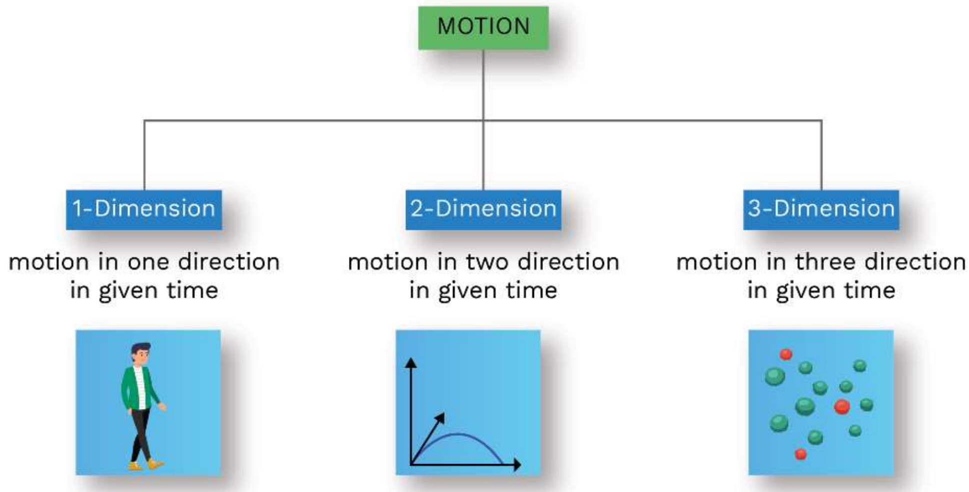
Rectilinear Motion

Rectilinear Motion



Rest & Motion:

- If position of particle does not change with time, known as 'rest'.
- If position of particle changes with time, known as 'motion'.
- Rest and motion are relative to observer, which depends on frame of reference.
- There is no meaning of absolute rest for motion. An object can be both at rest or in motion simultaneously for different observer.

Frame of reference:

- It is the reference w.r.t. which position or motion of particle is defined.
- If not specified we consider ground as frame of reference.

Distance:

Length of actual path travelled by the body in given time interval.

- Actual length of path.
- Scalar quantity.
- Can't be negative.
- Unit : meter.
- Distance is path dependent.

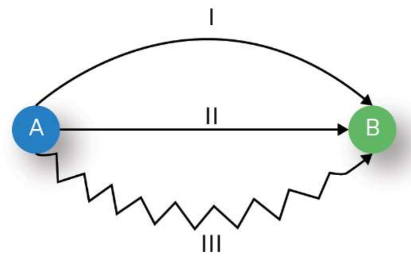
$(\text{Distance})_{I} \neq (\text{Distance})_{II} \neq (\text{Distance})_{III}$

Key Points

- Position
- Rest and motion
- Frame of reference

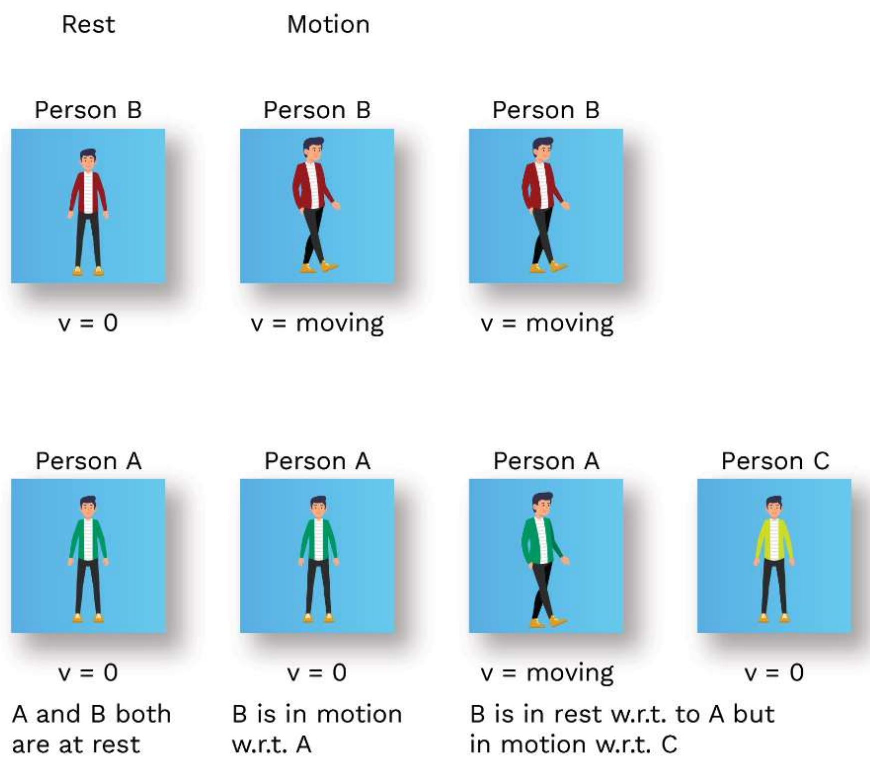
Definition

Frame of reference is the reference with respect to which position or motion of particle is defined.

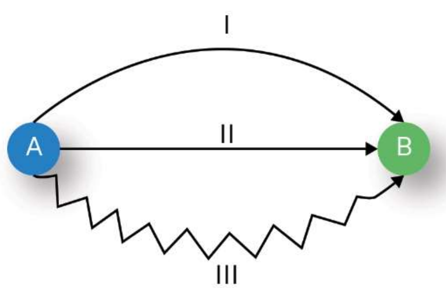


Definition

Distance: Length of actual path travelled by the body in given time interval.



Displacement:



Shortest distance between initial and final position.

- Vector quantity.
- Its direction is from initial to final position.
- Can be positive, negative or zero.
- Path independent.

$(\text{Displacement})_I = (\text{Displacement})_{II} = (\text{Displacement})_{III}$

Definition

Displacement: Shortest distance between initial and final position.

Relation between distance and displacement:

- Distance $\geq |\overrightarrow{\text{Displacement}}|$
- $\dfrac{\text{Distance}}{|\overrightarrow{\text{Displacement}}|} \geq 1$
- $\dfrac{|\overrightarrow{\text{Displacement}}|}{\text{Distance}} \leq 1$

Concept Reminder:

Distance $\geq |\overrightarrow{\text{Displacement}}|$

$\dfrac{\text{Distance}}{|\overrightarrow{\text{Displacement}}|} \geq 1$

$\dfrac{|\overrightarrow{\text{Displacement}}|}{\text{Distance}} \leq 1$

S.no.	Distance	S.no.	Displacement
(i)	Scalar quantity.	(i)	Vector quantity (direction from initial to final position).
(ii)	Depends on path.	(ii)	Depends on initial and final position.
(iii)	For a moving body it always increases.	(iii)	For a moving body it can increases or decreases.
(iv)	For a moving body it is always positive, never be negative or zero.	(iv)	For a moving body it can be positive, negative or zero.
(v)	If distance travelled is zero then body must be at rest.	(v)	If displacement is zero then body either is at rest or passing through its initial position.
(vi)	There can be infinite values of distance between two fixed points.	(vi)	There is only one unique value of displacement between two fixed points.

Displacement vector (In terms of position vector):

According to triangle law of vector addition.

