Sound Waves

Sound Waves

Sound Waves

Sound is the type of longitudinal waves. In general majority of the longitudinal waves are termed as the sound waves. Sound is produced by the vibrating source, like when a gong of the bell is struck with the hammer, sound is produced. The vibrations produced by the gong are propagated through the air, Through air these vibrations reach to ear and ear drum is set into the vibrations and these vibrations are communicated to the human brain. By touching gong of bell by hand we can feel the vibrations.

KEY POINTS

- Sound waves
- Infrasonic waves
- Audible waves
- Ultrasonic waves

Classification of sound wave

Sound waves can be classified in the three groups according to their range of frequency.

Infrasonic Waves

Longitudinal waves having frequencies below 20 Hz are called infrasonic waves. They cannot be heard by human beings. They are produced during earthquakes. Infrasonic waves can be heard by snakes.

Concept Reminder

Sound waves can be classified in three groups according to their range of frequencies

1. Infrasonic waves
2. Audible waves
3. Ultrasonic waves

Audible Waves

Longitudinal waves having frequencies lying between 20-20,000 Hz are called audible waves.

Ultrasonic Waves

Longitudinal waves having frequencies above 20,000 Hz are called ultrasonic waves. They are produced and heard by bats. They have a large energy content.

- **Equations of sound wave**

A longitudinal mechanical wave can be described in two ways :

(A) Displacement wave form

(B) Pressure wave form

(A) **Displacement wave form :-** When sound wave is described in term of longitudinal displacement suffered by particles of the medium, it is called displacement wave.

Which can be given by $y = A\sin(\omega t - kx)$

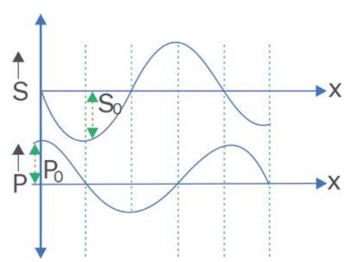
(B) Pressure wave form:- When sound wave is described in term of excess pressure generated due to compression and rarefaction called pressure wave.

Which can be given by $\Delta P = P_0 \cos(\omega t - kx)$

$P_0 = ABK$

where A = displacement of amplitude,
B = Bulk modulus,
K = propagation constant,
P_0 = Amplitude of pressure wave

- The phase difference between pressure wave form and displacement wave form is 90° and path difference is $\frac{\lambda}{4}$. Displacement will be maximum when pressure is minimum and vice versa.
- When we consider the interference of sound as pressure wave then there is no change in phase when reflected from a "rigid boundary" but have a phase change of π when reflected from free end.



This is in contrast to reflection of displacement wave which have a phase change of π from a "rigid-end" and no change in phase from "free-end".

- A sound sensor-e.g., ear, mike or listener, observer detects change in pressure. So, in this case we prefer pressure wave.

Concept Reminder

1. Displacement waveform
   $y = A\sin(\omega t - kx)$
2. Pressure waveform
   $\Delta P = P_0 \cos(\omega t - kx)$

Concept Reminder

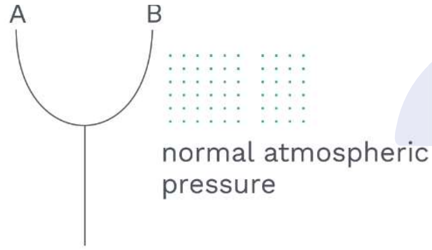
The phase difference between pressure wave form and displacement wave form is 90° and path difference is $\frac{\lambda}{4}$. Displacement will be maximum when pressure is minimum and vice versa.

- If detector is displacement sensor, we will prefer displacement wave.
- In stationary wave at the place of displacement node, pressure antinodes will form and vice versa.

Propagation of Sound Waves

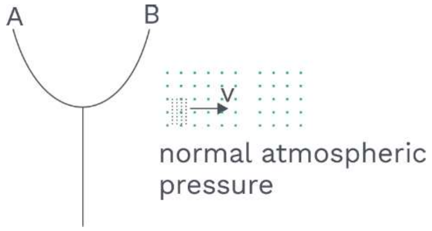
Sound is the mechanical three dimensional and longitudinal wave that is created by the vibrating source such a guitar string, the human vocal cords, the prongs of a tuning fork or diaphragm of a loudspeaker. Being a mechanical wave, sound needs medium having properties of the inertia and elasticity for its propagation. Sound waves propagate in any medium through a series of periodic compressions and rarefactions of pressure, which is produced by the vibrating source.

Consider a tuning fork producing sound waves.



Undisturbed turning fork

When Prong B moves outward towards right it compresses the air in front of it, causing the pressure to rise slightly. The region of increased pressure is called a compression pulse and it travels away from the prong with the speed of sound



Concept Reminder

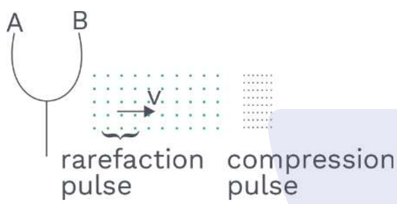
Sound waves propagate in any medium through a series of periodic compressions and rarefactions of pressure, which is produced by the vibrating source.

Rack your Brain

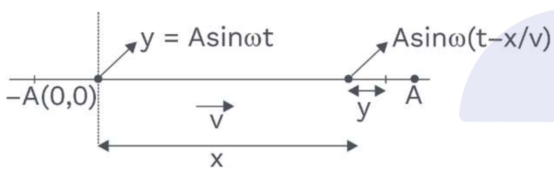
The fundamental frequency of a closed organ pipe of length 20 cm is equal to the second overtone of an organ pipe open at both the ends. The length of organ pipe open at both the ends is

(1) 140 cm (2) 80 cm
(3) 100 cm (4) 120 cm

After producing the compression pulse, the prong B reverses its motion and moves inward. This drags away some air from the region in front of it, causing the pressure to dip slightly below the normal pressure. This region of decreased pressure is called rarefaction pulse. Following immediately behind the compression pulse, the rarefaction pulse also travels away from the prong with the speed of sound.



A longitudinal wave in a fluid is described either in teams of the longitudinal displacements suffered by particles of medium.



Consider the wave going in the 'x'-direction in a fluid. Suppose that at a time t, the particle at the undisturbed position x suffers a displacement y in the x-direction.

$$y = A\sin\omega\left(t - \frac{x}{v}\right) \quad \text{... (i)}$$

Position of the any particle from the origin at any time = x + y

x = Distance of the mean position of the particle from the origin.

y = Displacement of the particle from its mean position.

General Equation :

$$(0, 0) \Rightarrow y = A\sin(\omega t + \phi)$$

$$(0, x) \Rightarrow y = A\sin[\omega(t - x/v) + \phi]$$

KEY POINTS

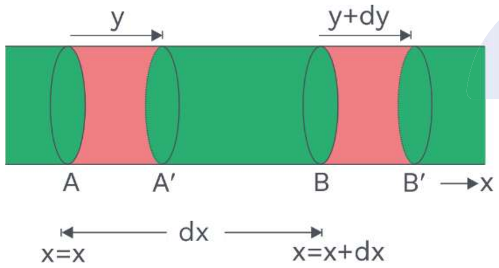
- Rarefaction
- Compression
- Pressure wave
- Longitudinal wave

Displacement wave $y = A\sin(\omega t - kx + \phi)$

- If we fix 'x' = x_0 then we are dealing with the particle whose mean position at distance x_0 from origin & this particle perform SHM of amplitude A with time period T and phase difference $= -kx + \phi$

Compression Waves

When the longitudinal wave propagated in the gaseous medium, it produces compression and rarefaction in medium periodically. Region where compression occurs, pressure is more than the normal pressure of the medium. Thus we can also describe longitudinal waves in a gaseous medium as pressure waves and these are also termed as compression waves in which the pressure at different point of medium also varies periodically with their displacements. Let us discuss the propagation of excess pressure in a medium in longitudinal wave analytically.



Consider a longitudinal wave propagating in positive x-direction as shown in figure. Figure shows a segment AB of the medium of width dx. In this medium let a longitudinal wave is propagating whose equation is given as

$$y = A\sin(\omega t - kx) \quad \text{... (1)}$$

Where 'y' is the displacement of medium particle situated at a distance 'x' from the origin, along the direction of propagation of the wave. In the figure shown AB is the medium segment whose a medium particle is at the position x = x and B is at x = x + dx at an instant. If after some time t

Concept Reminder

When a longitudinal wave propagated in a gaseous medium, it produces compression and rarefaction in the medium periodically. The region where compression occurs, the pressure is more than the normal pressure of the medium

medium particle at 'A' reaches to a point 'A' which is displaced by y and the medium particle at b reaches to point 'B' which is at a displacement y + dy from B. Here dy is given by equation

$$dy = -Ak\cos(\omega t - kx)dx$$

Here due to the displacement of section AB to A'B' the change in volume of it's section is given as

$$dV = -S\,dy \qquad [S \rightarrow \text{Area of cross-section}]$$

$$= SA\,k\cos(\omega t - kx)dx$$

The volume of section AB is V = S dx
Thus volume strain in section AB is

$$\frac{dV}{V} = \frac{-SAk\cos(\omega t - kx)dx}{Sdx}$$

or $$\frac{dV}{V} = -Ak\cos(\omega t - kx)$$

If B is the bulk modulus of medium, then the excess pressure in the section AB can be given as

$$\Delta P = -B\left(\frac{dV}{V}\right) \quad \text{... (2)}$$

$$\Delta P = BAk\cos(\omega t - kx)$$

or $$\Delta P = \Delta P_0\cos(\omega t - kx) \quad \text{... (3)}$$

Here ΔP_0 is the pressure amplitude at a medium particle at position x from origin and ΔP is excess pressure at that point. Equation shown that the excess varies periodically at every point of medium with pressure amplitude ΔP_0, which is given as

$$\Delta P_0 = BAk = \frac{2\pi}{\lambda}AB \quad \text{... (4)}$$

Equation shown is also termed as equation of pressure wave in the gaseous medium. We can also see that the pressure wave differs in phase is $\frac{\pi}{2}$ from the displacement wave and pressure maxima occurs where displacement is zero and displacement maxima occur where pressure is at

Concept Reminder

In pressure wave equation
$\Delta P = \Delta P_0\cos(\omega t - kx)$

ΔP_0 is the pressure amplitude at a medium particle at position x from origin and ΔP is the excess pressure at that point.

$$\Delta P_0 = BAk = \frac{2\pi}{\lambda}AB$$

its normal level. Remembers that pressure maxima implies that pressure at a point is the pressure amplitude times more or less then normal pressure level of the medium.

Ex. A sound wave of the wavelength 40 cm travels in the air. If difference between the maximum and minimum pressures at a given point is the 2.0×10^{-3} N/m^2, find out amplitude of vibration of particles in medium. The bulk modulus of the air is 1.4×10^5 N/m^2

Sol. The pressure amplitude is

$$p_0 = \frac{2.0 \times 10^{-3} N/m^2}{2} = 10^{-3} N/m^2$$

The displacement amplitudes s_0 is given by

$$p_0 = Bk\,s_0$$

$$\text{or, } s_0 = \frac{p_0}{Bk} = \frac{p_0 \lambda}{2\pi B} = \frac{10^{-3} N/m^2 \times (40 \times 10^{-2} m)}{2 \times \pi \times 14 \times 10^4 N/m^2}$$

$$= \frac{100}{7\pi} Å = 4.6Å$$

Density Wave

In this section we will find out relation between pressure wave and density wave. According to definition of bulk modulus (B),

$$B = \left(-\frac{dP}{dV/V}\right)$$

Further, $\text{Volume} = \frac{\text{mass}}{\text{density}}$

or $V = \frac{m}{\rho}$ or $dV = -\frac{m}{\rho^2}.d\rho = -\frac{V}{\rho}.d\rho$

or $\frac{dV}{V} = -\frac{d\rho}{\rho}$

Substituting in Eq. (i), we get

$$d\rho = \frac{\rho(dP)}{B} = \frac{dP}{v^2} \qquad \left(\frac{\rho}{B} = \frac{1}{v^2}\right)$$

Or this can be written as,

Concept Reminder

Bulk modulus is defined as

$$B = -\frac{dP}{dV/V}$$

$$\Delta\rho = \frac{\rho}{B}.\Delta P = \frac{1}{v^2}\Delta P$$

So, this relation relates the pressure equation with density equation. For example, if

$$\Delta P = (\Delta P)_m \sin(kx - \omega t)$$

then $\Delta\rho = (\Delta\rho)_m \sin(kx - \omega t)$

where, $(\Delta\rho)_m = \frac{\rho}{B}.(\Delta P)_m = \frac{(\Delta P)_m}{v^2}$

Thus, density equation is in phase with pressure equation and this is 90° out of phase with the displacement equation.

Velocity And Acceleration of particle :

General equation of wave is given by

$$y = A\sin(\omega t - kx)$$

$$v_P = \frac{\partial y}{\partial t} = A\,\omega\cos(\omega t - kx) \quad \text{... (1)}$$

$$a_P = \frac{\partial^2 y}{\partial t^2} = -A\omega^2 \sin(\omega t - kx) \quad \text{... (2)}$$

$$\frac{\partial y}{\partial x} = -Ak\cos(\omega t - kx) \quad \text{... (3)}$$

Here $\boxed{\frac{\partial y}{\partial x} = \text{slope of } (y, x) \text{ curve}}$

Now again differentiate equation-3

$$\frac{\partial^2 y}{\partial x^2} = -Ak^2 \sin(\omega t - kx) \quad \text{... (4)}$$

from eq. (2) & (4)

$$\boxed{\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}}$$

Velocity of Sound/Longitudinal Waves In Solids

Consider a section AB of the medium as shown in figure(a) of cross-sectional area S. Let A and B be two cross section as shown. Let in this medium sound propagation is from left to the right. If wave source is at the origin O and when it oscillates, the oscillations at the point propagate along rod.

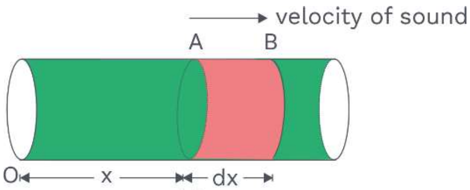
Concept Reminder

- Displacement of particle in wave
  $y = A\sin(\omega t - kx)$
- Velocity of particle in wave
  $v_P = A\omega\cos(\omega t - kx)$
- Acceleration of particle in wave
  $a_P = -A\omega^2 \sin(\omega t - kx)$

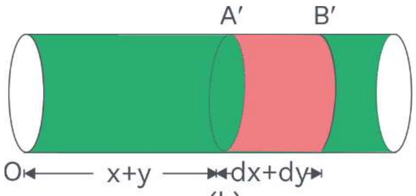
Concept Reminder

General Equation of wave in differential form is

$$\boxed{\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}}$$



(a)



(b)

Here, we say an elastic wave has propagated along the rod with the velocity determined by physical properties of medium. Due to oscillations say a force F is developed at the every point of medium which produces a stress in the rod and is the cause of strain or propagation of disturbance along rod. This stress at the any cross-sectional area can be given as

$$\text{Stress } S_1 = \frac{F}{S} \qquad \text{... (1)}$$

If we consider the section AB of medium at a general instant of time t. The end A is at a distance x from 'O' and 'B' is at a distance 'x + dx' from 'O'. Let in time dt due to oscillations, medium particles at a are displaced along the length of medium by y and those at B by y + dy. The resulting position of section and A’ and B’ shown in figure 'b', Here we can say that the section AB is deformed (elongated) by a length 'dy'. Thus strain produced in it is

$$\text{Strain in section AB} \qquad E = \frac{dy}{dx} \qquad \text{... (2)}$$

If Young’s modulus of the material of medium is Y, we have

$$\text{Young's Modulus } Y = \frac{\text{Stess}}{\text{Strain}} = \frac{S_1}{E}$$

Rack your Brain

The number of possible natural oscillations of air column in a pipe closed at one end of length 85 cm whose frequencies lie below 1250 Hz are (velocity of sound = 340 ms $^{-1}$)

(1) 4
(2) 5
(3) 7
(4) 6

From equation (1) and (2), we have $Y = \frac{F / A}{dy / dx}$

A is area of cross-section

or $F = YA\frac{dy}{dx}$ ... (3)

If net force acting of secting 'AB' is 'dF' then it is given as:

$dF = dma$ ...(4)

Where dm is the mass of section AB and a be its acceleration, which can be given as for a medium of density ρ.

$$dm = \rho A dx \quad \text{and} \quad a = \frac{d^2y}{dt^2}$$

From equation (4), we have $dF = (\rho A dx)\frac{d^2y}{dt^2}$

or $\frac{dF}{dx} = \rho A \frac{d^2y}{dt^2}$ ... (5)

From equation (3) on differentiating w.r.t to 'x', we can write

$$\frac{dF}{dx} = YS\frac{d^2y}{dx^2} \quad ...(6)$$

From equation '5' & '6' we get,

$$\frac{d^2y}{dt^2} = \frac{Y}{\rho}\frac{d^2y}{dx^2} \quad ....(7)$$

Equation '7' is the differential form of wave equation, comparing it with previous equation we get the wave velocity in medium can be given as

$$v = \sqrt{\frac{Y}{\rho}}$$

Similar to case of a solid in fluid, instead of Young's Modulus we use Bulk modulus of the medium hence velocity of the longitudinal waves in a fluid medium is given as

$$v = \sqrt{\frac{B}{\rho}}$$

Concept Reminder

Speed of longitudinal wave in solid medium is

$$v = \sqrt{\frac{Y}{\rho}}$$

Concept Reminder

Speed of longitudinal wave in fluid is

$$v = \sqrt{\frac{B}{\rho}}$$

Where B is the Bulk modulus of medium. For a gaseous medium bulk modulus is defined as

$$B = \frac{dP}{(-dV/V)} \quad \text{or} \quad B = -V\frac{dP}{dV}$$

Newton's formula

For any medium $\boxed{v_L = \sqrt{\frac{E}{\rho}}}$ where E $\to$ elastic modulus of medium, $\rho \to$ density of medium.

(A) In solid medium

$$\boxed{v = \sqrt{\frac{E_{solid}}{\rho}} = \sqrt{\frac{Y}{\rho}}}$$

where Y = Young's modulus.

(B) In liquid medium :

$$\boxed{v = \sqrt{\frac{E_{Liquid}}{\rho}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{K}{\rho}}}$$

where B = Bulk modulus or volumetric elastic modulus.

So $\Rightarrow$

E_{solid}	>>	E_{liquid}	>>	E_{gas}
ρ_{solid}	>	ρ_{liquid}	>	ρ_{gas}
v_{solid}	>	v_{liquid}	>	v_{gas}
$\downarrow$		$\downarrow$		$\downarrow$
Example: Soft iron		water		air
Speed of sound: $\simeq 5050\,m/sec$		$\cong 1450 m/sec$		$\simeq 330\,m/sec$

So,

E_{Quartz}	>	E_{Fe}	>	E_{Cu}
v_{Quartz}	>	v_{Fe}	>	v_{Cu}
$\downarrow$		$\downarrow$		$\downarrow$
$\cong 6000m/sec$		$\cong 5000m/sec$		$\cong 3500m/sec$

- Maximum speed of sound is in diamond about ($\cong$6500 m/sec) then in Quartz (6000 m/sec)
- In gaseous medium speed of sound is maximum in H_2 medium. $v_{H_2} \cong 1234\,m/sec$

Speed of Longitudinal Wave (Sound wave) in Gaseous medium.

Elastic modulus of gaseous medium

$$\boxed{E_{gas} = \frac{\text{stress}}{\text{strain}} = \frac{\Delta P}{\left(\frac{-\Delta V}{V}\right)} = -V\left(\frac{\Delta P}{\Delta V}\right)}$$

Concept Reminder

The absence of matter produces no sound because the sound vibrations needs a medium to pass through.

E_{gas}

Isothermal
PV = constant
Diff. w.r.t. volume

$$P + V\frac{\Delta P}{\Delta V} = 0$$

$$\boxed{E_{iso} = V\frac{\Delta P}{\Delta V} = P}$$

Adiabatic
PV^{γ} = constant

$$\boxed{E_{adia} = \gamma P}$$

Newton's Formula for Velocity of Sound in Gases

Newton assumed that during sound propagation temperature of medium remains constant hence stated that propagation of sound in the gaseous medium is an isothermal phenomenon, thus Boyle's law can be applied in the process. So for the section of medium we use

$$PV = \text{constant}$$

Differentiating we get

$$PdV + V\,dP = 0$$

or $$-V\frac{dP}{dV} = P$$

or bulk modulus of medium can be given as

$$B = P \text{ (Pressure of medium)}$$

Newton found that during isothermal propagation of the sound in a gaseous medium, bulk modulus of the medium is equal to the pressure of medium, hence sound velocity in a gaseous medium can given as;

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{P}{\rho}} \quad \text{... (1)}$$

From gas law we have $\frac{P}{\rho} = \frac{RT}{M}$ ... (2)

From (1) & (2) we have $v = \sqrt{\frac{RT}{M}}$ ... (3)

From the expression in equation (1) the sound velocity in air at the normal atmospheric pressure 'P' = 1.01×10^5 Pa is

(Density of air at NTP is $\rho = 1.293\,kg/m^3$)

Concept Reminder

- According to Newton propagation of sound in gaseous medium is isothermal propagation.
- Newton's formula

$$v = \sqrt{\frac{P}{\rho}}$$

$$v = \sqrt{\frac{1.01 \times 10^5}{1.293}} = 279.49\,m/s$$

But experimental value of velocity of sound determined from various experiments gives velocity of sound at NTP, 332 m/s. Therefore there is a difference of about 52 m/s between theoretical and experimental value. This large difference can't be attributed to experimental errors. Newton was unable to explain error in his formula. This correction was explained by the French Scientist Laplace.

Laplace Correction

Laplace explained that when sound waves propagated in the gaseous medium. There is compression and rarefaction in particles of the medium. Where there is 'compression', particles come near to the each other and heated up, where there is rarefaction, medium expands and there is the fall of the temperature. Therefore, the temperature of medium at the every point does not remain constant so process of sound propagation is not 'isothermal'. The total quantity of heat of system as whole remains the constant. medium does not gain or loose they any heat to surrounding. Thus in the gaseous medium sound propagation is the 'adiabatic process'. For adiabatic process the relation in pressure and volume of a section of medium can be given as

$$PV^{\gamma} = \text{constant} \qquad \text{... (1)}$$

Here $\gamma = \frac{C_p}{C_v}$, ratio of specific heats of the medium.

Differentiating equation (1) we get,

$$dPV^{\gamma} + \gamma V^{\gamma-1}PdV = 0$$

or $$dP + \gamma\frac{PdV}{V} = 0$$

Concept Reminder

- According to Laplace propagation of sound wave is a adiabatic process
- Laplace's correction $v = \sqrt{\frac{\gamma P}{\rho}}$

or $-V\frac{dP}{dV} = \gamma P$

Bulk modulus of medium $B = \gamma P$

Thus, Laplace found that during 'adiabatic' propagation of sound, the Bulk modulus of gaseous medium is equal to product of ratio of specific heats and pressure of medium. Thus, velocity of the sound propagation can given as;

$$v = \sqrt{\frac{\gamma B}{\rho}}$$

From gas law $v = \sqrt{\frac{\gamma RT}{M}}$

From the above equation we find sound velocity in the air at NTP, we have

Normal atmospheric pressure P= 1.01×10^5 Pa

Density of air at NTP ρ = 1.293 kg/m^3

Ratio of specific heat of air $\gamma = \frac{C_p}{C_v} = 1.42$

$$\Rightarrow \quad v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{1.42 \times 1.01 \times 10^5}{1.293}} = 333.04\,m/s$$

This value is in agreement with experimental value.

