{g}$$

Range:

$$R = \vec{s_x} = \vec{u_x}t + \frac{1}{2}\vec{a_x}t^2, \quad R = u\cos\theta T$$

Maximum time when strike to the ground.

1. $t_1 = \sqrt{\frac{2h}{g}}$

2. $t_2 = \sqrt{\frac{2(H-h)}{g}}$

$$T = t_1 + t_2 = \sqrt{\frac{2h}{g}} + \sqrt{\frac{2(H-h)}{g}}$$

$$\frac{dT}{dh} = \frac{d}{dh}\left(\sqrt{\frac{2h}{g}}\right) + \frac{d}{dh}\left(\sqrt{\frac{2(H-h)}{g}}\right)$$

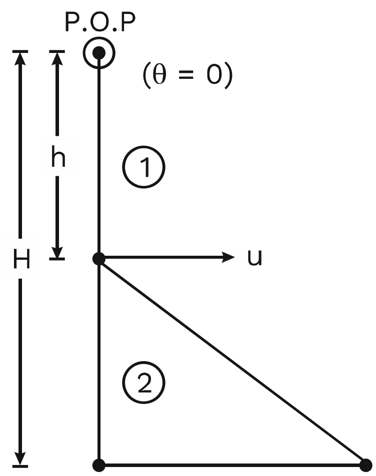
$$\frac{dT}{dh} = \sqrt{\frac{2}{g}}\left(\frac{d\sqrt{h}}{dh} + \frac{d\sqrt{H-h}}{dh}\right)$$

$$\frac{dT}{dh} = \sqrt{\frac{2}{g}}\left(\frac{1}{2\sqrt{h}} + \frac{1(-1)}{2\sqrt{H-h}}\right)$$

$$\because \quad \frac{dT}{dh} = 0$$

Concept Reminder

If an object is dropped from horizontally flying aeroplane the object falls in a straight line vertically down with respect to pilot/plane. But with respect to ground the obj9ect executes projectile motion.



$$\therefore \qquad 0=\sqrt{\frac{2}{g}}\left(\frac{1}{2\sqrt{h}}-\frac{1}{2\sqrt{H-h}}\right)$$

$$\frac{1}{2}\left(\sqrt{\frac{2}{gh}}\right)^2=\frac{1}{2}\left(\sqrt{\frac{2}{g(H-h)}}\right)^2$$

$$\frac{2}{gh}=\frac{2}{g(H-h)}$$

$$H-h=h$$

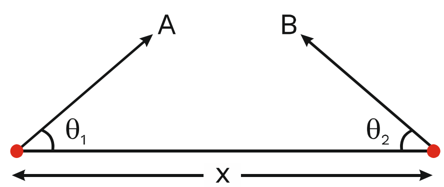
$$2h=H$$

$$h=\frac{H}{2}$$

- **Collision of two projectile :-**

Check:-

1. Step - I: $(u_A)_y=(u_B)_y$

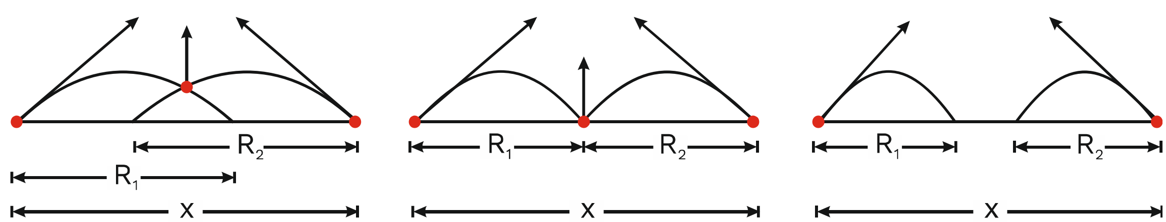


2. Step - II:

(i) $x<R_1+R_2$

(ii) $x=R_1+R_2$

(iii) $x>R_1+R_2$



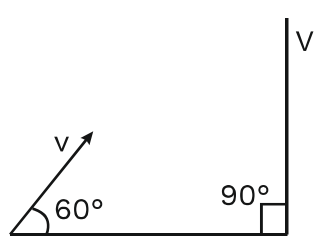
Both called the v and V:-

$$(U_A)_y=(U_B)_y$$

$$v\sin 60^\circ=V\sin 90^\circ$$

$$v\left(\frac{\sqrt{3}}{2}\right)=V$$

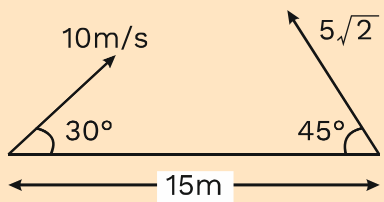
$$\boxed{\frac{v}{V}=\frac{2}{\sqrt{3}}}$$