Initial position vector

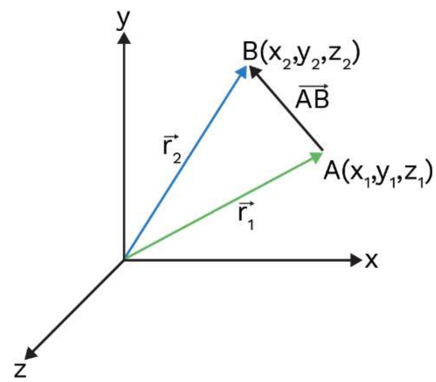
$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

Final position vector

$\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

$\vec{r}_1 + \overrightarrow{AB} = \vec{r}_2 \quad \Rightarrow \quad \overrightarrow{AB} = \vec{r}_2 - \vec{r}_1$

$$\boxed{\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}}$$



Q1 **Athlete completes one round of a circular track. Which track radius R in 40 sec. Find out his displacement at the end of 2 min 20 sec ?**

(1) Zero **(2) 2 R**

(3) 2 πR **(4) 7 πR**

A1 40 sec $\rightarrow$ 1 cycle

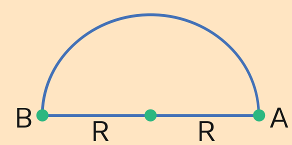
2 min + 20 sec = 140 sec

Number of cycle = $\frac{140}{40} = 3.5$

After 3.5 cycle athlete will reach from one point of diameter to another point.

So, displacement = 2r

Q2 **A particle start moving from point A and reaches at point B along a semi-circular path then find out distance and displacement.**



A2 Displacement = 2R

Distance L = $\frac{2\pi R}{2}$

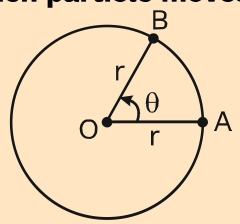
Length or arc = angle × radius = πR

Key Points

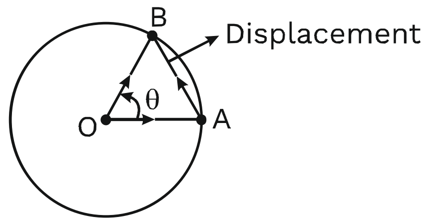
- Distance
- Displacement

Rectilinear Motion

Q3 **When particle moves from A to B then find out displacement.**



A3 Distance or arc = angle × radius = θ . r



$|\overrightarrow{OA}| = r$

$|\overrightarrow{OB}| = r$

$|\overrightarrow{AB}| = \overrightarrow{OB} - \overrightarrow{OA}$

$|\overrightarrow{AB}| = |\overrightarrow{OB} - \overrightarrow{OA}|$

$$AB = \sqrt{r^2 + r^2 - 2r^2 \cos\theta}$$

$$AB = \sqrt{2r^2(1 - \cos\theta)}$$

$$AB = \sqrt{2r^2\left(1 - 1 + \sin^2\frac{\theta}{2}\right)}$$

$$AB = \sqrt{4r^2 \sin^2\frac{\theta}{2}}$$

$$\Rightarrow \quad AB = 2r\sin\frac{\theta}{2}$$

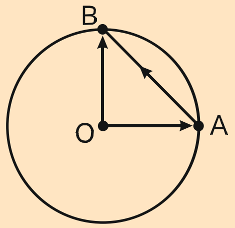
Q4 **In the following figure find out distance and displacement.**

(i) 1/4th cycle or quarter

(ii) ½ circle

(iii) Full cycle

(iv) At angle 60°



A4 (i) Distance = Angle × radius = $\frac{\pi}{2} \times r = \frac{\pi r}{2}$

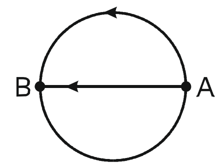
Displacement = $2r\sin\frac{\theta}{2}$ = 2r sin 45

$$= 2r \times \frac{1}{\sqrt{2}} = \sqrt{2}\,r$$

(ii) ½ circle

Distance = πR,

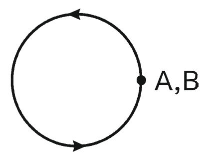
Displacement = 2R



(iii) Full cycle

Distance = 2 πR,

Displacement = 0



(iv) At angle 60°

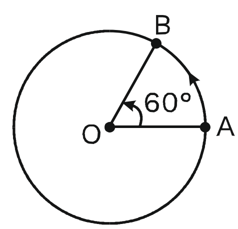
distance = angle × radius

$$\frac{\pi}{3} \times R = \frac{\pi R}{3}$$

displacement = $\sqrt{R^2 + R^2 - 2R^2 \cos 60^\circ}$

or $= 2R \sin \frac{60^\circ}{2}$

= 2R × — = R, displacement = R



Q5 **x = 4 – 6t + t², 'x' in meter and 't' is in second, then the find distance and displacement?**
(time t = 0 to t = 4 sec)

(1) 10 m, – 8 m **(2) 8 m, – 10 m**

(3) 20 m, 14 m **(4) None of these**

A5 $x = 4 - 6t + t^2$

$v = \frac{dx}{dt}$, $v = 0 - 6 + 2t$

v = 2t – 6

check velocity at zero

2t – 6 = 0,

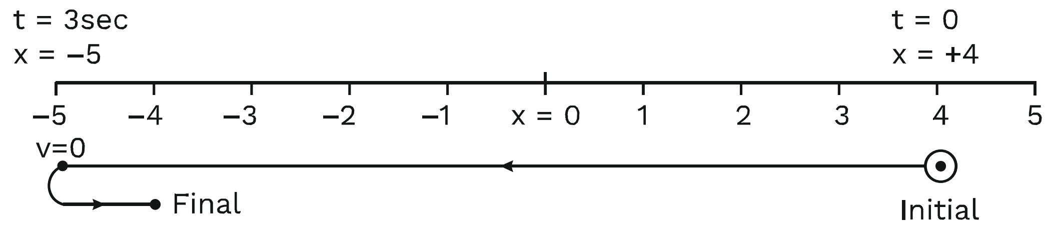
⇒ t = 3 sec, means body turn on t = 3 sec

at t = 3 sec, v will be zero.

t = 0, x_i = + 4m

t = 3 sec, x = – 5m

t = 4 sec, x = – 4m

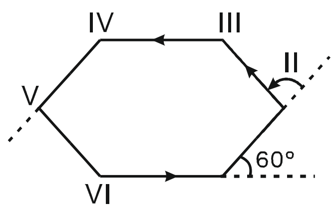


distance = 4 + 5 + 1 = 10 m

displacement = $x_F - x_i$ = – 4 – (4) = – 8 m

Q6 **Motorcyclist follow straight line 'l' meter and turn left side by 60° and repeat it then find (i) distance and (ii) displacement just after 1st, 2nd and 5th turn ?**

A6



1st turn

distance = l

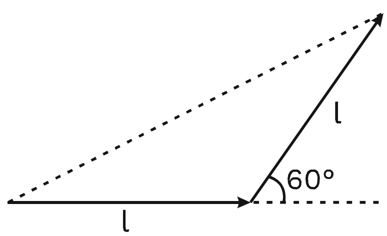
displacement = l

After 2nd turn

distance = l + l ⇒ 2l

displacement = $\sqrt{l^2 + l^2 + 2l^2 \cos 60°}$

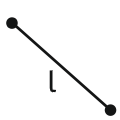
displacement = $\sqrt{3}\,l$



After 5th turn

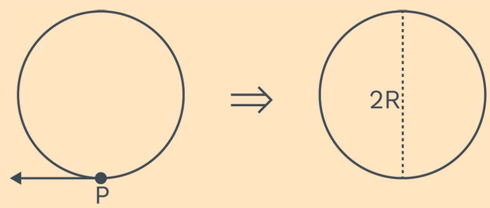
distance = l + l + l + l + l = 5 l

displacement =

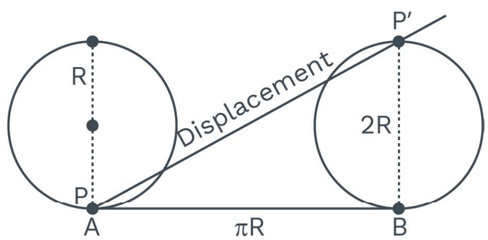


displacement = l

Q7 **Wheel of radius R is rolling in forward direction (+x) axis. Find displacement of P when it reach to the highest point ?**



