Surface Tension & Viscosity

DISCLAIMER

Surface Tension & Viscosity

INTRODUCTION

Surface tension is basically a property of the liquid. The liquid surface acts like a stretched elastic membrane which has a natural tendency to contract and tends to have minimum possible area. This property of liquid is known as surface tension.

KEY POINTS

- Surface tension
- Intermolecular forces
- Cohesive force
- Adhesive force

INTERMOLECULAR FORCES

Forces of attraction or repulsion acting among the molecules are known as intermolecular forces. The nature of intermolecular force is electromagnetic.

There are two types of intermolecular attractive forces.

(a) COHESIVE FORCE:

The force of attraction between molecules of the same substance is called cohesive force. In case of the solids, the force of cohesion is very large and because of this solids have definite shape and size. On other hand, the force of cohesion in case of liquids is weaker than the solids. Hence liquids do not have definite shape but have definite volume. The force of cohesion is negligible in the case of gases. Because, gases have neither fixed shape nor volume.

Example:

- Two drops of liquid coalesce into one when brought in mutual contact because of cohesive force.
- It is difficult to separate two sticky plates of glass wetted with water because of large force has to be applied against cohesive force between the molecules of water.

It is very difficult to break the drop of mercury into small droplets because of large cohesive force between mercury molecules.

Definitions

Cohesive Force: The force of attraction between the molecules of the same substance is called cohesive force.

Definitions

Adhesive Force: The force of attraction between molecules of different substances is called adhesive force.

(b) ADHESIVE FORCE:

The force of attraction between the molecule of different substances is called the adhesive force Examples.

- Adhesive force enable us to write on the black board with the chalk.
- Adhesive force help us to write on the paper with ink.
- Large force of adhesion between the cement and bricks help us in construction work.
- Fevicol and gum are used in gluing two surfaces together because of adhesive force.
- Intermolecular forces are different from gravitational forces in the sense that the former does not obey inverse square law.
- The distance upto which these forces remains effective, is called molecular range. This distance is nearly 10^{-9} m. Within this limit the forces increases very rapidly as the distance decreases.
- Molecular range depends on the nature of the substance.

EXPLANATION OF SURFACE TENSION (MOLECULAR THEORY OF SURFACE TENSION)

Laplace explained the phenomenon of surface tension on the basis of intermolecular forces. According to him surface tension is a molecular phenomenon and its root cause is electromagnetic force. He explained the cause of surface tension as described below. If distance between two molecules is less than molecular range ($\approx 10^{-9}$ m) then they attract each other but if distance is more than this the attraction becomes negligible. If a sphere of radius C with a molecule at centre is drawn, then only those molecules which are enclosed within this sphere can attract or be attracted by the molecule at the centre of the sphere. This sphere is called sphere of molecular activity or sphere of influence. In order to understand the tension acting at the

Concept Reminder

It is very difficult to break a drop of mercury into small droplets because of large cohesive force between mercury molecules.

Concept Reminder

- Adhesive force helps us to write on the paper with ink.
- Large force of adhesion between cement and bricks helps us in construction work.

Rack your Brain

When there is no external force, the shape of a liquid drop is determined by

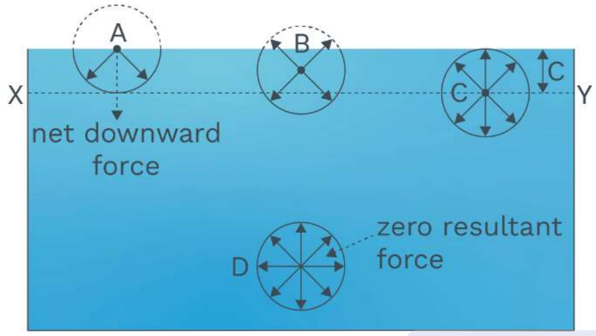
(1) Surface tension of the liquid

(2) Density of liquid

(3) Viscosity of liquid

(4) Temperature of air only

free surface of liquid, let us consider four liquid molecules A, B, C and D along with their spheres of molecular activity.



(a) According to figure sphere D is completely inside liquid. So molecule is attracted equally in all directions and hence resultant cohesive force is equal to zero.

(b) According to figure, sphere of molecule C is just below the liquid surface. So resultant cohesive force is equal to zero.

(c) The molecule B which is a little below the liquid surface is attracted downwards due to excess of molecules present below. Hence the resultant cohesive force is acting downwards.

(d) Molecule A is situated at the surface so that its sphere of molecular activity is half outside the liquid and half inside. Only lower portion has liquid molecules. Hence it experience a maximum downward force. Thus all the molecules situated between surface and a plane XY, distant 'C' below the surface, experience the resultant downward cohesive force.

When surface area of the liquid is increased molecules from the interior of the liquid rises from the surface. As these molecules reach close to the surface, work is done against the downward cohesive force. This work is stored in the molecules in form of

Concept Reminder

Some small things can float on a surface because of surface tension, even though they normally could not float. Some insects (e.g. water striders) can run on the surface of water because of this.

potential energy. Thus potential energy of the molecules lying close to the surface is greater than that of the molecules in the interior liquid. A system is in the 'stable equilibrium' when potential energy is minimum. Hence in order to have minimum potential energy the liquid surface tend to have minimum number of molecules. In other words any surface tends to contract to a minimum possible area. This tendency is exhibited as surface tension.

Effects of surface tension:

- Small liquid drops and soap bubbles are spherical.
- The hair of the brush remain separated from each other inside water, but when the brush is taken out, the hairs stick together.
- Floatation of needle on water.
- Formation of lead shots.
- Dirty clothes become clean in hot detergent solution in comparison to pure water at room temperature.

Concept Reminder

Because of surface tension rain water forms beads on the surface of a waxy surface such as a leaf.

Dependency of Surface Tension

On Cohesive Force: Those factors which increase the cohesive forces between molecule increases the surface tension and those which decrease cohesive force between the molecules hence decrease the surface tension.

On Impurities: If the impurity is completely soluble then on dissolving it in the liquid, its surface tension increases. e.g., on dissolving ionic salts in small quantities in a liquid, its surface tension increases. If the impurity is partially soluble in the liquid then its surface tension decreases because adhesive force between the insoluble impurity molecules and liquid molecules decreases cohesive force effectively, e.g.

On mixing detergent in the water its surface tension decreases.

Concept Reminder

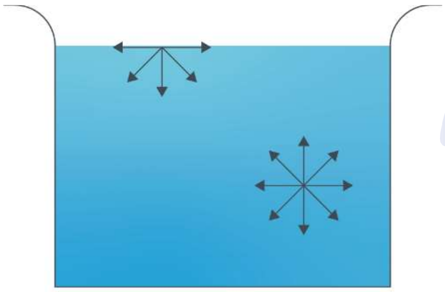
Surface tension depends upon-

(i) Cohesive force
(ii) Impurities
(iii) Temperature
(iv) Contamination

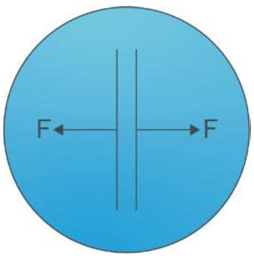
Temperature: On increasing the temperature surface tension decreases. At critical temperature and boiling point it becomes zero.
Note: Surface tension of water is maximum at 4°C.
On Contamination: Dust particles or lubricating materials on the liquid surface decreases its surface tension.

SURFACE TENSION

The property of the liquid at rest due to which its free surface tries to have minimum surface area and behaves as if it were under tension somewhat like the stretched elastic membrane is called surface tension.
The molecule of the liquid exert attractive forces on the each other. There is zero net force on the molecule inside the volume of the liquid.



But a surface molecules is drawn into the volume. Thus, the liquid tends to minimize its surface area, just as a stretched membrane does.



Surface tension of the liquid is measured by the force acting per unit length on either side of an

Definitions

The property of a liquid at rest due to which its free surface tries to have minimum surface area and behaves as if it were under tension somewhat like a stretched elastic membrane is called surface tension.

imaginary line drawn on the free surface of the liquid, direction of this force being perpendicular to the line and tangential to the free surface of the liquid. So if F is the force acting on one side of imaginary line of length L. then

$$T = \left(\frac{F}{L}\right)$$

Regarding surface tension it is worth noting that:

- It depends only on the nature of liquid and is independent of the area of surface or length of line considered.
- It is a scalar as it has a unique direction which is not to be specified.
- It has dimension $[MT^{-2}]$ and SI units N/m while CGS unit dyne/cm, so that one MKS unit of surface tension = 10^3 dyne/cm.
- Surface tension of a liquid decreases with rise in temperature.
- The surface tension of a liquid is very sensitive to impurities on the surface (called contami-nation) and decreases with contamination of surface.
- In case of the soluble impurities surface tension may increase or decrease depending on the nature of the impurity. Usually, highly soluble salt such as sodium chloride increases the surface tension while sparingly soluble salt such as soap decreases surface tension.

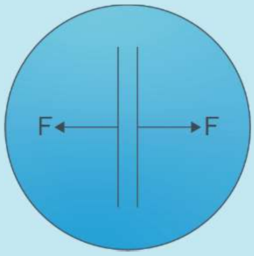
Ex. A stick of length 5 cm is floating on the surface of the water. If one side of it has the surface tension of 70 dyne/cm and on the other side by keeping a piece of camphor the surface tension is reduced to 50 dyne/cm, then find out the resultant force (in dyne) acting on the stick.

Sol. $F_{net} = F_1 - F_2$

$= (T_1 - T_2)l = (70 - 50)5$

$= 100$ dyne

Concept Reminder



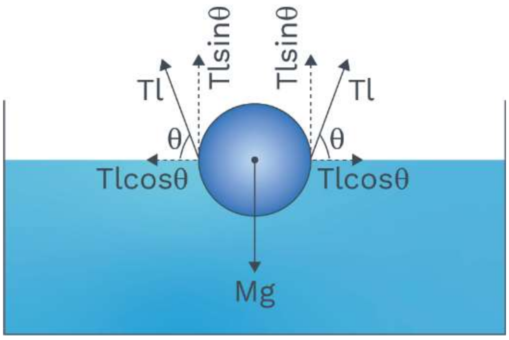
Surface tension of a liquid is measured by the force acting per unit length on either side of an imaginary line drawn on the free surface of liquid-

$$T = \left(\frac{F}{L}\right)$$

Rack your Brain

The wettability of a surface by a liquid depends primarily on:
(1) Surface tension
(2) Density
(3) Angle of contact between the surface and the liquid
(4) Viscosity

Important examples based on Surface Tension:



When a needle floats on the liquid surface then

$2Tl \sin \theta = Mg$

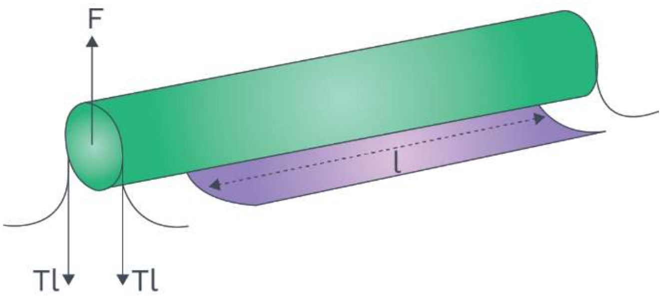
Special Case: Maximum mass of needle which can float on the liquid surface is when.

$\theta = 90°$

$\therefore \quad 2Tl = M_{max}g$

$\therefore \quad \boxed{M_{max} = \frac{2T\ell}{g}}$

- If the needle is lifted from the liquid surface then required excess force will be $F_{excess} = 2Tl$
  Minimum force required
  $F_{min} = Mg + 2Tl$

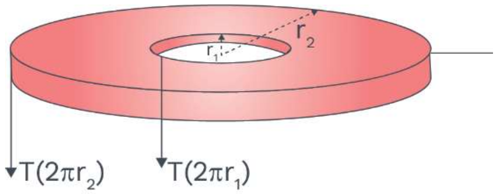


- Required excess force for a circular thick ring (or annular ring) having internal and external radii r_1 and r_2 is dipped in and taken out from liquid.
  $F_{excess} = F_1 + F_2 = T(2\pi r_1) + T(2\pi r_2)$
  $= 2\pi T(r_1 + r_2)$.

Concept Reminder

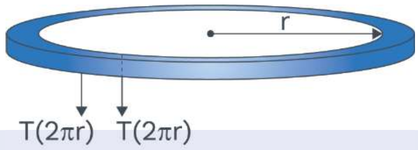
Maximum mass of needle which can float on liquid surface is,

$M_{max} = \frac{2T\ell}{g}$



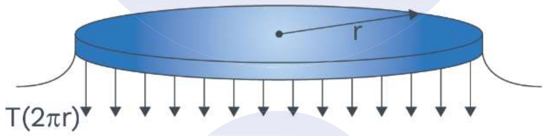
- Required excess force for a circular ring ($r_1 = r_2 = r$)

  $F_{excess} = 2\pi T(r + r) = 4\pi rT.$



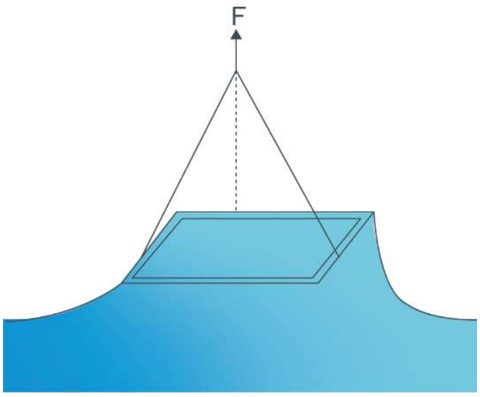
- Required excess force for a circular disc ($r_1 = 0$, $r_2 = r$)

  $F_{excess} = 2\pi rT$



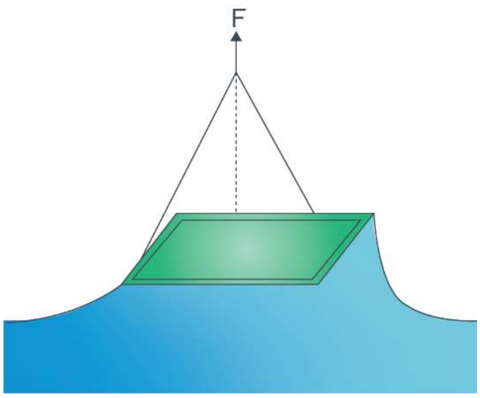
- Required excess force for a square frame of side l

  $F_{excess} = 8T \times l$



- Required excess force for a square lamina of side l

  $F_{excess} = 4\ T \times l$



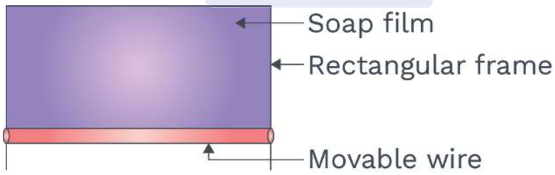
- Required excess force for a rectangular frame of length l and width b

  $F_{excess} = 4T(l + b)$

- Required excess force for a rectangular lamina of length l and width b

  $F_{excess} = 2T(l + b)$

Ex. If a soap film is formed on the frame then find the radius of the wire to maintain equilibrium. (Surface tension of soap solution = 5×10^{-3} N/m and Density of wire $= \frac{5}{\pi} kg/m^3$, g = 10 m/s^2)



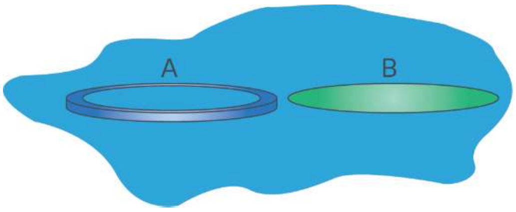
Sol. At equilibrium

$Mg = 2Tl$

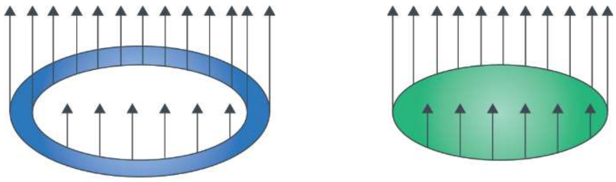
$\rho(\pi r^2 l)g = 2Tl$

$r = \sqrt{\frac{2T}{\pi\rho g}} = 1.4 \times 10^{-2} m = 1.4 \text{ cm}$

Ex. A rigid ring A and a thin rigid disc B both made of same material, when gently placed on water, just manage to float due to surface tension as shown in the figure. Both the ring and the disc have same radius. What can you conclude about their masses?



Sol.



For ring

$2T(2\pi R) = M_R g$ ...(i)

For disc

$T(2\pi R) = M_D g$ ...(ii)

From eq. (i) & (ii)

$\boxed{M_R = 2M_D}$

Ring has double the surface length in contact with liquid than that of disc. So ring has double the mass compared to the mass of disc.

Ex. Find the maximum weight of the needle which can float on the water having surface tension 0.073 N/m length of needle is 1 cm.

Sol. $W_{max} = 2Tl = 2\ (0.073)\ (1 \times 10^{-2})$

$= 1.46 \times 10^{-3}$ N

SURFACE ENERGY

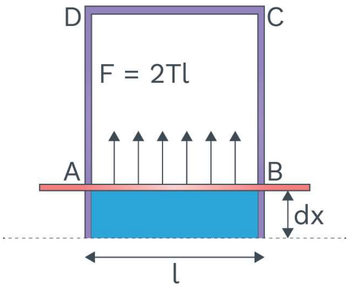
According to the molecular theory of surface tension the molecules on the surface have certain additional energy due to their position. This additional energy of the surface is called ' Surface energy".

Let a liquid film be formed on a wire frame and a straight wire of length l can slide on this wire frame as shown in figure. The film has two surfaces and both the surfaces are in contact with the sliding wire and hence, exert force of

Definitions

According to molecular theory of surface tension the molecules on the surface have certain additional energy due to their position. This additional energy of the surface is called ' Surface energy".

surface tension on it. If T be the surface tension of the solution, each surface will pull the wire parallel to itself with a force Tl. Thus, the net force on the wire due to both the surfaces is 2Tl.



Apply an external force 'F' equal and opposite to it to keep the wire in equilibrium.
Thus, F = 2Tl.
Suppose, the wire is moved through small distance 'dx', then work done by the force is,

$$dW = F\,dx = (2Tl)dx$$

But (2l) (dx) is the total increase in area of both the surfaces of the film. Let it be dA, then

$$dW = T\,dA$$

$\Rightarrow$ $T = dW/dA$

Thus, the surface tension 'T' can also be defined as the work done in increasing the surface area by unity.
Further, since there is no change in kinetic energy, work done by the external force is stored as potential energy of the new surface.

$$T = \frac{dU}{dA} \quad [\text{as } dW = dU]$$

Special Cases:

- Work done (surface energy) in formation of a drop of radius r = Work done against surface tension
  W = Surface tension (T) × change in area
  $\Delta A = T \times 4\pi r^2 = 4\pi r^2 T$

Work done (surface energy) in blowing of a soap bubble of radius r:
$W = T \times \Delta A$

Concept Reminder

The surface tension T can also be defined as the work done in increasing the surface area by unity.

