Units & Dimensions

DISCLAIMER

Units & Dimensions

Physical Quantities

All the quantities which are used in order to describe the laws of physics are known as physical quantities.

Physical quantities are of three types

- Fundamental or Basic quantities
- Derived Quantities
- Supplementary Quantities

Definitions

All the quantities which are used to describe the laws of physics are known as physical quantities.

Fundamental (Basic) Quantities:

These are the elementary quantities which covers the entire span of physics.

Any other quantities can be derived from these.

All the basic quantities are chosen such that they should be different, that means independent of each other. (i.e., distance (d), time (t) and velocity (v) cannot be chosen as basic quantities (because they are related as $v = \frac{d}{t}$). An International Organization named CGPM: General Conference on weight and Measures, has chosen seven physical quantities as basic or fundamental.

KEY POINTS

- Physical quantity
- Fundamental quantity
- Derived quantity
- Supplementary quantity

- Length (L)
- Time (T)
- Mass (M)
- Temperature (K)
- Electrical current (A)
- Luminous Intensity (Cd)
- Amount of Substance (mol)

Derived Quantities

The physical quantities which can be expressed in the terms of basic quantities (M, L,T....) are called derived quantities.

For example,

$$\text{Speed} = \frac{\text{Distance}}{\text{time}} \quad \text{and} \quad \text{Density} = \frac{\text{mass}}{\text{volume}}$$

Supplementary quantities:

Besides, seven fundamental quantities two supplementary quantities are also defined. They are

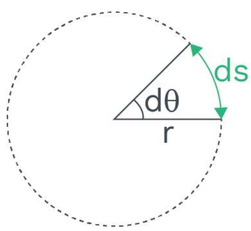
- Plane angle (The angle between two lines)
- Solid angle

The remaining two SI units (Supplementary Units) and their respective quantities are defined as follows:

(i) **Plane Angle :** The angle subtended by an arc of a circle at its centre is called a plane angle. Thus plane angle $d\theta = \frac{ds}{r}$

SI unit of plane angle is radian which is represented as rad.

One radian is defined as the plane angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.



The total plane angle which is subtended by a circle at its centre is 2π radian. Another unit in which angle is measured, is degree.

$$1^\circ = \frac{\pi}{180}\text{rad}$$

One degree is also equal to 60 minutes and 1 minute is equal to 60 seconds

$$1^\circ = \frac{\pi}{180}\text{rad} = (60)' = (3600)''$$

Concept Reminder

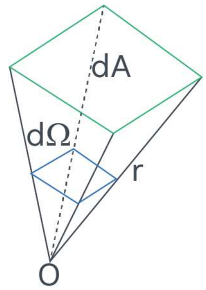
$$1^\circ = \frac{\pi}{180}\text{rad} = (60)' = (3600)''$$

Definitions

One radian is defined as the plane angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.

(ii) **Solid Angle :** The angle subtended by a given surface area of a spherical surface at its centre is called a solid angle. Mathematically it is the ratio of the intercepted area dA of the spherical surface to the square of its radius r, as shown.

Thus solid angle $d\Omega = \frac{dA}{r^2}$



SI Unit of a solid angle is steradian and is represented as 'sr'.

One steradian is defined as the solid angle subtended at the centre of a sphere by a surface of the sphere equal in area to that of a square, having each side equal to the radius of the sphere.

Units of Physical Quantities

The chosen standard reference of measurement in multiples of which, a physical quantity is expressed is called the unit of that quantity.

- **System of Units:**

(i) **FPS or British Engineering system –** In this system mass, length & time are taken as fundamental quantities and their base units are foot (ft), pound (lb) and second (s) respectively.

(ii) **CGS or Gaussian system:** In this system the fundamental quantities are mass, length & time and their respective units are centimeter (cm), gram (g) and second (s).

Definitions

The chosen reference standard of measurement in multiples of which, a physical quantity is expressed is called the unit of that quantity.

(iii) MKS system: In this system also, the fundamental quantities are mass, length & time and their fundamental units are metre (m), kilogram (kg) and second (s) respectively.

(iv) International system (SI) of units: This system is the modification of MKS system & so it is also known as the Rationalized MKS system. Besides these three base units of MKS system four fundamental and two supplementary units are also included in this system.

SI BASE QUANTITIES AND THEIR UNITS			
S.No.	**PHYSICAL QUANTITY**	**UNIT**	**SYMBOL**
1.	Length	Meter	m
2.	Mass	Kilogram	kg
3.	Time	Second	s
4.	Temperature	Kelvin	K
5.	Electric Current	Ampere	A
6.	Luminous intensity	Candela	cd
7.	Amount of substance	Mole	mol

Apart from these seven base fundamental units, there are two supplementary units used

1. The plane angle measured in radian.

2. The solid angle measured in steradian.

SI Prefix:

Suppose distance between SIKAR to JAIPUR is 3000 m. so $d = 3000\,m = 3 \times \boxed{1000}\,m$ (1000 → kilo (k))

= 3 km (here 'k' is the prefix used for 1000 (10^3))

Suppose thickness of a wire is 0.05 m

$d = 0.05 = 5 \times \boxed{10^{-2}}\,m$ (10^{-2} → centi (c))

Concept Reminder

Magnitude of physical quantity = Numerical value × Unit

= 5 cm (here 'c' is the prefix used to (10^{-2}))
Similarly, the magnitude of physical quantities varies over a wide range. So, in order to express very large magnitude as well as very small magnitude more compactly, "CGPM" recommended so standard prefixes for certain powers of 10.

Concept Reminder

Distance travelled by light in vacuum in a year is known as one light year.
Light year = 9.46×10^{15} m

Practical Units of Length

1. Light year = 9.46×10^{15} m
2. Parsec = 3.084×10^{16} m
3. Fermi = 10^{-15} m
4. Angstron (Å) = 10^{-10} m
5. Micron/Micrometer = 10^{-6} m
6. Nano meter = 10^{-9} m
7. Picometer = 10^{-12} m
8. Actometer = 10^{-18} m
9. Astronomical unit (A.U.) = 1.496×10^{11} m
10. Otto meter = 10^{-21} m

Concept Reminder

The average distance of sun from earth is known as astronomical unit.
1 A.U = 1.496×10^{11} m

POWER TO 10	PREFIX	SYMBOL	POWER TO 10	PREFIX	SYMBOL
10^{18}	exa	E	10^{-1}	deci	d
10^{15}	peta	P	10^{-2}	centi	c
10^{12}	tera	T	10^{-3}	milli	m
10^{9}	giga	G	10^{-6}	micro	μ
10^{6}	mega	M	10^{-9}	nano	n
10^{3}	kilo	k	10^{-12}	pico	p
10^{2}	hecto	h	10^{-15}	femto	f
10^{1}	deca	da	10^{-18}	atto	a

Some Important Practical Units

S.No.	QUANTITY	UNIT
1.	Mass	Solar mass = 2×10^{30} kg Dalton = 1.66×10^{-27} kg Chander Shekhar = 1.4 times of mass of sun
2.	Pressure	Pascal = 1 N/m^2 Bar = 10^5 N/m^2
3.	Area	Barn = 10^{-28} m^2
4.	Radio activity	Becquerel
5.	Radiation doze for cancer	Roentgen
6.	Time	Shake = 10^{-8} sec

Classification of Units

The units of the physical quantities can be classified as follows:

(i) **Fundamental or base units:** The units of the fundamental quantities are called base units. In SI there are seven base units.

(ii) **Derived units:** The units of the derived quantities or the units that can be expressed in terms of the base units are called derived units.
e.g., unit of speed

$$\frac{\text{unit of distance}}{\text{unit of time}} = \frac{\text{metre}}{\text{second}} = m/s$$

Some derived units are named in honor of the great scientists.
e.g., unit of force - Newton (N), unit of the frequency is hertz (Hz), etc.

BASE QUANTITY			SI UNITS
	NAME	SYMBOL	DEFINITION
Length	Meter	m	The metre is the length of the path traveled by light in vacuum during a time interval of 1/299, 792, 458 of a second (1983)
Mass	Kilogram	kg	The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at International Bureau of Weights and Measures, at Sevres, near Paris, France. (1889)
Time	Second	s	The second is the duration of 9, 192, 631, 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom (1967)
Electric Current	Ampere	A	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} Newton per metre of length. (1948)
Thermodynamic Temperature	Kelvin	K	The kelvin, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (1967)
Amount of Substance	Mole	mol	The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. (1971)
Luminous Intensity	Candela	cd	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian (1979).

(iii) **Supplementary units**: In International System (SI) of units two supplementary units are also defined as radian (rad) for plane angle and steradian (sr) for solid angle.

- **Radian**: 1 radian is the angle subtended at the centre of a circle by an arc equal in the length to the radius of the circle.
- **Steradian**: 1 steradian is a solid angle subtended at the centre of the sphere, by that surface of the sphere which is equal in the area to the square of the radius of the sphere.