Keywords

- Collision

Q18 **Find out condition for collision (collision will occur or not) ?**



A18 **1.** $(u_A)_y = (u_B)_y$

$10 \sin 30° = 5\sqrt{2} \sin 45$

$\frac{10}{2} = \frac{5\sqrt{2}}{\sqrt{2}}$

$\boxed{5 = 5}$

2. $R_1 = \frac{2u^2 \sin\theta \cos\theta}{g}$

$R_2 = \frac{2 \times u^2 \sin\theta \cos\theta}{g}$

$R_1 = 2 \times \frac{100}{10} \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$R_2 = \frac{2 \times 25 \times 2}{10} \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

$R_1 = 5\sqrt{3}$

$R_2 = \frac{50}{10} = 5$

$R_1 = 1.732 \times 5,$ $\qquad R = 5$

$R_1 = 8.660$

$R_1 + R_2 = 8.6 + 5 = 13.6$

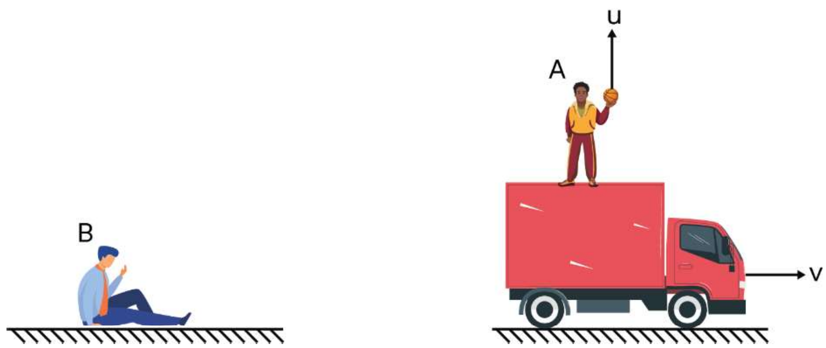
$\boxed{R_1 + R_2 < x}$

Collision not occur.

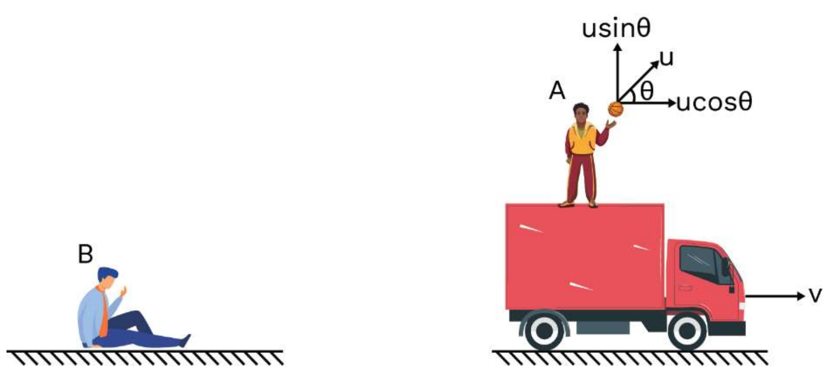
IMPORTANT CONCEPT

Projection from a moving Platform:-

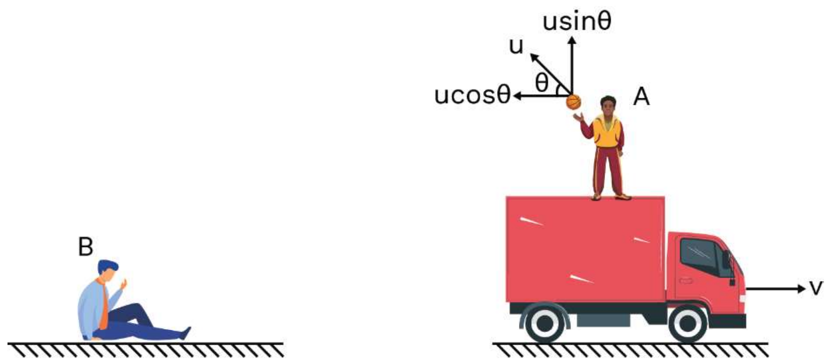
Case (1) : When a ball is thrown upward from a truck moving with an uniform speed, then observer A standing in the truck, will see the ball moving in straight vertical line (upward & downward). The observer B sitting on road will see the ball moving in a parabolic path. The horizontal speed of the ball is equal to the speed of the truck.



