Relative Motion

DISCLAIMER

Relative Motion

RELATIVE MOTION

Motion is a property of the object under study as well as the observer. It is always relative; Motion is always defined with respect to an observer or reference frame.

Definitions

Relative position is the position of a particle with respect to the another particle.

Frame of Reference

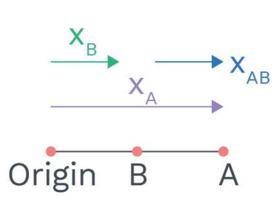
Motion of an object can be observed only if it changes its position with respect to some other body. Therefore, for motion to be observed there must be an object, which is changing its position with respect to another object and a person who is observing its motion. A person observing its motion is called observer. An observer for the purpose of investigation must own a clock to measure time and a point in the space attached with the other object as origin and a set of coordinate axis. The two things (the time measured by the clock and the coordinate system) are collectively called reference frame.

We can say that motion of the moving body is expressed in terms of its position coordinates changing with time.

RELATIVE MOTION IN ONE DIMENSION:

Relative Position:

It is defined as the position of an object w.r.t. observer. It means if position of A with respect to origin is x_A and B with respect to origin is x_B then position of A with respect to B" x_{AB} is



$x_{AB} = x_A - x_B$

KEY POINTS

- Relative motion
- Frame of reference
- Relative position
- Relative velocity

Relative Velocity:

Definition:

Relative velocity of a particle A with respect to B means the velocity of A as observed by B if b is considered to be at rest.

Relative velocity in one dimension:

If x_A is the position of A with respect to ground, x_B is position of B with respect to ground and x_{AB} is position of A with respect to B then

v_A = velocity of A with respect to ground = $\frac{dx_A}{dt}$

v_B = velocity of B with respect to ground = $\frac{dx_B}{dt}$

and v_{AB} = velocity of A with respect to

$$B = \frac{dx_{AB}}{dt} = \frac{d}{dt}\left(x_A - x_B\right)$$

Thus, $v_{AB} = v_A - v_B$

Note: Velocity of an object with respect to itself is always zero.

Relative Acceleration:

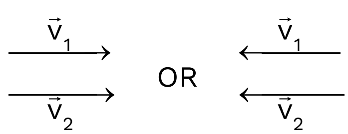
It is defined as rate of change of relative velocity with respect to time.

$$a_{AB} = \frac{d\,(v_{AB})}{dt} = \frac{dv_A}{dt} - \frac{dv_B}{dt} = a_A - a_B$$

NOTE: Relative = Actual – Reference

For same direction.

When two particles are moving in the same direction, then magnitude of their relative velocity is equal to the difference between their individual speeds.



$$|\vec{v}_{12}| = |\vec{v}_{21}| = v_1 - v_2$$

For opposite directions:

When two particles are moving in the opposite directions, then magnitude of their relative

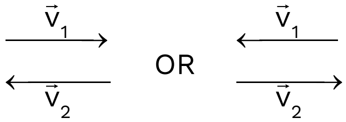
Rack your Brain

Preeti reached the metro station and found that the escalator was not working. She walked up the stationary escalator in time t_1. On other days, if she remains stationary on the moving escalator, then the escalator takes her up in time t_2. Then calculate time taken by her to walk up on the moving escalator.

Definitions

Relative velocity of a particle A with respect to B is defined as the velocity with which A appears to move if B is considered to be at rest.

velocity is always equal sum of their individual speeds.

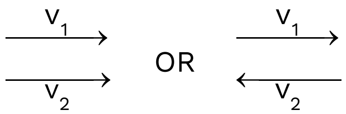


$| \vec{v}_{12} | = | \vec{v}_{21} | = v_1 + v_2$

Numerical approach-

1. When two particles are moving along a straight line with constant speeds then their relative acceleration must be zero and, in this condition, relative velocity is the ratio of relative displacement to time.

   v_1 = constant, v_2 = constant.



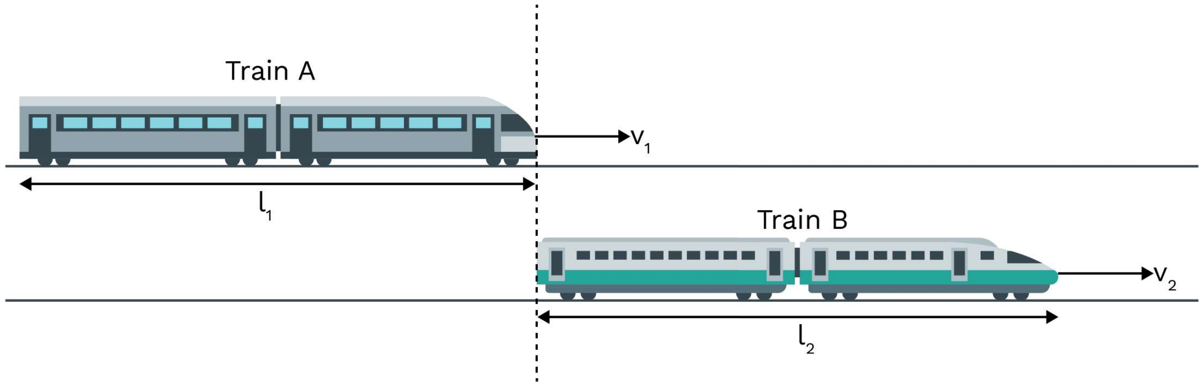
when $a_{rel.} = 0$

$$v_{rel.} = \frac{x_{Relative}}{time}$$

Concept Reminder

- $$v_{rel.} = \frac{x_{Relative}}{time}$$

A. When train overtake to another train

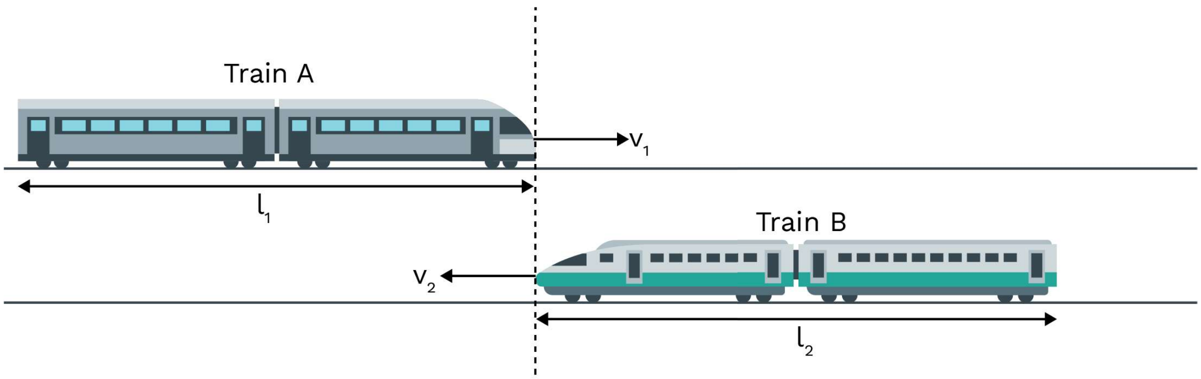


If $v_1 > v_2$ and same direction than train A overtake to train B in time t is

$$v_{overtake} = \frac{l_1 + l_2}{v_1 - v_2}$$

B. When Train cross-each other: -

When train travelled in opposite direction



Time taken to cross each other-

$$\boxed{t_{cross} = \frac{l_1 + l_2}{v_1 + v_2}} \qquad \boxed{t_{overtake} > t_{cross}}$$

Concept Reminder

- $\boxed{t_{overtake} = \frac{l_1 + l_2}{v_1 - v_2}}$
- $\boxed{t_{cross} = \frac{l_1 + l_2}{v_1 + v_2}}$
- $\boxed{t_{overtake} > t_{cross}}$

Ex. Two train of same length 75 m are moving with speed 12 m/s and 13 m/s on parallel tracks. Find out time of crossing if they move in

(a) Same direction

(b) Opposite direction

Sol. When same direction

$$t_{overtake} = \frac{l_1 + l_2}{v_1 - v_2} = \frac{75 + 75}{13 - 12} = \frac{150}{1} = 150\,\text{sec}.$$

When opposite direction

$$t_{cross} = \frac{l_1 + l_2}{v_1 + v_2} = \frac{75 + 75}{13 + 12} = \frac{150}{25}$$

$$t_{cross} = 6\,\text{sec}.$$

Ex. A train of length 500 m is moving with speed 30 m/s along east. A bird is moving just above train with speed 20 m/s in opposite direction. Then find time taken by the bird to cross the train.

Sol. $t_{cross} = \dfrac{l}{v_{rel.}} = \dfrac{500}{30 + 20}$

$t_{cross} = \dfrac{500}{50} = 10\,\text{sec}.$

2. When two particles move in such
A way that their relative acceleration is non-zero but constant then we apply equation of motion in the relative form.