KEY POINTS

- Radian
- Steradian

(iv) Practical units: Due to the fixed sizes of the SI units, some practical units are also defined for both the fundamental & derived quantities. e.g., light year (ly) is a practical unit of distance (a fundamental quantity) & horsepower (hp) is a practical unit of power (a derived quantity). The practical units may or may not belong to a particular system of units but it can be expressed in any system of the units.

e.g., 1 mile = 1.6 km = 1.6×10^3 m = 1.6×10^5cm.

(v) Improper units: These are the units which are not of the same nature as that of the physical quantities for which they are being used. e.g., kg - wt. is an improper unit of weight. Here 'kg' is a unit of mass, but it is used to measure weight (force).

Concept Reminder

While defining a unit for a physical quantity the following characteristics are considered.

(1) well defined
(2) Invariability
(3) Accessibility
(4) Convenience in use

UNITS OF SOME PHYSICAL QUANTITIES IN DIFFERENT SYSTEMS				
TYPE OF PHYSICAL QUANTITY	**PHYSICAL QUANTITY**	**CGS (ORIGINATED IN FRANCE)**	**MKS (ORIGINATED IN FRANCE)**	**FPS (ORIGINATED IN FRANCE)**
Fundamental	Length	cm	m	ft
	Mass	g	kg	lb
	Time	s	s	s
Derived	Force	dyne	newton (N)	poundal
	Work or Energy	erg	joule (J)	ft - poundal
	Power	erg/s	watt (W)	ft - poundal/s

Dimensions and dimensional formula

All the physical quantities of our interest can be derived from the base quantities. "The power (exponent) of base quantity which enters into the expression of a physical quantity, is called the dimension of the quantity in that base quantity. To make it clear, consider the physical quantity force.

FUNDAMENTAL QUANTITY	DIMENSIONS
Length	L
Mass	M
Time	T
Temperature	K
Electric Current	A
Luminous intensity	cd
Amount of substance	mol

Force = mass × acceleration

$$= \text{mass} \times \frac{\text{length / time}}{\text{time}}$$

= mass × length × $(\text{time})^{-2}$

So, the dimensions of the force are 1 in mass, 1 in length and –2 in time. Thus

$[\text{Force}] = [MLT^{-2}]$

Similarly, energy has the dimensional formula given by

$[\text{Energy}] = [ML^2T^{-2}]$

i.e., energy has dimensions, 1 in mass, 2 in length and -2 in time.

- Such an expression for a physical quantity in terms of base quantities is called dimensional formula.

Important Concepts: -

(1) Only same physical quantities can be added

Definitions

The dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity.

or subtracted
e.g. $x + y$
$[x] = [y]$

(2) If physical quantities are multiplied or divided, then a new physical quantity is formed.
e.g. $F = ma$

$\boxed{xy}$ $\boxed{\frac{x}{y}}$

(3) Dimensionless Quantities
(i) Ratio of physical quantities with same dimensions.
e.g. $\mu = \frac{c}{v}$

(ii) All mathematical constants.
e.g., $\pi(3.14), e(2.7), 4, 11, 94$ etc.

(iii) All standard mathematical functions and their inputs (exponential, logarithmic, inverse trigonometric & trigonometric).

$$e^{(x)},\ \log(2ay),\ \sin^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right),\ \sin(ax+b)$$

(4) Principle of homogeneity:- Each term of both sides in an equation has same dimensions.
$xy + pq = z$
$[xy] = [pq] = [z]$

Ex. Find dimension of a, b, c, where x is position and t is time.

(1) $F = \frac{a}{x} + bt$

$[F] = \left[\frac{a}{x}\right] = [bt]$

$[MLT^{-2}] = \left[\frac{a}{x}\right] = [bt]$

$[a] = [ML^2T^{-2}]$
$[bt] = [F]$

Concept Reminder

A dimensionally consistent equation need not be correct physically.

KEY POINTS

- Dimension
- Dimensional formula
- Principle of homogeneity

$\Rightarrow \quad [b] = [MLT^{-2}] / [T]$

$\Rightarrow \quad [b] = [MLT^{-3}]$

(2) $V = ax + \dfrac{b}{c - t}$

$[V] = [ax] = \left[\dfrac{b}{c-t}\right]$

$\left[\dfrac{b}{c-t}\right] = [V]$, $[ax] = [V]$

$[c] = [t] = [T]$, $[a] = [T^{-1}]$

$\left[\dfrac{b}{T}\right] = [V]$

$[b] = [LT^{-1}] \times [T]$

$[b] = [L]$

(3) $V = ae^{-bt}$

$[a] = [V] = [LT^{-1}]$

Input = $-bt$ = Dimensionless(DL)

bt = DL

$[bt] = [M^0L^0T^0]$

$[b]\ [T] = [M^0L^0T^0] \Rightarrow [b] = [M^0L^0T^{-1}]$

(4) $x = a\sin(bt - cx)$

$[a] = [x] = [L]$

Input = bt − cx = DL $\Rightarrow$ $[Cx] = 1$

and $\quad [C]\ [L] = 1$

$[bt] = 1 \quad [C] = [L^{-1}]$

$[b]\ [T] = 1$

$[b] = [T^{-1}]$

(4) $\left(P + \dfrac{a}{v^2}\right)(v - b) = RT$

$\left[\dfrac{a}{v^2}\right] = [P]$ and $[b] = [v]$

$\dfrac{[a]}{[L^6]} = [ML^{-1}T^{-2}]$ and $[b] = [L^3]$

$\Rightarrow \quad [a] = [ML^{-1}T^{-2}] \times [L^6]$

$\Rightarrow \quad [a] = [ML^5T^{-2}]$

Rack your Brain

The velocity v of a particle at time t is given by $v = at + \dfrac{b}{t + c}$ where a, b and c are constants. Find dimension of a, b and c.

Uses of dimensional analysis

(i) To convert a physical quantity from one system of the units to another:

It is based on the fact that, the numerical value × unit = constant

So, on changing unit, the numerical value will also get changed. If n_1 and n_2 are the numerical values of a given physical quantity and u_1 and u_2 be the units respectively in two different systems of units, then

$$n_1u_1 = n_2u_2$$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

Here

n_1= numerical value in I system
n_2= numerical value in II system
M_1= unit of mass in I system
M_2= unit of mass in II system
L_1= unit of length in I system
L_2= unit of length in II system
T_1= unit of time in I system
T_2= unit of time in II system

Concept Reminder

$$n_1u_1 = n_2u_2$$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

Ex. Convert 1 newton (SI unit of the force) into dyne (CGS unit of force).

Sol. The dimensional equation of force is

$[F] = [M^1L^1T^{-2}]$

Therefore, if n_1, u_1, and n_2, u_2 corresponds to SI & CGS units respectively, then

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-2} = 1$$

$$\left[\frac{kg}{g}\right]\left[\frac{m}{cm}\right]\left[\frac{s}{s}\right]^{-2} = 1 \times 1000 \times 100 \times 1 = 10^5$$

$\therefore$ 1 Newton = 10^5 dyne.

Ex. Convert 1 J energy to a system where

$$\left.\begin{array}{r}\text{unit of mass} = 100\text{ gm}\\ \text{length} = 10\text{ cm}\\ \text{time} = 5\text{ sec}\end{array}\right]\text{system (2)}$$

Sol. Energy = $[M\,L^2T^{-2}]$

$a = 1,\ b = 2,\ c = -2$

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$$

$$= \left(\frac{1000\,gm}{100\,gm}\right)\left(\frac{100\,cm}{10\,cm}\right)^2\left(\frac{1}{5}\right)^{-2}$$

$$= (10)(10)^2\left(\frac{1}{5}\right)^{-2} = (10)\times(100)\times(5)^2 = 25000$$

Ex. A calorie is the unit of heat or energy & it equals about 4.2 J, where 1 J = 1 kg m^2/s^2. Suppose we employ a system of the units in which the unit of mass equals 'α' kg, the unit of length which is equals β metre, the unit of time is γ second. Show that a calorie has magnitude 4.2 $\alpha^{-1}\beta^{-1}\gamma^2$in terms of the new units.

Sol. 1 cal = 4.2 kg m^2s^{-2}

SI	New system
n_1= 4.2	n_2= ?
M_1= 1 kg	M_2= α kg
L_1= 1 m	L_2= β metre
T_1= 1 s	T_2= γ second

Dimensional formula of energy is $[ML^2T^{-2}]$

Comparing with $[M^aL^bT^c]$, we find out that a = 1, b = 2, c = –2

Now, $n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$

$$= 4.2\left[\frac{1\,kg}{\alpha\,kg}\right]^1\left[\frac{1\,m}{\beta\,m}\right]^2\left[\frac{1\,s}{\gamma\,s}\right]^{-2} = 4.2[\alpha^{-1}\beta^{-2}\gamma^2]$$

(ii) To check dimensional correctness of a given physical relation:

It is based on the principle of homogeneity, which states that in a given physical relation is dimensionally correct, if the dimensions of the various terms on either side of the relation are same.

Ex. Let us check the dimensional correctness of this relation v = u + at.

