Wave Optics

DISCLAIMER

Wave Optics

Nature of Light:-

Newton's Corpuscular Theory of Light:-

This theory was given by Newton.

- **Characteristics of the Theory:-**
  - **(i)** Extremely minute, elastic particles and very light has been uniformly emitted by all light source (luminous bodies) in all directions which are known as corpuscles.
  - **(ii)** When these corpuscles strike the retina of our eye then they produce the sensation of vision.
  - **(iii)** The velocity of these corpuscles in vacuum is 3×10^8 m/s.
  - **(iv)** The different colours of light are due to different size of these corpuscles.
  - **(v)** The rest mass of these corpuscles is zero.
  - **(vi)** The velocity of these corpuscles in an isotropic medium is same in all directions but it changes with the change of medium.
  - **(vii)** These corpuscles travel in straight lines.
  - **(viii)** These corpuscles are invisible.

- **The phenomena explained by this theory**
  - **(i)** Reflection and refraction of light.
  - **(ii)** Rectilinear propagation of light.
  - **(iii)** Existence of energy in light.

- **The phenomena not explained by this theory:-**
  - **(i)** Interference, polarisation, double refraction, diffraction and total internal reflection.
  - **(ii)** Velocity of light being more in rarer medium than that in a denser medium.
  - **(iii)** Compton effect and photoelectric effect

Huygen's Wave theory of light

Huygen produced this theory in a hypothetical medium known as luminiferrous ether.

KEY POINTS

- Corpuscle
- Interference
- Diffraction
- Polarisation
- Total internal reflection
- Photoelectric effect
- Compton effect

Concept Reminder

Ether is that imaginary medium which prevails in all space and is isotropic, perfectly elastic and mass-less

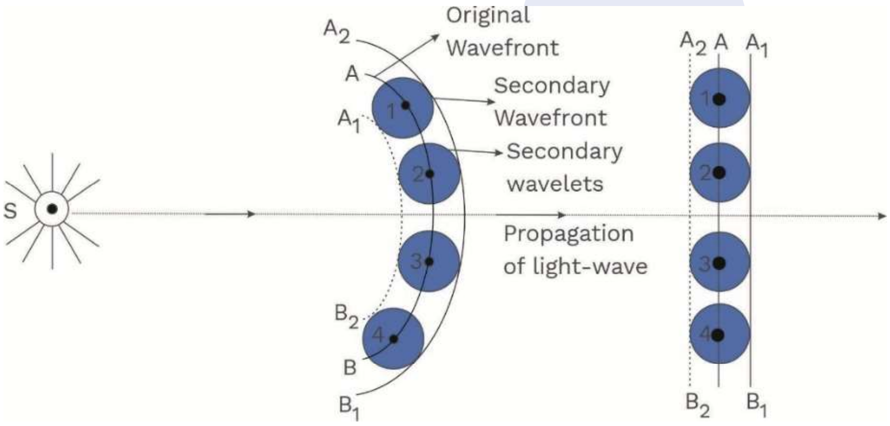
Ether is that imaginary medium which prevails in all space and is perfectly elastic, isotropic and mass-less.
The velocity of light in medium is uniform but changes with change of medium.

Huygen's Principle :
The various postulates are :

1. All source of light is a centre of disturbance from which waves spread in all directions. All particles equidistant from the source and vibrating in same phase lie on the surface known as wavefront.
2. Wave propagates perpendicular to wavefront
3. Each ray take equal time to reach from one wavefront to another wavefront
4. Each point on a wavefront is source of new disturbance which generates secondary wavelets. These wavelets are spherical & travel with the velocity of light in all directions in that medium.

Concept Reminder

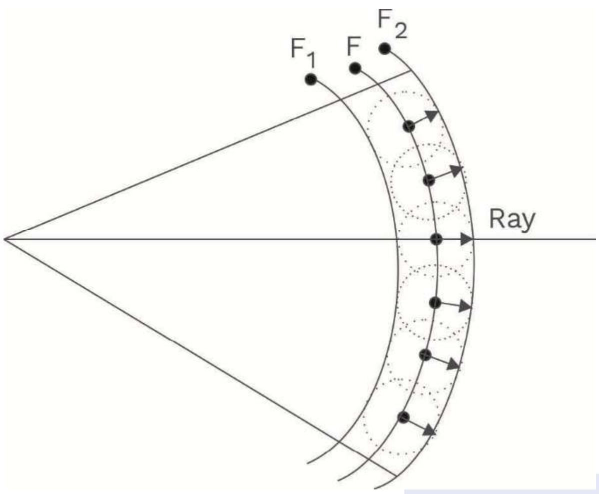
Every point on a wavefront is a source of new disturbance which produces secondary wavelets. These wavelets are spherical & travel with the speed of light in all directions in that medium



5. Only forward envelope encircling the tangents at the secondary wavelets for any instant gives the new position of wavefront. There is no reverse flow of energy when a wave travels in the forward direction.

Concept Reminder

According to Huygen's theory, There is no backward flow of energy when a wave travels in the forward direction.



The phenomena explained by this theory

(i) Reflection, refraction, diffraction, interference.

(ii) Rectilinear propagation of light.

(iii) Speed of light in rarer medium being more than that in denser medium.

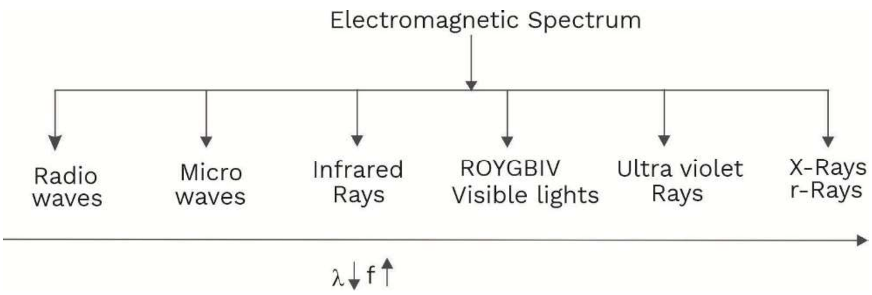
Phenomena not explained by this theory

(i) Photoelectric effect and Raman effect.

Concept Reminder

The speed of light in vacuum doesn't depend on nature of the source, direction of propagation, motion of the source or observer wavelength and intensity of the wave.

Electromagnetic Spectrum :



Visible light is that part of E.M. spectrum which is visible to us

Light can be studied under two sections.

1. Geometrical optics (If the dimensions of the body is greater as compared to wavelength of light)

2. Wave optics (If the dimensions of the body is comparable to wavelength of light.)

KEY POINTS

- Wavefronts
- Wavelets

Wave Front

- Wave front is a locus of particles which has same phase.
- Direction of propagation of wave is vertical to wave front.

Each particle of a wave front act as a new source & is known as secondary wavelet.

Types of Wave front

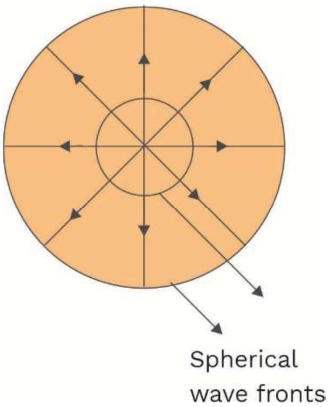
The shape of wave front is dependent upon the shape of the light source from the wave front generates. On this basis we can classify wave front in three types.

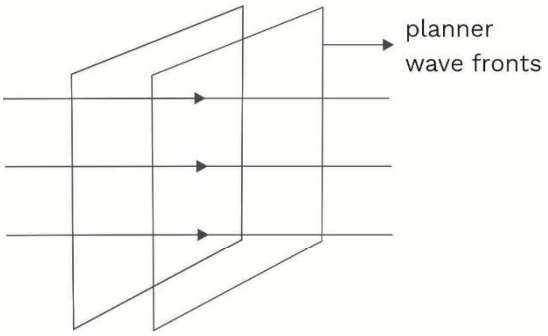
Shape of wave fronts vary from source to source.

Point source → Spherical Wave fronts

Distant Parallel Rays → Planar wave front

Line source → Cylindrical wave fronts





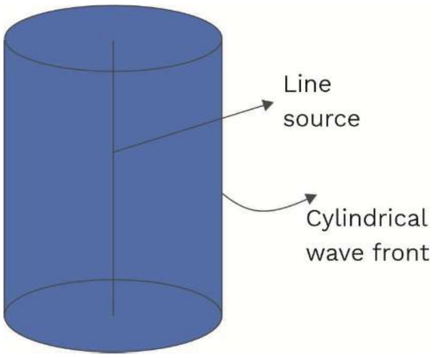
Concept Reminder

We use geometrical optics if a >> λ and wave optics if a << λ. where,
a = dimension of body
λ = wavelength of light

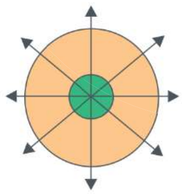
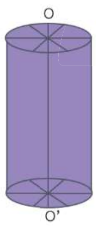
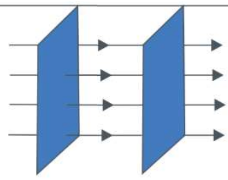
Rack your Brain

At the first minimum adjacent to the central maximum of a single-slit diffraction pattern, the phase difference between the Huygen's wavelet from the edge of the slit and the wavelet from the mid-point of the slit is

(1) $\pi/8$ radian
(2) $\pi/4$ radian
(3) $\pi/2$ radian
(4) π radian



Variation of amplitude and intensity with distance for three wavefront

S.No	Wavefront	Shape of light source	Diagram of shape of wavefront	Variation of amplitude (A) with distance	Variation of intensity (I) with distance
1.	Spherical	Point source		$A \propto \frac{1}{d}$ or $A \propto \frac{1}{r}$	$I \propto \frac{1}{r^2}$
2.	Cylindrical	Linear source or slit		$A \propto \frac{1}{\sqrt{d}}$ or $A \propto \frac{1}{\sqrt{r}}$	$I \propto \frac{1}{r}$
3.	Plane	Extended large source of point source situated at very large distance		A = constant	I = constant

Characteristic Of Wave front

(a) The phase difference between several particles on the wave front is zero.

(b) These wave fronts travel with the velocity of light in all the directions in an isotropic medium.

(c) A point source of light will always give rise to a spherical wave front in the isotropic medium.

(d) In an isotropic medium it will travels with different speed in different directions.

(e) Normal to the wave front shows a ray of light.

(f) It always propagate in the forward direction in the medium.

Interference Of Light

When two light waves of equal frequency with uniform phase difference superimpose over each other, then the resultant amplitude (or intensity) in region of superimposition will be different from amplitude (or intensity) of individual waves. This modification in intensity in region of superposition is known as interference.

(a) Constructive interference

When resultant intensity is more than the addition of two individual wave intensities [$I > (I_1 + I_2)$], then the interference is called constructive.

(b) Destructive interference

If the resultant intensity is less than the addition of two individual wave intensities [$I < (I_1 + I_2)$], then the interference is known as destructive.

There is no breaking of the law of conservation of energy in interference. In this case, the energy from the points of minimum energy shifts to the points of maximum energy.

Types Of Sources

- **Coherent Sources**

Two sources are will be called coherent if they emit light waves of the same frequency and start with equal phase or have a uniform phase difference. They are obtained from a single source.

Definitions

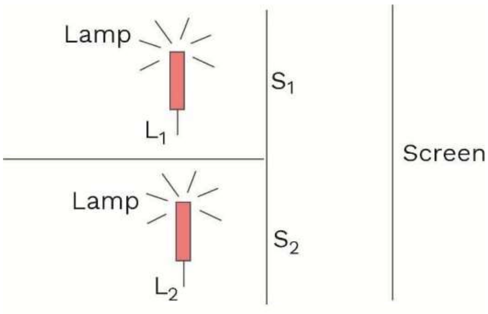
When two light waves of same frequency with constant phase difference superimpose over each other, then the resultant amplitude (or intensity) in the region of superimposition is different from the amplitude (or intensity) of individual waves This modification in intensity in the region of superposition is called interference

Concept Reminder

Reflection and refraction arise through interaction of incident light with the atomic constituents of matter. Atoms may be viewed as oscillators, which take up the frequency of the external agency (light) causing forced oscillations. The frequency of light emitted by a charged oscillator equals its frequency of oscillation. Thus, the frequency of scattered light equals the frequency of incident light.

Note : Laser is a source for monochromatic light waves of high degree of coherence.

- **Incoherent sources**



independent source
(Non-coherent source)

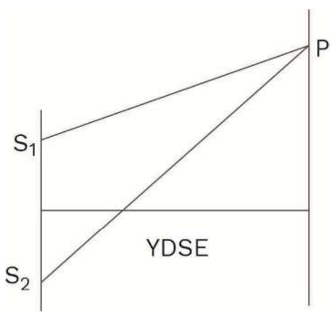
Two independent monochromatic sources, emits waves of same frequency. But the waves are not in same phase. So, they are said to be incoherent. This is because, atoms can't emit light waves in equal phase and these sources are known to be incoherent sources. By using the two independent laser beams it has been possible to record the interference pattern.

Methods Of Obtaining Coherent Source:-

- **Division of wave front:-**

In this method of obtaining coherent source, the wave front is divided into the two or more parts by reflection or refraction using mirrors, lenses or prisms.

Illustration : Young's double slit experiment (YDSE). Lloyd's single mirror method and Fresnel's Biprism.



Rack your Brain

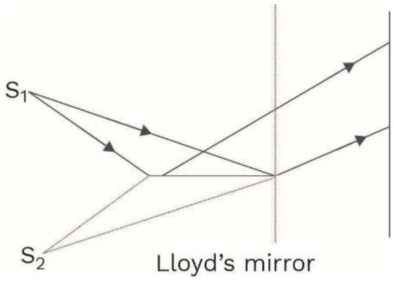
In a double slit experiment, the two slits are 1 mm apart and the screen is placed 1 m away. A monochromatic light of wavelength 500 nm is used. What will be the width of each slit for obtaining ten maxima of double slit within the central maxima of single slit pattern?

(1) 0.02 mm (2) 0.2 mm

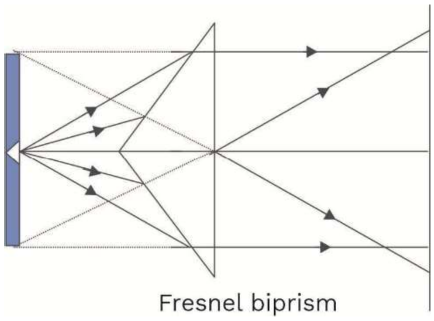
(3) 0.1 mm (4) 0.5 mm

Concept Reminder

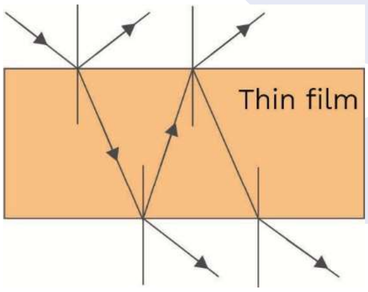
When light travels from a rarer to a denser medium, it loses some speed but it doesn't imply a reduction in the energy carried by the light wave

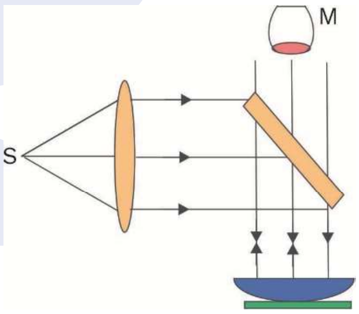


Lloyd's mirror


Fresnel biprism

- **Division of amplitude**
  The amplitude of incoming beam is divided into two or more parts by partial refraction or reflection and these divided parts travels into different paths and at last brought together to produce interference.
  Illustration : The brilliant colour seen in a thin film of transparent material like soap film, oil film, Michelson's Interferon Meter, Newton's ring etc.





Condition for sustained interference:
To get stationary interference pattern, the following conditions should be fulfilled :

(a) The two sources must be coherent, that means they must vibrate in the equal phase or there must be a uniform phase difference between them.

(b) The two sources must emit continuous waves of equal wavelength and frequency.

(c) The separation between the two coherent sources should be small.

(d) The distance of the screen from two sources should be large.

(e) For the good contrast between maxima and minima, the amplitude of the two interfering waves must be as nearly same as possible and background should be dark.

(f) For a large number of fringes in field of view, the sources should be narrow and monochromatic.

KEY POINTS
- Coherent
- Incoherent
- Constructive interference
- Destructive interference

Analysis Of Interference Of Light:-

Let two waves having amplitude a_1 and a_2 and same frequency, and constant phase difference ϕ superpose.

Let their displacement are :

$y_1 = a_1 \sin\omega t$ and $y_2 = a_2 \sin(\omega t + \theta)$

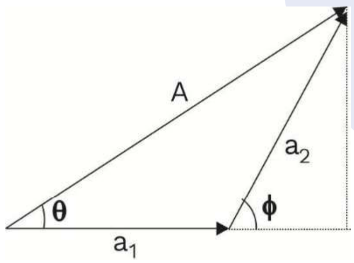
$y = y_1 + y_2 = A\sin(\omega t + \phi)$

Where A = Amplitude of resultant wave

θ = New initial phase angle

Phasor Diagram

By right angle triangle:



$$A^2 = (a_1 + a_2\cos\phi)^2 + (a_2\sin\phi)^2$$

Resultant amplitude $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$

Phase angle $\theta = \tan^{-1}\left(\dfrac{a_2\sin\phi}{a_1 + a_2\cos\phi}\right)$

Intensity $\propto$ (Amplitude)^2 $\Rightarrow I \propto A^2 \Rightarrow I = KA^2$

so $I_1 = Ka_1^2$ & $I_2 = Ka_2^2$

$\therefore \quad I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

here, $2\sqrt{I_1}\sqrt{I_2}\cos\phi$ is defined as interference factor.

- If distance of a source from the two points 'A' and 'B' is x_1 and x_2 then

Path difference $\delta = x_2 - x_1$

Phase difference $\phi = \dfrac{2\pi}{\lambda}(x_2 - x_1) \Rightarrow \phi = \dfrac{2\pi}{\lambda}\delta$

Rack your Brain

For a parallel beam of monochromatic light of wavelength λ, diffraction is produced by a single slit whose width a is of the order of the wavelength of the light. If D is the distance of the screen from the slit, the width of the central maxima will be

(1) $\dfrac{2D\alpha}{\lambda}$ (2) $\dfrac{2D\lambda}{\alpha}$

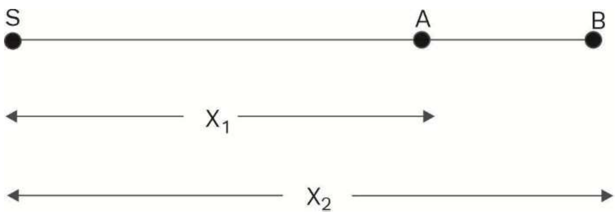
(3) $\dfrac{D\lambda}{\alpha}$ (4) $\dfrac{D\alpha}{\lambda}$

Concept Reminder

If the distance of a source from two points A and B is x_1 and x_2 then Path difference $\delta = x_2 - x_1$

Phase difference

$\phi = \dfrac{2\pi}{\lambda}(x_2 - x_1) \Rightarrow \phi = \dfrac{2\pi}{\lambda}\delta$



Time difference $\Delta t = \frac{\phi}{2\pi} t$

$$\frac{\text{Phase difference}}{2\pi} = \frac{\text{Path difference}}{\lambda} = \frac{\text{Time difference}}{T}$$

$$\Rightarrow \frac{\phi}{2\pi} = \frac{\delta}{\lambda} = \frac{\Delta t}{T}$$

Conditions for having constructive and destructive Interference

Constructive Interference

When both waves are in same phase then phase difference is an even multiple of $\pi \Rightarrow \phi = 2n\pi ; n = 0, 1, 2$ ......