Now at any temperature 'T' °C velocity of sound

$$v_t = \sqrt{\frac{\gamma R(273 + t^\circ)}{M}} = \sqrt{\frac{\gamma R273}{M}}\left(1 + \frac{t}{273}\right)^{1/2}$$

$$\boxed{v_t = v_0\left(1 + \frac{t}{546}\right)}$$

Factor affecting the speed of sound in a gaseous medium :-

Rack your Brain

If we study the vibration of a pipe open at both ends, then the following statement is not true:

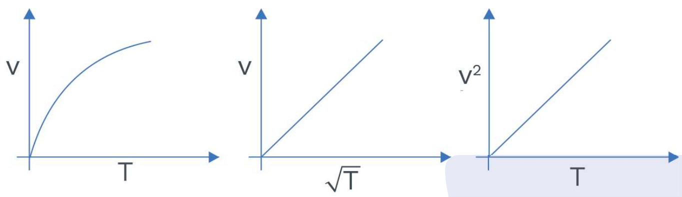
(1) Odd harmonics of the fundamental frequency will be generated
(2) All harmonics of he fundamental frequency will be generated
(3) Pressure change will be maximum at both ends
(4) Open end will be antinode

(1) Effect of Temperature (T).

We know that $v = \sqrt{\dfrac{\gamma RT}{M_w}}$

Here, γ, R and M_w = const, then $v \propto \sqrt{T}$

$$\boxed{\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}}$$



- $v \propto \sqrt{T} \rightarrow \boxed{\dfrac{\Delta v}{v} \times 100 = \dfrac{1}{2}\dfrac{\Delta T}{T} \times 100}$ valid upto 5% change. (small changes)
- Speed of sound in a gaseous medium at t°C temperature, $v_t \propto \sqrt{t + 237}$ ... (1)

  Speed of sound in a gaseous medium at 0°C temperature, $v_t \propto \sqrt{273}$ ... (2)

  From equation (1)/(2)

$$\frac{v_t}{v_0} = \left(\frac{t + 273}{273}\right)^{\frac{1}{2}} = \left(1 + \frac{t}{273}\right)^{\frac{1}{2}}$$

$$\Rightarrow v = v_0\left(1 + \frac{t}{273}\right)^{\frac{1}{2}}$$

$$\begin{bmatrix} \text{Binomial exp}^n : - \\ (1 + x)^n \cong 1 + nx \text{ if } x << 1 \end{bmatrix}$$

If t <<< 273, then

$$\boxed{v_t = v_0\left(1 + \frac{t}{546}\right)} \rightarrow \text{valid upto } 30°C.$$

- **Only for air medium-** v_0 = 330 m/sec

Concept Reminder

$$\boxed{v_t = v_0\left(1 + \frac{t}{546}\right)}$$

$\boxed{v_t = v_0 + 0.61t}$ only for air medium

Both valid upto 30°C.

$$v_t = v_0 + \frac{v_0 \times t}{546} \Rightarrow v_t = v_0 + \frac{330}{546} t \Rightarrow \boxed{v_t = v_0 + 0.61t}$$

$\Rightarrow$ upto 30°C

Note :- If temperature is increased by 1°C then speed of sound increases by 0.61 m/s (only for air)

(2) Effect of Pressure :-

At constant temperature there is no effect of pressure change on speed of sound in gaseous medium.

at const. temp. $\Rightarrow$ PV = const. $\Rightarrow$ $P \propto \frac{1}{V}$

$$v \propto \frac{1}{\rho} \Rightarrow \boxed{P \propto \rho}$$

$$v = \sqrt{\frac{\gamma P}{\rho}}$$ then v = const.

when pressure (P) changes then ρ changes and velocity (v) remains constant.

Note:- If in a question both temperature and pressure are changing then to solve question we take effect of temperature only.

Concept Reminder

At constant temperature there is no effect of pressure change on speed of sound in gaseous medium.

At const. temp. $\Rightarrow$ PV = const

$P \propto \frac{1}{V}$

(3) Effect of Nature of Gaseous Medium:-

We know that $v = \sqrt{\frac{\gamma RT}{M_w}}$, then $\boxed{v \propto \sqrt{\frac{\gamma}{M_w}}}$,

$$\frac{v_1}{v_2} = \sqrt{\frac{\gamma_1 \times (M_w)_2}{\gamma_2 \times (M_w)_1}}$$

$n \rightarrow$ no. of moles

Poly atomic $\rightarrow \gamma = \frac{4}{3} = 1.33$,

$$(M_w)_{mix} = \frac{n_1 M_{w_1} + n_2 M_{w_2}}{n_1 + n_2},$$

$$\gamma_{mix} = \frac{C_{P_{mix}}}{C_{V_{mix}}} = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 C_{V_1} + n_2 C_{V_2}}$$

(4) Effect of Density (Humidity)

We know that $v = \sqrt{\frac{\gamma P}{\rho}} \Rightarrow v \propto \frac{1}{\sqrt{\rho}}$

Now $\rho_{dry\,air} > \rho_{humid\,air}$

So, $\boxed{v_{humid\,air} > v_{dry\,air}}$

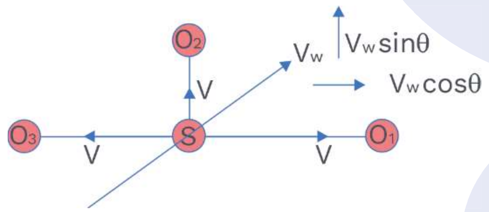
We know that $\boxed{\rho \propto M_w}$ at NTP

$(M_w)_{dry\,air} = 28.8$ g/mol

$(M_w)_{humid\,air}$ = dry air + H_2O(Between 18 and 28.8)

Means $(M_w)_{humid\,air} < (M_w)_{dry\,air}$ So $\boxed{\rho_{humid\,air} < \rho_{dry\,air}}$

(5) Effect of Wind (V_w):-



$(V_{eff})_{O_1} = V + V_w \cos\theta$

$(V_{eff})_{O_2} = V + V_w \sin\theta$

$(V_{eff})_{O_3} = V - V_w \cos\theta$

(6) Relation between v_{rms} and v_{sound}

$v_{rms} = \sqrt{\frac{3RT}{M}}$ and $v_{sound} = \sqrt{\frac{\gamma RT}{M}}$

$\boxed{\frac{v_{rms}}{v_{sound}} = \sqrt{\frac{3}{\gamma}}}$ But $1 < \gamma < 2$ so $\boxed{v_{rms} > v_{sound}}$

(7) Speed of sound is not affected by elements of wave such as amplitude, frequency, wavelength etc. and characteristics of wave such as loudness, pitch and quality. It is affected by medium and medium properties such as elasticity, density, inertia etc.

Concept Reminder

The loudest sound naturally occurs on Earth is the sound coming from a volcano eruption. The Krakatoa eruption in 1883 was recorded to have the loudest sound produced in the world.

Ex. In a gaseous medium on increase in temperature by 900 K speed of sound becomes double of its initial value. Find initial temperature in °C.

Sol. $\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} \Rightarrow \frac{x}{2x} = \sqrt{\frac{T}{T+900}} \Rightarrow 4T = T + 900$

$\Rightarrow T = 300\,K = (300 - 273)°C = 27°C$

Ex. If pressure of gaseous medium increase upto 4 times of its initial value at constant volume. Find effect on speed of sound.

Sol. $v \propto \sqrt{P} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{P}{4P}} \Rightarrow v_2 = 2v_1$

Ex. If change in temperature of any gaseous medium is 2%. Find out percentage change in velocity of sound.

Sol. $\frac{\Delta v}{v} \times 100 = \frac{1}{2} \times \frac{\Delta T}{T} \times 100 = \frac{1}{2} \times 2 = 1\%$

Ex. In gaseous medium v of sound is 1112 m/s at 10°C. Find out v at 5°C.

Sol. $v_t = v_0\left(1 + \frac{t}{546}\right) \Rightarrow 1112 = v_0\left(1 + \frac{10}{546}\right)$

$v_{5°C} = v_0\left(1 + \frac{5}{546}\right)$

On dividing $\frac{v_{5°C}}{1112} = \frac{546+5}{546+10} \Rightarrow \frac{v_{5°C}}{1112} = \frac{551}{556}$

$\Rightarrow v_{5°C} = 551 \times 2 = 1102\,m/s$

Ex. At NTP, speed of sound in air medium is 330 m/s. Find out speed of sound in same medium at 819°C.

Sol. $\frac{330}{v_2} = \sqrt{\frac{T_1}{T_2}} \Rightarrow \frac{330}{v_2} = \sqrt{\frac{273}{1092}} \Rightarrow v_2 = 660\,m/s$

Ex. Find out the ratio of speed of sound in H_2 & He at same temperature.

Sol. $\frac{v_{H_2}}{v_{He}} = \sqrt{\frac{\gamma_{H_2}}{\gamma_{He}} \times \frac{(M_w)_{He}}{(M_w)_{H_2}}} = \sqrt{\frac{7}{5} \times \frac{3}{5} \times \frac{4}{2}} = \sqrt{\frac{42}{25}}$

Ex. Find out distance travelled by longitudinal wave in solid in 2 s, if $Y = 2.2 \times 10^{11} N/m^2$, and $\rho = 8.8 \times 10^3$ kg / m^3

Sol. $v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2.2}{8.8} \times \frac{10^{11}}{10^3}} = \sqrt{\frac{10^8}{4}} \Rightarrow v = \frac{10^4}{2}$

$\Rightarrow v = 5 \times 10^3 m/s$

$\because d = v \times t$

$\therefore d = 2 \times 5 \times 10^3 = 10^4 m$

Ex. 2 mediums are given in which one is dry helium and other is humid helium. Find in which speed of sound is more

Sol. (M_w) of dry He = 4

(M_w) of humid He = 4 to 18

$(M_w)_{dry\ He} < (M_w)_{humid\ He}$

$(\rho \propto M_w)$

$\rho_{dry He} < \rho_{humid He}$

$v_{dry(He)} > v_{humid(He)}$

Ex. The equation of a sound wave in the air is given by

p = (0.02) sin [(3000) t – (9.0) x], where all the variables are in S.I. units.

(a) Find out 'frequency', 'wavelength' and speed of sound wave in air.

(b) If the equilibrium pressure of air is 1.0 × 10^5 N/m^2, what are the maximum and minimum pressures at a point as wave passes through that point?

Sol. (a) Comparing with the standard form of a travelling wave

$$p = p_0 \sin[\omega(t - x/v)]$$

we see that $\omega = 3000\, s^{-1}$. The frequency is

$$f = \frac{\omega}{2\pi} = \frac{3000}{2\pi} Hz$$

Also from the same comparison, $\omega / v = 9.0\, m^{-1}$.

or, $$v = \frac{\omega}{9.0\, m^{-1}} = \frac{3000 s^{-1}}{9.0\, m^{-1}} = \frac{1000}{3} m/s^{-1}$$

The wavelength is $\lambda = \frac{v}{f} = \frac{1000/3}{3000/2\pi} = \frac{2\pi}{9} m$

(b) The pressure amplitude is p_0 = 0.02 N/m². Hence, maximum and minimum pressures at the point in the wave motion will be $(1.01 \times 10^5 \pm 0.02)\,N/m^2$

Ex. A wave of wavelength '4' mm is produced in air and it travels at a speed of '300' m/s. Will it be audible?

Sol. From the relation $v = n\lambda$, the frequency of the wave is $n = \frac{v}{\lambda} = \frac{300\,m/s}{4 \times 10^{-3}m} = 75000\,Hz.$

This is much above audible range. It is the ultrasonic wave and will not be audible to the humans, but it will be audible to bats.

Ex. The constant 'γ' for oxygen as well as for hydrogen is '1.40'. If speed of sound in oxygen is 450 m/s, what will be the speed of hydrogen at the same temperature and pressure?

Sol. $v = \sqrt{\frac{\gamma RT}{M}}$

since temperature, T is constant,

$$\therefore \quad \frac{v_{(H_2)}}{v_{(O_2)}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32}{2}} = 4$$

$$\Rightarrow \quad v_{(H_2)} = 4 \times 450 = 1800 \text{ m/s}$$

Intensity Of Sound Waves :

Like any other progressive wave, sound waves also carry energy from the one point of space to other.

This energy can be used to do the work, for example, forcing the 'eardrums' to vibrate or in the extreme case of the sonic boom created by the supersonic jet, can even cause glass windows to crack.

The amount of the energy carried per unit time by a wave is called its power and power per unit 'A' area held perpendicular to direction of energy flow is called intensity.

For a sound wave travelling along the positive x-axis described by the equation.

$$s = s_0 \sin(\omega t - kx + \phi)$$

$$P = p_0 \cos(\omega t - kx + \phi)$$

$$\frac{\delta s}{\delta t} = \omega s_0 \cos(\omega t - kx + \phi)$$

Concept Reminder

The amount of energy carried per unit time by a wave is called its power and power per unit area held perpendicular to the direction of energy flow is called intensity.

Instantaneous power $P = F.v = pA \frac{\delta s}{\delta t}$

$$P = p_0 \cos(\omega t - kx + \phi)A\omega s_0 \cos(\omega t - kx + \phi)$$

$$P_{average} = < P >$$

$$= p_0 A\omega s_0 < \cos^2(\omega t - kx + \phi) >$$

$$= \frac{p_0 A\omega s_0}{2} \Rightarrow v = \sqrt{\frac{B}{\rho}}$$

$$B = \rho v^2 \Rightarrow p_0 = Bks_0 = \rho v^2 ks_0$$

$$P_{average} = \frac{1}{2}\omega p_0 A\left(\frac{p_0}{\rho v^2 k}\right) = \frac{p_0^2 A}{2\rho v} = \frac{\rho A v \omega^2 s_0^2}{2}$$

Total energy transfer = $P_{av} \times t = \frac{\rho A v \omega^2 s_0^2}{2} \times t$

Average intensity = average power / area
The average intensity at position x is given by

$$< I > = \frac{1}{2}\frac{\omega^2 s_0^2 B}{v} = \frac{p_0^2 v}{2B} \quad \text{... (1)}$$

Substituting $B = \rho v^2$, intensity can also be expressed as

$$\boxed{I = \frac{p_0^2}{2\rho v}} \quad \text{... (2)}$$

Note :

- If source is a point source then $I \propto \frac{1}{r^2}$ and $s_0 \propto \frac{1}{r}$ and $s = \frac{a}{r}\sin(\omega t - kr + \theta)$
- If a sound source is line source then $I \propto \frac{1}{r}$ and $s_0 \propto \frac{1}{\sqrt{r}}$ and $s = \frac{a}{\sqrt{r}}\sin(\omega t - kr + \theta)$

Rack your Brain

Sound waves travel at 350 m/s through a warm air and at 3500 m/s through brass. The wavelength of a 700 Hz acoustic wave as it enters brass from warm air

(1) Decreases by a factor 20
(2) Decreases by a factor 10
(3) Increases by a factor 20
(4) Increases by a factor 10

Concept Reminder

- If the source is a point source then $I \propto \frac{1}{r^2}$ and $s_0 \propto \frac{1}{r}$ and $s = \frac{a}{r}\sin(\omega t - kr + \theta)$
- If a sound source is a line source then $I \propto \frac{1}{r}$ and $s_0 \propto \frac{1}{\sqrt{r}}$ and $s = \frac{a}{\sqrt{r}}\sin(\omega t - kr + \theta)$

Ex. The pressure amplitude of a sound wave from a radio receiver is (2.0×10^{-3} N/m^2) and the intensity at a point is (10^{-6} W/m^2). If by turning the "Volume" knob the pressure amplitude is increased to (3×10^{-3} N/m^2), evaluate intensity.

Sol. The intensity is proportional to the square of the pressure amplitude.

Thus, $\frac{I'}{I} = \left(\frac{p_0'}{p_0}\right)^2$

or $I' = \left(\frac{p_0'}{p_0}\right)^2 I = \left(\frac{3}{2.0}\right)^2 \times 10^{-6} W/m^2 \qquad = 2.25 \times 10^{-6} W/m^2$

Ex. A microphone of cross-sectional area '0.40' cm^2 is placed in front of the small speaker emitting π W of sound output. If the distance between microphone and the speaker is 2.0 m, how much energy falls on the microphone in 5.0 s?

Sol. The energy emitted by speaker in one second is π J. Let us consider a sphere of radius 2.0 m centered at speaker. The energy π J falls normally on the total surface of this sphere in one second. The energy falling on the area 0.4 cm^2 of the microphone in '1' second

$$= \frac{0.4\,cm^2}{4\pi(2.0)^2} \times \pi J = 2.5 \times 10^{-6} J/s$$

The energy falling on the microphone in 5.0 second is

2.5×10^{-6} J $\times 5 = 12.5\ \mu J$.

Ex. Find the amplitude of vibration of particles of air through which a sound wave of intensity 8.0×10^{-6} W/m^2 and frequency 5.0 kHz is passing. Density of air = '1.2 kg/m^3' and speed of sound in air = '330 m/s'.

Sol. The relation between intensity of sound and displacement amplitude is

$I = \frac{\omega^2 s_0^2 B}{2v}$, where $B = \rho v^2$ and $\omega = 2\pi\nu$

$I = 2\pi^2 s_0^2 \nu^2 \rho_0 v$

or, $s_0^2 = \frac{I}{2\pi^2 \nu^2 \rho_0 v}$

$$= \frac{8.0 \times 10^{-6} W/m^2}{2\pi^2 \times (25.0 \times 10^6 s^{-2}) \times (1.2 kg/m^3) \times (330\,m/s)}$$

or $s_0 = 6.4\,nm$

Characteristics of Sound

Sound is characterised by the following three parameters :

Loudness :- The quality of sound on the basis of which, sound is said to be high or low. It depends on :

(1) Shape & size of the source

(2) Intensity of sound

$\Rightarrow$ According to Weber - Fechner the loudness of a sound of intensity I is given by: $L \propto \log_{10} I$

Which is called Weber-Fechner law [unit of L is 'sone'. It also measured in decibel]

So, $\Delta L = L_2 - L_1 = 10 \log_{10} \frac{I_2}{I_1}$

For zero decibel Loudness, Intensity is called threshold of hearing. ($I_0 = 10^{-12}$ W/m^2)

In decibel the loudness of a sound of intensity I is given by $L = 10 \log_{10} (I/I_0)$

Where I_0 represents the threshold of hearing at 0 dB loudness level.

The loudness of a roaring lion is more than the sound produced by a mosquito.

Pitch : It is the sensation received by the ear due to frequency and is characteristic which distinguishes a shrill sound from a grave sound. As pitch depends on frequency, higher the frequency higher will be the pitch and shriller will be the sound. For example;

(1) The buzzing of a bee or humming of a mosquito has high pitch but low loudness while the roar of a lion has large loudness but low pitch.

(2) Due to more harmonics usually the pitch of female voice is higher than male.

Quality (timbre) : It is the sensation received by the ear due to 'waveform'.

Two sounds of same intensity and frequency as shown in figure will produce different sensation

Concept Reminder

Sound is characterised by the following three parameters :

1. Loudness
2. Pitch
3. Quality

Concept Reminder

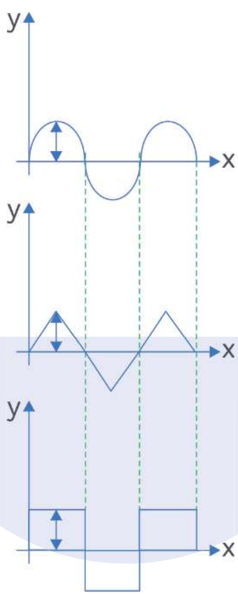
In decibel the loudness of a sound of intensity I is given by $L = 10 \log_{10} (I/I_0)$

Where I_0 represents the threshold of hearing at 0 dB loudness level

Concept Reminder

When sound waves bounced off on objects instead of absorbing the sound waves, you can hear echoes produced. The echo is a reflection of the sound waves produced because of the bouncing off of waves.

on the ear if their waveforms are different. Now as waveform depends on overtones present, quality of sound depends on number of overtones, i.e., harmonics present and their relative intensities.



Ex. If intensity is increased by a factor of 20, by how many decibels is the intensity level increased.

Sol. Let initial intensity by I and the intensity level be β_1 and when the intensity is increased by '20 times', the intensity level increases to β_2.

Then $\beta_1 = 10 \log (I / I_0)$

and $\beta_2 = 10 \log (20I / I_0)$

Thus $\beta_2 - \beta_1 = 10 \log (20I / I)$

$= 10 \log 20$

$= 13 \text{ dB}$

Ex. A bird is singing on a tree. A person approaches the tree and perceives that the intensity has increased by 10 dB. Find the ratio of initial and final separation between the man and the bird.

Sol. $\beta_1 = 10 \log \frac{I_1}{I_0}$

$\beta_2 = 10 \log \frac{I_2}{I_0} \Rightarrow \beta_2 - \beta_1 = 10 \log \frac{I_2}{I_1}$

$$10 = 10\log_{10}\left(\frac{I_2}{I_1}\right) \Rightarrow \frac{I_2}{I_1} = 10^1 = 10$$

for point source $I \propto \frac{1}{r^2} \Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{I_2}{I_1}} = \sqrt{10}$

Ex. The sound level at a point is increased by '40 dB'. By what factor is pressure amplitude increased ?

Sol. The sound level in dB is

$$\beta = 10\log_{10}\left(\frac{I}{I_0}\right)$$

If β_1 and β_2 are the sound levels and I_1 and I_2 are the intensities in the two cases,

$$\beta_2 - \beta_1 = 10\left[\log_{10}\left(\frac{I_2}{I_0}\right) - \log_{10}\left(\frac{I_1}{I_0}\right)\right]$$

or, $40 = 10\log_{10}\left(\frac{I_2}{I_1}\right)$ or, $\frac{I_2}{I_1} = 10^4$.

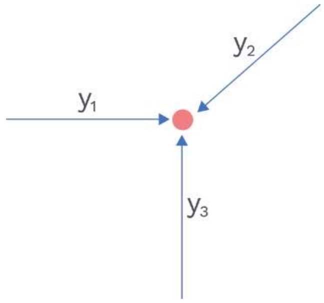
As the intensity is proportional to 'square' of the pressure amplitude,

we have $\frac{p_{02}}{p_{01}} = \sqrt{\frac{I_2}{I_1}} = \sqrt{10000} \approx 100$

Superposition of Waves :-

The phenomena of super mixing or intermixing of two or more than two waves and produce a new wave during propagation in the medium is known as superposition.

(1) In superposition phenomena the resultant displacement of medium particles is the vector addition of the individual displacements due to individual waves.



$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3$$

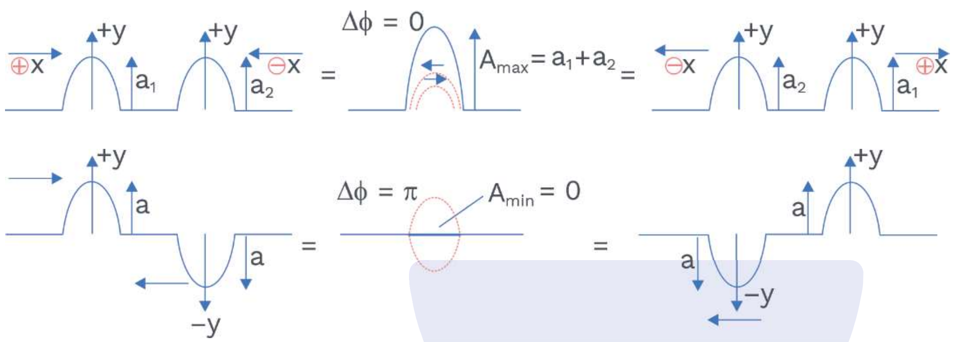
It is known as principle of superposition.