$a_{AB} = a_A - a_B$
$= 0 - a = -a$
So, a_{AB} = constant

Equation of Motion (Relative):

$v_{rel} = u_{rel} + a_{rel}\, t$

$s_{rel} = u_{rel}\, t + \frac{1}{2} a_{rel}\, t^2$

$v^2_{rel} = u^2_{rel} + 2a_{rel}\, s_{rel}$

$s_{rel} = \frac{1}{2}(u_{rel} + v_{rel})\, t$

Relative speed:
Relative speed = |Relative velocity|

$V_{rel} = |\vec{v}_{rel}|$ If both object moves in a straight line than relative speed can be obtained as

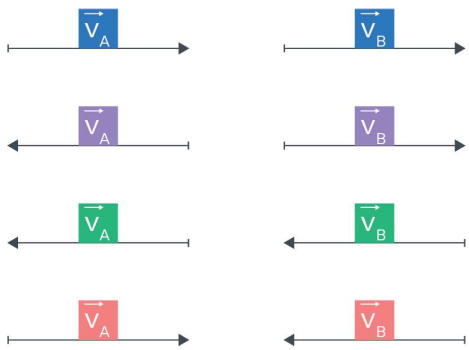
Rack your Brain

A train of 150 metre length is going towards north direction at a speed of 10m/s. A parrot flies at the speed of 5m/s towards south direction parallel to the railways track. Then calculate time taken by the parrot to cross the train.

Rack your Brain

A bus is moving with a speed of 10 ms^{-1} on a straight road. A scooterist wishes to overtake the bus in 100 s. If the bus is at a distance of 1 km from the scooterist, with what speed should the scooterist chase the bus?

Case – 1 Case – 2



$$\left|\vec{v}_{AB}\right| = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$$

If $\theta = 0° \rightarrow$ (Same direction)

$v_{AB} = v_A - v_B$

$\theta = 180° \rightarrow$ (Opposite direction)

$v_{AB} = v_A + v_B$

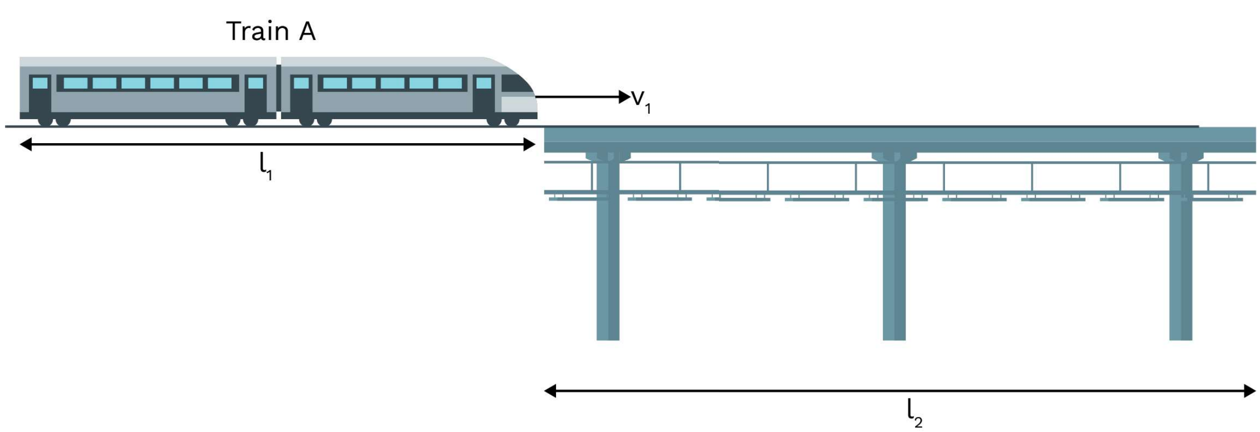
Concept Reminder

- $v_{rel} = u_{rel} + a_{rel}\, t$
- $s_{rel} = u_{rel}\, t + \frac{1}{2} a_{rel}\, t^2$
- $v^2_{rel} = u^2_{rel} + 2a_{rel}\, s_{rel}$
- $s_{rel} = \frac{1}{2}(u_{rel} + v_{rel})\, t$

KEY POINTS

- Relative speed
- Relative acceleration

Train Bridge Problem:



Time taken by train to cross whole bridge

$$t = \frac{l_1 + l_2}{v_{train}} = \frac{\text{total distance covered by train}}{v_{train}}$$

Ex. A train 175 m long crosses a bridge which is 225 m long in 80 seconds. What is the speed of the train?

Sol. Length of train, l_1 = 175 m

Length of the bridge l_2 = 225 m

$$t = \frac{l_1 + l_2}{v_{train}} = \frac{\text{total distance}}{v_{train}}$$

$$80 = \frac{175 + 225}{v} \Rightarrow v = \frac{400}{80}$$

v = 5 m/sec.

Ex. A 175 m long train is travelling along a straight track with a velocity 72 km/h. A bird is flying parallel to the train in the opposite direction with a velocity 18 km/h. The time taken by the bird to cross the train is-

(1) 35 sec. (2) 27 sec.
(3) 11.6 sec. (4) 7 sec.

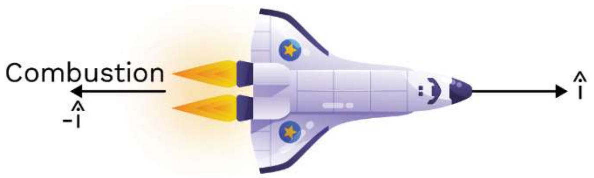
Sol. (4) $t = \dfrac{175}{\left(72 \times \dfrac{5}{18}\right) + \left(18 \times \dfrac{5}{18}\right)}$

$t = \dfrac{175}{20 + 5}$; $\Rightarrow$ t = 7 sec.

Ex. A Jet aeroplane is travelling at the speed of 500 km/hr and eject its products of combustion at speed of 1500 km/hr relative to the jet plane. The speed of the products of combustion with respect to an observer on the ground is:

(1) 500 km/hr (2) 1000 km/hr
(3) 1500 km/hr (4) 200 km/hr

Sol. (2)



$(\vec{v}_{Jet})_g$ = +500 km/hr

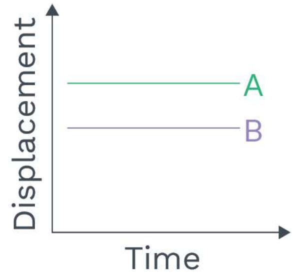
$-1500\hat{j} = (\vec{v}_c)_g - 500\hat{i}$

$(\vec{v}_c)_g = 500\hat{i} - 1500\hat{i}$; $(\vec{v}_c)_g = -1000\hat{i}$

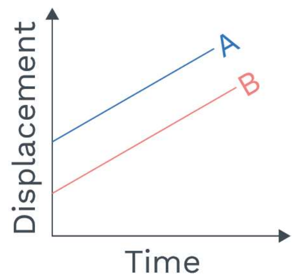
$\left|(\vec{v}_c)_g\right|$ = 1000 km/hr

Ex. Which one of the following represent displacement – time graph of two object A and B moving with zero relative velocity?

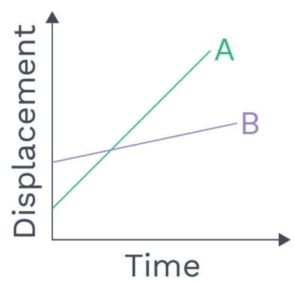
(1)



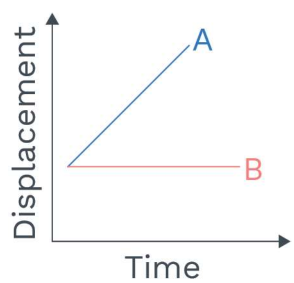
(2)



(3)



(4)



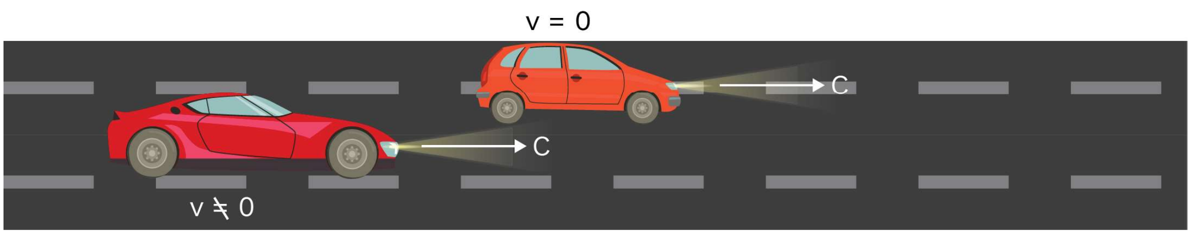
Sol. (2) $\vec{v} = \frac{\overrightarrow{ds}}{dt}$

$$\left(\frac{\overrightarrow{ds}}{dt}\right)_A = \left(\frac{\overrightarrow{ds}}{dt}\right)_B \qquad \text{(For option (2))}$$

$$\vec{v}_A = \vec{v}_B$$

$$\vec{v}_{AB} = 0$$

Ex. In Figure, one Car is at rest and velocity of light from head light is C, then velocity off light from head light for the moving car at velocity v, would be –



(1) C + v (2) C – v
(3) C × v (4) C

Sol. (4) $C = 3 \times 10^8$ m/s

$v << C$

$\left.\begin{matrix} C + v \\ C - v \end{matrix}\right]$ is irrelevant.

Ex. Two train are moving with equal speed in opposite direction along 2 parallel railway tracks. If wind is blowing with speed u along the track so that the relative velocities of the train w.r.t. the wind are in the ratio 1 : 2, then the speed of each train must be –

(1) 3u (2) 2u
(3) 5u (4) 4u

Sol. (1) $v \rightarrow$ Speed of train

$u \rightarrow$ Speed of wind

$$\frac{v - u}{v + u} = \frac{1}{2}$$

$v = 3u$

Rack your Brain

A police jeep is chasing with velocity of 45 km/h a thief in another jeep moving with velocity 153 km/h. Police fires a bullet with muzzle velocity of 180 m/s, then calculate velocity with which it will strike the car of the thief.