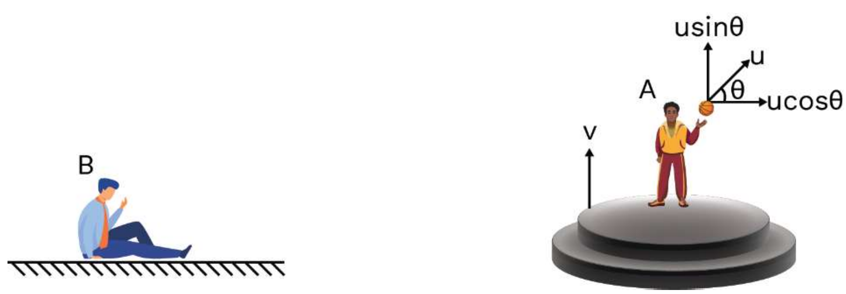
Case (2) : When a ball is thrown at some angle 'θ' in the direction of motion of the truck horizontal & vertical component of ball's velocity w.r.t. observer A standing on the truck is u cos θ and u sin θ respectively. Horizontal & vertical component of ball's velocity w.r.t. observer B sitting on the ground is u_x = ucos θ + v and u_y = usin θ respectively.



Case (3) : When a ball is thrown at some angle 'θ' in the opposite direction of motion of the truck horizontal & vertical component of ball's velocity w.r.t. observer A standing on the truck is u cos θ and u sin θ respectively. Horizontal & vertical component of ball's velocity w.r.t. observer B sitting on the ground is u_x = ucos θ – v and u_y = usin θ respectively.



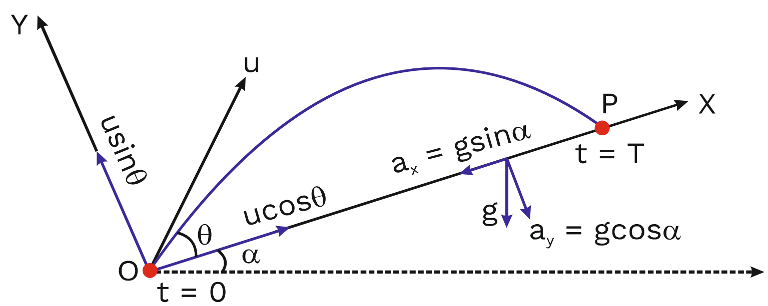
Case (4) : When a ball is thrown at some angle 'θ' from a platform moving with speed v upwards horizontal & vertical component of ball's velocity w.r.t. observer A standing on the moving platform is ucos θ and usinθ respectively. Horizontal & vertical component of ball's velocity w.r.t. observer B sitting on the ground is u_x = ucosθ and u_y = usinθ + v respectively.



Case (5) : When a ball is thrown at some angle 'θ' from a platform moving with speed v downwards horizontal & vertical component of ball's velocity w.r.t. observer A standing on the moving platform is ucosθ and usinθ respectively. Horizontal & vertical component of ball's velocity w.r.t. observer B sitting on the ground is u_x = ucosθ and u_y = usinθ – v respectively.

Projectile Motion on an Inclined Plane:-

Let a particle be projected up with a speed u from an inclined plane which makes an angle α with the horizontal velocity of projection makes an angle θ with the inclined plane. We have taken reference x-axis in the direction of plane.



Hence the component of initial velocity parallel and perpendicular to the plane are equal to $u\cos\theta$ and $u\sin\theta$ respectively.

$$\text{i.e. } u_{\parallel} = u\cos\theta \text{ and } u_{\perp} = u\sin\theta$$

The component of g along the plane is $g\sin\alpha$ and perpendicular to the plane is $g\cos\alpha$ as shown in the figure

$$\text{i.e. } a_{\parallel} = -g\sin\alpha \text{ and } a_{\perp} = g\cos\alpha$$

Therefore the particle decelerates at a rate of $g\sin\alpha$ as it moves from O to P.

