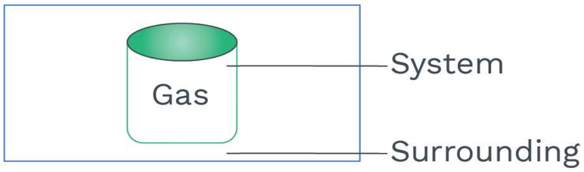
Thermodynamics

DISCLAIMER

Thermodynamics

Branch of physics which deals with the inter conversion of heat energy and any other form of energy.

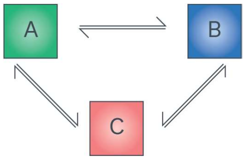


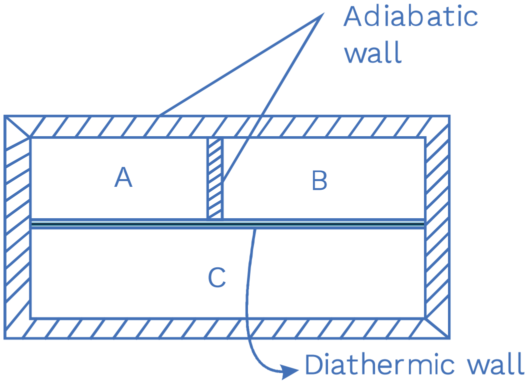
Type of System:-

1. Open system: It exchange both matter and energy with surrounding.
2. Closed system: It exchange only energy not matter with surrounding.
3. Isolated system: It exchange neither energy nor matter.

Zeroth Law of Thermodynamics (ZLOT):-

According to this law if system A and B are in thermal equilibrium and B and C are in thermal equilibrium then A and C will also be in thermal equilibrium.





* ZLOT defines temperature only

Examples:
Open system: Boiling of water in open container
Closed system: Boiling of water in close container.
Isolated system: Thermos.

Concept Reminder

Thermal Equilibrium:
If two bodies of different temperature comes in contact then there is transfer of heat between bodies till temperature of both bodies becomes equal. This is called thermal equilibrium.

Key Points

- Open system
- Close system
- Isolated system
- Thermal equilibrium

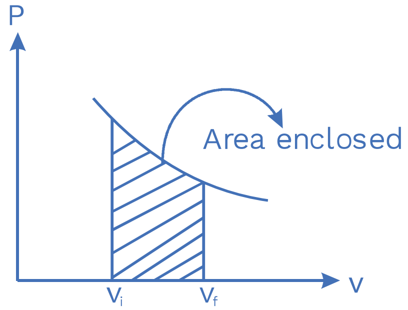
Equation of state (for ideal gases) :-

The relation between the thermodynamics variables (P, V, T) of the system is called equation of state.

The equation of state for an ideal gas of n moles is given by $\boxed{PV = nRT}$

Work Done by an Ideal Gas:-

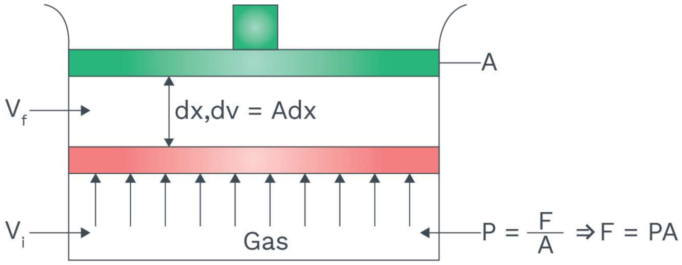
* Work done by Gas is Defined by Volume:-



Concept Reminder

Area under P-V.
Curve = Work done during process

$$W = \int_{V_1}^{V_2} P \cdot dV$$



$$W = \int \vec{F} \cdot \vec{ds} \qquad \left[\because P = \frac{F}{A} \Rightarrow F = P \times A\right.$$

$$\Rightarrow \quad W = \int PA\,dx \qquad \left. F = \text{Force, } P = \text{Pressure, } A = \text{Area}\right]$$

$$\Rightarrow \quad \boxed{W = \int_{v_i}^{v_f} P \cdot dv}$$

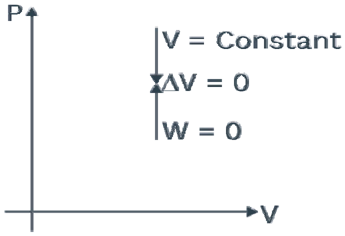
$$W = P\left[V\right]_{V_i}^{V_f} = P\Delta V$$

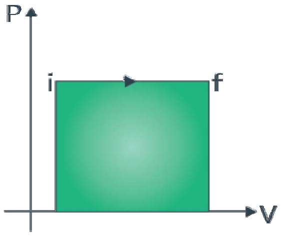
$$\Rightarrow \quad \boxed{W = P \cdot \Delta V}$$

at V = constant then $\Delta V = 0$, W = 0.

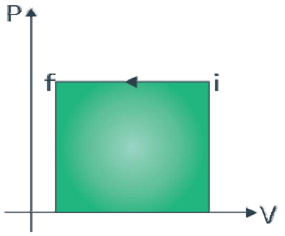
At P = constant then $\begin{bmatrix} PV = nRT \\ V \propto T \end{bmatrix}$

* Area covered by P - V curve on volume axis is equal to work.





V↑, ΔV = +ve, W = +ve
work done by gas



V↓, ΔV = –ve, W = –ve
work done by gas

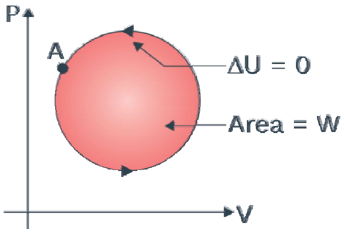
Cycle Process:-

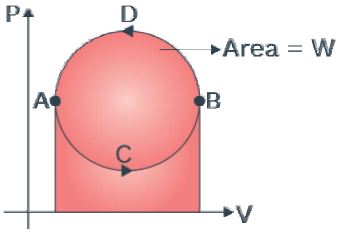
- * In a process initial and final stage are same then this process is called cyclic process.
- * In cyclic process change in internal energy is always zero.
- * Area enclosed by cyclic process = ΔW.

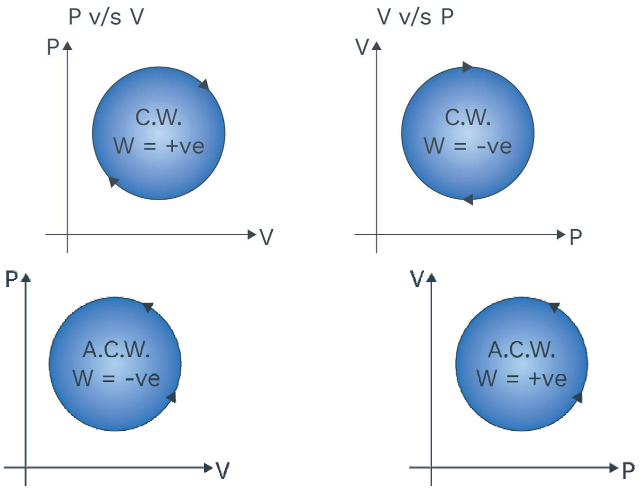
In PV graph direction	**In VP graph direction**
Clockwise direction	Clockwise direction
⇒ ΔW = +ve	⇒ ΔW = –ve
Anticlockwise direction	Anticlockwise direction
⇒ ΔW = –ve	⇒ ΔW = +ve

Key Points

- ◆ Ideal gas
- ◆ Cyclic process
- ◆ Internal energy

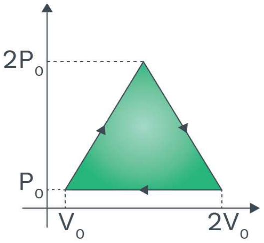






Ex. Find out the work done in given cyclic process

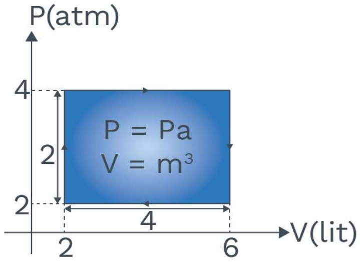
(i)



Sol. Work = Area (P v/s V)

$$W = +\frac{1}{2}P_0V_0$$

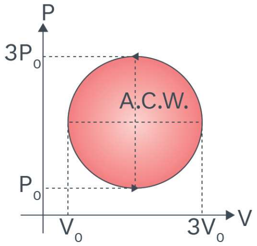
(ii)



Sol. $W = +ve\ (4 \times 10^{-3} \times 2 \times 10^{5})$

$W = +ve \Rightarrow 800\ J$

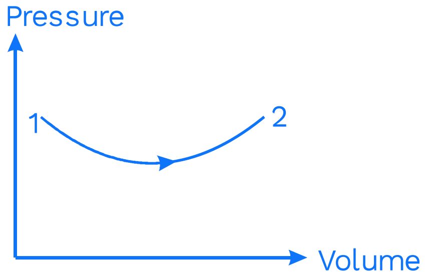
(iii) Ellipse consider ⇒ Unit does not same.



Sol. $\Delta W = \left[\frac{P_2 - P_1}{2}\right]\left[\frac{V_2 - V_1}{2}\right]$

$W = -\pi P_0 V_0$

Ex. Consider the process for a system shown in figure. During the process, the work done by system.

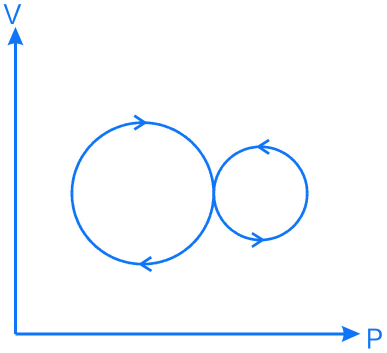


(1) Continuously increases
(2) Continuously decreases
(3) First increases then decreases
(4) First decreases then increases

Sol. $W = \int PdV$

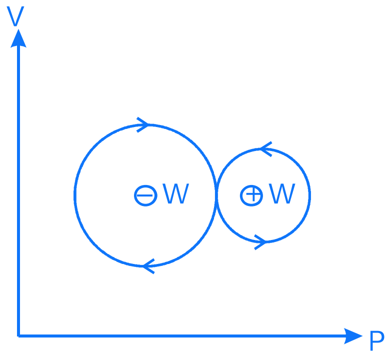
As volume is continuously increasing. so, work done will keep on increasing.

Ex. The net work done in the given process is



(1) Positive (2) Negative
(3) Zero (4) None of these

Sol.