- Path difference is an even multiple of $\frac{\lambda}{2}$

$$\because \quad \frac{\phi}{2\pi} = \frac{\delta}{\lambda} \Rightarrow \frac{2n\pi}{2\pi} = \frac{\delta}{\lambda}$$

$$\Rightarrow \delta = 2n\left(\frac{\lambda}{2}\right) \Rightarrow \delta = n\lambda \text{ (where n = 0, 1, 2...)}$$

- When time difference is even multiple of $\frac{T}{2}$

$$\therefore \quad \Delta t = 2n\left(\frac{T}{2}\right)$$

- In the above condition the resultant amplitude and intensity each maximum.

$$A_{max} = (a_1 + a_2)$$

$$\Rightarrow \boxed{I_{max} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} = (\sqrt{I_1} + \sqrt{I_2})^2}$$

Destructive Interference

When both waves are in opposite phase. So phase difference is odd multiple of π.

$$\phi = (2n-1)\pi ; n = 1, 2 \ldots$$

When path difference is odd multiple of $\frac{\lambda}{2}$, $\delta = (2n-1)\frac{\lambda}{2}$, $n = 1, 2 \ldots$

- When time difference is odd multiple of $\frac{T}{2}$,

  $\Delta t = (2n - 1)\frac{T}{2}$, $(n = 1, 2 ....)$

  In the above condition the resultant amplitude and intensity of wave will be minimum.

  $$A_{min} = (a_1 - a_2) \Rightarrow \boxed{I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2}$$

Note :

- Interference follows law of conservation of energy.
- Average Intensity $I_{av} = \frac{I_{max} + I_{min}}{2} = I_1 + I_2 = a_1^2 + a_2^2$
- Intensity $\propto$ width of slit $\propto$ (amplitude)^2

  $$I \propto w \propto a^2 \Rightarrow \frac{I_1}{I_2} = \frac{w_1}{w_2} = \frac{a_1^2}{a_2^2}$$

- $\frac{I_{max}}{I_{min}} = \left[\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right]^2 = \left[\frac{a_1 + a_2}{a_1 - a_2}\right]^2 = \left[\frac{a_{max}}{a_{min}}\right]^2$
- Fringe visibility $V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100\%$ when $I_{min} = 0$ then fringe visibility is maximum that means when both slits are of same width the fringe visibility is best and 100%.

Concept Reminder

- Condition of constructive interference

(i) phase = $2n\pi$ difference

(ii) path difference = $n\lambda$ where n = 0, 1, 2, 3 ....

- condition of destructive interference

(i) phase difference = $(2n - 1)\pi$

(ii) path difference = $(2n - 1)\frac{\lambda}{2}$

where n = 1, 2, 3 ......

Ex. If two waves represented by $y_1 = 4\sin\omega t$ & $y_2 = 3\cos\omega t$ interfere at a point. Calculate the amplitude of the resulting wave

Sol. Resultant amplitude (A) = $\sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi} = \sqrt{4^2 + 3^2 + 2(4)(3)\cos 90^\circ} = 5\,\text{Unit}$

Ex. Two waves having the intensities in the ratio of 9 : 1 produce interference. The ratio of max. to min. intensity is equal to

Sol. Intensity ratio $\frac{I_1}{I_2} = \frac{9}{1}$

$\therefore$ Amplitude ratio $\frac{a_1}{a_2} = \frac{3}{1}$

If a_1 = 3 units, then a_2 = 1 unit.

At maxima $a_{max} = a_1 + a_2 = 4$

At minima, $a_{min} = a_1 - a_2 = 2$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{(a_{max})^2}{(a_{min})^2} = \frac{16}{4} = \frac{4}{1}$$

Ex. The interference pattern is observed intensity ratio between the bright and dark fringes as nine. Find the ratio of (a) intensities, and (b) amplitudes of the two interfering waves?

Sol. In case of interference, we have

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

(a) $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$

and $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = (\sqrt{I_1} - \sqrt{I_2})^2$

Here, $\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2} = \frac{9}{1}$ or $\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} = \frac{3}{1}$

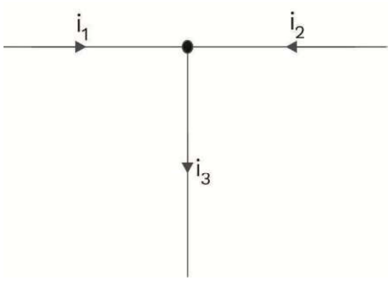
By componendo and dividendo, we get

$$\frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{3+1}{3-1} \text{ or } \frac{I_1}{I_2} = \frac{4}{1} = 4$$

(b) Now, as $I \propto A^2$,

$$\frac{I_1}{I_2} = \left[\frac{A_1}{A_2}\right]^2, \left[\frac{A_1}{A_2}\right]^2 = 4 \text{ or } \frac{A_1}{A_2} = 2$$

Ex. If $i_1 = 3\sin\omega t$ and $i_2 = 4\cos\omega t$, find i_3 which is given by $i_3 = i_1 + i_2$



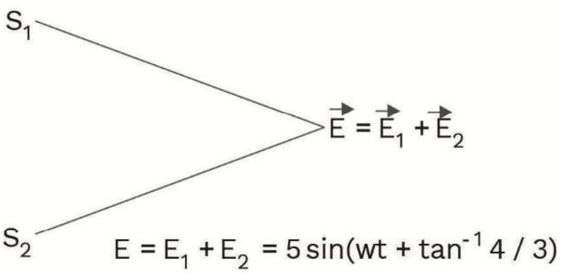
Sol. $i_3 = i_1 + i_2$

$$= 3\sin\omega t + 4\sin(\omega t + \frac{\pi}{2})$$

$$= 5\sin(\omega t + \tan^{-1}\frac{4}{3})$$

Ex. S_1 & S_2 are two source of light which produce individually disturbance at point P given by $E_1 = 3\sin\omega t$, $E_2 = 4\cos\omega t$. Suppose $\vec{E}_1$ & $\vec{E}_2$ to be along the same line, find the result of their superposition.

Sol.



Ex. Light from two source, each of same frequency and travelling in equal direction, but with intensity in the ratio 4 : 1 interfere. Calculate ratio of maximum to minimum intensity.

Sol. $$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right) = \left(\frac{\sqrt{\frac{I_1}{I_2}}+1}{\sqrt{\frac{I_1}{I_2}}-1}\right) = \left(\frac{2+1}{2-1}\right)^2 = 9:1$$

Ex. Find the max. intensity in case of interference of 'n' identical waves each of intensity I_0 if the source is (a) coherent and (b) incoherent.

Sol. The resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

(a) The sources are known as coherent if they have constant phase difference between them. Then intensity will be maximum when $\phi = 2n\pi$; the sources are in same phase.

Thus $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$

Similarly, for n identical waves,

$$I_{max} = (\sqrt{I_0} + \sqrt{I_0} + ...)^2 = n^2 I_0$$

(b) The incoherent sources have phase difference that changes randomly with time

Thus $[\cos\phi]_{av} = 0$

Hence $I = I_1 + I_2$

Hence for 'n' identical waves,

$$I = I_0 + I_0 + ... = nI_0$$

Young's Double Slit Experiment (Y.D.S.E.) :

In 1802, the Thomas Young purposed a method to generate a stationary interference pattern. This was based upon the division of a single wave front into 2 ; these 2 wave fronts acted as if they emanated from 2 sources having a fixed phase relationship. Hence when they were permitted to interfere, stationary interference pattern was observed.

Concept Reminder

In the photon picture of light, for a given frequency, intensity of light is determined by the number of photons per unit area.

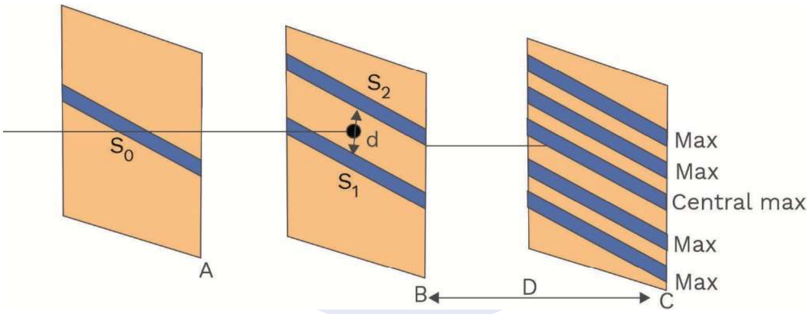


Figure: Young's Arrangement to generate stationary interference pattern by the division of the wave front S_0 into S_1 and S_2

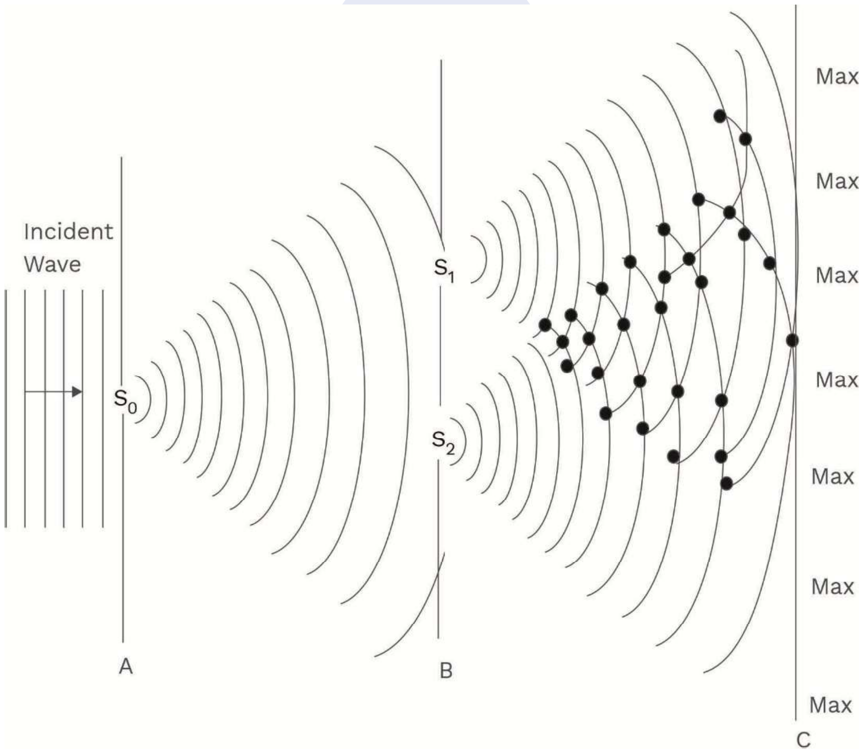
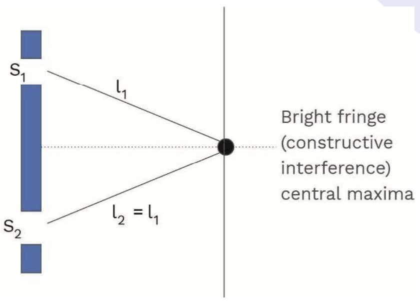
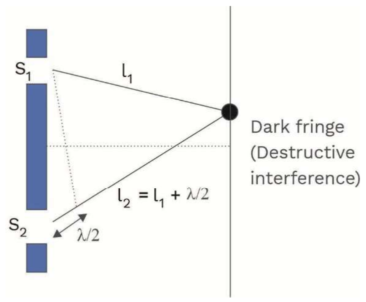
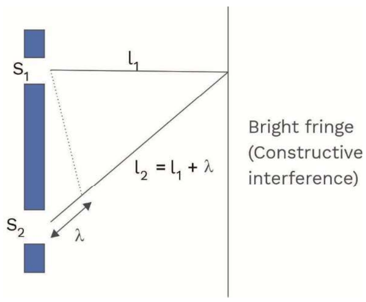


Figure : In Young's interference experiment, light diffracted from pinhole S_0 encounters pinholes S_1 and S_2 in screen B. Light diffracted from these two pinholes overlaps in the region between screen B and viewing screen C, producting an interference pattern on screen C.

The geometry of experiment is simple Parallel wave front of a monochromatic wave are incident on two identical narrow slits, each of width a separated by a distance d. The slit widths & their separation are of the order of the wavelength of the incident monochromatic light. Monochromatic light after passing through two slits S_1 & S_2 acts as coherent sources of light waves that interfere constructively & destructively at different point on the screen to produce a interference pattern.







KEY POINTS

- Young's double slit interference.
- Maxima
- Minima
- Fringe width

Concept Reminder

Astronomers call the increase in wavelength due to doppler effect as red shift since a wavelength in the middle of the visible region of the spectrum moves towards the red end of the spectrum. When waves are received from a source moving towards the observer, there is an apparent decrease in wavelength, this is referred to as blue shift.

Rack your Brain

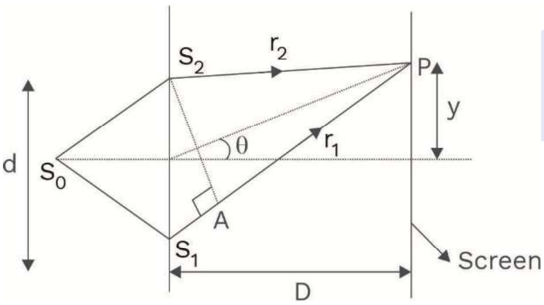
Two slits in Youngs experiment have widths in he ratio 1 : 25. The ratio of intensity at the maxima and minima in the interference pattern, $\frac{I_{max}}{I_{min}}$ is

(1) $\frac{4}{9}$ (2) $\frac{9}{4}$

(3) $\frac{121}{49}$ (4) $\frac{49}{121}$

Analysis of Interference Pattern:

We have insured in the above arrangement that the light wave passing through S_1 is in phase with that passing through S_2. However the wave reaching P from S_2 may not be in phase with the wave reaching P from S_1, because the latter must travel a longer path to reach P than the former. We have already discussed the phase-difference arising due to path difference. if the path difference is equal to zero or is an integral multiple of wavelengths, the arriving waves are exactly in phase and undergo constructive interference. If the path difference is an odd multiple of half a wavelength, the arriving waves are out of phase and undergo fully destructive interference. Thus, it is the path difference Δx, which determines the intensity at a point P.



Path difference

$$\Delta x = S_1P - S_2P = \sqrt{\left(y + \frac{d}{2}\right)^2 + D^2} - \sqrt{\left(y - \frac{d}{2}\right)^2 + D^2} \quad \text{... (1)}$$

Approximation I :

For D >> d, we can estimate rays $\vec{r}_1$ and $\vec{r}_2$ as being approximately parallel, at angle θ to the principle axis.

Now, $S_1P - S_2P = S_1A = S_1S_2 \sin\theta$

$\Rightarrow$ path difference = $d \sin\theta$ ... (2)

Concept Reminder

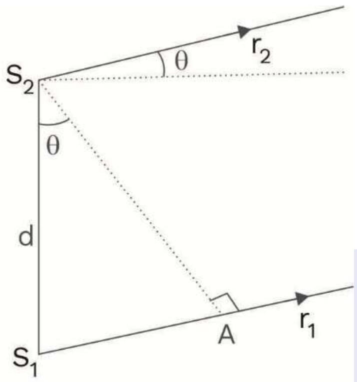
In YDSE

Path difference = $\frac{yd}{D}$

Approximation II :

Later if θ is small, i.e., y << D, $\tan\theta = \frac{y}{D}$

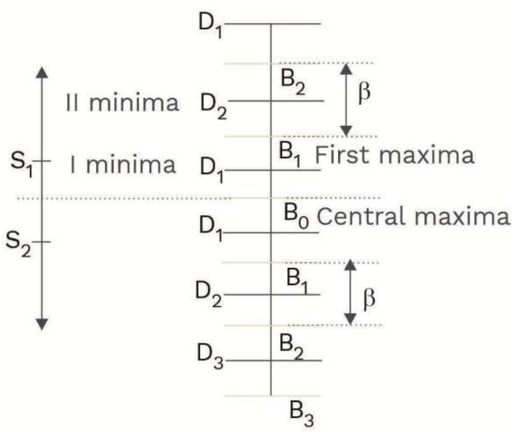
and hence, path difference = $\frac{dy}{D}$ ... (3)



for maxima (constructive interference)

$$\Delta x = \frac{dy}{D} = n\lambda$$

$$\Rightarrow \; y = \frac{n\lambda D}{d}, n = 0, \pm 1, \pm 2, \pm 3 \quad \text{... (4)}$$



Here n = 0 corresponds to central maxima
n = ± 1 correspond to the 1^{st} maxima
n = ± 2 correspond to the 2^{nd} maxima and later on.

for minima (destructive interference)

$$\Delta x = \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2} \pm\frac{5\lambda}{2}$$

Concept Reminder

In YDSE, the position of maxima is $\frac{\lambda D}{D}$ where n = 0, +1, +2, + 3 ......

Concept Reminder

In YDSE, position of minima is $y = \frac{(2n-1)\lambda D}{2d}$

where n = 1, 2, 3 ....

$$\Rightarrow \quad \Delta x = \begin{cases} (2n-1)\dfrac{\lambda}{2} & n = 1, 2, 3...... \\ (2n+1)\dfrac{\lambda}{2} & n = -1, -2, -3..... \end{cases}$$

consequently,

$$y = \begin{cases} (2n-1)\dfrac{\lambda D}{2d} & n = 1, 2, 3...... \\ (2n+1)\dfrac{\lambda D}{2d} & n = -1, -2, -3..... \end{cases} \quad ... (5)$$

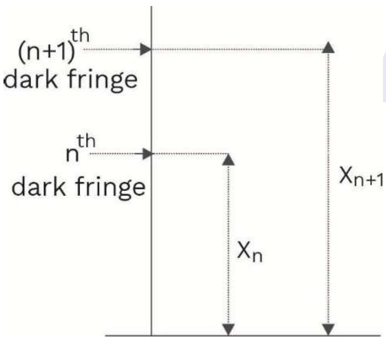
Here $n = \pm 1$ corresponds to first minima,
$n = \pm 2$ corresponds to second minima and so on.

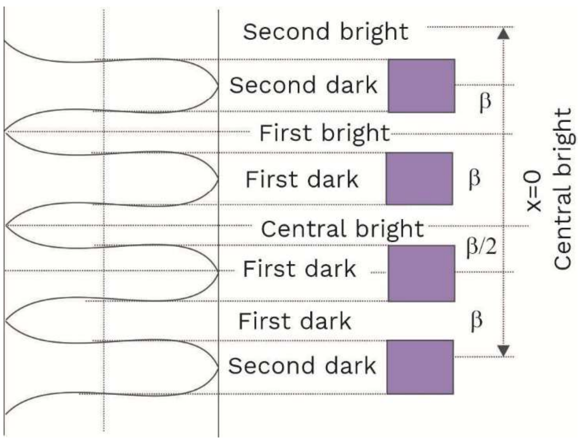
Fringe Width

The distance between 2 successive bright or dark fringe is known as fringe width.

$$\beta = x_{n+1} - x_n = \frac{(n+1)D\lambda}{d} - \frac{nD\lambda}{d}$$

Fringe Width $\beta = \dfrac{D\lambda}{d}$





Concept Reminder

The distance between two successive bright or dark fringe is known as fringe width.

$$\beta = x_{n+1} - x_n = \frac{(n+1)D\lambda}{d} - \frac{nD\lambda}{d}$$

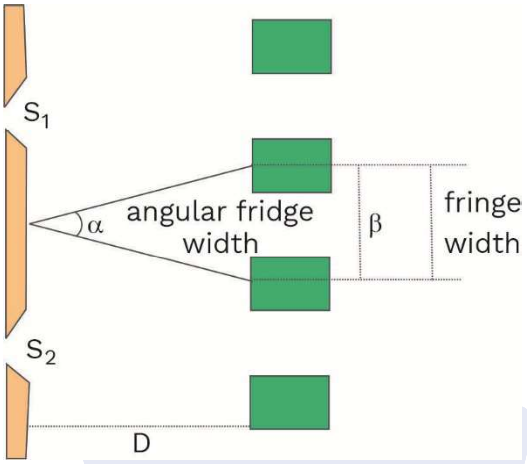
Fringe Width $\beta = \dfrac{D\lambda}{d}$

Rack your Brain

In the Young's double-slit experiment, the intensity of light at a point on the screen where the path difference is λ is K, (λ being the wavelength of light used). The intensity at a point where the path difference is $\lambda/4$, will be

(1) K
(2) $\dfrac{K}{4}$
(3) $\dfrac{K}{2}$
(4) Zero

Angular Fringe Width



$\alpha = \frac{\beta}{D}, \alpha = \frac{\lambda}{d}$ $\left[\because \frac{\beta}{D} = \frac{\lambda}{d}\right]$

- The distance between n^{th} bright fringe and the central bright fringe $x_n = \frac{n\lambda D}{d} = n\beta$
- The distance between n_1 and n_2 bright fringe $x_{n_2} - x_{n_1} = n_2\frac{\lambda D}{d} - n_1\frac{\lambda D}{d} = (n_2 - n_1)\beta$
- The distance of m^{th} dark fringe from central fringe $x_m = \frac{(2m-1)D\lambda}{2d} = \frac{(2m-1)\beta}{2}$
- The distance of n^{th} bright fringe from m^{th} dark fringe $x_n - x_m = n\frac{D\lambda}{d} - \frac{(2m-1)D\lambda}{2d} = n\beta - \frac{(2m-1)\beta}{2}$

$$x_n - x_m = \left[n - \frac{(2m-1)}{2}\right]\beta$$

Ex. In a Y.D.S.E. 2 narrow slits at 0.8 mm are illuminated by a source of yellow light (λ = 5893 Å). At what distance the adjacent bright bands in the interference pattern observed on a screen 2 m away?