Definitions

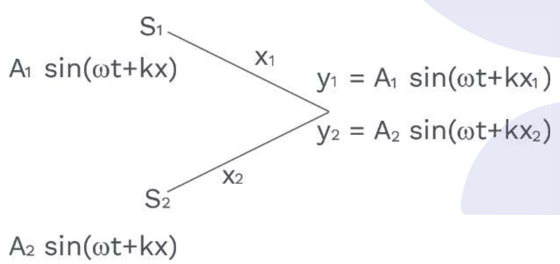
The phenomena of super mixing or intermixing of two or more than two waves and produce a new wave during propagation in the medium is known as superposition.

Sound Waves

(2) In superposition phenomena the existence of one wave is not affected due to presence or absence of another wave in the medium, waves independently propagate in the medium.



Mathematical Analysis



Interference implies superposition of the waves. Whenever two or more than two waves superimpose each other at some position then resultant displacement of particle is given by vector sum of individual displacements. Let the two waves coming from sources 'S_1' & 'S_2' be

$$y_1 = A_1 \sin(\omega t + kx_1)$$

$$y_2 = A_2 \sin(\omega t + kx_2) \text{ respectively.}$$

Due to superposition

$$y_{net} = y_1 + y_2$$

$$y_{net} = A_1 \sin(\omega t + kx_1) + A_2 \sin(\omega t + kx_2)$$

Phase difference between y_1 & $y_2 = k(x_2 - x_1)$

Rack your Brain

A tuning fork of frequency 512 Hz makes 4 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per sec when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was

(1) 508 Hz (2) 510 Hz

(3) 514 Hz (4) 516 Hz

i.e., $\Delta\phi = k(x_2 - x_1)$

As $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$ (where Δx = path difference & $\Delta\phi$ = phase difference)

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$\Rightarrow A_{net}^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

$$\therefore I_{net} = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi \quad (\text{as } I \propto A^2)$$

When the two displacements are in phase, then the resultant amplitude will be sum of the two amplitude & I_{net} will be maximum, this is known as constructive interference. For I_{net} to be maximum

$\cos\phi = 1 \Rightarrow \phi = 2n\pi$ where n = {0,1,2,3,4,5..........}

$$\frac{2\pi}{\lambda}\Delta x = 2n\pi \Rightarrow \Delta x = n\lambda$$

For constructive interference

$$I_{net} = (\sqrt{I_1} + \sqrt{I_2})^2$$

When $I_1 = I_2 = I$

$$I_{net} = 4I$$

$$A_{net} = A_1 + A_2$$

When the superposing waves are in opposite phase, the resultant amplitude is the difference of two amplitudes & I_{net} is minimum; this is known as the destructive interference.

For I_{net} to be minimum, $\cos\Delta\phi = -1$

$\Delta\phi = (2n + 1)\pi$ where n = {0,1,2,3,4,5..........}

$$\frac{2\pi}{\lambda}\Delta x = (2n + 1)\pi$$

$$\Rightarrow \Delta x = (2n + 1)\frac{\lambda}{2}$$

For destructive interference

$$I_{net} = (\sqrt{I_1} - \sqrt{I_2})^2$$

If $I_1 = I_2$

$$I_{net} = 0$$

Concept Reminder

$A_{net}^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$

$I_{net} = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$

(as $I \propto A^2$)

Concept Reminder

For constructive interference

$I_{net} = (\sqrt{I_1} + \sqrt{I_2})^2$

When $I_1 = I_2 = I$

$I_{net} = 4I$

Concept Reminder

For destructive interference

$I_{net} = (\sqrt{I_1} - \sqrt{I_2})^2$

If $I_1 = I_2$

$I_{net} = 0$

$$A_{net} = A_1 - A_2$$

Generally, $I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

If $I_1 = I_2 = I$

$$I_{net} = 2I + 2I\cos\phi$$

$$I_{net} = 2I(1+\cos\phi) = 4I\cos^2\frac{\Delta\phi}{2}$$

$$\boxed{I_{net} = 4I\cos^2\frac{\Delta\phi}{2}}$$

$$\boxed{\text{Ratio of } I_{max} \ \& \ I_{min} = \frac{(\sqrt{I_1}+\sqrt{I_2})^2}{(\sqrt{I_1}-\sqrt{I_2})^2}}$$

Concept Reminder

$$\boxed{I_{net} = 4I\cos^2\frac{\Delta\phi}{2}}$$

$$\boxed{\text{Ratio of } I_{max} \ \& \ I_{min} = \frac{(\sqrt{I_1}+\sqrt{I_2})^2}{(\sqrt{I_1}-\sqrt{I_2})^2}}$$

Longitudinal Standing Waves

Two longitudinal waves of same frequency and amplitude travelling in opposite directions interfere to produce the standing wave. If two interfering waves are given by;

$p_1 = p_0\sin(\omega t - kx)$ and $p_2 = p_0\sin(\omega t + kx + \phi)$

then the equation of the resultant standing wave would be given by

$$p = p_1 + p_2 = 2p_0\cos(kx + \frac{\phi}{2})\sin(\omega t + \frac{\phi}{2})$$

$$\Rightarrow \ p = p_0'\sin(\omega t + \frac{\phi}{2}) \qquad \text{... (1)}$$

This is equation of SHM in which the amplitude p_0' depends on position as

$$p_0' = 2p_0\cos(kx + \frac{\phi}{2}) \qquad \text{... (2)}$$

Points where the pressure remains permanently at its average value, i.e., pressure amplitude is '0' is called a pressure node and condition for the pressure node would be given by

$$p_0' = 0$$

i.e. $\cos(kx + \frac{\phi}{2}) = 0$

Concept Reminder

When sound waves bounced off on objects instead of absorbing the sound waves, you can hear echoes produced. The echo is a reflection of the sound waves produced because of the bouncing off of waves.

i.e. $kx + \frac{\phi}{2} = 2n\pi \pm \frac{\pi}{2}$, n = 0, 1, 2 ......

Similarly, points where pressure 'amplitude' is maximum is called a pressure 'antinode' and condition for a pressure antinode would be given by;

$$p_0^{'} = \pm 2p_0$$

i.e. $\cos(kx + \frac{\phi}{2}) = \pm 1$

or $(kx + \frac{\phi}{2}) = n\pi$, n = 0, 1, 2, ...

Note :

- Note that a pressure node in a standing wave would correspond to a displacement antinode and a pressure anti-node would correspond to the displacement node.
- (when we label eq^n. as SHM, what we mean that excess pressure at any point varies simple harmonically. If sound waves were represented in terms of displacement waves, then the equation of standing wave corresponding to would be

$$s = s_0^{'} \cos(\omega t + \frac{\phi}{2});\ \text{where}\ s_0^{'} = 2s_0 \sin(kx + \frac{\phi}{2})$$

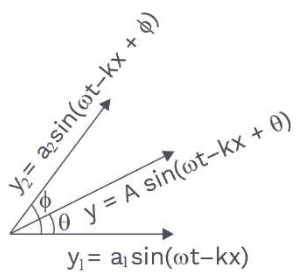
This is easily observed to be an equation of 'SHM'. It represents the medium particles moving simple harmonically about their mean position at x.

Let two wave is :-

$$y_1 = a_1 \sin(\omega t - kx)$$

$$y_2 = a_2 \sin(\omega t - kx + \phi)$$

then resultant displacement of medium particle

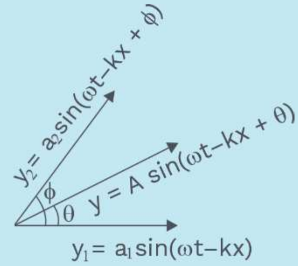


Concept Reminder

A pressure node in a standing wave would correspond to a displacement antinode and a pressure anti-node would correspond to a displacement node.

Concept Reminder

Resultant displacement of medium particle



$$\boxed{y = A \sin(\omega t - kx + \theta)}$$

$\boxed{y = A\sin(\omega t - kx + \theta)}$ resultant wave equation

where $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$

and $\tan\theta = \dfrac{a_2\sin\phi}{a_1 + a_2\cos\phi}$

Phenomena Based On Superposition :-

(1) Interference

(1) When two coherent waves of same frequency propagate in same direction and superimpose over one another then the intensity of resultant wave becomes maximum at some points and at some points it becomes minimum. This phenomenon of intensity variation w.r.t. position is known as interference.

(2) It is based on energy conservation principle means total energy of the waves remains constant in the medium. Here only redistribution of energy takes place.

(3) Here intensity is the function of position mean $\boxed{I = f(x)}$.

Definitions

When two coherent waves of same frequency propagate in same direction and superimpose over one another then the intensity of resultant wave becomes maximum at some points and at some points it becomes minimum. This phenomenon of intensity variation w.r.t. position is known as interference.

Note :-

Same amplitude of waves is not an essential condition for interference phenomena.

Coherent Waves :-

Waves for which phase difference remains constant w.r.t. time are known as coherent waves these are produced by coherent source.

Note: Two independent source cannot be coherent.

Mathematical Analysis :-

Let two wave is :-

$$y_1 = a_1\sin(\omega t - kx_1 + \phi_1)$$

$$y_2 = a_2\sin(\omega t - kx_2 + \phi_2)$$

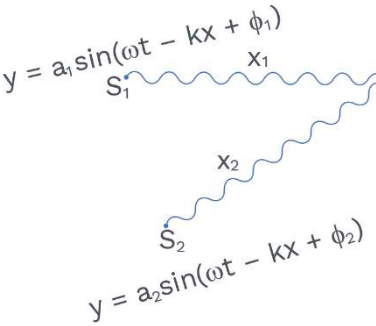
$$\Delta\phi = k(x_2 - x_1) + \phi_1 - \phi_2$$

If both source have same phase initially then

$$\phi_1 = \phi_2$$

$$\Delta\phi = K(x_2 - x_1)$$

$$\boxed{\Delta\phi = \frac{2\pi}{\lambda}\Delta x}$$



$y_r = y_1 + y_2$

$y_1 = a_1 \sin(\omega t - kx_1 + \phi_1)$

$y_2 = a_2 \sin(\omega t - kx_2 + \phi_2)$

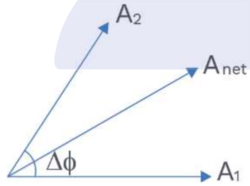
Concept Reminder

- Superposition
- Interference
- Constructive interference
- Destructive interference

Resultant amplitude at point P

$$A_{net} = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos\Delta\phi}$$

Both sources are coherent and wave travel in same medium then



$$I_1 = Ka_1^2$$

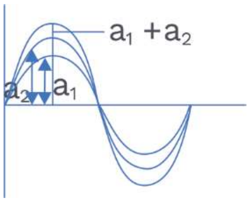
$$I_2 = Ka_2^2$$

$$I_{net} = Ka_{net}^2$$

then $I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi$

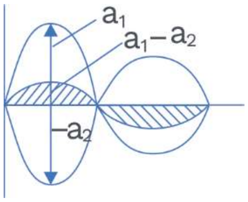
Constructive Interference (C.I.)

- Waves superimposed over one another in same phase



Destructive Interference (D.I.)

waves superimposed in opposite phase.



- Resultant intensity and amplitude are maximum

These are minimum

- $\cos\phi = +1 \Rightarrow$ then $\phi = 0, 2\pi, 4\pi, ..2n\pi$

$\Delta x = 0, \lambda, 2\lambda.......n\lambda$

$(n = 0, 1, 2...)$

$\cos\phi = -1$

$\Rightarrow$ then $\phi = \pi, 3\pi, 5\pi...(2n+1)\pi$

$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}.....(2n+1)\frac{\lambda}{2} n = (0, 1, 2...)$

- $A_{max} = a_1 + a_2$

$A_{min} = a_1 - a_2$

- $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$

$= \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$

$= \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

Some Important Formulae :-

- $\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$
- $\frac{I_{max}}{I_{min}} = \frac{A_{max}^2}{A_{min}^2} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$
- $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2$
- $\frac{I_1}{I_2} = \left(\frac{\sqrt{I_{max}} + \sqrt{I_{min}}}{\sqrt{I_{max}} - \sqrt{I_{min}}}\right)^2$
- $I_{avg} = \frac{I_{max} + I_{min}}{2} = \frac{I_1 + I_2 + 2\sqrt{I_1 I_2} + I_2 + I_2 - 2\sqrt{I_1 I_2}}{2} = I_1 + I_2$

Note :- for n sources

$$\boxed{I_{avg} = I_1 + I_2 + I_3 + ...I_n}$$

Not divided by no. of sources

- **Degree of interference pattern or degree of hearing (f%)**

$$\boxed{f\% = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100}$$

If f = 100% $\rightarrow$ then interference is perfect or best means contrast or clear.

Concept Reminder

- $\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$
- $\frac{I_{max}}{I_{min}} = \frac{A_{max}^2}{A_{min}^2} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$
- $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2$
- $\frac{I_1}{I_2} = \left(\frac{\sqrt{I_{max}} + \sqrt{I_{min}}}{\sqrt{I_{max}} - \sqrt{I_{min}}}\right)^2$
- $I_{avg} = \frac{I_{max} + I_{min}}{2}$

$= \frac{I_1 + I_2 + 2\sqrt{I_1 I_2} + I_2 + I_2 - 2\sqrt{I_1 I_2}}{2}$

$I_{avg} = I_1 + I_2$

- **Condition for perfect interference :-**

(1) f = 100%
(2) $I_{min} = 0$
(3) $I_1 = I_2 = I$
(4) $a_1 = a_2 = a$
(5) $A_{max} = a + a = 2a$
(6) $A_{min} = a - a = 0$
(7) $I_{max} = 4I$

Ex. If $\frac{I_{max}}{I_{min}} = \frac{36}{25}$ then find $\frac{I_1}{I_2}$ and $\frac{a_1}{a_2}$

Sol. $\frac{I_1}{I_2} = \left(\frac{\sqrt{36}+\sqrt{25}}{\sqrt{36}-\sqrt{25}}\right)^2 = \left(\frac{6+5}{6-5}\right)^2 = \frac{121}{1}$

$$\frac{36}{25} = \left(\frac{a_1+a_2}{a_1-a_2}\right)^2$$

$$\frac{6}{5} = \frac{a_1+a_2}{a_1-a_2} \Rightarrow 6a_1 - 6a_2 = 5a_1 + 5a_2 \Rightarrow \frac{a_1}{a_2} = 11$$

Ex. Intensities of 2 waves are I_1 and I_2 where $I_1 = I_2 = I_0$ and phase difference between them ϕ. Find out intensity of resultant wave.

Sol. $I_{res} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

$$I = I_0 + I_0 + 2I_0\cos\phi$$

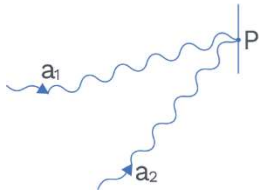
$$I = 2I_0 + 2I_0\cos\phi \qquad \therefore \cos 2\theta = 2\cos^2\theta - 1$$

$$I = 2I_0(1+\cos\phi)$$

$$I = 2I_0\left(1 + 2\cos^2\frac{\phi}{2} - 1\right) = 4I_0\cos^2\left(\frac{\phi}{2}\right)$$

Ex. Amplitude of 2 waves are a_1 and a_2, where $a_1 = a_2 = a$ and both are superimposed at P where path difference between then is $\frac{\lambda}{4}$. Find amplitude of resultant wave.

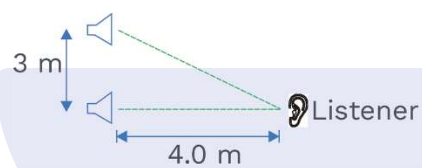
Sol. $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\frac{\pi}{2}} = \sqrt{a^2 + a_2^2} = \sqrt{2}a$



$$\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda}$$

$$\Delta\phi = 2\pi \times \frac{\pi}{4} = \frac{\pi}{2}$$

Ex. Two loudspeakers as shown in fig. below separated by a distance 3 m, are in phase. Assume that the amplitudes of the sound from the speakers is approximately same at the position of a listener, Who is at a distance 4.0 m in front of one of the speakers. For what frequencies does the listener hear minimum signal? Given that speed of sound in air is '330 ms^{-1}'.



Sol. The distance of the listener from the second speaker = $\sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5\,m$

path difference = (5 – 4.0) m = 1 m

For fully destructive interference 1 m = (2n + 1) $\frac{\lambda}{2}$

Hence $\lambda = 2 / (2n + 1)\,m$

The corresponding frequencies are given by

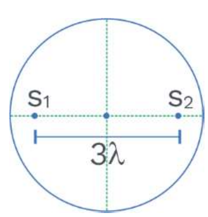
n = [330 × (2n + 1)]/2 s^{-1}, for n = 0, 1, 2, 3, 4, ...

= 165 (2n + 1) s^{-1}, for n = 0, 1, 2, 3, 4, ...

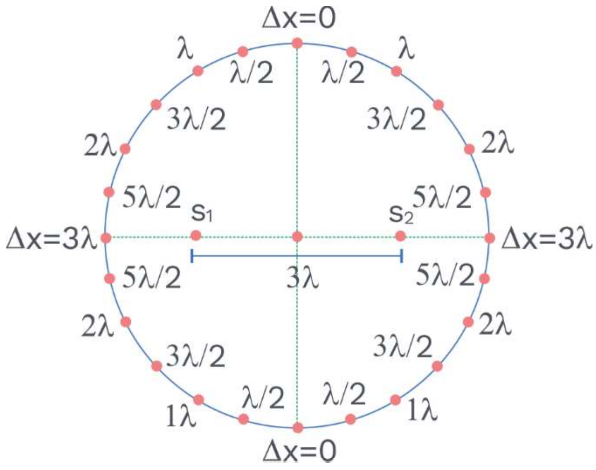
Therefore the frequencies for which the listener would hear a minimum intensity 165 Hz. 495 Hz,

Ex. Two sound source of S_1 and S_2 of coherent waves are separated by distance of 3λ and detector (D) revolve in circular path around them. Find out number of maxima and minima detected by detector is/are.

Sol.



($\therefore$ S_1 and S_2 are at equal distance from centre)



Beats :-

When two wave of slightly different frequency propagate is same direction and superimpose over one another then the intensity of resultant wave charges periodically w.r.t. time at a particular position.

This periodic variation is intensity w.r.t. time at a particular position is known as beat phenomena.

Interference

Let two coherent waves

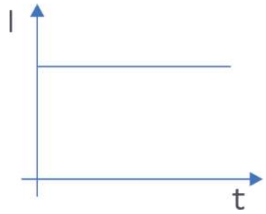
$y_1 = a_1 \sin(\omega t - kx)$

$y_2 = a_2 \sin(\omega t - kx + \phi)$

$\Delta\phi = \phi \rightarrow \text{const. (time independent)}$

then intensity of resultant wave

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\underset{\text{time independent}}{\underset{\uparrow}{\cos\phi}}$$



*** Static interference**

* stable intensity pattern

* permanent intensity pattern

$\Delta\phi = (\omega_2 - \omega_1)t$

Beats

Let two waves-

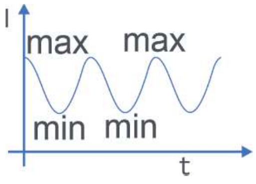
$y_1 = a_1 \sin \omega_1 t$

$y_2 = a_2 \sin \omega_2 t$

$\boxed{\Delta\phi = (\omega_2 - \omega_1)t}$ (time dependent)

then resultant intensity

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\underset{\text{time dependent}}{\underset{\uparrow}{\cos(\omega_2 - \omega_1)t}}$$



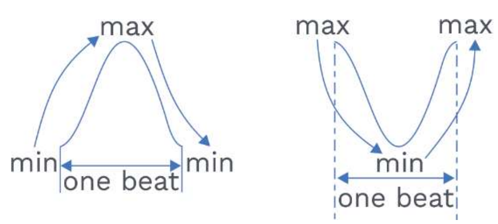
*** Dynamic interference**

* unstable intensity pattern

* temporary intensity pattern

$$t = 0 \Rightarrow \Delta\phi = 0 \Rightarrow I_{max}$$
$$t = \uparrow \Rightarrow \Delta\phi = \pi \Rightarrow I_{min}$$
$$t = \uparrow \Rightarrow \Delta\phi = 2\pi \Rightarrow I_{max}$$

- **One Beat:-** One time increment and one time decrement in intensity is known as one beat.



Note:- One maxima and minima are present in one beat.

- **Beat Time period (T_B)**
Time taken to complete one beat or time interval during which phase difference changes by an amount of 2π is known as beat time period.

$\boxed{\Delta\phi = (\omega_2 - \omega_1)t}$ if $t = 0 \Rightarrow \Delta\phi = 0$

$t = T_B \Rightarrow \Delta\phi = 2\pi$ $2\pi = (\omega_2 - \omega_1)T_B$

$2\pi = 2\pi(n_2 - n_1)T_B$ $\boxed{T_B = \frac{1}{n_2 - n_1} = \frac{1}{\Delta n} = \frac{1}{b}}$

Beat time period (T_B) = $\frac{1}{\text{Beat frequency (b)}}$

Beat frequency (b) $= \Delta n = n_2 - n_1$ = no. of beats produced per sec = $\frac{1}{\text{Beat time period } (T_B)}$

Mathematical Analysis :
Whenever two sources of sound that have almost same frequency sounded together, an interesting phenomenon occurs. A sound with the frequency average of the two is heard and loudness of sound repeatedly grows and decay, rather than being constant. Such a repeated variation in the amplitude of sound, called 'beats'.

Definitions

One time increment and one time decrement in intensity is known as one beat.

Definitions

Time taken to complete one beat or time interval during which phase difference changes by an amount of 2π is known as beat time period.

Concept Reminder

Beat time period (T_B) = $\frac{1}{\text{Beat frequency (b)}}$

Beat frequency (b) $= \Delta n = n_2 - n_1$ = no. of beats produced

If frequency of one of source is changed, there is a corresponding change in the rate at which amplitude varies. This rate is called the beat frequency. As frequencies come closer together, the beat frequency becomes 'slower'. A musician can tune a guitar to the another source by listening for beats while increasing or decreasing tension in each string, eventually the beat frequency becomes very low so that the effectively no beats can heard and the two sources are then in the tune.