In V-P graph
W_1 = –ve (cw)
W_2 = +ve (acw)
W_{net} = –ve

* **First Law of Thermodynamics [FLOT] (Based on Conservation of Energy):-**

The change in internal energy of a system is equal to difference of heat added to the system and work done by the system

$\boxed{\Delta U = \Delta R - \Delta W}$ $\boxed{\Delta Q = \Delta U + \Delta W}$

$\boxed{\Delta Q = \Delta U + W}$

Path function
$\Delta Q = nC\Delta T$

State function
$[\Delta U = nC_v\Delta T]$

Path function
$[W=\int P.dv = nR\Delta T]$

- FLOT follow conservation of energy and defined by Joule's law.
- FLOT is applicable in all states solid, liquid and gas.
- All quantities, ΔQ, ΔU and ΔW have a same unit.

Note :- Change in internal energy is a state function is independent from path.

$\Delta U = nC_V\Delta T$ for all process.

$T\uparrow \Rightarrow \Delta U(+ve)$

$T\downarrow \Rightarrow \Delta U(-ve)$

* Sign convection of first law of thermodynamics.

ΔU → +ve, ΔT = +ve, T↑
ΔU → -ve, ΔT = -ve, T↓
ΔU → 0, ΔT = 0, T = constant

W → +ve, ΔV = +ve, V↑ = constant
→ Work done by gas (Expansion)

W → -ve, ΔV = -ve, V↓ = constant
→ W.D. on gas (Compression)

Definition

Internal energy: Internal energy of a system is the energy possessed by the system due to molecular motion and molecular configuration.

The energy due to molecular motion is called internal kinetic energy U_K and that due to molecular configuration is called internal potential energy U_P, i.e., total internal energy $U = U_K + U_P$.

For an ideal gas, as there is no molecular attraction $U_P = 0$ and $U = U_K = 3/2\ \mu RT$ and change in internal energy $\Delta U = \frac{3}{2}\mu R\,\Delta T$.

Key Points

- State function
- Heat energy

ΔQ
- +ve, Heat absorb to gas
  → Absorbed/supplied to the system
- -ve, Heat taken from gas
  → Released/rejected by the system.

Ex. For an ideal gas 200 cal heat is given and work done is 200 J. Then find out change in internal energy.

Sol. $\Delta Q = \Delta U + \Delta W.$

$$\Delta U = \Delta Q - W$$

$$\Delta U = (200 \times 4.2)\,J - (200\,J)$$

$$\Delta U = 840 - 200$$

$$\Delta U = +640\,J$$

Then temperature will increase.

Ex. For an ideal gas in isothermal compression work done is 50 J, then find out the how much heat absorbed or released.

Sol. $\Delta Q = \Delta U + W$

$$\Delta Q = \Delta U + W\,,\ \ \Delta U = 0$$

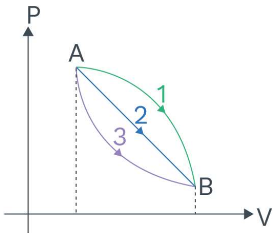
$\Delta W = -50$ J

50 Joule heat is released.

Isothermal T = 0; $\Delta U = 0$

Compression W = -ve.

Ex. In given indicating curve for given process compare the change in internal energy, work and heat.



Sol. 1. $\Delta U_1 = \Delta U_2 = \Delta U_3$

[ΔU is a state function and in given all process initial and final state are same.]

Rack your Brain

If the amount of heat given to a system is 35 J and the amount of work done on the system is 15 J, then calculate change in internal energy of the system.

2. Work = Area [P v/s V]

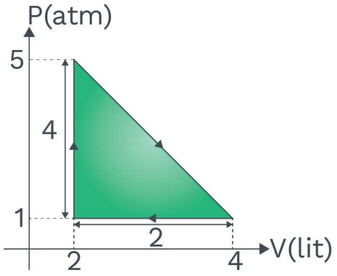
$A_1 > A_2 > A_3 \;||\; W_1 > W_2 > W_3$

3. $\Delta Q = \Delta U + W,\ \Delta U = 0$

$\Delta Q \propto W$

$Q_1 > Q_2 > Q_3$

Ex. Find out that how much heat is absorbed and released in given cyclic process -



Sol. $\Delta Q = \Delta U + \Delta W,\ \Delta U = 0$

$$\Delta Q = \Delta W = \frac{1}{2}\left(2\times10^{-3}\right)\left(4\times10^{5}\right)$$

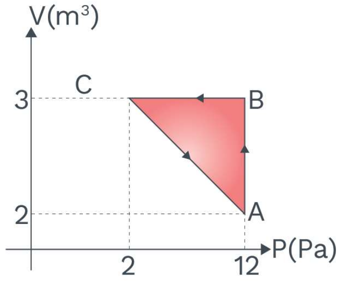
Cyclic process, $\Delta U = 0$

$\Delta Q = +400J$ (because all units are consider in SI)

Rack your Brain

When an ideal monoatomic gas is heated at constant pressure, find the fraction of heat energy supplied which increases the internal energy of gas.

Ex. Find out the work done in CA process in given cyclic process when heat is given to gas in complete process 5 J is.



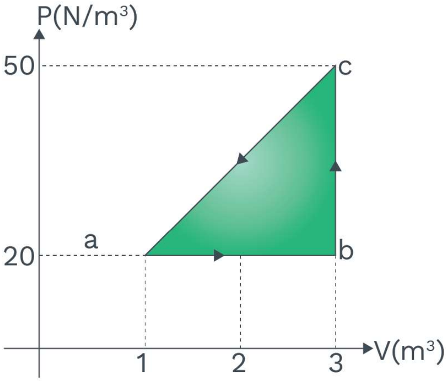
Sol. $\Delta Q = \Delta U + W$ here $\Delta U = 0$

$\Delta Q = W_{AB} + W_{BC} + W_{CA}$, $W_{BC} = 0$

$+5J = 12 \times 1 + W_{CA}$

$W_{CA} = 5 - 12 = -7J$

Ex. A thermodynamical process abca is plotted on a PV indicator diagram such that internal energy at a & b are zero and 20 J respectively. If heat supplied along bc is 30 J, then find



Sol. (i) $U_c = ?$

$$\Delta Q_{bc} = \Delta W_{bc} + U_c - U_b$$

$$\Delta Q_{bc} = \Delta W_{bc} + U_c - U_b$$

$$\Rightarrow \quad U_c = 30 + 20 = 50J$$

(ii) $\Delta W_{ab} = ?$

$$\Delta W_{ab} = 20 \times (3 - 1) = 40\text{ J}$$

(iii) $\Delta W_{bc} = ?$

$$\Delta W_{bc} = 0$$

(iv) $\Delta Q_{ab} = ?$

$$\Delta Q_{ab} = \Delta W_{ab} + U_b - U_a$$

$$\Rightarrow \quad \Delta Q_{ab} = 40 + 20 - 0 = 60\text{ J}$$

(v) $\Delta W_{abca} = ?$

$$\Delta W_{abca} = -\frac{1}{2}(2)(30) = -30\text{ J}$$

(vi) $\Delta W_{ba} = ?$

$$\Delta W_{ba} = -40\text{ J}$$

(vii) $\Delta U_{abca} = ?$

$$\Delta U_{abca} = 0$$

(viii) $\Delta W_{ac} = ?$

$$\Delta W_{ac} = 40 + 30 = 70\text{ J}$$

(ix) $\Delta Q_{ac} = ?$

$$\Delta Q_{ac} = \Delta W_{ac} + U_c - U_a = 70 + 50 - 0 = 120\,\text{J}$$

Ex. If Q amount of heat is given to a diatomic ideal gas in process in which the gas perform a work $\frac{2Q}{3}$ on its surrounding. Find the molar heat capacity (in terms of R) for the process.

Sol. According to FLOT

$$Q = W + \Delta U$$

$$Q = \frac{2}{3}Q + nC_v\Delta T$$

$$\frac{Q}{3} = n \times \frac{5}{2}R\Delta T$$

$$nC\Delta T = \frac{15}{2}nR\Delta T$$

$$\boxed{c = \frac{15}{2}R}$$

Different Types of Process:-

In thermodynamical process we follow 10 questions, as followings:

1. Which thermodynamic parameter is constant.
2. Which gas law is applied.
3. Form of gas equation.
4. Work-done in process.
5. Apply FLOT.
6. PV indicator diagram.
7. Slope of PV curve.
8. Specific condition required for process
9. Special example.
10. Comments/Remarks.

- **Isochoric process:**

1. Volume = Constant, $\Delta V = 0$
2. Gay-lussac's Law
3. $PV = nRT \Rightarrow P \propto T$
4. $\Delta W = 0$
5. FLOT: $\Delta Q_v = dU = nC_v dT$

$$C_v = \frac{R}{\gamma - 1}$$

Rack your Brain

During an isothermal expansion, a confined ideal gas does –150J then find the amount of heat.

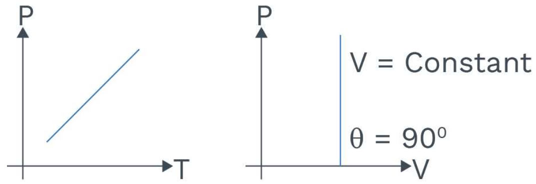
Definition

Thermodynamical process in which volume is constant during process is called isochoric process.

Key Points

- Isochoric process
- Gay-lussac's law

6. P v/s V



7. Slope of P-V diagram:

$$\text{slope} = \tan\theta = \tan 90^\circ = \infty$$

$$\text{slope} = \frac{dP}{dV} = \infty_{(\text{max.})}$$

8. Closed container
9. Heating of solid and liquid
10. $\Delta W = 0$, $dU = nC_v dT$

$\Rightarrow$ In isochoric process given heat can be convert in internal energy.

Specific heat : $[C_V]$

$$C_v = \frac{fR}{2} = \frac{R}{\gamma - 1}$$

$\Rightarrow$ Bulk modulus of elasticity (β)

$$\beta = -\frac{V \cdot \Delta P}{\Delta V \times 0} = \infty$$

- **Isobaric Process:-**

1. Pressure (P) = constant
2. Charles Law
3. $PV = nRT \Rightarrow V \propto T$
4. Work : $W = \int_{V_1}^{V_2} PdV \Rightarrow \quad P\Delta V = nR\Delta T$

$$\boxed{\Delta Q = \Delta U + W}$$

5. FLOT

$$\Delta Q = \Delta U + \Delta W \qquad \Delta Q \neq 0; \Delta U \neq 0; \Delta W \neq 0$$

$$C_P = C_V + R = \frac{\gamma R}{\gamma - 1}$$

$$\therefore \quad C_V = \frac{R}{\gamma - 1}$$

Concept Reminder

In isochoric process slope of P-V curve = ∞, work done during process = 0

$\Delta U = nC_V \Delta T$

Key Points

- Bulk modulus
- Specific heat
- Isobaric
- Charle's law

Definition

Thermodynamical process in which pressure is constant during process is called isobaric process.