KEY POINTS

- Surface energy

Concept Reminder

Work done in blowing of small bubble of radius r_1 to large bubble of radius r_2.

$$W = 8\pi T(r_2^2 - r_1^2)$$

or $W = T \times 2 \times 4\pi r^2$

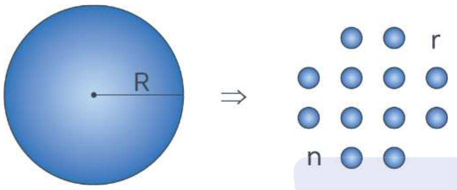
$= 8\pi r^2 T$ [∵ soap bubble has two surfaces]

- Work done in blowing of small bubble of radius r_1 to large bubble of radius r_2.

$W = T \times \Delta A$

or $T \times 2 \times (4\pi r_2^2 - 4\pi r_1^2) = 8\pi T(r_2^2 - r_1^2)$

Splitting of big drop into smaller droplets.



If a big a drop is split into smaller droplets then in this process volume of liquid always remain conserved. Let the big drop have a radius R. It is splitted into n smaller drops of radius r then by conservation of volume

(i) $\frac{4}{3}\pi R^3 = n\left[\frac{4}{3}\pi r^3\right]$

$\Rightarrow \quad n = \left[\frac{R}{r}\right]^3 \Rightarrow r = \frac{R}{n^{1/3}}$

(ii) Initial surface area = $4\pi R^2$

and final surface area = $n(4\pi r^2)$

Therefore initial surface energy

$E_i = 4\pi R^2 T$

and final surface energy

$E_f = n(4\pi r^2 T)$

Change in area

$\Delta A = n4\pi r^2 - 4\pi R^2 = 4\pi (nr^2 - R^2)$.

Therefore the amount of surface energy absorbed i.e.

$\Delta E = E_f - E_i = 4\pi T(nr^2 - R^2)$

∴ Magnitude of work done against surface tension i.e.

$W = 4\pi(nr^2 - R^2)T$

$W = 4\pi T(nr^2 - R^2) = 4\pi R^2 T(n^{1/3} - 1)$

Rack your Brain

The work done in blowing a soap bubble of 10 cm radius is (surface tension of soap solution is 0.03 N/m).

(1) 37.68×10^{-4} J
(2) 75.36×10^{-4} J
(3) 126.82×10^{-4} J
(4) 75.36×10^{-3} J

$$= 4\pi R^2 T\left[\frac{R}{r} - 1\right]$$

$$\Rightarrow \quad W = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right]$$

In this process temperature of the system decreases as energy gets absorbed during the increase surface area.

$$W = Jms\,\Delta\theta = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right]$$

$$\Rightarrow \quad \Delta\theta = \frac{4\pi R^3 T}{\frac{4}{3}\pi R^3 J\rho s}\left[\frac{1}{r} - \frac{1}{R}\right] = \frac{3T}{J\rho s}\left[\frac{1}{r} - \frac{1}{R}\right]$$

Where ρ = liquid density, s = specific heat of liquid.

Thus, in this process area increases, surface energy increases, internal energy decreases, temperature decreases, and energy is absorbed.

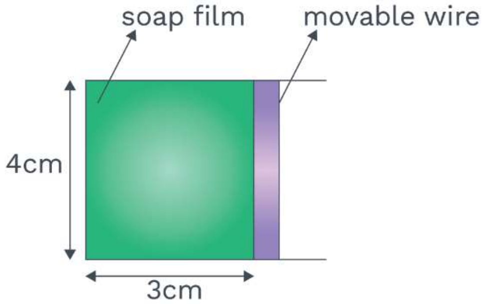
Concept Reminder

Decrease in temperature when a large drop of radius R splits into n numbers of small drops of radius r.

$$\Delta\theta = \frac{3T}{J\rho S}\left(\frac{1}{r} - \frac{1}{R}\right)$$

Ex. If work done required to displace a movable wire by 2 cm is 6 × 10⁻⁴ J. Then find out surface tension of the soap solution.

Sol. W = T(2ΔA)



$$\Rightarrow \quad T = \frac{W}{2\Delta A} = \frac{6 \times 10^{-4}}{2[20 \times 10^{-4} - 12 \times 10^{-4}]}$$

$$= \frac{6}{2 \times 8} = \frac{3}{8} = 0.375 \text{ N/m}$$

Ex. Calculate work done required to increase the area of a rectangular film of liquid from (4 cm × 5 cm) to (5 cm × 6 cm). (surface tension of the liquid is 0.3 N/m)

Sol. $W = T(2\Delta A)$

$$= 0.3 \times 2 \times (30 \times 10^{-4} - 20 \times 10^{-4})$$

$$= 6 \times 10^{-4}\ J$$

Ex. If a big drop of radius R spitted into 27 identical small droplets. Then find out work done in this process.

Sol. $W = 4\pi R^2 T\ [n^{1/3} - 1]$

$$W = 4\pi R^2 T\ [(27)^{1/3} - 1] = 8\pi R^2 T$$

Ex. If a drop of mercury having radius 1 cm is broken into 10^6 identical small droplets. If surface of mercury is $35 \times 10^{-3}\ N/m^2$, then find out work done in this process.

Sol. $W = 4\pi R^2 T\ [n^{1/3} - 1]$

$$= 4 \times \frac{22}{7} \times (10^{-4} \times 35 \times 10^{-3})(100 - 1)$$

$$= 4.35 \times 10^{-3}\ J$$

Ex. A big drop of radius 1 cm is converted into 1000 small identical droplets. If surface tension of liquid is 70 dyne/ cm. Then find out:

(1) Radius of small drop.

(2) Work done required.

(3) Ratio of final surface energy to initial surface energy.

Sol. (1) $\frac{R}{r} = n^{1/3} \Rightarrow \frac{1\text{cm}}{r} = 10 \Rightarrow r = 0.1\text{ cm}$

(2) $\text{W.D.} = 4\pi R^2 T(n^{1/3} - 1)$

$$= 4 \times \frac{22}{7}(1)^2 \times 70 \times (10 - 1) = 7920\text{ erg}$$

(3) $\frac{SE_f}{SE_i} = \frac{n \times 4\pi r^2 T}{4\pi R^2 T} = n^{1/3} = 10$

Ex. Find out the percentage loss in surface energy when 10^6 small identical droplets are combined to form a big drop.

Sol. Percentage loss

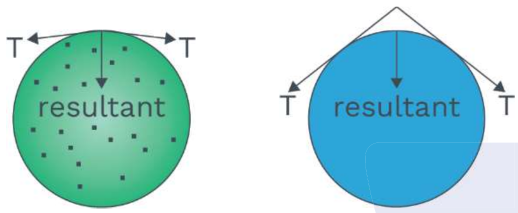
$$= \frac{SE_i - SE_f}{SE_i} \times 100 = \left(1 - \frac{SE_f}{SE_i}\right) \times 100$$

$$= \left(1 - \frac{4\pi R^2 T}{n \times 4\pi r^2 T}\right) \times 100 = \left(1 - \frac{1}{n^{1/3}}\right) \times 100 = \left(1 - \frac{1}{100}\right) \times 100 = 99\%$$

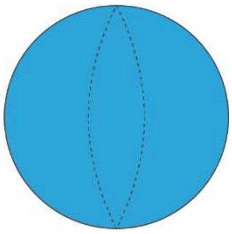
Excess pressure inside a curved liquid surface

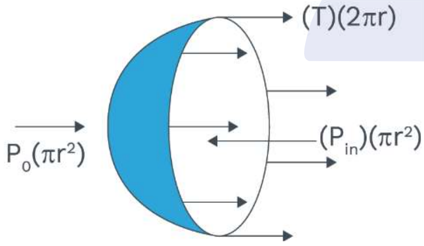
The pressure on the concave side of curved liquid surface is greater than that on the convex side. Therefore a pressure difference exists across two sides of a curved surface. This pressure difference is called excess pressure.

EXCESS PRESSURE INSIDE A LIQUID DROP



So the pressure of water inside will be greater than the outside atmospheric pressure. This extra pressure is called excess pressure. To find excess pressure, make free body diagram of the half part. The forces on this hemisphere are:





(i) Pushing force on the left half liquid due to right half liquid will be $(P_{in})(\pi r^2)$.

(ii) Pushing force due to atmospheric pressure will be (P_0) × (facing area) = $P_0(\pi r^2)$.

(iii) Surface tension force on left half surface due to right half surface will be $(T)(2\pi r)$.

As drop is in equilibrium:

$$(P_{in})(\pi r^2) = P_0(\pi r^2) + (T)(2\pi r)$$

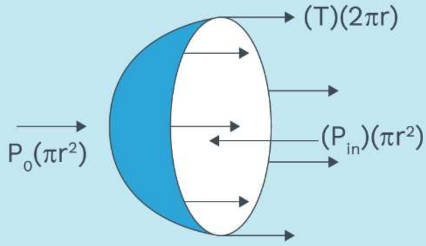
$$\Rightarrow \quad P_{in} = P_0 + \frac{2T}{r}$$

Definitions

The pressure on the concave side of curved liquid surface is greater than that on the convex side. Therefore a pressure difference exists across two sides of a curved surface. This pressure difference is called excess pressure.

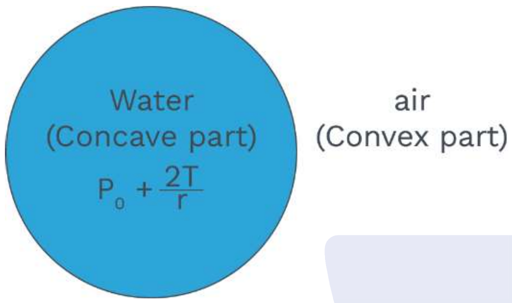
Concept Reminder

Excess pressure inside a drop-

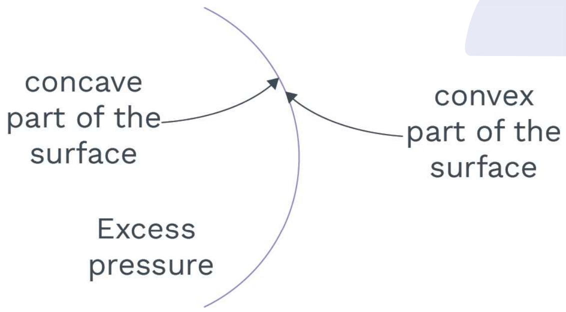


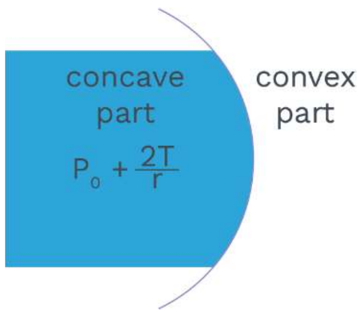
$$\Delta P = \frac{2T}{r}$$

Here, $\frac{2T}{r}$ is called excess pressure. So pressure inside the drop will be greater than pressure outside the drop by $\frac{2T}{r}$.



Generally, we can say that pressure at concave part will be greater than pressure at convex part by $\frac{2T}{r}$ where r is radius of curvature of the surface between them.





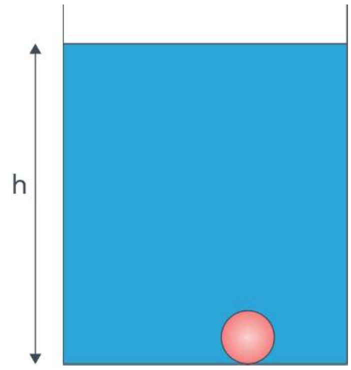
Concept Reminder

Generally, we can say that pressure at concave part will be greater than pressure at convex part by $\frac{2T}{r}$ where r is radius of curvature of the surface between them.

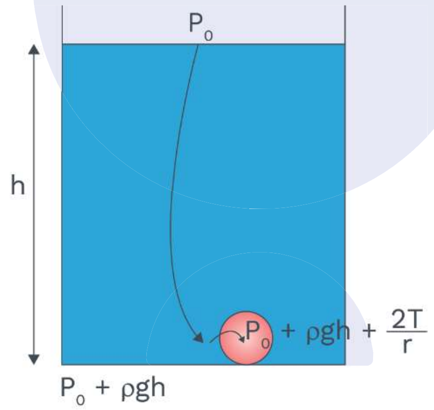
Rack your Brain

If the excess pressure inside a soap bubble is balanced by an oil column of height 2 mm, then find the surface tension of soap solution.
(r = 1 cm, density of oil = 0.8 g/cm^3)

Ex. A small air bubble of radius r is at depth ‘h’. Find the pressure inside the bubble.



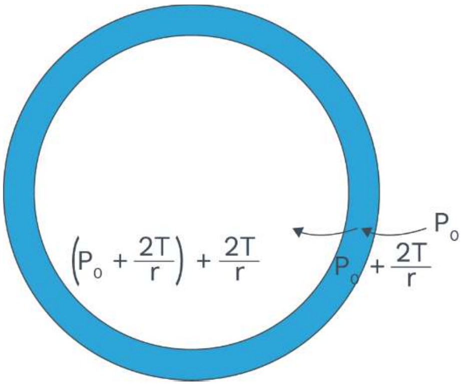
Sol. Air inside the bubble is on concave side.



∴ Pressure inside bubble

$$= P_0 + \rho gh + \frac{2T}{r} \quad (P_0 = \text{atmospheric pressure})$$

Excess pressure inside a liquid bubble kept in air:



So pressure inside the liquid bubble = $P_0 + \frac{4T}{r}$

So excess pressure inside the liquid bubble $= \frac{4T}{r}$

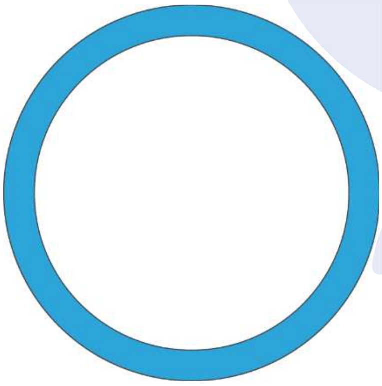
Alternative method :

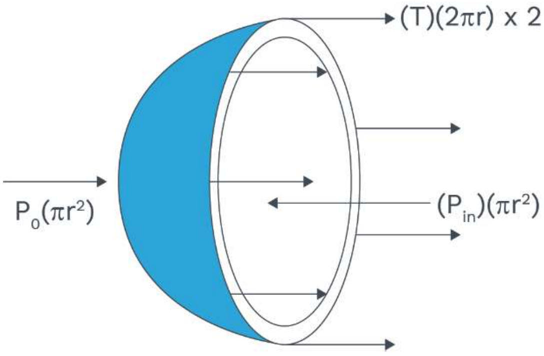
Draw f.b.d. of half part of the bubble. The force on this hemisphere are:

(i) Pushing force on the left half liquid due to right half liquid will be $(P_{in})(\pi r^2)$.

(ii) Pushing force due to atmospheric pressure will be (P_0) × (facing area) = $P_0(\pi r^2)$.

(iii) Surface tension force on both inner and outer surface will be $(T)(2\pi r) \times 2$.





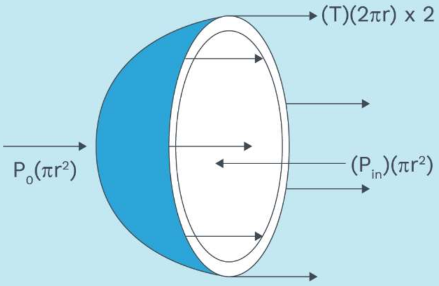
Bubble is in equilibrium:

$$(P_{in})(\pi r^2) = P_0(\pi r^2) + (T)(2\pi r) \times 2$$

$$\Rightarrow \quad P_{in} = P_0 + \frac{4T}{r}$$

Concept Reminder

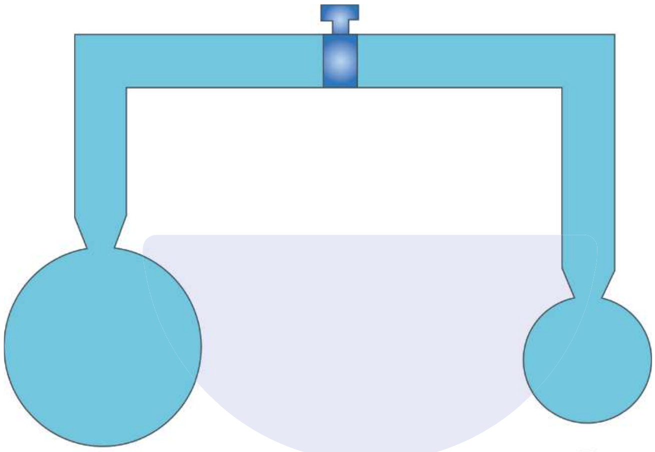
Excess pressure inside a bubble



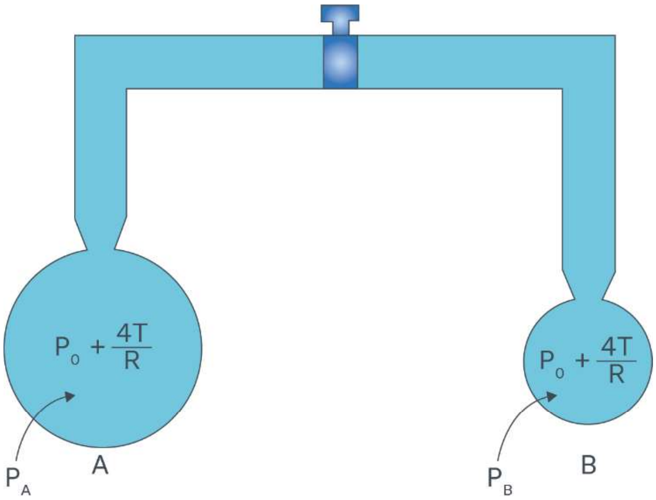
$$\Delta P = \frac{4T}{r}$$

So, pressure excess inside a liquid bubble = $\frac{4T}{r}$

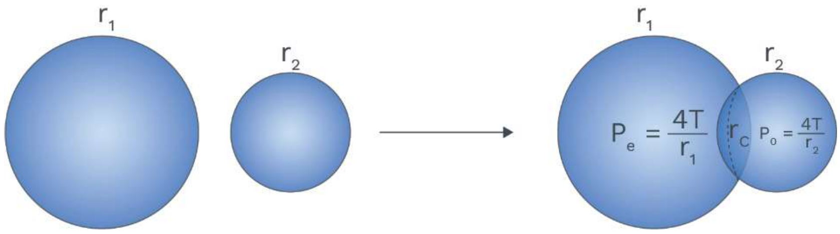
Ex. Two soap bubbles are formed on the ends of the tube closed with or value as shown. If valve is opened, in which direction will the air flow?