We can also explain phenomenon of the beat mathematically. Let us consider the two superposing waves having frequencies 'n_1' and 'n_2' then their respective equations of oscillation are

$$y_1 = A \sin 2\pi n_1 t \quad \text{... (1)}$$

and

$$y_2 = A \sin 2\pi n_2 t \quad \text{... (2)}$$

On superposition at a point the displacement of the medium of particle is given as;

$$y = y_1 + y_2$$

$$y = A \sin 2\pi n_1 t + A \sin 2\pi n_2 t$$

$$y = 2A \cos 2\pi \left(\frac{n_1 - n_2}{2}\right) t \sin 2\pi \left(\frac{n_1 + n_2}{2}\right) t \quad \text{... (3)}$$

$$y = R \sin 2\pi \left(\frac{n_1 + n_2}{2}\right) t \quad \text{... (4)}$$

There equation (4) gives the displacement of medium particle where superposition takes place, it shows that the particle executes SHM with frequency $\frac{n_1 + n_2}{2}$, average of the two superposing frequencies and with amplitude R which varies with time, given as

$$R = 2A \cos 2\pi \left(\frac{n_1 - n_2}{2}\right) t \quad \text{... (5)}$$

Here R becomes maximum when

Rack your Brain

The driver of a car travelling with speed 30 m/sec towards a hill sounds a horn of frequency 600 Hz. If the velocity of sound in air is 330 m/s, the frequency of reflected sound as heard by driver is:

(1) 555.5 Hz (2) 720 Hz
(3) 500 Hz (4) 550 Hz

KEY POINTS

- Beats
- Beat frequency
- Beat time period

$$\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t = \pm 1$$

or $$2\pi\left(\frac{n_1 - n_2}{2}\right)t = N\pi \qquad [N \in I]$$

or $$t = \frac{N}{n_1 - n_2}$$

or at time $$t = 0, \frac{1}{n_1 - n_2}, \frac{2}{n_1 - n_2}, ...$$

At above instants the sound of the 'maximum loudness' is heard. Similarly, we can find time instants when the loudness of sound is minimum, it occurs when;

$$\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t = 0$$

or $$2\pi\left(\frac{n_1 - n_2}{2}\right)t = (2N + 1)\frac{\pi}{2} \qquad [N \in I]$$

or $$t = \frac{2N + 1}{2(n_1 - n_2)}$$

or at times instants $$t = \frac{1}{2(n_1 - n_2)}, \frac{3}{2(n_1 - n_2)}, ...$$

Here we can see that these time instants are exactly lying in the middle of the instants when the loudest sound is heard. Thus on the superposition of the above two frequencies at the medium particle, the sound will be increased, decreasing, increasing and decreasing and so on. This effect is called the beats. Here time between two successive maximum or minimum sounds is called the beat period, which is given as

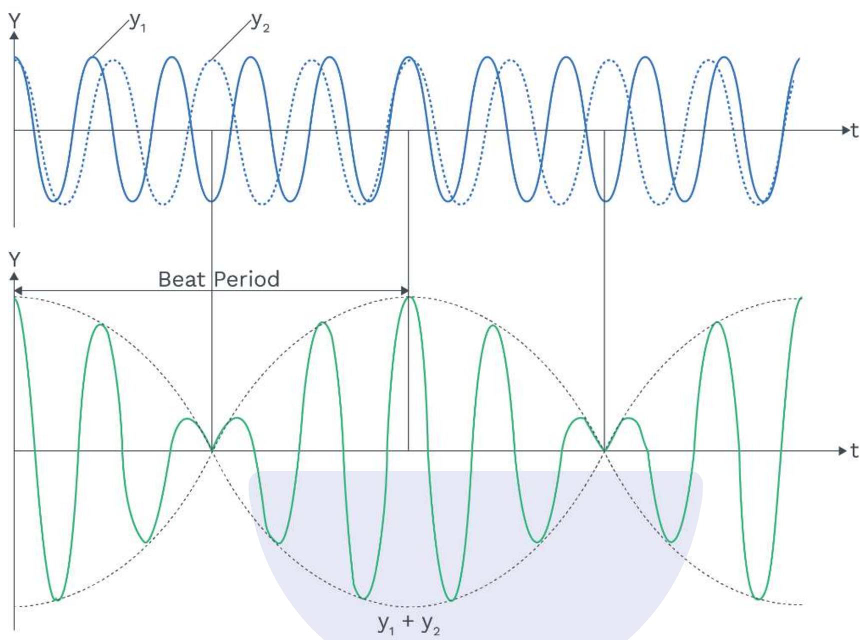
Beat Period T_B = time between two successive maxima = time between two successive minima

$$= \frac{1}{n_1 - n_2}$$

Thus beat frequency or number of beats heard per second can be given as

$$f_B = \frac{1}{T_B} = n_1 - n_2$$

The superposition of the two waves of slightly different frequencies is graphically shown in the figure. The resulting envelope of wave formed after superposition is also shown in figure 'b'. Such a wave when propagates, produces the “beat” effect at medium particles.



Important Points :-

- Beat frequency $\Rightarrow$ Frequency of Intensity Variation
- Frequency of resultant wave $n_{avg} = \frac{n_1 + n_2}{2}$
- Frequency of resultant amplitude variation $n_{amp} = \frac{n_1 - n_2}{2} = \frac{b}{2}$

Ex. Two vibrating tuning forks produce the progressive waves given by $y_1 = 4\sin(500\pi t)$ and $y_2 = 2\sin(506\ \pi t)$. These tuning forks held near the ear of the person. The person will hear the beat/s with intensity ratio between maxima and minima equal to β. Find the value of $\beta - \alpha$.

Sol. $y_1 = 4\sin(500\pi t)$, $y_2 = 2\sin(506\pi t)$

Number of beats = 253 – 250 = 3 beat/sec

$\Rightarrow \quad \alpha = 3$ beats/sec

$$\text{As } I_1 = 16 \text{ and } I_2 = 4 \Rightarrow \frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$$

$$\Rightarrow \left(\frac{4+2}{4-2}\right)^2 = \left(\frac{6}{2}\right)^2 = 9 \Rightarrow \beta = 9$$

So, $\beta - \alpha = 9 - 3 = 6$

- **Vibration of tuning fork** : when tuning fork is sounded by striking its one end on rubber pad, then the ends of prongs vibrate in and out while the stem vibrates up and down or vibration of the prongs are transverse and that of the stem is longitudinal. Generally tuning fork produces fundamental tone.

Ex. Tuning fork (T.K.) having n = 300 Hz produces 5 beats/sec. with another T.F. If impurity is added on the arm of known tuning fork number of beats decreases then find frequency of unknown T.F.?

Sol. $295 \xleftarrow[\text{wax is added}]{300} \quad 305$

If it would be 305 Hz, beats would have increased but with 295 Hz beats decreases so answer is 295 Hz.

Ex. 41 tuning forks are arranged in a series in such a way that each T.F. produce 3 beats with its neighbouring T.F. If the frequency of last is 3 times of first then find the frequency of 1st 11th 16th 21st & last T.F.

Sol. $n_1 = n$ (let) So $n_{41} = 3n$ (according to Que.)

$n_2 = n + b$

$n_3 = n + 2b$

$n_4 = n + 3b$

$n_{41} = n + 40\,b \quad \Rightarrow \quad n = 60$ Hz

$n_{11} = n + 10\,b = 90$ Hz, $n_{16} = n + 15\,b = 105$ Hz

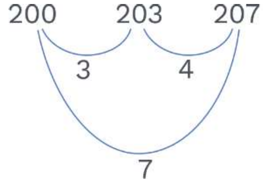
$n_{21} = n + 20b = 120$ Hz

Ex. Three tuning forks of frequencies 200, 203 and 207 Hz are sounded together. Find out the beat frequency.

Sol. $\frac{1}{3} \quad \frac{2}{3} \quad \left(\frac{3}{3}\right)$

$\frac{1}{4} \quad \frac{2}{4} \quad \frac{3}{4} \quad \left(\frac{4}{4}\right)$

$\frac{1}{7} \quad \frac{2}{7} \quad \frac{3}{7} \quad \frac{4}{7} \quad \frac{5}{7} \quad \frac{6}{7} \quad \left(\frac{7}{7}\right)$



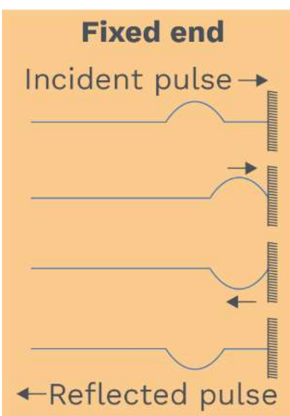
Divide 1 second into 3, 4 or 7 equal divisons

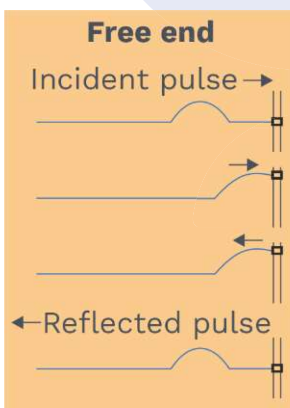
Eliminate common time instants. Total Maxima in one second 3 + 3 + 6 = 12

REFLECTION OF WAVES, STATIONARY WAVES, STANDING WAVES IN STRINGS AND ORGAN PIPE

Reflection and transmission of waves

Whenever a travelling wave reaches a boundary, part or all of the wave will be reflected. For example, consider a pulse travelling on a string fixed at one end (figure). When the pulse reaches the fixed wall, it will be reflected. Since the support attaching the string to the wall is assumed to be rigid, it does not transmit any part of disturbance to wall. Note that the reflected pulse is inverted. This can be explained by the Newton's third law, the support must exert an equal and opposite reaction force on the string. This downward forces cause the pulse to invert upon reflection.

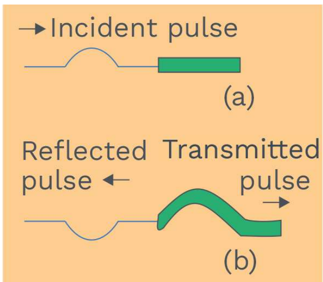


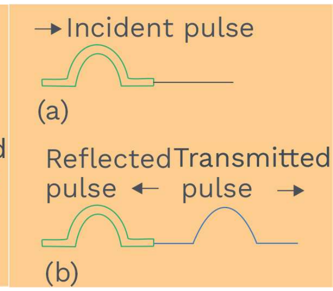


Now consider another case where the pulse arrives at the end of a string that is free to move vertically. The tension at free end is maintained by tying the string to a ring of negligible mass that to slide vertically on a smooth post. Again, the pulse will be reflected, but this time its displacement is not inverted. As the pulse reaches the post, it exerts a force on the free end, causing the ring to accelerate upward.

Concept Reminder

- Reflection
- Transmission
- Fixed end
- Free end
- Rigid end





In the process, the ring "overshoots" the height of the incoming pulse and is then returned to its original position by the downward component of the tension.

Finally, we may have a situation in which the boundary is intermediate between these two extreme cases, that is, one in which the boundary is neither rigid nor free. In this case, part of the incident energy is transmitted and part is reflected. For instance, suppose a light string is attached to a heavier string as shown in figure. When a pulse travelling on the light reaches the junction, part of it is reflected and inverted, and part of it is transmitted to the heavier string. As one would expect, the reflected pulse has a smaller amplitude than the incident pulse, since part of the incident energy is transferred to the pulse in the heavier string. The inversion in the reflected wave is similar to the behaviour of a pulse meeting a rigid boundary, when it is totally reflected.

When a pulse travelling on a heavy string strikes the boundary of a lighter string, as shown in figure, again part is reflected and part is transmitted. However, in this case the reflected pulse is not inverted. In either case, the relative height of the reflected and transmitted pulses depend on the relative densities of the two string. Thus, the speed of a wave on a string increases as the density of the string decreases. That is, a pulse travels more slowly on a heavy string than on the light string, if both are under same tension. The

Concept Reminder

In space, no sound is produced because of the absence of corners and obstacles, where sound waves are supposed to bounce off.

Concept Reminder

When a wave pulse travels from medium A to medium B and $v_A > v_B$ (B is denser than A), the pulse will be inverted upon reflection following general rules apply to reflected waves :

(1) When a wave pulse travels from medium A to medium B and $v_A > v_B$ (B is denser than A), the pulse will be inverted upon reflection.

(2) When a wave pulse travels from medium A to medium B and $v_A < v_B$ (A is denser than B), it will not be inverted upon reflection.

Concept Reminder

When a wave pulse travels from medium A to medium B and $v_A < v_B$ (A is denser than B), it will not be inverted upon reflection

Stationary Waves

(1) When two identical progressive waves (transverse or longitudinal) propagating in opposite direction superimpose in a bounded medium (having boundaries) the resultant wave is called stationary wave or standing wave.

(2) Stationary wave pattern may be obtained only and only in limited region.

(3) We can obtain two same type of progressive waves, only & only by method of reflection.

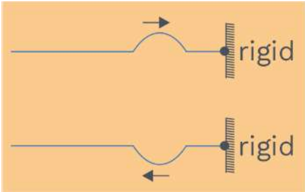
(4) According to the nature of reflected surface, reflection are of two types –

Concept Reminder

When two identical progressive waves (transverse or longitudinal) propagating in opposite direction superimpose in a bounded medium (having boundaries) the resultant wave is called stationary wave or standing wave.

(a) Rigid End

In such type of reflection incident and reflected waves have phase difference of π and direction of propagation are opposite.



incident wave $y_1 = a\sin(\omega t - kx)$

reflected wave $y_2 = a\sin(\omega t + kx + \pi)$

or $y_2 = -a\sin(\omega t + kx)$

$$y = y_1 + y_2$$

$$y = a\{\sin(\omega t - kx) - \sin(\omega t + kx)\}$$

Rack your Brain

Which one of the following statements is true?

(1) Both light and sound waves in air are transverse

(2) The sound waves in air are longitudinal while the light waves are transverse

(3) Both light and sound waves in air are longitudinal

(4) Both light and sound waves can travel in vacuum

after solving

$$y = -2a \sin kx \cos \omega t$$

$$y = -A \cos \omega t \text{ where } A = 2a \sin kx$$

at $x = 0, A = 0$

$$y_1 = A \sin[kx - \omega t]$$

$$y_2 = A \sin[kx + \omega t]$$

$$y = y_1 + y_2$$

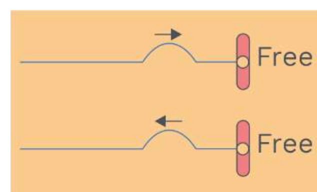
$$\boxed{y = 2A \sin kx \cos \omega t}$$

(b) Free End

In such type of reflection incident and reflected waves are in phase and direction of propagation are opposite.

incident wave $y_1 = a \sin(\omega t - kx)$

reflected wave $y_2 = a \sin(\omega t + kx)$



From superposition of wave

$$y = y_1 + y_2$$

$$y = a\{\sin(\omega t - kx) + \sin(\omega t + kx)\}$$

After solving

$$y = 2a \cos kx \sin \omega t$$

$$y = A \sin \omega t \text{ where } A = 2a \cos kx$$

So $x = 0$ and $A = 2a$

in the above equation Amplitude = 2A sin kx

If **(1)** $x = 0$

$y = 0$(Node)

(2) $x = \lambda / 8$ $\quad y = 2A \sin \frac{2\pi}{\lambda} \frac{\lambda}{8} \cos \omega t = \sqrt{2} A \cos \omega t$

(3) $A = \lambda$ $\quad y = 2A \sin \frac{2\pi}{\lambda} \frac{\lambda}{4} \cos \omega t$

$= 2A \cos \omega t$ (Anti – node)

(4) $x = 3\lambda / 8$ $\quad y = 2A \sin \frac{2\pi}{\lambda} \frac{9\lambda}{8} \cos \omega t$

$= \sqrt{2} A \cos \omega t$

(5) $x = \lambda / 2$ $\quad y = 2A \sin \frac{2\pi}{\lambda} \frac{\lambda}{2} \cos \omega t$

$= 0$ (Node)

(6)	$x = 5\lambda / 8$	$y = -\sqrt{2}A \cos \omega t$
(7)	$x = 3\lambda / 4$	$y = -2A \cos \omega t$(A.N.)
(8)	$x = 7\lambda / 8$	$y = -\sqrt{2}A \cos \omega t$
(9)	$x = \lambda$	$y = 0$

Special Properties of Stationary Wave Pattern

- **Zero wave velocity :** No transfer of energy between two points, particle velocity is non zero but wave velocity is zero.
- Position of antinodes & nodes in this pattern remains fix.
- The particles between two consecutive nodes vibrate in same phase while medium particles nearby of any node on both sides always vibrate in opposite phase.
- All medium particles doing simple harmonic vibrations have identical time period but different vibration Amplitude and because of this their maximum velocity at mean position is different
- All medium particles pass through their mean position simultaneously but with different maximum velocity.
- All medium particles pass their mean position in their one complete vibration two times hence stationary wave
- pattern is obtained as straight line twice in its one complete cycle.
- In this pattern, at antinode position, displacement and velocity is maximum, but wave strain is minimum.

  Strain = slope of stationary wave pattern $\left(\frac{dy}{dx}\right)$

  At node position displacement and velocity is minimum but wave strain is maximum.
- Amplitude of incident wave > Amplitude of reflected wave

  For node $a_1 - a_2 \Rightarrow$ minima

  For antinode $a_1 + a_2 \Rightarrow$ maxima

Concept Reminder

The particles between two consecutive nodes vibrate in same phase while medium particles nearby of any node on both sides always vibrate in opposite phase.

Concept Reminder

Transverse stationary waves → Musical instruments based on wire (sonometer).
Longitudinal stationary waves → Musical instruments based on air (resonance tube).

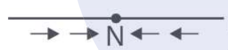
- For any wave each and every reflecting surface have some absorptive power and due to this the energy, intensity & amplitude of reflected wave is always less compared to that of incident wave. Two waves differ in their amplitude having same frequency and wavelength and propagate in reverse or opposite direction always give stationary wave pattern by their superposition.
- According to nature of superposing waves stationary waves are of two types –

  Transverse stationary waves → Musical instruments based on wire (sonometer).

  Longitudinal stationary waves → Musical instruments based on air (resonance tube).
- Only applied to longitudinal stationary wave

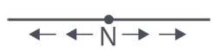
KEY POINTS

- Stationary wave
- Sonometer
- Resonance tube



Pressure ↑ (compression) density ↑

If medium particles move in this way



Pressure ↓ (Rarefaction) density ↓

at antinode → Pressure & density constant so variations minimum.

at node → Pressure & density variations maximum.

- $E_{gas} = \dfrac{\text{change in pressure}}{\text{volumetric strain}} = \dfrac{dP}{\left(\dfrac{dy}{dx}\right)}$ = const. then $dP \propto \text{strain}\left(\dfrac{dy}{dx}\right)$

So, strain is maximum at node positions and minimum at antinode positions.

Mathematical Analysis :-

$y_{air} = a\sin(\omega t - kx)$

Rigid end

$y_R = -a\sin(\omega t + kx)$

$\vec{y}_{SW} = \vec{y}_{in} + \vec{y}_R$

$y_{SW} = a[\sin(\omega t - kx) - \sin(\omega t + kx)]$

$y_{SW} = -(2a\sin kx)\cos\omega t$

(2a sin kx) → amplitude; cos ωt → SHM

Free end

$y_R = a\sin(\omega t + kx)$

$\vec{y}_{SW} = \vec{y}_{in} + \vec{y}_R$

$y_{SW} = a[\sin(\omega t - kx) + \sin(\omega t + kx)]$

$y_{SW} = (2a\cos kx)\sin\omega t$

(2a cos kx) → amplitude; sin ωt → SHM

$$y_{air} = a\cos(\omega t - kx)$$

Rigid end

$y_R = -a\cos(\omega t + kx)$

$\vec{y}_{SW} = \vec{y}_{in} + \vec{y}_R$

$y_{SW} = a[\cos(\omega t - kx) - \cos(\omega t + kx)]$

$y_{SW} = 2a \sin kx \sin \omega t$

amplitude SHM

Free end

$y_R = a\cos(\omega t + kx)$

$\vec{y}_{SW} = \vec{y}_{in} + \vec{y}_R$

$y_{SW} = a[\cos(\omega t - kx) + \cos(\omega t + kx)]$

$y_{SW} = 2a \cos kx \sin \omega t$

amplitude SHM

Results :-

(1) In stationary wave equation if sin and cos both terms are present then incident and reflecting wave are of sin type otherwise cos type.

sin cos, cos sin ⟩ sin type sin sin, cos cos ⟩ cos type

(2) If reflection is from rigid end then incident and reflecting wave are in opposite phase so node is formed at that end and sinkx term is present in equation of stationary wave.

Concept Reminder

If reflection is from rigid end then incident and reflecting wave are in opposite phase so node is formed at that end and sinkx term is present in equation of stationary wave.

(3) If reflection is from free end than incident and reflecting wave are in same phase and so antinode is formed at that end and coskx term is present.

(i) if $x = 0 \Rightarrow y = 0 \Rightarrow$ Node $\Rightarrow$ Rigid end

(ii) if $x = 0 \Rightarrow y \neq 0 \Rightarrow$ antinode $\Rightarrow$ Free end

Ex. The equation of a progressive wave is given by $y = 0.09\sin 8\pi\left(t - \frac{x}{20}\right)$. After reflection of this wave from rigid end the amplitude becomes $\frac{2}{9}$ of initial amplitude. Find out equation of reflected wave.

Sol. $y_R = -0.02\sin 8\pi\left(t + \frac{x}{20}\right)$

Ex. If equation of standing wave is given by $y = 5\cos(60\pi t)\cos\left(\frac{\pi}{6}x\right)$ where x and y are in cm and t in second. Find

(1) General equation of standing wave

(2) Type of reflector
(3) Type of incident and reflected wave
(4) Equation of incident and reflected wave
(5) Amplitude at (x) position
(6) Amplitude at AN and N
(7) Distance between 2 successive N
(8) Distance from reflection where particle has minimum vibration.
(9) Speed of incident and reflected wave
(10) Speed of standing wave

Sol. **(1)** $y = 2A\cos\omega t\cos kx$

$2A = 5, A = 2.5\,cm, \omega = 60\pi$

$k = \frac{\pi}{6} = \frac{2\pi}{\pi} \Rightarrow \lambda = 12\,cm$

(2) Free end

(3) cos type

(4) $y_i = 2.5\cos\left(60\pi t - \frac{\pi}{6}x\right)$;

$y_R = 2.5\cos\left(60\pi t - \frac{\pi}{6}x\right)$

(5) $A = 5\cos\left(\frac{\pi}{6}x\right)$

(6) $A_N = 5 \times 0 = 0, A_{AN} = 5 \times 1 = 5\,cm$

(7) $N - N = \frac{\lambda}{2} = \frac{12}{2} = 6\,cm$

(8) $N - AN = \frac{\lambda}{4} = \frac{12}{4} = 3\,cm$

(9) $v = \frac{\omega}{k} = \frac{60\pi \times 6}{\pi} = 360\ cm/s$

(10) $v_{sw} = 0$

Ex. If equation of standing wave is given by $y_{sw} = 8\sin(30\pi t)\sin\left(\frac{\pi}{3}x\right)$ then find out :

(1) General equation of standing wave
(2) Type of reflector
(3) Type of incident and reflected wave
(4) Equation of incident and reflected wave
(5) Amplitude at (x) position
(6) Amplitude at AN and N

(7) Distance between 2 successive N
(8) Distance from reflection where particle has maximum vibration.
(9) Speed of incident and reflected wave
(10) Speed of standing wave

Sol. **(1)** $y = 2A \sin \omega \sin kx$

(2) Rigid end

(3) cos type

(4) $y_i = 4\cos\left[30\pi t - \frac{\pi}{3}x\right], \quad y_R = -4\cos\left[30\pi t + \frac{\pi}{3}x\right]$

(5) $A = 8\sin\left(\frac{\pi}{3}x\right)$

(6) $A_N = 8 \times 0 = 0\,(\text{minimum}),$

$A_N = 8 \times 1 = 8\,(\text{maximum})$

(7) $N - N = \frac{\lambda}{2} = \frac{6}{2} = 3$

(8) $N - AN = \frac{\lambda}{4} = \frac{6}{4} = 1.5\,\text{cm}$

(9) $v = \frac{\omega}{k} = \frac{30\pi}{\pi} \times 3 = 90\,\text{cm/s}$

(10) $v_{sw} = 0$

Definitions

Fundamental Frequency:- Minimum possible frequency of sound produced by a source is known as fundamental frequency

Application of Stationary Waves

Longitudinal Stationary Wave
- Organ pipes
- Resonance tube exp.