Rack your Brain

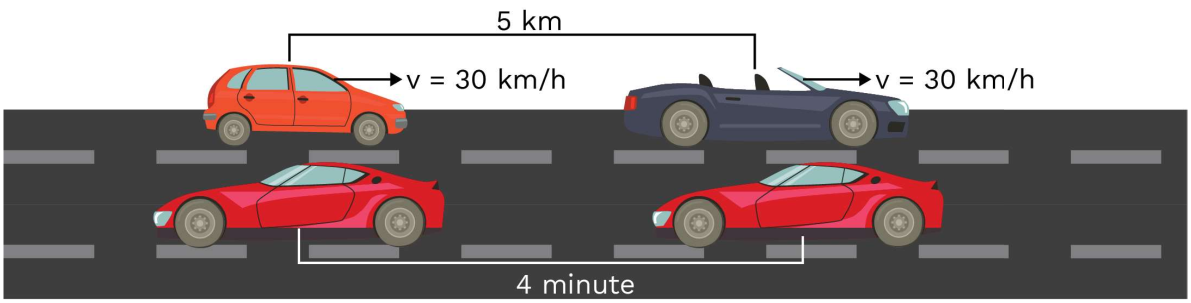
Two trains, each 50 m long are travelling in opposite direction with velocity 10 m/s and 15 m/s. Then calculate time of crossing.

Ex. Two Cars are moving in the exactly same direction with the same speed 30 km/hr. They are at a distance of 5 km. The magnitude of velocity of a car moving in the opposite direction. If this car meets these two cars at an interval of 4 min., will b:

(1) 40 km/hr (2) 45 km/hr

(3) 30 km/hr (4) 15 km/hr

Sol. (2)



$$t = \frac{\text{distance}}{\text{relative velocity}}$$

$$\frac{4}{60}\text{hr} = \frac{5\,\text{km}}{(30+u)}\text{km/hr}$$

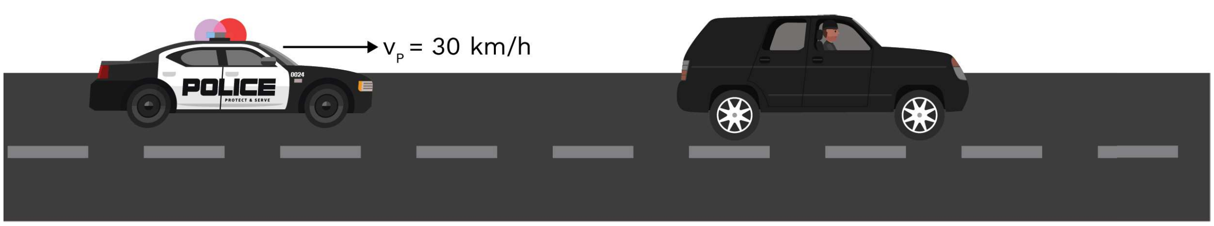
u = 45 km/hr

Ex. A Police Car moving on a highway at 30 km/hr fires a bullet on a speeding Car of thieves at 192 km/hr. If the muzzle speed of bullet is 150 m/s. Calculate the speed with which the bullet strike the Car of the thieves–

(1) 95 m/s (2) 105 m/s

(3) 180 m/s (4) 192 m/s

Sol. (2)



$$\left(v_p\right)_g = 30 \times \frac{5}{18} = \frac{25}{3}\ m/s$$

$$\left(v_T\right)_g = 192\ km/hr = 192 \times \frac{5}{18} m/s = \frac{160}{3} m/s$$

$$v_{BT} = ?$$

$$v_{BP} = 150\ m/s$$

$$v_{BT} = \left(v_B\right)_g - \left(v_T\right)_g$$

$$v_{BP} = v_B - v_P$$

$$v_B = v_{BP} + v_P$$

$$v_B = 150 + \frac{25}{3}$$

$$v_B = \frac{475}{3}\ m/s$$

$$v_{BT} = \left(v_B\right)_g - \left(v_T\right)_g$$

$$= \frac{475}{3} - \frac{160}{3}$$

$$v_{BT} = 105\ m/s$$

Ex. Let $\vec{r}_1(t) = 3t\hat{i} + 4t^2\hat{j}$ and $\vec{r}_2(t) = 4t^2\hat{i} + 3t\hat{j}$ represent the position of particles 1 and 2, respectively as function of time t; $\vec{r}_1(t)$ and $\vec{r}_2(t)$ are in meter and t in second. The relative speed is the two particles at the instant t = 1s will be –

(1) 1 m/s (2) $3\sqrt{2}$ m/s

(3) $5\sqrt{2}$ m/s (4) $7\sqrt{2}$ m/s

Sol. (3) $\vec{v}_{12} = \vec{v}_1 - \vec{v}_2$

$$\vec{v}_1 = \frac{d\vec{r}_1}{dt} = 3\hat{i} + 8t\hat{j}$$

$$\left(\vec{v}_1\right)_{t=1} = 3\hat{i} + 8\hat{j}$$

$$\vec{v}_2 = \frac{d\vec{r}_2}{dt} = 8t\hat{i} + 3\hat{j}$$

$$\left(\vec{v}_2\right)_{t=1} = 8\hat{i} + 3\hat{j}$$

Rack your Brain

Let $\vec{r}_1(t) = 3\hat{i} + 5t^2\hat{j}$ and $\vec{r}_2(t) = 4\hat{i} + 6t\,\hat{j}$ represent the positions of particles 1 and 2, respectively as function of time t; $\vec{r}_1(t)$ and $\vec{r}_2(t)$ are in meter and t in second. Then calculate relative speed of the two particles at the instant t = 3 s.

$$\vec{v}_{12} = \left(3\hat{i} + 8\hat{j}\right) - \left(8\hat{i} + 3\hat{j}\right)$$

$$\vec{v}_{12} = -5\hat{i} + 5\hat{j}$$

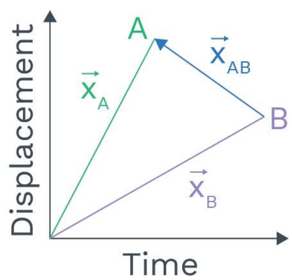
$$\left|\vec{v}_{12}\right| = \sqrt{25 + 25} = 5\sqrt{2}\ \text{m / s}$$

⇒ **Relative motion in two Dimension:**

$\vec{x}_A$ = Position of A with respect to O

$\vec{x}_B$ = Position of B with respect to O

$\vec{x}_{AB}$ = Position of A with respect to B



$$\vec{x}_{AB} = \vec{x}_A - \vec{x}_B$$

$$\frac{d\left(\vec{x}_{AB}\right)}{dt} = \frac{d\left(\vec{x}_A\right)}{dt} - \frac{d\left(\vec{x}_B\right)}{dt}$$

$$\Rightarrow \quad \vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

$$\frac{d\left(\vec{v}_{AB}\right)}{dt} = \frac{d\left(\vec{v}_A\right)}{dt} - \frac{d\left(\vec{v}_B\right)}{dt}$$

$$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$$

Ex. A man is moving in his Car with speed of 10 km/hr due east. A train appears to him moving along north with speed $10\sqrt{3}$ km/hr then find out actual velocity of train.

Sol. $\vec{v}_M = 10\hat{i}$

$$\vec{v}_{TM} = 10\sqrt{3}\,\hat{j} = \vec{v}_T - \vec{v}_M$$

$$\vec{v}_T = 10\hat{i} + 10\sqrt{3}\,\hat{j}$$

$$\vec{v}_T = 10\sqrt{3}\hat{j} + 10\hat{i}$$

$$\left|\vec{v}_T\right| = \sqrt{\left(10\sqrt{3}\right)^2 + \left(10\right)^2} = \sqrt{400}$$

Concept Reminder

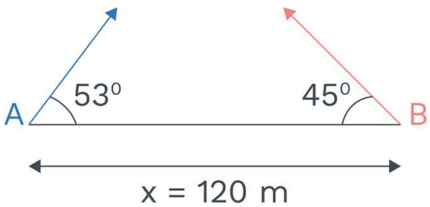
- $\vec{x}_{AB} = \vec{x}_A - \vec{x}_B$
- $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$
- $\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$

$$\left|\vec{v}_T\right| = 20 \text{ km / hr}$$

$$\tan\theta = \frac{10\sqrt{3}}{10} = \sqrt{3}$$

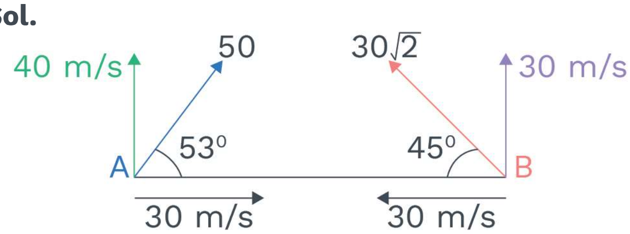
$$\vec{v}_T = 20 \text{ km / hr}$$ due East 60° North

Ex. Two particle A and B are projected from ground as shown in figure.



Then find out time after which their relative displacement become zero.

Sol.



$$t = \frac{x}{v_{relative}} = \frac{120}{30+30}$$

$$t = \frac{120}{60}$$

$$t = 2 \text{ sec.}$$

Ex. Two particles A and B are present at a distance 5 km from each other such that B is present to the south of A. Both A and B starts moving with same speed 10 km/hr along east and north respectively, then find out:

(A) $\vec{v}_{AB}$

(B) Minimum distance between A and B during motion.