A7



$L = 2\pi R$ (total circumference)

$$AB = \frac{2\pi R}{2} = \pi R$$

$$\text{displacement } PP' = \sqrt{(\pi R)^2 + (2R)^2} = R\sqrt{\pi^2 + 4}$$

Q8 **If $x = t^3 - 6t^2 + 9t$ then distance $t = 0$ to $t = 5$ sec?**

A8

= —— = − + =

$t^2 - 4t + 3 = 0$, $t = 1$, $t = 3$ sec

(i) $t = 0$, $x_i = 0$

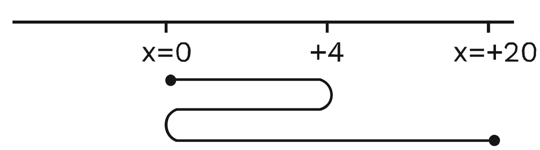
(ii) $t = 1$, $x = +4$

(iii) $t = 3$, $x = 0$

(iv) $t = 5$, $x = +20$

distance = 4 + 4 + 20 = 28 m

displacement = 20 − 0 = 20 m



Q9 **The position vector of a particle is determined by the expression $\vec{r} = 3t^2\hat{i} + 4t^2\hat{j} + 7\hat{k}$. The displacement traversed in first 10 sec is**

(1) 100 m **(2) 150 m** **(3) 300 m** **(4) 500 m**

A9

$\overrightarrow{\Delta r} = \vec{r}_f - \vec{r}_i$

$(\vec{r}_i)_{t=0} = 7\hat{k}$, $(\vec{r}_F)_{t=10} = 300\hat{i} + 400\hat{j} + 7\hat{k}$

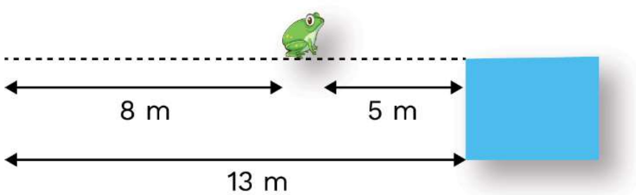
$\overrightarrow{\Delta r} = 300\hat{i} + 400\hat{j}$,

$|\Delta r| = \sqrt{(300)^2 + (400)^2} = 500\,m$

Q10 **A frog walking in a narrow lane takes 5 leaps forward and 3 leaps backward, then again 5 leap forward and 3 leaps backward, and so on. Each leap is 1 m long and requires 1 sec. Determine how long the frog takes to fall in a pit 13 m away from the starting point.**

(1) 35 sec **(2) 36 sec** **(3) 37 sec** **(4) 38 sec**

A10



5 forward + 3 backward = 8 step $\rightarrow$ 2 meter

8 step $\rightarrow$ 2 meter
16 step $\rightarrow$ 4 meter
24 step $\rightarrow$ 6 meter
32 step $\rightarrow$ 8 meter
32 step + 5 step = 37 step
8m + 5m = 13 meter

$\because$ 1 step $\rightarrow$ 1 sec
37 step $\rightarrow$ 37 sec

Speed

Change of distance w.r.t. time.

- Unit: m/sec.
- It is a scalar quantity.
- Speed cannot be negative.
- Speed = $\frac{\text{distance}}{\text{time}}$.
- There are four types of speed.

(i) Instantaneous speed: It is a speed of particular instant of time.

$$v_{ins} = \frac{ds}{dt}$$

(ii) Uniform speed: If body covers equal distance in equal time interval.

(iii) Non-uniform speed: If body covers unequal distance in equal time interval or equal distance in unequal time interval or unequal distance in unequal time interval then speed is non-uniform.

(iv) Average speed: $v_{avg} = \frac{\text{Total distance}}{\text{Total time}}$

Note: For a moving body distance never decreases but average speed can decreases.

Average speed (For numericals):

Case-I: If particle travels S_1, S_2, ... S_n distances in time t_1, t_2, ... t_n respectively.

$$< v > = \frac{S_1 + S_2 + ... + S_n}{t_1 + t_2 + ... + t_n}$$

Case-II: If particles travels S_1, S_2, ... S_n, with speeds v_1, v_2, ... v_n respectively.

$$< v > = \frac{S_1 + S_2 + ... + S_n}{\frac{S_1}{v_1} + \frac{S_2}{v_2} + ... \frac{S_n}{v_n}}$$

Case-III: If particle travels with speeds v_1, v_2, ... v_n in time interval t_1, t_2, ... t_n respectively.

$$< \quad > = \frac{\quad + \quad + \quad +}{\quad + \quad + \quad +}$$

Definitions

Change in distance w.r.t. time is called 'speed'.

Rack your Brain

A particle covers half of its total distance with speed v_1 and rest half distance with speed v_2, calculate its average speed during the complete journey.

Concept Reminder:

For 'n' equal interval of distance

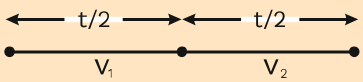
$$v_{avg} = \frac{n}{\frac{1}{v_1} + \frac{1}{v_2} + ... \frac{1}{v_n}}$$

Harmonic mean.

Key Points

- Speed
- Uniform speed
- Non-uniform speed
- Average speed
- Instantaneous speed

Q11 **A person moves first half time of his motion with speed v_1 and remaining half time with speed v_2. Then find out average speed.**



A11

$$v_{avg} = \frac{s_1 + s_2}{t} = \frac{v_1 \times \frac{t}{2} + v_2 \times \frac{t}{2}}{t} \Rightarrow v_{avg} = \frac{v_1 + v_2}{2}$$

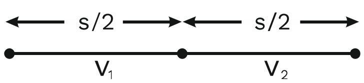
Note: For 'n' equal time interval.

$$\boxed{v_{avg} = \frac{v_1 + v_2 + v_3 + \ldots\ldots + v_n}{n}}$$ Arithmetic mean.

Q12 **A particle covers first half distance of its motion with speed v_1 and remaining half distance with speed v_2. Find out average speed.**

A12

$$v_{avg} = \frac{s}{t_1 + t_2} = \frac{s}{\frac{s}{2v_1} + \frac{s}{2v_2}} = \frac{2v_1 v_2}{v_1 + v_2}$$



Note: For 'n' equal interval of distance

$$\boxed{v_{avg} = \frac{n}{\frac{1}{v_1} + \frac{1}{v_2} + \ldots\ldots \frac{1}{v_n}}}$$ Harmonic mean.

Definitions

Rate of change in displacement with respect to time is called 'velocity'.

Velocity

Change of displacement with respect to time.