Sol. Radius of curvature of smaller bubble 'B' is less so pressure inside the smaller bubble will be more $\left(P_0 + \frac{4T}{r}\right)$. Air will flow from high pressure to the low pressure, so it will flow from smaller bubble to bigger bubble. The small bubble will become smaller and the big bubble will grow bigger.



Ex. Two soap bubble of radius r_1 and r_2 combine. Find the radius of curvature of the common surface separating them.



Sol. The excess pressure across the common surface directed towards the centre of smaller bubble

$$\frac{4T}{r_2} - \frac{4T}{r_1} = \frac{4T}{r_c}$$

r_c = radius of curvature of resultant to surface

$$\Rightarrow \quad \frac{1}{r_c} = \frac{1}{r_2} - \frac{1}{r_1} \quad \Rightarrow r_c = \frac{r_1 r_2}{r_1 - r_2}$$

Pressure inside a charged bubble:

Consider a charged bubble of radius R, surface tension 'T' and surface charge density 'σ'.The total surface tension force for the each surface (inner and outer) is T (2πR) for a total of 2T (2πR). Force due to inside pressure (P_{in}) is $P_{in}\ \pi R^2$ and due to outside pressure (P_0) is $P_0\pi R^2$.

$$2T(2\pi R) + P_0\pi R^2 = P_{in}\pi R^2 + \frac{\sigma^2}{2\varepsilon_0}\pi R^2$$

$$P_{in} = P_0 + \frac{4T}{R} - \frac{\sigma^2}{2\varepsilon_0}$$

Ex. A soap bubble of radius 'r' and surface tension constant 'T' is given a charge, so that it develops a surface charge density σ. Due to charge, the radius of the soap bubble expands isothermally to double its radius then find 'σ'. (atmospheric pressure = P_0)

Sol. Initial pressure inside the bubble

$$P_i = P_0 + \frac{4T}{r}$$

Now a uniform surface charge in given to the bubble.

The surface tension imparts a pulling force, which increases pressure inside the bubble $\left(\text{by } \frac{4T}{r}\right)$.

The charges given to the surface will repel each other. So due to the given charge, pressure inside the bubble will decrease (pressure on a charged surface $\frac{\sigma^2}{2\varepsilon_0}$).

So, final pressure inside the bubble

$$P_f = P_0 + \frac{4T}{r_f} - \frac{\sigma^2}{2\varepsilon_0}$$

As the temperature of the gas inside the bubble is kept constant so,

$$P_iV_i = P_fV_f$$

$$\left(P_0 + \frac{4T}{r}\right)\left(\frac{4}{3}\pi r^3\right) = \left(P_0 + \frac{4T}{r_f} - \frac{\sigma^2}{2\varepsilon_0}\right)\left(\frac{4}{3}\pi r_f^3\right)$$

Given, $r_f = 2r$

So, we get $\sigma = \sqrt{\left(7P_0 + \frac{12T}{r}\right)\frac{\varepsilon_0}{4}}$

Ex. A minute spherical air bubble is rising slowly through a column of the mercury contained in a deep jar. If the radius of bubble at a depth of 100 cm is 0.1 mm, calculate its depth where its radius becomes 0.126 mm, given that the surface tension of the mercury is 567 dyne/cm. Consider that the atmospheric pressure is 76 cm of mercury.

Sol. The total pressure inside bubble at depth h_1 is (P_0 is atmospheric pressure)

$$= (P_0 + h_1\rho g) + \frac{2T}{r_1} = P_1$$

and the total pressure inside the bubble at depth h_2 is

$$= (P_0 + h_2\rho g) + \frac{2T}{r_2} = P_2$$

Now, according to Boyle's Law (Bubble rising slowly, the process in isothermal)

$$P_1V_1 = P_2V_2 \text{ where } V_1 = \frac{4}{3}\pi r_1^3$$

and $V_2 = \frac{4}{3}\pi r_2^3$

Hence, we get

$$\left[(P_0 + h_1\rho g) + \frac{2T}{r_1}\right]\frac{4}{3}\pi r_1^3 = \left[(P_0 + h_2\rho g) + \frac{2T}{r_2}\right]\frac{4}{3}\pi r_2^3$$

or, $$\left[(P_0 + h_1\rho g) + \frac{2T}{r_1}\right]r_1^3 = \left[(P_0 + h_2\rho g) + \frac{2T}{r_2}\right]r_2^3$$

Given that:

$h_1 = 100$ cm, $r_1 = 0.1$ mm $= 0.01$ cm,

$r_2 = 0.126$ mm $= 0.0126$ cm,

$T = 567$ dyne/cm,

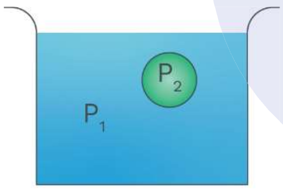
$P = 76$ cm of mercury.

Substituting all the values, we get

$h_2 = 9.48$ cm.

Note:

(1) If we have an air bubble surrounded by a liquid, a single surface is formed. There is air on the concave side and liquid on the convex side. The pressure in the concave side (that is in the air) is greater than the pressure in the convex side (that is in the liquid) by an amount $\frac{2T}{R}$.



Concept Reminder

Separation of oil and water is caused by a tension in the surface between dissimilar liquids. This type of surface tension is called "interface tension".

$\therefore \quad P_2 - P_1 = \frac{2T}{R}$

The above expression has been written by assuming P_1 to be constant from all sides of the bubble. For small size bubbles this can be assumed.

(2) From the above discussion, we can make a general statement. The pressure on the concave side of a spherical liquid surface is greater than the convex side by $\frac{2T}{R}$.

(3) For any curved surface excess pressure on the concave side $= T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ where R_1 & R_2 are radius of curvature of the surface in two perpendicular direction of instead of liquid surface, liquid film is given then above expression will be

$$P = 2T\left(\frac{1}{R_1} + \frac{1}{R_2}\right).$$

For spherical curved surface R_1, R_2.

Ex. What should be the pressure inside a small air bubble of 0.1 mm radius situated just below the water surface. Surface tension of water $= 7.2 \times 10^{-2}$ N/m and atmospheric pressure $= 1.013 \times 10^5$ N/m^2.

Sol. Surface tension of water

$T = 7.2 \times 10^{-2}$ N/ m

Radius of air bubble R = 0.1 mm = 10^{-4} m

The excess pressure inside air bubble is given by,

$$P_2 - P_1 = \frac{2T}{R}$$

∴ Pressure inside the air bubble,

$$P_2 = P_1 + \frac{2T}{R}$$

Substituting the values, we have

$$P_r = (1.013 \times 10^5) + \frac{2 \times 7.2 \times 10^{-2}}{10^{-4}}$$

$= 1.027 \times 10^5$ N/m^2.

Ex. Prove that if two bubbles of radii r_1 and r_2 coalesce isothermally in vacuum then the radius of the new bubble will be $r = \sqrt{r_1^2 + r_2^2}$.

Sol. When two bubbles coalesce then total number of molecules of air will remain same and temperature will also remain constant

so, $n_1 + n_2 = n$

$\Rightarrow$ $P_1V_1 + P_2V_2 = PV$

$$\Rightarrow \frac{4T}{r_1}\left(\frac{4}{3}\pi r_1^3\right) + \frac{4T}{r_2}\left(\frac{4}{3}\pi r_2^3\right) = \frac{4T}{r}\left(\frac{4}{3}\pi r^3\right)$$

$$\Rightarrow r = \sqrt{r_1^2 + r_2^2}$$

Rack your Brain

A certain number of spherical drops of a liquid of radius r coalesce to form a single drop of radius R and volume V. If T is the surface tension of the liquid, then

(1) Energy = $4VT\left(\frac{1}{r} - \frac{1}{R}\right)$ is released

(2) Energy = $3VT\left(\frac{1}{r} + \frac{1}{R}\right)$ is absorbed

(3) Energy = $3VT\left(\frac{1}{r} - \frac{1}{R}\right)$ is released

(4) Energy is neither released nor absorbed

Ex. If the excess pressure inside a liquid drop is 3 times of excess pressure of another liquid drop made from same liquid. Find ratio of their volumes.

Sol. $\left(\frac{2T}{r_1}\right) = 3\left(\frac{2T}{r_2}\right)$

$$\frac{r_1}{r_2} = \frac{1}{3}$$

$$\therefore \frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = \frac{1}{27}$$

Ex. If the excess pressure inside a soap bubble is 8 mm of water column and radius is 0.1 mm. Find the surface tension of bubble.

Sol. $\frac{4T}{r} = h\rho_w g$

$$\frac{4T}{0.1\times10^{-3}} = (8\times10^{-3})(10^3)(10)$$

$T = 2 \times 10^{-3}$ N/m.

Ex. If the excess pressure inside two soap bubbles are 1.02 atm and 1.03 atm respectively. Find ratio of their volumes.

Sol. $\frac{(P_{ex})_1}{(P_{ex})_2} = \frac{(P_{in} - P_{out})_1}{(P_{in} - P_{out})_2} = \frac{1.02-1}{1.03-1} = \frac{2}{3}$

$$\frac{\frac{4T}{r_1}}{\frac{4T}{r_2}} = \frac{2}{3} \Rightarrow \frac{r_1}{r_2} = \frac{3}{2}$$

$$\therefore \quad \frac{V_1}{V_2} = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$$

Ex. The excess pressure inside an air bubble of radius r just below the surface of water is P_1. The excess pressure inside a drop of the same radius just outside the surface is p_2. If T is surface tension, then find the relation between p_1 and p_2.

Sol. Excess pressure in air bubble just below the water surface.

$$P_1 = \frac{2T}{r}$$

Excess pressure inside a drop, $P_2 = \frac{2T}{r}$

So, $P_1 = P_2$

Ex. The excess pressure because of the surface tension inside a spherical drop is 9 unit. If twenty seven such drops combine then find the excess pressure due to surface tension inside the larger drop.

Sol. $R = (n)^{1/3} \cdot r = 3r$

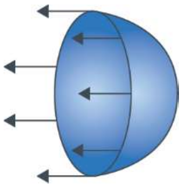
$$P_{ex} \propto \frac{1}{r}$$

$$\Rightarrow \quad \frac{(P_{ex})_{big}}{(P_{ex})_{small}} = \frac{r}{R} = \frac{1}{3}$$

$P_{ex} = 3$ unit

Ex. If a section of liquid drop (of radius R) through its centre is considered then find the force on one half due to surface tension (T).

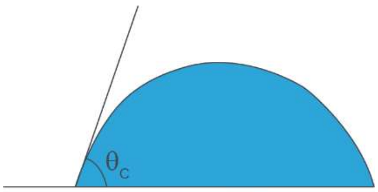
Sol. Surface tension = T



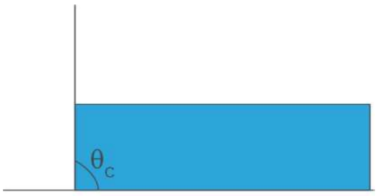
Force due to surface tension will be = $T(2\pi r)$ = $2\pi rT$.

Angle of contact (θ_c):

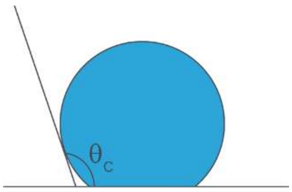
The angle enclosed between tangent plane at liquid surface and the tangent plane at the solid surface at the point of contact inside the liquid is known as the angle of contact. The angle of contact depends on the nature of the solid and liquid in contact.



$\theta_c < 90^o$



$\theta_c = 90^o$



$\theta_c > 90^o$

Definitions

The angle enclosed between the tangent plane at the liquid surface and the tangent plane at the solid surface at the point of contact inside the liquid is defined as the angle of contact.

Some important angle of contact-

(i) Pure water and glass → 90°

(ii) Ordinary water and glass → 8° to 18°

(iii) Water and silver → 90°

(iv) Mercury and glass → 138°

Note:

(i) Effect of Temperature on angle of contact:
On increasing temperature surface tension decreases, thus cos θ_c increases

$\left[\because \cos\theta_c \propto \frac{1}{T}\right]$ and θ_c decreases.

So, on increasing temperature, θ_c decreases.

(ii) Effect of Impurities on angle of contact:

(a) Soluble impurities increase surface tension, so cos θ_c decreases and angle of contact θ_c increases.

(b) Partially soluble impurities decrease the surface tension, so angle of contact θ_c decreases.

(iii) Effect of Water Proofing Agent:
Angle of contact increases due to the presence of water proofing agent. It changes from acute to obtuse angle.

Shape of Liquid Surface (Shape of meniscus)

When a liquid is brought in contact with a solid surface, the surface of the liquid becomes curved near the place of contact. The shape of the surface (concave or convex) depends upon the relative magnitude of cohesive force between the liquid molecules and the adhesive force between the molecules of the liquid and the solid.

The free surface of a liquid which is near the walls of a vessel and which is curved because of surface tension is known as meniscus. The cohesive force acts at an angle of 45° from liquid surface whereas the adhesive force acts at right angles to the solid surface.

Concept Reminder

Some important angle of contact-

(i) Pure water and glass → 90°

(ii) Ordinary water and glass → 8° to 18°

(iii) Water and silver → 90°

(iv) Mercury and glass → 138°

KEY POINTS

- Excess pressure
- Angle of contact
- Meniscus

Rack your Brain

With an increase in temperature, surface tension of liquid: (except molten copper and cadmium)

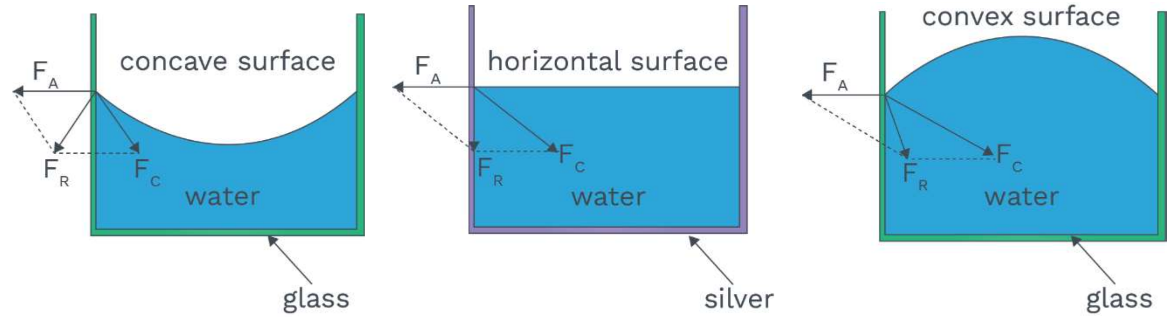
(1) Increases

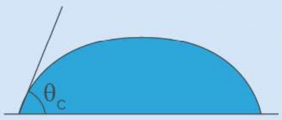
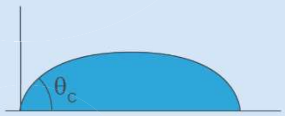
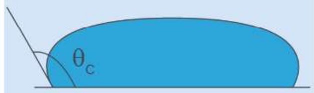
(2) Remain same

(3) Decreases

(4) First decreases then increases

The relation between the shape of liquid surface, cohesive/adhesive forces, angle of contact, etc are summarized in the table below:



Shape of meniscus	Concave	Plane	Convex
Angle of contact	$\theta_C < 90^o$ (Acute angle)	$\theta_C = 90^o$ (Right angle)	$\theta_C > 90^o$ (Obtuse angle)
Shape of liquid drop			
Level of liquid	Liquid rises up	Liquid neither rises nor falls	Liquid falls
Wetting property	Liquid wets the solid surface	Liquid does not wet the solid surface	Liquid does not wet the solid surface
Example	Glass – Water	Silver – Water	Glass – Mercury

Capillary Tube and Capillarity:

A glass tube with a fine bore and open at both ends is known as a capillary tube. The property by virtue of which a liquid rises or gets depressed in a capillary tube is known as capillarity. Rise or fall of liquid in tubes of narrow bore (capillary tube) is called capillary action.

Example:

(i) Kerosene oil in lanterns rise upward due to capillarity.

(ii) Working of fountain pen's nib split due to capillarity.

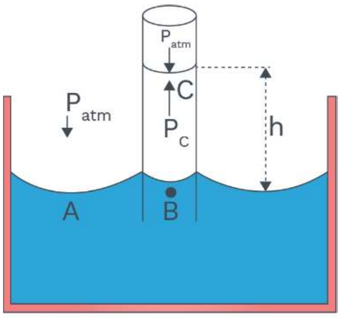
Definitions

Capillary tube: A glass tube with a fine bore and open at both ends is known as a capillary tube.

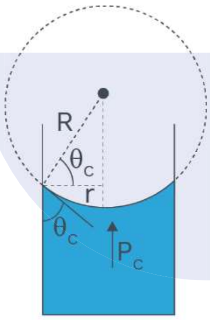
Calculation of Capillary Rise

(i) Pressure Balance Method:

When a capillary tube is first dipped in a liquid as shown in the figure, the liquid climbs up the walls curving the surface. Let the radius of the meniscus be 'R' and the radius of the capillary tube be r. Angle of contact is θ_C, surface tension is T, density of liquid is r and the liquid rises to a height h.



R = Radius of the meniscus



$R = \dfrac{r}{\cos\theta}$

Now let us consider two points A and B at the same horizontal level as shown. By Pascal's law

$P_A = P_B \Rightarrow P_A = P_C + \rho gh$

Now, point C is on the curved meniscus which has P_{atm} and P_C as the pressures on its concave and convex sides respectively.

$$\therefore \quad P_{atm} = \left(P_{atm} - \frac{2T}{R}\right) + h\rho g$$

$$\Rightarrow \quad h = \frac{2T}{R\rho g} = \frac{2T\cos\theta_c}{r\rho g}$$

(ii) Force Balance Method:

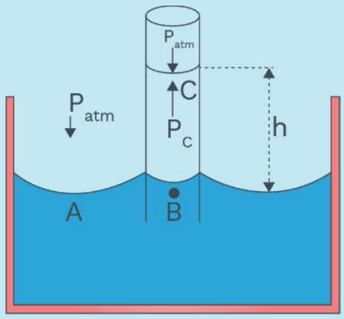
The liquid continues to rise in capillary tube until the weight of the liquid column becomes equal to force due to surface tension. On liquid force due to surface tension = $(2\pi r)\,T\cos\theta_C$.