Sol. Here 'u' represents the initial velocity, 'v' represents the final velocity, 'a' the uniform acceleration & 't' the time.

Dimensional formula of 'u' is given as $[M^0LT^{-1}]$

Dimensional formula of 'v' is given as $[M^0LT^{-1}]$

Dimensional formula of 'at' is given as $[M^0LT^{-2}][T] = [M^0LT^{-1}]$

Here the dimensions of every term in the given physical relation are same, hence the given physical relation is dimensionally correct.

(iii) To establish a relation between the different physical quantities:
If we know the various factors on which a physical quantity depends, then we can find out a relation among the different factors by using principle of homogeneity.

Ex. Let us find out an expression for the time period 't' of a simple pendulum. The time period t may possibly depend upon (i) mass m of the bob of the pendulum, (ii) length l of pendulum, (iii) acceleration due to gravity g at the place where the pendulum is suspended.

Sol. Let (i) $t \propto m^a$ (ii) $t \propto l^b$ (iii) $t \propto g^c$

Combining all the three factors, we get

$$t \propto m^a\, l^b\, g^c$$

$$\Rightarrow t = Km^a\, l^b\, g^c \qquad ...(i)$$

where K is a dimensionless constant of the proportionality.

Writing down the dimensions on the either side of equation (i), we get

$[T] = [M^a][L^b][LT^{-2}]^c = [M^a L^{b+c} T^{-2c}]$

Comparing dimensions, a = 0, b + c = 0, – 2c = 1

a = 0, c = – 1/2, b = 1/2

From equation (i)

$$t = Km^0 l^{1/2} g^{-1/2} \text{ or } t = K\left(\frac{l}{g}\right)^{1/2} = K\sqrt{\frac{l}{g}}$$

The value of K, as found by the experiment or mathematical investigation, comes out to be 2π.

$$\therefore \quad t = 2\pi\sqrt{\frac{l}{g}}$$

Limitations of this method

- In Mechanics the formula for a physical quantity which depends on more than three physical quantities cannot be derived. It can only be checked.

Concept Reminder

Principle of homogeneity states that a given physical relation is dimensionally correct if dimensions of various terms on either side of relation are same.

Rack your Brain

The density of a material in CGS system of units is $4\,gcm^{-3}$. In a system of units in which unit of length is 10 cm and unit of mass is 100 g. Find the value of density of material.

Concept Reminder

If we write kinetic energy as $K = \frac{2}{5}mv^2$ then it is dimensionally correct but physically incorrect.

- If the dimensions are given, physical quantity may or may not be unique as many physical quantities have same dimensions.
- It gives no information whether the physical quantity is a scalar or a vector.

DIMENSIONS	QUANTITY
$[M^0L^0T^{-1}]$	Frequency, angular frequency, angular velocity, velocity gradient and decay constant
$[M^1L^2T^{-2}]$	Work, internal energy, potential energy, kinetic energy, torque
$[M^1L^{-1}T^{-2}]$	Pressure, stress, Young's modulus, bulk modulus, modulus of rigidity, energy density
$[M^1L^1T^{-1}]$	Momentum, Impulse
$[M^0L^1T^{-2}]$	Acceleration due to gravity, gravitational field intensity
$[M^1L^1T^{-2}]$	Thrust, force, weight, energy gradient
$[M^1L^2T^{-1}]$	Angular momentum, Planck's constant
$[M^1L^0T^{-2}]$	Surface tension, surface energy (energy per unit area)
$[M^0L^0T^0]$	Strain, refractive index, relative density, angle, solid angle, distance gradient, relative permittivity (dielectric constant), relative permeability.
$[M^0L^2T^{-2}]$	Latent heat, gravitational potential
$[ML^2T^{-2}\theta^{-1}]$	Thermal capacity, gas constant, Boltzmann constant, entropy
$[M^0L^0T^1]$	$\sqrt{l/g}$, $\sqrt{m/k}$, $\sqrt{R/g}$ where l = length, g = acceleration due to gravity, m = mass, k = spring sonstant, R = radius of Earth
$[M^0L^0T^1]$	L/R, $\sqrt{LC}$, RC where L = inductance, R = resistance, C = capacitance
$[ML^2T^{-2}]$	I^2Rt, $\frac{V^2}{R}t$, VIt, qV, LI^2, $\frac{q^2}{C}$. CV^2 where I = current, t = time, q = charge, L = inductance, C = capacitance, R = resistance

- This method can only be used if the dependency is of the multiplication type. The formulae which contains exponential, trigonometrical and logarithmic functions cannot be derived using this method. The formulae containing more than one term which are added or subtracted like $s = ut + \frac{1}{2}at^2$ also cannot be derived.
- The relation derived using this method gives no information about the dimensionless constants.

Errors In Measurements

Significant Figures or Digits

Significant figures (SF) in a measurement are the figures or the digits that are known with certainty plus one that is uncertain (i.e., Last digit).

Significant figures in a measured value of the physical quantity which tells the number of the digits on which we have confidence.
Larger the number of the significant figures obtained in a measurement, greater is its accuracy & vice versa.

Definitions

Significant figures (SF) in a measurement are the figures or digits that are known with certainty plus one that is uncertain (i.e., Last digit).

Rules to find out the number of the significant figures(SF)

I Rule: All non-zero digits are significant e.g., 1984 has 4 SF.

II Rule: All the zeros in between two non-zero digits are significant. e.g., 10806 has 5 SF.

III Rule: All the zeros to the left of the first non-zero digit are not significant. e.g.,00108 has 3 SF.

IV Rule: If the number is less than 1, then the zeros on the right of decimal point but to the left of first non-zero digit are not significant. e.g., 0.002308 has 4 SF.

V Rule: The trailing zeros (zeros to the right of last non-zero digit) in a number with a decimal point are significant. e.g., 01.080 has 4 SF.

VI Rule: The trailing zeros in the number without any decimal point, may not be significant e.g., 010100 has3 SF.

VII Rule: When the number is expressed in the exponential form, the exponential term does not affect number of S.F. For example, in $x = 12.3 = 1.23 \times 10^1 = 0.123 \times 10^2 = 0.0123 \times 10^3 = 123 \times 10^{-1}$ each term has 3 SF only.

Rack your Brain

Which pairs do not have equal dimensions?
(1) Energy and torque
(2) Force and impulse
(3) Angular momentum and Planck's constant
(4) Elastic modulus and pressure.

Rules for the arithmetical operations with the significant figures

I Rule: In the addition or subtraction, the number of the decimal places in the result should be equal to the number of the decimal places of that term in the operation which contain lesser number of the decimal places. For e.g., $12.587 - 12.5 = 0.087 = 0.1$ (the second term contain lesser i.e., one decimal place)

II Rule: In the multiplication or division, the number of the significant figures in the product or quotient is same as the smallest number of SF in any of the factors. e.g., $2.4 \times 3.65 = 8.8$.

UNITS AND DIMENSIONS OF SOME PHYSICAL QUANTITIES		
PHYSICAL QUANTITY	**UNIT**	**UNIT DIMENSION**
Density	kg/m^3	M/L^3
Force	Newton (N)	ML/T^2
Work	Joule (J)(= N - m)	ML^2/T^2
Energy	Joule (J)	ML^2/T^2
Power	Watt (W) (=J/s)	ML^2/T^3
Momentum	kg - m/s	ML/T
Gravitational constant	Nm^2/kg^2	L^3/MT^2
Angular velocity	radian/s	T^{-1}
Angular acceleration	radian/s^2	T^{-2}
Angular momentum	kg - m^2/s	ML^2/T
Moment of inertia	kg - m^2	ML^2
Torque	N - m	ML^2/T^2
Angular frequency	radian/s	T^{-1}
Frequency	hertz (Hz)	T^{-1}
Period	s	T
Surface Tension	N/m	M/T^2
Coefficient of viscosity	N - s/m^2	ML/T
Wavelength	m	L
Intensity of wave	W/m^2	M/T^3
Temperature	Kelvin (K)	K
Specific heat capacity	J/kg - K	L^2/T^2K
Stefan's constant	W/m^2 - K^4	M/T^3K^4
Heat	J	ML^2/T^2
Thermal conductivity	W/m - K	ML/T^3K
Current density	A/m^2	A/L^2
Electrical conductivity	1/Ω - m(= mho/m)	A^2T^3/ML^3
Electric dipole moment	C-m	LAT
Electric field	V/m (=N/C)	ML/AT^3
Potential (voltage)	volt (V) (= J/C)	ML^2/AT^3
Electric flux	V - m	ML^3/AT^3
Capacitance	farad (F)	A^2T^4/ML^2
Electromotive force	volt (V)	ML^2/AT^3
Resistance	ohm (Ω)	ML^2/A^2T^3
Permittivity of space	C^2/N-m^2(=F/m)	A^2T^4/ML^3
Permeability of space	N/A^2	ML/A^2T^2
Magnetic field	tesla (T) (=Wb/m^2)	M/AT^2
Magnetic flux	weber (Wb)	ML^2/AT^2
Magnetic dipole moment	N - m/T	AL^2
Inductance	henry (H)	ML^2/A^2T^2

Ex. Write down the number of the significant figures in the following.