Rack your Brain

A train is moving towards east and a car is along north, both with same speed. What is the observed direction of car to the passenger in the train?

Rack your Brain

A ship A is moving Westwards with a speed of 10 kmh^{-1} and a ship B 100km South of A, is moving Northwards with a speed of 10kmh^{-1}. Calculate the time after which the distance between them becomes shortest.

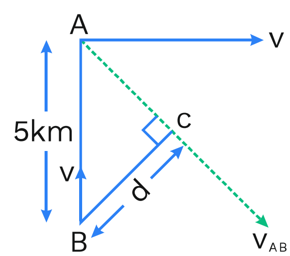
(C) Time after which distance between A and B is minimum.

Sol. (A) $\vec{v}_{AB} = 10\hat{i} - 10\hat{j}$ km / hr

$$|\vec{v}_{AB}| = \sqrt{(10)^2 + (10)^2} = 10\sqrt{2} \text{ km / hr},$$

due SE $(\theta = 45°)$

(B) Minimum distance ΔABC: -



$$\sin 45° = \frac{d}{5}$$

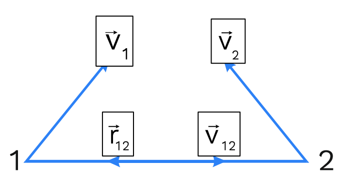
$$\frac{1}{\sqrt{2}} = \frac{d}{5} \Rightarrow d = \frac{5}{\sqrt{2}} \text{ km}$$

(C) $$t = \frac{\vec{d}}{\vec{v}_{AB}} = \frac{\frac{5}{\sqrt{2}}}{10\sqrt{2}} = \frac{5}{20} = \frac{1}{4} \text{ hrs}$$

$$= \frac{1}{4} \times 60 = 15 \text{ min}.$$

$\Rightarrow$ **Condition for collision for two projectiles: -**

If relative acceleration of particle is zero, then for collision relative velocity of particles must be along the line joining them.



$$\hat{r}_{12} = -\vec{v}_{12}$$

$$\hat{r}_{12} = \vec{v}_{21}$$

$$\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} = \frac{\vec{v}_2 - \vec{v}_1}{|\vec{v}_2 - \vec{v}_1|}$$

Ex. A body is dropped from an aeroplane which is moving horizontally with constant speed u at some height. What will be the path of body.

(1) with respect to observer present of ground

(2) with respect to observer present inside a plane.

Sol. (1) Velocity of aeroplane transfer to body and relative g act perpendicular then body path is parabola.

(2) When observer inside plane then net relative velocity is zero then body path observe straight line.

Ex. A train is moving horizontally with constant speed. A bob inside a train project a stone vertically upwards. What will be the path of stone?

(1) w.r.t. to a boy present on ground

(2) w.r.t. to a boy inside a train

Sol. (1) When observer present on ground then train velocity transfer to stone and velocity and acceleration perpendicular then path is parabolic.

(2) When observer inside train then relative velocity is zero and stone travel in straight line.

Ex. Two projectiles are projected from ground from same point with speed u_1 and u_2 at and angles θ_1 & from horizontal respectively.

Then what will be the path of one projectile w.r.t. another during the motion.

Sol. $\vec{u}_{12} = \vec{u}_1 - \vec{u}_2 \neq 0$

$\vec{a}_{12} = \vec{a}_1 - \vec{a}_2 = 0$

$\vec{u}_{12} = \text{constant} \qquad \left[\because \ \vec{a}_{12} = 0\right]$

Path will be straight line.

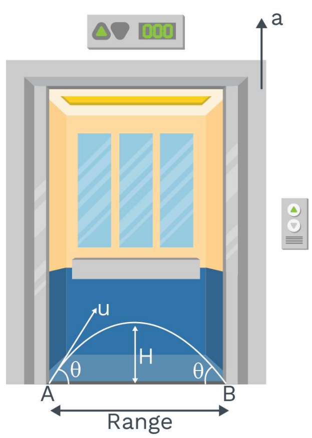
⇒ **Relative Motion in Lift:**

Projectile motion in a lift moving with acceleration 'a' upwards-

(1) In the reference frame of lift, acceleration of a freely falling object is g + a

(2) Velocity at maximum height = $u\cos\theta$

(3) $T = \dfrac{2u\sin\theta}{(g+a)}$



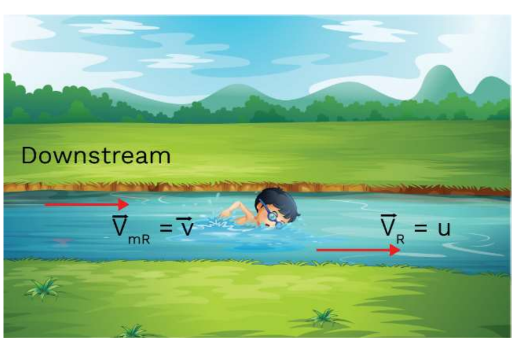
(4) Maximum height (H) = $\dfrac{u^2 \sin^2 \theta}{2(g+a)}$

(5) Range (R) = $\dfrac{u^2 \sin 2\theta}{(g+a)}$

⇒ **Relative Motion in River flow:**
A man can swim relative to water with velocity $\vec{V}_{mR}$ and water is flowing relative to ground with velocity $\vec{V}_R$, velocity of man relative to ground $\vec{V}_m$ will be given by

$\vec{V}_{mR} = \vec{V}_m - \vec{V}_R$ $\qquad$ $\vec{V}_m = \vec{V}_{mR} + \vec{V}_R$

⇒ **River problem in one-dimension:**
Velocity of river is u and velocity of man in still water is v.
(i) **Down stream**
If the swimmer is in the direction of flow of water.



Concept Reminder

Time of flight

$(T) = \dfrac{2u\sin\theta}{(g+a)}$

Maximum height

$(H) = \dfrac{u^2 \sin^2 \theta}{2(g+a)}$

Range (R) = $\dfrac{u^2 \sin 2\theta}{(g+a)}$

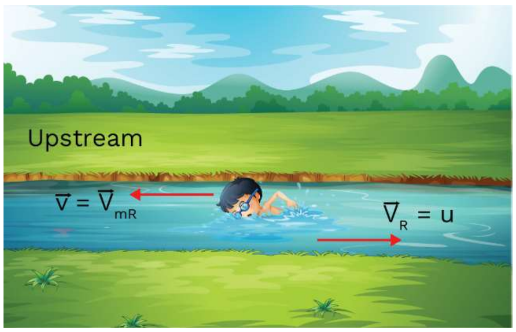
Rack your Brain

A boat crosses a river from port A to port B, which are just on the opposite side. The speed of the water is V_W and that of boat is V_B relative to still water. Assume $V_B = 2V_W$. What is the time taken by the boat, if it has to cross the river directly on the AB line.

$\vec{V}_m = \vec{V}_{mR} + \vec{V}_R$

$V_m = u + v$

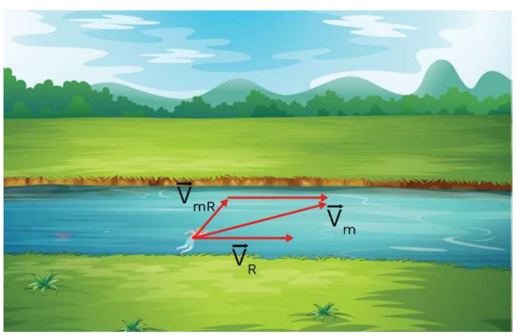
⇒ **Upstream:**
If the swimmer is in the direction opposite to the flow of water.



$\vec{V}_m = \vec{V}_{mR} + \vec{V}_R$

$V_m = v - u$

⇒ **River problem in two dimension:**
If the man is crossing the river, $\vec{v}$ and $\vec{V}_R$ are non collinear then use vector algebra.



$\vec{V}_{mR} = \vec{V}_m - \vec{V}_R$

$\vec{V}_m = \vec{V}_{mR} + \vec{V}_R$

$\because \vec{V}_{mR} = \vec{v}$

$\therefore \vec{V}_m = \vec{v} + \vec{V}_R$

Concept Reminder

Down Stream :
- $\vec{V}_m = \vec{V}_{mR} + \vec{V}_R$
- $V_m = u + v$

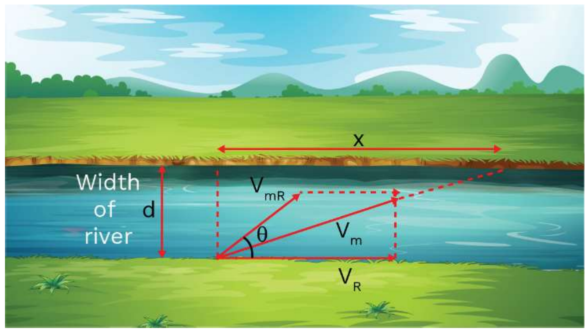
Up Stream :
- $\vec{V}_m = \vec{V}_{mR} + \vec{V}_R$
- $V_m = v - u$

KEY POINTS

- River flow
- Upstream
- Downstream

Rack your Brain

A boat is moving with a velocity $3\hat{i} + 4\hat{j}$ with respect to ground. The water in the river is moving with a velocity $-3\hat{i} - 4\hat{j}$ with respect to ground. Calculate the relative velocity of boat with respect to water.



$$\vec{V}_m = \left(V_{mR}\cos\theta\,\hat{i} + V_{mR}\sin\theta\hat{j}\right) + V_R\,\hat{i}$$

$$\vec{V}_m = \left(V_{mR}\cos\theta + V_R\right)\hat{i} + V_{mR}\sin\theta\,\hat{j}$$

TO CROSS A RIVER

Minimum distance of approach OR Cross the River along shortest possible path OR Cross the river and reach a point just opposite to the starting path	Cross the River in shortest possible time

⇒ **Crossing the River in shortest time:**

$V_{mR}\sin\theta$ is the component of velocity of man in the direction perpendicular to the river flow, which is reason to crossing the river.
Time to cross the river is t.

$$t = \frac{d}{V_{mR}\sin\theta}$$

Where d is width of the river

⇒ **Drift:**

It is known as the displacement of man in the direction of river flow.