- Unit: m/sec.
- It is a vector quantity.
- Velocity can be +ve, –ve or zero.
- Velocity = $\dfrac{\text{displacement}}{\text{time}}$

Concept Reminder:
Instantaneous velocity:

- $\vec{V}_{inst.} = \dfrac{d\vec{s}}{dt}$
- Slope of tangent displacement-time curve denote instantaneous velocity $\vec{V}_{inst.}$

Types of velocity:

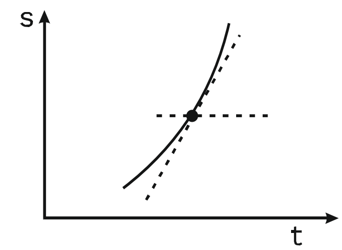
(i) Instantaneous velocity [$\vec{V}_{inst.}$]

Velocity at particular time.

$$\vec{V}_{inst.} = \frac{\vec{ds}}{dt}$$

$$|\vec{V}_{inst.}| = \frac{ds}{dt}$$

Direction of instantaneous velocity is always tangential to path followed by particle.



Q13 **If $\vec{s} = -(t^2 + t)\hat{i} + 1$ meter and t in sec. Find speed and velocity after 1 sec?**

A13

$$\vec{V}_{inst} = \frac{d}{dt}[(-t^2 - t)\hat{i} + 1]$$

$$\vec{V}_{inst} = (-2t - 1)\hat{i}$$

at t = 1 sec

$$\vec{V}_{inst} = (-2 \times 1 - 1)\hat{i} \quad \Rightarrow \vec{V}_{inst} = -3\hat{i}$$

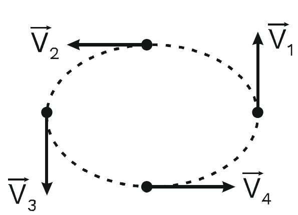
$$|\vec{V}_{inst}| = |-3\hat{i}| \quad \Rightarrow \vec{V}_{inst} = 3\,m/s$$

(ii) Uniform velocity: (magnitude + direction) both should be same.

(iii) Non-uniform velocity: If either magnitude or direction of velocity or both changes.

Examples:

Horizontal circular motion:



$$|\vec{V}_1| = |\vec{V}_2| = |\vec{V}_3| = |\vec{V}_4|$$

but direction changed

$\rightarrow \neq$ $\rightarrow \neq$

$\Rightarrow$ $\boxed{\rightarrow \neq}$

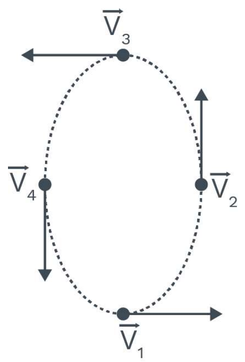
Rack your Brain

The displacement x of a particle of mass m moving in one dimension under the action of a force, is related to time t by $t = \sqrt{x} + 3$, calculate the displacement of the particle when the velocity is zero.

Magnitude and direction both changed (Vertical circular motion)

$$|\vec{V}_1| \neq |\vec{V}_2| \neq |\vec{V}_3| \neq |\vec{V}_4|$$

magnitude is changing due to gravitational force.



(iv) Average velocity

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{\Delta\vec{s}}{\Delta t}$$

when (unique direction + straight line) given then

$$|\overrightarrow{\text{velocity}}| = \text{speed}$$

because $(\text{distance} = |\overrightarrow{\text{displacement}}|)$

Concept Reminder:
When (unique direction + straight line) given

$|\overrightarrow{\text{velocity}}| = \text{speed}$

because

$(\text{distance} = |\overrightarrow{\text{displacement}}|)$

Its direction is along net displacement.

Q14 **A car completes its journey in a straight line in three equal parts with speed V_1, V_2, V_3 respectively. The average speed V is given by**

(1) $\frac{V_1 + V_2 + v_3}{3}$ **(2)** $3\sqrt{V_1V_2V_3}$

(3) —— — — — **(4)** —— — — —

A14

$$V_{avg} = \frac{\text{Total distance}}{\text{Total time}} = \frac{\frac{S}{3} + \frac{S}{3} + \frac{S}{3}}{\frac{S}{3V_1} + \frac{S}{3V_2} + \frac{S}{3V_3}}$$

$$V_{avg} = \frac{1}{\frac{1}{3V_1} + \frac{1}{3V_2} + \frac{1}{3V_3}},$$

Key Points

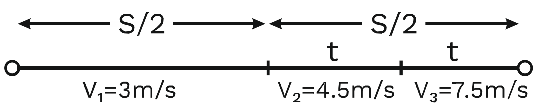
- Velocity
- Uniform velocity
- Non-uniform velocity
- Average velocity
- Instantaneous velocity

$$V_{avg} = \frac{3}{\frac{1}{V_1} + \frac{1}{V_2} + \frac{1}{V_3}}$$

Q15 **A particle moving in a straight line covers it's half the distance with speed of 3 m/s. The other half of the distance is covered in two equal time intervals with speed of '4.5 m/s' & '7.5 m/s' respectively. Average speed of the particle during this motion is**

(1) 4.0 m/s **(2) 5.0 m/s** **(3) 5.5 m/s** **(4) 4.8 m/s**

A15



$$V' = \frac{V_2 + V_3}{2} = \frac{4.5 + 7.5}{2} = 6\,m/s$$

$$V_{avg} = \frac{2V_1V'}{V_1 + V'} = \frac{2 \times 3 \times 6}{9} = 4\,m/s$$

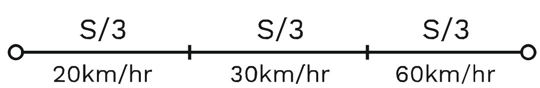
Rack your Brain

If the velocity of the particle is $v = At + Bt^2$, where A and B are constants, then calculate distance travelled by it between 1 s and 2 s.

Q16 **A car covers 1/3 part of total distance with a speed of 20 km/hr and second 1/3 part with a speed of 30 km/hr and last 1/3 part with a speed of 60 km/hr. The average speed of the car is**

(1) 55 km/hr **(2) 30 km/hr** **(3) 45 km/hr** **(4) 37.3 km/hr**

A16



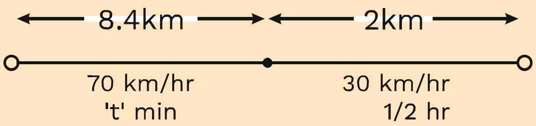
$$V_{avg} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{S}{t_1 + t_2 + t_3} = \frac{S}{\frac{S}{3 \times 20} + \frac{S}{3 \times 30} + \frac{S}{3 \times 60}}$$

V_{avg} = 30 km/hr

Q17 **Vishnu drive a car at a speed of 70 km/hr in a straight road for 8.4 kilometer & then the car runs out of petrol. Vishnu walk for 30 min to reach a petrol pump at a distance of 2 kilometer. The average velocity, V_{avg} from the beginning of your drive till you reach the petrol pump is**

(1) 16.8 km/hr **(2) 35 km/hr** **(3) 64 km/hr** **(4) 18.6 km/hr**



A17

$$t = \frac{8.4}{70} = 0.12\,\text{hr}$$

$$V_{avg} = \frac{\text{Total distance}}{\text{Total time}} = \frac{10.4\ \text{km}}{0.12 + 0.5}$$

$$V_{avg} = 16.8\ \text{km/hr}$$

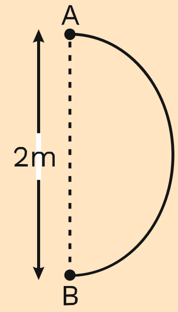
Q18 **After 1 sec particle reach from A to B along circumference then find**

(i) Average velocity

(ii) Average speed

(1) π, 2 **(2) 2, π**

(3) π, π **(4) None of these**



A18 Here R = 1 m

(i) Average velocity

$$= \frac{\text{Total displacement}}{\text{Total time}} = \frac{2R}{1}$$

Average velocity = 2 m/s

(ii) Average speed

$$= \frac{\text{Total distance}}{\text{Total time}} = \frac{\pi R}{1} = \frac{\pi \times 1}{1}$$

Average speed = π m/s

Rack your Brain

A car moves a distance of 200m. It covers the first half of the distance at speed 40km/h and the second half of distance at speed v. The average speed is 48 km/h, then calculate value of v.