Sol. Given: $d = 0.8 \times 10^{-3}m$, $\lambda = 5893\,\text{Å} = 5893 \times 10^{-10}m$, $D = 2m$

The fringe width : $\beta = \frac{\lambda D}{d} = \frac{5893 \times 10^{-10} \times 2}{0.8 \times 10^{-3}}m = 1.47\,mm$

Ex. Laser light of 630 nm wavelength incident on a set of slits generates an interference pattern in which the fringes are separated by 8.1 mm. A second laser light produces an interference pattern in which the fringes are separated by 7.2 mm. Calculate the wavelength of the second light.

Sol. Fringe separation is given by $\beta = \frac{\lambda D}{d}$ i.e., $\frac{\beta_2}{\beta_1} = \frac{\lambda_2}{\lambda_1}$

$$\Rightarrow \lambda_2 = \frac{\beta_2}{\beta_1} \times \lambda_1 = \frac{7.2}{8.1} \times 630 = 560\,\text{nm}$$

Ex. A Y.D.S.E. arrangement generates interference fringes for sodium light (λ = 5890 Å) which are 0.20° apart. Find the angular fringe separation if the entire arrangement is immersed in water? (Refractive index of water = 4/3)

Sol. The wavelength of light in water is $\lambda_w = \frac{\lambda}{\mu}$

Angular fringe-width in air, $(\theta_0)_a = \frac{\lambda}{d}$

Angular fringe-width in water, $(\theta_0)_w = \frac{\lambda_w}{d}$

$$\therefore \quad (\theta_0)_w = \frac{\lambda_w}{\theta} \times \frac{\lambda}{\mu d} = \frac{(\theta_0)_a}{\mu}$$

$$= \frac{0.20^\circ}{4/3} = 0.15^\circ$$

Ex. In a Young's double slit experiment, two wavelengths of 500 nm and 700 nm were used. What is the minimum distance from the central maximum where their maxima's coincide again? Take D/d = 10^3. Symbols have their usual meanings.

Sol. $n_1 \frac{D}{d}\lambda_1 = n_2 \frac{D}{d}\lambda_2$ (for the maxima to coincide)

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{7}{5}$$

$$n_1 = 7, n_2 = 5$$

$$\therefore \quad \text{Minimum distance } n_1 \frac{D}{d}\lambda_1 = 7 \times 10^3 \times 5 \times 10^{-7}$$

$$= 35 \times 10^{-4}\,\text{m} = 3.5\,\text{mm}$$

Ex. In Young's double slit experiment, D = 1 m, d = 1 mm and $\lambda = 1/2$ mm

(i) Calculate the distance between the first and central maxima on the screen.

(ii) Calculate the number of maxima and minima obtained on the screen.

Sol. D >> d

Hence $\Delta P = d\sin\theta$

$\frac{d}{\lambda} = 2,$

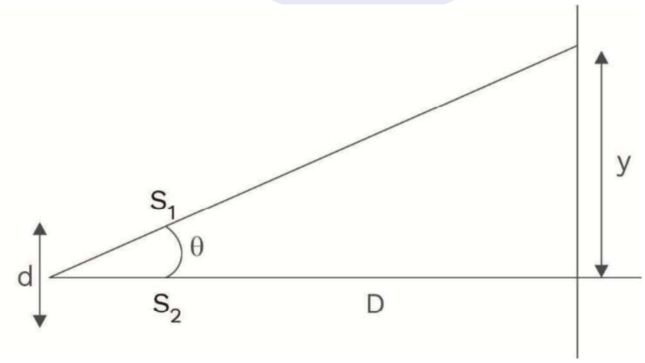
clearly, $n << \frac{d}{\lambda} = 2$ is not possible for any value of n.

Hence $\Delta p = \frac{dy}{D}$ cannot be used

for Ist maxima, $\Delta p = d\sin\theta = \lambda$

$\Rightarrow \quad \sin\theta = \frac{\lambda}{d} = \frac{1}{2} \quad \Rightarrow \quad \theta = 30°$

Hence, $y = D\tan\theta = \frac{1}{\sqrt{3}}$ meter



(ii) Maximum path difference

$\Delta P_{max} = d = 1mm$

$\Rightarrow$ Highest order maxima, $n_{max} = \left[\frac{d}{\lambda}\right] = 2$

and highest order minima $n_{min} = \left[\frac{d}{\lambda} + \frac{1}{2}\right] = 2$

Total no. of maxima = $2n_{max} + 1 = 5$

(for central maxima 1 is added)

Total no. of minima $= 2n_{min} = 4$

Ex. Monochromatic light of wavelength 5000 Å is used in Y.D.S.E, with slit-width, d = 1 mm, distance between screen and slits, D = 1 m. If the intensity at the two slits are $I_1 = 4I_0$, $I_2 = I_0$, find

(i) fringe width β

(ii) distance of fifth minima from the central maxima on the screen

(iii) Intensity at $y = \frac{1}{3}$ mm

(iv) Distance of the 1000th maxima

(v) Distance of the 5000th maxima

Sol. **(i)** $\beta = \frac{\lambda D}{d} = \frac{5000 \times 10^{-10} \times 1}{1 \times 10^{-3}} = 0.5\,\text{mm}$

(ii) $y = (2n - 1) \times \frac{\lambda D}{2d}, n = 5 \Rightarrow y = 2.25\,\text{mm}$

(iii) At $y = \frac{1}{3}\text{mm}, y << D$

Hence $\Delta p = \frac{dy}{D}$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta p = 2\pi\frac{dy}{\lambda D} = \frac{4\pi}{3}$$

Now resultant intensity

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi$$

$$= 4I_0 + I_0 + 2\sqrt{4I_0^2}\cos\Delta\phi = 5I_0 + 4I_0\cos\frac{4\pi}{3} = 3I_0$$

(iv) $\frac{d}{\lambda} = \frac{10^{-3}}{0.5 \times 10^{-6}} = 2000$

n = 1000 is not << 2000

Hence now $\Delta p = d\sin\theta$ must be used

Hence, $d\sin\theta = n\lambda = 1000\,\lambda$

$\Rightarrow$ $\sin\theta = 1000\frac{\lambda}{d} = \frac{1}{2} \Rightarrow \theta = 30°$

$y = D\tan\theta = \frac{1}{\sqrt{3}}$ meter

(vi) Highest order maxima

$$n_{max} = \left[\frac{d}{\lambda}\right] = 2000$$

Hence, n = 5000 is not possible.

Intensity :

Assume the electric field components of the light waves arriving at point P (in the Figure) from the 2 slits S_1 and S_2 vary with time as

$$E_1 = E_0 \sin \omega t$$

and $E_2 = E_0 \sin(\omega t + \phi)$

Here $\phi = k\Delta x = \frac{2\pi}{\lambda}\Delta x$

Concept Reminder

The Doppler effect for light is very important in astronomy. It is the basis for the measurements of the radial velocities of distant galaxies.

and we have assumed that the intensity of the two slits S_1 and S_2 are same (say I_0); hence waves have same amplitude E_0 then the resultant electric field at point P is given by,

$$E = E_1 + E_2 = E_0 \sin \omega t + E_0 \sin(\omega t + \phi)$$

$$= E_0 ' \sin(\omega t + \phi ')$$

where $E_0'^2 = E_0^2 + E_0^2 + 2E_0.E_0 \cos\phi = 4E_0^2 \cos^2 \phi / 2$

Hence the resultant intensity at point P,

$$I = 4I_0 \cos^2 \frac{\phi}{2} \qquad \text{... (2)}$$

$$I_{max} = 4I_0 \quad \text{when } \frac{\phi}{2} = n\pi, \qquad n = 0, \pm 1, \pm 2.......$$

$$I_{min} = 0 \text{ when} \frac{\phi}{2} = \left(n - \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2.......$$

Here $\phi = k\Delta x = \frac{2\pi}{\lambda}\Delta x$

If $D >> d$, $\qquad \phi = \frac{2\pi}{\lambda} d \sin\theta$

If $D >> d$ & $\qquad y << D, \phi = \frac{2\pi}{\lambda} d \frac{y}{D}$

However, if the two slits were of distinct intensities I_1 and I_2,

say $\qquad E_1 = E_{01} \sin \omega t$

and $\qquad E_2 = E_{02} \sin(\omega t + \phi)$

then resultant field at point P,

$$E = E_1 + E_2 = E_0 \sin(\omega t + \phi)$$

Where $\qquad E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01} E_{02} \cos\phi$

Hence resultant intensity at point P,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$$

Ex. A beam of light consisting of 2 wavelengths, 6500 Å and 5200 Å, is used to get interference fringes in a Young's double slit experiment. If the distance between the slits is 2 mm and plane of the slits and the screen is 120 cm apart then,

(a) Calculate the distance of the 3rd bright fringe on the screen from the central max. for the wavelength 6500 Å

(b) Find the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

Sol. Given : $\lambda_1 = 6500\,\text{Å} = 6500 \times 10^{-10}\,\text{m}$,

$\lambda_2 = 5200\,\text{Å} = 5200 \times 10^{-10}\,\text{m}$

$d = 2\,\text{mm} = 2 \times 10^{-3}\,\text{m}, D = 120\,\text{cm} = 1.2\,\text{m}$

(a) $y_n = \dfrac{n\lambda D}{d}$

$\therefore \quad y_3 = \dfrac{3 \times 6500 \times 10^{-10} \times 1.2}{2 \times 10^{-3}} = 1.17\,\text{mm}$

(b) The least distance from the central max. where the bright fringes because of both the wavelength coincide corresponds to that value of n for which

$n_1\lambda_1 = n_2\lambda_2$

or $\quad \dfrac{n_2}{n_1} = \dfrac{6500}{5200} = \dfrac{5}{4}$

$\therefore \quad n_1 = 4$

$\therefore \quad y = \dfrac{n\lambda D}{d} = \dfrac{4 \times 6500 \times 10^{-10} \times 1.2}{2 \times 10^{-3}}$

$= 1.56 \times 10^{-3}\,\text{m} = 1.56\,\text{mm}$

Ex. In a Young's double-slit experiment the distance between the slits is 1 mm and distance of the screen from slits is 1 m. If light of wavelength 6000 Å is used, find the distance between the second dark fringe and the fourth bright fringe.

Sol. The position of the 2nd dark fringe,

$y_2(\text{dark}) = (2n - 1)\dfrac{\lambda}{2}\dfrac{D}{d}$

$= (4 - 1)\dfrac{\lambda}{2}\dfrac{D}{d} = \dfrac{3}{2}\dfrac{\lambda D}{d}$

The position of the 4th bright fringe,

$y_2(\text{bright}) = \dfrac{n\lambda D}{d} = \dfrac{4\lambda D}{d}$

Therefore, the distance,

$$= y_4(\text{bright}) - y_2(\text{dark}) = \left(4 - \frac{3}{2}\right)\frac{\lambda D}{d}$$

$$= \frac{5}{2} \times \frac{6000 \times 10^{-10} \times 1}{10^{-3}} = 1.5 \times 10^{-3} = 1.5\,\text{mm}$$

Ex. In a YDSE bi-chromatic light of wavelengths 400 nm and 560 nm are used. The slits are 0.1 mm apart and plane of the slits and the screen is 1 m apart. Find the minimum distance between two successive regions of complete darkness.

Sol. Let n^{th} minima of 400 nm coincides with m^{th} minima of 560 nm then

$$(2n - 1)400 = (2m - 1)560$$

$$\Rightarrow \quad \frac{2n-1}{2m-1} = \frac{7}{5} = \frac{14}{10} = \frac{21}{15}$$

i.e. 4^{th} minima of 400 nm coincides with 3^{rd} minima of 560 nm.

The location of this minima is

$$= \frac{7(1000)(400 \times 10^{-6})}{2 \times 0.1} = 14\,\text{mm}$$

Next, 11^{th} minima of 400 nm will coincide with 8^{th} minima of 560 nm

Location of this minima is

$$= \frac{21(1000)(400 \times 10^{-6})}{2 \times 0.1} = 42\,\text{mm}$$

$\therefore$ Required distance = 28 mm

Ex. In a Young's double-slit experiment β is the fringe width and I_0 is the intensity at the central bright fringe. At a distance 'y' from the central bright fringe, find the intensity

Sol. The fringe width is given as $\beta = \dfrac{\lambda D}{d}$

The path difference for the given point is

$$\Delta x = y\frac{d}{D}$$

$\therefore$ phase difference, $\Delta\phi = \dfrac{2\pi}{\lambda}\Delta x$

If a is the amplitude due to each slit, at the screen,

$$I_0 = k(2a)^2 = 4ka^2$$

where k is a constant.

For a phase difference of $\Delta\phi$, the amplitude is

$$A = 2a\cos(\Delta\phi / 2)$$

Therefore, the intensity at the given point,

$$I = kA^2 = k\,4a^2\cos^2\left(\frac{\Delta\phi}{2}\right) = 4ka^2\cos^2\left(\frac{2\pi}{2\lambda}\Delta p\right)$$

$$= I_0\cos^2\left(\frac{\pi}{\lambda}\frac{yd}{D}\right) = I_0\cos^2\left(\frac{\pi y}{\beta}\right) \qquad \left\{\because \beta = \frac{\lambda D}{d}\right\}$$

Ex. In a Young's double-slit experiment using a monochromatic light the fringe width is found to be 2.66 mm. If the whole set up is submerged in water of refractive index 1.33, the new fringe-width will be

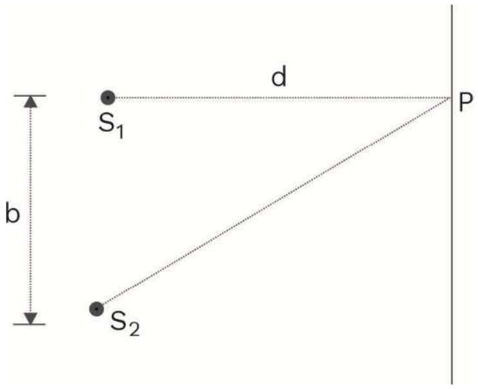
Sol. Wavelength in water is given as $\lambda' = \frac{\lambda}{\mu}$

The fringe width in water will become

$$\beta' = \frac{\lambda' D}{d} = \frac{\lambda}{\mu}\frac{D}{d} = \frac{1}{\mu}\left(\frac{\lambda D}{d}\right) = \frac{\beta}{\mu} = \frac{2.66}{1.33} = 2\,\text{mm}$$

Ex. The white light is used to illuminate the 2 slits in a Y.D.S.E. The separation between the slits is 'b' and the screen is at a distance 'd' (>>b) from the slits. At a certain point on the screen directly in front of one of the slits, certain wavelengths are missing. Some of these missing wavelengths will be

Sol. Path difference = $S_2P - S_1P$



From figure, $(S_2P)^2 - (S_1P)^2 = b^2$

or $\quad (S_2P - S_1P)(S_2P + S_1P) = b^2$

or $\quad (S_2P - S_1P) = \frac{b^2}{2d}$

For dark fringes, $\frac{b^2}{2d} = (2n-1)\frac{\lambda}{2}$

For n = 1, $\frac{b^2}{2d} = \frac{\lambda}{2}$ or $\lambda = \frac{b^2}{d}$

For n = 2, $\frac{b^2}{2d} = \frac{3\lambda}{2}$ or $\lambda = \frac{b^2}{3d}$

NOTE :

- If the whole apparatus is submerged in a liquid of refractive index μ, then wavelength of light $\lambda' = \frac{\lambda}{\mu}$ since $\mu > 1$ so $\lambda' < \lambda \Rightarrow$ wavelength will decrease. Hence fringe width $(\beta \propto \lambda)$ will decrease so fringe width in liquid $\beta' = \beta / \mu$. Angular width will also decrease.
- With increase in distance between the slit and the screen 'D', angular width of maxima does not change, fringe width β increases linearly with 'D' but the intensity of fringes will decrease.
- If the additional phase difference of π is created in one of the wave then the central fringe becomes dark.
- When the wavelength λ_1 is used to get a fringe n_1. At the same point wavelength λ_2 is required to obtain a fringe n_2 then $n_1\lambda_1 = n_2\lambda_2$
- When waves from two coherent sources S_1 and S_2 interfere in space the shape of the fringe is hyperbolic with foci at S_1 and S_2.

Concept Reminder

If the whole apparatus is immersed in a liquid of refractive index μ then wavelength and fringe width both decreases.

$$\lambda' = \frac{\lambda}{\mu}$$

and $\beta' = \frac{\beta}{\mu}$

Geometrical Path & Optical Path:

Actual distance travelled by the light in a medium is known as geometrical path (Δx). Assume a light wave given by the equation

$$E = E_0 \sin(\omega t - kx + \phi)$$

If the light travels by Δx, its phase changes by $k\Delta x = \frac{\omega}{v}\Delta x$, where ω is the frequency of light which doesn't depend on the medium, but v, the speed of light depends on the medium as $v = \frac{c}{\mu}$

Consequently, change in phase,

$$\Delta\phi = k\Delta x = \frac{\omega}{c}(\mu\Delta x)$$

It is clear that a wave which travels a distance

Concept Reminder

A wave travelling a distance Δx in a medium of refractive index μ suffers the same phase change as when it travels a distance $\mu\Delta x$ in vacuum. i.e., a path length of Δx in medium of refractive index μ is equivalent to a path length of $\mu\Delta x$ in vacuum.

Δx in a medium of refractive index μ suffers the equal phase change as when it travels a distance $\mu\Delta x$ in vacuum. i.e., a path length of Δx in medium of refractive index μ is equivalent to a path length of $\mu\Delta x$ in vacuum.