Drift = $u_x.t$

$$\text{Drift }(x) = \left(V_{mR}\cos\theta + V_R\right).\frac{d}{V_{mR}\sin\theta}$$

$$(x) = \frac{(V_{mR}\cos\theta + V_R).d}{V_{mR}\sin\theta}$$

⇒ **Shortest path and Minimum drift:**

If a man wants to cross the river such that his "displacement should be minimum". It means he intends to reach just opposite point across the river.

For minimum drift condition

$$x_{min} = 0$$

$$(V_{mR}\cos\theta + V_R).\frac{d}{V_{mR}\sin\theta} = 0$$

$$V_{mR}\cos\theta + V_R = 0$$

$$\cos\theta = -\frac{V_R}{V_{mR}}$$

$\cos\theta$ is –ve, $\therefore \theta > 90°$

For minimum drift the man must swim at some angle α with the perpendicular in backward direction.

$$\sin\alpha = \frac{V_R}{V_{mR}}$$

$$\theta = \cos^{-1}\left(\frac{-V_R}{V_{mR}}\right)$$

$$\left|\frac{V_R}{V_{mR}}\right| \le 1$$

$$V_R \le V_{mR}$$

Condition for minimum drift is zero.

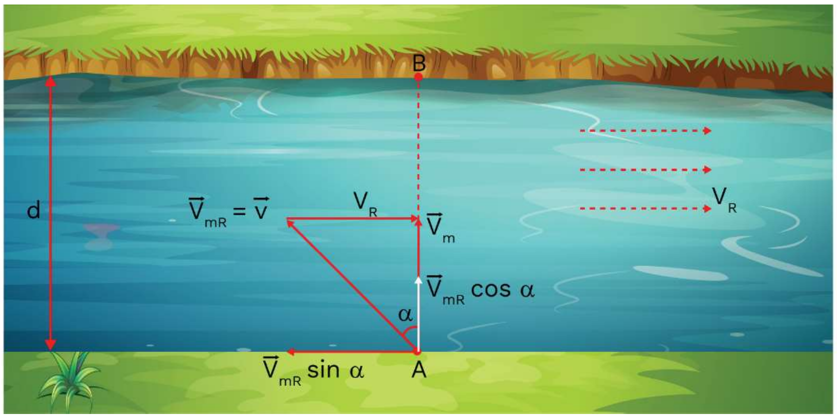
If $V_R > V_{mR}$ then it is not possible to have zero drift.

Concept Reminder

$$(x) = \frac{(V_{mR}\cos\theta + V_R).d}{V_{mR}\sin\theta}$$

KEY POINTS

- Drift



Such that its resultant velocity $\vec{V}_m = \vec{V}_{mR} + \vec{V}_R$ is in the direction of displacement AB. To reach at B (drift is zero)

$V_{mR} \sin\alpha = V_R$

$$\sin\alpha = \frac{V_R}{V_{mR}}$$

Time to cross the river along the shortest path

$$t = \frac{d}{V_{mR}\cos\alpha} = \frac{d}{\sqrt{V_{mR}^2 - V_R^2}} = \frac{d}{\sqrt{v^2 - V_R^2}}$$

$\because \vec{V}_{mR} = \vec{v}$

Ex. A motorboat covers a given distance in 6 hours moving down stream along river. It takes 10 hours to cover the same distance moving upstream. The time it takes to cover the same distance in still water is-

(1) 9 hours (2) 7.5 hours
(3) 6.5 hours (4) 8 hours

Sol. (2)

$$t = \frac{d}{v+u} \quad .....(i)$$

$$t' = \frac{d}{v-u} \quad .....(ii)$$

$$t = \frac{d}{v} \quad .....(iii)$$

From eq^n (i)

$$6 = \frac{d}{v + u}$$

$$d = 6v + 6u \qquad .....(iv)$$

From eqⁿ (ii)

$$10 = \frac{d}{v - u}$$

$$d = 10v - 10u \qquad .....(v)$$

From eqⁿ (iv) and (v)

$$6v + 6u = 10v - 10u$$

$$16u = 4v$$

$$u = \frac{v}{4}$$

From eqⁿ (iv)

$$d = 6v + 6\left(\frac{v}{4}\right)$$

$$6\left[v + \frac{v}{4}\right]$$

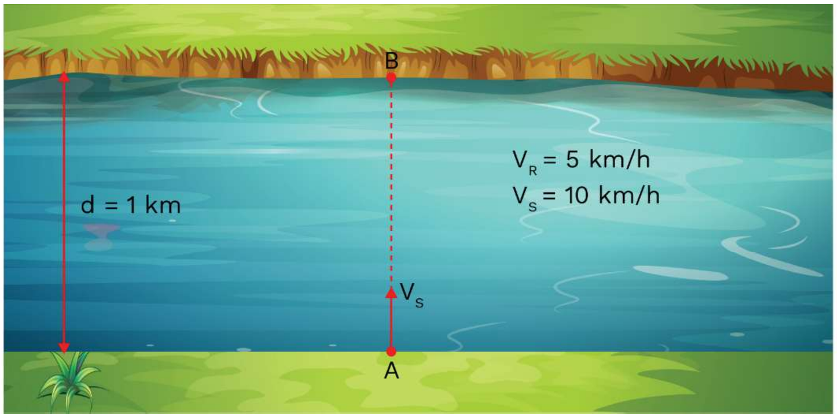
$$d = 6\left(\frac{5v}{4}\right)$$

From eqⁿ (iii)

$$t = \frac{6\left(\frac{5v}{4}\right)}{v} = \frac{30}{4} = \frac{15}{2}$$

t = 7.5 hours

Ex.

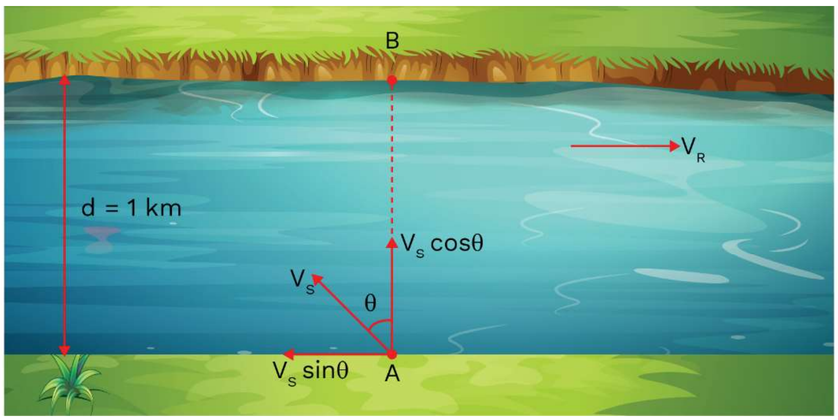


If swimmer want to reach just opposite point of river bank or along shortest path cross the river than

(i) t = ?

(ii) In which direction he should have to swim?

Sol.



$$V_S \sin\theta = V_R$$

$$\sin\theta = \frac{V_R}{V_S} = \frac{5}{10}$$

$$\sin\theta = \frac{1}{2}$$

$\theta = 30°$ with width

Angle with downstream = 90° + 30° = 120°

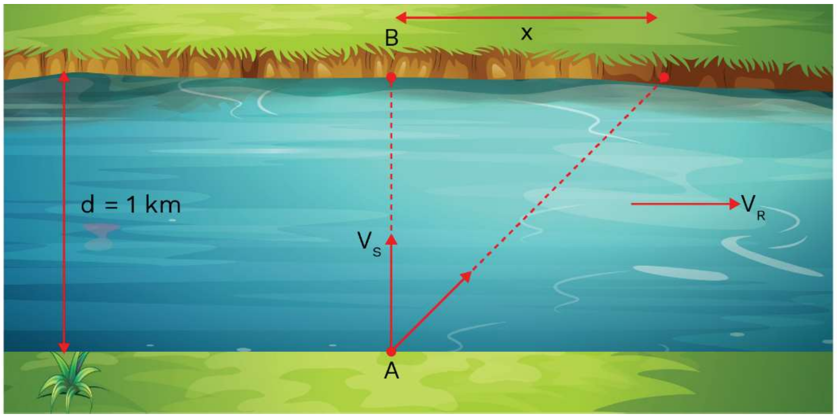
Angle with upstream = 90° – 30° = 60°

$$t = \frac{d}{V_S \cos\theta} = \frac{d}{\sqrt{V_S^2 - V_R^2}}$$

$$t = \frac{1}{\sqrt{(10)^2 - (5)^2}} = \frac{1}{5\sqrt{3}} \text{ hr}$$

$$t = \frac{1}{5\sqrt{3}} \text{ hr}$$

Ex. In given figure if swimmer want to cross the river in the shortest time then calculate t_{min} (V_s = 10 km per/hr)



Sol. Angle with downstream or upstream $= \frac{\pi}{2}$

Angle with width of river = 0°

$$t_{min} = \frac{d}{V_S} = \frac{1\text{ km}}{10\text{ km / hr}}$$

$$t_{min} = \frac{1}{10}\text{ hr}$$

$$t_{min} = 6\text{ min}$$

Rack your Brain

A boat is sent across a river with a velocity of 8 km/hr. If the resultant velocity of boat is 10 km/hr, then calculate velocity of the river?