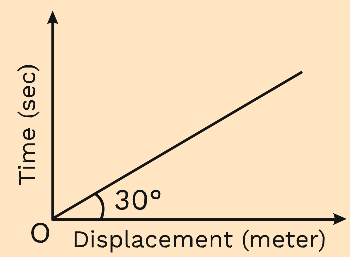
Q19 **From the following displacement-time find out the velocity of a moving body**

(1) $\frac{1}{\sqrt{3}}$ **m/s**

(2) 3 m/s

(3) $\sqrt{3}$ **m/s**

(4) $\frac{1}{3}$ **m/s**



A19

$$V = \frac{ds}{dt} = \tan\theta$$

here $\tan\theta = \frac{dt}{ds} = \tan 30^\circ$

$$\frac{ds}{dt} = \frac{1}{\tan 30^\circ}$$

$$\Rightarrow |\vec{V}| = \frac{1}{(1/\sqrt{3})}$$

$$|\vec{V}| = \sqrt{3}\ m/s$$

Rack your Brain

A bus travelling the first one-third distance at a speed of 10 km/h, the next one-third at 20 km/h and at last one-third at 60 km/h, then calculate average speed of the bus.

Q20 **The position of a particle $\vec{r}$ (in meter) at a time t sec is given by the relation $\vec{r} = (3t\hat{i} - t^2\hat{j} + 4\hat{k})$. Calculate the magnitude of velocity of the particle after 5 sec.**

(1) 3.55 **(2) 5.03** **(3) 8.75** **(4) 10.44**

A20

$$\vec{v} = \frac{d\vec{r}}{dt} = 3\hat{i} - 2t\hat{j}$$

$$\vec{v} \text{ at } t = 5 \text{ sec};\ (\vec{v}) = 3\hat{i} - 2\times 5\hat{j}$$

$$\vec{v} = 3\hat{i} - 10\hat{j}$$

$$|\vec{v}| = \sqrt{9 + 100} = \sqrt{109}$$

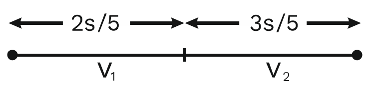
$$|\vec{v}| = 10.44 \text{ m/s}$$

Q21 **If a car covers $\frac{2}{5}$th of the total distance with v_1 speed and $\frac{3}{5}$th distance with v_2 then average speed is**

(1) $\frac{1}{2}\sqrt{v_1 v_2}$ (2) $\frac{v_1 + v_2}{2}$ (3) $\frac{2v_1 v_2}{v_1 + v_2}$ (4) $\frac{5v_1 v_2}{3v_1 + 2v_2}$

A21

$$v_{avg} = \frac{\text{Total distance}}{\text{Total time}} = \frac{s}{\frac{2s}{5v_1} + \frac{3s}{5v_2}}$$



$$v_{avg} = \frac{5v_1 v_2}{2v_2 + 3v_1}$$

Relation between average speed and average velocity:

Distance ≥ |Displacement|

$$v_{avg} \geq |\vec{v}_{avg}|$$

Note:

(i) Time average velocity

If v = f(t)

$$\langle v \rangle = \frac{\int_{t_1}^{t_2} v \cdot dt}{\int_{t_1}^{t_2} dt}$$

(ii) Space average velocity

If v = f(x)

$$\langle v \rangle = \frac{\int_{x_1}^{x_2} v \cdot dx}{\int_{x_1}^{x_2} dx}$$

Concept Reminder:

- $v_{avg} \geq |\vec{v}_{avg}|$
- $\langle v \rangle = \frac{\int_{t_1}^{t_2} v \cdot dt}{\int_{t_1}^{t_2} dt}$

Q22 **If velocity of particle is given by v = (2t + 3). Where 't' is time. Find average velocity for interval 0 ≤ t ≤ 3 sec.**

A22

$$\langle v\rangle = \frac{\int_0^3 v\cdot dt}{\int_0^3 dt} = \frac{\int_0^3 (2t+3)dt}{\int_0^3 dt} = \frac{\left[2\times\frac{t^2}{2}+3t\right]_0^3}{[t]_0^3}$$

$$= \frac{9+9}{3} = 6\ m/s$$

Q23 **If v = cos x, find average velocity for interval $0 \le x \le \frac{\pi}{2}$.**

A23

$$\langle v\rangle = \frac{\int_0^{\pi/2} v\cdot dx}{\int_0^{\pi/2} dx} = \frac{\int_0^{\pi/2} \cos x\cdot dx}{\int_0^{\pi/2} dx} = \frac{[\sin x]_0^{\pi/2}}{\left[\frac{\pi}{2}-0\right]} = \frac{2}{\pi}$$

Acceleration

- Rate of change in velocity.
- Unit: m/sec^2.
- It is a vector quantity.
- It can be +ve, –ve or zero.
- Acceleration = $\frac{\text{Change in velocity}}{\text{Time}}$.

Types of Acceleration:

(i) **Instantaneous acceleration:** It is a velocity of particular instant of time.

$$\vec{a}_{ins} = \frac{d\vec{v}}{dt}$$

(ii) **Uniform acceleration:** Magnitude as well as direction of acceleration remain same.

(iii) **Non-uniform acceleration:** Either magnitude or direction of acceleration or both changes.

Definitions

Rate of change in velocity is called 'acceleration'.

Rack your Brain

A particle of unit mass undergoes one-dimensional motion such that its velocity varies according to $v(x) = \beta x^{-2n}$, where β and n are constants and x is the position of the particle. Then the calculate acceleration of the particle as a function of x.

(iv) Average acceleration ($\vec{a}_{avg}$)

$$\vec{a}_{avg} = \frac{\text{Change in velocity}(\Delta\vec{v})}{\text{Total time}(\Delta t)}$$

Direction of average acceleration is along change in velocity.

$\vec{a}_{ins} = \frac{d\vec{v}}{dt}$, when $\vec{v}$ in function of time.

$\vec{a}_{ins} = \frac{d\vec{v}}{dt}\cdot\frac{d\vec{s}}{d\vec{s}} = \vec{v}\cdot\frac{d\vec{v}}{d\vec{s}}$, when $\vec{v}$ is function of position.

$$\boxed{\vec{a} = \frac{d\vec{v}}{dt} = \vec{v}\cdot\frac{d\vec{v}}{d\vec{s}} = \frac{d^2\vec{s}}{dt^2}}$$

Important points:

- When particle moves with constant velocity then its acceleration will be zero.
- If particle move with constant velocity then

  $v_{avg} = |\vec{v}_{avg}| = v_{inst} = |\vec{v}_{inst}|$.
- A particle may have variable velocity but constant speed. Ex- uniform circular motion.
- Acceleration which oppose the motion of body is known retardation.
- Increment or decrement in speed depends on direction of $\vec{v}$ and $\vec{a}$.

Case-I: If $\vec{v}$ and $\vec{a}$ are same direction then speed increases.

(i) $\xrightarrow{\vec{v} = +ve}$

$\xrightarrow{\vec{a} = +ve}$

(ii) $\xleftarrow{\vec{v} = -ve}$

$\xleftarrow{\vec{a} = -ve}$

Case-II: If $\vec{v}$ and $\vec{a}$ are in opposite direction then speed decreases.