Concept Reminder

For isobaric process:

V/T = constant

Work done during process

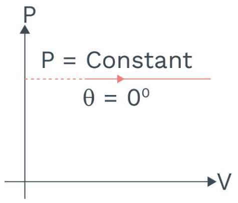
$W = P\Delta V = nR\Delta T$

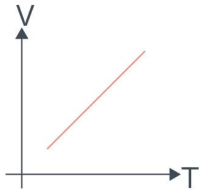
$$C_P = C_V + R = \frac{\gamma R}{\gamma - 1}$$

Bulk Modulus = 0

Slope = 0

6. P v/s V





7. Slope (m):-

$$m = \frac{dP}{dV} = \tan\theta$$

$$m = \frac{dP}{dV} = 0 \ \text{(minimum)}$$

8. Examples:-

- Constant pressure (open container)
- All phase conversion (melting of solid)

9. $(\Delta W)_P = nRdT = nR(T_2 - T_1) = P(V_2 - V_1)$

$(\Delta Q)_P = nC_PdT$

$(\Delta U)_P = nC_VdT$

⇒ In isobaric process $1/\gamma$ part of given heat is convent into internal energy and $1-1/\gamma$ part convent into work done.

⇒ **Specific heat:-**

$$C_P = C_V + R$$

⇒ **Bulk modulus:-**

$$\beta = \frac{-V\Delta P}{\Delta V} = 0, \qquad (\Delta P = 0)$$

Isothermal Process:-

⇒ Container is made by conducting walls and process runs slowly-slowly.

1. T = constant, $\Delta T = 0$
2. Boyle's law

$$\left[P \propto \frac{1}{V}\right]$$

3. PV = constant ⇒ $P_1V_1 = P_2V_2$

Rack your Brain

4.0 g of a gas occupies 22.4 litres at NTP. The specific heat capacity of the gas at constant volume is 5.0 JK^{-1}. If the speed of sound in this gas at NTP is 952 ms^{-1}, then find the heat capacity at constant pressure.
(Take gas constant R = 8.3 JK^{-1} mol^{-1})

Definition

The process in which temperature is constant is called isothermal process.

Concept Reminder

Isothermal Process:
Slow process
State equation
PV = constant
Work done
W = 2.303 nRT $\log_{10}(V_2/V_1)$
= 2.303 nRT $\log_{10}(P_1/P_2)$
$\Delta U = 0, \Delta T = 0$

$$\text{Slope} = -\frac{P}{V}$$

4. Work :

$$W = \int_{V_i}^{V_f} P\,dV \qquad \left| \begin{aligned} &PV = nRT \\ &P = \frac{nRT}{V} \end{aligned} \right.$$

$$= \int_{V_i}^{V_f} \frac{nRT}{V}\,dV$$

$$= nRT\left[\log_e V\right]_{V_i}^{V_f} = nRT[\log_e V_f - \log_e V_i]$$

$$\boxed{W = nRT\log_e \frac{V_f}{V_i}}$$

$$\Rightarrow \boxed{W = nRT\log_e \frac{P_i}{P_f}}$$

$$\boxed{W = 2.303\,nRT\log_{10} \frac{V_f}{V_i}}$$

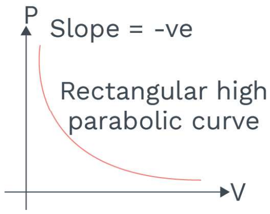
$$\Rightarrow \boxed{W = 2.303\,nRT\log_{10} \frac{P_i}{P_f}}$$

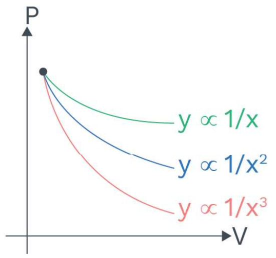
$\Delta Q = \Delta U + W \qquad \because \qquad T = \text{Constant}$

$\therefore \quad \Delta Q = W \qquad \therefore \qquad \Delta T = 0,\ \Delta U = 0,$

5. FLOT : $\Delta Q = \Delta W$, [dU = 0 ; only for ideal gas]
[Isothermal process is practically impossible for ideal gas]

6. P v/s V





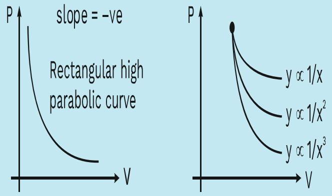
Rack your Brain

If γ denotes the ratio of two specific heats of a gas, then calculate the ratio of slopes of adiabatic and isothermal PV curves at their point of intersection.

Key Points

- Isothermal
- Slow process
- Boyle's law

Concept Reminder



In inverse relation it power is increase then slope of curve is also increase.

7. Slope (m):

$$m = \frac{dP}{dV}$$

PV = constant

Differentiate w.r.t. V

$$d(PV) = d(nRT)$$

$$PdV + VdP = 0$$

$$VdP = -PdV$$

$$\frac{dP}{dV} = -\frac{P}{V} \qquad \text{(slope = –ve)}$$

8. Condition for isothermal process:-
   (i) Container should be heat conducting
   (ii) Process should be carried out slowly
9. All phase conversions are isothermal process.
10. $\Delta W = 2.303nRT\log_{10}\left(\frac{V_2}{V_1}\right)$

$$= \Delta W = 2.303nRT\log_{10}\left(\frac{P_1}{P_2}\right)$$

⇒ In isothermal process given complete heat can be convert into work

⇒ Specific heat :-

$$C_V = \frac{\Delta Q}{n\Delta T} = \frac{\Delta Q}{0} = \infty$$

⇒ **Bulk Modulus (β) :**

$$\beta = \frac{-VdP}{dV} = -V\left(\frac{-P}{V}\right) = P$$

- **Adiabatic Process / Isentropic Process :-**

1. Heat (Q) = constant ⇒ $\Delta Q = 0$;

$$S(\text{entropy}) = \text{constant}$$

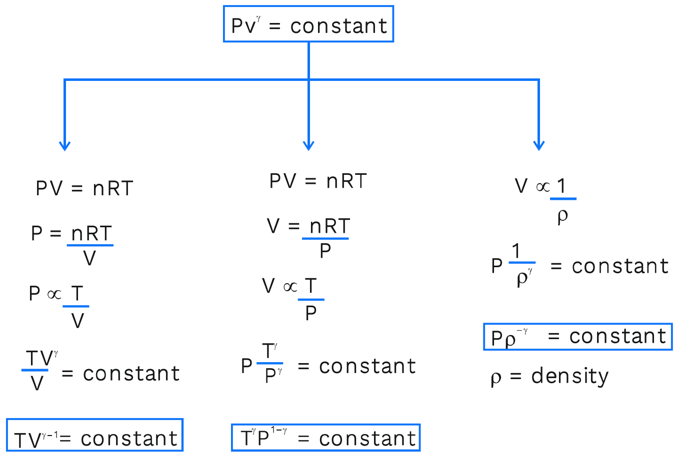
2. Poisson law
3. $\boxed{PV^{\gamma} = \text{constant}} \Rightarrow \boxed{P \propto \frac{1}{V^{\gamma}}}$

Key Points

- Adiabatic
- Poisson law
- Entropy

Definition

The theromodynamical process in which change in heat during process is zero is called adiabatic process.



$\boxed{Pv^{\gamma} = \text{constant}}$

$PV = nRT$

$P = \frac{nRT}{V}$

$P \propto \frac{T}{V}$

$\frac{TV^{\gamma}}{V} = \text{constant}$

$\boxed{TV^{\gamma-1} = \text{constant}}$

$PV = nRT$

$V = \frac{nRT}{P}$

$V \propto \frac{T}{P}$

$P\frac{T^{\gamma}}{P^{\gamma}} = \text{constant}$

$\boxed{T^{\gamma}P^{1-\gamma} = \text{constant}}$

$V \propto \frac{1}{\rho}$

$P\frac{1}{\rho^{\gamma}} = \text{constant}$

$\boxed{P\rho^{-\gamma} = \text{constant}}$

ρ = density

4. $\Delta Q = \Delta U + W\,,\ \Delta Q = 0$

$W = -\Delta U = -nC_v\Delta T$

$$W = \frac{-nR}{\gamma - 1}(T_f - T_i) = \frac{P_iV_i - P_fV_f}{\gamma - 1}$$

5. FLOT :

$\Delta Q = 0$

$dW = -dU = -nC_vdT$

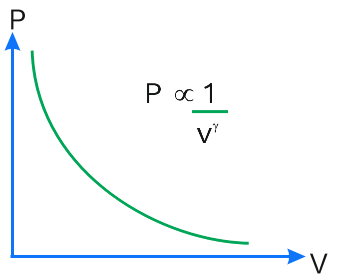
$dU = nC_vdT$

$$C_v = \frac{R}{\gamma - 1}$$

$dT = T_2 - T_1$

$$dW = \frac{-nR}{\gamma - 1}(T_2 - T_1) = \frac{nR}{\gamma - 1}(T_1 - T_2)$$

6.



Concept Reminder

Adiabatic process: $\Delta Q = 0$

State equation: $PV\gamma = C$

$TV\gamma^{-1} = C$

$T^{g}P^{1-}\gamma = C$

$P\rho^{-}\gamma = C$

Work done:

$$W = \frac{P_iV_i - P_fV_f}{\gamma - 1} = \frac{-nR}{\gamma - 1}\left(T_f - T_i\right)$$

$$\text{Slope} = -\frac{\gamma P}{V}$$

Rack your Brain

During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. Then find the ratio C_p/C_v for the gas.

7. PV^{γ} = const.

Differentiate w.r.t. (V)

$$\frac{d}{dV}\left(PV^{\gamma}\right) = 0$$

$$P(\gamma V^{\gamma-1}) + V^{\gamma}\frac{dP}{dV} = 0$$

$$V^{\gamma}\frac{dP}{dV} = -\gamma PV^{\gamma-1}$$

$$\frac{dP}{dV} = \frac{-\gamma PV^{\gamma-1}}{V^{\gamma}} \Rightarrow \frac{dP}{dV} = -\gamma\frac{P}{V}$$

(slope of adiabatic) = γ (slope of isothermal)

Concept Reminder

Slope of adiabatic process
= γ × slope of isothermal process.

8. Conditions for adiabatic process-

- Container should be perfectly heat insulated.
- Process must be carried out rapidly, faster or suddenly.