(a) 165 (b) 2.05

(c) 0.005 (d) 26900 kg

Sol. (a) 165 3SF(following rule I)

(b) 2.05 3SF (following rules I & II)

(c) 0.005 1SF(following rules I & IV)

(d) 26900 kg 5SF (see rule VI)

Rounding Off

To represent the result of any computation which is containing more than one uncertain digit, it is then rounded off to an appropriate number of significant figures.

Rules for rounding off the numbers:

I Rule: If the digit which is to be rounded off is more than 5, then the preceding digit is increased by one.

e.g., $6.87 \approx 6.9$

II Rule: If the digit which is to be rounded off is less than 5, then the preceding digit is left unchanged.

e.g., $3.94 \approx 3.9$

III Rule: If the digit which is to be rounded off is 5 then the preceding digit is increased by one whereas if it is odd & is left unchanged if it is an even. e.g., $14.35 \approx 14.4$ and $14.45 \approx 14.4$

Ex. Following values can be then rounded off to four significant figures as follows:

(a) $36.879 \approx 36.88$

($\because 9 > 5 \therefore 7$ is then increased by one i.e. I Rule)

(b) $1.0084 \approx 1.008$

($\because 4 < 5 \therefore 8$ is then left unchanged i.e., II Rule)

(c) $11.115 \approx 11.12$

($\because$ last '1' before 5 is odd it is increased by one i.e., III Rule)

Concept Reminder

A choice of change in different units does not change the number of significant figures in a measurement.

11.1250 $\approx$ 11.12
($\because$ '2' before 5 is even it is left unchanged i.e., III Rule)
(d) 11.1251 $\approx$ 11.13
($\because$ 51 > 50 $\therefore$ 2 is increased by 1 i.e. I Rule)

Order of Magnitude

Order of the magnitude of a quantity is the power of 10, which is required to represent that quantity. This power is being determined after rounding off the value of the quantity properly. In order to round off, the last digit is simply ignored if it is less than 5 & is increased by one if it is 5 or more than 5.

- When any number is divided by 10^x (where x is the order of the number) the result will always lie between 0.5 and 5 i.e., $0.5 \leq N/10^x < 5$

Ex. Order of the magnitude of the following values can be determined as follows:

(a) $49 = 4.9 \times 10^1 \approx 10^1$
Order of magnitude = 1
(b) $51 = 5.1 \times 10^1 \approx 10^2$
$\therefore$ Order of magnitude = 2
(c) $0.049 = 4.9 \times 10^{-2} \approx 10^{-2}$
$\therefore$ Order of magnitude = –2
(d) $0.050 = 5.0 \times 10^{-2} \approx 10^{-1}$
$\therefore$ Order of magnitude = –1

(e) $0.051 = 5.1 \times 10^{-2} \approx 10^{-1}$
$\therefore$ Order of magnitude = –1

Accuracy, Precision of Instruments & Errors in Measurement

Accuracy & Precision: The result of every measurement by any measuring instrument contains some uncertainty. This uncertainty in the measured value is called error. Every calculated quantity which is based on the measured value, also has an error. Every measurement is limited by reliability of measuring instrument and skill of the person making the measurement. If we repeat a particular measurement again, we usually do not get precisely the same result as each of the result is being subjected to some experimental error. This flow in measurement can be described in the terms of accuracy and precision. The accuracy in the measurement is a measure of how close the measured value is to that of the true value of the quantity. Whereas precision tells us the resolution or limit the quantity is to be measured, we can illustrate the difference between accuracy & precision with the help of an example. Let us suppose the true value of a certain length is 1.234 cm. In one experiment, measurement is made using a measuring instrument of resolution 0.1 cm, the measured value is found to be 1.1cm, while in another experiment using a measuring device of greater resolution of 0.01m, the length is determined to be 1.53cm. The first measurement has more accuracy (as it is closer to the true value) but less precision (as resolution is only 0.1 cm), while the second measurement is less accurate but more precise.

KEY POINTS

- Significant figures
- Order of magnitude
- Accuracy
- Precision

Errors

The difference between the true value & the measured value of the quantity is known as the error in the measurement.

Errors may result from different sources and are usually classified as follows:-

Systematic or Controllable Errors

Systematic errors are the type of errors whose causes are known. They can be either positive or negative. Because of the known causes these errors can be minimised. Systematic errors can further be classified into the three categories:

(i) **Instrumental errors**: - This error is caused due to the imperfect design or the erroneous manufacture or misuse of the measuring instrument. These can be reduced by using the more accurate instruments.

(ii) **Environmental errors**: - These errors are due

Definitions

- The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity.
- Precision tells us to what resolution or limit the quantity is measured.

to the changes in the external environmental conditions such as temperature, pressure, humidity, dust, vibrations or magnetic & electrostatic fields.

(iii) **Observational errors: -** These error arises due to the improper setting of the apparatus or carelessness in taking observations.

Random Errors: -

These errors are due to the unknown causes. Therefore, they occur irregularly & are variable in the sign and magnitude. Since the causes of these errors are not known precisely, they cannot be completely eliminated. For example, when the same person repeats same observation in the same conditions, he may get different readings at different times.

Random errors can be then reduced by repeating the observations a large number of times & taking the arithmetic mean of all observations. This final mean value would be very close to the most accurate reading.

Note:- If the number of the observations is made n times, then the random error reduces to $\left(\frac{1}{n}\right)$ times.

Example:- If the random error in the arithmetic mean of 100 observations is 'x' then the random error in the arithmetic mean of 500 observations will be $\frac{x}{5}$.

Gross Error:-

Gross error arise due to human carelessness and mistakes in taking reading or calculating & recording the measurement results.

For example:-

(i) Reading the instrument without proper initial settings.

KEY POINTS

- Errors
- Systematic Errors
- Random Errors

Concept Reminder

An error can arise due to the limitation of the measuring instrument or due to the skill of the person making the measurement.

Rack your Brain

If force (F), velocity (v) and time (T) are taken as fundamental units, then find out dimension of mass.

(ii) Taking observations wrongly without taking the necessary precautions.
(iii) Committing mistakes in recording the observations.
(iv) Putting improper values of observations in the calculations.

These errors can be then minimised by increasing the sincerity & alertness of the observer.

Representation of Errors

Errors can be expressed in following ways:-

Absolute Error (Δa): The difference between the true value & the individual measured value of the quantity is called the absolute error of the measurement.

Suppose a physical quantity is being measured 'n' times & the measured values are a_1, a_2, a_3......... a_n. The arithmetic mean (a_m) of all these values is

$$a_m = \frac{a_1 + a_2 + a_3 + ...a_n}{n} = \frac{1}{n}\sum_{i=1}^{n} a_i$$

If true value of the quantity is not given, then the mean value (a_m) can be taken as true value. Then the absolute errors in the measured values by an individual are –

$$\Delta a_1 = a_m - a_1$$

$$\Delta a_2 = a_m - a_2$$

....

....

$$\Delta a_n = a_m - a_n$$

The arithmetic mean of all the absolute errors present is defined as final or the mean absolute error $(\Delta a)_m$ or $\overline{\Delta a}$ of the value of the physical quantity a

$$(\Delta a)_m = \frac{|\Delta a_1| + |\Delta a_2| + ... + |\Delta a_n|}{n} = \frac{1}{n}\sum_{i=1}^{n} |\Delta a_i|$$

So, if the measured value of the quantity be 'a'

KEY POINTS

- Gross Error
- Absolute Error

Concept Reminder

Random Error

$$\propto \frac{1}{\text{No. of observation}}$$

and the error in measurement be Δa, then the true value (a_t) can be written as $a_t = a \pm \Delta a$.

Relative or Fractional Error: It is defined as ratio of the mean absolute error $((\Delta a)_m$ or $\overline{\Delta a})$ to the true value or the mean value $(a_m$ or $\bar{a})$ of the quantity measured.

Relative or fractional error

$$= \frac{\text{Mean absolute error}}{\text{Mean value}} = \frac{(\Delta a)_m}{a_m} \text{ or } \frac{\overline{\Delta a}}{\bar{a}}$$

When the relative error is expressed in the percentage, it is known as the percentage error,

percentage error = relative error × 100

or percentage error =

$$= \frac{\text{Mean absolute error}}{\text{true value}} \times 100\% = \frac{\overline{\Delta a}}{a_m} \times 100\%$$

Ex. In an experiment if refractive index of H_2O is found 1.32, 1.34, 1.36, 1.38 respectively. Find

(i) Mean value

(ii) Mean absolute error

(iii) Relative error

(iv) Percentage error

Sol.