Time of flight :

We know for oblique projectile motion $T = \dfrac{2u\sin\theta}{g}$

or we can say $T = \dfrac{2u_{\perp}}{a_{\perp}}$

$\therefore$ Time of flight on an inclined plane $T = \dfrac{2u\sin\theta}{g\cos\alpha}$

Maximum height :

We know for oblique projectile motion $H = \dfrac{u^2\sin^2\theta}{2g}$

or we can say $H = \dfrac{u_{\perp}^2}{2a_{\perp}}$

$\therefore$ Maximum height on an inclined plane $H = \dfrac{u^2 \sin^2\theta}{2g\ \cos\alpha}$

Horizontal range:

For one dimensional motion $s = ut + \dfrac{1}{2}at^2$

Horizontal range on an inclined plane $R = u_{||}T + \dfrac{1}{2}a_{||}T^2$

$$R = u\ \cos\ \theta T - \frac{1}{2}g\sin\alpha\, T^2$$

$$R = u\ \cos\theta\left(\frac{2u\sin\theta}{g\cos\alpha}\right) - \frac{1}{2}g\sin\alpha\left(\frac{2u\sin\theta}{g\cos\alpha}\right)^2$$

By solving $R = \dfrac{2u^2}{g}\dfrac{\sin\theta\cos(\theta+\alpha)}{\cos^2\alpha}$

(i) Maximum range occurs when $\theta = \dfrac{\pi}{4} - \dfrac{\alpha}{2}$

(ii) The maximum range along the inclined plane when the projectile is thrown upwards is given by $R_{max} = \dfrac{u^2}{g(1+\sin\alpha)}$

(iii) The maximum range along the inclined plane when the projectile is thrown downwards is given by $R_{max} = \dfrac{u^2}{g(1-\sin\alpha)}$

EXAMPLE

Q1 **A particle will project with 20 m/s at angle 53° with horizontal find out time at which its velocity will make an angle 45° will horizontal.**

Sol: $t_1 = \frac{v\cos\theta}{g}[\tan\theta - \tan\alpha]$

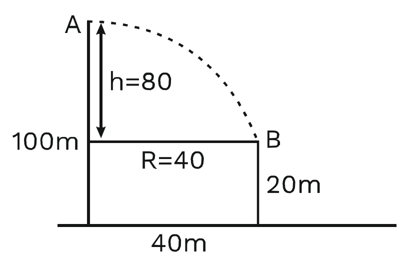
$$= \frac{20\cos 53^\circ}{10}[\tan 53^\circ - \tan 45^\circ]$$

$$= \frac{2\times 3}{5}\left[\frac{4}{3} - 1\right] = \frac{6}{5}\times\frac{1}{3} = \frac{2}{5}$$

$$t_2 = \frac{2\times 3}{5}\left[\frac{4}{3} + 1\right] = \frac{6}{5}\times\frac{7}{3} = \frac{14}{5}$$

Q2 **In the following diagram find out velocity of ball with which it will project horizontally from top of tower A to reach a top of tower B.**

Sol:



$R = u\sqrt{\frac{2h}{g}}$

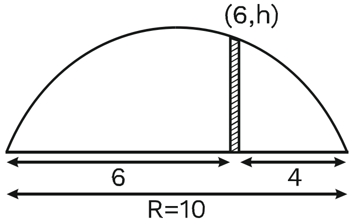
$40 = u\sqrt{\frac{2\times 80}{10}}$

$40 = 4\ u$

$u = 10$

Q3 **A ball with throw from ground at angle 45° then it will just pass top of the tower which is located at the distance of 6 m from projection point and the ball will hit the ground at the distance of 4m ahead of tower. Find out height of the tower.**

Sol:



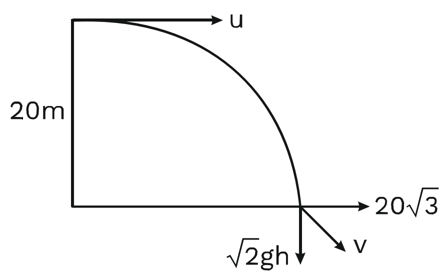
$$y = x\tan\theta\left[1-\frac{x}{R}\right]$$

$$h = 6\times 1\left[1-\frac{6}{10}\right]$$

$$h = \frac{6\times 4}{10} = \frac{12}{5}$$

Q4 **A particle will throw horizontally from height 20 m with $20\sqrt{3}$ m/sec then find out magnitude and angle with horizontal of hitting velocity.**

Sol:



$\Rightarrow \quad |\vec{v}| = 40$

$\alpha = 30°$

Q5 **Particle of man 'm' will project for parabolic motion. Find out change in momentum from t = 0 to t.**

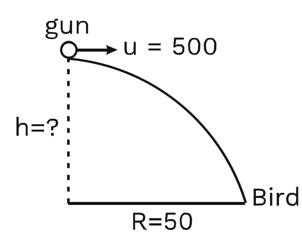
Sol:

$$\vec{F} = \frac{\Delta\vec{P}}{\Delta t}, \qquad -mg\,\hat{j} = \frac{\Delta\vec{P}}{t}$$

$$\Delta\vec{P} = -mgt\,\hat{j}$$

Q6 **A bird is sitting at the horizontal distance of 50 m from hunter find out the height above the bird at which he will place a gun and fire horizontally to hit the bird. It speed of bullet at the instant of firing is 500 m/sec.**