9. Examples for adiabatic process-

- Propagating of sound wave in gaseous medium.
- Sudden blast of tyre, tube (cooling effect).
- If CO_2 gas cylinder is suddenly opened then conversion of CO_2 gas into dry ice (cooling effect).
- Continuous hammering (heating effect).
- Pump barrel becomes hot during pumping of air into tyre tube.
- Free expansion of ideal and real gases.

10. From FLOT: 0 = du + DW

Key Points

- Sudden process
- Real gases
- Ideal gas
- Expansion
- Compression

$$\boxed{\Delta W = -\Delta U}$$

$V\uparrow \Rightarrow$ Adiabatic expansion
$\Rightarrow \Delta W = +ve$
$\Rightarrow \Delta U = -ve$
$U\downarrow \Rightarrow T\downarrow$ (Adiabatic cooling)

$V\downarrow \Rightarrow$ Adiabatic compression
$\Rightarrow \Delta W = -ve$
$\Rightarrow \Delta U = +ve$
$U\uparrow \Rightarrow T\uparrow$ (Adiabatic heating)

Ex. 1 mole ideal gas is at 27°C temperature. If at same temperature. It is expanded 1l to 2l. then find out the work done.

Sol. $\Delta W = nRT\log_e \frac{V_f}{V_i}$

$$\Rightarrow \quad \Delta W = 1 \times \frac{25}{3} \times 300 \times 2.303 \times \log_{10}\left(\frac{2}{1}\right)$$

$$\Rightarrow \quad \Delta W = 1 \times 2500 \times 2.303 \times 0.3$$

$$\Rightarrow \quad \Delta W \approx 1750 \text{ J}$$

Ex. An ideal gas is suddenly compressed, then it's volume is change 2 V_0 to $V_0/2$. then find out ratio of pressure (g = 1.5)

Sol. Suddenly compressed → Adiabatic process.

$$PV^{\gamma} = \text{constant}$$

$$P \propto \frac{1}{V^{\gamma}}$$

$$\Rightarrow \quad \frac{P_i}{P_f} \propto \left(\frac{V_f}{V_i}\right)^{\gamma}$$

$$\Rightarrow \quad \frac{P_i}{P_f} \propto \left[\frac{V_0}{4V_0}\right]^{3/2}$$

$$\frac{P_i}{P_f} \propto \left(\frac{1}{2^2}\right)^{3/2} \quad \Rightarrow \quad \frac{P_i}{P_f} \propto \frac{1}{8}$$

Ex. An ideal gas is at initial temperature of 27°C, if it is suddenly compress than its volume is changed 1/8 times of initial then. find out the final temperature of gas $\left(\gamma = \frac{5}{3}\right)$

Sol. $TV^{\gamma-1} = \text{constant}$

$$T_1 \propto \frac{1}{V^{\gamma-1}}$$

$$\frac{T_i}{T_f} \propto \left(\frac{V_f}{V_i}\right)^{\gamma-1}$$

$$\frac{300}{T_f} = \left(\frac{1}{8}\right)^{\frac{5}{3}-1}$$

$$\frac{300}{T_f} = \left(\frac{1}{2^3}\right)^{\frac{2}{3}}$$

Rack your Brain

Air is expanded from 50 litre to 150 litre at 2 atmospheric pressure (1 atm pressure = 10^5 kgm^{-2}). Calculate the external work done.

$$T_f = 1200\,K$$

$$T_f = 927°C$$

Ex. An ideal gas is at initial temperature of 27°C, If it is suddenly compress then it's pressure is made 8 times of initial, then find out final temperature of gas. $[\gamma = 1.5]$

Sol. $T^{\gamma}P^{1-\gamma} = \text{constant}$

$$\left(\frac{T_i}{T_f}\right)^{\gamma} = \left(\frac{P_f}{P_i}\right)^{1-\gamma} \Rightarrow \left(\frac{300}{T_f}\right)^{3/2} = (8)^{-1/2}$$

$$\left(\frac{300}{T_f}\right)^{3/2} = \left(\frac{1}{2^3}\right)^{1/2} \Rightarrow \left(\frac{300}{T_f}\right)^{3/2} = \left(\frac{1}{2}\right)^{3/2}$$

$T_f = 600$ K.
$T_f = 327°C$

Ex. For a diatomic gas in Adiabatic process ratio of density is 32. then find out ratio of pressure.

Sol. $PV^{\gamma} = \text{constant}$

$$P\left(\frac{M}{\rho}\right)^{\gamma} = \text{constant}$$

$$V = \frac{M}{\rho},\ V \propto \frac{1}{\rho}$$

$$P \propto (\rho)^{\gamma}$$

$$\frac{P_i}{P_f} = \left(\frac{32}{1}\right)^{7/5}$$

$$\frac{P_i}{P_f} = \left(\frac{2^5}{1}\right)^{7/5} \qquad \Rightarrow \frac{P_i}{P_f} = \frac{128}{1}$$

Ex. A monoatomic gas is at pressure P and Volume V expanded isothermally to a volume 2V and after the adiabatically expanded to volume 16 V, then find out the final pressure of gas.

Sol. $P, V \xrightarrow{\text{isothermally}} \left(\frac{P}{2}, 2V\right)$

PV = Constant

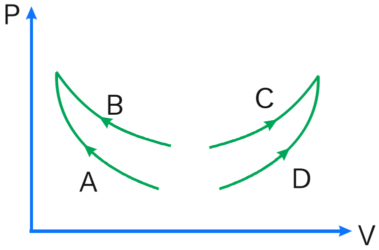
$$P \propto \frac{1}{V}$$

$$\left(\frac{P}{2}, 2V\right) \xrightarrow{\text{Aidabatically}} \left(P', 16V\right)$$

$$\Rightarrow \quad \frac{P_i}{P_f \times 2} = \left(\frac{16V}{2V}\right)^{\gamma}$$

$$\frac{P}{2P_f} = (8)^{\gamma}, \; P_f = \frac{P}{64}$$

Ex. Identify the curves for adiabatic and isothermal process.



Sol.

A → Adiabatic

B → Isothermal

C, D → Polytrophic

→ $(\text{Slope})_A > (\text{Slope})_B$

Slope of isothermal and adiabatic is negative.

Ex. The work 146 J is performed in order to compress 1 mole of gas Adiabatically and in this process the temperature of the gas is increase by 7°c then gas is

(1) Monoatomic
(2) Diatomic
(3) Linear polyatomic
(4) Mixture of mono and diatomic gas

Sol. (2)

$$W = \frac{nR(T_i - T_f)}{\gamma - 1} \qquad [T_f - T_i] = 7$$

$$\Rightarrow \quad 146 = \frac{1 \times 25 \times 7}{3(\gamma - 1)} = \frac{25 \times 7}{3(\gamma - 1)}$$

$$\Rightarrow \quad \frac{58.31 + 146}{146} = \gamma$$

$$\Rightarrow \quad \gamma = 1.4$$

$\gamma = 1.4$ for diatomic gas.

Ex. For an ideal gas in adiabatic process pressure is proportional to cube of temperature then find out the γ.

Sol. $P \propto T^3$

$$\frac{T^{\gamma}}{P^{\gamma-1}} = \text{constant}$$

$$P^{\gamma-1} = T^{\gamma}$$

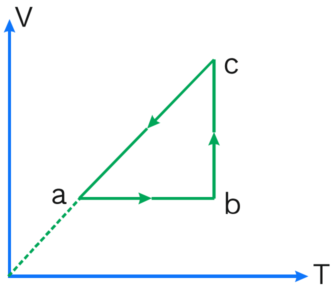
$$P^{\frac{\gamma-1}{\gamma-1}} = T^{\frac{\gamma}{\gamma-1}}$$

$$\therefore \quad \frac{\gamma}{\gamma-1} = 3$$

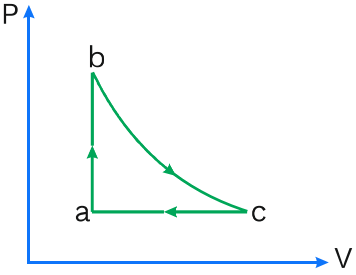
$$\Rightarrow \quad \gamma = 3\gamma - 3$$

$$\Rightarrow \quad \gamma = \frac{3}{2}$$

Ex. Convert the given V-T diagram into PV diagram



Sol.



$a \rightarrow b; V = \text{constant}\ ;\ P \propto T$

$T \uparrow; P \uparrow$

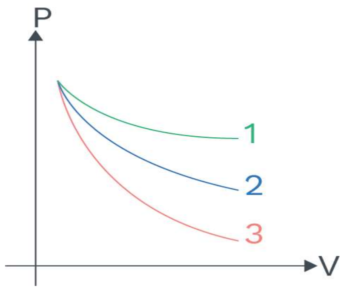
$b \rightarrow c; T = \text{constant}\ ;\ P \propto \frac{1}{V}$

$V \uparrow; P \downarrow$

$c \rightarrow a; P = \text{constant}\ ;\ V \propto T$

$V \downarrow; T \downarrow$

Ex. In given indicating curve which curve represent the He gas, H_2 gas and CO_2 in adiabatic process.



Sol. for monoatomic (He) $\gamma = \frac{5}{3} = 1.7$

for diatomic (H_2) $\gamma = \frac{7}{35} = 1.33$

for triatomic (CO_2) $\gamma = \frac{9}{7} = 1.28$

$$\boxed{P \propto \frac{1}{V^{\gamma}}}$$

when $\gamma \uparrow$; slope $\uparrow$

curve-1 $\rightarrow$ CO_2
curve-2 $\rightarrow$ H_2
curve-3 $\rightarrow$ He

- **Polytropic Process**

⇒ A process different to standard process then it is called polytropic process.

$$\text{Standard process} \rightarrow \boxed{P \propto T, V \propto T, P \propto \frac{1}{V}, P \propto \frac{1}{V^{\gamma}}}$$

$$\boxed{PV^{m} = \text{constant}}$$

condition $\rightarrow$ $\boxed{m \neq 0,\ m \neq 1,\ m \neq \gamma}$

⇒ Slope :

$$\frac{dP}{dV} = -\frac{mP}{V}$$

⇒ Bulk modulus :

$\beta = -mP$

Rack your Brain

Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2V. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

⇒ work :

$$W = \frac{nR(T_i - T_f)}{m-1}$$

⇒ Specific heat

$$\boxed{C = C_V - \frac{R}{m-1}}$$

Ex. For a ideal gas follows the given process in which P is directly proportional to $V^2 \left(P \propto V^2\right)$ then find out specific heat of gas.