Definitions

Capillarity: The property by virtue of which a liquid rises or gets depressed in a capillary tube is known as capillarity.

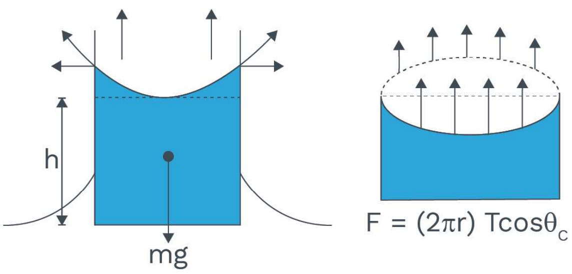
Concept Reminder

Rise in capillary tube-



R = Radius of the meniscus

$$h = \frac{2T}{R\rho g} = \frac{2T\cos\theta_c}{r\rho g}$$



In equilibrium: force due to S.T = weight of rise liquid

$$(2\pi r)\ T\cos\theta_c = mg$$

$$(2\pi r)\ T\cos\theta_c = mg$$

$$h = \frac{2T\cos\theta_c}{r\rho g}$$

Concept Reminder

Teflon coating is done on surface of a non stick pan because it increases the angle of contact from acute to obtuse.

Zurin's Law:

The height of rise of liquid in the capillary tube is inversely proportional to the radius of the capillary tube, if T, θ, ρ and g are constant $h \propto \frac{1}{r}$ or rh = constant. It implies that liquid will rise more in capillary tube of less radius and vice versa.

Important points:

(i) For pure water and clean glass capillary $\theta_c \approx 0° \Rightarrow$ Radius of meniscus = radius of capillary.

(ii) If angle of contact θ_c is acute then cos θ_c is positive, so h is positive and liquid rises. If θ_c is obtuse then cos θ_c is negative, so h is negative, therefore liquid gets deprersed.

(iii) Inside a satellite, water will rise upto the top level but will not overflow. Radius of curvature (R') increases in such a way that final height h' is reduced and given by $h' = \frac{hR}{R'}$. (It is in accordance with Zurin's law).

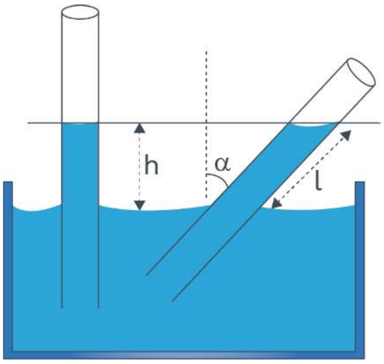
(iv) If a capillary tube is dipped into a liquid and tilted at an angle a from vertical then the

Concept Reminder

The height of rise of liquid in a capillary tube is inversely proportional to the radius of the capillary tube,

Rh = constant

vertical height of the liquid column remains same whereas the length of liquid column in the capillary tube increases.



Ex. Calculate the height to which water will rise in a capillary tube of diameter 14.6×10^{-3} m. [Given: Surface tension of water is 0.073 N/m, angle of contact is 0°, density of water is 1000 kg/m^3, g = 10 m/s^2]

Sol. $h = \dfrac{2T\cos\theta}{r\rho \cdot g}$

$$h = \frac{2 \times 0.073 \times \cos 0^\circ}{(7.3 \times 10^{-3})(10^3)(10)} = 2 \times 10^{-3} \text{m} = 2 \text{ mm}$$

Ex. A capillary tube of radius r can support a liquid of weight 6.28×10^{-4} N. If the surface tension of the liquid is 2×10^{-2} N/m then find:

(1) Radius of capillary tube.

(2) Internal circumference of capillary tube.

Sol. **(1)** $T(2\pi r) = mg$

$$r = \frac{mg}{2\pi T} = \frac{6.28 \times 10^{-4}}{2 \times 3.14 \times 2 \times 10^{-2}}$$

$= 5 \times 10^{-3}$ m = 5 mm

(2) Internal circumference = $2\pi r$

$= 2 \times 3.14 \times 5 = 31.4$ mm

Ex. Water rised to a height of 10mm in a capillary. If the radius of the capillary is made half of its previous value then what is the new value of the capillary rise ?

Sol. $\dfrac{h_2}{h_1} = \dfrac{r_1}{r_2} = \dfrac{r}{(r/2)} = 2$

Hence $h_2 = 2h_1 = 2 \times 10$ mm = 20 mm.

Ex. Water rises in a vertical capillary tube upto a length of 10cm. If the tube is inclined at 45° with the vertical then find the length of water risen in the tube.

Sol. $l \cos\theta = h$

$$\therefore \quad \ell = \frac{h}{\cos 45^\circ} = \frac{10}{\left(\frac{1}{\sqrt{2}}\right)} = 10\sqrt{2} \text{ cm}$$

Ex. In the capillary tube experiment, a vertical 30 cm long capillary is dipped in water. The water rises upto a height of 10cm due to the capillary action. If this experiment is conducted in the freely falling elevator, what will be the length of water column.

Sol. Liquid will rise upto top of the tube

$\therefore \quad h' = 30$ cm

Ex. Two capillary tubes of a radius r_1 and r_2 are vertically immersed in the same liquid, then find:

(1) Ratio of height of liquid rise

(2) Ratio of mass of liquid rise

(3) Ratio of potential energy of liquid rise

Sol. **(1)** By Zurin's law-

$h_1r_1 = h_2r_2$

$$\therefore \quad \frac{h_1}{h_2} = \frac{r_2}{r_1}$$

(2) $$\frac{m_1}{m_2} = \frac{\rho \times V_1}{\rho \times V_2} = \frac{\rho(\pi r_1^2)h_1}{\rho(\pi r_2^2)h_2} = \frac{r_1}{r_2}$$

(3) When water rise upto a height h then mass of liquid rise $m = (\pi r^2 h)\rho$

$\because$ Total mass be located at centre of mass, then potential energy, $U = mg\,\frac{h}{2}$

$$\frac{(P.E.)_1}{(P.E.)_2} = \frac{m_1 g\left(\frac{h_1}{2}\right)}{m_2 g\left(\frac{h_2}{2}\right)} = \left(\frac{m_1}{m_2}\right)\left(\frac{h_1}{h_2}\right) = 1$$

Ex. A U-tube is supported with its vertical limbs and is partially filled with water. If internal diameters of the limbs are 1cm and 0.5 cm respectively. What will be the difference in heights of water in the two limbs? (Surface tension of the water is 70 dyne/cm, density of water is 1 gm/cc, g = 1000 cm/s^2)

Sol. Let h_1 and h_2 be the height of water columns in the limbs of radii r_1 and r_2.

Therefore difference in height = $h_2 - h_1$

$$= \frac{2T\cos\theta_c}{r_1\rho g} - \frac{2T\cos\theta_c}{r_2\rho g}$$

$$= \frac{2T\cos\theta_c}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \frac{2(70)(1)}{(1)(1000)}\left(\frac{1}{\frac{1}{2}} - \frac{1}{\frac{0.5}{2}}\right) = 0.28 \text{ cm}$$

Ex. In a cylindrical vessel at its bottom a round hole of diameter 1 mm is drilled and water is filled in it. Calculate the maximum height to which water can be filled in it without leakage. (Surface tension of water is 75×10^{-3} N/m, density of water is 1000 kg/m^3 and angle of contact is 0°)

Sol. Water will start leaking out when $(h\rho g)(\pi r^2) \geq T.2\pi r$

$$h \geq \frac{2T}{\rho g r}$$

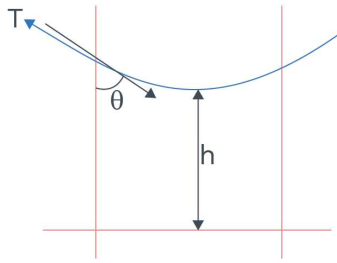
$$\therefore \quad h = \frac{2 \times 75 \times 10^{-3}}{1000 \times 10 \times 0.5 \times 10^{-3}}$$

$= 3 \times 10^{-2}$ m $= 3$ cm

Alternate Method:

As it can be seen from figure that T sin θ cancels out:

The force due to T cos θ balances the weight of liquid (mg = rVg)



vol. of the curve is negligible

$\therefore$ vol. of liquid in $\pi r^2 h$

$$T\cos\theta \times 2\pi r = \rho \pi r^2 hg$$

$$\Rightarrow \quad h = \frac{2T\cos\theta}{r\rho g}$$

Rack your Brain

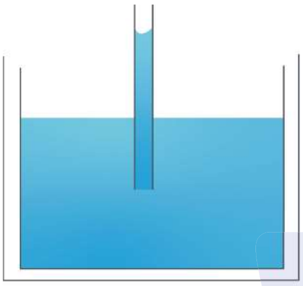
When a glass capillary tube of radius 0.015 cm is dipped in water, the water rises to a height of 15 cm within it. Assuming contact angle between water and glass to be 0°, the surface tension of water is [ρ_{water} = 1000 kg m^{-3}, g = 9.81 ms^{-2}]

(1) 0.11 Nm^{-1}
(2) 0.7 Nm^{-1}
(3) 0.072 Nm^{-1}
(4) None of these

This is the desired result and from this it is clear that:

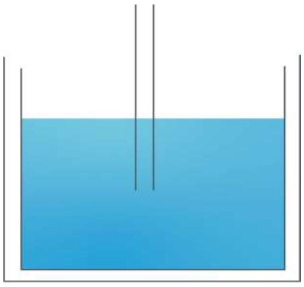
(1) The capillarity depends on the nature of the liquid and solid both, i.e., on T, ρ, θ and R. If $\theta > 90°$, i.e., meniscus is convex, h will be negative, i.e., the liquid will descends in the capillary as actually happens in case of mercury in a glass tube. However, if $\theta = 90°$, i.e., meniscus is plane, h = 0 and so no capillarity.

(i) $\theta < 90°$



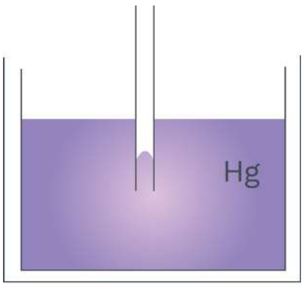
(A)

(ii) $\theta = 90°$



(B)

(iii) $\theta > 90°$



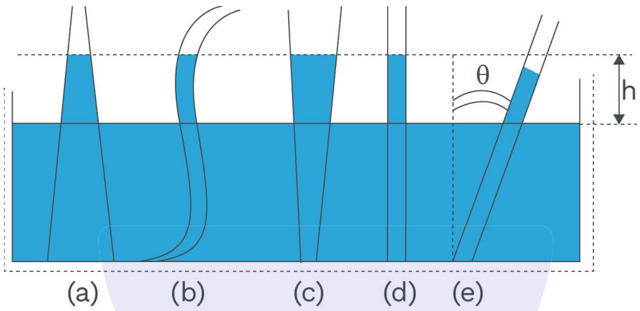
(C)

Concept Reminder

(i) If $\theta < 90°$ meniscus is concave h will be positive.
(ii) If $\theta = 90°$ meniscus is plane g = 0.
(iii) If $\theta > 90°$ meniscus is convex h will be negative.

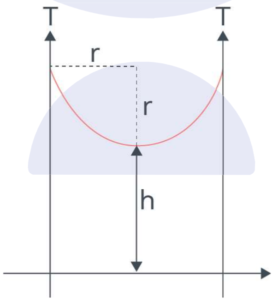
(2) For a given liquid and solid at a given place as ρ, T, θ and g are constant, hr = constant. It means lesser the radius of capillary greater will be the rise and vice-versa.

(3) Here it is important to note that in equilibrium the height h is independent of the shape of capillary if the radius of meniscus remains the same. This is why the vertical height h of a liquid column in capillaries of different shapes and sizes will be same if the radius of meniscus remains the same and also the vertical height of the liquid in a capillary does not change, when it is inclined to the vertical. (figure shown)



(4) In Case of glass and water $\theta = 0°$

Here force due to surface tension balances the weight of the liquid ($\rho \times V \times g$)



volume of the liquid = $\pi r^2 h + \pi r^3 - \frac{2}{3}\pi r^3$

where $\pi r^3 - \frac{2}{3}\pi r^3$ is the volume of the curve which is not negligible in this case.

$$\therefore \quad T \cdot 2\pi r = \rho\left(\pi r^2 h + \pi r^3 - \frac{2}{3}\pi r^3\right)g$$

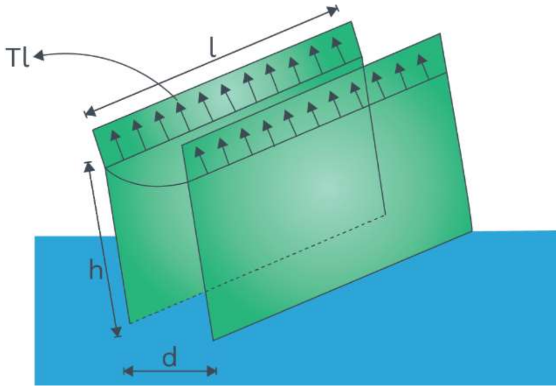
$$2T = rh\rho g + \frac{1}{3}\pi r^2 \rho g$$

Concept Reminder

If two parallel plates with the spacing d are placed in water reservoir, then height or rise.

$$h = \frac{2T}{\rho d g}$$

(5) If two parallel plates with the spacing d are placed in water reservoir, then height or rise.



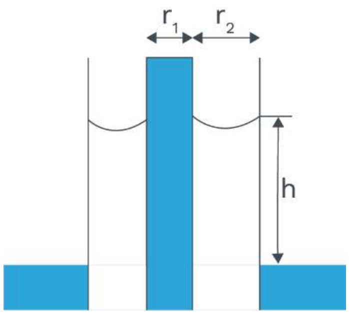
$$2T\ell = \rho \ell h d g$$

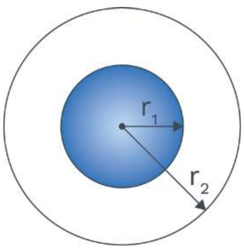
$$\Rightarrow \quad h = \frac{2T}{\rho d g}$$

(6) If two concentric tubes of radius r_1 and r_2 (inner one is solid) are placed in water reservoir, then height of rise?

$$\Rightarrow \quad T[2\pi r_1 + 2\pi r_2] = [\pi r_2^2 h - \pi r_1^2 h]\rho g$$

$$h = \frac{2T}{(r_2 - r_1)\rho g}$$





Rack your Brain

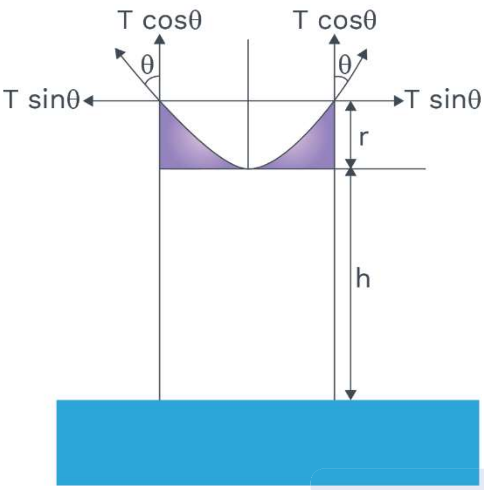
Water rises to a height h in capillary tube. If the length of capillary tube above the surface of water is made less than h, then:

(1) Water does not rise at all

(2) Water rises upto the tip of capillary tube and then starts overflowing like a fountain

(3) Water rises upto the top of capillary tube and stays there without overflowing

(4) Water rises upto a point a little below the top and stays there



(7) If weight of the liquid in the meniscus is to be consider:

$$T\cos\theta \times 2\pi r = \left[\pi r^2 h + \frac{1}{3}\pi r^2 \times r\right]\rho g$$

$$\left[h + \frac{r}{3}\right] = \frac{2T\cos\theta}{r\rho g}$$

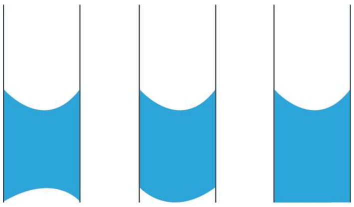
(8) When capillary tube (radius, r) is in vertical position, the upper meniscus is concave and pressure due to surface tension is directed vertically upward and is given by $p_1 = \frac{2T}{R_1}$ where R_1 = radius of curvature of upper meniscus.

The hydrostatic pressure $p_2 = h\rho g$ is always directed downwards.

If $p_1 > p_2$ i.e. the resulting pressure is directed upward. For equilibrium, the pressure due to the lower meniscus should be downward. This makes the lower meniscus concave downward (fig a). The radius of lower meniscus R_2 can be given by $\frac{2T}{R_2} = (p_1 - p_2)$.

Concept Reminder

Capillary action allows a liquid to rise upwards against the force of gravity. It is an important natural phenomenon on which is the functioning principle of several natural and artificial processes.


(a) (b) (c)

If $p_1 < p_2$ i.e. resulting pressure is directed downward for the equilibrium, the pressure due to the lower meniscus should be upward. This makes the lower meniscus convex upward (fig. b) The radius of the lower meniscus can be given by

$$\frac{2T}{R_2} = p_2 - p_1.$$

If $p_1 = p_2$, then is no resulting pressure.

Then, $p_1 - p_2 = \frac{2T}{R_2} = 0$ or $R_2 = \infty$ i.e. lower surface will be flat (figure c)

Concept Reminder

In some pairs of materials, including mercury and glass, the intermolecular forces within the liquid (mercury), exceed those between the solid (glass) and the liquid (mercury). In such cases, capillary action tends to work in reverse.

Ex. A drop of the water volume 0.05 cm^3 is pressed between the two glass-plates, as a consequence of which, it spreads and fills an area of 40 cm^2. If surface tension of the water is 70 dyne/ cm, find the normal force required to separate out the two glass plates in newton.

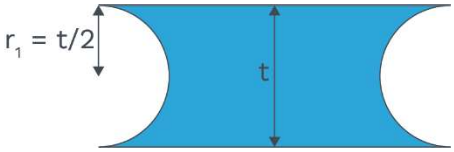
Sol. Pressure inside film is less than outside by an amount,

$$P = T\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$$

Where r_1 and r_2 are the radii of curvature of the meniscus.