⇒ **Wind airplane problems:**

This is similar as swimmer-river problems. The difference is that boat is replaced by aeroplane and river is replaced by wind.

Velocity of aeroplane with respect to wind

$$\vec{V}_{aw} = \vec{V}_a - \vec{V}_w$$

$$\vec{V}_a = \vec{V}_{aw} + \vec{V}_w$$

$\vec{V}_a$ = velocity of aeroplane w.r.t. ground

$\vec{V}_w$ = velocity of wind.

Ex. An aeroplane is flying from A to B along straight line and then back again. The relative speed with respect to the wind is V. The wind blows perpendicular to line AB with speed v. The distance between A and B is ℓ. Find out total time for the round trip.

Sol. For no drift

$$V\sin\theta = v$$

$$\sin\theta = \frac{v}{V}$$

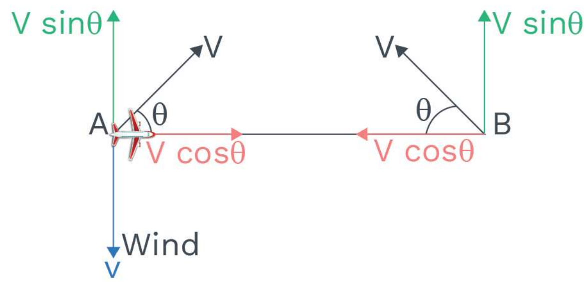
$$t = t_{AB} + t_{BA}$$

$$t = \frac{\ell}{V\cos\theta} + \frac{\ell}{V\cos\theta}$$

$$t = \frac{2\ell}{V\cos\theta} = \frac{2\ell}{V\sqrt{1-\sin^2\theta}}$$

$$t = \frac{2\ell}{V\sqrt{1-\frac{v^2}{V^2}}} = \frac{2\ell V}{V\sqrt{V^2-v^2}}$$

$$t = \frac{2\ell}{\sqrt{V^2-v^2}}$$



Rain-man problem:

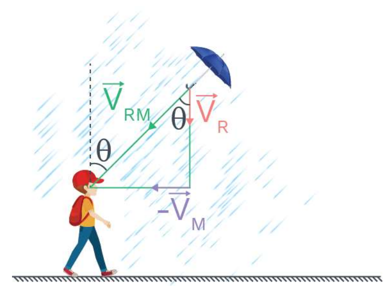
(i) If rain is falling vertically downward with a velocity $\vec{V}_R$ and an observer is moving horizontally with velocity $\vec{V}_m$ the velocity of rain relative to observer will be

$$\vec{V}_{Rm} = \vec{V}_R - \vec{V}_m$$

$$\vec{V}_{Rm} = -\vec{V}_R\hat{j} - \vec{V}_m\hat{i}$$

Which by law of vector addition has magnitude.

$$V_{Rm} = \sqrt{V_R^2 + V_m^2}$$



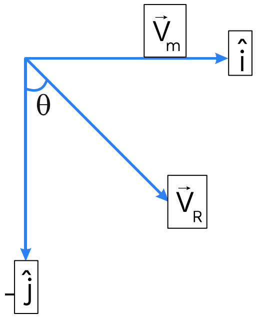
The direction of $\vec{V}_{Rm}$ is such that it makes an angle θ with the vertical given by $\theta = \tan^{-1}\left(\frac{V_m}{V_R}\right)$ as shown in figure.

(ii) If rain is already falling at an angle θ with the vertical with a velocity $\vec{V}_R$ and an observer is moving horizontally with speed $\vec{V}_m$ finds that the raindrops are hitting on his head vertically downwards

Here $\vec{V}_{Rm} = \vec{V}_R - \vec{V}_m$

$$\vec{V}_{Rm} = \left(V_R\sin\theta - V_m\right)\hat{i} - V_R\cos\theta\,\hat{j}$$

Now for rain to appear falling vertically, the horizontal component of $\vec{V}_{Rm}$ should be zero.



$$\vec{V}_R \sin\theta - V_m = 0$$

$$\sin\theta = \frac{V_m}{V_R}$$

$$\left|\vec{V}_{Rm}\right| = V_R \cos\theta = V_R\sqrt{1-\sin^2\theta}$$

$$= V_R\sqrt{1-\frac{V_m^2}{V_R^2}}$$

$$V_{Rm} = \sqrt{V_R^2 - V_m^2}$$

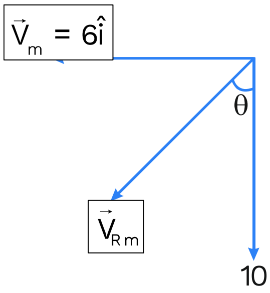
Ex. Rain is falling vertically with velocity 10 m/s and a man is moving with velocity 6 m/s. Find the angle at which the man should hold his umbrella to avoid getting wet.

Sol. $\vec{V}_{rain} = -10\hat{j}$

$\vec{V}_{man} = 6\hat{i}$

$$\tan\theta = \frac{6}{10} = \frac{3}{5}$$

$$\theta = \tan^{-1}\left(\frac{3}{5}\right)$$



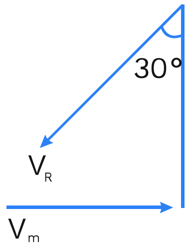
Ex. Raindrops are falling at angle 30° from vertical with speed 20 m/s. A man moving horizontal feels that the raindrops are hitting him vertically downwards then find out:

(i) V_m (ii) V_{Rm}

Sol. (i) $V_m = V_R \sin 30°$

$$V_m = 20\sin 30° = 20 \times \frac{1}{2}$$

$$V_m = 10 \text{ m/s}$$



(ii) $V_{Rm} = \sqrt{V_R^2 - V_m^2}$

$$= \sqrt{(20)^2 - (10)^2}$$

$$= 10\sqrt{3} \text{ m/s}$$

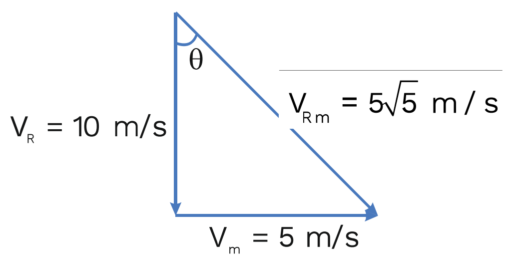
Ex. A man is moving with velocity of 5 m/s observes rain falling vertically downward with velocity 10 m/s. Find the magnitude of velocity and direction of the rain with respect to ground.

Sol. $V_{Rm} = 10 \text{ m/s}, \qquad V_m = 5 \text{ m/s}$

$\vec{V}_{Rm} = \vec{V}_R - \vec{V}_m$

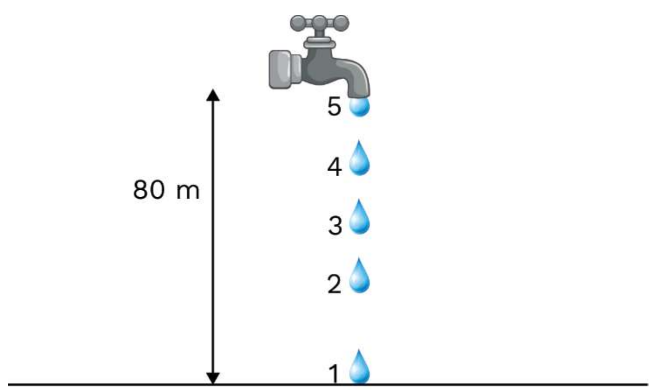
$\vec{V}_R = \vec{V}_{Rm} + \vec{V}_m$

$\vec{V}_R = \sqrt{(10)^2 + (5)^2} = \sqrt{125} = 5\sqrt{5} \text{ m/s}$



$\tan\theta = \frac{5}{10} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right)$

Ex.



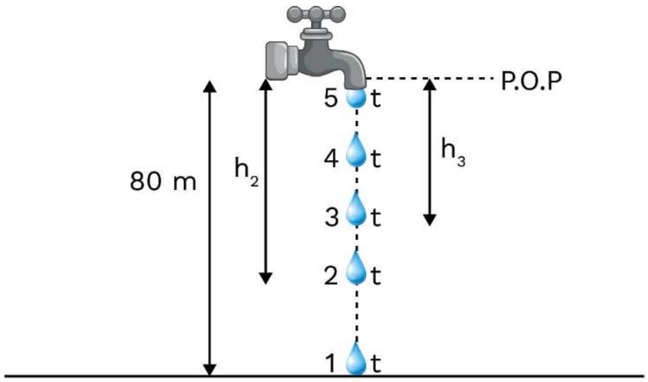
Water drops at regular interval-what is difference between height of 3rd drop and height of 2nd drop?