Sol. $P \propto V^2 \Rightarrow \frac{P}{V^2} = \text{constant.}$

$$\Rightarrow \quad PV^{-2} = \text{constant}, \qquad \because \quad PV^m = \text{constant}$$

$$\therefore \quad m = -2$$

$$C = C_V - \frac{R}{m-1} = \frac{3R}{2} - \frac{R}{-2-1}$$

$$C = \frac{9R + 2R}{6} = \frac{11R}{6}$$

Key Points

- Polytropic

Ex. Two mole diatomic gas is heated such that it's temperature is increased by 10°C, then find out total heat supplied for given process.

Sol. $P \propto V$

$$\Rightarrow \quad PV^{-1} = \text{constant}, \qquad \because \; PV^m = \text{constant}$$

$$\therefore \quad m = -1$$

$$C = C_V - \frac{R}{m-1}$$

$$C = \frac{5R}{2} - \frac{R}{-1-1} = \frac{5R}{2} + \frac{R}{2} \Rightarrow C = 3R$$

and

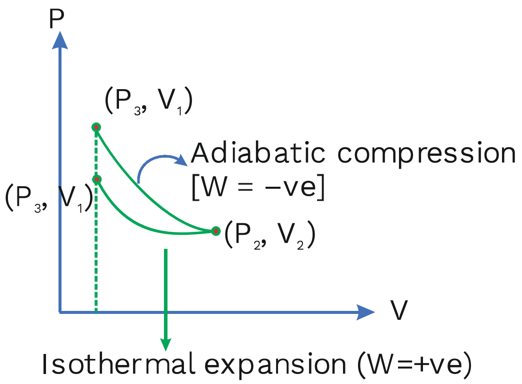
$$\Delta Q = nC\Delta T = 2(3R)\,10 = 120 \text{ cal.}$$

Note :

In adiabatic expansion temperature is decreased but in Adiabatic compression, temperature is increase.

Ex. An ideal gas is isothermally expanded from $(P_1, V_1) \rightarrow (P_2, V_2)$ and after then adiabatically compressed from $(P_2V_2 \rightarrow P_3V_1)$ then find out the nature of work and compare the pressure.

Sol.

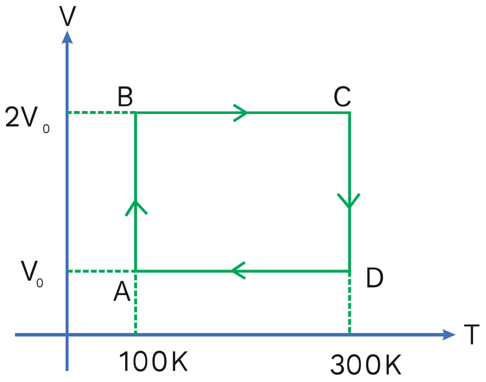


[W = –ve or W < 0]

& $P_3 > P_1$

Ex. For 2 mole gas find out the work done in given cyclic process.

Sol.



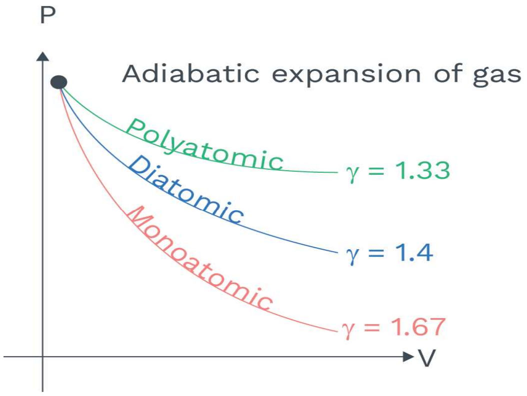
$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$W = 2(2)100\log_e \frac{2V_0}{V_0} + 0 + 2(2)(300)\log_e \frac{V_0}{2V_0} + 0$$

$$W = 400(0.7) + 1200(0.7)$$

$$W = 280 - 840 = -560\,\text{cal}$$

Note : For different type of gas PV curve in adiabatic process.



General expression for $C(C_P \text{ or } C_V)$ in the process PV^x = constant

$$\boxed{C = \frac{R}{\gamma - 1} + \frac{R}{1 - x}}$$

for isobaric process, P = const. so x = 0

$$\therefore \qquad C = C_P = \frac{R}{\gamma - 1} + R = C_V + R$$

for isothermal process, PV = constant, so x = 1

$$\therefore \qquad C = \infty$$

for adiabatic process ; $PV\gamma$ = constant, so $x = \gamma$

Concept Reminder

Specific heat:

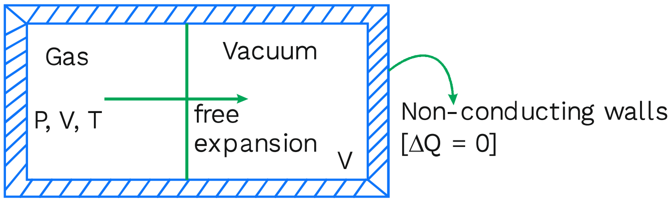
$$\boxed{C = \frac{R}{\gamma - 1} + \frac{R}{1 - x}}$$

⇒ Values of f, U, C_V, C_P and γ for different gases are shown in table below.

Atomicity of gas	**f**	**C_V**	**C_P**	**γ**
Monoatomic	3	$\frac{3}{2}R$	$\frac{5}{2}R$	$\frac{5}{3} = 1.67$
Diatomic, Triatomic and Triatomic linear (at normal temperature)	5	$\frac{5}{2}R$	$\frac{7}{2}R$	$\frac{7}{5} = 1.4$
Polyatomic, Triangular Non-linear	6	$\frac{6}{2}R = 3R$	$\frac{8}{2}R = 4R$	$\frac{4}{3} = 1.33$

Free Expansion

⇒ If gas molecules are expanded without any resistive force, then this expansion is called free expansion of gas.



Condition for free expansion.

- Expansion of gas against Vacuum
- Container should be heat insulated

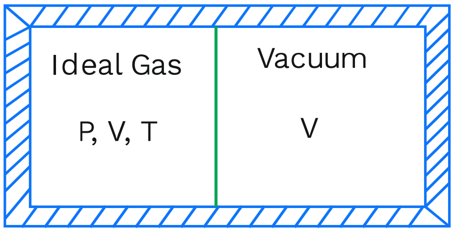
Definition

If gas molecules are expanded without any resistive force, then this expansion is called free expansion of gas.

Ideal Gas	**Real Gas**
$\Delta Q = 0 \rightarrow \text{Adiabatic}$	$\Delta Q = 0 \rightarrow \text{Adiabatic}$
$\Delta W = 0 \rightarrow [\text{intermolecular}]$	$\Delta W \neq 0 \rightarrow [\text{intermolecular}]$
$\Delta U = 0 \Rightarrow U = \text{constant},\ \Delta T = 0 \Rightarrow T = \text{constant}$	$\Delta U \neq 0$

- For ideal gas (Isothermal & adiabatic)
  U = constant $\Rightarrow$ T = constant
  Temperature of ideal gas is constant in free expansion process.
- For ideal gas [only adiabatic]
  Temperature of Vander-wall gas decrease in free expansion because the thermal internal energy decreases.

Ex. If ideal gas expands against vacuum, insulated container then what will be final volume, final pressure and final temperature?



Sol. (i) $T' = T$ (ii) $V' = 2V$

(iii) $P' = \dfrac{P}{2}\ \left[P \propto \dfrac{1}{V}\right]$

Drawback of FLOT

1st Law of Thermodynamics:

- No information about what part of heat converted into mechanical work.
- Does not give proper direction of heat flow.

Rack your Brain

A gas expands 0.25 m^3 at constant pressure 10^3 N/m^2, then calculate work done.

Key Points

- Vander-wall gas
- Free expansion

Rack your Brain

A refrigerator works between 4°C and 30°C. It is required to remove 600 calories of heat every second in order to keep the temperature of the refrigerated space constant. Calculate the power.

2^{nd} Law of Thermodynamics

→ **Kelvin Plank Statement :**

This statement say that it is impossible that an engine can have 100% efficiency because it is impossible to absorb complete heat from source and to convert it into work without giving to sink.

Clausius Statement :

This statement state that without any external agency heat can not flow from cold body to hot body

Heat Engine

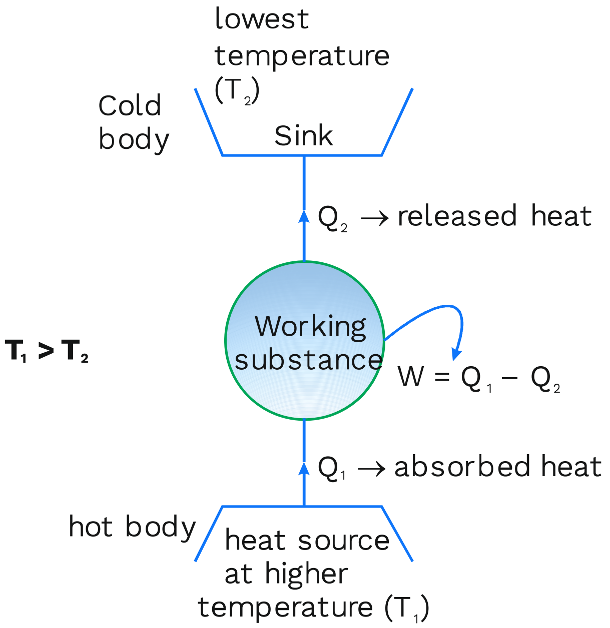
→ An engine which convert heat energy into mechanical energy is know as heat engine.

Definition

An engine which convert heat energy into mechanical energy is know as heat engine.

Type of Heat Engine

1. **External combustion engine :**
   In this engine combustion of fuel take place outside the engine.
   Ex. steam engine
2. **Internal combustion engine :**
   In this engine combustion of fuel take place inside the engine.
   Ex. Diesel engine and petrol engine.



Rack your Brain

Calculate the efficiency of an ideal heat engine working between the freezing point and boiling point of water.

Efficiency of Heat Engine:

$$\eta = \frac{\text{work}}{\text{heat given}} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

$$\Rightarrow \quad \frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

Maximum efficiency

$$\eta = \left(\frac{T_1 - T_2}{T_1}\right) \times 100\,\%$$

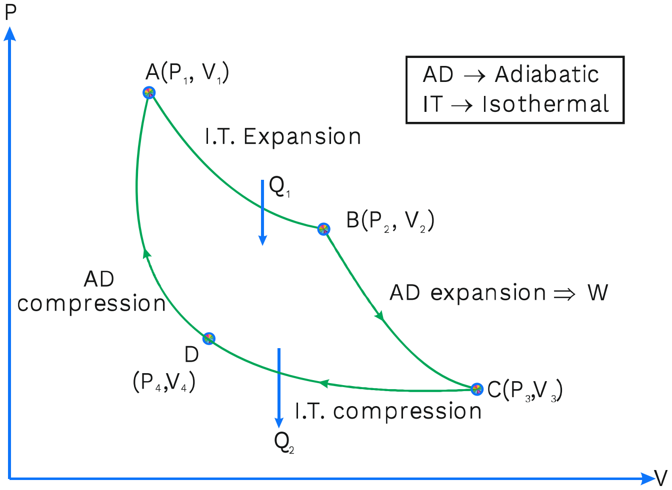
$$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$$

Note : (1) If $T_1 = \infty\,K$ or $T_2 = 0\,K$ then $\eta = 100\%$ (that is practically impossible)

(2) Efficiency depends on temperature of source (T_1K) and temperature of sink (T_2K)

Source and sink are high thermal capacity body.