Here $r_1 = \frac{t}{2}$ and $r_2 = \infty$

Then the force required to separate two glass plates, between which a liquid film is enclosed (figure) is,



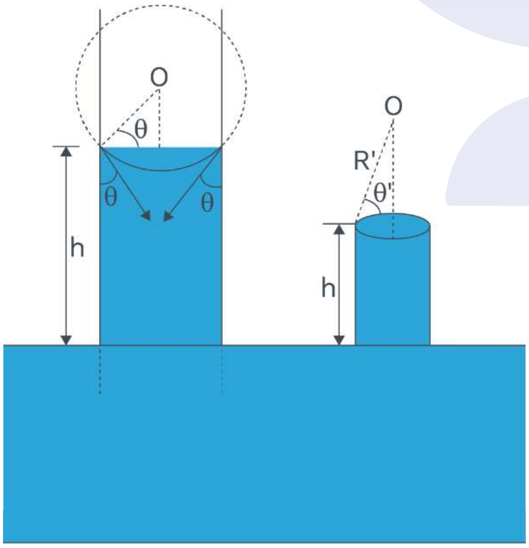
$$F = P \times A = \frac{2AT}{t},$$

Where 't' is the thickness of the film, A = area of film.

$$F = \frac{2A^2T}{At} = \frac{2A^2T}{V}$$

$$F = \frac{2 \times (40 \times 10^{-4})^2 \times (70 \times 10^{-3})}{0.05 \times 10^{-6}} = 45\ N$$

CAPILLARY RISE IN A TUBE OF INSUFFICIENT HEIGHT



We know, the height through which a liquid rises in capillary tube of radius 'r' is given by

$$\therefore \quad h = \frac{2T}{R\rho g} \text{ or } hR = \frac{2T}{\rho g} = \text{constant}$$

When the capillary tube is cut and its length is less than h (i.e. h'), then the liquid rises upto the top of the tube and spreads in such a way that the radius (R') of the liquid meniscus

Concept Reminder

Capillary action is an interesting phenomenon that helps plants raise water without requiring motion.

Concept Reminder

When the capillary tube is cut and its length is less than h (i.e. h'), then the liquid rises upto the top of the tube and spreads in such a way that the radius (R') of the liquid meniscus increases and it becomes more flat.

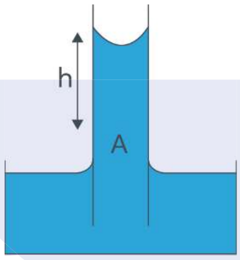
increases and it becomes more flat so that hR = h'R' = Constant. Hence the liquid does not overflow.

If $h' < h$ then $R' > R$

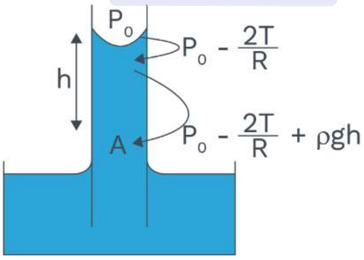
or $\frac{r}{\cos\theta'} > \frac{r}{\cos\theta}$

$\Rightarrow \cos\theta' < \cos\theta \Rightarrow \theta' > \theta$

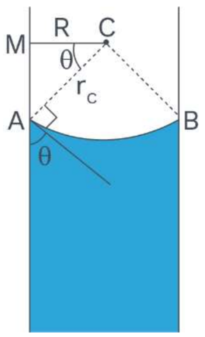
Ex. Water is filled in the capillary tube of radius 'R'. If the surface of water is hemispherical ($\theta = 0$), then find pressure at a point 'A' which is at h depth below the surface.



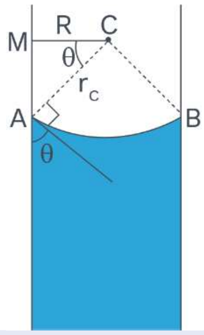
Sol. Water is on convex side. So pressure of water just below the surface will be less by $\frac{2T}{R}$. So pressure at point A is $P_0 - \frac{2T}{R} + \rho gh$. Here surface of water was hemispherical (contact angle $\theta = 0$) so radius of curvature of the surface = radius of the tube = R.



Ex. In the previous question, suppose contact angle is not zero, but it is θ (the surface not hemispherical) now find pressure at point A.



Sol. Draw normal (radial lines) at point A and B of periphery. The point (C) where radial lines meet is called centre of curvature. If contact angle is θ, from ΔACM, $r_c = R \sec\theta = R/\cos\theta$.



So radius of curvature of the surface $r_c = R \sec\theta$.

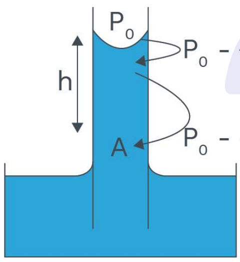
Point to remember:

If the liquid surface is hemispherical ($\theta = 0$) then

$r_c = R$

If liquid surface is not hemispherical ($\theta \neq 0$) then

$r_c = R \sec\theta$.



$$P_0 - \frac{2T}{r_c} = P_0 - \frac{2T}{R\sec\theta} = P_0 - \frac{2T\cos\theta}{R}$$

$$P_0 - \frac{2T}{R\sec\theta} + \rho gh = P_0 - \frac{2T\cos\theta}{R} + \rho gh$$

So, pressure at A is $P_0 - \dfrac{2T\cos\theta}{R} + \rho gh$.

Practical Applications of the Capillarity:

1. The oil in a lamp rises in wick by the capillary action.

2. The tip of nib of a fountain pen is split up, to make a narrow capillary so that the ink rises upto the tip of nib continuously.

3. Sap and water rise upto top of leaves of the tree by capillary action.

4. If one end of the towel is dipped into a bucket of water and the other end hanging over the bucket the towel soon becomes wet throughout because of capillary action.

5. Ink is absorbed by the blotter because of capillary action.

6. Sandy soil is more dry than the clay. It is because the capillaries between the sand particles are not so fine as to draw the water up by the capillaries. And so surface remains dry.

7. The moisture rises in the capillaries of the soil to the surface, where it evaporates. To preserve the moisture in the soil, capillaries have to be. broken up. This is done by the ploughing and levelling the fields.
8. Bricks are porous and have capillaries hence soak water.

Ex. A capillary of internal radius 4 mm, is dipped in water. To how much height, will the water rise in the capillary. (T_{water} = 70 × 10^{-3} N/m, g = 10 m/sec^2, r_{water} = 10^3 kg/m^3, contact angle $\theta \to 0$).

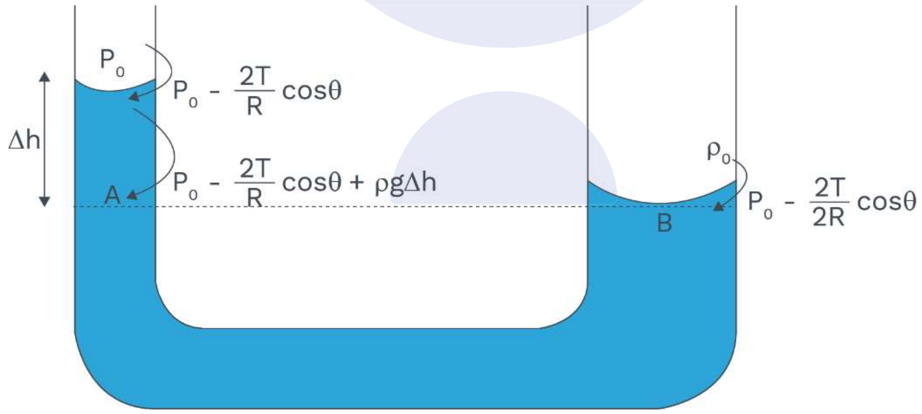
Sol. Capillary rise

$$h = \frac{2T}{\rho g R}\cos\theta = \frac{2 \times 70 \times 10^{-3}}{10^3 \times 10 \times 4 \times 10^{-3}} \quad ...(i)$$

h = 3.5 mm

Ex. In the U-tube, radius of one arm is 'R' and the other arm is '2R'. Find the difference in the water level if contact angle is θ = 60° and surface tension of water is T.

Sol. Balancing pressure at points A and B situated in same horizontal level.



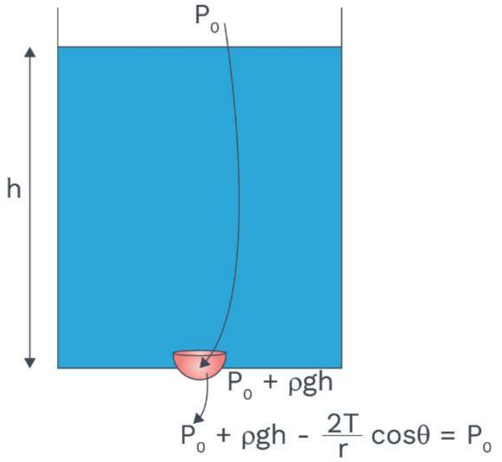
$$P_0 - \frac{2T}{R}\cos\theta + \rho g \Delta h = P_0 - \frac{2T}{2R}\cos\theta$$

Here, θ = 60°,

Solving we get $\Delta h = \dfrac{T}{2\rho g R}$

Ex. There is a small hole of diameter 0.1 mm at the bottom of a large container. To what minimum height we can fill water in it, so that water doesn't come out of hole. (T_{water} = 75 × 10^{-3} N/m, r_{water} = 10^3 kg/m^3, g = 10 m/sec^2)

Sol. The lower surface of water, which will try to come out will be hemi-spherical. Pressure just outside the spherical surface is:



$$P_0 + \rho gh - \frac{2T}{r}\cos\theta = P_0$$

$$\Rightarrow \quad \rho gh = \frac{2T}{r}\cos\theta$$

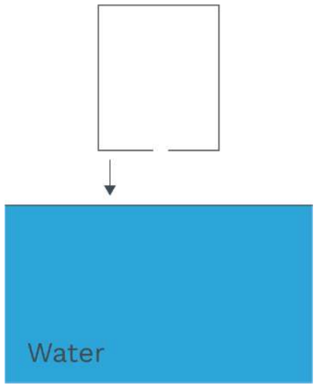
$$\Rightarrow \quad h = \frac{2T}{\rho gr}\cos\theta$$

$$\Rightarrow \quad (h)_{max} = \frac{2T}{\rho gr}(\cos\theta)_{max} \text{ and } (\cos\theta)_{max} = 1$$

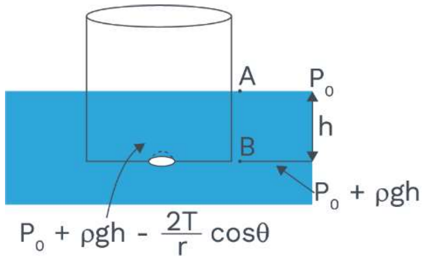
We get,

$(h)_{max} = 0.3 \text{ m} = 30 \text{ cm}$

Ex. An empty container has a circular hole of radius 'r' at its bottom. The container is pushed into the water very slowly. For what depth the lower surface of container (from surface of water) can be pushed into water such that water does not flow into the container? (Surface tension of water = T, density of water = ρ)



Sol. Let the container is dipped to depth h, so the contact angle becomes q



$$P_A = P_0 \Rightarrow P_B = P_0 + \rho gh$$

$$P_C = P_0 + \rho gh - \frac{2T}{r}\cos\theta = P_0$$

$$\Rightarrow \quad \cos\theta = \frac{\rho ghr}{2T} \le 1$$

$$h \le \frac{2T}{\rho gr} \Rightarrow h_{max} = \frac{2T}{\rho gr}$$

SOME OTHER APPLICATIONS OF SURFACE TENSION:-

(i) The wetting property is made use of in the detergents and waterproofing. When the detergent materials are added to the liquids, the angle of contact decreases and hence wettability increases. On the other hand, when the water proofing material is added to a fabric, it increases the angle of contact, making the fabric water-repellant.

(ii) The antiseptic ointments have very low value of the surface tension. The low value of surface tension prevents the formation of drops that may otherwise block the entrance to skin or a wound. Due to low surface tension the antiseptic ointments spreads properly over the wound. The lubricating oils and paints also have the low surface tension. So they may spread properly.

Concept Reminder

Paper towels have small pores present in them. When they come in contact with a liquid, capillary action allows the liquid to move up into the towel, thus allowing the paper towels to soak the liquid.

(iii) Surface tension of the all lubricating oils and paints is kept low so that they can spread over a large area.

(iv) Oil spreads over the surface of the water because the surface tension of oil is less than the surface tension of cold water.

(v) A sea tide heights can be reduced by pouring oil on its surface.

Ex. A barometer contains 2 uniform capillaries of radii 1.44×10^{-3} m and 7.2×10^{-4} m. If the height of the liquid in narrow tube is 0.2 m more than that in the wide tube, find the true pressure difference. Density of the liquid = 10^3 kg/m^3, surface tension = 72 × 10^{-3} N/m and g = 9.8 m/s^2.

Sol. Let the pressure in wide and narrow capillaries of radii r_1 and r_2 respectively be P_1 and P_2. Then pressure just below the meniscus in the wide and narrow tubes respectively are $\left(P_1 - \frac{2T}{r_1}\right)$ and $\left(P_2 - \frac{2T}{r_2}\right)$; $\left[\text{excess pressure} = \frac{2T}{r}\right]$.

Difference in these pressures

$$= \left(P_1 - \frac{2T}{r_1}\right) - \left(P_2 - \frac{2T}{r_2}\right) = h\rho g$$

∴ True pressure difference = $P_1 - P_2$

$$= h\rho g + 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= 0.2 \times 10^3 \times 9.8 + 2 \times 72 \times 10^{-3}\left[\frac{1}{1.44 \times 10^{-3}} - \frac{1}{7.2 \times 10^{-4}}\right]$$

= 1.86 × 10^3 = 1860 N/m^2.

Alternative: Due to different curvatures at meniscus, the difference in excess pressures causes error. Hence, true pressure = $\rho gh + 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$, etc.

Ex. A liquid of the specific gravity 1.5 is observed to rise 3.0 cm in a capillary tube of radius 0.25 mm and the liquid wets the surface of the tube. Determine the excess pressure inside a spherical bubble of 1.0 cm diameter blown from the same liquid. Angle of contact = 0°.

Sol. The surface tension of the liquid is

$$T = \frac{rh\rho g}{2} = \frac{(0.025\ \text{cm})(3.0\ \text{cm})(1.5\ \text{gm/cm}^3)\ (980\ \text{cm/sec}^2)}{2}$$

= 55 dyne/cm.

Hence excess pressure inside a spherical bubble

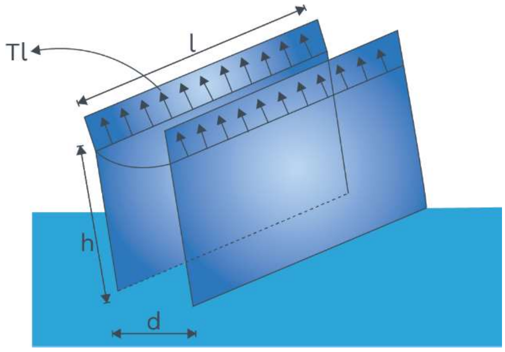
$$p = \frac{4T}{R} = \frac{4 \times 55\ \text{dyne/cm}}{(0.5\ \text{cm})} = 440\ \text{dyne/cm}^2$$

Ex. Two parallel plates which are separated by a very small distance d, are dipped in water. To how much height will the water raise between the plates (Assume contact angle $\theta \to 0$)

Sol. Lets draw free body diagram of the water raised up. Forces on it are:

(i) The plates pull the surface in upward direction with a force $2T\ell$.

(ii) The weight of raised water = $(\rho)(\ell hd)g$
For equilibrium, forces should be balanced.



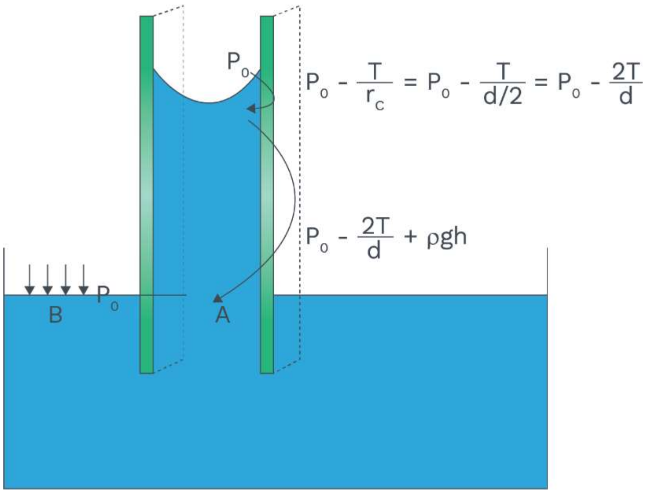
$$2T\ell = (\rho)(\ell hd)g \quad \Rightarrow h = \frac{2T}{\rho gd}$$

Also $\frac{T}{d/2} = \rho gh$; so we can say that excess pressure due to cylindrical surface $= \frac{T}{d/2} = \frac{T}{r_c}$.

Excess pressure due to spherical surface = $\frac{2T}{r_c}$

Excess pressure due to cylindrical surface = $\frac{T}{r_c}$

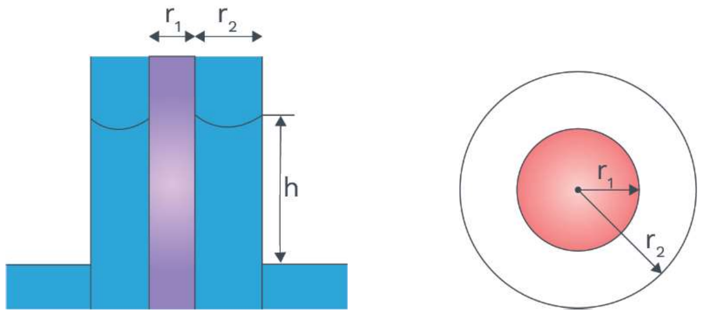
Alternative method:



$$\Rightarrow \quad P_0 - \frac{2T}{d} + \rho gh = P_0 \quad \Rightarrow h = \frac{2T}{\rho gd}$$

Ex. A thin capillary of inner radius r_1 and outer radius r_2 (The inner tube is solid) is dipped in water. To how much height will the water raise in the tube? (Assume contact angle $\theta \rightarrow 0$)

Sol. Applying force balance



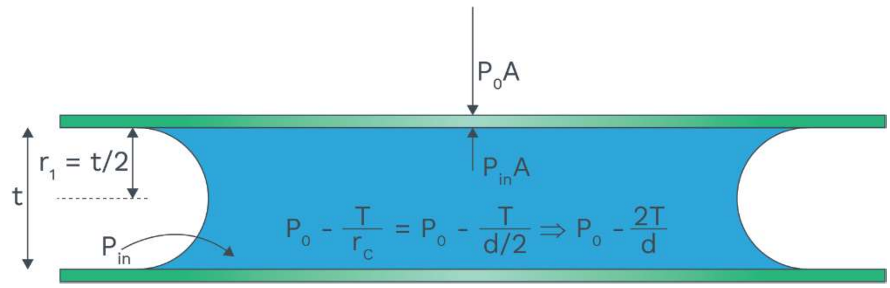
$$T[2\pi r_1 + 2\pi r_2] = [\pi r_2^2 - \pi r_1^2]\rho gh$$

$$\Rightarrow \quad h = \frac{2T}{(r_2 - r_1)\rho g}$$

Ex. A drop of the water volume 0.05 cm^3 is pressed between the two glass-plates, as a result of which, it spreads and occupies an area of 40 cm^2. If the surface tension of the water is 70 dyne/cm, find the normal force required to separate out the two glass plates in newton.