The quantity $\mu\Delta x$ is called the optical path length of light, $\mu\Delta x_{opt}$. And in terms of optical path length, the phase difference can be given as,

$$\Delta\phi = \frac{\omega}{c}\Delta x_{opt} = \frac{2\pi}{\lambda_0}\Delta x_{opt} \qquad \text{... (1)}$$

where λ_0 is wavelength of light in vacuum

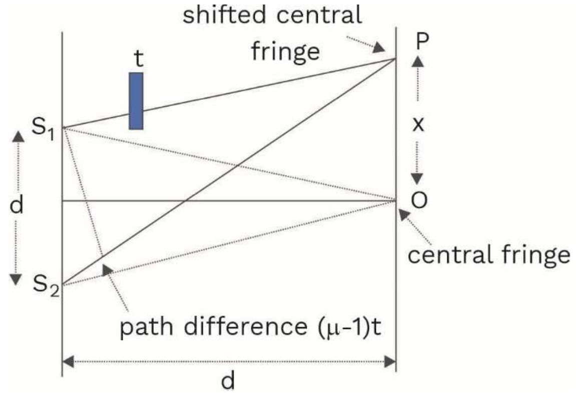
However, in terms of geometrical path length Δx ,

$$\Delta\phi = \frac{\omega}{c}(\mu\Delta x) = \frac{2\pi}{\lambda}\Delta x \qquad \text{... (2)}$$

where λ is wavelength of light in the medium $(\lambda = \frac{\lambda_0}{\mu})$.

Effect Of Thin Films:

When a glass plate of thickness 't' and refractive index μ is placed in front of one of the slits in YDSE then the central fringe shifts towards that side in which glass plate is placed because of the extra path difference introduced by the glass plate. In the path S_1P distance travelled by wave in air = $(S_1 P - t)$



Rack your Brain

In Young's double-slit experiment, if the distance between the slits is halved and the distance between the slits and the screen is doubled, the fringe width becomes

(1) Half

(2) Double

(3) Four times

(4) Eight times

Concept Reminder

If the two sources are coherent (i.e., if the two needles are going up and down regularly) then the phase difference ϕ at any point will not change with time and we will have a stable interference pattern; i.e., the positions of maxima and minima will not change with time.

Distance travelled by a wave in the sheet = t
Time taken by the light to reach up to point P will be same from S_1 and S_2

$$\Delta x = S_2P - (S_1P - t + \mu t)$$

$$\Delta x = (S_2P - S_1P) - (\mu - 1)t$$

$$\Delta x = d\sin\theta - (\mu - 1)t$$

$$\Delta x = \frac{dy}{D} - (\mu - 1)t$$

$$\Rightarrow \quad y = \Delta x \frac{D}{d} + (\mu - 1)\frac{tD}{d}$$

at C.B.F $\Delta x = 0$

$$y = \frac{(\mu - 1)tD}{d}$$

Path difference = $(\mu - 1)t$

$\Rightarrow$ Phase difference $\phi = \frac{2\pi}{\lambda}(\mu - 1)t$

Number of fringes displaced = $\frac{(\mu - 1)t}{\lambda}$

Distance of shifted fringe from central fringe

$$\frac{d(\mu - 1)t}{D} \qquad \left[\because \frac{xd}{D} = (\mu - 1)t\right]$$

$$\therefore \quad x = \frac{\beta(\mu - 1)t}{\lambda} \text{ and } \beta = \frac{D\lambda}{d}$$

Note :

- If a glass plate of refractive index μ_1 and μ_2 having same thickness t is placed in the path of rays coming from S_1 and S_2 then path difference $x = \frac{D}{d}(\mu_1 - \mu_2)t$
- Distance of displaced fringe from central fringe $x = \frac{\beta(\mu_1 - \mu_2)t}{\lambda}$

$$\because \quad \frac{\beta}{\lambda} = \frac{D}{d}$$

Concept Reminder

If a glass plate of refractive index μ_1 and μ_2 having same thickness t is placed in the path of rays coming from S_1 and S_2 then path difference $x = \frac{D}{d}(\mu_1 - \mu_2)t$

Ex. In the YDSE, λ = 6000 Å, D = 2 m, d = 6 mm. If a film of refractive index 1.5 is introduced in front of the lower slit, then the 3^{rd} maxima shifts to the origin.

(a) Find the thickness of the film.

(b) Find the position of the fourth maxima.

Sol. (a) The position of 3rd maxima on the screen is given as

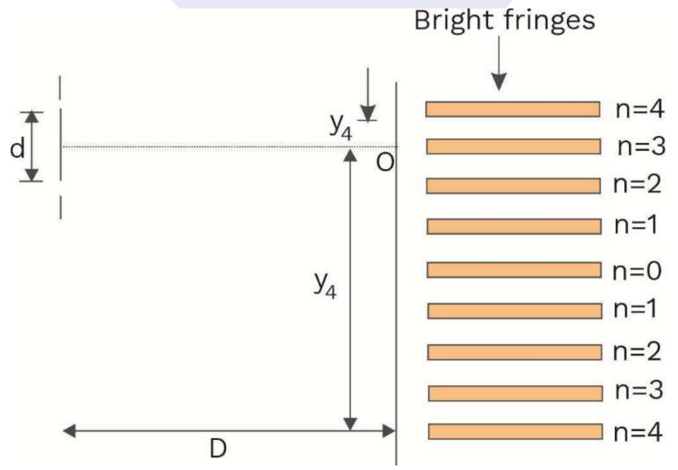
$$y_3 = \frac{n\lambda D}{d} = \frac{3\lambda D}{d}$$

Since this maxima shifts to origin, the lateral shift due to introduction of film in lower slit is

$$\frac{D}{d}(\mu - 1)t = \frac{3\lambda D}{d}$$

or $$t = \frac{3\lambda}{\mu - 1} = \frac{3 \times 6000 \times 10^{-10}}{1.5 - 1} = 3.6\,\mu m$$

(b) There are 2 fourth maxima – one above and the other below the central max., as shown.



$$y_4 = 1 \times \beta = \frac{\lambda D}{d} = \frac{6000 \times 10^{-10} \times 2}{6 \times 10^{-3}} = 0.2\,mm$$

$$y_4' = -7 \times \beta = \frac{7\lambda D}{d} = -7 \times 0.2\,mm = -1.4\,mm$$

Ex. In YDSE with d = 1 mm & D = 1 m slabs of thickness $1\mu m$ & 0.5 μm, Refractive index 3 & 2 are respectively introduced in front of upper & lower slit respectively. Find the shift in fringe pattern.

Sol. $$x = \frac{\left((3-1)1 - (2-1)\frac{1}{2}\right) \times 10^{-6}}{10^{-3}} = 1.5 \text{ mm upward}$$

Colour In Thin Films:

When the white light is incident on a thin film (for example oil film on surface of water or a soap bubble) then the interference will take place between the waves reflected from its 2 surfaces and waves refracted through it.

The intensity becomes max. and min. as a result of interference and colours are seen.

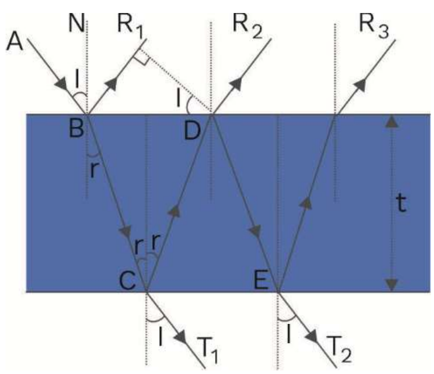
(i) The source of light source must be an extended source.

(ii) The colours achieved in reflected and transmitted light are mutually complementary.

(iii) The colours obtained in thin films are because of interference whereas those achieved in prism are because of dispersion.

Interference Due To Thin Films:

Assumed a thin transparent film of thickness 't' and refractive index μ. Suppose a ray of light AB be incident on the film at 'B'. At 'B', a part of light is reflected along BR_1, and a part of light refracted along BC. At 'C' a part of light is reflected along CD and a part of light transmitted along CT_1. At 'D', a part of light is refracted along DR_2 and a part of light is reflected along DE. Thus interference in this film takes place because of reflected light in between BR_1 and DR_2 also in transmitted light in between CT_1 and ET_2. Here coherent sources are obtained by division of amplitude.



Concept Reminder

If we use two sodium lamps illuminating two pinholes we will not observe any interference fringes. This is because of the fact that the light wave emitted from an ordinary source (like a sodium lamp) undergoes abrupt phase changes in times of the order of 10^{-10} seconds. Thus the light waves coming out from two independent sources of light will not have any fixed phase relationship and would be incoherent.

Rack your Brain

If two waves, each of intensity I_0, having the same frequency but differing by a constant phase angle of 60°, superimposing at a certain point in space, then the intensity of the resultant wave is

(1) $2I_0$ (2) $3I_0$

(3) I_0 (4) $4I_0$

Reflected System

The path difference between BR_1 and DR_2 is $x = 2\mu t\cos r$. Reflection from the surface of the denser medium involves the additional phase difference of π or path difference $\lambda / 2$. Therefore the effective path difference between BR_1 and DR_2 is.

$\Rightarrow \; x' = 2\mu t\cos r - \lambda / 2$

Maximum or constructive Interference occurs when path difference between the light waves is $n\lambda$.

$$2\mu t\cos r - \lambda / 2 = n\lambda \Rightarrow 2\mu t\cos r = n\lambda + \lambda / 2$$

So, the film will appear bright if

$$2\mu t\cos r = (2n+1)\lambda / 2 \quad (n = 0, 1, 2, 3 .....)$$

- **Form minima or destructive interference :**

When path difference is odd multiple of $\frac{\lambda}{2}$

$$\Rightarrow \; 2\mu t\cos r - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

So, the film will appear dark if $2\mu t\cos r = n\lambda$

- **For transmitted system**

Since no additional path difference between transmitted rays CT_1 and ET_2. So, the net path difference between them is $2x = \mu t\cos r$

For maxima $2\mu t\cos r = n\lambda, n = 0, 1, 2........$

For minima $2\mu t\cos r = (2n+1)\frac{\lambda}{2}, n = 0, 1, 2........$

Ex. In a Young's double slit experiment with d = 1 mm and D = 1 m, slabs of $(t = 1\mu m, \mu = 3)$ and $(t = 0.5\,\mu m, \mu = 2)$ are introduced in front of upper and lower slit respectively. Calculate the shift in the fringe pattern.

Sol. Optical path for light coming from the upper slit S_1 is

$$S_1P + 1\mu m(2-1) = S_2P + 0.5 m$$

Similarly the optical path for light coming from S_2 will be

Path difference : $\Delta p = (S_2P + 0.5\ \mu m) - (S_1P + 2\,\mu m)$

$$= (S_2P - S_1P) - 1.5\,\mu m$$

$$= \frac{yd}{D} - 1.5\,\mu m$$

for central bright fringe $\Delta p = 0$

$$\Rightarrow \quad y = \frac{1.5\,\mu m}{1mm} \times 1m = 1.5\,mm$$

So the whole pattern is shifted by 1.5 mm upwards.

Ex. Interference fringes were produced by Y.D.S.E. the wavelength of the light used is 6000 Å. The distance between the 2 slits is 2 mm. The slits and screen are 10 cm apart. When a transparent plate of 0.5 mm thickness is placed over one of slits, the fringe pattern is displaced by 5 mm. Find refractive index of material of the plate.

Sol. Here $d = 2\,mm = 2 \times 10^{-3} m$, $D = 10\,cm = 0.10\,m$

$t = 0.5\,mm = 0.5 \times 10^{-3} m$, $\Delta x = 5\,mm = 5 \times 10^{-3} m$,

$\lambda = 6000Å = 6000 \times 10^{-10} m$

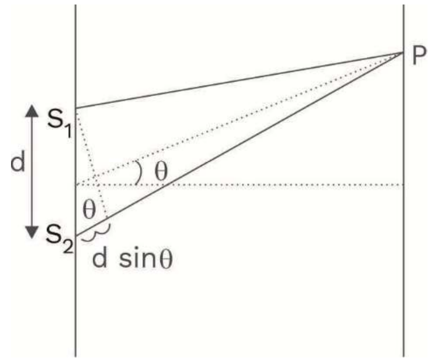
As $\quad x_0 = \frac{D}{d}(\mu - 1)t$

$$\therefore \quad \mu - 1 = \frac{x_0 . d}{D \times t} = \frac{5 \times 10^{-3} \times 2 \times 10^{-3}}{0.10 \times 0.5 \times 10^{-3}} = 0.2$$

or $\quad \mu = 1 + 0.2 = 1.2$

Ex. In a YDSE light of wavelength λ = 5000 Å is used, which arises in the phase from the two slits at a distance d = 3 × 10^{-7} m apart. A transparent sheet of refractive index n = 1.17, thickness t = 1.5 × 10^{-7} m is placed over one of slits. Where does the central maxima of the interference now appear?

Sol. The path difference introduced because of introduction of the transparent sheet is given by $\Delta x = (\mu - 1)t$. If the central maxima occupies of nth fringe,



then $(\mu - 1)t = n\lambda = d \sin\theta$

$$\sin\theta = \frac{(\mu - 1)t}{d} = \frac{(1.17 - 1) \times 1.5 \times 10^{-7}}{3 \times 10^{-7}} = 0.085$$

Hence the angular position of central maxima is

$$\theta = \sin^{-1}(0.085) = 4.88°$$

For small angles $\sin\theta \simeq \theta \simeq \tan\theta$

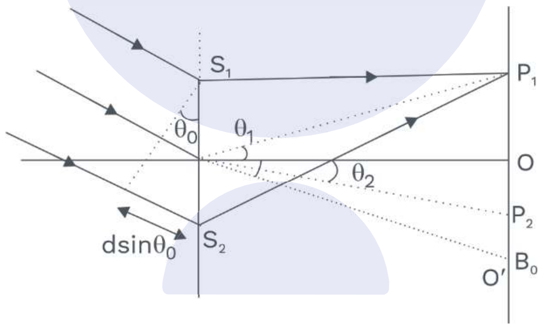
As $\tan\theta = \frac{y}{D}$

so $\frac{y}{D} = \frac{(\mu-1)t}{d}$

Shift of central maxima is $Y = \frac{D(\mu-1)t}{d}$. This formula can be used if 'D' is given.

YDSE With Oblique Incidence:

In YDSE, a ray is incident on the slit at an inclination of θ_0 to axis of symmetry of the experimental set-up for the points above the central point on screen, (say for P_1)



$$\Delta p = d\sin\theta_0 + (S_2P_1 - S_1P_1)$$

$\Rightarrow$ $\Delta p = d\sin\theta_0 + d\sin\theta_0$ (If d << D)

For point O, $\Delta p = d\sin\theta_0$ (because $S_2O = S_1O$)

and for points below 'O' on the screen, (let for P_2)

$$\Delta p = |(\sin\theta_0 + S_2P_2) - S_1P_2|$$

$$= |(\sin\theta_0 - (S_1P_2 - S_2P_2)|$$

$\Rightarrow$ $\Delta p = |d\sin\theta_0 - d\sin\theta_2|$ (if d << D)

We get the central maxima at a point where, $\Delta p = 0$

$(d\sin\theta_0 - d\sin\theta_2) = 0$

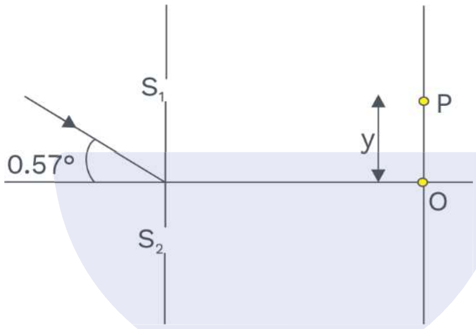
or $\theta_2 = \theta_0$

This relates to the point O' in the diagram

Hence we have finally for path difference.

$$\Delta p = \begin{cases} d(\sin\theta_0 + \sin\theta) \rightarrow \text{for points above O} \\ d(\sin\theta_0 - \sin\theta) \rightarrow \text{for points between O \& O'} \\ d(\sin\theta_0 + \sin\theta) \rightarrow \text{for points below O'} \end{cases}$$

Ex. In a YDSE D = 1 m, d = 1 mm light of 500 nm wavelength is incident at an angle of 0.57° with respect to the axis of symmetry of experimental set up. If centre of symmetry of screen is 'O' as shown.



(i) find the position of central maxima

(ii) Intensity at point 'O' in terms of intensity of central maxima I_0

(iii) Number of maxima lying between 'O' and the central maxima.

Sol. (i) $\theta = \theta_0 = 0.57°$

$\Rightarrow$ $y = -D\tan\theta \simeq -D\theta = -1 \text{ meter} \times \left(\frac{0.57}{57}\text{rad}\right)$

$\Rightarrow$ $y = -1\text{cm}$

(ii) for point 0, $\theta = 0$

Hence,

$\Delta p = d\sin\theta_0 = d\theta_0 = 1\text{mm} \times (10^{-2}\text{rad})$

$= 10,000\,\text{nm} = 20 \times (500\,\text{nm})$

$\Rightarrow$ $\Delta p = 20\lambda$

Hence point 'O' corresponds to 20th maxima

$\Rightarrow$ intensity at O = I_0

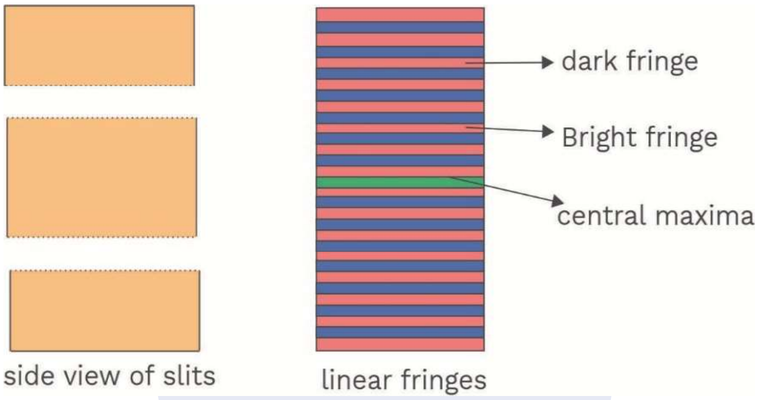
(iii) 19th maxima lie between central maxima and O, excluding maxima at O and central maxima.

Concept Reminder

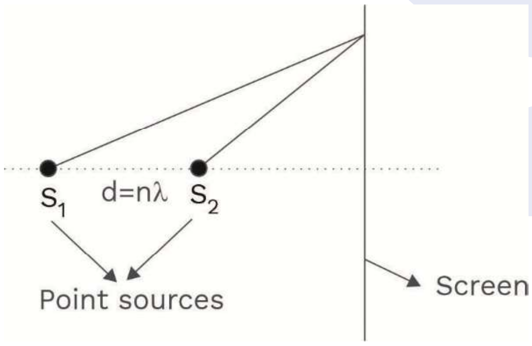
English physicist, physician and Egyptologist. Young worked on a wide variety of scientific problems, ranging from the structure of the eye and the mechanism of vision to the decipherment of the Rosetta stone. He revived the wave theory of light and recognised that interference phenomena provide proof of the wave properties of light.

Shape Of Interference Pattern :

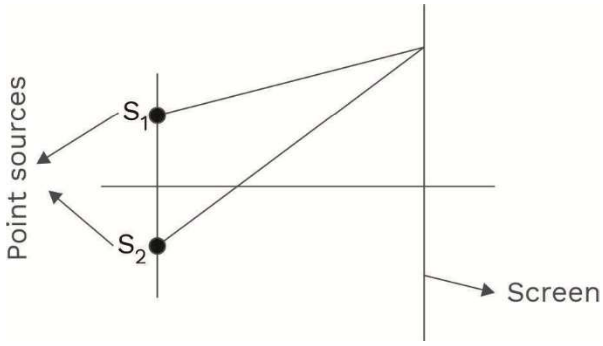
1. Shape of the Pattern when interference takes due to waves produced by two slits.



2. Shape of Pattern when interference takes place due to waves produced by two-point sources (where line of sources is perpendicular to screen).



3. Shape of the Pattern when interference takes place due to waves produced by two-point sources (where the line of sources is parallel to the screen).