(i) Mean value

$$a_m = \frac{a_1 + a_2 + a_3 + a_4}{4}$$

$$= \frac{1.32 + 1.34 + 1.36 + 1.38}{4} = \frac{5.40}{4} = 1.35$$

(ii) Mean absolute error

$$\Delta a_m = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + |\Delta a_4|}{4}$$

$$a_m - a_1 = \Delta a_1 \Rightarrow 1.35 - 1.32 = 0.03 = \Delta a_1$$

$$a_m - a_2 = \Delta a_2 \Rightarrow 1.35 - 1.34 = 0.01 = \Delta a_2$$

$$a_m - a_3 = \Delta a_3 \Rightarrow 1.35 - 1.36 = -0.01 = \Delta a_3$$

Definitions

The difference between the true value and the individual measured value of the quantity is called the absolute error of the measurement.

- Relative Error
- Percentage Error

$$a_m - a_4 = \Delta a_4 \Rightarrow 1.35 - 1.38 = -0.03 = \Delta a_4$$

$$\Delta a_m = \frac{0.03 + 0.01 + 0.03 + 0.01}{4}$$

$$= 0.02$$

(iii) Relative error

$$\frac{\Delta a_m}{a_m} = \frac{0.02}{1.35} = 0.014$$

(iv) Percentage error

Relative error × 100 = 0.014 × 100 = 1.4%

$$\mu = \left(a_m \pm \Delta a_m\right) = \left(1.35 \pm 0.02\right)$$

$$\mu = \left(a_m \pm \%(\Delta a_m / a_m)\right)$$

$$= (1.35 \pm 1.4\%)$$

Propagation of the Errors in Mathematical Operations

Rule I: The maximum absolute error present in the sum or the difference of two quantities is equal to the sum of the absolute errors in the individual quantities.

If X = A + B or X = A – B & if $\pm\Delta A$ and $\pm\Delta B$ represent the absolute errors in A and B respectively, then the maximum absolute error in X is $\Delta X = \Delta A + \Delta B$ and Maximum percentage error $= \frac{\Delta X}{X} \times 100$.

The result will be written as $X \pm \Delta X$ (in terms of absolute error) or $X \pm \left(\frac{\Delta X}{X} \times 100\right)\%$ (in terms of the percentage error)

Rule II: The maximum fractional or the relative error in the product or quotient of the quantities is equal to the sum of the fractional or relative errors in the individual quantities.

If X = AB or $X = \frac{A}{B}$, then $\frac{\Delta X}{X} = \pm\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$

Concept Reminder

1. Relative or fractional error

$$= \frac{\text{Mean absolute error}}{\text{Mean value}}$$

$$= \frac{(\Delta a)_m}{a_m} \text{ or } \frac{\overline{\Delta a}}{\overline{a}}$$

2. Percentage error

$$= \frac{\text{Mean absolute error}}{\text{true value}} \times 100\%$$

$$= \frac{\overline{\Delta a}}{a_m} \times 100\%$$

Rule III: The maximum fractional error present in a quantity raised to a power (n) is n times the fractional error in the quantity itself, i.e.

If $X = A^n$, then $\frac{\Delta X}{X} = n(\frac{\Delta A}{A})$

If $X = A^p B^q C^r$, then $\frac{\Delta X}{X} = [p(\frac{\Delta A}{A}) + q(\frac{\Delta B}{B}) + r(\frac{\Delta C}{C})]$

If $X = \frac{A^p B^q C^r}{D^s}$ then

$$\frac{\Delta X}{X} = \left[p\left(\frac{\Delta A}{A}\right) + q\left(\frac{\Delta B}{B}\right) + r\left(\frac{\Delta C}{C}\right) + s\left(\frac{\Delta D}{D}\right)\right]$$

Ex. The initial & final temperatures of water as recorded by the observer are (40.6 ± 0.2) °C and(78.3 ± 0.3) °C. Calculate the rise in temperature with the proper error limits.

Sol. Given $\theta_1 = (40.6 \pm 0.2)$ °C and $\theta_2 = (78.3 \pm 0.3)$ °C

Rise in temp.

$\theta = \theta_2 - \theta_1 = 78.3 - 40.6 = 37.7°C.$

$\Delta\theta = \pm(\Delta\theta_1 + \Delta\theta_2)$

$= \pm (0.2 + 0.3) = \pm 0.5°C$

∴ Rise in the temperature = (37.7 ± 0.5) °C

Ex. A thin copper wire having length L, which increases in length by 2% when heated from T_1 to T_2. If a copper cube of side 10 L is heated from T_1 to T_2 what will be the change in percentage of the following:

(i) Area of one face of cube &.

(ii) Volume of the cube.

Sol. (i) Area

$A = 10L \times 10L = 100L^2 \Rightarrow A \propto L^2$

% Change in area

$= \frac{\Delta A}{A} \times 100 = 2 \times \frac{\Delta L}{L} \times 100 = 2 \times 2\% = 4\%$

Concept Reminder

(1) $X = A \pm B$

$\Delta X = \Delta A + \Delta B$

(2) $X = AB$ or $X = \frac{A}{B}$

$\frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$

(ii) Volume

$V = 10L \times 10L \times 10L = 1000L^3 \Rightarrow V \propto L^3$

% Change in volume

$= \frac{\Delta V}{V} \times 100 = 3\frac{\Delta L}{L} = 3 \times 2\% = 6\%$

Ex **(1)** $R_1 = (\underset{R_1}{24} \pm \underset{\Delta R_1}{0.5})\,\Omega$

$R_2 = (\underset{R_2}{8} \pm \underset{\Delta R_2}{0.3})\,\Omega$

Both are connected in series. Find equivalent resistance with absolute error.

$\Delta X = \Delta A + \Delta B$

$R_{eq} = R_1 + R_2 = 24 + 8 = 32$ W

$\Delta R_{eq} = \Delta R_1 + \Delta R_2 = 0.5 + 0.3 = 0.8$ W

$R_{eq} = (32 \pm 0.8)$ W

(2) $V = (\underset{V}{8} \pm \underset{\Delta V}{0.5})\,v$

$I = (\underset{I}{2} \pm \underset{\Delta I}{0.2})A$

Find R in Ω with relative error, % & absolute error $R = \frac{V}{I}$

$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{0.5}{8} + \frac{0.2}{2} = 0.16$

% Error = 0.16 × 100 = 16%

Absolute Error $\Delta R = (0.16)$ R

= (0. 16) 4

= 0.64 Ω

(3) $[X] = [M^a L^b T^c]$

% Error in $M = \alpha$

$L = \beta$

$T = \gamma$

Find % error in X

(% Er X) = a (% Er M) + b(% Er L) + c(% Er T)

$= \alpha a + \beta b + \gamma c$

(4) $P = \frac{a^2 b^2}{c}$

% Error in $a = \pm 1\%$

$b = \pm 2\%$

$c = \pm 3\%$

Find % error in P, which quantity contributing maximum error.

$P = \frac{a^2b^2}{c}$

(% Er) = 2(% Er a) + 2(% Er b) + (% Er c)
= 2(1) + 2(2) + 3
= 2 + 4 + 3
= 9%

(5) A wire having mass $(\overset{m}{0.3} \pm \overset{\Delta m}{0.003})\text{gm}$, radius $(\overset{r}{0.5} \pm \overset{\Delta r}{0.005})\text{mm}$ and length $(\underset{l}{6} \pm \underset{\Delta l}{0.06})\text{cm}$. Find% error in density $d = \frac{m}{\pi r^2 l}$

$$\frac{\Delta d}{d} = \frac{\Delta m}{m} + \frac{2\Delta r}{r} + \frac{\Delta l}{l}$$

$$\frac{\Delta d}{d} = \frac{0.003}{0.3} + 2 \times \frac{0.06}{0.5} + \frac{0.06}{6}$$

= 0.01 + 0.020 + 0.01
= 0.04

% error = 0.04 × 100 = 4%

(6) If mass and velocity of a particle changes by 1% and 2% respectively. Find % change in K.E.

$KE = \frac{1}{2}mv^2$

(% Error k) = (% Er in m) + 2(% Er in v)
% Δ k = (%Δm) + 2(%Δv)
= 1% + 2(2%) = 5%
Only applicable if change is < 5%

Ex. The least count of a stopwatch is $\frac{1}{5}$ second. The time of 20 oscillations of the pendulum is measured to be 25 seconds. What is the percentage error in the measurement of time?

Sol. Error in measuring $25\,s = \frac{1}{5}s = 0.2\,\text{sec}$.

$\therefore$ percentage error = $\frac{0.2}{25} \times 100 = 0.8\%$

Note: The final absolute error in such type of questions is taken equal to the least count of the measuring instrument.