Sol. Pressure inside the surface

$$P_{in} = P_0 - \frac{T}{r_c} = P_0 - \frac{T}{t/2} = P_0 - \frac{2T}{t}$$



So, net inwards force = $P_0A - P_{in}A$

$$= \left(P_0 - \frac{2T}{t}\right)A - P_0A = \frac{2TA}{t}$$

Here volume between the plates V = A × t

$$\Rightarrow \quad t = \frac{V}{A}$$

Putting the value of t,

$$F = \frac{2A^2T}{V} = \frac{2\times(40\times10^{-4})^2\times(70\times10^{-3})}{0.05\times10^{-6}} = 45\ N$$

So this much force is required to separate the plates.

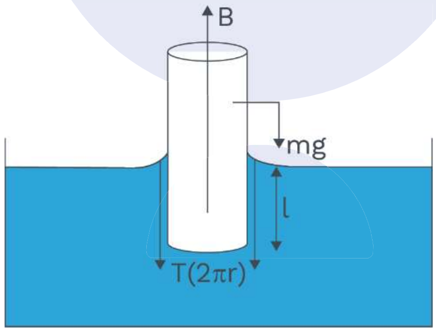
Ex. A glass plate whose dimensions are follows length 10 cm, breadth 1.54 cm and thickness 0.20 cm weighs 8.2 gm in the air. It is held vertically with long side horizontal and the lower half under water. Determine the apparent weight of the plate. Surface tension of the water = 73 dyne per cm, g = 980 cm/sec^2.

Sol. The forces acting on the plate are

(i) Buoyancy force of water acting upward

$$B = \rho_1 V_{sub} g = 1\times\frac{1.54\times10\times0.2}{2}\times980$$

$= 1509.2$ dyne



(ii) Weight of the glass plate acting downward

= (8.2) × 980 dyne

(iii) Force of surface tension acting downward;

surface tension × line of contact

$= 2\ (\ell + b)T = 2\ (10 + 0.2\ )\ 73 = 1489.2$

So net downward force

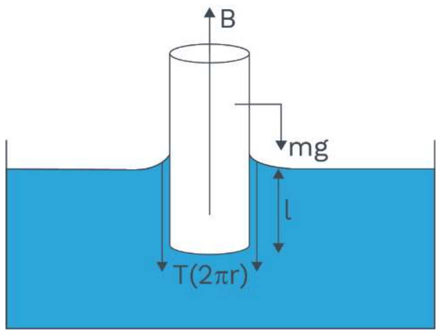
= mg + (surface tension force) – B

= (8.2) × 980 + 1489.2 – 1509.2

= 8016.008 dyne = 8.1796 gm force

Ex. A glass tube of the circular cross-section is closed at the one end. This end is weighted and the tube floats vertically in the water, heavy end down. How far below the water surface is end of the tube? Given: Outer radius of tube 0.14 cm, mass of weighted tube 0.2 gm, surface tension of water 73 dyne/cm and g = 980 cm/sec^2.

Sol. Let ℓ be the length of tube inside water. The forces acting on the tube are:



(i) Buoyancy force of water acting upward

$$B = \pi r^2 \ell \times 1 \times 980 = \frac{22}{7} \times (0.14)^2 \ell \times 980$$

$$B = 60.368 \ell \text{ dyne}$$

(ii) Weight of the system acting downward
= mg = 0.2 × 980 = 196 dyne.

(iii) Force of surface tension acting downward,
surface tension × line of contact

$$= 2\pi r T = 2 \times \frac{22}{7} \times 0.14 \times 73 = 64.24 \text{ dyne.}$$

Since tube is in equilibrium, the upward force is balanced by downward forces.
So, 60.368 ℓ = 196 + 64.24 = 260.24.

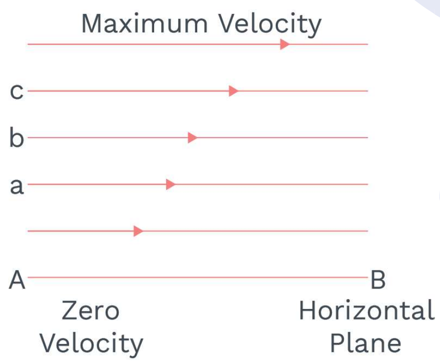
$$\therefore \quad \ell = \frac{260.24}{60.368} = 4.31 \text{ cm}$$

VISCOSITY:-

When a solid body slides over the another solid body, a frictional-force start to act between them. This force opposes the relative motion of bodies. Similarly, when a layer of the liquid slides over another layer of same liquid, a frictional-force acts between them which opposes the relative motion between the layers. This force is called 'internal frictional-force'. This is due to inter molecular forces.

Consider a liquid is flowing in streamlined motion on a fixed horizontal surface AB (Fig.). The layer of a liquid which is in such with the surface is at rest, while the velocity of the other layers increases with the distance from the fixed surface. In the Figure., the lengths of arrows represent the increasing velocity of layers. Thus there is a relative motion between the adjacent layers of the liquid.

Let us consider three parallel layers 'a', 'b' and 'c'. Their velocities are in increasing order. The layer 'a' tends to retard the layer 'b', while 'b' tends to retard 'c'. Thus each layer tends to decrease velocity of layer above it. Similarly, each layer tends to increase velocity of layer below it. This means that in between the any two layers of the liquid, internal tangential forces act which try to destroy the relative motion between layers. These forces are known as 'viscous forces'. If the flow of liquid is to be maintained, an external force have to applied to overcome dragging viscous forces. In the absence of external force, the viscous forces would soon bring the liquid to the rest. The property of the liquid because of which it opposes relative motion between its adjacent layers is defined as 'viscosity'.



The property of the viscosity is seen in the following illustrations:

(i) A stirred liquid, when left, comes to the rest on account of viscosity. Thicker liquids like honey, coal tar, glycerine, etc. have a larger viscosity than thinner ones like water. If we pour coal tar and water on a table, the coal tar will stop to flow soon while the water will flow upto quite a large distance.

(ii) If we pour the water and honey in separate funnels, the water comes out readily from the hole in funnel while honey takes enough time to do so. This is because the honey

Definitions

Viscosity: The property of the liquid by virtue of which it opposes the relative motion between its adjacent layers is known as 'viscosity.'

is much more viscous than the water. As honey tends to flow down under the gravity, the relative motion between its layers is opposed strongly.

(iii) We can walk fast in the air, but not in the water. The reason is again the viscosity which is very small for the air but comparatively much larger for the water.

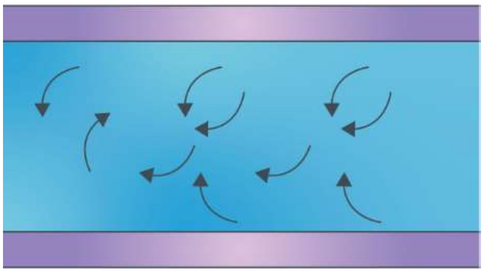
(iv) The cloud particles fall down very slowly due to the viscosity of the air and hence appear floating in the sky.

Viscosity plays an important role only when there is a relative motion between the layers of same material. That's why it does not act in the solids.

FLOW OF LIQUID IN A TUBE: CRITICAL VELOCITYL:-

When a liquid flows in the tube, the viscous forces oppose flow of liquid. Hence the pressure difference is applied between the ends of the tube which maintains the flow of liquid. If all particles of the liquid going through a particular point in tube move along the same path, the flow" of the liquid is called 'stream-lined flow'.

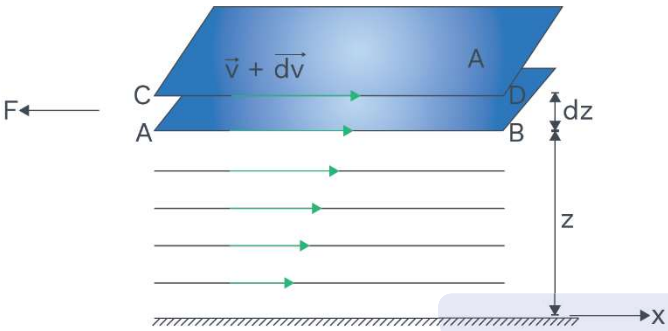
This occurs only when the velocity of flow of the liquid is below a certain limiting value called 'critical velocity'. When the velocity of flow exceeds the critical velocity, the flow is no longer stream-lined but becomes turbulent. In this type of flow, the motion of the liquid becomes zig-zag and eddy-currents are developed in it as shown in Figure.



Reynold observed that the critical velocity for a liquid flowing in a tube is $v_c = R\eta/\rho a$. where ρ is density and η is viscosity of the liquid, a is radius of the tube and R is 'Reynold's number' (whose value for a narrow tube and for the water is about 1000). When the velocity of the flow of liquid is less than the critical velocity, then the flow of liquid is controlled by viscosity, the density having no effect on it.

But when the velocity of the flow is larger than the critical velocity, then the flow is mainly governed by the density, the effect of the viscosity becoming less important. It is because of this reason that when a volcano erupts, then the lava coming out of it flows speedly inspite of being very thick (of large viscosity).

Newton's law of viscosity



The rate of change of velocity with distance perpendicular to the direction of flow i.e. $\frac{\Delta v_x}{\Delta y}$, is called velocity gradient.

According to Newton, the viscous force F acting between two adjacent layers of a liquid flowing in streamlined motion depends upon the following two factors:

(i) F ∝ contact area of the layers i.e.

$F \propto A$

(ii) F ∝ velocity gradient between the layers i.e.

$$F \propto \frac{\Delta v_x}{\Delta y}$$

Combining (i) and (ii)

$$F \propto A\frac{\Delta v_x}{\Delta y}$$

$$\Rightarrow \boxed{F = \eta A\frac{\Delta v_x}{\Delta y}}$$

where h is a constant called coefficient of viscosity of the liquid.

Coefficient of viscosity

$$\eta = \frac{F / A}{v / \ell} = \frac{\text{shear stress}}{\text{strain rate}}$$

Concept Reminder

Newton's law of viscosity:

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$F \propto A$

(ii) F ∝ velocity gradient between the layers i.e.

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$$\Rightarrow \boxed{F = \eta A\frac{\Delta v_x}{\Delta y}}$$

Where η is a constant called coefficient of viscosity of the liquid.

SI Unit: $N\text{-}sm^{-2}$ = Pa-s = poiseuille (PI) = deca poise
CGS Unit: dyne-s/cm^2 = poise;
1 decapoise = 10 poise.
Dimensions: $[M^1L^{-1}T^{-1}]$

Important points:

(i) Viscosity of fluid depends only on the nature of fluid and is independent of area considered or velocity gradient.

(ii) Thin liquids like water, alcohol are less viscous than thick liquids like blood, glycerin, honey.

(iii) Viscosity of liquid is much greater (about 100 times more) than that of gases.
Viscosity of water $\approx$ 0.01 poise, Viscosity of air $\approx$ 200 mili poise

Ex. There is a 3mm thick layer of glycerine between a plate of area $10^{-3}m^2$ and a large plate. If the coefficient of viscosity of glycerine is 2 kg/ms, then what force is required to move the smaller plate with a velocity of 12 cm/s.

Sol. $F = \eta A\frac{\Delta v}{\Delta y} = \frac{2\times10^{-3}\times12\times10^{-2}}{3\times10^{-3}} = 0.08\ N$

Ex. The velocity of water in a river is 15 m/s near the surface. If the river is 5m deep, find the shearing stress between the horizontal layers of water. The coefficient of viscosity of water = 10^{-2} poise.

Sol. Velocity gradient

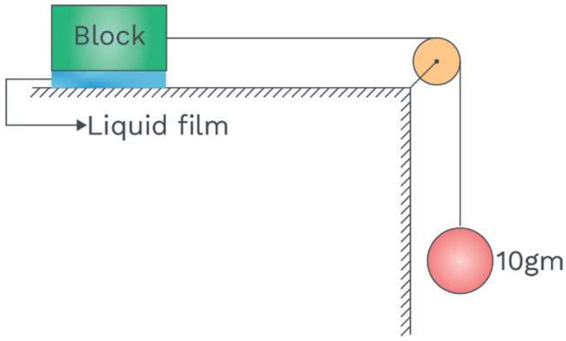
$$\frac{\Delta v}{\Delta y} = \frac{15}{5} = 3\ s^{-1}$$

Shearing stress =

$$\frac{F}{A} = \eta\cdot\frac{\Delta v}{\Delta y} = 10^{-3}\times3 = 3\times10^{-3}\ N/m^2$$

Ex. Thickness of liquid film is 0.02 cm and surface area of block is 10^4 cm^2. When mass 10 gm is released then block moves towards right with constant velocity 2 cm/sec. Then find out viscosity coefficient of the liquid. [g = 1000 cm/s^2]

Sol. $F = \eta A\frac{\Delta v}{\Delta y} \Rightarrow \eta = \frac{F}{A}\cdot\frac{\Delta y}{\Delta v}$

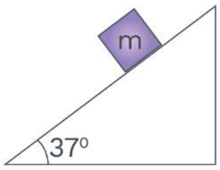


> **KEY POINTS**
> - Stokes law
> - Terminal velocity

$$\eta = \frac{(10 \times 1000)}{10^4} \times \frac{0.02}{2}$$

$\eta = 0.01$ poise

Ex. A cubical block of side 'a' and density 'ρ' slides over a fixed inclined plane with constant velocity 'v'. There is a thin film of the viscous fluid of thickness 't' between the plane and the block. Then find the coefficient of viscosity of the thin film.



Sol. $$mg\sin 37^\circ = \eta A \frac{\Delta v}{\Delta y}$$

$$\Rightarrow \frac{g}{2}(a^3\rho) \times \frac{3}{5} = \eta \cdot a^2 \cdot \frac{v}{t}$$

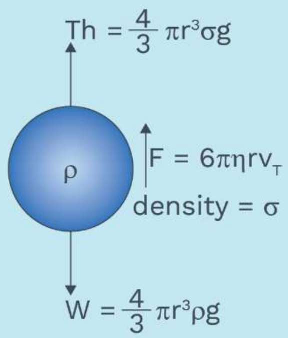
$$\Rightarrow \eta = \frac{3\rho agt}{5v}$$

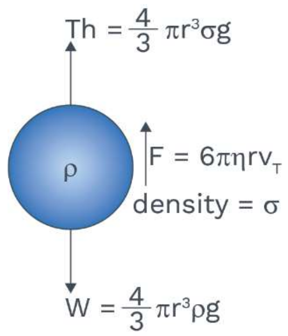
Stokes Law and Terminal velocity

Stokes Law:

Stoke showed that if a small sphere of radius r is moving with a velocity v through a homogeneous stationary medium (liquid or gas), of viscosity η then the viscous force acting on the sphere is $F_v = 6\pi\eta rv$.

> **Concept Reminder**
>
> **Stokes Law:**
>
> $F_v = 6\pi\eta rv$





Terminal Velocity:

When a solid sphere falls in a liquid, its accelerating velocity is controlled by the viscous force due to liquid and hence it attains a constant velocity which is known as the terminal velocity (v_T).

As shown in the diagram when the body moves with constant velocity i.e. terminal velocity (with no acceleration) the net upward force (upthrust Th + viscous force F_v) balances the downward force (weight of the body W).

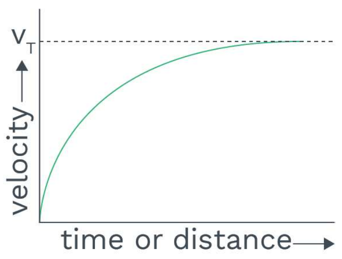
Therefore Th + F_v = W

$$\frac{4}{3}\pi r^3\rho g + 6\pi\eta r v_T = \frac{4}{3}\pi r^3\rho g$$

$$v_T = \frac{2}{9}\frac{r^2(\rho-\sigma)}{\eta}g$$

where r = radius of body
ρ = density of body
σ = density of medium
η = coefficient of viscosity.

Graph:



Definitions

Terminal Velocity:

When a solid sphere falls in a liquid, its accelerating velocity is controlled by the viscous force due to liquid and hence it attains a constant velocity which is known as the terminal velocity (v_T).

Concept Reminder

$$v_T = \frac{2}{9}\frac{r^2(\rho-\sigma)}{\eta}g$$

where r = radius of body
ρ = density of body
σ = density of medium
η = coefficient of viscosity.

Graph:

The variation of the velocity with time (or distance) is shown in the adjacent graph.

Some applications of terminal velocity:

(a) Actual velocity of rain drops is very small in comparison to the velocity which would have acquired by a body falling freely from the height of clouds.

(b) Descent of a parachute with moderate velocity.

(c) Determination of electronic charge in Milikan's oil drop experiment.

Ex. Two balls made of same material having masses m and 8m respectively. Then find ratio of their terminal velocity in same liquid.

Sol. $v_T \propto r^2$

$$\Rightarrow \quad \frac{v_{T_1}}{v_{T_2}} = \left(\frac{r_1}{r_2}\right)^2$$

$$\Rightarrow \quad \frac{v_{T_1}}{v_{T_2}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Since: $m \propto r^3$

$$\frac{m_1}{m_2} = \left(\frac{r_1}{r_2}\right)^3$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \left(\frac{m}{8m}\right)^{1/3} = \frac{1}{2}$$

Ex. If terminal speed of a sphere of the gold is 0.4 m/s in a viscous liquid. Then find the terminal speed of a sphere of copper of the same size in the same liquid.

Given:

ρ_{gold} = 13 g/cc

ρ_{copper} = 8 g/cc

ρ_{liquid} = 3 g/cc

Sol. $v_T \propto (\rho_b - \rho_\ell)$

Concept Reminder

Actual velocity of rain drops is very small in comparison to the velocity which would have acquired by a body falling freely from the height of clouds.

Rack your Brain

64 identical rain drops are moving in air with constant velocity 10 m/s. If they combined to form a big drop. Then find constant velocity of big drop.

$$\Rightarrow \quad \frac{v_{T_1}}{v_{T_2}} = \frac{\rho_{gold} - \rho_{liquid}}{\rho_{copper} - \rho_{liquid}}$$

$$\Rightarrow \quad \frac{0.4}{v_{T_2}} = \frac{13-3}{8-3} \Rightarrow v_{T_2} = 0.2\ m/s$$

Ex. An air bubble of 1 cm radius is rising at a steady rate of 2.00 mm/sec through a liquid of density 1.5 gm per cm^3. Neglect density of air. If g is 1000 cm/sec^2, then find the coefficient of viscosity of the liquid:

Sol. $v_T = \frac{2}{9}\frac{r^2\rho_L g}{\eta}$

$$\eta = \frac{2}{9}\frac{r^2\rho_L g}{v_T} = \frac{2}{9}\frac{(1)^2 \times 1.5 \times 1000}{(2\times 10^{-1})}$$

$= 1.66 \times 10^3$ poise

Ex. A ball rises to surface at a constant velocity in a liquid whose density is 6 times than that of the material of the ball. Find the ratio of the force of friction acting on the rising ball and its weight.