$h_2 - h_3 = ?$

Sol. $\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$-h_2 = 0 - \frac{1}{2}g(3t)^2$

$h_2 = \frac{1}{2}g(9t^2) \qquad .....(1)$



$$-h_3 = -\frac{1}{2}g\left(2t^2\right)$$

$$h_3 = \frac{1}{2}g\left(4t^2\right) \qquad .....(2)$$

from equation (1) and (2)

$$h_2 - h_3 = \frac{9}{2}gt^2 - 2gt^2 = \frac{5}{2}gt^2$$

Ex. A ball is thrown downwards with a velocity of 20 m/s from the top of a building of height 150 m and at same time another ball is thrown with velocity of 30 m/s vertically upward from the bottom of this building. Find the time at which both the balls will meet. ($g = 10 m/s^2$)

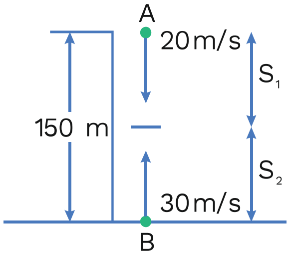
Sol. $S_1 = 20t + 5t^2$

$S_2 = 30t - 5t^2$

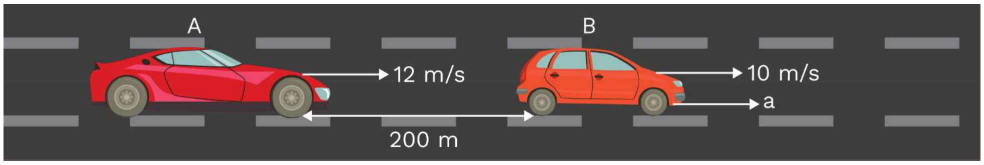
$S_1 + S_2 = 150$

$\Rightarrow \quad 150 = 50\ t$

$\Rightarrow \quad t = 3\ s$



Ex. A and B are two cars which are moving in similar direction on a straight single lane road with velocities 12m/s & 10m/s respectively. When the separation between the two was 200m B started accelerating to avoid collision. What is the minimum acceleration of car B so that they don't collide.



Sol. Acceleration of car A w.r.t. car B

$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B = \vec{a}_A - \vec{a}_B = 0 - a = (-a)$

$\vec{u}_{AB} = \vec{u}_A - \vec{u}_B = 12 - 10 = 2\,m/s.$

The collision can be avoided if relative velocity becomes zero just at the moment the two cars meet each other.

i.e. $v_{AB} = 0$ When $s_{AB} = 200$

Now $v_{AB} = 0$, $\vec{u}_{AB} = 2$, $\vec{a}_{AB} = -a$ and $s_{AB} = 200$

$\therefore\ v_{AB}^2 - u_{AB}^2 = 2a_{AB}s_{AB}$

$\Rightarrow\ 0 - 2^2 = -2 \times a \times 200$

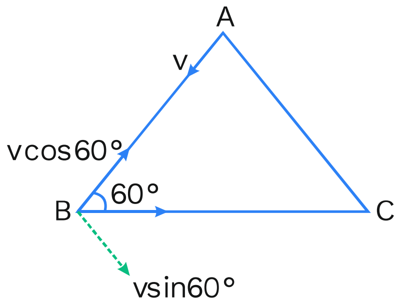
$\Rightarrow\ a = \frac{1}{100}m/s^2 = 0.01m/s^2 = 1cm/s^2.$

$\therefore$ Minimum acceleration needed by car B = 1 cm/s^2

Ex. Three persons A, B, C are standing at the vertices of an equilateral triangle of side a. If they start moving simultaneously with same speed are such that A always face towards B, B always face towards C and C always face towards A then find out

(1) Position where all they meet

(2)



(3) Time after which they will meet

(4) Draw the trajectory

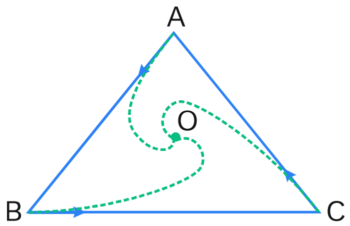
Sol. (1) They will meet at centroid of triangle

(2) By relative motion

$$V_{AB} = v - (-v\cos 60°) = \frac{3v}{2}$$

$$t = \frac{a}{\frac{3v}{2}} = \frac{2a}{3v}\,sec$$

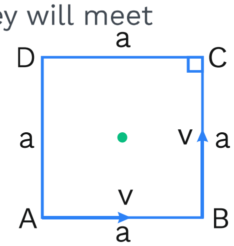
(3)



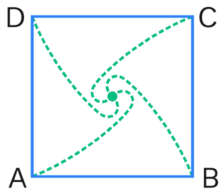
Ex. Four persons A, B, C & D are standing at the corner of a square of side a. If they start moving simultaneously with same speed are such that A always face towards B, B always face towards C, C always face towards D and D always face towards A then find out-

(1) Position where all they meet

(2) Time after which they will meet



Sol. (1) They will meet at centre of square



(2) Relative velocity of B along A is:

$V_{BA} = V$

time taken by person B to which person A is

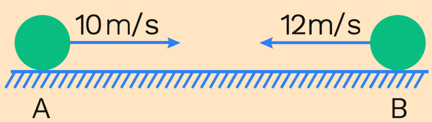
$$t = \frac{a}{V}$$

EXAMPLES

Q1 **An object A is moving with velocity 5 m/s and B is moving with velocity 20 m/s in the same direction.**
(Position x-axis)
(i) Find velocity of B w.r.t. A.
(ii) Find velocity of A w.r.t. B.

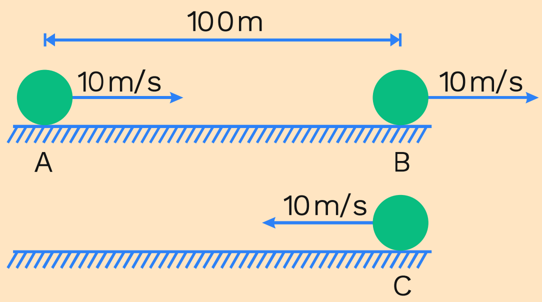
Sol: (i) $v_B = +20$ m/s
$v_A = +5$ m/s, $v_{BA} = v_B - v_A = +15$ m/s
(ii) $v_B = +20$ m/s, $v_A = +5$ m/s;
$v_{AB} = v_A - v_B = -15$ m/s

Q2 **A particle A is moving with velocity 10 m/s towards right and another particle B is moving at speed of 12 m/s towards left. Find their velocity of approach.**



Sol: $v_A = +10, v_B = -12 \Rightarrow v_{AB} = v_A - v_B$
$v_{AB} = 10 - (-12) = 22 \text{ m/s}$
Since separation is decreasing hence
$v_{app} = |v_{AB}| = 22 \text{ m/s}$

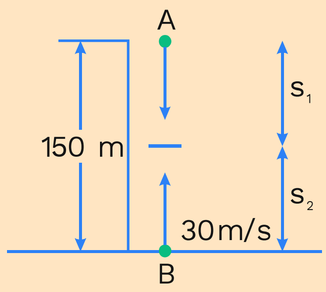
Q3 **A particle A is moving with a velocity of 10 m/s towards right, particle B is moving at a velocity of 10 m/s towards right and another particle C is moving with velocity of 10 m/s towards left. The distance between A and B is 100 m. Find out the time interval between C meeting B and C meeting A.**



Sol: $t = \dfrac{\text{seperation between A and C}}{v_{app} \text{ of A and C}}$

$= \dfrac{100}{10-(-10)} = 5 \text{ sec}$

Q4 **A ball is thrown downwards with a speed of 20 m/s from the top of a building 150 m high and at same time another ball is thrown vertically upwards with a velocity of 30 m/s from the foot of the building. Find the time after which both balls will meet.**
($g = 10 \text{ m/s}^2$)

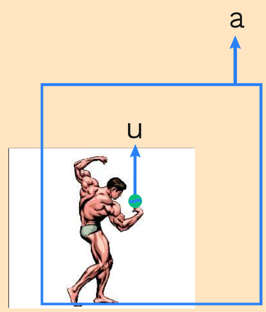


Sol: $s_1 = 20t + 5t^2$

$s_2 = 30t - 5t^2$

$s_1 + s_2 = 150 \quad \Rightarrow \quad 150 = 50t \quad \Rightarrow \quad t = 3s$

Q5 **A lift is moving up with acceleration a. A person inside the lift throws the ball upwards with a velocity u relative to hand.**
(a) Find out the time of flight of the ball.
(b) Find out the maximum height reached by the ball in the lift?



Sol: (a) $\vec{a}_{BL} = \vec{a}_B - \vec{a}_L = g + a$

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}_{BL}t^2$$

$$0 = ut - \frac{1}{2}(g + a)T^2 \qquad \therefore \qquad T = \frac{2u}{(g + a)}$$

(b) $v^2 - u^2 = 2\ as$

$$0 - u^2 = -2(g + a)H$$

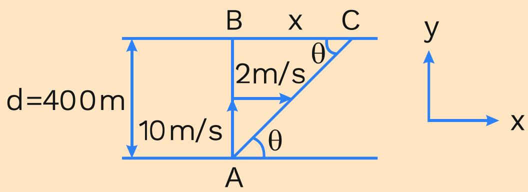
$$H = \frac{u^2}{2(g + a)}$$

Q6 **A 400 m wide river is flowing at a rate of 2.0 m/s. A boat is sailing with a velocity of 10 m/s with respect to the water, in a direction perpendicular to the river.**

(a) What is the time taken by the boat to reach the opposite bank?

(b) How far from the point directly opposite to the starting point does the boat reach the opposite bank.

(c) In what direction does the boat actually move.