Least Count:

The smallest value of the physical quantity which can be measured accurately with an instrument is called its least count (L. C.).

$$\text{Accuracy} \propto \frac{1}{\text{L.C.}}$$

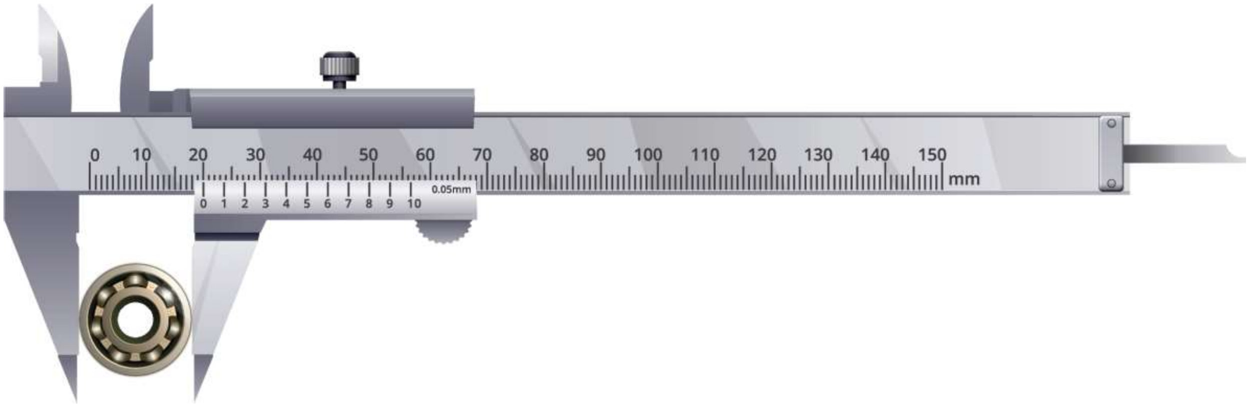
Least Count of Vernier Calipers: Suppose the size of one main scale division (M.S.D.) is M units & that of one vernier scale division (V. S. D.) is V units. Also let the length of 'a' the main scale divisions is equal to the length of 'b' vernier scale divisions.

$$aM = bV \Rightarrow V = \frac{a}{b}M$$

$$\therefore \quad M - V = M - \frac{a}{b}M = \left(\frac{b-a}{b}\right)M$$

$$[M \rightarrow MSD, V \rightarrow VSD]$$

$$\boxed{L.C. = M - V = \left(\frac{b-a}{b}\right)M}$$



$$\boxed{\text{Reading} = (\text{Main Scale Reading}) + (\text{Vernier Scale Reading} \times LC)}$$

$$aM = bV \Rightarrow V = \frac{a}{b}M$$

$$= \left(\overset{1M-IV}{1-\frac{a}{b}}\right)M$$

$$\boxed{LC = \left(\frac{b-a}{b}\right)M}$$

a = No. of division on Main scale.

b = No. of division on Vernier scale.

M = 1 Main Scale division.

V = 1 Vernier scale division.

Ex. 10 M = 1 cm

$$M = \frac{1}{10} = 0.1cm$$

b = 20

a = 18

$$L.C. = \left(\frac{b-a}{b}\right)M$$

$$= \left(\frac{20-18}{20}\right)\times 0.1$$

$$= \left(\frac{20-18}{20}\right)\times \frac{1}{10}cm$$

$$= \frac{1}{100}cm = 0.01cm$$

Ex. The main scale division of vernier calliper is equal to 1 mm. 19 division of main scale are equal in length to 20division of vernier scale. In measuring diameter, the main scale read 35 division and 4th division of vernier scale coincide with main scale. Find

(i) Least count

(ii) Diameter

Sol. M = 1 mm

a = 19

b = 20

MSR = 35 division = 35 M = 35 mm

VSR = 4

Definitions

The smallest value of a physical quantity which can be measured accurately with an instrument is called its least count (L. C.).

(i)

$$LC = \frac{b-a}{b} \times M = \frac{20-19}{20} \times 35\,mm = 0.05\,mm$$

(ii) Diameter

$\Rightarrow$ MSR + (VSR × LC)

= 35 mm + (4 × 0.05 mm)

= 35.2 mm

Concept Reminder

For Vernier Calipers

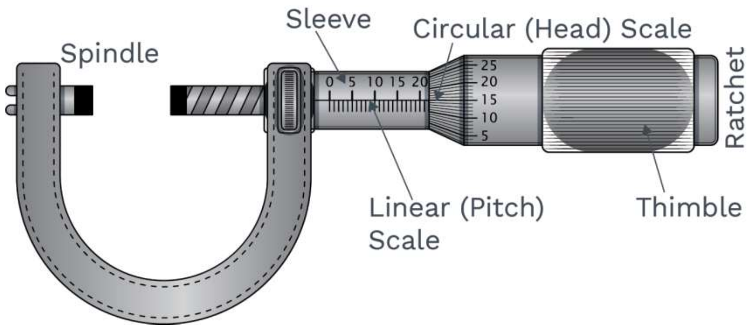
- Reading = (Main Scale Reading) + (Vernier Scale Reading × LC)
- $$LC = \left(\frac{b-a}{b}\right)M$$

a = No. of division on Main scale.

b = No. of division on Vernier scale.

M = 1 Main Scale division.

Least Count of screw gauge or spherometer



$$\text{Least Count} = \frac{\text{Pitch}}{\text{Total no. of divisions on the circular scale}}$$

where the pitch is defined as the distance moved by the screw head when the circular scale is given one complete rotation. i.e.,

$$\text{Pitch} = \frac{\text{Distance moved by the screw on the linear scale}}{\text{Number of full rotations given}}$$

Note: With decrease in the least count of measuring instrument, the accuracy of measurement increases and the error in the measurement decreases.

Ex. 1 cm on the main scale of the vernier callipers is divided into ten equal parts. If 20 divisions of the vernier scale coincide with 18 small divisions of the main scale. Determine the least count of the callipers?

Sol. 20 division of vernier scale = 18 division of main scale

$$\Rightarrow 1\text{VSD} = \left(\frac{18}{20}\right)\text{MSD} = 0.9\,\text{MSD}$$

Least count = 1 MSD – 1 VSD = 1 MSD – 0.9 MSD = 0.1 MSD

$$= 0.1 \times 0.1 \text{ cm} = 0.01 \text{ cm} \left(\because 1\text{MSD} = \frac{1}{10}\text{cm} = 0.1\text{cm}\right)$$

Ex. The n^{th} division of main scale coincides with $(n + 1)^{th}$ division of vernier scale. Given one main scale division is equal to 'a' unit. Find the least count of the vernier.

Sol. (n + 1) divisions of vernier scale = n divisions of main scale

$$1 \text{ vernier division} = \frac{n}{n+1} \text{ main scale division}$$

Least count = 1 MSD – 1 VSD

$$\left(1-\frac{n}{n+1}\right)\text{MSD} = \left(\frac{1}{n+1}\right)\text{MSD} = \frac{a}{n+1}$$

Ex. A spherometer has 100 equal divisions marked along the periphery of its disc, and one full rotation of the disc advances on the main scale by 0.01 cm. Find the least count of the system.

Sol. Given Pitch = 0.01 cm

Concept Reminder

$$\text{Least Count} = \frac{\text{Pitch}}{\text{Total no. of divisions on the circular scale}}$$

where pitch is defined as the distance moved by the screw head when the circular scale is given one complete rotation. i.e.,

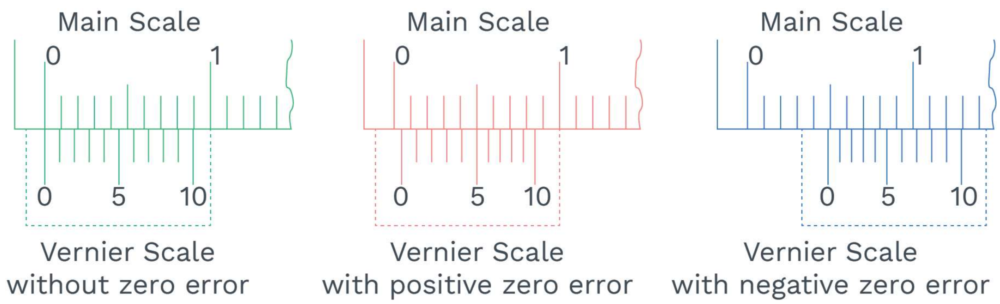
$$\text{Pitch} = \frac{\text{Distance moved by the screw on the linear scale}}{\text{Number of full rotations given}}$$

∴ Least count

$$= \frac{\text{Pitch}}{\text{Total no. of divisions on the circular scale}}$$

$$= \frac{0.01}{100}\text{cm} = 10^{-4}\text{cm}$$

Zero Error Vernier Scale



The zero error is always subtracted from the reading to get the corrected value.

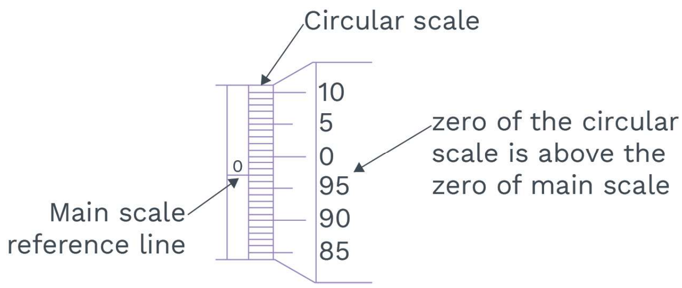
If the zero error is positive, its value is calculated as we take any normal reading.