Carnot Heat Engine :-



AB → Isothermal Expansion (IT Exp.)
BC → Adiabatic Expansion (AD Exp.)
CD → Isothermal Compression (IT comp.)
DA → Adiabatic Compression (AD comp.)

⇒ It is based on Carnot cycle and it have maximum efficiency.

Concept Reminder

Efficiency of heat engine

$$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$$

If $T_1 = \infty$ or $T_2 = 0$
then $\eta = 100\%$

Key Points

- Heat engine
- Efficiency
- Carnot engine

Rack your Brain

A carnot engine, having an efficiency of $\eta = 1/10$ as heat engine, is used as a refrigerator. If the work done on the system is 10 J, then calculate amount of energy absorbed from the reservoir at lower temperature.

1. Carnot engine is an ideal engine.
2. In a Carnot engine four parts.
    - Source
    - Sink
    - Working substance
    - Stand
3. In Carnot engine ideal gas is used as working substance.
4. Stand is non-conducting body it is used in Adiabatic process.
5. It is a hypothetical engine because working substance is an ideal gas.
6. It's efficiency is maximum but not 100%.
7. This cycle process have two isothermal and two adiabatic process.

⇒ **Work done by Carnot Engine :**

$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$W = nRT_1 \log_e \frac{V_2}{V_1} + \frac{nR(T_1 - T_2)}{\gamma - 1} + nRT_2 \log_e \frac{V_4}{V_3} + \frac{nR(T_2 - T_1)}{\gamma - 1}$$

$$W = nRT_1 \log_e \frac{V_2}{V_1} + \frac{nR(T_1 - T_2)}{\gamma - 1} + nRT_2 \log_e \frac{V_4}{V_3} - \frac{nR(T_1 - T_2)}{\gamma - 1}$$

In complete Carnot cycle work done in Adiabatic process is **zero**.

So, $T_1V_2^{\gamma-1} = T_2V_3^{\gamma-1}$ and $T_1V_1^{\gamma-1} = T_2V_4^{\gamma-1}$

$$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

So work done

$$\boxed{W = nRT_1 \log_e \frac{V_2}{V_1} + nRT_2 \log_e \frac{V_4}{V_3}}$$

$$W = nR\left(T_1 - T_2\right)\log_e \frac{V_2}{V_1}$$

$$= 2.303\, nR\left(T_1 - T_2\right)\log_e \left(\frac{V_2}{V_1}\right)$$

⇒ **Efficiency :**

$$\eta = \frac{\text{output}}{\text{input}} \Rightarrow \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$\boxed{\eta = 1 - \frac{Q_2}{Q_1}} = \boxed{1 - \frac{T_2}{T_1}}$$

$$\eta\% = \left(1 - \left(\frac{T_2}{T_1}\right)\right) \times 100\%$$

Some Important Points :

- For 100% efficiency, $T_2 = 0$ K or $T_1 = \infty$ K

  → Carnot engine can not have 100% efficiency because for 100% efficiency $T_2 = 0$ (sink) and $T_1 = (\infty$ K). (source) i.e. is impossible.
- For maximum efficiency T_2/T_1 is minimum
- $\eta_{max} = 1 - \left(\frac{T_2}{T_1}\right)_{min.}$
- $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

Rack your Brain

A carnot engine operating between temperature T_1 and T_2 has efficiency 1/6. When T_2 is lowered by 62 K, its efficiency increases to 1/3. Then calculate T_1 and T_2.

Carnot Theorem :

According to this theorem **maximum** efficiency engine can not based on irreversible cycle it is based on Reversible cycle.

Ex. A heat engine is working at 27°C temperature and –123°C. Find efficiency of engine.

Sol. $\eta = 1 - \frac{T_2}{T_1}$

$$\Rightarrow \quad \eta = 1 - \frac{150}{300} = 1 - \frac{1}{2} = \frac{1}{2} = 0.5$$

$$\Rightarrow \quad \eta = 0.5$$

$$\Rightarrow \quad \%\eta = 50\%$$

Ex. A Carnot engine efficiency is 1/6. If temperature of sink is reduced 62°C/K then efficiency is made 2 times of initial then find out the temperature of source and sink.

Sol. $\eta = \frac{1}{6}$

$$\eta = 1 - \frac{T_2}{T_1} \Rightarrow \frac{1}{6} = 1 - \frac{T_2}{T_1}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{5}{6}$$

62°C temperature is decreases of sink then efficiency

$$\eta' = 2\eta = 2 \times \frac{1}{6} = \frac{1}{3}$$

$$\Rightarrow \quad \eta' = 1 - \frac{T_2 - 62}{T_1} = 1 - \frac{T_2 - 62}{T_1}$$

$$\Rightarrow \quad \frac{1}{3} = 1 - \frac{T_2}{T_1} + \frac{62}{T_1}$$

$$\Rightarrow \quad \frac{62}{T_1} = \frac{1}{3} - 1 + \frac{5}{6} = \frac{2-6+5}{6} = \frac{1}{6}$$

$$\Rightarrow \quad \frac{62}{T_1} = \frac{1}{6}$$

T_1 = 372 K.

$$\frac{T_2}{T_1} = \frac{5}{6} \Rightarrow \frac{T_2}{372} = \frac{5}{6}$$

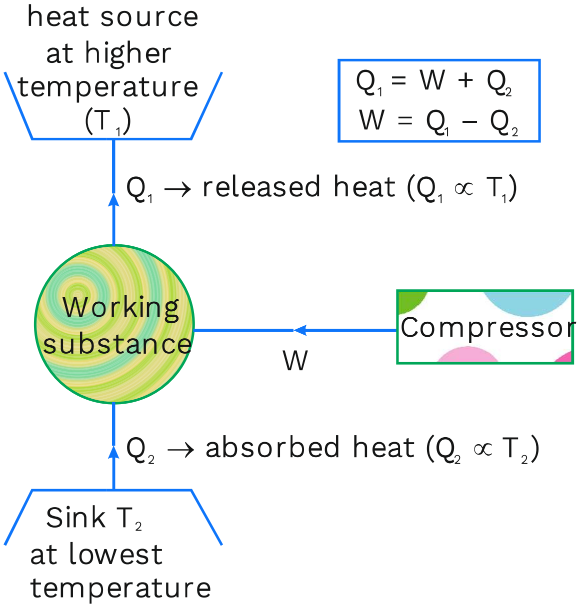
T_2 = 310 K

A.C. & Refrigerator

→ This instrument is work opposite to Carnot engine.

→ In this instrument released heat is always greater than absorb heat.

→ In a close room. If door of Refrigerator is open then temperature of room is increase.



Rack your Brain

The temperature inside a refrigerator is t_2°C and the room temperature is t_1°C. Then calculate the amount of heat delivered to the room for each joule of electrical energy consumed ideally.

Carnot Refrigerator:

$$\boxed{Q_1 = W + Q_2}$$

$$\boxed{W = Q_1 - Q_2}$$

In Carnot refrigerator heat is absorbed from a cold body and some external work is to be done on refrigerator and the total heat is given out at higher temperature.

Refrigerator is just opposite to heat engine so coefficient of performance (COP),

$$\beta = \frac{\text{heat absorbed}}{\text{work}} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

$$\boxed{\text{COP}(\beta) = \frac{T_2}{T_1 - T_2}}$$

Relation between η (Carnot engine efficiency) and β

$$\boxed{\eta = \frac{1}{1+\beta}} \text{ or } \boxed{\beta = \frac{1}{\eta} - 1}$$

Ex. In Carnot engine efficiency is 10% is used as a compressor in a Refrigerator then work done is 20 J, then find out how much heat absorb.

Sol. $\beta = \frac{1-\eta}{\eta} = \frac{1-0.1}{0.1} = 9$

$$\because \quad \beta = \frac{Q_2}{W}$$

$$9 = \frac{Q_2}{20}$$

$$Q_2 = 180\,\text{J}.$$

Ex. A Carnot engine having an efficiency of $\frac{1}{10}$ as heat engine is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is :

Sol. $\eta = \frac{1}{10}$

$$\beta = 1 - \frac{1}{\eta} \Rightarrow \beta = 9 = \frac{Q_2}{W}$$

$$Q_2 = 9 \times 10 \Rightarrow 90 \text{ J}$$

[Heat absorbed at lower temperature]

$Q_1 = 90 + 10 \Rightarrow Q_1 = 100$ J

[Heat released at higher temperature].

Concept Reminder

$$\text{COP } (\beta) = \frac{T_2}{T_1 - T_2}$$

$$\eta = \frac{1}{1+\beta}$$

Key Points

- Carnot refrigerator
- Coefficient of performance

EXAMPLES

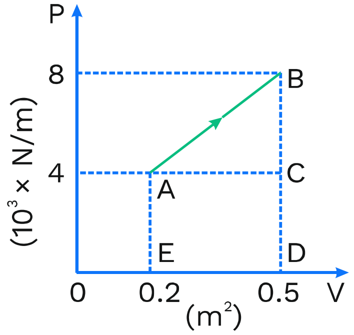
Q1 **The pressure in monoatomic gas increases linearly from** $4 \times 10^{5}\text{Nm}^{-2}$ **to** $8 \times 10^{+5}\text{Nm}^{-2}$ **when its volume increases from 0.2m³ to** $0.5\,\text{m}^{3}$ **Calculate.**

(i) Work done by the gas,
(ii) Increase in the internal energy,
(iii) Amount of heat supplied,
(iv) Molar heat capacity of the gas.