Rack your Brain

In an interference experiment monochromatic light is replaced by white light, we will see

(1) Uniform illumination on the screen

(2) Uniform darkness on the screen

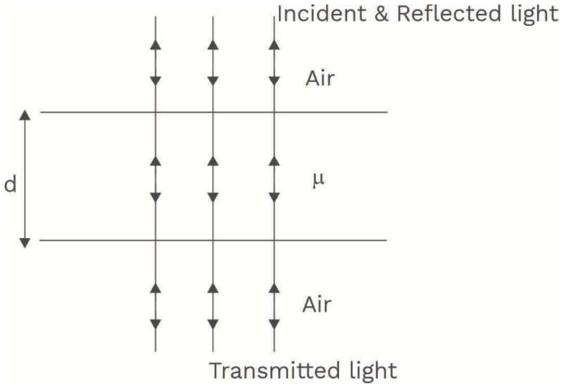
(3) Equally spaced white and dark bands

(4) A few coloured bands and then uniform illumination

Ex. White light, with a constant intensity across the visible wavelength range 430–690 nm, is perpendicularly incident on the water film, of index of refraction $\mu = 1.33$ and thickness d = 320 nm, that is suspended in air. At what wavelength λ the light reflected by the film brightest to an observer is brightest.

Sol. This situation is as that of Figure shown, for which equation written below gives the interference maxima.

$2\mu d = (m + \frac{1}{2})\lambda$ for constructive interference.



Solving for λ and putting the given data, we get

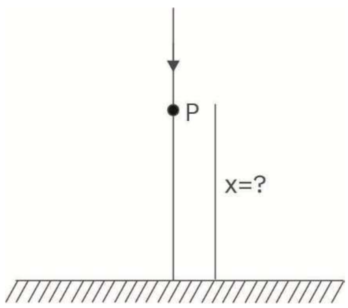
$$\lambda = \frac{2\mu d}{m + 1/2} = \frac{(2)(1.33)(320\,nm)}{m + 1/2} = \frac{851\,nm}{m + 1/2}$$

for m = 0, this give us λ = 1700 nm, which is in infrared region. For m = 1, we find I = 567 nm, which is the yellow-green light, near middle of the visible spectrum. For m = 2, λ = 340 nm, which is the ultraviolet region. So wavelength which light seen by the observer is brightest is

$$\lambda = 567\,nm$$

Note: When the light gets reflected from a denser medium, then there is an abrupt phase change of π no phase change occurs when reflection takes place from rarer medium

Ex. Find the minimum value of 'x' for which a maxima is obtained at 'P'.



Sol. For maxima, $\Delta x = \lambda$ (because x should be minimum)

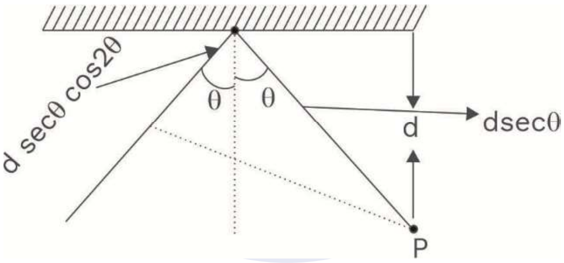
Path difference between the direct & reflected ray

$= x + \frac{\lambda}{2} + x$ (due to reflection, a phase change of π or path change of $\frac{\lambda}{2}$ takes place)

$\therefore \quad \lambda = x + \frac{\lambda}{2} + x$

$2x = \frac{\lambda}{2} \Rightarrow x = \frac{\lambda}{4}$

Ex. Find the value of θ for which a maxima is obtained at P.



Sol. For maxima at P, $\Delta x = \lambda$

Path difference between direct & reflected ray $= d\sec\theta\cos 2\theta + \frac{\lambda}{2} + d\sec\theta = \lambda$

$d\sec\theta(1+\cos 2\theta) = \frac{\lambda}{2}$

$d\sec\theta(2\cos^2\theta) = \frac{\lambda}{2}$

$\cos\theta = \frac{\lambda}{4d} \Rightarrow \theta = \cos^{-1}\left(\frac{\lambda}{4d}\right)$

Ex. White light may be considered to have λ from 4000 Å to 7500 Å. If an oil film has thickness 10^{-6} m, deduce the wavelengths in the visible region for which the reflection along the normal direction will be (i) weak, (ii) strong. Take μ of the oil as 1.40.

Sol. The condition for dark fringe or weak reflection when seen in reflected light is $2\mu t\cos r = n\lambda$, where n is an integer.

For normal incidence, r = 0 and cos r = 1

so that $\quad 2\mu t = n\lambda$ or $\lambda = \frac{2\mu t}{n}$

Substituting the values of μ and t, we get

$$\lambda = \frac{2 \times 1.4 \times 10^{-6}}{n} = \frac{28 \times 10^{-7}}{n} m$$

For values of n < 4 or n > 7, the values of λ do not lie in the visible range 4000 Å to 7500 Å. But for values of n = 4, 5, 6, 7, the following wavelengths lie in the visible region :

(i) $\lambda = \frac{28 \times 10^{-7}}{4} = 7.0 \times 10^{-7} m = 7000 \text{Å}$

(ii) $\lambda = \frac{28 \times 10^{-7}}{5} = 5.6 \times 10^{-7} m = 5600 \text{Å}$

(iii) $\lambda = \frac{28 \times 10^{-7}}{6} = 4.667 \times 10^{-7} m = 4667 \text{Å}$

(iv) $\lambda = \frac{28 \times 10^{-7}}{7} = 4.0 \times 10^{-7} m = 4000 \text{Å}$

The condition for bright fringe or strong reflection is

$$2\mu t = \frac{(2n+1)\lambda}{2} \text{ or } \lambda = \frac{4\mu t}{(2n+1)}$$

Substituting the values of μ and t, we get

$$\lambda = \frac{4 \times 1.4 \times 10^{-6}}{2n+1} = \frac{56 - 10^{-7}}{2n+1} m$$

For values of n < 4 or n_2 > 6, the values of λ do not lie in the visible range. But for n = 4, 5, 6 the following wavelengths lie in the visible range

(i) $\lambda = \frac{56 \times 10^{-7}}{2 \times 4 + 1} = 6.222 \times 10^{-7} m = 6222 \text{Å}$

Uses Of Interference Effect

Thin layer of oil on water and soap bubbles show different colours due to interference of waves reflected from two surfaces of their films. Here we get two coherent beams by division of amplitude making use of partial reflection and partial refraction

Uses :

- Used to determine the wavelength of light precisely.
- Used to determine refractive index or thickness of transparent sheet.
- Used in holography to produce 3-D images.

Ex. Light of the wavelength 6000 Å is incident on thin glass plate of refractive index 1.5 such that the refraction angle into the plate is 60°. Find the smallest thickness of plate which will make it appear dark by reflection.

Sol. $2\mu t \cos r = n\lambda$

$$t = \frac{n\lambda}{2\mu\cos r} = \frac{1 \times 6 \times 10^{-7}}{2 \times 1.5 \times \cos 60}$$

$$= \frac{6 \times 10^{-7}}{1.5} = 4 \times 10^{-7} m$$

Ex. Light is incident on the glass plate (μ = 1.5) such that the angle of refraction is 60°. Dark band is observed corresponding to wavelength of 6000 Å. If thickness of glass plate is 1.2×10^{-3} mm. Calculate the order of the interference band for reflected system.

Sol. $\mu = 1.5, r = 60°, \lambda = 6000\,Å = 6 \times 10^{-7} m$

$t = 1.2 \times 10^{-3} mm = 1.2 \times 10^{-6} m$

For dark band in the reflected light $2\mu t \cos r = n\lambda$

$$n = \frac{2\mu t \cos r}{\lambda} = \frac{2 \times 1.5 \times 1.2 \times 10^{-6} \times \cos 60°}{6 \times 10^{-7}}$$

$$= \frac{2 \times 1.5 \times 1.2 \times 10^{-6} \times \frac{1}{2}}{6 \times 10^{-7}} = 3$$

Thus third dark band is observed.

Ex. A parallel beam of the white light is incident normally on a water film 1×10^{-4} cm thick. Find wavelength in the visible range (400 nm –700 nm) which are strongly transmitted by the film. (μ_{water} = 4/3)

Sol. $2\mu t = n\lambda$

$$\frac{2 \times \frac{4}{3} \times 10^{-6}}{n} = \lambda$$

$$\frac{80}{3n} \times 10^{-7} = \lambda$$

when $n = 4, \lambda = 666\,nm$

$n = 5, \lambda = 533\,nm$

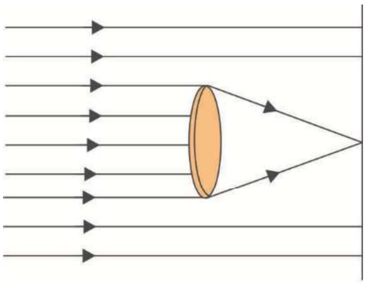
$n = 6, \lambda = 444\,nm$

Diffraction of Light:

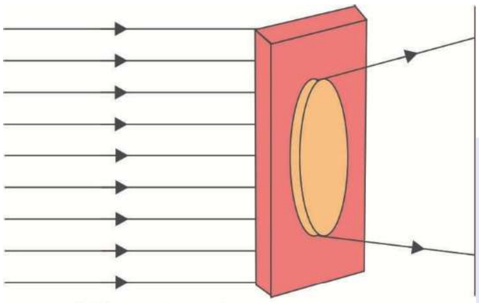
Bending of the light rays from sharp edges of an opaque obstacle or aperture and its spreading in the geometrical shadow region is defined as diffraction of light or deviation of light from its rectilinear propagation tendency is defined as diffraction of light.

Definitions

Diffraction
Bending of light rays from sharp edges of an opaque obstacle or aperture and its spreading in the geometrical shadow region is defined as diffraction of light



diffraction from obstacle



diffraction from aperture

Diffraction was discovered by Grimaldi.

- Theoretically explained by Fresnel
- Diffraction is possible in all the type of waves means mechanical or electromagnetic waves shows diffraction.

Diffraction depends on two factors:

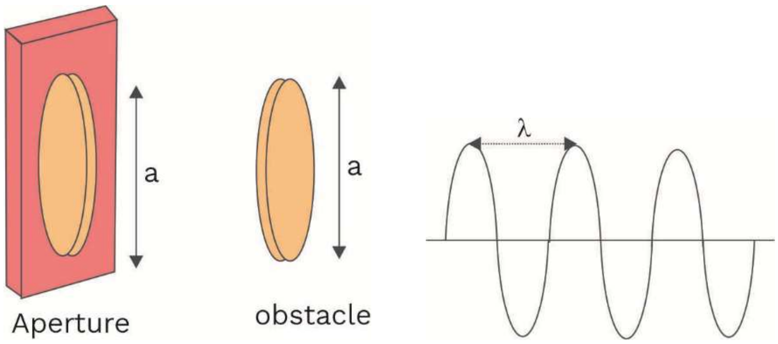
(i) Size of obstacles or aperture

(ii) Wavelength of wave

Concept Reminder

Condition of diffraction is size of obstacle or aperture should be nearly equal to wavelength of light

i.e., $\frac{a}{\lambda} \simeq 1.$

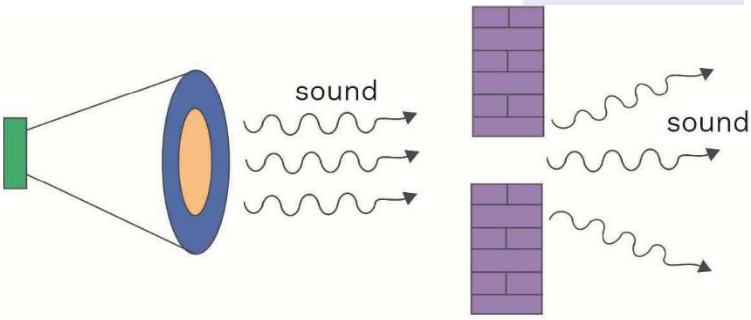


Condition of diffraction: The size of obstacle or aperture should be nearly equal to the wavelength of light

i.e. $\lambda \simeq a$ $\frac{a}{\lambda} \simeq 1$

If the size of obstacle is much greater than wavelength of light, then the rectilinear motion of the light is observed.

- It is practically observed when the size of aperture or obstacle is greater than 50 λ. The obstacle or aperture does not shows diffraction.
- Wavelength of light is of the order 10^{-7} m. In general obstacle of this wavelength is not present so the light rays does not show the diffraction and it appears to travel in straight line Sound wave shows more diffraction as compared to the light rays because wavelength of sound is large (16 mm to 16 m). So, it is generally diffracted by objects of our daily life.
- Diffraction of ultrasonic waves is also not observed easily because their wavelength is of order of about 1 cm. Diffraction of the radio waves is very conveniently observed because of its very large wavelength (2.5 m to 250 m). X-rays can be diffracted easily by crystals. It was discovered by Lave.



Diffraction of sound from a window

Fraunhoffer Diffraction

In the Fraunhoffer diffraction both the source and the screen are effectively at infinite distance from diffracting device and the pattern is the image of source modified by diffraction effects. Diffraction at single slit, diffraction grating and double slit are the example of Fraunhoffer diffraction.

Concept Reminder

Diffraction is a general characteristic exhibited by all types of waves, be it sound waves, light waves, water waves or matter waves. Since the wavelength of light is much smaller than the dimensions of most obstacles; we do not encounter diffraction effects of light in everyday observations

KEY POINTS

- Diffraction
- Geometrical shadow
- Fraunhoffer diffraction
- Diffraction grating

Fraunhoffer Diffraction Due To Single Slit:

AB is single slit of width 'a', the plane wave front is incident on a slit AB. The secondary wavelets coming from every part of AB reach axial point 'O' in equal phase forming central maxima. The intensity of the central maxima is max. in this diffraction. where represents direction of n^{th} minima, Path difference

for n^{th} minima a

(if is small)

- When the path difference between secondary wavelets coming from A and B is or or even multiple of then minima occurs.

  For minima where n = 1, 2, 3 ...

- When the path difference between secondary wavelets coming from A and B is or odd multiple of then maxima occurs

  For maxima where n = 1, 2, 3, ...

- Alternate ordered minima and maxima occurs on the both sides of the central maxima.

Concept Reminder

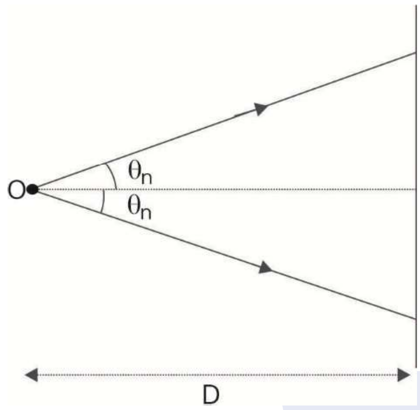
No one has ever been able to define the difference between interference and diffraction satisfactorily. It is just a question of usage, and there is no specific, important physical difference between them.

For n^{th} minima

If the distance of n^{th} minima from the central maxima = x_n

distance of slit from screen = D , width of slit = a

The path difference $\delta = a\sin\theta_n = \frac{2n\lambda}{2} \Rightarrow \sin\theta_n = \frac{n\lambda}{a}$



In $\Delta POP'$; $\tan\theta_n = \frac{x_n}{D}$

If θ_n is small $\sin\theta_n \approx \tan\theta_n \approx \theta_n$

$x_n = \frac{n\lambda D}{a} \Rightarrow \theta_n = \frac{x_n}{D} = \frac{n\lambda}{a}$ First minima occurs on both sides of central maxima.

For first minima $x = \frac{D\lambda}{a}$ and $\theta = \frac{x}{D} = \frac{\lambda}{a}$

- Linear width of central maxima $w_x = 2x \Rightarrow w_x = \frac{2D\lambda}{a}$
- Angular width of central maxima $w_\theta = 2\theta = \frac{2\lambda}{a}$

Special Case:
The lens L_2 is shifted very near to the slit AB. In this case distance between the slit and the screen will be nearly equal to focal length of the lens L_2 (i.e. $D \approx f$)

$$\theta_n = \frac{x_n}{f} = \frac{n\lambda}{a}$$

$$\Rightarrow x_n = \frac{n\lambda f}{a}$$

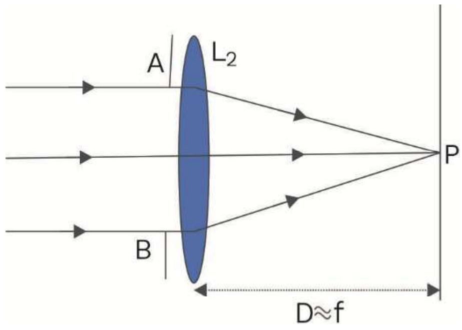
Rack your Brain

Waves that cannot be polarised are
(1) Light waves
(2) Electromagnetic waves
(3) Transverse waves
(4) Longitudinal waves

Concept Reminder

(i) Linear width of central maxima $w_x = \frac{2D\lambda}{a}$

(ii) Angular width of central maxima $w_\theta = 2\theta = \frac{2\lambda}{a}$



Concept Reminder

- Fringe width in diffraction is
  $\beta = x_{n+1} - x_n = (n+1)\frac{\lambda D}{a} - \frac{n\lambda D}{a} = \frac{\lambda D}{a}$

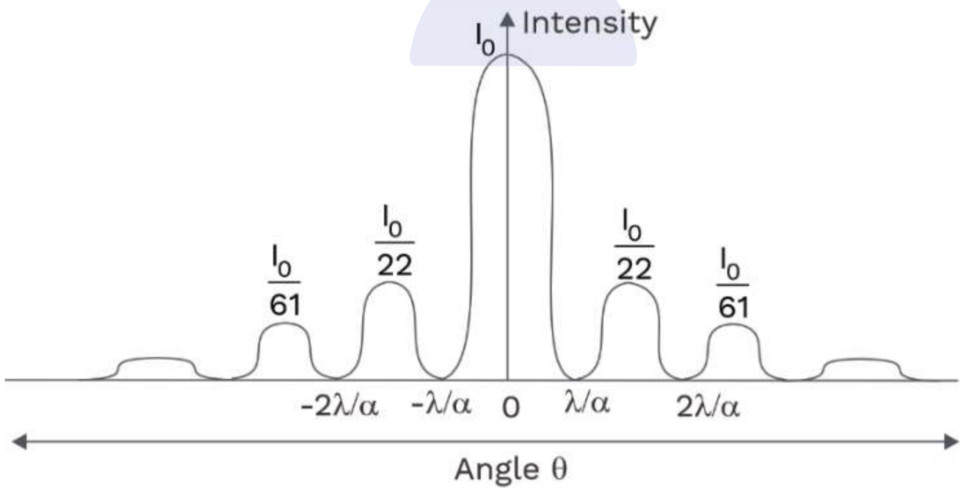
$w_x = \frac{2\lambda f}{a}$ and angular width of central maxima $w_\theta = \frac{2x}{f} = \frac{2\lambda}{a}$

Fringe width : Distance between 2 consecutive maxima (bright fringe) or minima (dark fringe) is cold fringe width. Fringe width of the central maxima is doubled then width of other maxima i.e.,

$$\beta = x_{n+1} - x_n = (n+1)\frac{\lambda D}{a} - \frac{n\lambda D}{a} = \frac{\lambda D}{a}$$

(other than central maxima)

Intensity curve of Fraunhoffer's diffraction



Ex. Light of 6000 Å wavelength is incident normally on the slit of width 24 × 10^{-5} cm. Calculate angular position of second minimum from central maximum?