Sol. Viscous force acting downward = effective force in upward direction

$= Th - W$

$= \rho_\ell vg - \rho_b vg$

$= (6\rho)vg - \rho vg = 5\rho vg$

$$\Rightarrow \quad \frac{F_v}{W} = \frac{5\rho vg}{\rho vg} = \frac{5}{1}$$

Ex. A tiny sphere of mass 2kg and density 5g/cc is dropped in a jar of glycerine of density 3g/cc. When the sphere acquires terminal velocity, then find the magnitude of the viscous force acting on the sphere.

Sol. $Th + F_v = W$

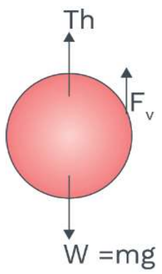
$\Rightarrow \quad F_v = W - Th = mg - \rho_\ell Vg$

Concept Reminder

Similarities: Viscosity and solid friction are similar as-

- Both oppose relative motion. Whereas viscosity opposes the relative motion between two adjacent liquid layers, solid friction opposes the relative motion between two solid layers.
- Both come into play, whenever there is relative motion between layers of liquid or solid surfaces as the case may be.
- Both are due to molecular attractions.

$$= mg - \rho_\ell \cdot \frac{m}{\rho_b} g$$



$$\Rightarrow \quad mg\left[1 - \frac{\rho_\ell}{\rho_b}\right] = 8\ N$$

SIMILARITIES AND DIFFERENCES BETWEEN VISCOSITY AND SOLID FRICTION

Similarities: Viscosity and solid friction are similar as-

1. Both oppose relative motion. Whereas viscosity opposes the relative motion between two adjacent liquid layers, solid friction opposes the relative motion between two solid layers.
2. Both come into play, whenever there is relative motion between layers of liquid or solid surfaces as the case may be.
3. Both are due to molecular attractions.

Differences between them are given below:

S. No.	Viscosity	Solid Friction
1.	Viscosity (or viscous drag) between layers of liquid is directly proportional to the area of the liquid layers.	Friction between two solids is independent of the area of solid surfaces in contact.
2.	Viscous drag is proportional to the relative velocity between two layers of liquid.	Friction is independent of the relative velocity between two surfaces.
3.	Viscous drag is independent of normal reaction between two layers of liquid.	Friction is directly proportional to the normal reaction between two surfaces in contact.

Some Applications of Viscosity:-

Knowledge of the viscosity of the various liquids and gases have been put to use in the daily life. Some applications of its knowledge are discussed as under:

1. As the viscosity of liquids vary with temperature, proper choice of lubricant is made depending upon season.
2. Liquids of high viscosity are used in shock absorbers and buffers at railway stations.

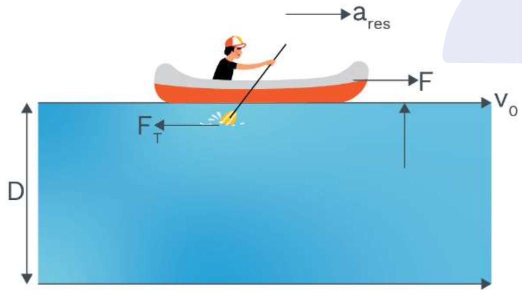
3. The phenomenon of viscosity of air and liquid is used to damp the motion of some instruments.
4. The knowledge of the coefficient of viscosity of organic liquids is used in determining the molecular weight and shape of the organic molecules.
5. It finds an important use in the circulation of blood through arteries and veins of human body.

Ex. A man is rowing a boat with a uniform velocity v_0 in a river. The contact area of boat is A and coefficient of viscosity is η. The depth of river is D. Find the force required to row the boat.

Sol. $F - F_T = ma_{res}$

As boat moves with constant velocity $a_{res} = 0$

$F = F_T$



But $F_T = \eta A \dfrac{dv}{dz}$,

But $\dfrac{dv}{dz} = \dfrac{v_0 - 0}{D} = \dfrac{v_0}{D}$

Then $F = F_T = \dfrac{\eta A v_0}{D}$

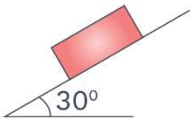
Ex. A cubical block (of side 2m) of mass 20 kg slides on inclined plane lubricated with the oil of viscosity $\eta = 10^{-1}$ poise with constant

Rack your Brain

If a ball of steel (density $\rho = 7.8$ g cm^{-3}) attains a terminal velocity of 10 cm s^{-1} when falling in a water (Coefficient of viscosity $\eta_{water} = 8.5 \times 10^{-4}$ Pa.s) then its terminal velocity in glycerine ($\rho = 1.2$ g cm^{-3}, $\eta = 13.2$ Pa.s) would be, nearly:

(1) 6.25×10^{-4} cm s^{-1}
(2) 6.45×10^{-4} cm s^{-1}
(3) 1.5×10^{-5} cm s^{-1}
(4) 1.6×10^{-5} cm s^{-1}

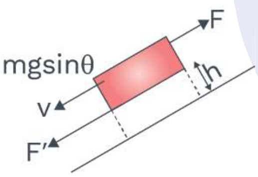
velocity of 10 m/sec. (g = 10 m/sec²). Find out the thickness of layer of liquid.



Sol. $F = F' = \eta A \frac{dv}{dz} = mg\sin\theta$

$\frac{dv}{dz} = \frac{v}{h}$

$\therefore \quad 20 \times 10 \times \sin 30° = \eta \times 4 \times \frac{10}{h}$



$h = \frac{40 \times 10^{-2}}{100} [\eta = 10^{-1} \text{ poise} = 10^{-2} \text{ N} - \text{sec m}^{-2}]$

$= 4 \times 10^{-3}$ m = 4 mm

Effect of Temperature on the Viscosity:

The viscosity of liquids decrease with increase in temperature and increase with the decrease in temperature. That is, $\eta \propto \frac{1}{\sqrt{T}}$. On the other hand, the value of viscosity of gases increases with the increase in temperature and vice-versa. That is, $\eta \propto \sqrt{T}$.

Important point

Thus, terminal velocity of the ball is directly proportional to the square of its radius.

Air bubble in water always goes up. It is because density of air (ρ) is less than the density of water (σ).

So, the terminal velocity for air bubble is Negative, which implies that the air bubble will go up. Positive terminal velocity means the body will fall down.

Concept Reminder

Effect of Temperature on the Viscosity:

The viscosity of liquids decrease with increase in temperature and increase with the decrease in temperature. That is, $\eta \propto \frac{1}{\sqrt{T}}$.

Concept Reminder

Effect of Temperature on the Viscosity of gases:

The value of viscosity of gases increases with the increase in temperature and vice-versa. That is, $\eta \propto \sqrt{T}$.

Ex. A spherical ball is moving with terminal velocity inside a liquid. Determine the relationship of rate of heat loss with the radius of ball.

Sol. Rate of heat loss = power

$= F \times v = 6\pi\eta r v \times v = 6\pi\eta r v^2$

$$= 6\pi\eta r\left[\frac{2}{9}\frac{gr^2(\rho_0 - \rho_\ell)}{\eta}\right]^2$$

Rate of heat loss $\propto r^5$.

Ex. A drop of water of radius 0.0015 mm is falling in air. If the coefficient of viscosity of air is 1.8×10^{-5} kg /(m-s), what will be the terminal velocity of the drop? (density of water = 1.0×10^3 kg/m^2 and g = 9.8 N/kg.) Density of air can be neglected.

Sol. By Stokes law, the terminal velocity of a water drop of radius r is given by

$$v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

where ρ is the density of the water, σ is the density of the air and η the coefficient of the viscosity of the air. Here σ is negligible and r = 0.0015 mm = 1.5×10^{-3} mm = 1.5×10^{-6} m. Substituting the values:

$$v = \frac{2}{9} \times \frac{(1.5 \times 10^{-6})^2 \times (1.0 \times 10^3) \times 9.8}{1.8 \times 10^{-5}}$$

$= 2.72 \times 10^{-4}$ m/s

Ex. A metallic sphere of radius 1.0×10^{-3} m and density 1.0×10^4 kg/m^3 enters a tank of water, after a free fall through a distance of h in the earth's gravitational field. If its velocity remains unchanged after entering water, determine the value of h. Given: coefficient of viscosity of water = 1.0×10^{-3} N-s/m^2, g = 10 m/s^2 and density of water = 1.0×10^3

Concept Reminder

Terminal velocity of the ball is directly proportional to the square of its radius.

Air bubble in water always goes up. It is because density of air (ρ) is less than the density of water (σ).

kg/m³.

Sol. The velocity attained by the sphere in falling freely from a height h is

$$v = \sqrt{2gh} \qquad ...(i)$$

This is the terminal velocity of the sphere in water. Hence by Stokes law, we have

$$v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta} \qquad ...(ii)$$

Where r is the radius of the sphere, ρ is the density of the material of the sphere σ (= 1.0×10^3 kg/m³) is the density of water and η is coefficient of viscosity of water.

$$\therefore \quad v = \frac{2 \times (1.0 \times 10^{-3})^2 (1.0 \times 10^4 - 1.0 \times 10^3) \times 10}{9 \times 1.0 \times 10^{-3}}$$

$$= 20 \text{ m/s}$$

From equation (i), we have

$$h = \frac{v^2}{2g} = \frac{20 \times 20}{2 \times 10} = 20 \text{ m}$$

Applications of Stokes' Formula:

(i) In determining the Electronic Charge by Millikan's Oil Drop Experiment: Stokes' formula is used in Millikan's method for determining the electronic charge. In this method the formula is applied for finding out the radii of small oil-drops by measuring their terminal velocity in air.

(ii) Velocity of Rain Drops: Rain drops are formed by the condensation of water vapour on dust particles. When they fall under gravity, their motion is opposed by the viscous drag in air. As the velocity of their fall increases, the viscous drag also increases and finally becomes equal to the effective force of gravity. The drops then attain a (constant) terminal velocity which is directly proportional to the square of the radius of the drops. In the beginning the raindrops are very small in size and so they fall with such a small velocity that they appear floating in the sky as cloud. As they grow in size by further condensation, then they reach the earth with appreciable velocity.

Concept Reminder

Applications of Stokes' Formula:

- In determining the Electronic Charge by Millikan's Oil Drop Experiment
- Velocity of Rain Drops
- Parachute

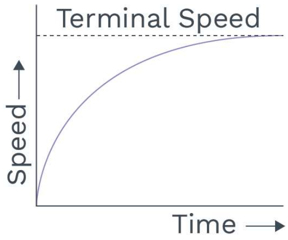
Definitions

Reynold's Number (R_e):

The type of flow pattern (streamline, laminar or turbulent) is determined by a non-dimensional number called Reynold's number (R_e). It is defined as

$$R_e = \frac{\text{Inertial force}}{\text{Viscous force}} = \frac{\rho v d}{\eta}$$

(iii) **Parachute:** When a soldier with a parachute jumps from a flying aeroplane, he descends very slowly in air.



In the beginning the soldier falls with gravity acceleration g, but soon the acceleration goes on decreasing rapidly when parachute is fully opened. Therefore, in the beginning the speed of the falling soldier increases somewhat rapidly but then very slowly. Due to the viscosity of air the acceleration of the soldier becomes ultimately zero and the soldier then falls with a constant terminal speed. Figure shown speed of the falling soldier with time.

Reynold's Number (R_e)

The type of flow pattern (streamline, laminar or turbulent) is determined by a non-dimensional number called Reynold's number (R_e). It is defined as

$$R_e = \frac{\text{Inertial force}}{\text{Viscous force}} = \frac{\rho v d}{\eta}$$

where ρ is the density of the fluid having viscosity η and flowing with a mean speed v. Here d denotes the lateral dimension of the obstacle or boundary of fluid flow.

Although there is no perfect demarcation for the value of R_e in case of laminar and turbulent flow but certain references take the value as:

R_e	< 1000	> 2000	BETWEEN 1000 TO 2000
Type of flow	Streamline or laminar	turbulent	unsteady

Upon increasing the speed of flow gradually transition from laminar flow to turbulent flow takes place at certain speed. This speed is called critical speed. For fluids lower density and higher viscosity with laminar flow is more probable.

Dependency of viscosity

(i) On Temperature of Fluid:

(a) Since cohesive forces decrease with increase in temperature. Viscosity of liquids decreases with a rise in temperature.

(b) Viscosity of gases is the result of diffusion of gas molecules from one moving layer to other. With an increase in temperature, the rate of diffusion increases. Consequently the viscosity increases. Thus, the viscosity of gases increases with the rise in temperature.

(ii) On Pressure of Fluid:

(a) Viscosity is normally independent of pressure. However, liquids under extreme pressure often undergo an increase in viscosity.

(b) Viscosity of gases is practically independent of pressure.

Concept Reminder

Dependency of viscosity:

- On Temperature of Fluid
- On Pressure of Fluid

Steady flow in capillary tube:

Poiseuille's Formula: In case of steady flow of liquid of viscosity (η) in a capillary tube of length (L) and radius (r) under a pressure difference (P) across it, the volume of liquid flowing per second is given by-

$$Q = \frac{dV}{dt} = \frac{\pi P r^4}{8 \eta L}$$

With the help of poiseuille's formula, coefficient of viscosity of a liquid can be determined.

Ex. A viscous liquid is flowing through two tubes which are connected in series. Their length are l and 4l and radius r and 2r respectively. Find the ratio of pressure difference across the first and second tubes.

Concept Reminder

Steady flow in capillary tube:

Poiseuille's Formula-

$$Q = \frac{dV}{dt} = \frac{\pi P r^4}{8 \eta L}$$

With the help of poiseuille's formula, coefficient of viscosity of a liquid can be determined.

Sol. $\dfrac{\pi P_1 r_1^4}{8\eta\ell_1} = \dfrac{\pi P_2 r_2^4}{8\eta\ell_2}$

$\Rightarrow \quad \dfrac{P_1 \times r^4}{\ell} = \dfrac{P_2 \times (2r)^4}{4\ell}$

$\Rightarrow \quad \dfrac{P_1}{P_2} = \dfrac{4}{1}$

Ex. When a viscous liquid flows at a rate Q through a tube of radius r placed horizontally, a pressure difference P develops across the ends of the tube. If the radius of the tube is doubled and rate of flow also doubled. Then find out the pressure difference across the ends of the tube.

Sol. $Q = \dfrac{\pi P r^4}{8\eta\ell}$

$\Rightarrow \quad \dfrac{Q_1}{Q_2} = \dfrac{P_1 r_1^4}{P_2 r_2^4}$

$\Rightarrow \quad \dfrac{Q}{2Q} = \dfrac{Pr^4}{P_2(2r)^4}$

$\Rightarrow \quad P_2 = \dfrac{P}{8}$

Rack your Brain

A small sphere of radius r falls from rest in a viscous liquid. As a result, heat is produced due to viscous force. The rate of production of heat when the sphere attains its terminal velocity, is proportional to:

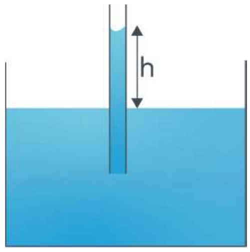
(1) r^3 (2) r^4

(3) r^5 (4) r^2

EXAMPLES

Q1 **A tube of 1 mm bore is dipped into a vessel containing a liquid of density 0.8 g/cm³, surface tension 30 dyne/ cm and angle of contact zero. Calculate the length which the liquid will occupy in the tube when the tube Is held**
(a) vertical
(b) inclined to the vertical at an angle of 30°.

Sol: (a) for vertically placed tube:

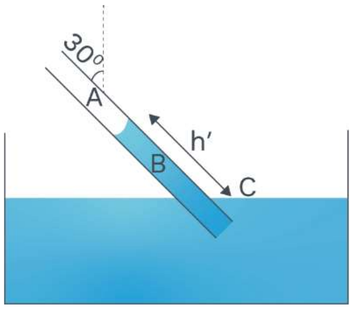


$$h = \frac{2T\cos\theta}{r\rho g}$$

Where r = 0.5 mm.

$$\text{So } h = \frac{2\times 30\,\frac{10^{-5}}{10^{-2}}\left(\frac{N}{m}\right)\times\cos\theta}{0.5\times 10^{-3}m\times 800\,\frac{Kg}{m^3}\times 9.8\left(\frac{m}{sec^2}\right)} = 1.51 \text{ cm.}$$

(b) for inclined tube:



We know that

$P_A = P_0$

$P_C = P_0$

and $P_B = P_0 - \frac{2T}{R}$

where P_0 = atmospheric pressure

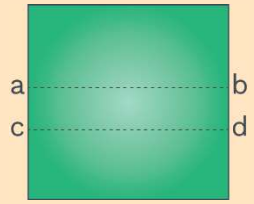
Equating pressure of the level in liquid : we get

$$P_0 - \frac{2T}{R} + \left(h'\cos 30^\circ\right)\rho g = P_0.$$

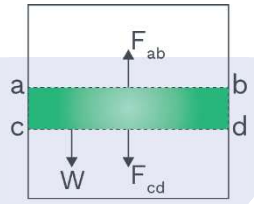
$$\Rightarrow \quad h' = \frac{2T}{\left(\cos 30^\circ\right)R\rho g} = \frac{2T\cos\theta}{\left(\cos 30^\circ\right)r\rho g} = \frac{h}{\cos 30^\circ}$$

Where h was found in part (a) so $h' = \frac{h}{\cos 30^\circ}$ = 1.749 cm.

Q2 **A soap film is stretched over a rectangular vertical wire frame as shown in the figure, what forces hold section abcd in equilibrium ?**



Sol:



The FBD of part abcd is as shown. The surface tension forces F_{ab} upward; F_{cd} downward and weight (w) of part abcd are holding the part abcd in equilibrium.

$F_{ab} - F_{cd} = w.$

Clearly $F_{ab} > F_{cd}$ and this is due to difference in concentration of soap solution in film.