Transverse Stationary Wave
- Sonometer wire
- Sonometer exp.

(1) **Fundamental Frequency:-** Minimum possible frequency of sound produced by a source is known as fundamental frequency.
(2) **Overtone Frequency (OT):-** Frequency higher than the fundamental frequency is known as overtone frequency.
(3) **Harmonic Frequency:-** Integer multiple frequency of fundamental frequency is known as harmonic frequency.

Definitions

Overtone Frequency (OT):- Frequency higher than the fundamental frequency is know as overtone frequency.

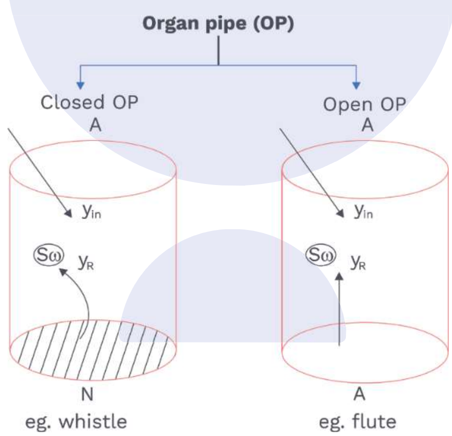
Example :-

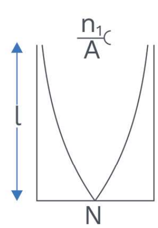
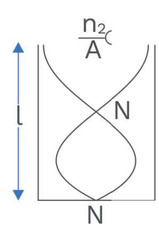
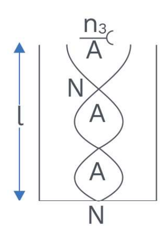
40Hz	60Hz	80Hz	100Hz	120Hz
fundamental frequency	1^{st}OT	2^{nd}OT	3^{rd}OT	4^{th}OT
or fundamental tone or I^{st} harmonic	×	2^{nd}harmonic	×	3^{rd}harmonic

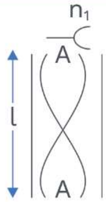
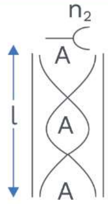
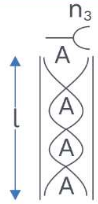
Longitudinal st. Wave.

Organ pipes :- These are used to produce sound from air.

ex → flute, whistle etc

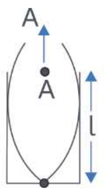


				from $n = \frac{v}{\lambda}$
	$l = \frac{\lambda_1}{4}$	$l = \frac{3\lambda_2}{4}$	$l = \frac{5\lambda_3}{4}$	
	$\lambda_1 = 4l$	$\lambda_2 = \frac{4l}{3}$	$\lambda_3 = \frac{4l}{5}$	
	$n_1 = \frac{v}{4l}$	$n_2 = \frac{3v}{4l}$	$n_3 = \frac{5v}{4l}$	
	n_C	$3n_C$	$5n_C$	
O.T.	0	1^{st}	2^{nd}	M^{th}
Harmonic	1	3	5	2M + 1
No. of Nodes	1	2	3	M + 1
No. of Antinodes	1	2	3	M + 1

	n_1	n_2	n_3	
				
	$l = \frac{\lambda_1}{4}$ $\lambda_1 = 2l$ $n_1 = \frac{v}{2l}$ n_0	$l = 2\left(\frac{\lambda_2}{2}\right)$ $\lambda_2 = \frac{2l}{2}$ $n_2 = \frac{2v}{2l}$ $2n_0$	$l = \frac{3\lambda_3}{2}$ $\lambda_3 = \frac{2l}{3}$ $n_3 = \frac{3v}{2l}$ $3n_0$	
O.T.	0	1^{st}	2^{nd}	M^{th} O.T
Harmonic	1	2	3	M + 1
No. of Nodes	1	2	3	M + 1
No. of Antinodes	2	3	4	M + 2 **End**

End connection (e) :-

Due to inertia position of antinode is slightly shifted above from open and of organ pipe. It is known as end connection(e) of organ pipe.



$$\boxed{\frac{\lambda}{4} = l + e}$$

$$\lambda = 4(l + e)$$

$$n = \frac{v}{\lambda} = \frac{v}{4(l + e)}$$

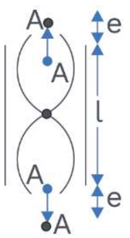
Definitions

Due to inertia position of antinode is slightly shifted above from open and of organ pipe. It is known as end connection(e) of organ pipe

$$\boxed{\frac{\lambda}{4} = l + e}$$

$$\lambda = 4(l + e)$$

$$n = \frac{v}{\lambda} = \frac{v}{4(l + e)}$$



$$\boxed{\frac{\lambda}{2} = l + 2e}$$

$$\lambda = 2(l + 2e)$$

$$n = \frac{v}{\lambda} = \frac{v}{2(l + 2e)}$$

$e \propto r$ where r = radius of organ pipe

$e = 0.6\ r$

$e = 0.3\ D$ (where D = Diameter of organ pipe)

$$\boxed{\frac{\lambda}{2} = l + 2e}$$

$$\lambda = 2(l + 2e)$$

$$n = \frac{v}{\lambda} = \frac{v}{2(l + 2e)}$$

$e \propto r$ when r = radius of organ pipe

$$\boxed{e = 0.6r}$$

$$\boxed{e = 0.3D}$$

D = Diameter of organ pipe

Rack your Brain

Two stationary sources each emit waves of wavelength λ. An observer moves from one source to another with velocity u. Then find the number of beats heard by him

Resonance Tube

This is an apparatus used to determine velocity of sound in air experimentally and also to compare frequencies of two tuning forks.

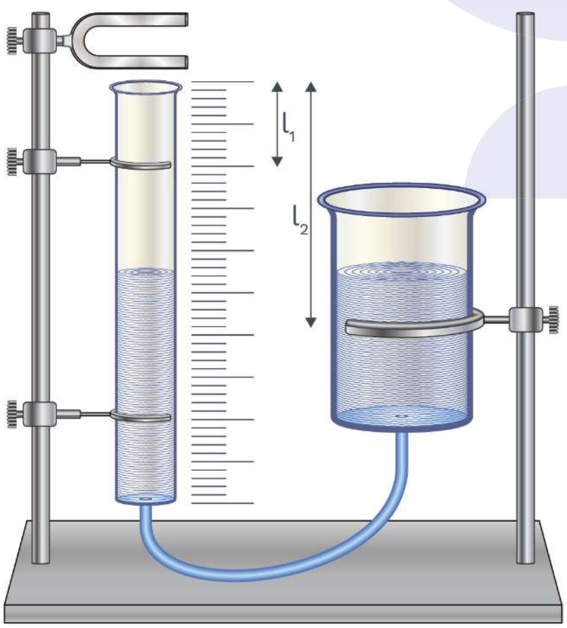


Figure shown in the setup of a resonance the experiment. There is a long tube T in which initially water is filled upto the top and the ear level can be change by moving a reservoir R up and down.

Concept Reminder

If we find two successive resonance lengths l_1 and l_2, we can get the wavelength of the wave as

$$l_2 - l_1 = \frac{\lambda}{2}$$

or $\lambda = 2(l_2 - l_1)$

A tuning fork of known frequency n_0 is struck gently on the rubber pad and brought near the open end tube T due to which oscillations are transferred to the air column in the tube above the water level. Now we gradually decrease water level in tube. This air column behaves like a closed organ pipe and the water level as closed end of pipe. As soon as water level reaches a position where there is a node of corresponding stationary wave, in air column, resonance takes place and maximum sound intensity is detected. Let at this position length of air column be l_1.
If the water level is further decreased, again maximum sound intensity is observed when the water level is at the another node i.e. at a length l_2 as shown in the figure. Here if we find two successive resonance lengths l_1 and l_2, we can get the wavelength of the wave as

$$l_2 - l_1 = \frac{\lambda}{2}$$

or $\lambda = 2(l_2 - l_1)$

Thus sound velocity in air given as;

$$v = n_0\lambda = 2n_0(l_2 - l_1)$$

Resonance Tube Experiment : -

(1) It is a variable length closed organ pipe. Its length can be varied by changing liquid level.

(2) Here function of liquid is only to provide rigid end, resonance condition is not affected by changing nature or density of liquid.

(3) Vibration produced in resonance tube are forced vibrations. Tuning fork work as a driver and tube works as a driven means vibrations are transferred from fork to tube.

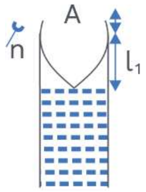
(4) It is based on resonance principle. Resonance, condition is a special condition of forced vibration in which frequency of driver is equal to one of the harmonic frequency of driven.

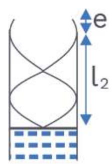
Concept Reminder

Resonance tube experiment is based on resonance principle.
Resonance, condition is a special condition of forced vibration in which frequency of driver is equal to one of the harmonic frequency of driven.

(5) At resonance condition is resonance tube, frequency of tuning fork is equal to one of the harmonic frequency of resonance tube and maximum energy is transferred from fork to tube and so intensity of produced sound is also become maximum.

Use :- It is used to calculate speed and wavelength of sound wave and end-correction of organ pipe.





$$\frac{\lambda}{4} = l_1 + e \ \ldots (1)$$

$$\frac{3\lambda}{4} = l_2 + e \ \ldots (2)$$

from (2) – (1)

$$\frac{3\lambda}{4} - \frac{\lambda}{4} = l_2 - l_1$$

$$\frac{\lambda}{2} = l_2 - l_1$$

$$\boxed{\lambda = 2(l_2 - l_1)}$$

$$\boxed{v = 2n(l_2 - l_1)}$$

from (2) / (1)

$$\frac{3\lambda/4}{\lambda/4} = \frac{l_2 + e}{l_1 + e}$$

$$\frac{3}{1} = \frac{l_2 + e}{l_1 + e}$$

$$\boxed{e = \frac{l_2 - 3l_1}{2}}$$

$$\text{if } e = 0 \text{ then} \Rightarrow l_2 = 3l_1$$

$$\text{if } e \neq 0 \text{ then} \Rightarrow l_2 > 3l_1$$

Ex. If fundamental frequency of COP and OOP are same. Find ratio of their length.

Sol. $n_C = n_0 \Rightarrow \frac{v}{4l_C} = \frac{v}{2l_0} \Rightarrow \frac{l_C}{l_0} = \frac{1}{2}$

Ex. Two COP's of length 30 cm and 31 cm are produce 5 b/s when vibrate together. Find out frequency of both pipes.

Sol. $n_C = \frac{v}{4l_C}$

$$n \propto \frac{1}{4} \Rightarrow \frac{n_1}{n_2} = \frac{l_2}{l_1} \Rightarrow \frac{n}{n+5} = \frac{30}{31}$$

$31n = 30n + 150 \Rightarrow n = 150$ Hz, $n + 5 = 155$ Hz

Ex. 3 successive overtones frequencies of an organ pipe is given by 360 Hz, 600 Hz, 840 Hz. Find out nature and fundamental frequency of organ pipe.

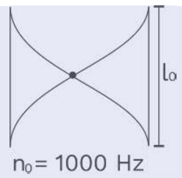
Sol. 360 Hz : 600 Hz : 840 Hz

$3 : 5 : 7 \Rightarrow$ COP

$$(2m+1)\frac{V}{4l} = \text{frequency of } m^{th} \text{ OT}$$

$$\frac{3V}{4l} = 360 \Rightarrow \frac{V}{4l} = 120\,\text{Hz}$$

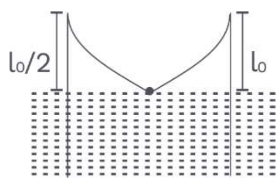
Ex. Initially frequency of an OOP is 1000 Hz. If 50% part OOP is dipped in water. Find new frequency of pipe.



Sol. $n_0 = \frac{v}{2l_0}$

$$1000 = \frac{v}{2l_0} \quad \dots (1)$$

$$n_C = \frac{v}{4l_0} \Rightarrow n_C = \frac{v \times 2}{4 \times l_0} \quad \dots (2)$$



$$\frac{1000}{n_C} = \frac{v}{2l_0} \times \frac{4l_0}{2v}$$

$$n_C = 1000\,\text{Hz}$$

Ex. If 2 OOP of length 25 cm and 25.5 cm are produce 6 b/s when vibrate together. Find velocity of sound.

Sol. $l_1 = 25\,\text{cm},\ l_2 = 25.5\,\text{cm}$

$$\frac{v}{2l_1} - \frac{v}{2l_2} = 6 \Rightarrow v\left(\frac{1}{2l_1} - \frac{1}{2l_2}\right) = 6$$

$$v\left(\frac{1}{2\times 25} - \frac{1}{2\times 25.5}\right) = 6 \Rightarrow v\left(\frac{1}{50} - \frac{1}{51}\right) = 6$$

$$\Rightarrow \quad v = \frac{50 \times 51 \times 6}{100} = 153\,\text{m/s}$$

Ex. If fundamental frequency of both pipes are 300 Hz. Find out frequency of COP and OOP corresponding to 8^{th} antinode.

Sol. For COP

$$\frac{v}{4}l = 300 = (2m+1)\left(\frac{v}{4l}\right)$$

Antinode $\Rightarrow$ $m + 1 = 8 \Rightarrow m = 7$

$= (2 \times 7 + 1) \times 300 = 4500$ Hz

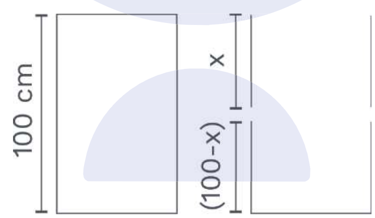
For OOP $\frac{v}{2l} = 300 = (m+1)\left(\frac{v}{2l}\right)$

$m + 2 = 8 \Rightarrow m = 6$

$= (6 + 1) \times 300 = 2100$ Hz

Ex. A COP of length =100 cm is divided into 2 unequal parts such as fundamental frequency of closed part is same as frequency of 1^{st} OT of open part. If speed of sound = 320 m/s. Find out length of both parts.

Sol. FF of COP = 1^{st} OT of OOP



$\Rightarrow \frac{v}{4l_C} = \frac{2v}{2l_0}$

$\Rightarrow l_0 = 4l_C$

$\Rightarrow x = 4(100 - x)$ $\Rightarrow x = 400 - 4x$

$\Rightarrow 5x = 400$ $\Rightarrow x = 80\,\text{cm} = l_0$

$\therefore l_C = 100 - x = 100 - 80 = 20\,\text{cm}$

Ex. A fork produce 5b/s with two COP of length to 30 cm and 31 cm separately. Find frequency of both pipes.

Sol. Let frequency of fork = n

$$n \propto \frac{1}{l} \Rightarrow \frac{n_1}{n_2} = \frac{l_2}{l_1} \Rightarrow \frac{n+5}{n-5} = \frac{31}{30}$$

$\Rightarrow 30n + 150 = 31n - 155$

$\Rightarrow n = 305$ Hz , $n + 5 = 310$ Hz, $n - 5 = 300$ Hz

Ex. Length of OOP is 44 cm and fundamental frequency is 340 Hz if velocity of sound is 340 m/s. Find end correction.

Sol. $n_0 = \frac{v}{2(l+2e)} \Rightarrow 340 = \frac{340 \times 100}{2(44+2e)}$

$\Rightarrow 44 + 2e = 50 \Rightarrow e = 3$

Ex. A fork of frequency 340 Hz is allowed to vibrate just above the open of resonance tube of length 120 cm. If velocity of sound = 340 m/s. Find (a) maximum number of resonance (b) maximum and minimum liquid level at resonance condition.

Sol. **(a)** $v = n\lambda$ Number of resonance = 2

$\lambda = 1m = 100\,cm$

$\frac{\lambda}{4} = \frac{100}{4} = 25\,cm$

$\frac{3\lambda}{4} = 75\,cm$

(b) Maximum liquid level = 120 – 25 = 95 cm

Minimum liquid level = 120 – 75 = 45 cm

Ex. A fork of frequency 330 Hz allowed to vibrate just above of tube of length 180 cm. If velocity of sound is '330 m/s'. Find: (a) maximum number of resonance (b) maximum and minimum liquid level at resonance condition.

Sol. **(a)** $v = n\lambda$

$\lambda = \frac{330}{330} = 1m = 100\,cm$

$\Rightarrow \frac{\lambda}{4} = \frac{100}{100} = 25\,cm$

$\Rightarrow \frac{3\lambda}{4} = 15\,cm$

Maximum number of resonance = 4

$\Rightarrow \frac{5\lambda}{4} = 125\,cm$

$\Rightarrow \frac{7\lambda}{4} = 175\,cm$

(b) Maximum liquid level = 180 – 25 = 155 cm

Minimum liquid level = 180 – 175 = 5 cm

Ex. 2 forks A and B produce 4 b/s when sounded together. A is in resonance with COP, l = 16 cm and B is in resonance with OOP of l = 32.5 cm. Find out frequency of both pipes.

Sol. $\because \quad n_2 - n_1 = 4$

$$n_1 = \frac{v}{4l_C} = \frac{v}{4 \times 16} \text{ and } n_2 = \frac{v}{2l_C} = \frac{v}{2 \times 32.5}$$

$$\therefore \quad n_2 - n_1 = 4 \Rightarrow \frac{v}{64} - \frac{v}{65} = 4 \Rightarrow v\left(\frac{1}{64} - \frac{1}{65}\right) = 4$$

$$\Rightarrow \quad v = 4 \times 64 \times 65$$

$$\therefore \quad n_1 = 260\,\text{Hz}, \; n_2 = 256\,\text{Hz}$$

Ex. A flute which we treat as an OOP of L = 60 cm. How far from the mouth piece a hole should be uncovered for fundamental frequency to be '330 Hz'. If speed of sound in air is '340 m/s', then also calculate fundamental frequency when all the holes are covered.

Sol. $n_0 = \frac{v}{2l_0} \Rightarrow 330 = \frac{340}{2l_0} \Rightarrow l_0 = \frac{17 \times 100}{33} = 51.5\,\text{cm}$

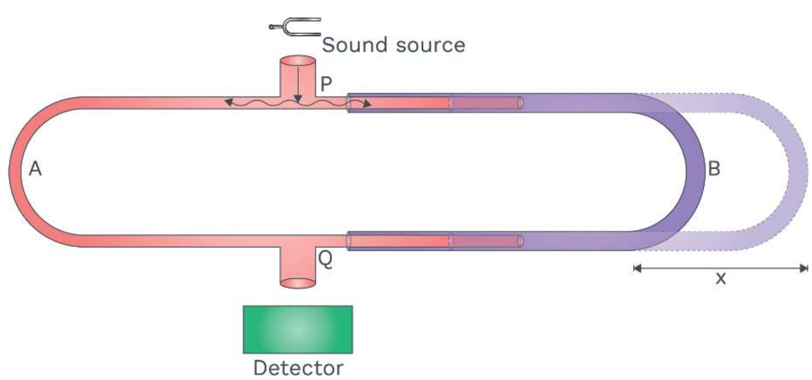
and $n_0 = \frac{v}{2l_0} = \frac{330 \times 100}{2 \times 60} = 283.33\,\text{Hz}$

Concept Reminder

A whale's voice travels at a maximum speed of 800 kilometres or 479 miles in the water. Dolphins hears sounds from 24 kilometres or 15 miles away underwater.

Quink's Tube

This is an apparatus used to demonstrate phenomenon of interference and also used to measure velocity of the sound in air. This is made up of two U-tubes 'A' and 'B' as shown in the figure. Here the tube 'B' can slide in and out from the tube 'A'. There are two openings 'P' and 'Q' in the tube 'A'. At opening 'P', a tuning fork or a sound source of known frequency n_0 is placed and at other opening a detector is placed to detect the resultant sound of interference occurred due to the superposition of two sound wave coming from the tubes 'A' and 'B'.



Initially tube B is adjusted so that detector detects a maximum. At this instant if length of paths covered by the two waves from P to Q from the side of A and side of B are l_1 are l_2 respectively then for constructive interference we must have

$$l_2 - l_1 = N\lambda \quad \text{... (1)}$$

If now tube B is further pulled out by a distance x so that next maximum is obtained and length of path from the side of B is l_2' then we have

$$l_2' = l_2 + 2x \quad \text{... (2)}$$

Where 'x' is the displacement of tube. For next constructive interference of sound at the point 'Q', we have

$$l_2' - l_1 = (N + 1)\lambda \quad \text{... (3)}$$

From equation (1), (2) and (3), we get

$$\text{or} \quad x = \frac{\lambda}{2} \quad \text{... (4)}$$

Thus by experiment we get wavelength of sound as for the two successive points of constructive interference, the path difference must be 'λ'. As the tube 'B' is pulled out by 'x', this introduces a path difference '2x' in the path of sound wave through tube 'B'. If the frequency of source is known, n_0, the velocity of sound in the air filled in tube can be given as;

$$v = n_0\lambda = 2n_0x \quad \text{... (5)}$$

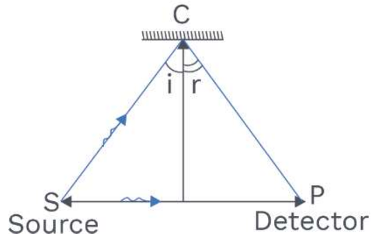
Ex. In the large room a person receives direct sound waves from the source '120 m' away from him. he also receives waves from same source which reach him, being reflected from '25 m' high ceiling at the point halfway between them. For which the wavelength will these two sound waves interfere constructively?

Sol. As shown in figure for reflection from the ceiling

Path SCP = SC + CP = 2SC $\quad$ [As $\angle i = \angle r, SC = CP$]

or $\quad$ Path SCP = $2\sqrt{60^2 + 25^2} = 130\,\text{cm}$

[As $\angle i = \angle r$, SC = CP]



So, path difference between interfering waves along path SCP, and SP,

$$\Delta x = 130 - 120 = 10\,m$$

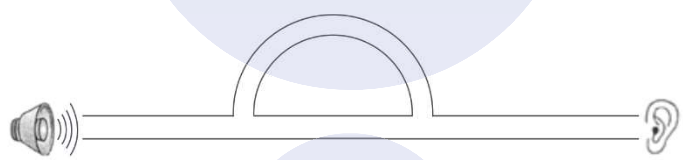
Now for constructive interference at P.

$\Delta x = n\lambda$ i.e, $10 = n\lambda$

or $\lambda = \frac{10}{n}$ with n = 1, 2, 3 ....

i.e. λ = 10 m, 5m, (10/3) m and so on

Ex. As shown a tube structure in which a sound signal is sent from the one end and is received at other end. The semicircular part has a radius of '20.0 cm'. The frequency of the sound source can be varied electronically between 1000 and 4000 Hz. Find the frequencies at which maxima of the intensity is detected. The speed of sound in air = '340 m/s'.



Sol. Sound wave reaches detector by the two paths simultaneously be straight as well as the semicircular track. Wave through the straight path travels a distance 'l_1 = 2 × 20 cm' and the wave through the curved part travels a distance $l_2 = \pi(20\,cm) = 62.8\,cm$

= 62.8 cm before they meet again and travel to receiver. The path difference between two waves received is, therefore,

$$\Delta l = l_2 - l_2 = 62.8\,cm - 40\,cm = 22.8\,cm = 0.228\,m$$

The wavelength of either wave is $\frac{v}{n} = \frac{340}{n}$. For constructive interference, $\Delta l = N\lambda$ where N is an integer

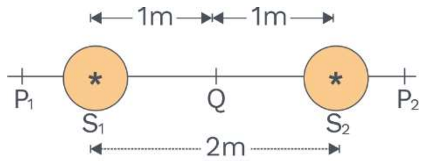
or, $0.228 = N\left(\frac{340}{n}\right)$

or, $n = N\left(\frac{340}{0.228}\right) = N(1491.2)\,Hz \simeq N(1490)\,Hz$

Thus, the frequencies within the specific range which cause maxima of intensity are 1490 Hz and 2980 Hz.