Sol. We are taking one mole of gas, we are given

$P_1 = 4 \times 10^5\ \text{Nm}^{-2}$, $P_2 = 8 \times 10^{+5}\ \text{Nm}^{-2}$, $V_1 = 0.2\ \text{m}^3$, $V_2 = 0.5\ \text{m}^3$

(i) Work done by the gas = Average pressure × Change in volume



$$\Delta W = \frac{1}{2}(AE + BD) \times AC = \frac{1}{2}(4 \times 10^5 + 8 \times 10^5) \times (0.5 - 0.2)$$

$$= \frac{1}{2} \times 12 \times 10^5 \times 0.3 = 1.8 \times 10^5\ \text{J}$$

(ii) Increase in internal energy

$$\Delta U = C_V (T_2 - T_1) = \frac{C_V}{R}(P_2V_2 - P_1V_1)$$

For monoatomic gas $C_V = \frac{3}{2}R$

$$\therefore\ \Delta U = \frac{3}{2}[(8 \times 10^5 \times 0.5) - (4 \times 10^5 \times 0.2)]$$

$$= \frac{3}{2}[4 \times 10^5 - 0.8 \times 10^5] = 4.8 \times 10^5\ \text{J}$$

(iii) Amount of heat supplied is

$$\Delta Q = \Delta U + \Delta W$$

$$= 3.2 \times 10^5 + 1.8 \times 10^5 = 5.0 \times 10^5\ \text{J}$$

(iv) Molar heat capacity of the gas C,

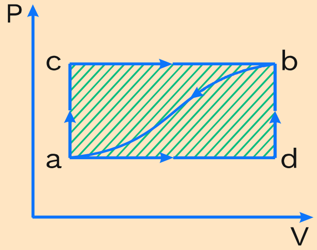
$$= \frac{Q}{\Delta T} = \frac{QR}{(P_2V_2 - P_1V_1)} = \frac{5 \times 10^5 \times 8.31}{1 \times 3.2 \times 10^5}$$

= 13.0 J/mole K

Q2 **When a system is taken from state a to state b, in figure along the path $a \to c \to b$, 60 J of heat flow into the system, and 30 J of work is done :**

(i) How much heat flows into the system along the path $a \to d \to b$ if the work is 10 J.

(ii) When the system is returned from b to a along the curved path, the work done by the system is –20 J. Does the system absorb or liberate heat, and how much?



Sol. For the path a, c, b, $\Delta U = \Delta Q - \Delta W$

$= 60 - 30 = 30$ J or $U_b - U_a = 30$ J

(i) Along the path a, d, b,

$Q = \Delta U + \Delta W = 30 + 10 = 40$ J

(ii) Along the curved path b, a, $Q = (U_a - U_b) + \Delta W$

$= (-30) + (-20) = -50$ J, heat flows out the system.

Q3 **5 moles of oxygen is heated at constant volume from 10°C to 20°C. What will be change in the internal energy of the gas? The gram molecular specific heat of oxygen at constant pressure, C_P = 8 cal/mole and R = 2 cal/mole °C.**

Sol. $C_V = C_P - R = 8 - 2 = 6$ cal/mole °C

$\therefore$ Heat absorbed by 5 moles of oxygen at constant volume

$Q = nC_V\Delta T = 5 \times 6\ (20 - 10) = 30 \times 10 = 300$ cal

At constant volume $\Delta V = 0 \quad \therefore \Delta W = 0$

$\therefore$ From first law of thermodynamics

$Q = \Delta U + W$

$\Rightarrow 300 = \Delta U + 0 \quad \Rightarrow \quad \Delta U = 300$ cal.

Q4 **At normal pressure and 0°C temperature the volume of 1 kg of ice is reduced by 91 cm^3 on melting. Latent heat of melting of ice is 3.4×10^5 J/kg. Calculate the change in the internal energy when 2kg of ice melts at normal pressure and 0°C. ($P=1.01 \times 10^5\ Nm^{-2}$)**

Sol. Heat energy absorbed by 2 kg of ice for melting

$Q = mL = 2 \times 3.4 \times 10^5 = 6.8 \times 10^5 J$

Change in volume of 2 kg of ice

$= 2 \times -91 = -182\ cm^3 = -182 \times 10^{-6}\ m^3$

$\therefore\ W = P\Delta V = 1.01 \times 10^5 \times (-182 \times 10^6) = -18.4\ J$

Since, work is done on ice so work W is taken -ve. Now from first law of thermodynamics

$Q = \Delta U + W \Rightarrow \Delta U = Q - W = 6.8 \times 10^5 - (-18.4)$

$= (6.8 \times 10^5 + 18.4) J$

Q5 **What amount of heat must be supplied to 2.0×10^{-2} kg of nitrogen (at room temperature) to raise the temperature by 45°C at constant pressure. Molecular mass of N_2 = 28, R = 8.3 J mol^{-1} K^{-1}.**

Sol. Here $m = 2 \times 10^{-2}$ kg,

$$\Rightarrow n = \frac{m}{M} = \frac{2 \times 10^{-2}}{28 \times 10^{-3}} = \frac{5}{7}\ \&\ C_P = \frac{7}{2}R$$

$$\therefore\ Q = nC_P\ \Delta T = \frac{5}{7} \times \frac{7}{2} \times 8.3 \times 45 = 933.75\,J$$

Q6 **Two moles of a gas at 127°C expand isothermally until its volume is doubled. Calculate the amount of work done.**

Sol. Given; $n = 2,\ T = 127 + 273 = 400K,\ \frac{V_2}{V_1} = 2$

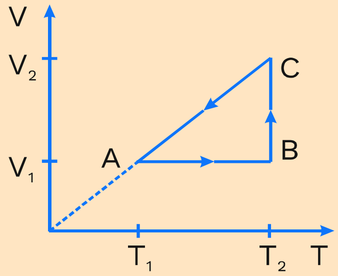
Work done in the process is,

$$W = nRT \ln\left(\frac{V_2}{V_1}\right)$$

$= 2 \times 8.3 \times 400 \times \ln 2$

$= 6640 \times 0.6931 \approx 4602J$

Q7 **Figure shows a process ABCA performed on an ideal gas. Find the net heat given to the system during the process.**



Sol. Since the process is cyclic, hence the change in internal energy is zero.
The heat given to the system is then equal to the work done by it.
The work done in part AB is $W_1 = 0$ (the volume remains constant). The part BC represents an isothermal process so that the work done by the gas during this part is $W_2 = nRT_2.\ln\frac{V_2}{V_1}$

During the part CA, $V \propto T$

So, V/T is constant and hence, $P = \frac{nRT}{V}$ is constant

The work done by the gas during the part CA is
$W_3 = P(V_1 - V_2) = nRT_1 - nRT_2 = -nR(T_2 - T_1)$.
The net work done by the gas in the process ABCA is

$$W = W_1 + W_2 + W_3 = nR\left[T_2 \ell n\frac{V_2}{V_1} - (T_2 - T_1)\right]$$

The same amount of heat is given to the gas.

Q8 **Two samples of a gas initially at same temperature and pressure are compressed from a volume V to V/2. One sample is compressed isothermally and the other adiabatically. In which sample is the pressure greater?**

Sol. Let initial volume, $V_1 = V$ and final volume, $V_2 = \frac{V}{2}$

Initial pressure, $P_1 = P$; final pressure, $P_2 = ?$
For isothermal compression

$$P_2V_2 = P_1V_1 \text{ or } P_2 = \frac{P_1V_1}{V_2} = \frac{PV}{V/2} = 2P$$

For adiabatic compression

$$P_2' V_2^{\gamma} = P_1V_1^{\gamma} \text{ or } P_2' = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = P\left(\frac{V}{V/2}\right)^{\gamma}$$

or $P_2' = 2^{\gamma}P$

Since $\gamma > 1 \therefore 2^{\gamma} > 2 \therefore P_2' > P_2$
Pressure during adiabatic compression is greater than the pressure during isothermal compression.

Q9 **A motor tyre pumped to a pressure of 3 atm. suddenly bursts. Calculate the fall in temperature due to adiabatic expansion. The temperature of air before expansion is 27°C. Given $\gamma = 1.4$.**

Sol. Initial pressure, $P_1 = 3$ atm ;

Final pressure, $P_2 = 1$ atm

Initial temperature, $T_1 = (27 + 273)\ \text{K} = 300\ \text{K}$

Final temperature, $= T_2$

Fall in temperature, $T_1 - T_2 = ?$

We know that $T_2^{\gamma} P_2^{1-\gamma} = T_1^{\gamma} P_1^{1-\gamma}$

$$\text{or}\quad \left(\frac{T_2}{T_1}\right)^{\gamma} = \left(\frac{P_1}{P_2}\right)^{1-\gamma} \quad \text{or}\quad \left(\frac{T_2}{300}\right)^{1.4} = \left(\frac{3}{1}\right)^{1-1.4} \quad \text{or}\quad \left(\frac{T_2}{300}\right)^{1.4} = \left(\frac{1}{3}\right)^{0.4}$$

or $T_2 = 219.2\ \text{K}$

So , $T_1 - T_2 = (300 - 219.2)\ \text{K} = 80.8\ \text{K}$

Q10 **Why it is cooler at the top of a mountain than at sea level?**

Sol. Pressure decreases with height. Therefore if hot air rises, it suffers adiabatic expansion.

From first law of thermodynamics $\Delta Q = \Delta U + \Delta W \Rightarrow \Delta U = -\Delta W \quad [\because \Delta Q = 0]$

This causes a decrease in internal energy and hence a fall of temperature.

Q11 **Calculate the work done when 1 mole of a perfect gas is compressed adiabatically. The initial pressure and volume of the gas are 10^5 N/m^2 and 6 litre respectively. The final volume of the gas is 2 liters. Molar specific heat of the gas at constant volume is $\frac{3R}{2}$ .Given[$(3)^{5/3} = 6.19$]**

Sol. For an adiabatic change $PV^{\gamma} = \text{constant} \Rightarrow P_1V_1^{\gamma} = P_2V_2^{\gamma}$

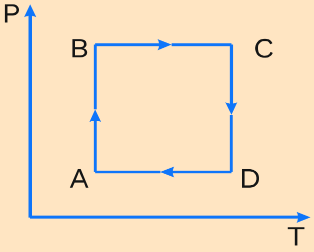
As molar specific heat of gas at constant volume $C_V = \frac{3}{2}R$

$$C_P = C_V + R = \frac{3}{2}R + R = \frac{5}{2}R \qquad \gamma = \frac{C_P}{C_V} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3}$$

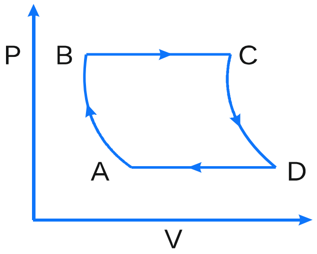
$$\therefore \quad P_2 = \left[\frac{V_1}{V_2}\right]^{\gamma} P_1 = \left[\frac{6}{2}\right]^{5/3} \times 10^5 = (3)^{5/3} \times 10^5 = 6.19 \times 10^5\ \text{N/m}^2$$

$$\text{Work done} \quad W = \frac{1}{1-\gamma}\left[P_2V_2 - P_1V_1\right]$$

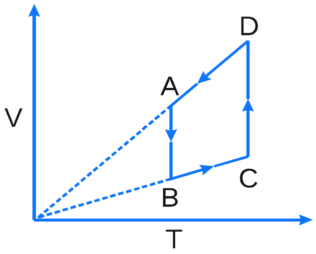
Q12 **Plot P-V, V-T graph corresponding to the P-T graph for an ideal gas shown in figure. Explain your answers.**



Sol.