(i) $\xrightarrow{\vec{v} = +ve}$

$\xleftarrow{\vec{a} = -ve}$

(ii) $\xleftarrow{\vec{v} = -ve}$

$\xrightarrow{\vec{a} = +ve}$

Key Points

- Acceleration
- Uniform acceleration
- Non-uniform acceleration
- Average acceleration
- Instantaneous acceleration

Concept Reminder:

$$\boxed{\vec{a} = \frac{d\vec{v}}{dt} = \vec{v}\cdot\frac{d\vec{v}}{d\vec{s}} = \frac{d^2\vec{s}}{dt^2}}$$

- The direction of motion of particle is decided by direction of instantaneous velocity.
- If initial velocity $(\vec{u})$ of particle is zero and acceleration is constant then its path must be a straight line.
- If initial velocity $(\vec{u})$ is non-zero and acceleration $(\vec{a})$ is constant then path may be straight line or parabola.
- For a moving body speed is always +ve, never be zero or –ve.
- For a moving body distance never decreases but average speed can decreases.

Equation of Motion:

- These are valid when acceleration is constant.

Scalar form	**Vector form**
$v = u + at$	$\vec{v} = \vec{u} + \vec{a}t$
$s = ut + \frac{1}{2}at^2$	$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
$v^2 = u^2 + 2as$	$\vec{v}\cdot\vec{v} = \vec{u}\cdot\vec{u} + 2\vec{a}\cdot\vec{s}$
$s_{n^{th}} = u + \frac{a}{2}(2n-1)$	$\vec{s}_{n^{th}} = \vec{u} + \frac{\vec{a}}{2}(2n-1)$

Here:

v = final velocity, s = displacement

u = initial velocity, t = time

a = constant acceleration,

$s_{n^{th}}$ = displacement in n^{th} sec.

Derivation:

(i) $a = \frac{dv}{dt} \Rightarrow \int_u^v dv = \int_0^t a\,dt \Rightarrow v - u = at$

$$\boxed{v = u + at}$$

(ii) $v = \frac{ds}{dt} \Rightarrow \int_0^s ds = \int_0^t v\,dt$

$$\Rightarrow \int_0^s ds = \int_0^t (u + at)dt$$

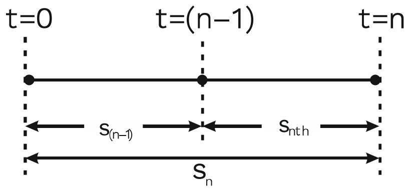
$$\boxed{s = ut + \frac{1}{2}at^2}$$

(iii) $a = v\frac{dv}{ds} \Rightarrow \int_u^v v\,dv = \int_0^s a\,ds \Rightarrow \left[\frac{v^2}{2}\right]_u^v = a[s]_0^s$

$$\boxed{v^2 = u^2 + 2as}$$

(iv) A particle have initial velocity u and constant acceleration a. Then displacement in n^{th} second.

$s_{n^{th}} = s_n - s_{(n-1)}$



$\because \quad s_n = un + \frac{1}{2}an^2$ ...(i)

$s_{(n-1)} = u(n-1) + \frac{a(n-1)^2}{2}$ ...(ii)

$\therefore \quad s_{n^{th}} = s_n - s_{(n-1)}$

$$\Rightarrow \boxed{s_{n^{th}} = u + \frac{a}{2}(2n-1)}$$

Graphs in 1-Dimenstion:

- **Position-time graph:**

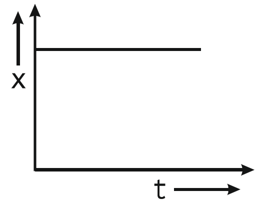
Slope of the tangent of this graph represent instantaneous velocity.

$\because \quad \tan\theta = \frac{\text{displacement}}{\text{time}} = \text{velocity}$

Area of x-t graph

$= \int x \cdot dt$ = no physical significance.

(i) When body is at rest



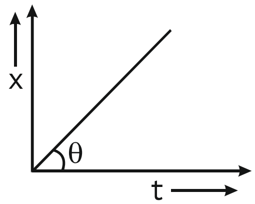
θ = 0°

tan θ = tan 0° = 0

velocity = 0

Acceleration = 0

(ii) When body is in uniform motion



θ = constant

tan θ = constant

velocity = constant

Acceleration = 0

Rack your Brain

A particle moves a distance x in time t according to equation $x = (t+5)^{-1}$. Calculate acceleration in term of velocity.

(iii) When body is in non-uniform motion

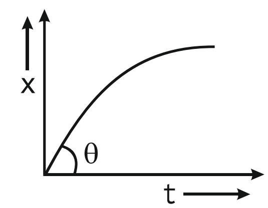
θ is decreasing with time

$\therefore$ tan θ is decreasing with time

$\therefore$ velocity is decreasing with time

Acceleration $\neq 0$

$a < 0$



(iv) When body is in non-uniform motion

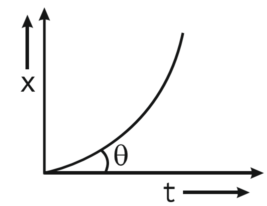
θ is increasing with time

$\therefore$ tan θ is increasing with time

$\therefore$ velocity is increasing with time

Acceleration $\neq 0$

$a > 0$



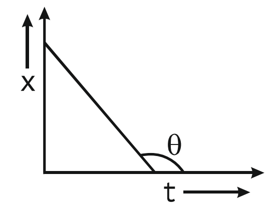
(v) when body is in uniform motion

$\theta > 90°$

tan θ = –ve

velocity = –ve but constant

Acceleration = 0



Velocity-time graph:

Slope of the tangent of v-t graph represents acceleration.

$\because \quad \tan\theta = \dfrac{\text{velocity}}{\text{time}} = \text{acceleration}$

Area of v-t graph = $\int v \cdot dt$

= displacement = change in position.

Concept Reminder:
Equation of Path:

- $\int x \cdot dt$ = no physical significance.
- $\int v \cdot dt$ = displacement = change in position.
- $\int a \cdot dt = \int dv = v_2 - v_1$ = change in velocity.

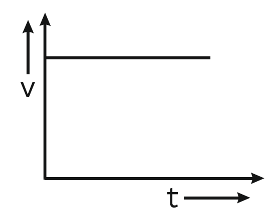
(i) Uniform motion

$\theta = 0°$

tan θ = tan 0° = 0

acceleration = 0

v = constant

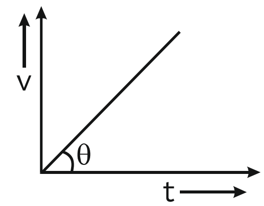


(ii) uniformly accelerated motion

θ = constant

$\tan\theta$ = constant

acceleration = constant



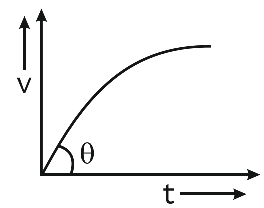
(iii) acceleration is decreasing with time

θ is decreasing with time

$\therefore$ $\tan\theta$ is decreasing with time

$\therefore$ acceleration is decreasing with time

acceleration goes on decreasing with time but it is not retardation.

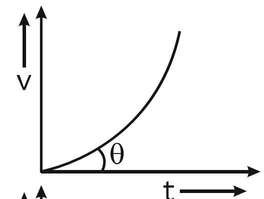


(iv) acceleration is increasing with time

θ is increasing with time

$\therefore$ $\tan\theta$ is increasing with time

$\therefore$ acceleration is increasing with time

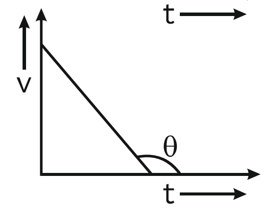


(v) Uniform retardation

$\theta > 90°$, $\tan\theta$ = –ve

acceleration = –ve but constant

constant or uniform retardation is acting on the body.