Sol. $\sin\theta = \frac{2\lambda}{a} = \frac{2 \times 6 \times 10^{-7}}{24 \times 10^{-7}} = \frac{1}{2}$

given $\lambda = 6 \times 10^{-7}\,m, a = 24 \times 10^{-5} \times 10^{-2}\,m$

$$\sin\theta = \frac{2\lambda}{a} = \frac{2\times 6\times 10^{-7}}{24\times 10^{-7}} = \frac{1}{2}$$

$\therefore \quad \theta = 30°$

Ex. Light of the wavelength 6000 Å is incident normally on the slit of width 0.2 mm. Find out the angular width of central maximum on a screen distance 9 m?

Sol. $\lambda = 6\times 10^{-7}$

$a = 2\times 10^{-4}\,m$

$$w = \frac{2\lambda}{a} = \frac{2\times 6\times 10^{-7}}{2\times 10^{-4}}\,\text{radian}$$

$= 6\times 10^{-3}\,\text{radian}$

Ex. Calculate half angular width of the central bright maximum in the Fraunhofer diffraction pattern of the slit of width 12×10^{-5} cm when slit is illuminated by monochromatic light of wavelength 6000 Å.

Sol. $\because \quad \sin\theta = \frac{\lambda}{a}$

$\theta =$ half angular width of the central maximum.

$a = 12\times 10^{-5}\,cm, \lambda = 6000Å = 6\times 10^{-5}\,cm$

$$\therefore \quad \sin\theta = \frac{\lambda}{a} = \frac{6\times 10^{-5}}{12\times 10^{-5}} = 0.50$$

$\Rightarrow \quad \theta = 30°$

Ex. Red light of 6500 Å wavelength from a distant source falls on a slit 0.50 mm wide. What is distance between 1st two dark bands on each side of central bright of diffraction pattern observed on the screen placed 1.8 m from slit.

Sol. Given that; $\lambda = 6500$ Å $= 65 \times 10^{-8}$ m, a = 0.5 mm = 0.5×10^{-3} m., D = 1.8 m.

Required distance between first two dark bands will be equal to width of central maxima.

$$w_x = \frac{2\lambda D}{a} = \frac{2\times 6500\times 10^{-10}\times 1.8}{0.5\times 10^{-3}}$$

$= 468\times 10^{-5}\,m = 4.68\,mm$

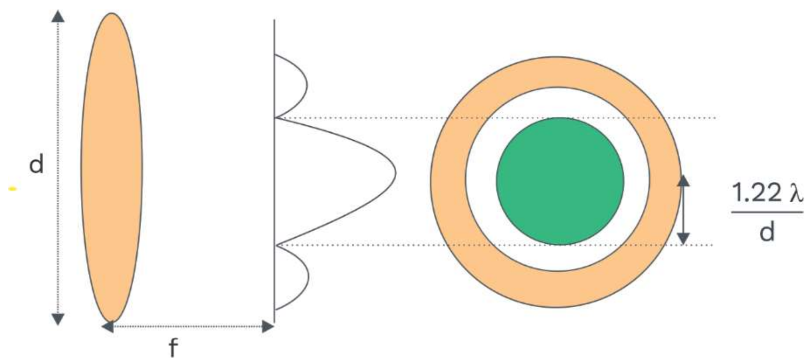
DIFFERENCE BETWEEN INTERFERENCE AND DIFFRACTION (FOR FRAUNHOFER SINGLE SLIT):

	Interference		Fraunhofer diffraction
(1)	it is the phenomenon of superposition of two waves coming from two different coherent sources.	(1)	It is the phenomenon of superposition of two waves coming from two.
(2)	In interference pattern, all bright lines are equally bright and equally spaces.	(2)	All bright lines are not equally bright and equally wide. Brightness and width goes on decreasing with the angle of diffraction.
(3)	All dark lines are totally dark.	(3)	Dark lines are perfectly dark. Their contrast with bright lines and width goes.
(4)	In interference bands are large in number	(4)	In diffraction bands are few in number.

Circular Aperture Diffraction

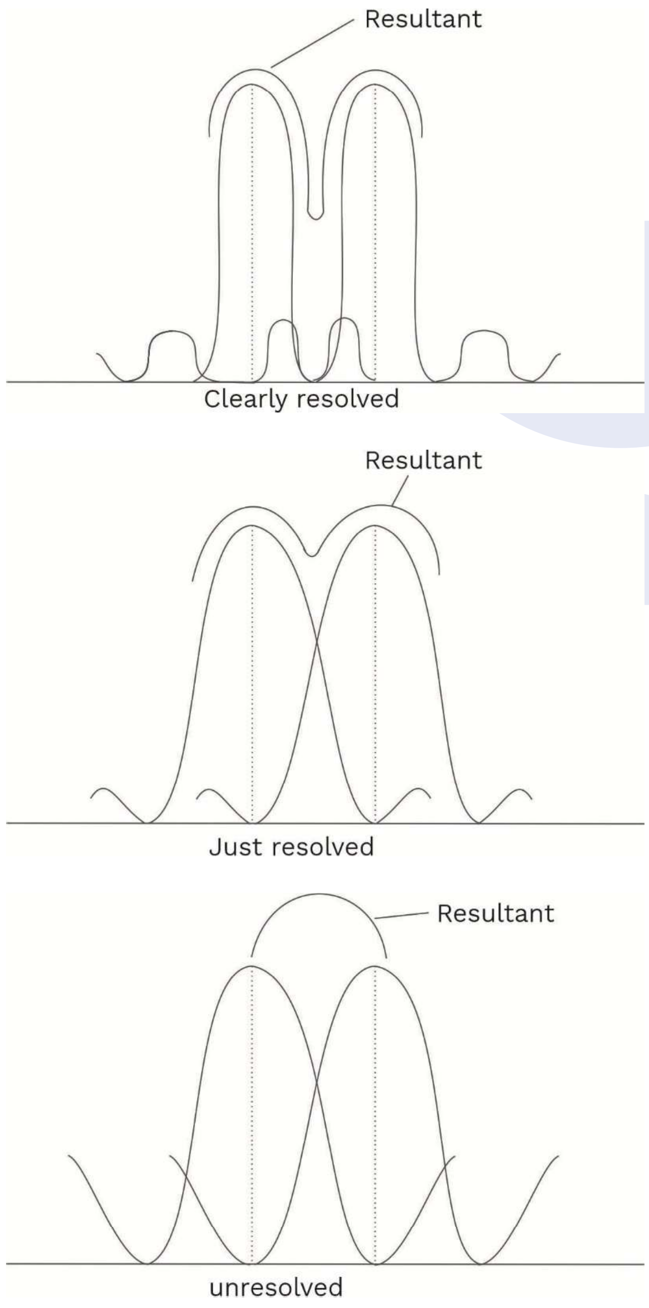
The diffraction pattern of circular disc shaped inter mediate dark & bright fringes with the central bright spot, formed when the light passes through a small circular aperture diffraction. This circular spot formed at the centre known as airy disc. The concentric rings will get fainter as it moves from the central spot. The first minima is given by $\theta = \frac{1.22\lambda}{d}$

d = diameter of aperture



Rayleigh's Criterion For Resolution

When the point source of the light is imaged by an optical system with the circular aperture image is an Airy disc. If the two points are very close, their Airy discs will overlap and we may not be able to resolve them, i.e., distinguish separate images.



Concept Reminder

In interference and diffraction, light energy is redistributed. If it reduces in one region, producing a dark fringe, it increases in another region, producing a bright fringe. There is no gain or loss of energy, which is consistent with the principle of conservation of energy.

Two points are just resolved by the optical system when central maximum of diffraction pattern due to one falls on the 1st minimum of diffraction pattern of other.

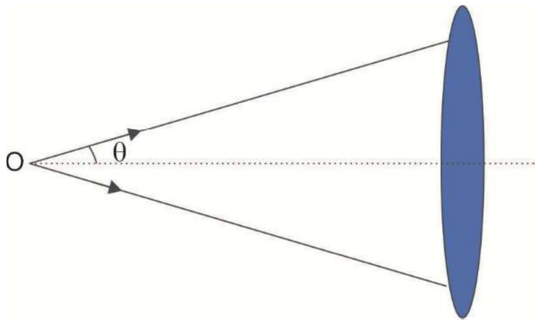
Resolution Limit

The limit of resolution is measure of ability of objective lens to separate in the image adjacent details that are present in object. It is the distance between two points in the object that are just resolved in the image.

Resolving Power (R.P.):

A large no. of images are formed as a consequence of light diffraction from the source. If 2 sources are separated such that their central maxima don't overlap, their images may be distinguished and are known to be resolved R.P. of the optical instrument is its ability to distinguish two neighbouring points. Reciprocal of resolving limit is called resolving power.

(1) Microscope : In reference to a microscope, minimum distance between two lines at which they are just distinct is called Resolving limit (RL) and it's reciprocal is called Resolving power (RP)



$$R.L. = \frac{1.22\lambda}{2\mu\sin\theta} \quad \text{and} \quad R.P. = \frac{2\mu\sin\theta}{1.22\lambda} \Rightarrow R.P. \propto \frac{1}{\lambda}$$

λ = Wavelength of light used to illuminate the object

KEY POINTS

- Rayleigh criteria
- Resolution limit
- Resolving power

Definitions

The limit of resolution is a measure of the ability of the objective lens to separate in the image adjacent details that are present in object.

Concept Reminder

For microscope
(i) Resolving limit
(ii) Resolving power

$$R.L. = \frac{1.22\lambda}{2\mu\sin\theta}$$

$$R.P. = \frac{2\mu\sin\theta}{1.22\lambda} \Rightarrow R.P. \propto \frac{1}{\lambda}$$

μ = Refractive index of the medium between object and objective,

θ = Half angle of the cone of light from the point object, $\mu \sin\theta$ = Numerical aperture.

(2) Telescope: Smallest angular separations $(d\theta)$ between two distant object, whose images are separated in the telescope is called resolving limit. So resolving limit and $d\theta = \frac{1.22\lambda}{a}$ resolving power

$$(RP) = \frac{1}{d\theta} = \frac{a}{1.22\lambda} \Rightarrow R.P. \propto \frac{1}{\lambda} \text{ where a = aperture of objective.}$$

Ex. Find limit of resolution of a microscope if an object of numerical aperture 0.10 is viewed by using light of wavelength 6000 Å :-

Sol. $R.L. = \frac{1.22\lambda}{2\mu\sin\theta} = \frac{1.22 \times 6 \times 10^{-7}}{2 \times 0.1} = 3.66 \times 10^{-6} m$

Ex. Find ratio of resolving powers of an optical microscope for two wavelengths $\lambda_1 = 4500$ Å & $\lambda_2 = 6000$ Å ?

Sol. $R.P. \propto \frac{1}{\lambda}$

$$\frac{R.P_1}{R.P_2} = \frac{6000}{4500} = \frac{4}{3}$$

Ex. The Hale telescope of Mount Palomar has a diameter of 200 inch. What is its limiting angle of resolution for 600 nm light?

Sol. Here; a = 200 inch = 200 × 2.54 cm = 508 cm = 5.08 m

λ = 600 nm = 600 × 10^{-9} m = 6.00 × 10^{-7} m

$$\Delta\theta = 1.22\left(\frac{\lambda}{a}\right) = 1.22\left(\frac{600 \times 10^{-7} m}{5.08}\right) = 1.44 \times 10^{-7} \text{ rad}$$

Validity of Geometrical Optics

When a slit of width d is illuminated by a parallel beam of light, the angular spread of diffracted light is approximately λ / d. Therefore, after travelling a distance D, the diffracted beam acquires a width $D\lambda / d$.

Geometrical optics is based on rectilinear propagation of light, which is just an approximation. We can say that geometrical optics is valid, if the width $D\lambda / d$ of the diffracted beam is less than the size of the slit, that is

$$\frac{D\lambda}{d} < d \qquad \text{or} \qquad D < \frac{d^2}{\lambda}$$

Ex. For what distance is the ray optics a good approximation, if the slit is 3 mm wide and the wavelength of light is 5000 Å?

Sol. $D < \frac{d^2}{\lambda} = \frac{(3 \times 10^{-3})^2}{5000 \times 10^{-10}} = 18\,m$

Thus, upto a distance of 18 m, we can assume rectilinear propagation of light to a good approximation.

Polarisation

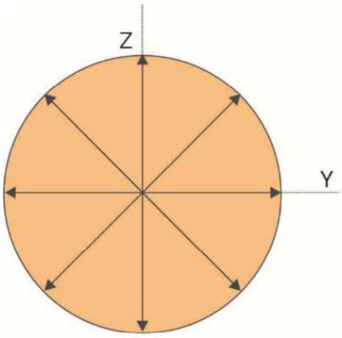
The phenomenon of restricting the vibration of light (electric vector) in a particular direction perpendicular to the direction of propagation of wave is called polarisation of light. In polarised light the vibration of the electric vector occur in a plane perpendicular to the direction of propagation of light and are confined to a single direction in the plane (do not occur symmetrically in all possible directions). After polarisation the vibrations become asymmetrical about the direction of propagation of light.

Concept Reminder

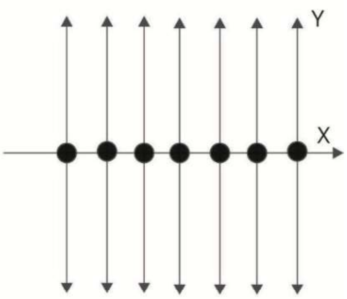
Polaroids can be used to control the intensity in sunglasses, windowpanes, etc. Polaroids are also used in photographic cameras and 3D movie cameras.

Unpolarised Light

An ordinary beam of light consists of a large number of waves emitted by the atoms of the light source. Each atom produces a wave with its own orientation of electric vector $\vec{E}$ so all direction of vibration of $\vec{E}$ are equally probable.



Unpolarised light propagating along . X-axis



Unpolarised light

The resultant electromagnetic wave is a superposition of waves produced by the individual atomic sources and it is called unpolarised light. In ordinary or unpolarised light, the vibrations of the electric vector occur symmetrically in all possible directions in a plane perpendicular to the direction of propagation of light.

KEY POINTS

- Polarisation
- Polaroid
- Polariser
- Analyser
- Tourmaline crystal
- Nicol Prism
- Plane of polarisation

Polariser

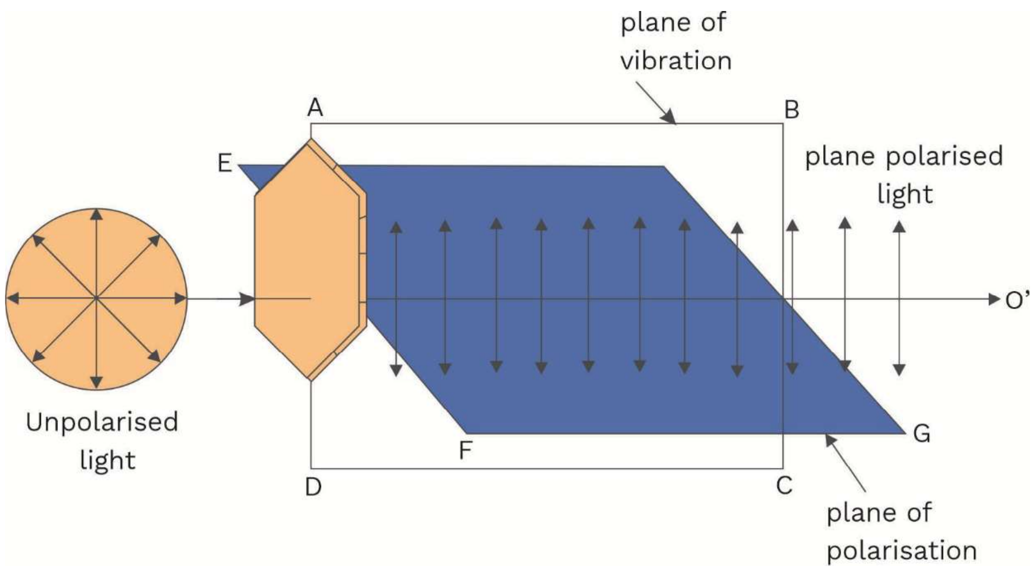
Tourmaline crystal: When light is passed through a tourmaline crystal cut parallel to its optic axis, the vibrations of the light carrying out of the tourmaline crystal are confined only to one direction in a plane perpendicular to the direction of propagation of light. The emergent light from the crystal is said to be plane polarised light.

Nicol Prism: A nicol prism is the optical device which can be used for production and detection of plane polarised light. It was invented by William Nicol in 1828.

Polaroid: A polaroid is a thin commercial sheet in form of the circular disc which makes use of property of selective absorption to produce an intense beam of plane polarised light.

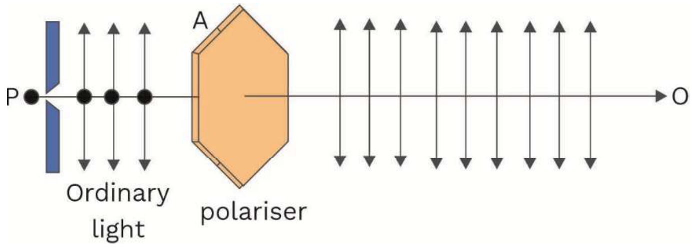
Plane Of Polarisation And Plane Of Vibration:

The plane in which the vibrations of light vector and the direction of propagation lie is called plane of vibration. A plane normal to the plane of vibration and in which no vibration takes place is known as plane of polarisation.



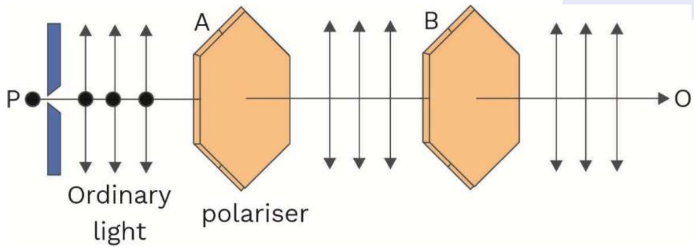
Experimental Demonstration Of Polarisation of Light

Take two tourmaline crystals cut parallel to their crystallographic axis (optic axis).

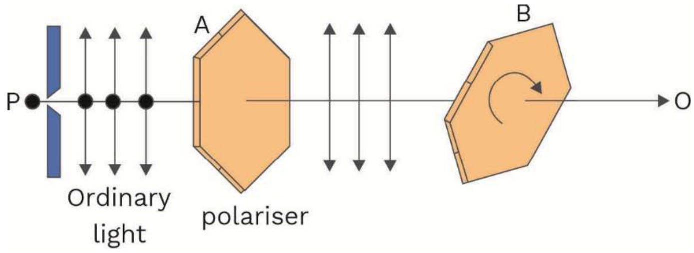


First hold the crystal A normally to the path of a beam of colour light. The emergent beam will be slightly coloured. Rotate the crystal A about PO. No change in the intensity or the colour of the emergent beam of light.

Take another crystal B and hold it in the path of the emergent beam of so that its axis is parallel to the axis of the crystal A. The beam of light passes through both the crystals and out coming light appears coloured.



Now, rotate the crystal 'B' about the axis 'PO'. It will be seen that intensity of the emergent beam decreases and when axes of both crystals are at right angles to each other no light comes out of the crystal B.