Ex. Two sources 'S_1' and 'S_2', separated by 2.0 m, vibrate according to equation y_1 = 0.03 $\sin \pi t$ and $y_2 = 0.02 \sin \pi t$ where y_1, y_2 and t are in M.K.S unit. They send out waves of velocity 1.5 m/s. Calculate amplitude of the resultant motion of particle co-linear with 'S_1' and 'S_2' and located at a point (a) to the right of S_2 (b) to the left of S_2 and (c) in the middle of S_1 & S_2.

Sol. The situation of shown in figure,



The oscillations y_1 and y_2 have amplitudes A_1 = 0.03 m and A_2 = 0.02 respectively.

The frequency of both sources in $n = \frac{\omega}{2\pi} = \frac{1}{2} = 0.5\,\text{Hz}$

Now wavelength of each wave $n = \frac{v}{n} = \frac{1.5}{0.5} = 3.0\,\text{m}$

(a) The path difference for all points P2 to the right of S_2 is

$\Delta = (S_1P_2 - S_2P_2) = S_1S_2 = 2\text{m}$

Phase difference $\phi = \frac{2\pi}{\lambda} \times$ Path difference

$= \frac{2\pi}{3} \times 2.0 = \frac{4\pi}{3}$

The resultant amplitude for this point is given by

$R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi}$

$= \sqrt{(0.03)^2 + (0.02)^2 + 2 \times 0.03 \times 0.02 \times \cos(4\pi / 3)}$

Solving we get R = 0.0265 m

(b) The path difference for all point P, to the left of S_1

$\Delta = (S_2P - S_1P) = S_1S_2 = 2.0\,\text{m}$

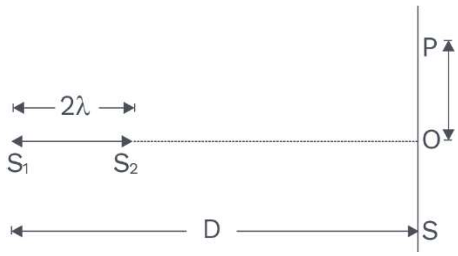
Hence the resultant amplitude for all points to the left of S_1 is also 0.0265 m

(c) For a point Q, between S_1 and S_2, the path difference is zero i.e., $\phi = 0$. Hence constructive interference take place at Q, thus amplitude at that point is maximum and given as;

$R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2}$

$= A_1 + A_2 = 0.03 + 0.02 = 0.05$ m

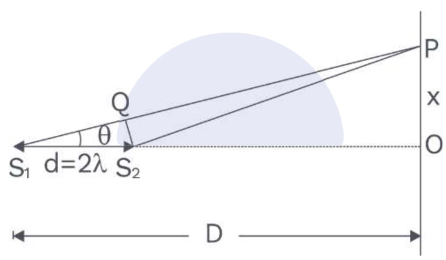
Ex. Two coherent narrow slits emitting of the wavelength λ in the same phase are placed parallel to each other at the small separation of 2λ. Sound is detected by moving a detector on screen S at a distance D ($>>\lambda$) from slit S_1 as shown in figure. Find the distance x such that the intensity at P is equal to the intensity at O.



Sol. When the detector is at O, we can see that the path difference in two waves reaching O is d = 2λ thus at O detector receives a maximum sound. When it reaches P and again there is a maximum sound detected at P the path difference between two waves must $\Delta = \lambda$. Thus shown figure the path difference at P can be given as

$$\Delta = S_1P - S_2P \simeq S_1Q$$

$$= d\cos\theta = 2\lambda\cos\theta$$



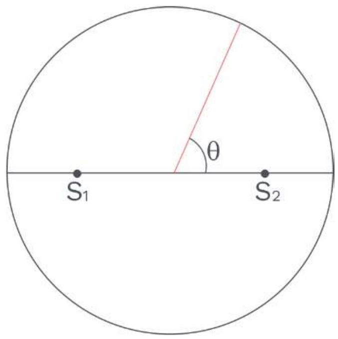
And we have at point 'P', path difference $\Delta = \lambda$. Thus, $\theta\Delta = 2\lambda\cos\theta = \lambda$

or, $\cos\theta = \frac{1}{2}$

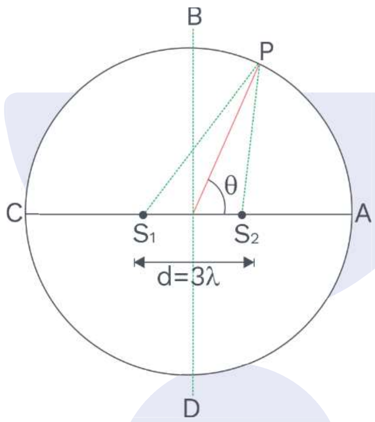
or, $\theta = \frac{\pi}{3}$

Thus the value of x can be written as x = D $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}D$

Ex. As shown in figure two coherent sources S_1 and S_2 which emit sound of wavelength λ in phase. The separation between the sources is 3λ. A circular wire of the large radius is placed in such a way that S_1S_2 lies in its plane and the middle point of S_1S_2 is at the centre of the wire. Find the angular position θ on wire for which constructive interference takes place.



Sol.



From previous question, we can say that for a point P on the circle shown in figure shown. The path difference in two waves at P is

$$\Delta = S_1P - S_2P = d\cos\theta = 3\lambda\cos\theta$$

We know for constructive interference at 'P'. Path difference must be integral multiple of wavelength λ. Thus for a maxima at P, we have,

$3\lambda\cos\theta = 0;$ $\theta = 3\lambda\cos\theta = \lambda;$

$3\lambda\cos\theta = 2\lambda;$ $3\lambda\cos\theta = 3\lambda;$

or $\theta = \frac{\pi}{2}$ or $\theta = \cos^{-1}\frac{1}{3}$

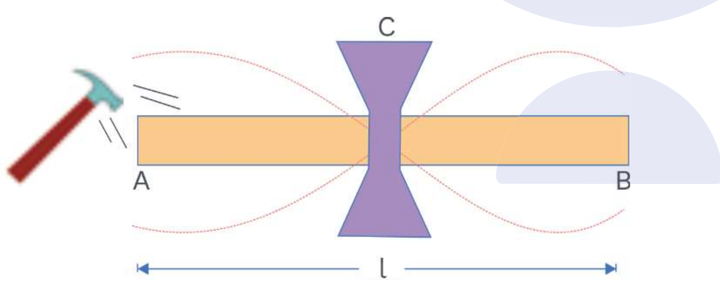
or $\theta = \cos^{-1}\frac{2}{3}$ or $\theta = 0$

There are four points A, B, C and D on circle at which $\theta = 0$ or $\frac{\pi}{2}$ and there are the two points in each quadrant at $\theta = \cos^{-1}\frac{1}{3}$ and $\theta = \cos^{-1}\frac{2}{3}$ at which constructive interference takes place. Thus there are total twelve points on circle at which maxima occurs.

Vibrations of Clamped Rod

We discussed the resonant vibrations of a string clamped at the two ends. Now we discuss oscillations of a rod clamped at the point on its length as shown in the figure. Figure shows the rod 'AB' clamped at the middle point. If we gently hit rod at its one end, it begin to oscillate and in natural oscillations rod vibrates at its lowest frequency and maximum wavelength, which we called the fundamental mode of oscillations. With the maximum wavelength when transverse stationary waves setup in rod, free ends vibrates as antinodes and the clamped end a node as shown in figure. Here if 'λ' be the wavelength of the wave, we have;

$$l = \frac{\lambda}{2} \quad \text{or} \quad \lambda = 2l$$



Thus the frequency of fundamental oscillations of the rod damped at mid point can be given as

$$n_0 = \frac{v}{\lambda} = \frac{l}{2l}\sqrt{\frac{Y}{\rho}} \quad \text{... (1)}$$

Where 'Y' is the Young's modulus of material of rod and ρ is the density of the material of rod. Next higher frequency at which rod vibrates will be then one when wave length is decreased to a value so that one node is inserted between mid-point and an end of rod as shown in figure.

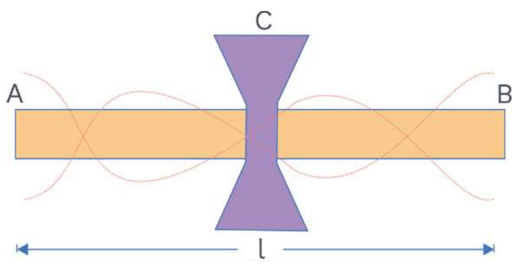
Rack your Brain

A car is moving towards a high cliff. The driver sounds a horn of frequency f. The reflected sound heard by the driver has frequency 2f if v the velocity of sound, then find the velocity of the car, in the same velocity units.

Concept Reminder

The frequency of fundamental oscillations of a rod damped at mid-point can be given as

$$n_0 = \frac{v}{\lambda} = \frac{l}{2l}\sqrt{\frac{Y}{\rho}}$$



In this case if λ be the wavelength of the waves in rod, we have

$$l = \frac{3\lambda}{2}$$

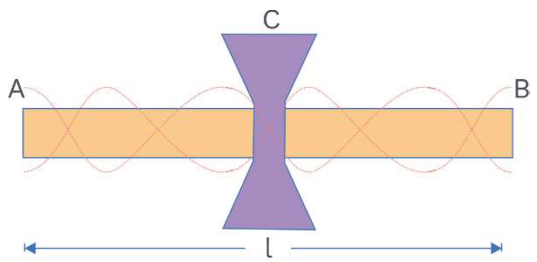
or $\lambda = \frac{2l}{3}$ ... (2)

Thus in this case the oscillation frequency of rod can be given as

$$n_1 = \frac{v}{\lambda} = \frac{3}{2l}\sqrt{\frac{Y}{\rho}} = 3n_0 \qquad \text{... (3)}$$

This is called first overtone frequency of damped rod or third harmonic frequency. Similarly, the next higher frequency oscillation i.e. second overtone of the oscillating rod can be shown in figure the shown. Here is 'λ' is the wavelength of wave then it can be given as

$$l = \frac{5\lambda}{2} \quad \text{or} \quad \lambda = \frac{2l}{5} \qquad \text{... (4)}$$



Thus frequency of oscillation of rod can be given as;

$$n_2 = \frac{v}{\lambda} = \frac{5}{2l}\sqrt{\frac{Y}{\rho}} = 5n_0 \qquad \text{... (5)}$$

Concept Reminder

When an external source of frequency matching with any of the harmonic of the damped rod then stationary waves are setup in the rod.

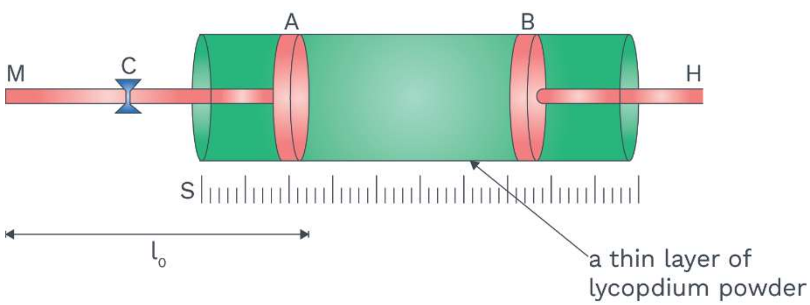
Thus, the second overtone frequency is fifth harmonic of fundamental oscillation frequency of rod. We can also see from above analysis that resonant frequencies at which stationary waves are setup in a damped rod are only odd harmonics of fundamental frequency.
Thus when an external source of frequency matching with any of the harmonic of the damped rod then stationary waves are setup in the rod.

Natural Oscillation of Organ Pipes

When we initiate some oscillations in an organ pipe, which harmonics are excited in the pipe depends on how initial disturbance is produced in it. For example, if you gently blow across the top of an organ pipe it resonates softly at its fundamental frequency. But if you blow must harder you hear the higher pitch of an overtone because the faster airstream higher the frequencies in exciting disturbance. This sound effect can also achieved by increasing air pressure to an organ pipe.

Kundt's Tube

This apparatus used to find velocity of sound in a gaseous medium or in the different materials. It consists of the glass tube as shown in the figure. one end of which a piston 'B' is fitted which is attached to the wooden handle 'H' and can be moved inside and outside tube and fixed, the rod 'M' of the required material is fixed at clamp 'C' in which velocity of sound is required, at one end of the rod a disc 'A' is fixed as shown.

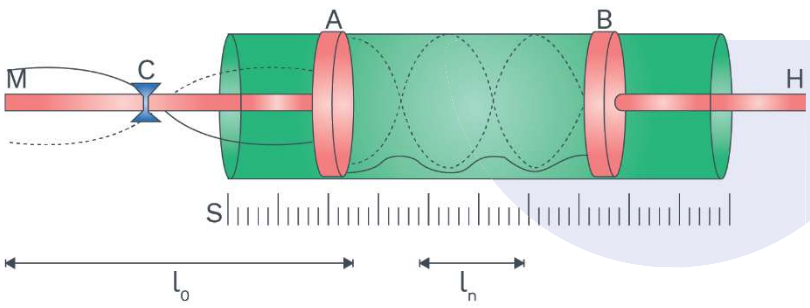


Rack your Brain

A whistle revolves in a circle with angular speed ω = 20 rad/s using a string of length 50 cm. If the frequency of sound from the whistle is 385 Hz, then what is the minimum frequency heard by an observer who is far away from the centre (v_{sound} = 340 m/s)?
(1) 385 Hz (2) 374 Hz
(3) 394 Hz (4) 333 Hz

In the tube air is filled at room temperature and a thin layer of lycopodium powder is put along the length of tube. It is very fine powder particles of which can be displaced by the air particles. When rod 'M' is gently rubbed with the resin cloth or hit gently, it starts oscillating in fundamental mode as shown in the figure, frequency of which can be given as;

$$n_{rod} = \frac{v}{\lambda} = \frac{1}{2l_0}\sqrt{\frac{Y}{\rho}} \qquad [\text{As } \lambda_0 = \frac{\lambda}{2}]$$



Doppler's Effect (Δn) :-

- Due to relative motion between source and observer observer observes different frequency from the frequency produced by a source.
- This change in frequency due to the relative motion between source and observer is known as Doppler's effect.
- This phenomena is related to change in frequency or change in pitch but not related to change in intensity.
- Sound waves, light waves, EM waves, micro waves, radio waves all show Doppler's Effect.

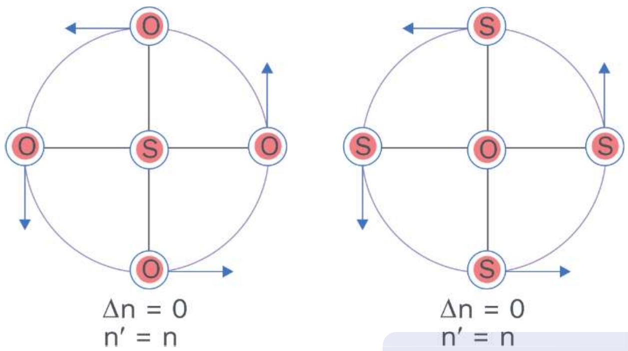
Doppler's Effect for Sound Waves :-
Condition for which sound wave does not show Doppler's effect:-

(1) When source and observer both are at rest.
(2) When source and observer both are moving with same speed in same direction.

Definitions

Doppler's Effect
Due to relative motion between source and observer observer observes different frequency from the frequency produced by a source. This change in frequency due to relative motion between source and observer is known as Doppler's effect

(3) Sound wave does not show Doppler's effect for transverse motion but light wave shows.



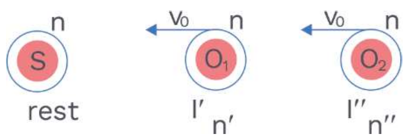
(4) When speed of source or observer is greater than speed of sound in the medium then Doppler's theory is not applicable.

Special Points :-

(1) When source or absorber or both move towards each other then apparent frequency is greater than actual frequency (n′ > n).

(2) When source or observer or both move away from each other then n′ < n.

(3) Change in frequency depends on relative motion between source and observer. It does not depends on distance between source and observer.

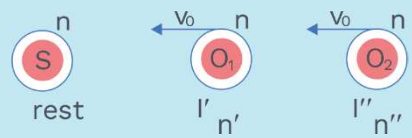


$\Rightarrow$ n' = n'' and l' > l''

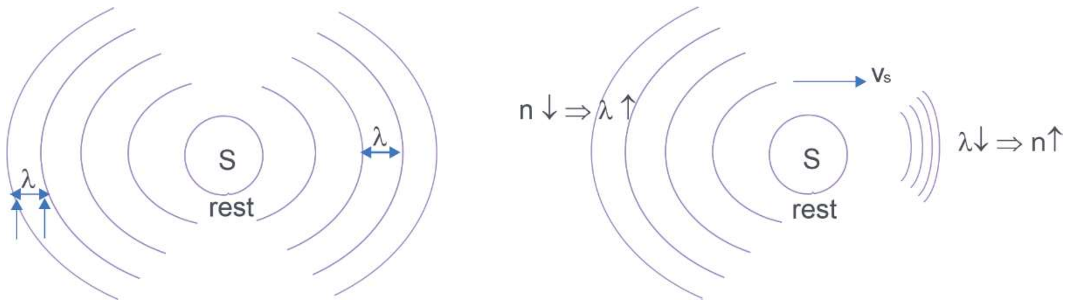
(4) Wavelength of wave is not affected by motion of observer it is affected by motion of source.

Concept Reminder

Change in frequency depends on relative motion between source and observer. It does not depends on distance between source and observer.



$\Rightarrow$ n' = n'' and l' > l''



Important Formula :-

$$n' = \left[\frac{v \pm v_0}{v \mp v_S}\right] n$$

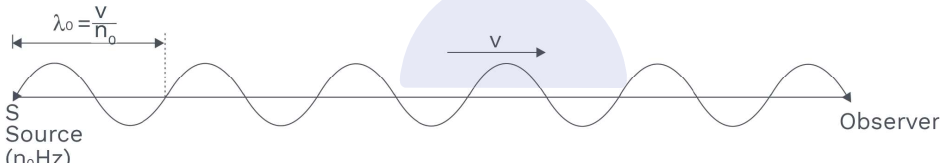
Concept Reminder

Dogs are capable of hearing sounds at a much higher frequency than humans can. They can hear sounds or noises humans cannot.

Stationary Source and Stationary Observer

Figure shown a stationary sources of frequency n_0 which produces sound waves in air of wavelength λ_0 given as

$$\lambda_0 = \frac{v}{n_0} \qquad \text{[v = speed of sound in air]}$$



Although sound waves are longitudinal wave here we represent sound weaves by transverse displacement curve as shown in the figure to understand the concept in the better way. As source produces waves, these waves travel towards, stationary observer O in the medium (air) with speed v and wavelength 'λ_0'. As observer is at rest here it will observe the same wavelength λ_0 is approaching it with speed v so it will listen the frequency 'n' given as;

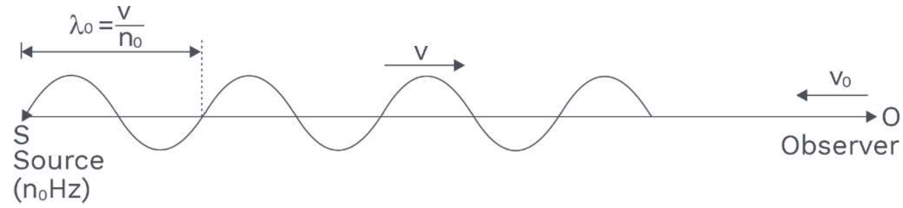
$$n = \frac{v}{\lambda_0} = n_0 \qquad \text{[same as that of source]} \qquad ...(1)$$

This is why when the stationary observer listen the sound from a stationary source of the sound, it detects same frequency sound which the source is producing. Thus, no Doppler effect takes place if there is no relative motion between the source and observer.

Stationary Source and Moving Observer

As shown in figure when a stationary sources of the frequency 'n_0' produces the sound waves which have wavelength in air given as;

$$\lambda_0 = \frac{v}{n_0}$$



These waves are travel toward moving observer with velocity v_0 towards source. When sound waves approach observer, it will receive the waves of wavelength λ_0 with speed '$v + v_0$'. Thus frequency of sound heard by observer can be given as

Apparent frequency $n_{ap} = \frac{v + v_0}{\lambda_0}$

$$= \frac{v + v_0}{\left(\frac{v}{n_0}\right)} = n_0\left(\frac{v + v_0}{v}\right) \quad \text{... (2)}$$

Similarly we can say that if the observer is receding away from the source the apparent frequency heard by the observer will be given as

$$n_{ap} = n_0\left(\frac{v - v_0}{v}\right) \quad \text{... (3)}$$

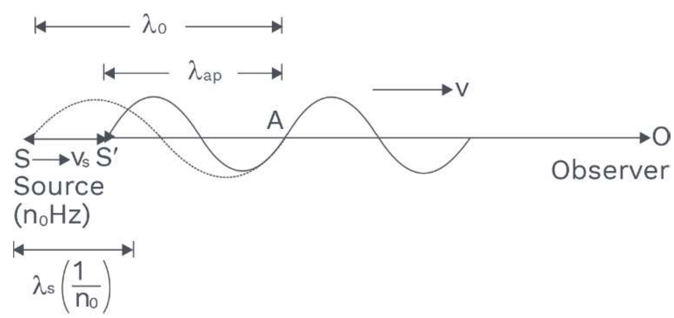
Concept Reminder

if the observer is receding away from the source the apparent frequency heard by the observer will be given as

$$n_{ap} = n_0\left(\frac{v - v_0}{v}\right)$$

Moving Source and Stationary Observer

As shown in the figure situation when a moving source S of frequency n_0 produces sound waves in medium and the waves travel toward the observer with velocity 'v'.



Here if we carefully look at the initial situation when source starts moving with velocity v_s as well as it starts producting waves. The period of one oscillation is $\left(\frac{1}{n_0}\right)$ sec and in this duration source emits one wavelength λ_0 in the direction of propagation of the waves with speed 'v', but in this duration the source will also move forward by the distance $v_s\left(\frac{1}{n_0}\right)$.

Thus the effective wavelength of emitted sound in air slightly compressed by this distance as shown in the figure. This is termed as apparent wavelength of sound in the medium by the moving source. This is given as;

Apparent wavelength $\lambda_{ap} = \lambda_0 - v_s\left(\frac{1}{n_0}\right)$ ... (1)

$$= \frac{\lambda_0 n_0 - v_s}{n_0} = \frac{n - v_s}{n_0}$$

This wavelength will approach observer with the speed 'v' (O is at rest). Thus frequency of sound heard by the observer can be given as;

Apparent frequency $n_{ap} = \frac{v}{\lambda_{ap}}$

$$= \frac{v}{(v - v_s)/n_0} = n_0\left(\frac{v}{v - v_s}\right) \quad \text{... (2)}$$

Similarly, if the source is receding away from observer, apparent wavelength emitted by the source in air toward observer will be slightly expanded and apparent frequency heard by the stationary observer can be given as

$$\boxed{n_{ap} = n_0\left(\frac{v}{v + v_s}\right)} \quad \text{... (3)}$$

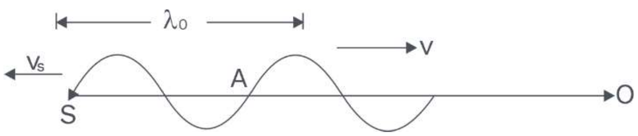
Concept Reminder

if source is receding away from observer, the apparent wavelength emitted by source in air toward observer will be slightly expanded and the apparent frequency heard by the stationary observer can be given as

$$\boxed{n_{ap} = n_0\left(\frac{v}{v + v_s}\right)}$$

Moving Source and Moving Observer

Let us consider the situation when both source and observer are moving in the same direction as shown in the figure at speeds 'v_s' and 'v_0' respectively.