Sol:



$$R = u\sqrt{\frac{2h}{g}}$$

$$50 = 500\sqrt{\frac{2h}{10}}$$

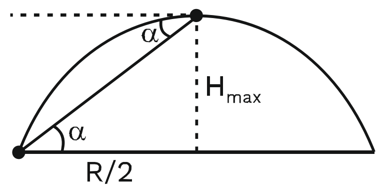
$$\frac{1}{10} = \sqrt{\frac{h}{5}}$$

$$\frac{1}{100} = \frac{h}{5}$$

$$h = \frac{1}{20}$$

Q7 **Particle will project for parabolic path then find out relation between elevation angle at top most point observed from projection point and angle of projection.**

Sol:



$$\tan\alpha = \frac{H_{max}}{R}\times 2, \qquad \frac{H_{max}}{R} = \frac{\tan\theta}{4}$$

$$\tan\alpha = \frac{\tan\theta \times 2}{4}, \quad \Rightarrow \quad \left[\tan\alpha = \frac{\tan\theta}{2}\right]$$

Q8 **At any point velocity of particle is $\vec{v} = \hat{i} + 3\hat{j}$ and acceleration is $2\hat{i} + \hat{j}$ then find out magnitude of rate of change of speed. [i.e. magnitude of velocity]**

Sol: i.e. component of $\vec{a}$ along $\vec{v}$ -

$$\left(\frac{\vec{a}\cdot\vec{v}}{v}\right)\hat{v} \rightarrow \frac{\text{non consider}}{\text{only magnitude}}, \left(\frac{\vec{a}\cdot\vec{v}}{v}\right)_{|\vec{v}|}$$

$$\Rightarrow \qquad |\vec{a}| = \frac{2+3}{\sqrt{10}} = \frac{5}{\sqrt{10}} = \sqrt{\frac{5}{2}}\,m/s^2$$

Q9 **If two particle having some mass and speed of projection will project for parabolic at any place (g = constant). Find out ratio of range for both if ratio of minimum kinetic energy is 1 : 2 and ratio of H_{max} is 3 : 1.**

Sol: u and g same

$R \propto \sin\theta \, . \cos\theta$

(i) $K_{min} \propto \cos^2\theta$

$\cos\theta \propto \sqrt{K_{min}}$

(ii) $H_{max} \propto \sin^2\theta$

$\sin\theta \propto \sqrt{H_{max}}$

$R \propto \sqrt{K_{min} \cdot H_{max}}$

$$\frac{R}{R} = \sqrt{\frac{1}{2} \times \frac{3}{1}}$$

$$\frac{R_1}{R} = \sqrt{\frac{3}{2}}$$

Mind Map
Horizontal Projectile Motion

Position at time t

$x = ut$

$y = 1/2\ gt^2$

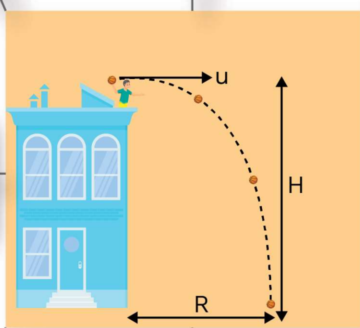
Velocity vector at time t

$\overline{v} = \overline{v}_x + \overline{v}_y$

$v = \sqrt{u^2 + g^2t^2}$

Equation of path of projectile motion

$y = gx^2/2u^2$

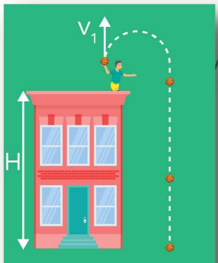


Horizontal range of projectile

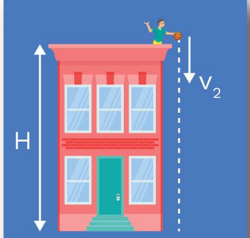
$R = u\sqrt{\frac{2H}{g}}$

Time to reach at ground

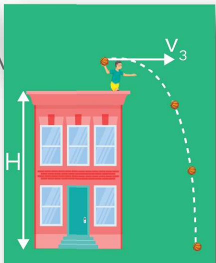
$t_m = \sqrt{\frac{2H}{g}}$



$v_1 = \sqrt{u^2 + 2gH}$



$v_2 = \sqrt{u^2 + 2gH}$



$v_3 = \sqrt{u^2 + 2gH}$