Q3 **A mercury drop of radius 1.0 cm is sprayed into 10^6 droplets of equal size. Calculate the energy expanded. (Surface tension of mercury = 32×10^{-2} N/m).**

Sol: We know that dw = T dA

$\Rightarrow \Delta W = T\, \Delta A$

$\therefore$ in drops, only one surface area is formed.

and $\Delta A = 10^6 \times 4\pi r^2 - 4\pi R^2.$

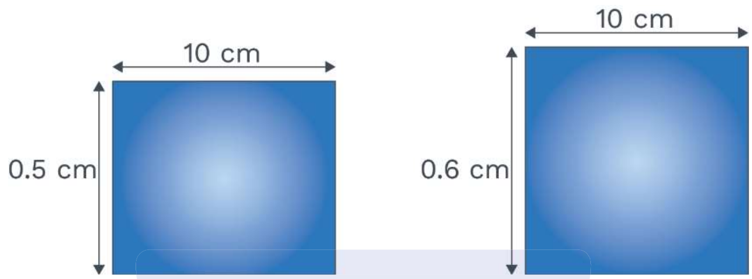
$= 4\pi r^2 (100 - 1)$

So $\Delta W = \left(32 \times 10^{-2} \frac{N}{m}\right) \times 4\pi \left(10^{-2} m\right)^2 [99]$

$= 3.978 \times 10^{-2}$ J.

Q4 **A film of water is formed between two straight parallel wires each 10 cm long and at separation 0.5 cm. Determine the work required to increase 1 mm distance between wires. Surface tension = 72×10^{-3} N/m.**

Sol:



The process is shown in the figure. As we have to produce 2 films; so

$\Delta W = (2\ \Delta A)\ T$

$= 2\left[10\,\text{cm} \times 0.6\,\text{cm} - 10\,\text{cm} \times 0.5\,\text{cm}\right] \times 72 \times 10^{-3}\ \frac{N}{m}$

$= 2 \times \left(10 \times 10^{-2} m\right)\left(0.1 \times 10^{-2} m\right) \times 72 \times 10^{-3}\ \frac{N}{m}$

$= 1.44 \times 10^{-5}$ J.

Q5 **The work done in blowing a bubble of volume V is W, then what is the work done in blowing a soap bubble of volume 2V ?**

Sol: Given $W = 2 \times (4\pi R^2) \times T$

Where $\frac{4}{3}\pi R^3 = V$ ...(i)

and we want of find $W' = 2 \times (4\pi)(R')^2 \times T$

where $\frac{4}{3}\pi (R')^3 = 2V$ ...(ii)

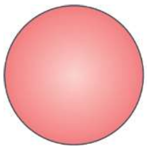
Dividing equations (i) and (ii)

$\frac{R}{R'} = \frac{1}{2^{\frac{1}{3}}} \Rightarrow R' = 2^{\frac{1}{3}} R.$

So $W' = 2T \times 4\pi \times 2^{\frac{2}{3}}.R^2 = 2^{\frac{2}{3}}\left(4\pi 2TR^2\right) = 2^{\frac{2}{3}}$ W.

Q6 **Find the excess pressure inside a drop of mercury of radius 2 mm, a soap bubble of radius 4 mm and an air bubble of radius 4 mm formed inside a tank of water. Surface tension of mercury is 0.465 N/m and soap solution and water are, 0.03 N/m and 0.076 N/m respectively.**

Sol: (a) drop of r = 2 mm.



$$P_{excess} = \frac{2T}{R} = \frac{2 \times 0.465}{2 \times 10^{-3} m}\frac{N}{m} = 465\frac{N}{m^2}$$

(b) Soap bubble has 2 films :

$$\text{So } P_{excess} = \frac{4T}{R} = \frac{4 \times 0.03\frac{N}{m}}{4 \times 10^{-3} m} = 30\frac{N}{m^2}$$

(c) As the air bubble is being formed inside a tank of water; so only one layer is formed.

$$P_{excess} = \frac{2T}{R} = \frac{2 \times 0.076\frac{N}{m}}{4 \times 10^{-3} m} = 38\frac{N}{m^2}.$$

Q7 **Two identical soap bubbles each of radius r and of the same surface tension T combine to form a new soap bubble of radius R. The two bubbles contain air at the same temperature. If the atmospheric pressure is p_0 then find the surface tension T of the soap solution in terms of p_0, r and R. Assume process is isothermal.**

Sol: Total number of moles of air in the two soap bubbles = number of moles of air in the resulting bubble.

$$\frac{2pv}{RT} = \frac{p'v'}{RT}$$

$$2pv = p'v'$$

$$2\left(p_0 + \frac{4T}{r}\right)\frac{4}{3}\pi r^3 = \left(p_0 + \frac{4T}{R}\right)\frac{4}{3}\pi R^3$$

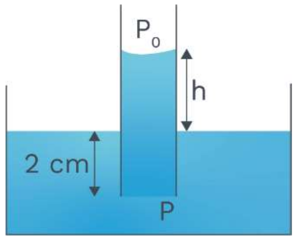
$$2\left(p_0 + \frac{4T}{r}\right)r^3 = \left(p_0 + \frac{4T}{R}\right)R^3$$

$$\therefore \quad T = \frac{p_0\left(R^3 - 2r^3\right)}{8r^2 - 4R^2} = \frac{p_0\left(R^3 - 2r^3\right)}{4\left(2r^2 - R^2\right)}.$$

Q8 **A capillary of 1 mm diameter, is dipped vertically in a pot of water. Find gauge pressure of the water in the tube 5.0 cm below the surface. Surface tension of water = 0.075 N/m. (take g = 9.8 m/s² and ρ_w = 1000 kg/m³).**

Sol: Assuming the contact angle to be 0°.

$$h = \frac{2T\cos\theta}{r\rho g} = \frac{2 \times 0.075 \frac{N}{m} \times 1}{0.5 \times 10^{-3} \times 1000 \frac{kg}{m^3} \times 9.8} = 3 \text{ cm.}$$



Now the pressure at point P (figure); 2 cm. below the water surface effectively will be.

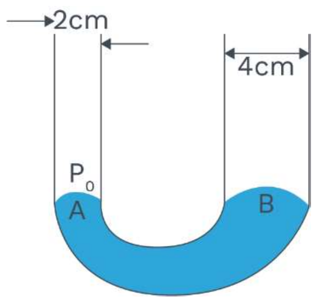
$\rho g \times 2$ cm = 196 N/m².

Q9 **The internal radius of one limb of a capillary U-tube is r_1 = 1 mm and the internal radius of the second limb is r_2 = 2 mm. The tube is filled with some mercury, and one of the limbs is connected to a vacuum pump. What will be the difference in air pressure when the mercury levels in both limbs are at the same height ? Which limb of the tube should be connected to the pump ? The surface tension and density of mercury are 480 dyn/ cm and 13.6 gm/ cm³ respectively. (assume contact angle to be θ = 180°) (g = 9.8 m/s²)**

Sol: Pressure difference = $\frac{2T}{r_1} - \frac{2T}{r_2}$ {$\because$ angle of contact is 180°}

$$\Rightarrow \rho_{Hg} h_{Hg} \times g = 2T\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

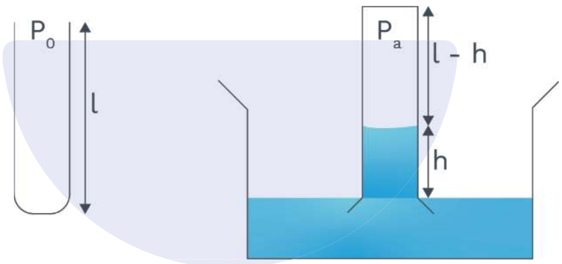
$$\Rightarrow h_{Hg} = \frac{2T}{\rho g}\left(\frac{r_2 - r_1}{r_1 r_2}\right) = 3.53 \text{ mm of Hg.}$$



As $P_A > P_B$, although they are at same height, hence the air above the point B has been evacuated. So the bigger limb of the tube should be connected to the pump.

Q10 **A capillary tube sealed at the top has an internal radius of r = 0.05 cm. The tube is placed vertically in water, with its open end dipped in water. What should be the length of such a tube for the water in it to rise in these conditions to a height h = 1 cm ? The pressure of the air is P_0 = 1 atm. = 76 cm of Hg, density of Hg = 13.6 g/cm³, g = 9.8 m/sec². The surface tension of water is σ = 70 dyn/cm.**
(assume temperature of air in the tube is constant).

Sol: Let the length of capillary = ℓ.



Now $P_a \times (\ell - h) = P_0 \times \ell$ ...(1)

and $P_a - \frac{2\sigma}{r} + \rho g h = P_0$

$\Rightarrow \frac{P_0 \ell}{\ell - h} - P_0 = \frac{2\sigma}{r} - \rho g h$

$\Rightarrow \frac{P_0}{\ell - h} = \frac{2\sigma}{rh} - \rho h$

$\Rightarrow \ell - h = \frac{P_0 rh}{2\sigma - \rho grh}$

$\Rightarrow \ell = h + \frac{P_0 rh}{2\sigma - \rho grh} \approx 556.55$ cm.

Q11 **The end of a capillary tube with a radius r is immersed into water. What amount of heat will be evolved when the water rises in the tube? If surface tension of water 'T' density of water = ρ.**

Sol: Here, the work done by surface tension force is being converted into gravitational potential energy and heat.

So $W_{F_s} = U_g + \text{heat}$

$\Rightarrow$ $(2\pi r)(T) \times (h) = mg\, h/2 + \text{heat}$ {h/2 because of P.E. of com.}

$$\Rightarrow \quad 2\pi T \times r \times \frac{2T}{r\rho g} = \frac{\left(\rho g \times \pi r^2 \times h\right) \times 2T}{r\rho g} \times \frac{1}{2} + \text{heat}$$

$$\text{heat evolved} = \frac{2\pi T^2}{\rho g}.$$

Q12 **An open capillary tube contains a drop of water. When the tube is in its vertical position, the drop forms a column with a length of**

(a) 2 cm (b) 4 cm (c) 2.98 cm.

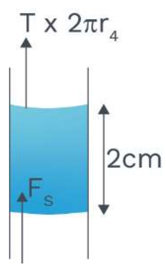
The internal diameter of the capillary tube is 1 mm. Determine the radii of curvature of the upper and lower meniscuses in each case. Consider the wetting to be complete. Surface tension of water = 0.073 N/m. (g = 9.8 m/s²).

Sol: As the wetting is complete. so upper meniscus radius will be same as radius of capillary in all cases.

$$r_4 = \frac{r_{\text{capillary}}}{2} = 0.5 \text{ mm.}$$

Now (a)

$F_s = T \times 2\pi rl.$



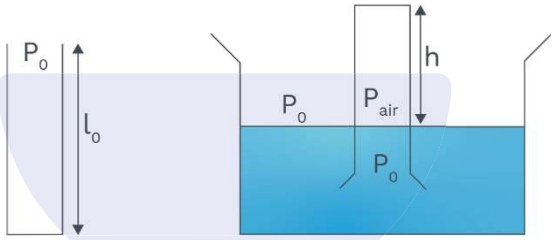
So balancing forces in vertical direction.

$$\rho \times \left(\pi r_c^2 \times h\right) \times g = T \times 2\pi r_l + T \times 2\pi r_c.$$

$$\text{Get, } r_1 = \frac{\rho h g r_c^2}{2T} - r_c.$$

Q13 **A glass capillary sealed at the upper end is of length 0.11 m and internal diameter 2×10^{-5} m. Tube is submerged vertically into a liquid of the surface tension of 5.06×10^{-2} N/m. To what length has capillary to be submerged so that the liquid level inside and outside capillary becomes the same. What will happen to water level inside capillary if the seal is now broken. Suppose isothermal condition in the tube. (Use g = 10 m/s²).**

Sol: Initially



Clearly: $P_{air} - P_0 = \frac{2\sigma}{r}$

∵ same level has pressure P_0 and by mole conservation.

$P_0 \times \ell_0 = P_{air} \times h$ ...(2)

$= \frac{P_0 \ell_0}{h} - P_0 = \frac{2\sigma}{r}$; $h = \frac{P_0 \ell_0}{\frac{2\sigma}{r} + P_0}$

⇒ h = 0.1 m.

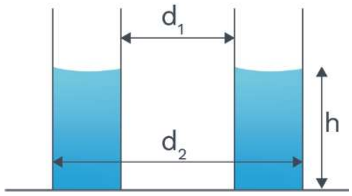
Hence the part inside the water is 0.11 – 0.1 = 0.01 m.

If seal is broken, the atmospheric air will be exerting the pressure which is lesser than P_{air}. Hence the liquid will rise up in the capillary.

Q14 **A glass rod of diameter d_1 = 1.5 mm is inserted symmetrically into a glass capillary with inside diameter d_2 = 2.0 mm. Then the whole arrangement is vertically oriented and brought in contact with the surface water. To what height will the liquid rise in the capillary. Surface tension of water = 73×10^{-3} N/m,**

Angle of contact = 0°. (use g = 9.8 m/s²).

Sol: Now there will be two meniscuses.
Net upward surface tension force is balanced by the weight of the column.



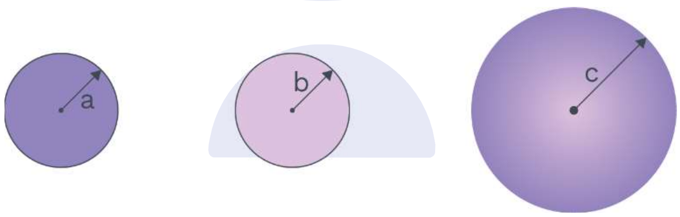
$$\Rightarrow \quad T \times 2\pi \frac{d_1}{2} + T \times 2\pi \frac{d_2}{2}$$

$$= \pi \times \left(\frac{d_2^2 - d_1^2}{4} \right) \times h\rho g$$

we get , $h = \dfrac{4T}{\rho g \left(d_2 - d_1\right)}$ = 6 cm.

Q15 **Under isothermal condition two soap bubbles of radii a and b coalesce to form a single bubble of radius c. If the external pressure is p_0, show that the surface tension $T = p_0\left(c^3 - a^3 - b^3\right) / 4\left(a^2 + b^2 - c^2\right)$.**

Sol:



$P_a = P_0 + \dfrac{4T}{a}$ and $P_b = P_0 = \dfrac{4T}{b}$ T = surface tension

$P_C = P_0 + \dfrac{4T}{a}$

Conserving moles:

$P_{\mathcal{A}} a^{\mathcal{A}} + P\ b \ = P\ .C$ putting the values

$P_0\left[a^3 + b^3 - c^3\right] + 4T\left[a^2 + b^2 - c^2\right] = 0$.

get $T = \dfrac{P_0\left(c^3 - a^3 - b^3\right)}{4\left(a^2 + b^2 - c^2\right)}$.

Q16 **A spherical ball of radius 3.0 × 10^{-4} m and density 10^4 kg/m³ falls freely under the gravity through a distance 'h' before entering a tank of the water. If after entering the water velocity of the ball doesn't change, find h. Viscosity of the water is 9.8 × 10^{-6} N-s/m². [g = 9.8 m/s²]**

Sol:

$$v = \frac{2}{9\eta} r^2 . (\rho_0 - \rho_w) g$$

$$= 180 \text{ m/sec.}$$

$$h = \frac{v^2}{2g} = \frac{32400}{2 \times 9.8} = \frac{81}{49} \times 10^3 \text{m}.$$

Q17 **A small sphere falls from the rest in the viscous liquid. Because of friction, heat is produced. Find the relation between the rate of the production of the heat and the radius of the sphere at the terminal velocity.**

Sol:

Terminal velocity $v_T = \frac{2r^2 g}{9\eta}(\rho_s - \rho_L)$

and viscous force $F = 6\pi\eta r v_T$

Viscous force is the only dissipative force. Hence,

$$\frac{dQ}{dt} = Fv_T = (6\pi\eta r v_T)(v_T) = 6\pi\eta r v_T^2$$

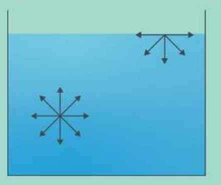
$$= 6\pi\eta r \left\{\frac{2}{9}\frac{r^2 g}{\eta}(\rho_s - \rho_L)\right\}^2 = \frac{8\pi g^2}{27\eta}(\rho_s - \rho_L)^2 r^5$$

$$\Rightarrow \frac{dQ}{dt} \propto r^5.$$

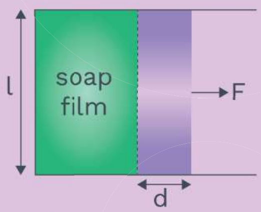
Mind Map

SURFACE TENSION (T)
The property of liquid by virtue of which free surface of liquid behaves like an elastic stretched membrane.

Surface Energy
The molecules at surface have extra energy as compared to molecules in the interior.
E = (Surface Tension x Surface Area)



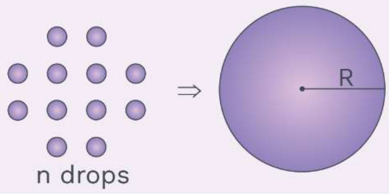
Work Done
W = Surface Tension x Increase in Area



W = T x (2ld)

Formation of bigger drop of radius R by n drops

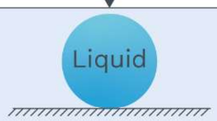
$$W = 4\pi TR^2 (n^{1/3} - 1)$$



Increase in temperature

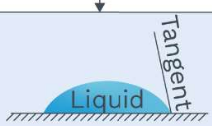
$$\Delta\theta = \frac{4\pi TR^2 (n^{1/3} - 1)}{mC}$$

Angle of Contact (θ)

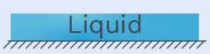


Contact angle - 180°
Not wetting

Contact angle > 90°
Low wetting



Contact angle < 90°
High wetting



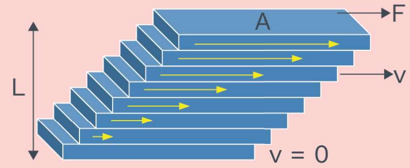
Contact angle - 0°
Perfect wetting

VISCOSITY
The property of fluid which give rise to frictional force when there is relative motion between the layers of a fluid is known as Viscosity

VISCOUS FORCE

$F = \eta A\left(\frac{v}{l}\right)$

η = Coefficient of viscosity



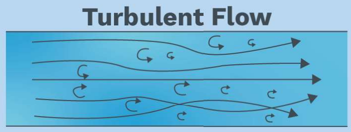
REYNOLD'S NUMBER

$R_e = \frac{\rho v d}{\eta}$

If $R_e < 1000 \Rightarrow$ Flow is steady
If $1000 < R_e < 2000 \Rightarrow$ Flow is unsteady
If $R_e < 2000 \Rightarrow$ Flow is turbulent

Laminar Flow

Turbulent Flow



STOKE'S LAW
Viscous force experienced by body moving through fluid.

$F = -6\pi\eta r v$

TERMINAL VELOCITY $v_T = \frac{2}{9}\frac{r^2}{\eta}(\rho - \sigma)$