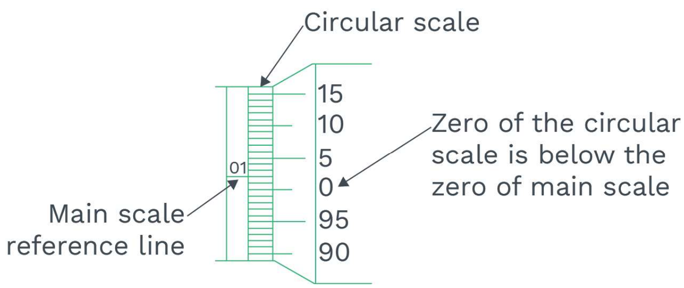
Negative zero error = – [Total no. of vsd – vsd coinciding] × L.C.

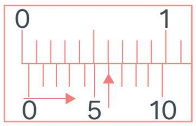
Positive zero error = – (vsd coinciding) × L.C.

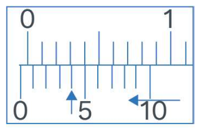
Zero Error in Screw Gauge

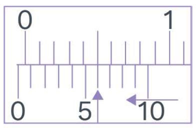
If there is no object in between the jaws (i.e., jaws are in contact), the screw gauge should give zero reading. But due to presence of extra material on jaws, even if there is no object, it gives some excess reading. This excess reading is called Zero error.

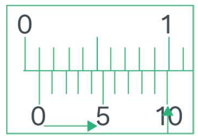












Ex. The two full turns of circular scale of screw gauge cover a distance of 1 mm on scale. The total division on circular scale is 50 further if found that screw gauge has a zero error of –0.01 mm. While measuring the diameter of thin wire a student notes the main scale reading of 2 mm and the no of circular scale division in line with the main scale as 25. The diameter of wire is

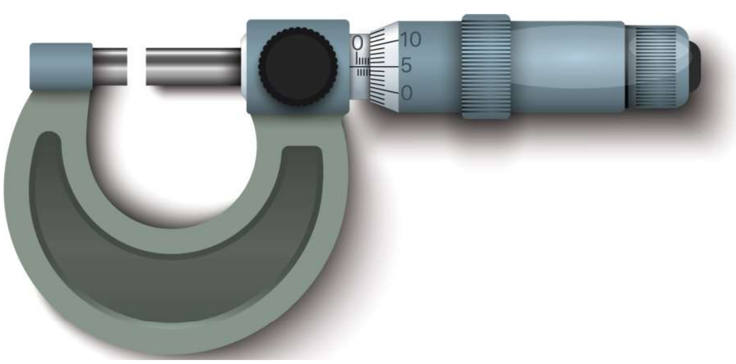
Sol. Reading = MSR + (CSR × L.C.) – ZE

$$= 2 + 25 \times \frac{0.5}{50} - (-0.01)$$

$$= 2 + 0.25 + 0.01$$

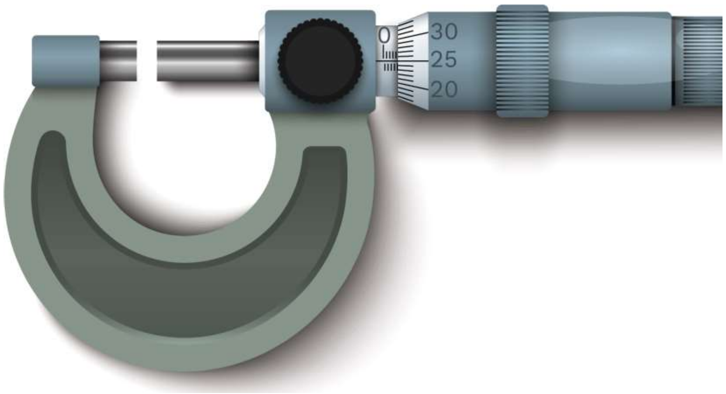
Reading = 2.26 mm

Ex. The number of circular divisions on the shown screw gauge is 50. It moves 0.5 mm on the main scale for one complete rotation. Main scale reading is 2. The diameter of the ball is:



Rack your Brain

A student measured the diameter of a small steel ball using a screw gauge of least count 0.001 cm. The main scale reading is 5 mm and zero of circular scale division above reference level. If screw gauge has zero error of –0.004 cm then find correct value of diameter of ball.



Sol. Leastcount $= \frac{0.5mm}{50} = 0.01mm$

Zero error= $.01 \times 5 = 0.05mm$

Reading= $(20 \times 0.5) + (25 \times .01) - (0.05) = 10.2$ mm

EXAMPLES

Q1 **The length & breadth of a rectangle are (5.7 ± 0.1) and (3.4 ± 0.2) cm. Calculate the area of the rectangle with error limits.**

Sol: Here $l = (5.7 \pm 0.1)\,\text{cm},\ b = (3.4 \pm 0.2)\,\text{cm}$

Area: $A = l \times b = 5.7 \times 3.4 = 19.38\,\text{cm}^2 = 19\,\text{cm}^2$

(rounding off to two significant figures)

$$\therefore \quad \frac{\Delta A}{A} = \pm\left(\frac{\Delta l}{l} + \frac{\Delta b}{b}\right) = \pm\left(\frac{0.1}{5.7} + \frac{0.2}{3.4}\right)$$

$$= \pm\left(\frac{0.34 + 1.14}{5.7 \times 3.4}\right) = \pm\frac{1.48}{19.38}$$

$$\Rightarrow \quad \Delta A = \pm\frac{1.48}{19.38} \times A = \pm\frac{1.48}{19.38} \times 19.38 = \pm 1.48 = \pm 1.5$$

(rounding off to two significant figures)

So, Area $= (19.0 \pm 1.5)\,\text{cm}^2$.

Q2 **The distance covered by a body in time (5.0 ± 0.6) s is (40.0 ± 0.4) m. Calculate the speed of the body and the percentage error in the speed.**

Sol: Here, $s = (40.0 \pm 0.4)\,\text{m}$ and $t = (5.0 \pm 0.6)\,\text{s}$

$$\therefore \quad \text{Speed } v = \frac{s}{t} = \frac{40.0}{5.0} = 8.0\,\text{ms}^{-1} \quad \left(\text{As } v = \frac{s}{t}\right)$$

$$\therefore \quad \frac{\Delta v}{v} = \frac{\Delta s}{s} + \frac{\Delta t}{t}$$

Here $\Delta s = 0.4$ m, $s = 40.0$ m, $\Delta t = 0.6$ s, $t = 5.0$ s

$$\therefore \quad \frac{\Delta v}{v} = \frac{0.4}{40.0} + \frac{0.6}{5.0} = 0.13$$

$$\Rightarrow \quad \Delta v = 0.13 \times 8.0 = 1.04$$

Hence, $v = (8.0 \pm 1.04)\,\text{ms}^{-1}$

$$\therefore \quad \text{Percentage error } = \left(\frac{\Delta v}{v} \times 100\right) = 0.13 \times 100 = 13\%.$$

Q3 **A screw gauge gives the following reading when used to measure the diameter of a wire. Main scale reading: 0 mm**
Circular scale reading: 52 divisions
Given that 1 mm on the main scale corresponds to 100 divisions of the circular scale.

Sol: Main scale reading = 0 mm

Circular scale reading = 52 divisions

$$\text{Least count} = \frac{\text{value of 1 main scale division}}{\text{Total divisions on circular scale}} = \frac{1}{100}\text{mm}$$

Diameter of wire = M.S.R + (C.S.R. × L.C)

$$= 0 + 52 \times \frac{1}{100}\text{mm} = 0.52\,\text{mm} = 0.052\,\text{cm}.$$

Q4 **A physical quantity is represented by $x = M^a L^b T^{-c}$. The percentage of errors in the measurements of mass, length and time are α%, β%, γ% respectively then the maximum percentage error is**

Sol: $$\frac{\Delta x}{x} \times 100 = a.\frac{\Delta M}{M} \times 100 + b.\frac{\Delta L}{L} \times 100 + c.\frac{\Delta T}{T} \times 100 = a\alpha + b\beta + c\gamma\ .$$

Q5 **Two resistors of resistance R_1 = (100 ± 3) Ω and $R_2 = (200 \pm 4)$ Ω are connected (a) in series, (b) in parallel. Find the equivalent resistance of the (a) series combination, (b) parallel combination.**

Sol: (a) The equivalent resistance of series combination

$$R = R_1 + R_2 = (100 \pm 3)\text{ohm} + (200 \pm 4)\text{ohm} = (300 \pm 7)\text{ohm}.$$

(b) The equivalent resistance of parallel combination

$$R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{200}{3} = 66.7\,\text{ohm}$$

Then, from $\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$

We get, $\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$

$$\Delta R' = \left(R'^2\right)\frac{\Delta R_1}{R_1^2} + \left(R'^2\right)\frac{\Delta R_2}{R_2^2}$$

$$= \left(\frac{66.7}{100}\right)^2 3 + \left(\frac{66.7}{200}\right)^2 4 = 1.8$$

Then, $R' = \left(66.7 \pm 1.8\right)$ ohm.