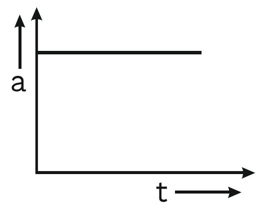
Acceleration-time graph:

Slope of the tangent of this graph represents jerk.

Area of a-t graph =

$$\int a \cdot dt = \int dv = v_2 - v_1 \text{ = change in velocity.}$$

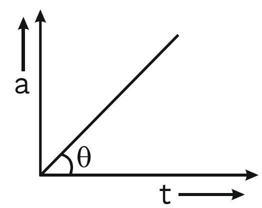
(i) Constant acceleration



Key Points

- Retardation

(ii) Uniformly acceleration

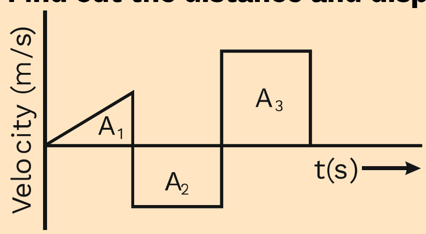


a ∝ t
uniformly increasing acceleration

Rack your Brain

The position x of a particle varies with time (t) as $x = at^2 - bt^3$. Calculate the time when acceleration will be zero.

Q24 **Find out the distance and displacement.**

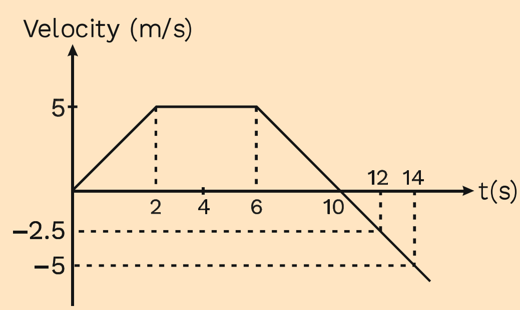


A24 $\int ds$ = Area of v-t curve.

Distance = $A_1 + A_2 + A_3$ → Always +ve
Distance = $A_1 - A_2 + A_3$ → Can be –ve

Q25 **The variation of velocity of a body moving along a straight line is shown in following figure. The distance travelled by the body in 12 sec is**

(1) 37.5 m
(2) 32.5 m
(3) 35.0 m
(4) None of these



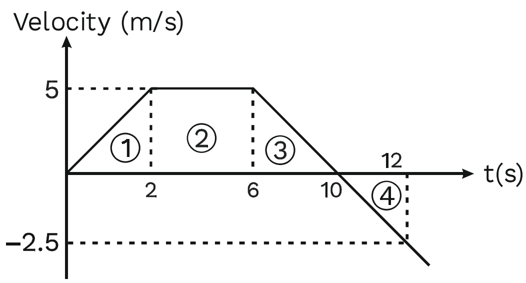
A25 $v = \frac{ds}{dt}$, $\int ds = \int v dt$

distance = $A_1 + A_2 + A_3 + A_4$

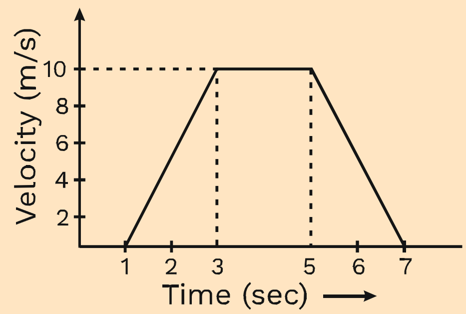
$A_1 = \frac{1}{2} \times 2 \times 5 = 5 \Rightarrow A_2 = 5 \times 4 = 20$

$A_3 = \frac{1}{2} \times 4 \times 5 = 10 \Rightarrow A_4 = \frac{1}{2} \times 2 \times \frac{5}{2} = 2.5$

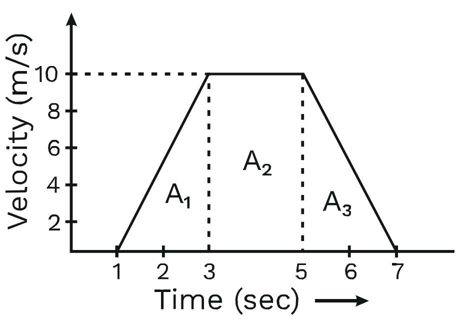
Distance = 5 + 20 + 10 + 2.5 = 37.5 m



Q26 **For the velocity-time graph shown in figure below the distance covered by the body in last two second of its motion is what fraction of the total distance covered by it in all the seven second**



A26 $\frac{\text{distance covered by last two second } (A_3)}{\text{Total distance } (A_1 + A_2 + A_3)}$

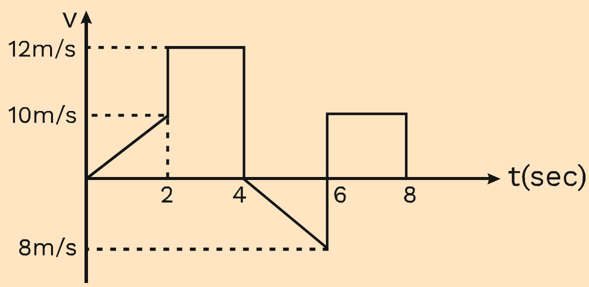


$\Rightarrow A_1 = \frac{1}{2} \times 2 \times 10 = 10$, $A_2 = 2 \times 10 = 20$,

$A_3 = \frac{1}{2} \times 2 \times 10 = 10$

$\Rightarrow \frac{A_3}{A_1 + A_2 + A_3} = \frac{10}{10 + 20 + 10} = \frac{1}{4}$

Q27 **Find distance and displacement in time interval 0 to 8 sec.**



A27 $A_1 = \frac{1}{2} \times 2 \times 10 = 10\,m$

$A_2 = 2 \times 12 = 24$ m

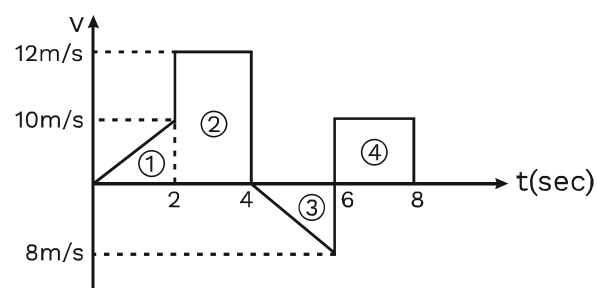
$A_3 = \frac{1}{2} \times 2 \times 8 = 8$ m

$A_4 = 2 \times 10 = 20$ m

Distance = $A_1 + A_2 + A_3 + A_4 = 62$ m

Displacement = $A_1 + A_2 - A_3 + A_4$ =

$10 + 24 - 8 + 20 = 46$ m

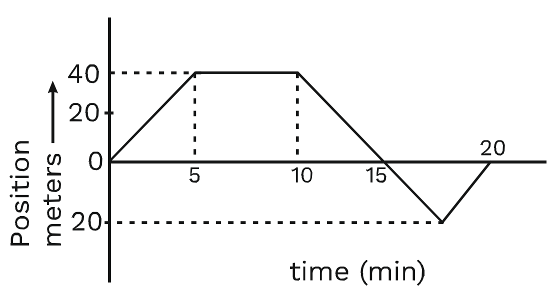


Q28 **Mahesh begins to walk eastward along a street in front of his house and the graph of Mahesh position from home is shown in the following figure. Mahesh's average speed for the whole time interval is equal to**

(1) 8 m/min (2) 6 m/min

(3) $\frac{8}{3}$ m/min (4) 2 m/min

A28



average speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{40 + 0 + 40 + 20 + 20}{20}$

$v_{avg} = 6$ m/min

Q29 **Equation of displacement for any particle is**
$S = 3t^3 + 7t^2 + 14t + 8$ m. Its acceleration at time t = 1 sec is.