In this case the apparent wavelength emitted by source behind it is given as

$$\lambda_{ap} = \frac{v + v_s}{n_0}$$

Now, this wavelength will approach the observer at relative speed $v + v_0$ thus the apparent frequency of sound heard by observer is given as

... (1)

Expression of apparent frequency given by equation, we can easily develop a general relation for finding the apparent frequency heard by a moving observer due to a moving source as

... (2)

Concept Reminder

a general relation for finding the apparent frequency heard by a moving observer due to a moving source as

Here + and – signs are chosen according to the direction of motion of source and observer. The sign convention related to the motion direction can be stated as :

(i) For both source and observer v_0 and v_s are taken in equation with –ve sign if they are moving in the direction of i.e. the direction of propagation of sound from source to observer.

(ii) For both source and observer v_0 and v_s are taken in equation (2) with +ve sign if they are moving in the direction opposite to i.e. opposite to the direction of propagation of sound from source to observer.

Doppler Effect in Reflected Sound

When a car is moving toward a stationary wall as shown in the figure. If the car sounds a horn, wave travels towards the wall and is reflected from wall. When the reflected wave is heard by driver, it appears to be of the relatively high pitch. If we wish to measure frequency of reflected sound then problem must be handled in two steps.

First we treat stationary wall as stationary observer and car as a moving source of sound of frequency 'n_0'. In this case the frequency received by the wall is given as;

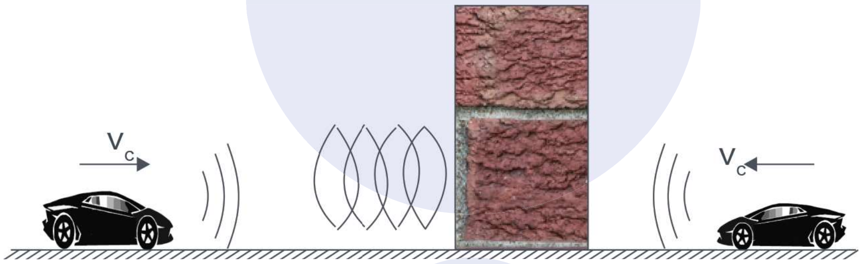
... (1)

Now, wall reflects this frequency and behaves like a stationary source of sound of frequency n_1 and car (driver) behave like a moving observer with velocity v_c. Here the apparent frequency heard by the car driver can be given as

$$n_{ap} = n_1\left(\frac{v + v_c}{v}\right)$$

$$= n_0\left(\frac{v}{v - v_c}\right) \times \left(\frac{v + v_c}{v}\right) = n_0\left(\frac{v + v_c}{v - v_c}\right) \quad \text{... (2)}$$

Same problem can also be solved in a different manner by using method of sound images. In this procedure we assume the image of the sound source behind the reflector. In previous example we can explain this by situation shown in figure.



Here we assume that the sound which is reflected by the stationary wall is coming from the image of car which is at the back of it and coming toward it with velocity v_c. Now the frequency of sound heard by car driver can directly be given as

$$\boxed{n_{ap} = n_0\left[\frac{v + v_c}{v - v_c}\right]} \quad \text{... (3)}$$

This method of images for solving the problems of Doppler effect is very convenient but is used only for the velocities of source and observer which are very small compared to the speed of sound and it should not used frequently when the reflector of sound is moving.

Concept Reminder

This method of images for solving problems of Doppler effect is very convenient but is used only for velocities of source and observer which are very small compared to the speed of sound and it should not be used frequently when the reflector of sound is moving.

Doppler's Effect for Accelerated Motion

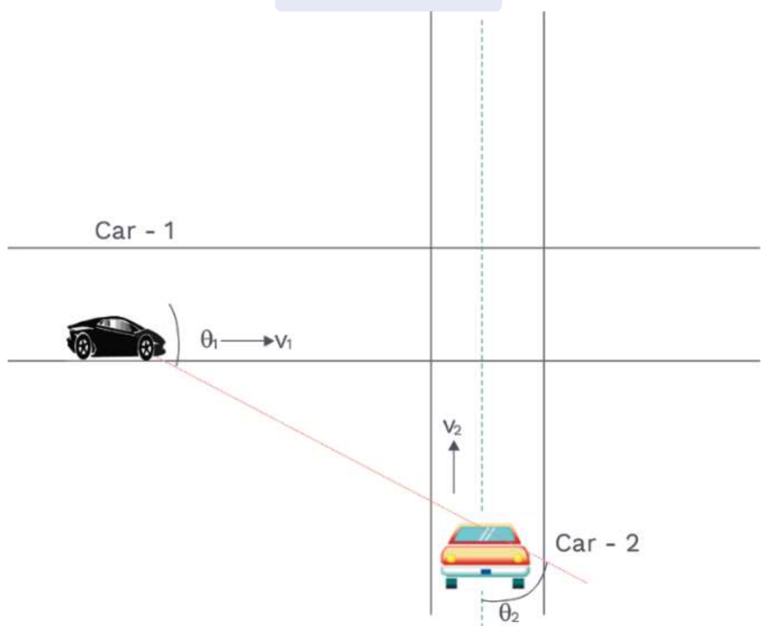
For the case of a moving source and a moving observer, we know apparent frequency observer can be given as;

$$n_{ap} = n_0 \left[\frac{v \pm v_0}{v \mp v_s} \right] \quad \dots (4)$$

Here v is the velocity of sound and v_0 and v_s are the velocity of observer and source respectively. When a source of observer has accelerated or retarded motion then in equation (4) we use that value of v_0 at which observer receives the sound and for source, we use that value of v_s at which it has emitted the wave.
The alternative method of solving this case is by the traditional method of compressing or expending wavelength of sound by motion of source and using relative velocity of sound with respect to observer

Doppler's Effect when Source and Observer are not in Same Line of Motion
Consider the situation shown in the figure. Two cars 1 & 2 are moving along perpendicular roads at speed v_1 and v_2. When car - 1 sound a horn of frequency n_0, it emits sound in all directions and say car - 2 is at the position, shown in figure. when it receives the sound. In such cases we use velocity components of the cars along the line joining the source and observer thus the apparent frequency of sound heard by car-2 can be given as

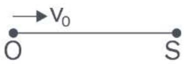
$$n_{ap} = n_0 \left[\frac{v + v_2 \cos\theta_2}{v - v_1 \cos\theta_1} \right]$$



Different Cases :

(I) 

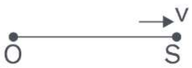
$f_{app} = f$

(ii) 

$f_{app} = \left(\frac{v + v_0}{v}\right) f$

(iii) 

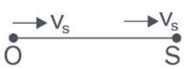
$f_{app} = \left(\frac{v - v_0}{v}\right) f$

(iv) 

$f_{app} = \left(\frac{v}{v + v_0}\right) f$

(v) 

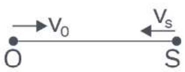
$f_{app} = \left(\frac{v}{v - v_s}\right) f$

(vi) 

$f_{app} = \left(\frac{v + v_0}{v + v_s}\right) f$

(vii) 

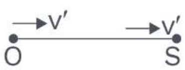
$f_{app} = \left(\frac{v - v_0}{v + v_s}\right) f$

(viii) 

$f_{app} = \left(\frac{v + v_0}{v - v_s}\right) f$

(ix) 

$f_{app} = \left(\frac{v - v_0}{v + v_s}\right) f$

(x) 

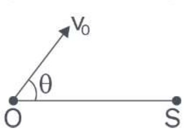
$f_{app} = f$

(xi) 

$f_{app} = f$

(xii) 

$f_{app} = f$

(xiii) 

$f_{app} = \left(\frac{v + v_0\cos\theta}{v}\right)$

Concept Reminder

Horror films like to use infrasound, which is below the range of human hearing. It creates shivering, anxiety, and even heart palpitations in humans when it is being played.

Different Examples:

(1) When source crosses the stationary observer.

$$n' = \left(\frac{v}{v - v_S}\right) n \quad ; \quad n'' = \left(\frac{v}{v + v_S}\right) n$$

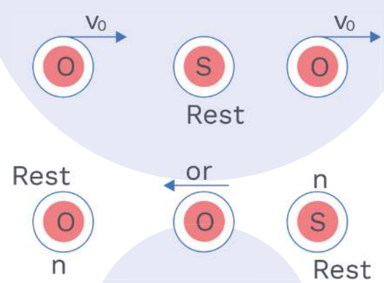
$$b = \Delta n = \left[\frac{v}{v - v_S} - \frac{v}{v + v_S}\right]n = \left[\frac{v^2 + vv_S - v^2 + vv_S}{v^2 - v_S^2}\right]n$$

$$b = \Delta n = \left[\frac{2vv_S}{v^2 - v_S^2}\right]n \text{ if } v_S << v \text{ then } v_S^2 \cong 0$$

$$b = \Delta n = \frac{2vv_S}{v^2}n$$

$$\boxed{b = \Delta n = \frac{2v_S}{v}n}$$

(2) When observer crosses the stationary source.

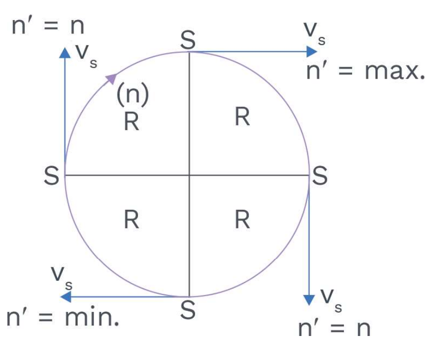


$$n' = \left(\frac{v + v_0}{v}\right)n \quad ; \quad n'' = \left[\frac{v - v_0}{v}\right]n$$

$$b = \Delta n = \left(\frac{v + v_0}{v} - \frac{v - v_0}{v}\right)n = \frac{2v_0}{v}n$$

$$\boxed{b = \Delta n = \frac{2v_0}{v}n}$$

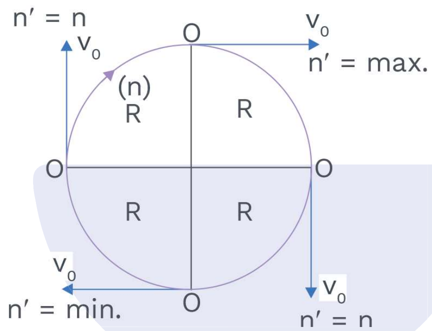
(3) If observer is stationary far away from circular path of source.



$$n'_{max.} = \left(\frac{v}{v - v_s}\right)n = \left(\frac{v}{v - \omega R}\right)n$$

$$n'_{max.} = \left(\frac{v}{v + v_s}\right)n = \left(\frac{v}{v + \omega R}\right)n \qquad (v_s = \omega R)$$

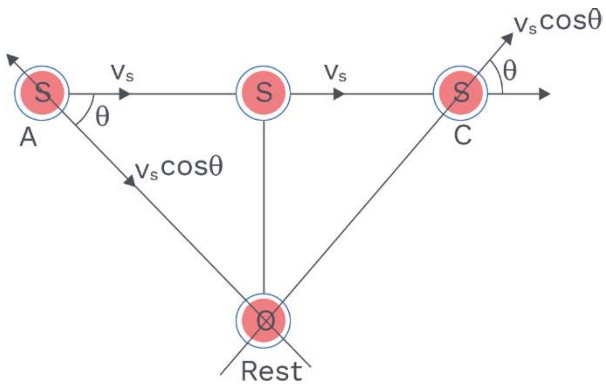
(4) If source is stationary far away from the circular path of the observer.



$$n'_{max.} = \left(\frac{v + v_0}{v}\right)n = \left(\frac{v + \omega R}{v}\right)n$$

$$n'_{min.} = \left(\frac{v - v_s}{v}\right)n = \left(\frac{v - \omega R}{v}\right)n \qquad (v_s = \omega R)$$

(5) Doppler's Effect For Transverse Motion:-

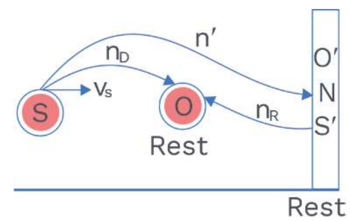


$$n'_A = \left[\frac{v}{v + v_s \cos\theta}\right]n \qquad \begin{matrix} n_B = n \\ \Delta n = 0 \end{matrix}, \qquad n'_C = \left[\frac{v}{v + v_s \cos\theta}\right]n$$

(6) Beat phenomena in Doppler's Effect:-

Case-I

When source, moved towards stationary target.

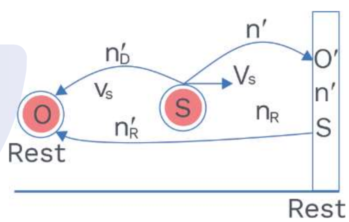


$$n'_D = \left(\frac{v}{v - v_s}\right)n$$

$$n' = \left(\frac{v}{v - v_s}\right)n$$

$$n'_R = n'$$

$$b = \Delta n = n'_R - n'_D = 0$$



$$n'_D = \left(\frac{v}{v + v_S}\right)n$$

$$n' = \left(\frac{v}{v - v_S}\right)n$$

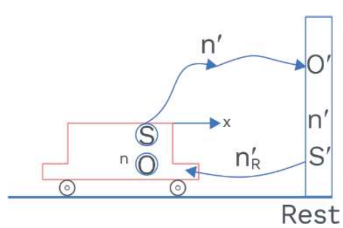
$$b = \Delta n = \left(\frac{v}{v - v_S} - \frac{v}{v + v_S}\right)n = \frac{2v_S vn}{v^2 - v_S^2}$$

$$b \cong \left(\frac{2v_S}{v}\right)n \qquad v >> v_s$$

Note : Stationary target behave as an observer for the incident sound and behave as a source for reflected sound.

Case-II : When source and observer both move towards stationary target.

(x is the speed of source and observer).



* $n'_D = n,$ * $n' = \left(\frac{v}{v - x}\right)n$

$$n'_R = \left(\frac{v + x}{v}\right)n', \qquad n'_R = \left[\frac{v + x}{v}\right]\left[\frac{v}{v - x}\right]n$$

$$\boxed{n'_R = \left(\frac{v + x}{v - x}\right)n}$$

$$\text{beats} = n'_R - n'_D = \frac{v + x}{v - x}n - n, \qquad \boxed{b = \frac{2x}{v - x}n}$$

Light Doppler Effect :-

Two basic difference between Doppler's effect for light waves and sound waves are.

(1) Sound waves do not show Doppler's effect for transverse motion but light waves show.

(2) Doppler effect for sound waves is an asymmetric phenomena but for light waves it is a symmetric phenomena.

$C = n\lambda$

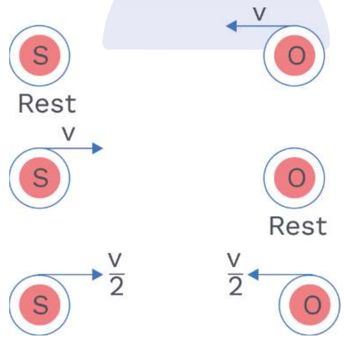
C = const, then $\boxed{n \propto \frac{1}{\lambda}}$

- Speed of light (c) works as infinity means $C \pm v_0 \cong C$ or $v_S \cong C$
- It is based on Einstein's theory of relativity.
- According to Einstein apparent frequency of light waves $n' = \sqrt{\frac{1 \pm \frac{v}{C}}{1 \mp \frac{v}{C}}}\, n$

when v is the relative speed between source and observer.

Case-I

When source or observer or both move towards each other with relative speed v.



$$n' = \sqrt{\frac{1 \pm \frac{v}{C}}{1 - \frac{v}{C}}}\, n$$

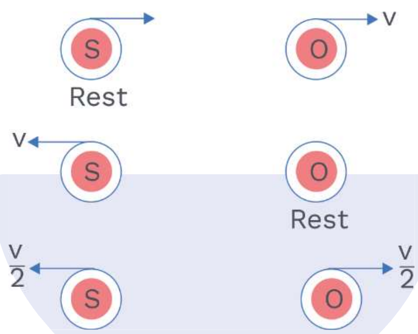
$$n' = \left(1 + \frac{v}{C}\right)^{\frac{1}{2}} \left(1 - \frac{v}{C}\right)^{-\frac{1}{2}} n$$, if v << C then

$$n' = \left(1 + \frac{v}{2C}\right)^{2} n, \qquad \boxed{n' = \left(1 + \frac{v}{C}\right) n}$$

For $\lambda' \to \lambda = \frac{C}{n'} = \frac{C}{\left(1+\frac{v}{C}\right)n}$, $\lambda' = \left(1+\frac{v}{C}\right)^{-1}\frac{C}{n}$

$$\boxed{\lambda' = \left(1-\frac{v}{C}\right)\lambda}, \quad \boxed{\Delta n = +\frac{v}{C}n}, \quad \boxed{\Delta\lambda = -\frac{v}{C}\lambda}$$

Case-II



$$\boxed{n' = \left[1-\frac{v}{C}\right]n}, \quad \boxed{\lambda' = \left[1+\frac{v}{C}\right]\lambda},$$

$$\boxed{\Delta n = \frac{-v}{C}n}, \quad \boxed{\Delta\lambda = \frac{+v}{C}\lambda},$$

$$\boxed{\Delta n = \pm\frac{v}{C}n}, \quad \boxed{\frac{\Delta n}{n}\times 100 = \frac{v}{C}\times 100},$$

$$\boxed{\Delta\lambda = \mp\frac{v}{C}\lambda}, \quad \boxed{\frac{\Delta\lambda}{\lambda}\times 100 = \frac{v}{C}\times 100}$$

$\Delta\lambda$ = Doppler displacement.

= Doppler shift.

= Change in wavelength.

= Shift of spectral line.

Spectrum

Spectrum of Visible Light

Applications of Doppler effect

(1) To explain the expansion of universe. (Red shifts)

(2) To find out velocity of any light source w.r.t. to us or earth.

(3) Spin Rotational speed of light source.
[Sun rotates on its own axis from East to West with 2 km/s]

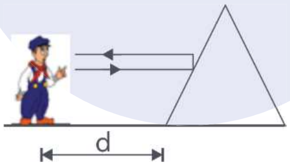
(4) Boarding of line spectrum can be explained by using LDE

Ex. A star which is emitting radiation at a wavelength of 5000 Å, is approaching the earth with a velocity of $1 \cdot 5 \times 10^3$ m/s. Calculate the change in wavelength of the radiation as received by the earth.

Sol. $\Delta\lambda = \frac{v}{c}\lambda = \frac{1.5 \times 10^3}{3 \times 10^8} \times 5000 = 0.025Å$

Echo :-

Reflected sound is known as Echo.



Time taken by a person to hear echo of his sound

$$t = \frac{2d}{v}$$

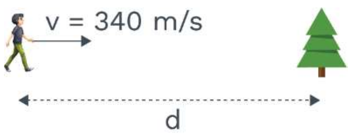
d $\rightarrow$ distance between source & obstacle

v $\rightarrow$ Velocity of sound.

Ex. A man standing on a cliff claps his hand and hears its echo after one second. If the sound in reflected from another mountain then find the distance between the man & reflection points (v_{sound} = 340 m/sec)

Sol. Let distance between cliff and mountain be d

$$1 = \frac{d}{340} + \frac{d}{340} \Rightarrow d = 170\,m$$



$$\text{distance} = \frac{340}{2} = 170\,m$$

Ex. A source of sound wave of frequency n travels with velocity v towards a large vertical plane wall. Sound is reflected from the wall. Speed of sound in medium is u (u >> v). Then what is the frequency of reflected wave.

Sol. Frequency of wave incident on wall is :

$n_1 = n\left(\frac{u}{u-v}\right)$ same frequency is reflected.

Ex. In above example, If an observer is also moving towards wall with velocity v behind the source. Then calculate the number of beats heard.

Sol. Frequency heard by observer directly from source is $n_2 = n\left(\frac{u+v}{u-v}\right) = n$

and frequency heard by observer of the wave reflected from wall.

$n_3 = n_1\left(\frac{u+v}{u}\right) = n\left(\frac{u+v}{u-v}\right)$

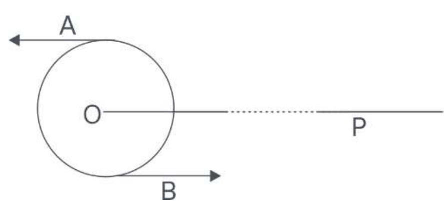
Number of beats

$= n_3 - n_2 = n\left(\frac{u+v}{u-v}\right) - n = n\left(\frac{2v}{u-v}\right) = \frac{2nv}{u-v}$

Ex. A whistle of frequency '540 Hz' is moving in a circle of radius '2 ft' at a constant angular speed of '15 rad/s'. What is the lowest and highest frequencies heard by the listener standing at the rest, a long distance away from centre of circle? Velocity of sound in air is 1100 ft/sec.

Sol. The whistle is moving along a circular path with constant angular velocity ω. The linear velocity of the whistle is given by

$v_S = \omega R$



where, 'R' is radius of the circle.

At points 'A' and 'B', the velocity 'v_S' of whistle is parallel to line 'OP'; i.e., with respect to observer at 'P', whistle has maximum velocity 'v_s' away from 'P' at point 'A', and towards 'P' at point 'B'. (Since distance OP is large compared to radius R, whistle may be assumed to be moving along line OP). Observer, therefore, listens maximum frequency when source is at B moving towards observer:

$$f_{max} = f\frac{v}{v - v_s}$$

where, 'v' is the speed of sound in air. Similarly, observer listens minimum frequency when source is at 'A', moving away from observer:

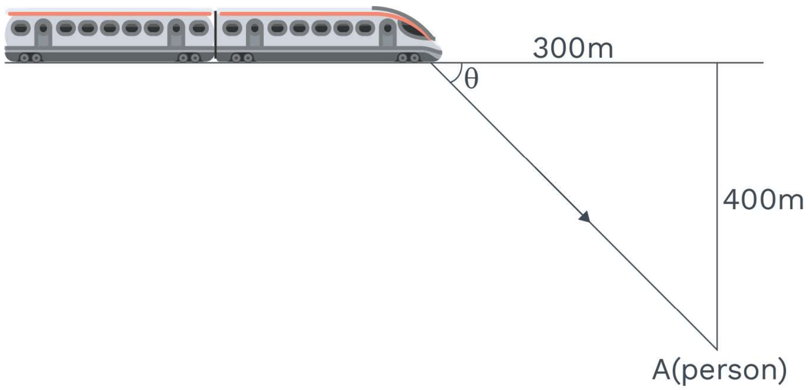
$$f_{min} = f\frac{v}{v + v_s}$$

If, f = 540 Hz, v_s = 2 ft × 15 rad/s = 30 ft/s, and v = 1100 ft/s,
we get (approx.)

f_{max} = 555 Hz and, f_{min} = 525 Hz.