Q6 **Let $\left[\varepsilon_0\right]$ denote the dimensional formula of permittivity of vacuum. If M is mass, L is length, T is time and A is electric current, then**

Sol: From coulomb's law $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{R^2}$

$$\varepsilon_0 = \frac{q_1 q_2}{4\pi F R^2}$$

Substituting the units

$$\varepsilon_0 = \frac{c^2}{N - m^2} = \frac{\left[AT\right]^2}{\left[MLT^{-2}\right]\left[L^2\right]}$$

$$= \left[M^{-1}L^{-3}T^4A^2\right].$$

Q7 **The dimensional formula of $\frac{a}{b}$ in the equation $P = \frac{a - ct^2}{bx}$ where P = pressure, x = displacement and t = time.**

Sol: $[P] = \left[\frac{a}{bx}\right] = \left[\frac{ct^2}{bx}\right]$

By principle of Homogeneity,

$\left[\frac{a}{bx}\right]$ should represent pressure

$\therefore \quad \left[\frac{a}{b}\right]\frac{1}{[L]} = \left[ML^{-1}T^{-2}\right] \Rightarrow \left[\frac{a}{b}\right] = \left[MT^{-2}\right].$

Q8 **Check whether relation $S = ut + \frac{1}{2}at^2$ is dimensionally correct or not, where symbols have their usual meaning.**

Sol: We have $S = ut + \frac{1}{2}at^2$. Checking the dimensions on both sides, LHS

$= [S] = \left[M^0L^1T^0\right],$

RHS $= [ut] + \left[\frac{1}{2}at^2\right] = \left[LT^{-1}\right][T] + \left[LT^{-2}\right]\left[T^2\right]$

$= \left[M^0L^1T^0\right] + \left[M^0L^1T^0\right] = \left[M^0L^1T^0\right]$

We find LHS = RHS.

Hence, the formula is dimensionally correct.

Q9 **If C is the velocity of light, h is Planck's constant and G is Gravitational constant are taken as fundamental quantities, then the dimensional formula of mass is**

Sol: $C = [LT^{-1}] \rightarrow (1); \quad h = [ML^2T^{-1}] \rightarrow (2)$

$$G = [M^{-1}L^3T^{-2}] \rightarrow (3)$$

Solving (2) and (3)

$$\frac{h}{G} = \left[\frac{ML^2T^{-1}}{M^{-1}L^3T^{-2}}\right] = [M^2L^{-1}T^1]$$

Substituting (1) in above

$$\frac{h}{G} = \frac{M^2}{C} \Rightarrow [M] = \left[h^{\frac{1}{2}}G^{\frac{-1}{2}}C^{\frac{1}{2}}\right].$$

Q10 **If E, M, J and G respectively denote energy, mass, angular momentum and universal gravitational constant, the quantity, which has the same dimensions as the dimensions of $\frac{EJ^2}{M^5G^2}$.**

Sol: D.F. of $\frac{EJ^2}{M^5G^2}$

Substituting D.F. of E, J, M, and G in above formula

$$= \frac{ML^2T^{-2}[ML^2T^{-1}]^2}{M^5[M^{-1}L^3T^{-2}]^2} = [M^0L^0T^0].$$

Q11 **In the equation $\left(\frac{1}{p\beta}\right) = \frac{y}{k_B T}$ where p is the pressure, y is the distance, k_B is Boltzmann constant and T is the temperature. Dimensions of β are**

Sol:

$$\frac{1}{p\beta} = \frac{y}{k_B T}$$

$$\text{Dimension of } [\beta] = \frac{[\text{Dimensional formula of } k_B][\text{Dimensional formula of } T]}{[\text{Dimensional formula of } p][\text{Dimensional formula of } y]}$$

$$= \frac{[ML^2T^{-3}][T]}{[ML^{-1}T^{-2}][L]} = [M^0L^2T^0]$$

$\therefore$ Dimensions of M, L, T in β are 0, 2, 0.

Q12 **A screw gauge is having 100 equal divisions & a pitch of length 1 mm is used to measure the diameter of a wire having length 5.6 cm. The main scale reading is 1 mm & 47thcircular division coincides with the main scale. Find the curved surface area of the wire in cm^2 to appropriate significant figures. (Use π = 22/7).**

Sol:

$$\text{Least Count} = \frac{1\text{mm}}{100} = 0.01\text{mm}$$

Diameter = MSR + CSR (LC) = 1 mm + 47 (0.01) mm = 1.47 mm.

$$\text{Surface area} = \pi Dl = \frac{22}{7} \times 1.47 \times 56\,\text{mm}^2$$

$$= 2.58724\,\text{cm}^2 = 2.6\,\text{cm}^2.$$

Q13 **In the Searle's experiment, the diameter of the wire as measured by the screw gauge having least count 0.001 cm is 0.050 cm. The length, measured by a scale having least count 0.1 cm, is 110.0 cm. When a weight of 50 N is being suspended from the wire, the extension is measured to be 0.125 cm by a micrometer having least count 0.001 cm. Find the maximum error present in the measurement of the Young's modulus of the material of the wire using the above data.**

Sol: Maximum percentage error in Y is given by

$$Y = \frac{W}{\frac{\pi D^2}{4}} \times \frac{L}{x} \Rightarrow \left(\frac{\Delta Y}{Y}\right) = 2\left(\frac{\Delta D}{D}\right) + \frac{\Delta x}{x} + \frac{\Delta L}{L}$$

$$= 2\left(\frac{0.001}{0.05}\right) + \left(\frac{0.001}{0.125}\right) + \left(\frac{0.1}{110}\right) = 0.0489.$$

Q14 **The side of a cube is measured by using vernier callipers (10 divisions of the vernier scale coincide with 9 visions of the main scale, where 1 division of main scale is 1 mm). The main scale reads 10 mm & the first division of the vernier scale coincides with the main scale. Mass of the given cube is 2.736 g. Find out the density of the cube in appropriate significant figures.**

Sol: Least count of vernier calipers

$$= \frac{1\,\text{division of main scale}}{\text{Number of divisions in vernier scale}} = \frac{1}{10} = 0.1\text{mm}$$

The side of cube = 10 mm + 1 × 0.1 mm = 1.01 cm

Now, density $= \frac{\text{Mass}}{\text{Volume}} = \frac{2.736\,\text{g}}{(1.01)^3\,\text{cm}^3} = 2.66\,\text{g cm}^{-3}$.

MIND MAP

PHYSICAL QUANTITY

A physical quantity is expressed as

$n \; u = \text{Constant}$

n → Numerical Value; u → Unit

SYSTEM OF UNITS

	LENGTH	MASS	TIME
CGS	Centimetre	Gram	Second
FPS	Foot	Pound	Second
MKS	Metre	Kilogram	Second

Fundamental Units

QUANTITY	UNIT
Length	Meter
Mass	Kilogram
Time	Second
Temperature	Kelvin
Electric Current	Ampere
Luminous intensity	Candela
Amount of substance	Mole

Derived Units

Units which can be derived using fundamental units are derived units

Like

SI unit of Velocity is m/s

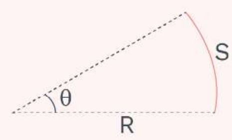
Supplementry Units

QUANTITY	UNIT
Plane Angle	Radian
Solid angle	Steradian

Plane Angle (θ)

$$\theta = \frac{\text{Arc Length}}{\text{Radius}}$$

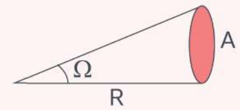
$$\theta = \frac{S}{R}$$



Solid Angle (Ω)

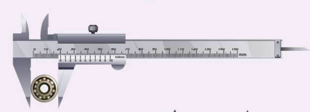
$$\Omega = \frac{\text{Area}}{(\text{Radius})^2}$$

$$\Omega = \frac{A}{R^2}$$



MEASUREMENT

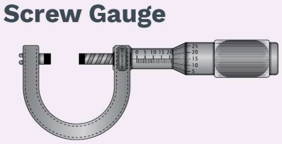
- **Errors Uncertainity in Measurement**
  - **Propagation of Errors**
    - **Addition or Subtraction**
      $Z = A \pm B$
      $\Delta Z = \Delta A + \Delta B$
    - **Multiplication or Division**
      $Z = \frac{A}{B}$ or $Z = AB$
      $\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$
  - **Representation of Errors**
    - Absolute Error (Δa)
    - Relative Error $\left(\frac{\Delta a}{a}\right)$
    - Percentage Error $\left(\frac{\Delta a}{a} \times 100\right)$
- **Measuring Instruments**
  - **Vernier Callipers**



    Least Count = $\left(\frac{b - a}{b}\right) M$

    b = Vernier Scale division equal to a main scale division and M is Main Scale division unit

  - **Screw Gauge**



    Least Count = $\frac{\text{Pitch}}{\text{(Total number of division on circular scale)}}$