Sol: (a) time taken to cross the river

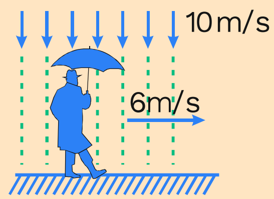
$$t = \frac{d}{v_y} = \frac{400\ \text{m}}{10\ \text{m/s}} = 40\,\text{s}$$

(b) drift (x) = $(v_x)(t)$ = (2 m/s) (40s) = 80 m

(c) Actual direction of boat,

$$\theta = \tan^{-1}\left(\frac{10}{2}\right) = \tan^{-1}5$$, (downstream) with the river flow.

Q7 **Rain is falling vertically and a man is moving with velocity 6 m/s. Find the angle at which the man should hold his umbrella to avoid getting wet.**

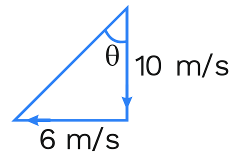


Sol: $\vec{v}_{rain} = -10\hat{j} \quad \Rightarrow \quad \vec{v}_{man} = 6\hat{j}$

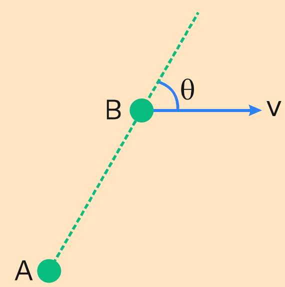
$\vec{v}_{r.w.r.t.man} = -10\hat{j} - 6\hat{i}$

$\tan\theta = \frac{6}{10} \quad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{3}{5}\right)$

Where θ is angle with vertical



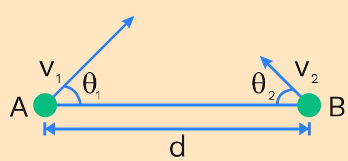
Q8 **Particle A is at rest and particle B is moving with constant velocity v as shown in the diagram at t = 0. Find their velocity of separation.**



Sol: $V_{BA} = v_B - v_A = v$

V_{sep} = component of v_{BA} along line AB = $v \cos\theta$

Q9 **Two particles A and B are moving with constant velocities v_1 and v_2. At t = 0, v_1 makes an angle θ_1 with the line joining A and B and v_2 makes an angle θ_2 with the line joining A and B.**



(i) Find out the condition for A and B to collide.

(ii) Find the time after which A and B will collide if separation between them is d at t = 0

Sol: (i) For A and B to collide, their relative velocity should be directed along the line joining them.

Therefore their relative velocity along the perpendicular to this line must be zero.

Thus $v_1 \sin\theta_1 = v_2 \sin\theta_2$.

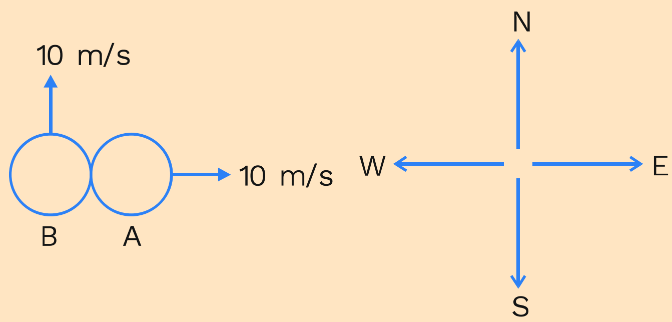
(ii) $v_{APP} = v_1 \cos\theta_1 + v_2 \cos\theta_2$

$$t = \frac{d}{v_{app}} = \frac{d}{v_1 \cos\theta_1 + v_2 \cos\theta_2}$$

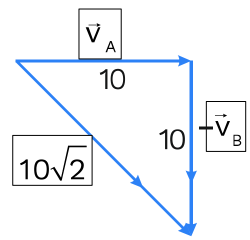
Q10 **A person can swim in still water at 5m/s. He moves in a river of velocity 3m/s, first down the stream and next same distance up the stream. The ratio of times taken are**

Sol: Speed of man in still water u=5m/s
Speed of stream v = 3m/s
From above Illustration We find that,
Time taken in Downstream is d/u+v
Time taken in Upstream is d/u–v
Hence, ratio of time taken is (u–v)/(u+v) = (5–3)/(5+3) = 1 : 4

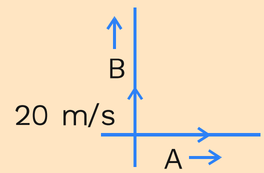
Q11 **Object A and B has velocities 10 m/s. A is moving along East while B is moving towards North from the same point as shown. Find velocity of A relative to B $(\vec{v}_{AB})$**



Sol: $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$
$\therefore |\vec{v}_{AB}| = 10\sqrt{2}$



Q12 **Consider the situation, shown in figure**



(i) Find out velocity of B with respect to A
(ii) Find out velocity of A with respect to B

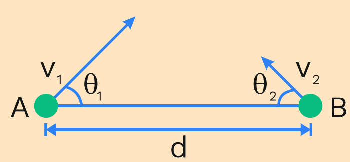
Sol: Velocity of B w.r.t. A is

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 20\hat{j} - 20\hat{i}$$

Velocity of A w.r.t. B is

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = 20\hat{i} - 20\hat{j}$$

Q13 **Two particles A and B are moving with constant velocities v_1 and v_2. At t = 0, v_1 makes an angle θ_1 with the line joining A and B and v_2 makes an angle θ_2 with the line joining A and B. Find out their velocity of approach.**



Sol: Velocity of approach is relative velocity along line AB

$$v_{APP} = v_1 \cos\theta_1 + v_2 \cos\theta_2$$

Q14 **A man can swim with velocity of 5 km/h in still water. A river 1 km wide flows at the rate of 3 km/h. The man wants to swim across the river directly opposite to the starting point.**

(a) Along what direction must the man swim?

(b) What should be his resultant velocity?

(c) How much time the man would take to cross?

Sol: (a) The velocity of man with respect to river v_{mR}= 5 km/hr, this is greater than the river flow velocity, therefore, he can cross the river directly (along the shortest path). The anlge of swim must be

$$\theta = \frac{\pi}{2} + \sin^{-1}\left(\frac{v_r}{v_{mR}}\right) = 90^\circ + \sin^{-1}\left(\frac{v_r}{v_{mR}}\right)$$

$= 90^\circ + \sin^{-1}\left(\frac{3}{5}\right) = 90^\circ + 37^\circ = 127^\circ$, with the river flow (upstream)

(b) Resultant velocity will be $v_m = \sqrt{v_{mR}^2 - v_R^2}$

$= \sqrt{5^2 - 3^2}$

= 4 km/hr

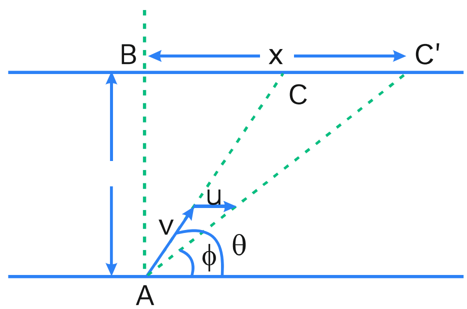
along the direction perpendicular to the river flow.

(c) time taken to cross the

$$t = \frac{\text{distance}}{\text{speed}} = \frac{1}{4}h$$

Q15 **A man wants to cross a river flowing with velocity u jumps at an angle θ w.r.t. the river flow. Find out the net speed of the man with respect to ground if he can swim with speed v. Also find how far from the point directly opposite to the starting point does the boat reach the opposite bank. In what direction should the boat actually move. If the width of the river is d.**

Sol:



Velocity of man $v_M = \sqrt{u^2 + v^2 + 2vu\cos\theta}$

$$\tan\phi = \frac{v\sin\theta}{u + v\cos\theta}$$

$$(v\sin\theta)t = d \quad \Rightarrow \quad t = \frac{d}{v\sin\theta}$$

$$x = (u + v\cos\theta)t = (u + v\cos\theta)\frac{d}{v\sin\theta}$$

Q16 **A swimmer capable of swimming with velocity 'v' relative to water jumps in a flowing river having velocity 'u'. The man swims a distance d (with respect to ground) down stream and comes back to the original position. What will be the time taken to complete this motion.**

Sol: Total time = time of swimming downstream + time of swimming upstream

$$t = t_{down} + t_{up} = \frac{d}{v+u} + \frac{d}{v-u} = \frac{2dv}{v^2 - u^2}$$

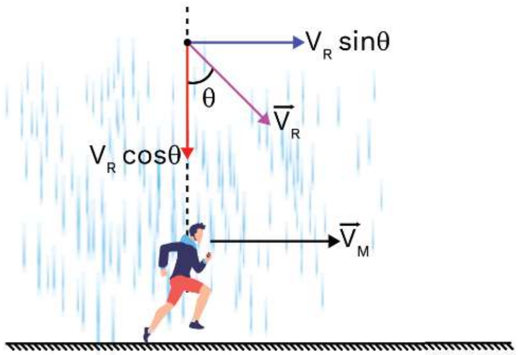
MIND MAP

MAN & RAIN

Due to wind $\vec{V}_R$ is tilted at an angle θ (w.r.t. vertical)

Condition for Vertical appearence (always above head)

If the horizontal velocity or component of $\vec{V}_R$ and horizontal component of $\vec{V}_M$ are equal



Condition for Vertical appearence

$$V_M = V_R \sin\theta$$

In such condition Velocity of rain relative to the moving man

$$V_{RM} = V_R \cos\theta \text{ (absolutely vertical)}$$

Relative Velocity

Velocity of rain relative to the man

$$\vec{V}_{rel} = \vec{V}_{RM} = \vec{V}_R - \vec{V}_M$$

$$= \vec{V}_R + (-\vec{V}_M)$$