Ex. A train approaching a railway crossing at a speed of '72 km/h' sounds a short whistle at frequency '640 Hz' when it is '1 km' away from crossing. The speed of sound in air is '330 m/s'. A road intersects crossing perpendicularly. What is the frequency heard by the person standing on the road at the distance of '1732 m' from the crossing.

Sol. The observer A is at rest with respect to the air and the source is travelling at a velocity of 72 km/h i.e., 20 m/s. As is clear from the figure, the person receives the sound of the whistle in a direction BA making an angle θ with the track where $\tan\theta = \frac{1732}{1000} = \sqrt{3}$, i.e. $\theta = 60°$. The component of velocity of the source (i.e., of the train) along this direction is 20 cos θ = 10 m/s. As the source is approaching person with this component, the frequency heard by the observer is



$$\upsilon' = \frac{v}{v - u\cos\theta}\upsilon = \frac{330}{330 - 10} \times 640\,\text{Hz} = 660\,\text{Hz}$$

EXAMPLES

Q1 **The pressure at a point varies from 99980 Pa to 100020 Pa due to the simple harmonic sound wave. Amplitude and the wavelength of the wave are 5 × 10⁻⁶ m and '40 cm' respectively. Find the bulk modulus of air**

Sol: $P_0 = B.\,k\,S_0$

$$B = \frac{P_0}{kS_0}$$

$$= \frac{P_0\lambda}{2\pi S_0}$$

$$= \frac{20 \times 0.4}{2\pi \times 5 \times 10^{-6}}$$

$$= \frac{8 \times 10^5}{\pi}\ \text{N/m}^2$$

Q2 **(a) Find out speed of sound in a mixture of 1 mol of helium and 2 mol of oxygen at 27°C.**

(b) If temperature is raised by 1K from 300 K. Find the percentage change in the speed of 'sound' in the gaseous mixture.

[Note : This can be done after studying heat.]

Sol:

(a) $$\gamma_{mix} = \frac{n_1C_{P_1} + n_2C_{P_2}}{n_1C_{V_1} + n_2C_{V_2}} = \frac{1 \times \frac{5}{2}R + 2 \times \frac{7}{2}R}{1 \times \frac{3}{2}R + 2 \times \frac{5}{2}R} = \frac{19}{13}$$

$$m_{mix} = \frac{n_1m_1 + n_2m_2}{n_1 + n_2} = \frac{1 \times 4 + 2 \times 32}{1 + 2} = \frac{68}{3}$$

$$v = \sqrt{\frac{\gamma_{mix}RT}{m_{mix}}} = \sqrt{\frac{19 \times 25 \times 300 \times 3}{13 \times 3 \times 68 \times 10^{-1}}} = 400.9\ \text{m/sec.}$$

(b) $$v = \sqrt{\frac{\gamma RT}{M}}$$

$$\Rightarrow \ell nv = \frac{1}{2}\ell n\frac{\gamma R}{M} + \frac{1}{2}\ell nT$$

$$\frac{1}{v}\frac{dv}{dT} = 0 + \frac{1}{2T}$$

$$\frac{dv}{v} \times 100 = \frac{1}{2}\frac{dT}{T} \times 100 = \frac{1}{2 \times 300} \times 100 = \frac{1}{6}\%.$$

Q3 **A gas mixture has 24 % of Argon, 32 % of oxygen, and 44 % of CO_2 by mass. Find the velocity of sound in the gas mixture at 27 °C. Given R = 8.4 S.I. units. Molecular weight of (Ar = 40), (O_2 = 32), (CO_2 = 44).**

$\gamma_{Ar} = 5/3$, $\gamma_{O_2} = 7/5$, $\gamma_{CO_2} = 4/3$.

Sol:

$$M_{mix} = \frac{M_{Total}}{\frac{M_{Ar}}{M_{Ar}} + \frac{M_{O_2}}{M_{O_2}} + \frac{m_{CO_2}}{M_{CO_2}}} = \frac{100}{\frac{24}{40} + \frac{32}{32} + \frac{44}{44}} = \frac{100}{2.6}$$

$$f_{mix} = \frac{n_1 f_1 + n_2 f_2 + n_3 f_3}{n_1 + n_2 + n_3} = \frac{0.6 \times 3 + 1 \times 5 + 1 \times 6}{2.6} = \frac{12.8}{2.6}$$

$$\gamma_{mix} = \frac{f_{mix} + 2}{f_{mix}} = \frac{\frac{12.8}{2.6} + 2}{\frac{12.6}{2.6}} = \frac{18}{12.8}$$

$$v = \sqrt{\frac{\gamma_{mix} RT}{M_{mix}}} = \sqrt{\frac{18 \times 8.4 \times 300}{12.8 \times \frac{100}{2.6} \times 10^{-3}}} = \sqrt{\frac{18 \times 8.4 \times 300 \times 2.6}{12.8 \times 100 \times 10^{-3}}}$$

$$= \sqrt{92137.6} = \approx 303.5 \text{ m/s.}$$

Q4 **Two sound waves one in air and other in fresh water are equal in the intensity.**

(a) Find the ratio of pressure amplitudes of the wave in water to that of wave in air.

(b) If pressure amplitudes of waves are equal then what will be the ratio of intensities of the waves.

[V_{sound} = 340 m/s in air and density of air = 1.25 kg/m³, V_{sound} = 1530 m/s in water, density of water = '1000 kg/m³'].

Sol: (a) $\frac{P_0^2}{2\rho V} = I$ Given, $I_w = I_{air}$

$$\left(\frac{P_0^2}{2\rho V}\right)_{Water} = \left(\frac{P_0^2}{2\rho V}\right)_{air}$$

$$\left(\frac{P_{0\ water}}{P_{0\ air}}\right)^2 = \frac{\rho_W V_W}{\rho_{air} V_{air}}$$

$$\frac{P_{0\ water}}{P_{0\ air}} = \sqrt{\frac{\rho_w V_w}{\rho_{air} V_{air}}} = \sqrt{\frac{1000}{1.25} \times \frac{1530}{340}} = 60$$

(b) $P_0 = \sqrt{2I\rho V}$

$$\left(\sqrt{2I\rho V}\right)_{water} = \left(\sqrt{2I\rho V}\right)_{air}$$

$$\frac{I_W}{I_{air}} = \frac{\rho_{air} V_{air}}{\rho_{water} V_{water}} = \frac{1}{3600}$$

Q5 **A point A is located at a distance r = 1.5 m from a point source of sound of frequency n = 600 Hz. The power of the source P = 0.80 W. Neglecting damping of the wave and assuming the velocity of sound in the air to be ($340\ ms^{-1}$). Find at the point A :**

(use $\rho_{air} = \frac{225\pi}{544}$ **kg** m^{-3}; $\pi^2 = \frac{100}{3 \times 3.375}$**)**

(a) Pressure oscillation amplitude $(\Delta p)_m$.
(b) The oscillation amplitude of particles of medium.

Sol:

$$P_0 = \sqrt{2I\rho V}$$

$$= \sqrt{2\left(\frac{P}{4\pi r^2}\right)\rho V} \qquad \left(\because I = \frac{P}{4\pi r^2}\right)$$

$$= \sqrt{\frac{2 \times 0.8 \times 225\pi \times 340}{544 \times 4\pi \times 1.5 \times 1.5}} = 5 N/m^2$$

$$S_0 = \frac{P_0}{Bk} = \frac{P_0}{\rho V^2\left(\frac{\omega}{V}\right)} = \frac{P_0}{2\pi\rho V n} \qquad \because B = \rho v^2,\ k = \frac{\omega}{v}$$

$$= \frac{5 \times 544}{2\pi \times 225\pi \times 340 \times 600}$$

$$= \frac{5 \times 544 \times 3 \times 3.375}{2 \times 225 \times 340 \times 600 \times 100} = 3 \times 10^{-6} m = 3\ \mu m.$$

Q6 **Two sources of the sound, 'S_1' and 'S_2', emitting waves of equal wavelength (2 cm), are placed with a separation of (3 cm) between them. A detector can be moved on a line parallel to S_1S_2 and at the distance of (24 cm) from it. Initially, the detector is equidistant from the two sources. Assuming that the waves emitted by the sources are in phase, find the minimum distance through which the detector should be shifted to detect the minimum of sound.**

Sol: Now new situation of detector is D’
so path difference for subsequent minima

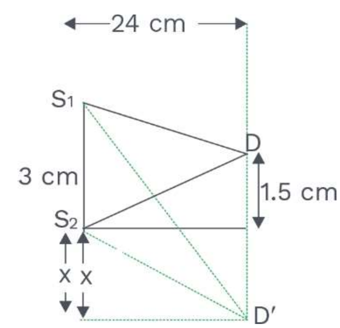
$$\Delta S = S_1D' - S_2D' = \frac{\lambda}{2}$$

$$= \sqrt{(3+x)^2 + 24^2} - \sqrt{x^2 + 24^2} = 1$$

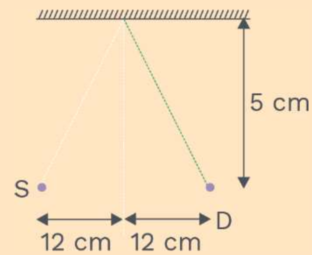
$\Rightarrow$ x = 7 cm.

So detector should be displaced by

7 + 1.5 = 8.5 cm.



Q7



A sound source, detector and the cardboard are arranged as shown in the figure. The wave is reflected from the cardboard at the line of symmetry of source and detector. Initially the path difference between the reflected wave and the direct wave is one third of the wavelength of sound. Find the minimum distance by which the cardboard should be moved upwards so that the both waves are in phase.

Sol: Initially path difference for same phase $2S_1 - 24 = \lambda/3$ ...(i)

$\Rightarrow$ $\lambda = 6$

If shifting is x for bring waves in phase $2S_1' - 24 = \lambda$ ...(ii)

subtract (ii) and (i)

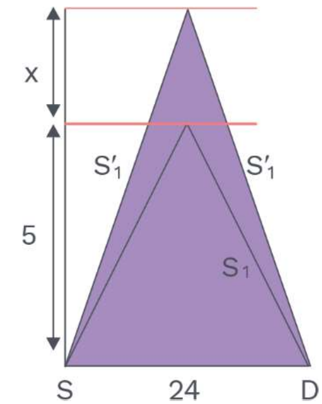
$$2S_1' - 2S_1 = \frac{2\lambda}{3}$$

$$S_1' - S_1 = \frac{\lambda}{3}$$

$$S_1' - S_1 = \frac{6}{3}$$

$$\sqrt{(5+x)^2 + 12^2} - \sqrt{5^2 + 12^2} = 2$$

On solving x = 4 cm.



Q8 **The equation of a longitudinal standing wave due to superposition of the progressive waves produced by two sources of sound is s = –20 sin (10π) × sin (100 πt) where 's' is the displacement from the mean position measured in mm, 'x' is in meters and is in seconds. The specific gravity of the medium is 10^{-3}. Density of water = 10^3 kg/m³. Find:**

(a) Wavelength, frequency and velocity of the progressive waves.

(b) Bulk modulus of the medium and 'pressure amplitude'.

(c) Minimum distance between 'pressure antinode' and 'displacement antinode'.

(d) Intensity at the displacement nodes.

Sol: (a) $f = \frac{100\pi}{2\pi} = 50$ Hz

$k = 10\pi \quad \Rightarrow \quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} \times 100 \text{ cm} = 20 \text{ cm}$

v = f λ = 50 × 0.2 = 10 m/s.

(b) $B = \rho\upsilon^2 = \left(10^{-3} \times 1000\right) \times 10^2$

$P_m = BK\,S_0 = 100 \times 10\pi \times 20 \times 10^{-3}$

$= 20\pi$ N/m².

(c) Distance between Pressure Antinode and displacement Node is

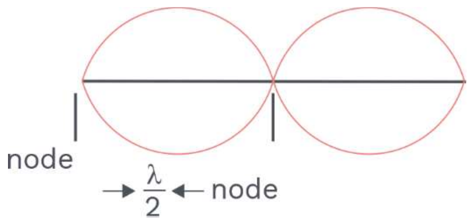
$\frac{\lambda}{4} = 5 \text{ cm} = 0.05 \text{ m}.$

(d) $I = \frac{P_m^2}{2\rho V} = \frac{(20\pi)(20\pi)}{2 \times 1 \times 10} = 20\,\pi^2$ W/m².

Q9 **In an organ pipe the distance between adjacent nodes is (4 cm). Find the frequency of source if the speed of sound in the air is 336 m/s.**

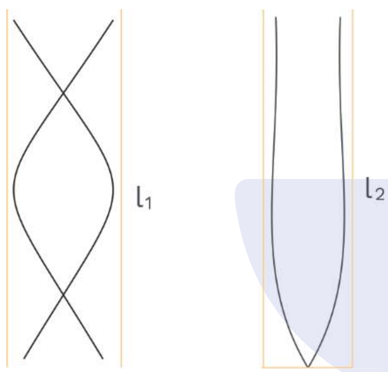
Sol: $\frac{\lambda}{2} = 4 \times 10^{-2}\text{m} \quad \Rightarrow \quad \lambda = 8 \times 10^{-2}$ m

$n = \frac{V}{\lambda} = \frac{336}{8 \times 10^{-2}} = 4200$ Hz = 4.2 KHz.



Q10 **A closed organ pipe of length l = 100 cm is cut into two unequal pieces. The fundamental frequency of the new closed organ pipe piece is found to be same as the frequency of first overtone of the open organ pipe piece. Determine the length of the two pieces and the fundamental tone of the open pipe piece. Take velocity of sound = 320 m/s.**

Sol:



$\frac{2\lambda_1}{2} = \ell_1$ $\qquad$ $\frac{\lambda_2}{4} = \ell_2$

$\lambda_1 = \frac{2\ell_1}{2} = \ell_1$ $\qquad$ $\lambda_2 = 4\ell_2$

$f_1 = f_2$

$\frac{V}{\ell_1} = \frac{V}{4\ell_2}$

$\ell_1 = 4\ell_2$ $\qquad$ $\ell_1 + \ell_2 = 100$

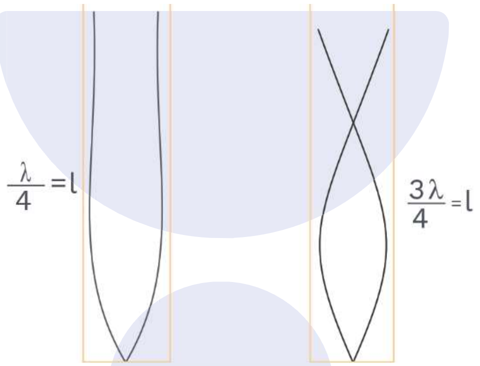
$5\ell_2 = 100$ $\qquad$ $\ell_2 = 20$ cm $\qquad$ $\ell_1 = 80$

Fundamental frequency of open pipe.

$$f = \frac{320}{\lambda} = \frac{320}{2 \times \ell_1} = \frac{320 \times 100}{2 \times 80} = 200 \text{ Hz}.$$

Q11 **Find the number of possible natural oscillations of air column in a pipe whose frequencies lie below 'f_0 = 1250 Hz'. The length of the pipe is 'l = 85 cm'. The velocity of sound is 'v = 340 m/s'.**
Consider the two cases :
(a) Pipe is closed from the one end
(b) Pipe is opened from both the ends.
The open ends of pipe are assumed to be the antinodes of the displacement.

Sol: $\frac{\lambda}{4} = \ell$ $\qquad$ $\frac{3\lambda}{4} = \ell$ and for n^{th} overtone $(2n+1)\frac{\lambda}{4} = \ell$.



$(2n+1)\frac{\lambda}{4} = \ell$ $\qquad$ $\lambda = \frac{4\ell}{(2n+1)}$

$$f = \frac{V(2n+1)}{4\ell} < 1250 \text{ Hz}$$

$$= \frac{340(2n+1) \times 100}{4 \times 85} < 1250$$

= (2n + 1) < 12.5 $\qquad$ n-overtone

2n < 11.5

$$n < \frac{11.5}{2}$$

n < 5.75 $\qquad$ n number of oscillation = 6

n = 0, 1, 2, 3, 4, 5

similarly for open organ pipe

$$f = \frac{V}{2\ell}(n+1) < 1250$$

$$\frac{340 \times 100}{2 \times 85}(n+1) < 1250$$

$$n+1<\frac{12.5}{2}$$

n + 1 < 6.25

n < 5.25

n = overtone

n = 0, 1, 2, 3, 4, 5

number of oscillation (6).

Q12 **Two identical piano wires have a fundamental frequency of 600 vib/sec, when kept under same tension. What fractional increase in tension of one wire will lead to occurrence of six beats per second when both the wires vibrate simultaneously.**

Sol:

$$f=\frac{1}{\lambda}\sqrt{\frac{T}{\mu}}$$

$$\ell n\ f=-\ell n\lambda+\frac{1}{2}\ell nT-\frac{1}{2}\ell n\ \mu$$

$$\frac{1}{f}\frac{df}{dT}=0+\frac{1}{2}\frac{1}{T}-0$$

$$\frac{df}{f}=\frac{1}{2}\frac{dT}{T}$$

$$\frac{dT}{T}=\frac{2df}{f}=\frac{2\times 6}{600}=\frac{2}{100}.$$

$$\%\ \text{change}=\frac{2}{100}\times 100=2\%.$$

Q13 **A metal wire of the diameter '1 mm', is held on two knife edges separated by the distance of '50 cm'. The tension in the wire is the '100 N'. The wire vibrating in its fundamental frequency and a vibrating tuning fork together produces the '5 beats per sec'. The tension in the wire is then reduced to the '81 N'. When the two are excited, beats are again at same rate. Calculate**

(a) The frequency of fork.

(b) The density of the material of wire.

Sol: $\frac{1}{2\ell}\sqrt{\frac{100}{\mu}} - n = 5$ ...(i)

$n - \frac{1}{2\ell}\sqrt{\frac{81}{\mu}} = 5$ ...(ii)

add equation (i) and (ii)

$\frac{1}{2\ell}\left[\frac{1}{\sqrt{\mu}}\right] = 10 \quad \Rightarrow \quad \sqrt{\mu} = \frac{1}{20\ell} = \frac{1}{20 \times 0.5} = \frac{1}{10} \quad \Rightarrow \quad \mu = 0.01$

$\rho\, \pi r^2 = 0.01$

$\rho = \frac{0.01}{\left(0.5 \times 10^{-3}\right)^2 \pi} = \frac{10^4}{0.5 \times 0.5\pi} = \frac{40}{\pi} \times 10^3 \ \text{kg/m}^3$

put $\mu = 0.01$ in equation ...(i)

$\frac{1}{2 \times 0.5}\sqrt{\frac{100}{0.01}} - n = 5$

$n = 100 - 5 = 95$ Hz.

Q14 **An observer rides with a sound source of frequency f and moving with velocity 'v' towards the large vertical wall. Considering the velocity of sound waves as 'c', find out:**

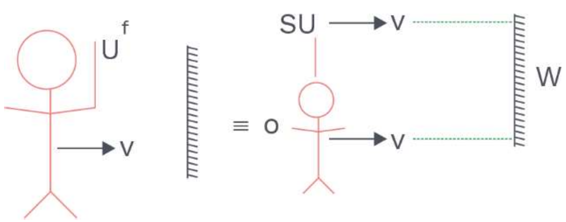
(i) The number of waves striking surface of wall per second

(ii) The wavelength of reflected wave

(iii) The frequency of the reflected wave as observed by the observer.

(iv) Beat frequency heard by the observer.

Sol:



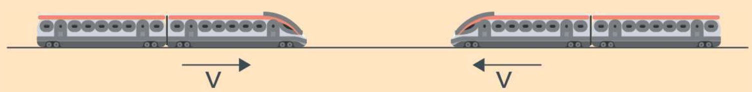
(i) $f' = f\frac{c}{c - v}$ Doppler effect between source and wall

(ii) $\lambda' = \frac{c}{f'} = \frac{c}{cf}\left(c - v\right) = \frac{c - v}{f}$

(iii) $f' = f\dfrac{c+v}{c-v}$ Doppler effect between wall and observer.

(iv) $f_{beat} = f'' - f = f\left[\dfrac{c+v}{c-v} - 1\right] = f\dfrac{2v}{c-v}$.

Q15 **Two trains move towards each other with same speed. Speed of the sound is '340 ms⁻¹'. If pitch of the tone of the whistle of one when heard on the other changes to '9/8 times', then speed of each train is.**



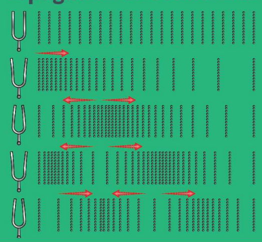
Sol: $n' = \left(\dfrac{V+V_s}{V-V_s}\right)n \Rightarrow \dfrac{9}{8}n = \left(\dfrac{V+V_s}{V-V_s}\right)n \Rightarrow \dfrac{9}{8} = \dfrac{V+V_s}{V-V_s}$

$9V - 9V_s = 8V + 8V_s \Rightarrow V = 17\,V_s \Rightarrow V_s = \dfrac{340}{17} = 20$ m/s.

Mind Map

SOUND WAVE

Propagation of Sound Wave



Velocity of Sound Wave

Newton's Formula

$V = \sqrt{\frac{P}{\rho}}$

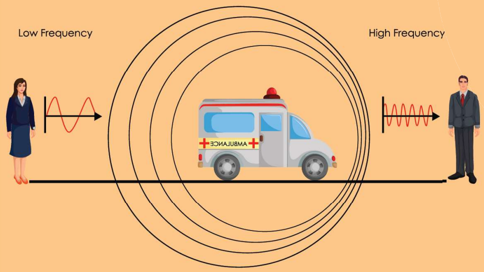
Laplace Correction

$V = \sqrt{\frac{rP}{\rho}}$

Doppler Effect

General formula

$f' = \frac{V \pm V_0}{V \mp V_S} f_0$



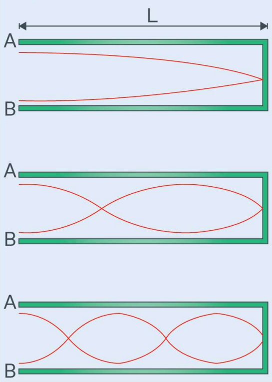
Superposition of Wave

Beats

Beat $\Delta f = |f_1 - f_2|$ frequency

Standing Waves

Organ pipe open at one end



First Harmonic = Fundamental

$L = \frac{1}{4}\lambda_1 \Rightarrow f_1 = \frac{v}{4L}$

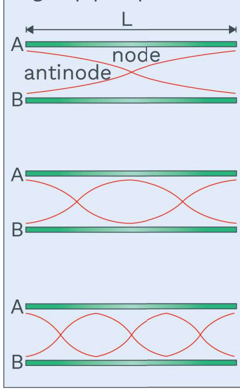
Third Harmonic

$L = \frac{3}{4}\lambda_3 \Rightarrow f_3 = \frac{3v}{4L} = 3f_1$

Fifth Harmonic

$L = \frac{5}{4}\lambda_5 \Rightarrow f_5 = \frac{5v}{4L} = 5f_1$

Organ pipe open at both end



First Harmonic - Fundamental

$L = \frac{1}{2}\lambda_1 \Rightarrow f_1 = \frac{v}{2L}$

Second Harmonic

$L = \lambda_1 \Rightarrow f_2 = \frac{v}{L} = \frac{2v}{2L}$

Third Harmonic

$L = \frac{3}{2}\lambda_3 \Rightarrow f_3 = \frac{3v}{2L}$