(P-V Curve)



(V-T Curve)

For process AB	T = constant	so $P \propto \frac{1}{V}$
For process BC	P = constant	so $V \propto T$
For process CD	T = constant	so $V \propto \frac{1}{P}$
For process DA	P = constant	so $V \propto T$

Q13 **Find the molar heat capacity (in terms of R) of a monoatomic ideal gas undergoing the process: $PV^{1/2}$ = constant ?**

Sol. $PV^{1/2} = K$

$V = \left(\frac{K}{P}\right)^2 \quad \dots \text{(i)}$

Using gas equation we have $P = \frac{nRT}{K^2} \times P^2$

$\therefore \quad P = \frac{K^2}{nRT}$, rearranging $\frac{K}{P} = \frac{nRT}{K}$

As $V = \left(\frac{K}{P}\right)^2 = \frac{n^2R^2T^2}{K^2}$

Differentiating w.r.t T, we get ; $dV = \frac{n^2R^2 2T.dT}{K^2}$

$$dW = P.dV = \frac{K^2}{nRT} \times \frac{n^2R^2.2T.dT}{K^2}$$

$dW = 2\ nRdT$

Now by, $dQ = dU + dW$

$$nCdT = \frac{n\,3RdT}{2} + 2n\ RdT$$

$$\therefore \quad C = \frac{7}{2}R$$

Q14 **Calculate the change in entropy to convert 1 gm ice at -10°C to steam at 100°C.**

Sol. -10°C ice → 0°C ice → 0°C water → 100°C water → 100°C steam

$$\Delta s = ms\log\frac{273}{263} + \frac{mL}{273} + ms\log\frac{373}{273} + \frac{mL}{373}$$

$$= 1 \times 0.5\log\frac{273}{263} + \frac{1 \times 80}{273} + 1 \times 1 \times \log\frac{373}{273} + \frac{1 \times 540}{373}$$

$\Delta s = 1.88$ cal/°C

Q15 **Calculate change in entropy to convert 1 gm water at 100°C to steam.**

Sol. $$\Delta s = \frac{mL}{T} = \frac{1 \times 540}{373} = 1.4 \text{ cal/°C}$$

Q16 **A Carnot engine working between 300 K and 600 K has a work output of 800 J per cycle. What is the amount of heat energy supplied to the engine from source per cycle?**

Sol. $W = 800$ J, $T_1 = 600$ K $\qquad T_2 = 300$ K

$$\therefore \quad \eta = 1 - \frac{T_1}{T_1} = \frac{W}{Q_1}$$

$$1 - \frac{300}{600} = \frac{800}{Q_1} \quad \text{or} \quad 0.5 = \frac{800}{Q_1}$$

Heat energy supplied by source $Q_1 = \dfrac{800}{0.5} = 1600$ Joule per cycle.

Q17 **The temperatures T_1 and T_2 of the two heat reservoirs in an ideal Carnot engine are 1500°C and 500°C respectively. Which of the following: increasing T_1 by 100°C or decreasing T_2 by 100°C would result in a greater improvement in the efficiency of the engine?**

Sol. The efficiency of a Carnot's engine is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

Given T_1 = 1500°C = 1500 + 273 = 1773 K
and T_2 = 500°C = 500 + 273 = 773 K.
When the temperature of the source is increased by 100°C, keeping T_2 unchanged, the new temperature of the source is

T'_1 = 1500 + 100 = 1600°C = 1873 K.
The efficiency becomes

$$\eta' = 1 - \frac{T_2}{T'_1} = 1 - \frac{773}{1873} = 0.59$$

On the other hand, if the temperature of the sink is decreased by 100°C, keeping T_1 unchanged, the new temperature of the sink is T'_2 = 500 – 100 = 400°C = 673 K.
The efficiency now becomes

$$\eta'' = 1 - \frac{T'_2}{T_1} = 1 - \frac{673}{1773} = 0.62$$

Since η'' is greater than η', decreasing the temperature of the sink by 100°C results in a greater efficiency than increasing the temperature of the source by 100°C.

Q18 **A heat engine operates between a cold reservoir at temperature T_2 = 300 K and a hot reservoir at temperature T_1.**
It takes 200 J of heat from the hot reservoir and delivers 120 J of heat to the cold reservoir in a cycle.
What could be the minimum temperature of hot reservoir?

Sol. W.D. by the engine in a cycle is
W = 200 – 120 = 80 J.

$$\eta = \frac{W}{Q} = \frac{80}{200} = 0.4$$

From Carnot's Theorem

$$0.4 \le 1 - \frac{T_2}{T_1} = 1 - \frac{300}{T_1} \text{ or } \frac{300}{T_1} \le 0.6 \text{ or } T_1 \ge \frac{300}{0.6}$$

or $T_1 \ge 500$K

Q19 **The temperature inside outside of refrigerator are 273 K and 303 K respectively. Assuming that the refrigerator cycle is reversible, calculate the heat delivered to surroundings for every joule of work done.**

Sol. Coefficient of performance of Carnot refrigerator $\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$

Here $T_2 = 273$ K, $T_1 = 303$ K; W = 1 Joule

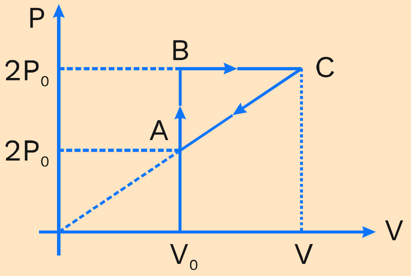
$\therefore \quad \frac{Q_2}{1} = \frac{273}{303 - 273} = \frac{273}{30}$; $Q_2 = \frac{273}{30} = 9.1$ Joule

Q20 **A refrigerator takes heat from water at 0°C and transfer it to room at 27°C. If 100 kg of water is converted in ice at 0°C then calculate the work done. (Latent heat of ice is 3.4×10^5 J/kg)**

Sol. $\text{C.O.P.} = \frac{T_2}{T_1 - T_2} = \frac{273}{300 - 273} = \frac{273}{27}$, $Q_2 = mL$

$W = \frac{mL}{COP} = \frac{100 \times 3.4 \times 10^5}{273/27} = \frac{100 \times 3.4 \times 10^5 \times 27}{273} = 3.36 \times 10^7$ J

Q21 **n moles of a diatomic gas has undergone a cyclic process ABC as shown in figure. Temperature at a is T_0.**



Find

(i) Volume at C.

(ii) Maximum temperature.

(iii) Total heat given to gas.

(iv) Is heat rejected by the gas, if yes how much heat is rejected.

(v) Find out the efficiency.

Sol. (i) Since triangle OAV_0 and OCV are similar therefore

$$\frac{2P_0}{V} = \frac{P_0}{V_0} \Rightarrow V = 2V_0$$

(ii) Since process AB is isochoric hence $\frac{P_A}{T_A} = \frac{P_B}{T_B}$

$$\Rightarrow T_B = 2T_0$$

Since process BC is isobaric therefore $\frac{T_B}{V_B} = \frac{T_C}{V_C}$

$$\Rightarrow T_C = 2T_B = 4\ T_0$$

(iii) Since process is cyclic ;

$Q = W = \text{area under the cycle} = \frac{1}{2}P_0V_0.$

(iv) Since U and W both are negative in process CA

$\therefore$ Q is negative in process CA and heat is rejected in process CA

$$Q_{CA} = W_{CA} + U_{CA}$$

$$= -\frac{1}{2}[P_0 + 2P_0]\ V_0 - \frac{5}{2}nR\ (T_c - T_a)$$

$$= -\frac{1}{2}\ [P_0 + 2P_0]\ V_0 - \frac{5}{2}\ nR\left(\frac{4P_0V_0}{nR} - \frac{P_0V_0}{nR}\right)$$

$= -\ 9P_0V_0$ = Heat injected.

(v) η = efficiency of the cycle

$$= \frac{\text{work done by the gas}}{\text{heat injected}} = \frac{P_0V_0/2}{Q_{\text{injected}}} \times 100$$

where

$$Q_{inj} = Q_{AB} + Q_{BC}$$

$$= \left[\frac{5}{2}nR(2T_0 - T_0)\right] + \left[\frac{5}{2}nR(2T_0) + 2P_0(2V_0 - V_0)\right]$$

$$= \frac{19}{2}P_0V_0.$$

Therefore $\eta = \frac{100}{19}\%$

Mind Map

Isobaric Process

Pressure = Constant

State Equation

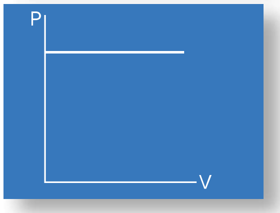
$V \propto T$

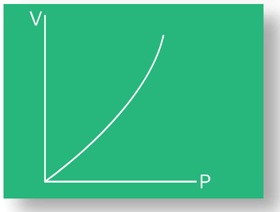
$\frac{V}{T} = C$

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$

Work done
$W = \int P dV = \int nR dT$

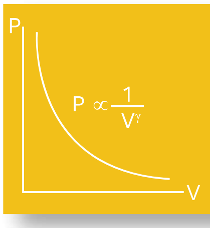
$\Delta Q = nC_P \Delta T$
$\Delta U = nC_V \Delta T$





Slope
$\frac{dP}{dV} = 0$

Adiabatic Process

- Quick Process
- Q = Constant
  $\Delta Q = 0$
- State Equation
  $PV^{\gamma} = C$
  $TV^{\gamma-1} = C$
  $P^{1-\gamma}T^{\gamma} = C$
  $PT^{\frac{\gamma}{1-\gamma}} = C$
- $\Delta U = -\Delta W$
- $W = \dfrac{P_2V_2 - P_1V_1}{1-\gamma}$
  $= \dfrac{nR(T_2 - T_1)}{1-\gamma}$
- 

- Slope $= \dfrac{dP}{dV} = -\gamma\dfrac{P}{V}$
  $= \gamma$ × Slope of isothermal process
- Cloud Formation
  - Large scale convention current
  - Part of carnot engine