Rack your Brain

In a Fraunhofer diffraction at a single slit of width d with incident light of wavelength 5500 Å, the first minimum is observed at angle of 30°. The first secondary maximum is observed at an angle equal to

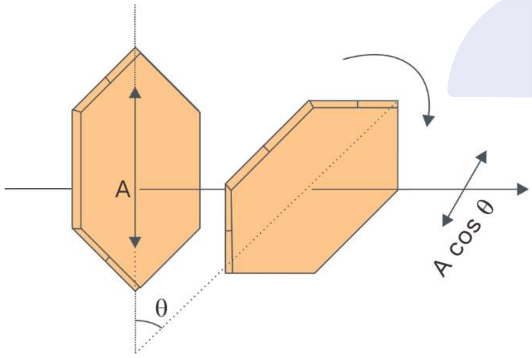
(1) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (2) $\sin^{-1}\left(\frac{1}{4}\right)$

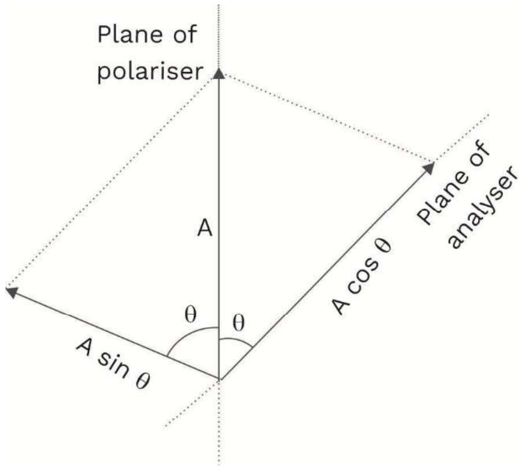
(3) $\sin^{-1}\left(\frac{3}{4}\right)$ (4) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

If the crystal 'B' is further rotated light reappears and the intensity becomes maximum again when their axis are parallel. This occurs after a further rotation of 'B' through 90°. This experiment confirms that light waves are transverse in nature. The vibrations in the light waves are perpendicular to direction of propagation of the the wave. First crystal 'A' polarises the light, so it is called polariser. 2nd crystal 'B' analyses the light whether it is polarised or not, so it is called analyser.

Law Of Malus

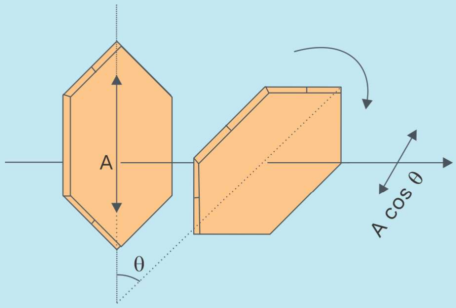
- When completely plane polarized light beam is incident on analyzer, then intensity of the emergent light changes as the square of cosine of the angle between the planes of transmission axis of the analyzer and the polarizer. $I \propto \cos^2\theta \Rightarrow I = I_0 \cos^2\theta$





Concept Reminder

According to Law of malus $I = I_0 \cos^2\theta$



(i) If $\theta = 0°$ then $I = I_0$ max. value (Parallel arrangement)

(ii) If $\theta = 90°$ then $I = 0$ min. value (Crossed arrangement)

- If plane polarized light of the intensity $I_0(= KA^2)$ is incident on a Polaroid and its vibrations of amplitude A make angle 'θ' with transmission axis, then component of vibrations parallel to transmission axis will be $A\cos\theta$ while perpendicular to it will be $A\sin\theta$.

  Polaroid will pass only those vibrations which are parallel to the transmission axis i.e. $A\cos\theta$,

  $\because \quad I_0 \propto A^2$

  So, the intensity of emergent light

  $I = K(A\cos\theta)^2 = KA^2\cos^2\theta$

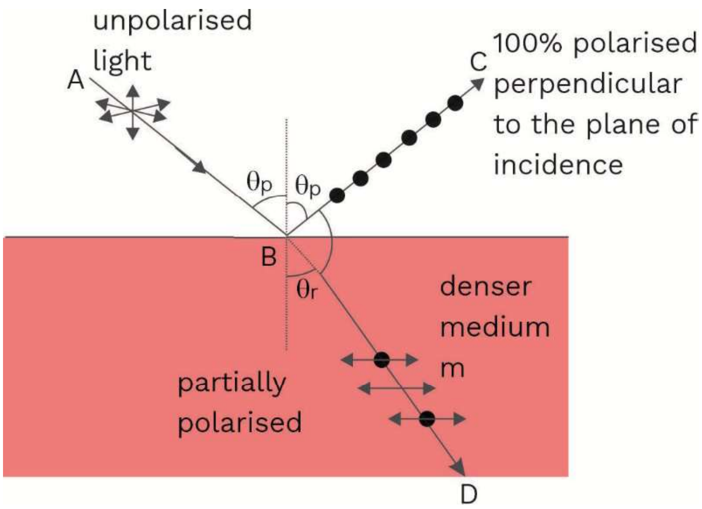
- If an unpolarised light is converted into the plane polarized light its intensity becomes half.
- If light of intensity I_1 emerging from one Polaroid called polarizer is incident on a second Polaroid (called analyzer) the intensity of light emerging from the second Polaroid is $I_2 = I_1\cos^2\theta$
- θ = angle between transmission axis of the two Polaroid's.

Methods of Obtaining Plane Polarised Light

Polarization by reflection: The simplest method to produce plane polarized light is by reflection. This method was discovered by Mauls in 1808. When beam of ordinary light is reflected from a surface, then the reflected light is partially polarized. (Degree of polarization of the polarized light in the reflected beam is greatest when it is incident at an angle called polarizing angle or Brewster's angle).

Concept Reminder

Polaroids can be used to control the intensity in sunglasses, windowpanes etc. Polaroids are also used in photographic cameras and 3D movie cameras.



- **Polarizing angle:** Polarizing angle is that angle of incidence at which the reflected light is completely plane polarization.
- **Brewster's Law:** When unpolarized light strikes at polarizing angle θ_P on an interface separating the rare medium from the denser medium of refractive index m, such that $\mu = \tan\theta_P$ then the reflected light (light in rare medium) is completely polarized. Also refracted and reflected rays are normal to each other. This relation is known as Brewster's law. The law state that the tan function of the polarizing angle of incidence of a transparent medium is equal to its refractive index $\mu = \tan\theta_P$

In case of polarisation by reflection:

(i) For $i = \theta_p$ refracted light is partially polarised.

(ii) For $i = \theta_p$ reflected and refracted rays are perpendicular to each other.

(iii) For $i < \theta_p$ or $i > \theta_p$ both reflected and refracted light become partially polarized.

According to Snell's saw $\mu = \dfrac{\sin\theta_p}{\sin\theta_r}$ ... (i)

But according to Brewster's law $\mu = \tan\theta_p = \dfrac{\sin\theta_p}{\cos\theta_p}$ ... (ii)

From equation (i) & (ii) $\dfrac{\sin\theta_p}{\sin\theta_r} = \dfrac{\sin\theta_p}{\cos\theta_p}$

$$\Rightarrow \sin\theta_r = \cos\theta_p$$

$\therefore \sin\theta_r = \sin(90° - \theta_p) \Rightarrow \theta_r = 90° - \theta_p$

or $\theta_p + \theta_r = 90°$

Thus refracted and reflected rays are mutually perpendicular.

By Refraction:

In this method, a pile of glass plates is made by taking 20 to 30 microscope slides and the light is made to be incident at polarising angle

Concept Reminder

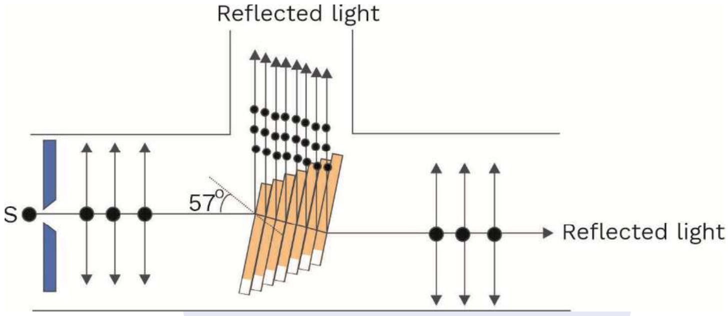
Brewster's Law

The law state that the tan function of the polarizing angle of incidence of a transparent medium is equal to its refractive index $= \tan\theta_p$

Concept Reminder

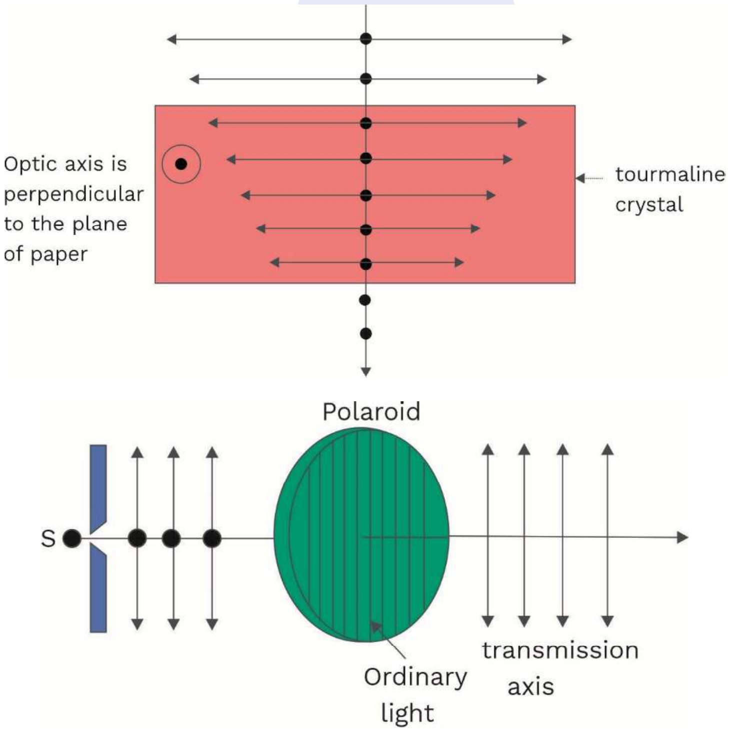
Most interference and diffraction effects exist even for longitudinal waves like sound in air. But polarisation phenomena are special to transverse waves like light waves.

57°. According Brewster's law, reflected light will be plane polarised with the vibrations perpendicular to plane of incidence and transmitted light will be partially polarised. Since in one reflection about 15% of light with vibration perpendicular to the plane of the paper is reflected therefore after going through a number of plates the emerging light will become plane polarised with the vibrations in the plane of paper.



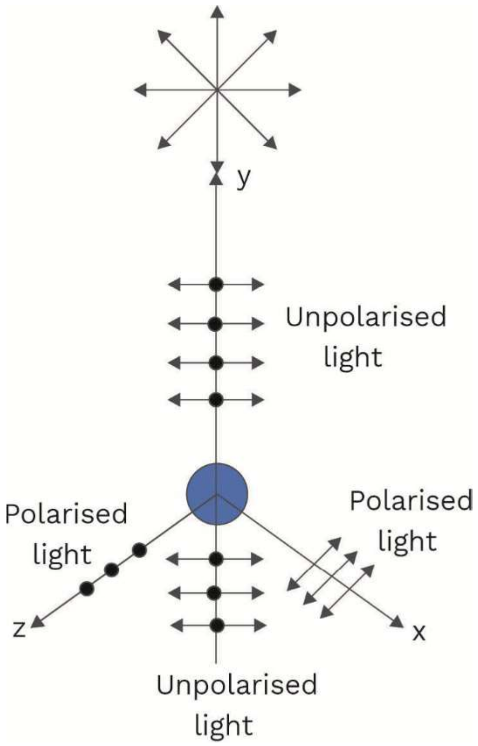
By Dichroism:

Some crystals such as the tourmaline and the sheets of iodosulphate of quinone have property of strongly absorbing light with vibrations perpendicular of a specific direction (called transmission axis) and transmitting light with vibration parallel to it. This selective absorption of the light is known as dichroism. So, if unpolarised light passes through proper thickness of these crystals, the transmitted light will plane polarised with vibrations parallel to transmission axis. Polaroid's work on this principle.



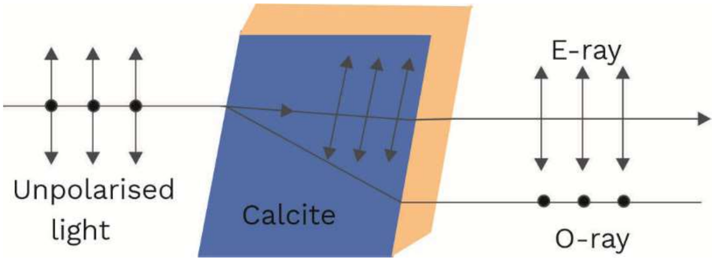
By Scattering :

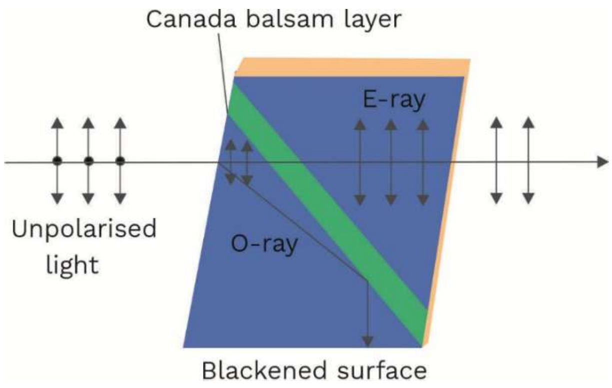
When the light is incident on small particles of dust, air molecules etc. (having lesser size as compared to the wavelength of light), then it is absorbed by the electrons and is re-radiated in all directions. The phenomenon is called as scattering. Light scattered in a direction at right angles to the incident light is always plane-polarised.



By Double Refraction:

It was found that in certain crystals such as calcite, quartz and tourmaline, etc., incident unpolarised light splits in into two light beams of equal intensities with perpendicular polarisations. One of the rays behaves as ordinary light and is called O-ray (ordinary ray) while the other does not obey laws of refraction and is called E-ray (extra-ordinary ray). This is why when an object is seen through these crystal is rotated, one image (due to E-ray) rotates around the other (due to O-ray).





By using the phenomenon of double refraction and isolating one ray from the other we can obtain plane polarised light which actually happens in a Nicol-prism. Nicol-prism is made up of calcite crystal and in it E ray is isolated from O-ray through the total internal reflection of O-ray at Canada balsam layer and then absorbing it at the blackened surface as shown in figure.

NOTE :

- Our eyes are nearly insensitive to polarisation of light, but this is not universally true for animals.
- If angle of incidence is 0° or 90° then the reflected beam is unpolarised.
- A Nicol prism is made by cutting a calcite crystal in a certain way.

Ex. For a given medium, the polarising angle is 60°. What will be the critical angle for this medium? Also find speed of light is medium.

Sol. Here $i_p = 60^\circ$

Thus, $= \tan i_p = \tan 60^\circ = \sqrt{3}$

If i_c is critical angle for the medium.

$$= \frac{1}{\sin i_c} \text{ or } \sin i_c = \frac{1}{} = \frac{1}{\sqrt{3}} \text{ or } \sin i_c = 0.5774$$

or $\quad i_c = 35^\circ 16'$

$$v = \frac{c}{} \Rightarrow v = \frac{3 \times 10^8}{\sqrt{3}} = \sqrt{3} \times 10^8 \text{ m / s}$$

Ex. Polarisation of light proves

(1) longitudinal wave nature of light

(2) transverse nature of light

(3) quantum nature of light

(4) corpuscular nature of light

Sol. As only transverse waves and not longitudinal waves are polarised so polarisation of light proves light waves are transverse waves.

Ex. Plane polarised light is passed through a Polaroid. On seeing through the Polaroid we find that when Polaroid is given one complete rotation about the direction of light, one of the following is observed

(1) the intensity of light gradually decreases to zero and remains at zero

(2) the intensity of light gradually increases to a maximum and remains at maximum

(3) there is no change in intensity

(4) the intensity of light is twice maximum and twice zero

Sol. Intensity after passing through Polaroid $I_0 = I\cos^2\theta$ as we given one rotation it becomes twice maximum and twice zero.

Ex. An unpolarised light is incident upon a glass plate of refractive index 1.48 at Brewster's angle and gets completely plane polarised. The angle of polarisation is (tan 56° = 1.48)

Sol. At Brewster's angle i_p of incident ray the refractive index $\mu = \tan i_p$

or $\quad i_p = \tan^{-1}(\mu) = \tan^{-1}(1.48) = 56°$

Ex. Two polaroids P_1 & P_2 are placed with their axis perpendicular to each other. Unpolarised light I_0 is incident on P_1. A third polaroid P_3 is kept in between P_1 & P_2 such that its axis makes an angle 30° with that of P_1.

Find the intensity of transmitted light through P_2?

Sol. $I = \frac{I_0}{2}\cos 30° \cos 60° = \frac{\sqrt{3}\, I_0}{8}$

EXAMPLES

Q1 **Two sources of intensity I and 4I are used in an interference experiment. Calculate the intensity at points where the waves from the two sources superimpose with a phase difference of**

(a) Zero

(b) $\frac{\pi}{2}$

(c) They meet at phase difference of π.

Sol: We know that $I_{res} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(\phi)$

(a) $I_{res} = I + 4I + 2\sqrt{I} \times 2\sqrt{I}\cos(0^\circ) = 9I$.

(b) $I_{res} = I + 4I + 2\sqrt{I} \times 2\sqrt{I}\cos\left(\frac{\pi}{2}\right) = 5I$

(c) $I_{res} = I + 4I + 2\sqrt{I} \times 2\sqrt{I}\cos(180^\circ) = I$.

Q2 **2 slits separated by a distance of 1 mm are illuminated with red light of wavelength 6.5×10^{-7} m. The interference fringes are seen on a screen placed 1 m apart from the slits. What is the distance between the 3^{rd} dark fringe and the 5^{th} bright fringe on the same side of the central maxima.**

Sol: As D >> d and λ << d.

Hence we can use $\beta = \frac{\lambda D}{d}$

So the distance between 5^{th} bright fringe and 3^{rd} dark fringe = $5\beta - (2\beta + \beta/2)$

$= \frac{5}{2}\beta = \frac{5}{2} \times \frac{6.5 \times 10^{-7} \times 1}{10^{-3}}$

= 1.625 mm.

Q3 **In a Y.D.S.E., the fringe width is observed of 0.4 mm. If the whole apparatus is submerged in water of refractive index (4/3) without changing the geometrical arrangement then find the new fringe width ?**

Sol: AS $\beta = \frac{\lambda D}{d} = 0.4 \text{ mm}$

$\Rightarrow$ by changing μ; λ becomes $\frac{\lambda}{\mu}$

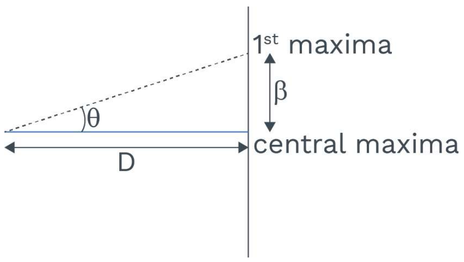
$\Rightarrow \beta' = \frac{3}{4} \times 0.4 = 0.3$ mm.

Q4 **Calculate the angular fringe width in a Y.D.S.E. with blue-green light of wavelength 6000 Å. When the slits is 3.0×10^{-3} m apart.**

As $\lambda << d$; $\Rightarrow$ we can use

Sol: $\beta = \frac{\lambda D}{d}$

So



$$\theta = \frac{\beta}{D} = \frac{\lambda}{d} = \frac{6000 \times 10^{-10}}{3 \times 10^{-3}} = 2 \times 10^{-4} \text{ radian.}$$

$$= \frac{180}{\pi} \times 2 \times 10^{-4} \text{ degree} = 0.011°$$

Wave Optics

Q5 **A source emitting 2 light waves of wavelengths of 580 nm and 700 nm respectively is used in a Y.D.S.E. The separation between slits is 0.20 mm and the interference is found on a screen placed at 150 cm from the slits. Calculate the linear separation between the 1st maximum (next to the central maximum) corresponding to the two wavelengths.**

Sol: As $\lambda << d$

$$\Rightarrow \quad \beta_1 = \frac{\lambda_1 D}{d} \text{ and } \beta_2 = \frac{\lambda_2 D}{d}$$

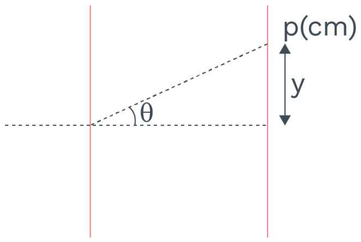
$$\text{So} \quad \beta_2 - \beta_1 = \frac{(\lambda_2 - \lambda_1)D}{d} = \frac{(700-580)\times 10^{-9} \times 1.5}{0.20 \times 10^{-3}} = 0.9 \text{ mm.}$$

Q6 **A flint glass and a crown glass are fitted on the 2 slits of a double slit method. The thickness of strips is 0.40 mm and the separation between slits is 0.12 cm. If the refractive index of crown glass and flint glass are 1.62 and 1.52 respectively for light of wavelength 480 nm which is applied in the experiment, then the interference is observed on a screen a distance one meter away.**
(a) Find fringe-width ?
(b) Find the distance from the geometrical centre where the nearest maximum be located?

Sol: (a) By inserting the strips, β does not change.

$$\beta = \frac{\lambda D}{d} = \frac{480 \times 10^{-9} \times 1\text{m}}{0.12 \times 10^{-2}} = 4.0 \times 10^{-4}.$$

(b) Lets try to observed the point at which the path difference is zero. (central maxima).



So at 'P' path difference due to extra travelling (i.e. d sinθ) must be same to optical path difference at the centre point path difference = $(\mu_2 - \mu_1)$
t = 83.33 λ

So as we move up the path difference will decrease so at height $\frac{\beta}{3}$ we will get maxima and at a distance $\frac{2\beta}{3}$ on lower side maxima will formed.

Q7 **Calculate the thickness of a plate which will produce a change in the optical path equal to 1/4 of the wavelength λ of the light passing through it normally. The refractive index of the plate is μ.**

Sol: Clearly, optical path difference

$$=(\mu-1)t=\frac{\lambda}{4} \Rightarrow t=\frac{\lambda}{4(\mu-1)}.$$

Q8 **A parallel beam of monochromatic light of wavelength λ is used in a Y.D.S.E. The slits are separated by a distance 'd' and the screen is put parallel to the plane of slits. The beam incidents at an angle of $\theta=\sin^{-1}\left(\frac{\lambda}{2d}\right)$ with the normal to plane of the slits. A transparent sheet with refractive index 'μ' and thickness $t=\frac{\lambda}{2(\mu-1)}$ is introduced Infront of one of the slit. Find the intensity at the geometrical centre.**

Sol: maximum

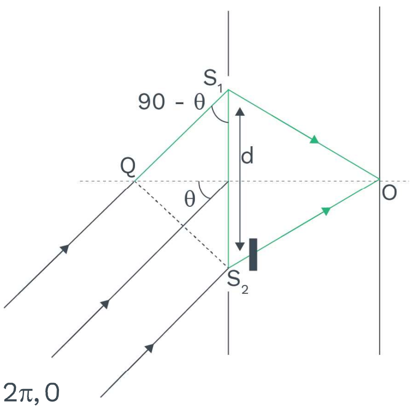
Clearly, at O, no difference of path due to inside motion of rays.

Only path difference = S_1Q

$= d\cos(90-\theta)$

$= d\sin\theta.$

$= d\times\frac{\lambda}{2d}=\frac{\lambda}{2}$



Net phase difference

$$\text{So } \Delta\phi=\frac{\lambda}{2}\times\frac{2\pi}{\lambda}\pm(\mu-1)t.\frac{2\pi}{\lambda}=\pi\pm(\mu-1)\frac{\lambda}{2(\mu-1)}\frac{2\pi}{\lambda}=2\pi,0$$

$\Rightarrow$ constructive interference and hence I_{res} = maximum.

Q9 **A soap film of thickness 0.3 μm appears dark when seen by the refracted light of wavelength 580 nm. Find the refraction index of the soap solution, if it is known to be between 1.3 and 1.5 ?**

Sol: The path difference is $2\mu t$.

Now for destructive interface it can be

$$2\mu t = \frac{\lambda}{4t}, \frac{3\lambda}{4t}, \frac{5\lambda}{4t} ....$$

$$= \frac{580 \times 10^{-9}}{4 \times 0.3 \times 10^{-6}}, \frac{3 \times 580 \times 10^{-9}}{4 \times 0.3 \times 10^{-6}} ....$$

= 0.4833, 3 × 0.4833 ....

So only $\mu = 3 \times 0.4833 = 1.45$ is the answer, $\{1.3 < \mu < 1.5\}$.

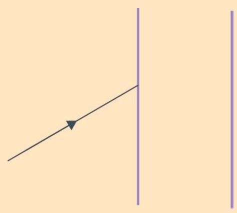
Q10 **A parallel beam of light of wavelength 560 nm falls on a thin film of oil of refractive index = 1.4. Find the minimum thickness of the film so that it weakly transmits the light ?**

Sol: Clearly for strong reflection, it should be destructive interference.

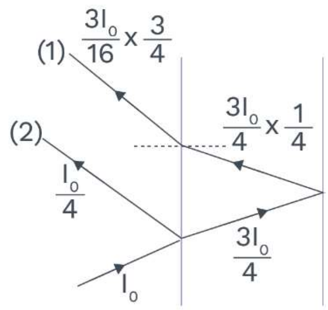
$$\Rightarrow \quad 2\mu t = \frac{\lambda}{2} \text{ \{for smallest thickness\}}$$

$$\Rightarrow \quad t = \frac{\lambda}{4\mu} = \frac{560 \times 10^{-9}}{4 \times 1.4} = 100 \text{ nm.}$$

Q11 **A narrow monochromatic beam of light of intensity I is incident on a glass plate as shown. An another identical glass plate is kept close to the 1st one and parallel to it. Each plate of glass reflects 25% of the light incident on it and transmits the remaining. Find the ratio of the minimum and the max. intensities in the interference pattern formed by 2 beams obtained after one reflection at each plate.**



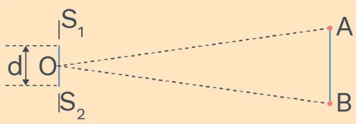
Sol: The shown diagram represents the reflection and refraction intensity.



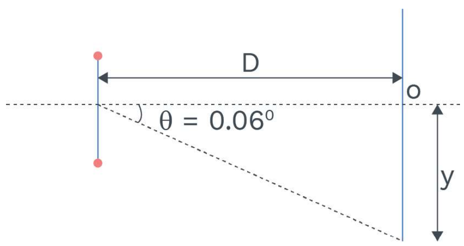
Hence the 2 waves interfering, 1 and 2, have intensities $\frac{I_0}{4}$ and $\frac{9I_0}{64}$.

$$\Rightarrow \frac{I_{min}}{I_{max}} = \frac{\left(\sqrt{\frac{I_0}{4}} - \sqrt{\frac{9I_0}{64}}\right)^2}{\left(\sqrt{\frac{I_0}{4}} + \sqrt{\frac{9I_0}{64}}\right)^2} = \frac{\left(\frac{1}{2} - \frac{3}{8}\right)^2}{\left(\frac{1}{2} + \frac{3}{8}\right)^2} = \frac{1}{49}.$$

Q12 **Figure shows two coherent sources $S_1 - S_2$ vibrating in equal phase. AB is a straight wire lying faraway from the sources S_1 and S_2. Let $\frac{\lambda}{d} = 10^{-3}$. $\angle BOA$ = 0.12°. Find the number of bright spots will be seen on the wire, including points A and B.**



Sol:



Say ‘n’ fringes are present in the region shown by ‘y’

$$\Rightarrow \quad y = n\beta = \frac{n\lambda D}{d}$$

$$\Rightarrow \quad \frac{y}{D} \approx \tan(0.06°) \approx \frac{0.06 \times \pi}{180} = \frac{n\lambda}{d}$$

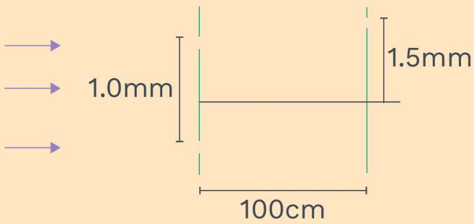
$$n = \frac{10^3 \times \pi}{180} \times 0.06 = \frac{\pi}{3} > 1.$$

Hence; only one maxima above and below point O. So total 3 bright spots will be present (including point 'O' i.e. the central maxima).

Q13 **White coherent light (400 nm-700 nm) is sent through the slits of a Y.D.S.E. (as shown). The slits are at 1 mm distance and the screen is 100 cm the slits. There is a hole in screen at 1.5 mm away (along the width of the fringes) from the central line.**

(a) Find the wavelength(s) for which there will be minima at that point ?

(b) Find the wavelength(s) which will have a maximum intensity ?



Sol: As $\lambda << d \quad \Rightarrow \quad \beta = \frac{\lambda D}{d}$ can be used

For minima

(a) $y = \left(n + \frac{1}{2}\right)\beta = \left(n + \frac{1}{2}\right)\frac{\lambda D}{d}$

$$\Rightarrow \quad \lambda = \frac{1.5 \times 10^{-3} \times 1 \times 10^{-3}}{1 \times \left(n + \frac{1}{2}\right)} = \frac{1.5 \times 10^{-6}}{n + \frac{1}{2}}$$

n = 1, $\lambda = \frac{2}{3} \times 1.5 \times 10^{-6}$ = 1000 nm

n = 2, $\lambda = \frac{2}{5} \times 1.5 \times 10^{-6}$ = 600 nm

n = 3, $\lambda = \frac{2}{7} \times 1.5 \times 10^{-6}$ = 428 nm

Putting integral values of 'n'

n = 1 ; λ = 1000 nm

n = 2 ; λ = 600 nm

n = 3 ; λ = 428 nm

So only λ = 428 nm and λ = 600 nm, will have minima at the hole. Hence they will be absent in the light coming out.

(b) 1.5 mm = n β

$$\Rightarrow \lambda = \frac{1.5 \times 10^{-3} \times 1 \times 10^{-3}}{n \times 100 \times 10^{2}}$$

for n = 1. λ = 1500 nm

for n = 2. λ = 750 nm

for n = 3. λ = 500 nm

Hence only λ = 500 nm will have maximum intensity.

Q14 **A beam of light consisting of 2 wavelengths, 6500 Å and 5200 Å is used to obtain slit experiment (1 Å = 10^{-10} m). The the slits are at a distance of 2.0 mm and the distance between the plane of the slits and the screen is 120 cm.**

(a) Find the distance of the third bright fringe on the screen from the central maximum for the wavelength 6500 Å.

(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide ?

(a) Clearly 3^{rd} bright fringe will be at y = 3β.

Sol:

$$= \frac{3 \times \lambda D}{d} = \frac{3 \times \left(6500 \times 10^{-10}\right) \times 1.2}{2 \times 10^{-3}} = 0.117 \text{ cm.} = 1.17 \text{ mm.}$$

(b) Say m^{th} bright fringe of 6500 Å coincides with n^{th} bright fringe of 5200 Å.

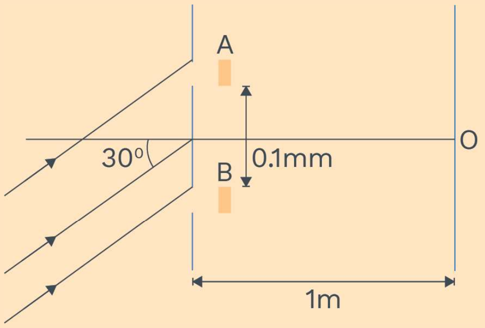
$\Rightarrow n\beta_1 = m\beta_2$

$$\Rightarrow \frac{n}{m} = \frac{5}{4}$$

Hence, for least distance, 5^{th} bright fringe of 5200 Å coincides with 4^{th} bright fringe of 6500 Å.

$$\Rightarrow y' = 4\beta_1 = \frac{4 \times \left(6500 \times 10^{-10}\right) \times 1.2}{2 \times 10^{-3}} = 0.156 \text{ cm} = 1.56 \text{ mm.}$$

Q15 **In a YDSE a parallel beam of light of wavelength 6000 Å is incident on the slits at angle of incidence 30°. 'A' and 'B' are two thin transparent films each of Refractive index 1.5. Thickness of A is 20.4 μm. Light coming through 'A' and 'B' have intensities I and 4I respectively on the screen. Intensity at point 'O' which is symmetric relative to the slits is 3I. The central maxima is above 'O'.**



(a) What is the maximum thickness of 'B' to do so.

Use the thickness of 'B' to be that found in part (a) answer in the following parts.

(b) Find fringe width, max. intensity and min. intensity on screen.

(c) Distance of nearest minima from 'O'.

(d) Find the intensity at 5 cm on either side of 'O'.

Sol: (a) For maximum thickness of B, the central maxima is just above O.

$\Rightarrow$ Phase difference at O must be = $\frac{2\pi}{3}$

$\{\because \cos\phi = -\frac{1}{2}$ for 3I intensity$\}$.

$\Rightarrow$ Path difference at $O = \frac{2\pi}{3} \times \frac{\lambda}{2\pi} = \frac{\lambda}{3}$

$\Rightarrow \; d\sin 30° + (\mu_A - 1) \times t_A - (\mu_B - 1)\, t_B = \lambda/3$

get t_B = 120 μm.

(b) Now t_B = 120 μm.

$$\beta = \frac{\lambda D}{d} = \frac{6000 \times 10^{-10} \times 1}{0.1 \times 10^{-3}} = 6 \text{ mm.}$$

$$I_{max} = \left(\sqrt{4I} + \sqrt{I}\right)^2 = 9I$$

$$I_{min} = \left(\sqrt{4I} - \sqrt{I}\right)^2 = I$$

(c) ΔP path difference at 'O'

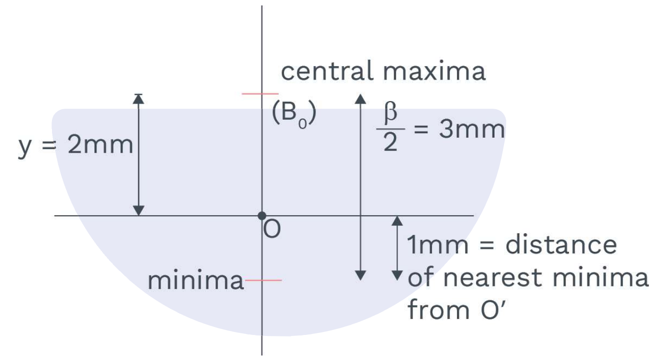
$$\Rightarrow \Delta P = d\sin\theta + (\mu_A - 1)t_A - (\mu_B - 1)t_B$$

= 0.1 sin 30° + (1.5 – 1) × 20.4 – (1.5 – 1) × 120

$\Delta P = 2 \times 10^{-7}$ m

$$\because \quad y = \frac{D}{d}\Delta P$$

$$\therefore \quad y = \frac{D}{d}\Delta P = \frac{1}{0.1\times 10^{-3}} \times 2 \times 10^{-7} \Rightarrow y = 2 \text{ mm.}$$

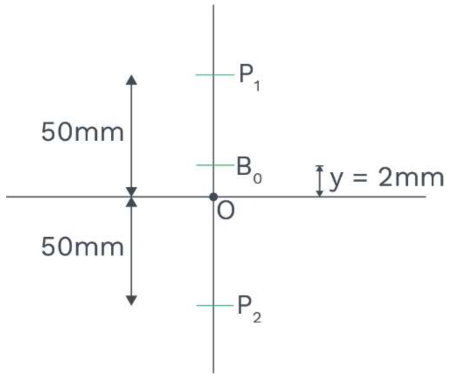


$$\because \quad \beta = \frac{\lambda D}{d} = 6 \text{ mm}$$

distance between maxima and minima = $\frac{\beta}{2}$ = 3 mm

distance of nearest minima from 'O' = $\frac{\beta}{6}$ = 1 mm

(d)



So phase difference at P_1

$$\Delta\phi = \frac{yd}{D} \times \frac{2\pi}{\lambda}$$

$$= \frac{48 \times 10^{-3} \times 10^{-4}}{1} \times \frac{2\pi}{6000 \times 10^{-10}}$$

$\Delta\phi = 16\pi$

Intensity at 'P_1'

$$I_1 = 4I + I + 2\sqrt{4I^2}\cos(16\pi)$$

$I_1 = 9I$

Intensity at point 'P_2'

Phase difference at 'P_2'

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta P$$

$$\Delta P = \frac{yd}{D} = \frac{52\,mm \times 0.1mm}{1m}$$

$$\Delta\phi = \frac{2\pi}{6000 \times 10^{-10}} \times \frac{52\,mm \times 0.1mm}{1m}$$

$$\Delta\phi = 52\frac{\pi}{3} = 16\pi + \frac{4\pi}{3}$$

$$I_2 = 4I + I + 2\sqrt{4I^2}\cos\left(16\pi + \frac{4\pi}{3}\right)$$

$$I_2 = 4I + I + 2\sqrt{4I^2}\cos\left(\frac{4\pi}{3}\right)$$

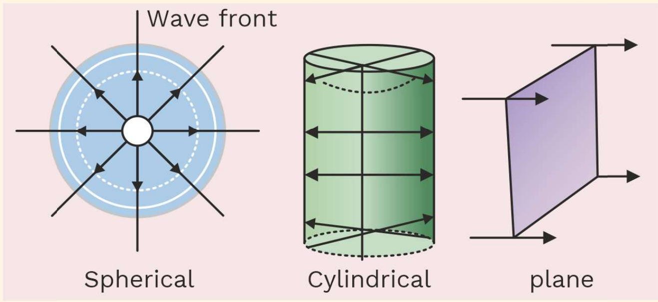
$$= 4I + I - 2 \times 2I\frac{1}{2}$$

$I_2 = 3I.$

MIND MAP

WAVE OPTICS

Huygen's wave theory

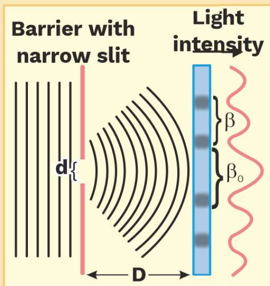


Interference

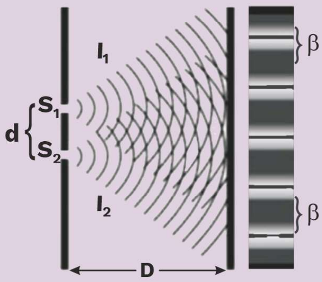
- Phase difference should not change with time → coherent source
- $A_{res}^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos\phi$
- $I_{res} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} \cos\phi$

Young's double slit experiment

- Fringe width $\beta = \frac{\lambda D}{d}$
- Position of bright fringe $x_n = \frac{n\lambda D}{d}$
- Position of dark fringe $x'_n = \frac{(2n-1)\lambda D}{d}$



Single Slit Diffraction



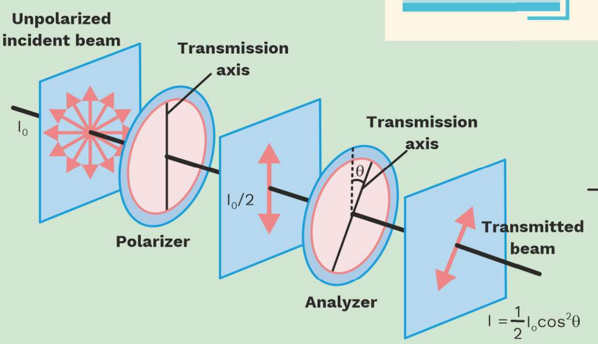
Diffraction

- Angular position of n^{th} minima $\theta_n = \frac{n\lambda}{d}$
- Angular position of n^{th} maxima $= \theta'_n = \frac{(2n+1)\lambda}{2d}$
- Width of central maxima $\beta_o = 2\beta = \frac{2D\lambda}{d}$

Malus law

$$I = \frac{I_0 \cos^2\theta}{2}$$

Polarisation



Brewster's angle

$$\theta_B = \tan^{-1}\frac{n_2}{n_1}$$