(1) 10 m/s² **(2) 16 m/s²**
(3) 25 m/s² **(4) 32 m/s²**

A29

$$V = \frac{dS}{dt} = 9t^2 + 14t + 14$$

$$a = \frac{dv}{dt} = 18t + 14$$

$t = 1$ sec $a = 32\ m/s^2$

Q30 **If the velocity of a particle is given by** $v = (180 - 16x)^{1/2}\ m/s$ **then its acceleration will be.**

(1) Zero **(2) 8 m/s²**
(3) – 8 m/s² **(4) 4 m/s²**

A30

$$a = \frac{dv}{dt} = v\frac{dv}{dx}$$

$v = (180 - 16x)^{1/2}$

$v^2 = 180 - 16x$

$$2v\frac{dv}{dx} = 0 - 16, \qquad v\frac{dv}{dx} = -8$$

$a = -8\ m/s^2$

Q31 **If** $V = \alpha\sqrt{x}$ **, V is velocity, x is position, a is +ve constant then a = ?**

A31

$V^2 = \alpha^2 x$

$$2V\frac{dV}{dx} = \alpha^2(1)$$

$$V \cdot \frac{dv}{dx} = \frac{\alpha^2}{2}$$

$$a = \frac{\alpha^2}{2}$$

Q32 **If $V = \sqrt{(108x - 9x^2)}$. Find acceleration 'a' after x = 3 cm.**

(1) 27 cm/s² **(2) 54 cm/s²**
(3) −27 cm/s² **(4) Zero**

A32

$a = V\dfrac{dV}{dx}$

$V^2 = 108x - 9x^2$

$2V\dfrac{dV}{dx} = 108 - 18x$

$V\dfrac{dV}{dx} = \dfrac{1}{2}[108 - 18 \times 3]$

$a = 54 - 27$

$a = 27 \text{ cm}/s^2$

Q33 **A body moves in a plane so that the displacement along the x and y axis are given by $x = 3t^3$ and $y = 4t^3$. The velocity of the body is.**

(1) 9t **(2) 15t**
(3) $15t^2$ **(4) $25t^2$**

A33

$V_x = \dfrac{dx}{dt}, \qquad V_x = \dfrac{d}{dt}(3t^3)$

$V_x = 9t^2$

$V_y = \dfrac{dy}{dt} = \dfrac{d}{dt}(4t^3) = 12t^2$

$\vec{V} = V_x\hat{i} + V_y\hat{j}$

$\vec{V} = 9t^2\hat{i} + 12t^2\hat{j}$

$|V| = \sqrt{(9t^2)^2 + (12t^2)^2}$

$V = 15t^2$

Q34 **A particle start from rest along straight line and moving from uniform acceleration. Find ratio of distance travelled.**
(i) After 1 sec, 2 sec, 3 sec
(ii) 1^{st}, 2^{nd}, 3^{rd} sec of motion.

A34 (i) $S = ut + \frac{1}{2}at^2$

$u = 0$

$S \propto t^2$, $S_1 : S_2 : S_3 =$

$(1)^2 : (2)^2 : (3)^2 = 1 : 4 : 9$

(ii) $S_{n^{th}} = u + \frac{a}{2}[2n - 1]$

$u = 0$, $S_{n^{th}} \propto (2n - 1)$

$S_1 : S_2 : S_3$

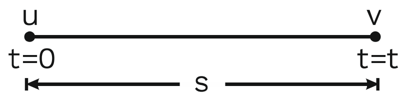
$(2 \times 1 - 1) : (2 \times 2 - 1) : (2 \times 3 - 1)$

$1 : 3 : 5$

Q35 **Particle have initial velocity u moving with constant acceleration. After time t its velocity becomes v. Then find out distance travelled in this duration.**

A35 $v = u + at$

$a = \frac{v - u}{t}$



$s = ut + \frac{1}{2}at^2$

$s = ut + \frac{1}{2}\left(\frac{v - u}{t}\right) \cdot t^2$

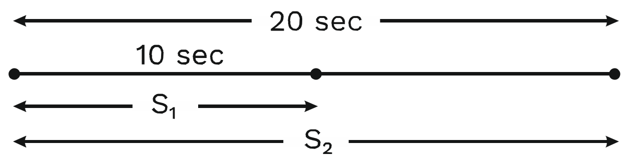
$$\boxed{s = \frac{(v + u)}{2} \cdot t}$$

Q36 **A particle starts its motion from rest under the action of a constant force. If the distance covered in first 10 seconds is S_1 and that covered in the first 20 second is S_2, then.**

(1) $S_2 = 2S_1$ **(2) $S_2 = 3S_1$**
(3) $S_2 = 4S_1$ **(4) $S_2 = S_1$**

A36 Force is constant.

$\vec{F} = m\vec{a}$ a is constant



$u = 0$

$S_1 = ut + \frac{1}{2}at^2$, $S_1 = \frac{1}{2}at^2$

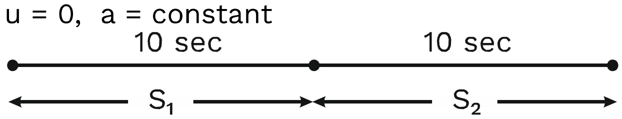
$S_1 = \frac{1}{2}a(10)^2$, $S_2 = \frac{1}{2}a(20)^2$

$\frac{S_2}{S_1} = \frac{4}{1}$

Q37 **A particle start its motion from rest under the action of a constant force. If the distance covered in first 10 second is S_1 and in next 10 second is S_2, then**

(1) $S_1 = S_2$ **(2) $S_2 = 3S_1$**
(3) $S_2 = 4S_1$ **(4) None of these**

A37 $u = 0$, a = constant



$S_1 = 0 + \frac{1}{2}a(10)^2$, $S_1 = \frac{1}{2}a(100)$ ...(i)

$S_1 + S_2 = 0 + \frac{1}{2}a(20)^2$, $S_1 + S_2 = \frac{1}{2}a(400)$ ...(ii)

$S_1 + S_2 = 4S_1 \Rightarrow S_2 = 3S_1$

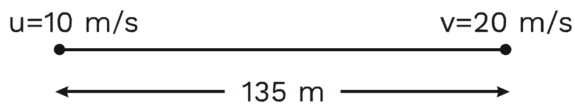
Q38 **A particle moves in a straight line with a constant acceleration. It changes its velocity from 10 m/s to 20 m/s while passing through a distance 135 m in t second. The value of t is**

(1) 12 **(2) 9**

(3) 10 **(4) 1.8**

A38 V = u + at

20 = 10 + at ...(i)



$v^2 = u^2 + 2as$

$$S = \frac{v^2 - u^2}{2a} \Rightarrow S = \frac{(20)^2 - (10)^2}{2a} = 135$$

$$a = \frac{400 - 100}{2 \times 135} = \frac{150}{135}$$

From equation (i)

$$20 = 10 + \frac{150}{135}t,$$

$$t = \frac{10 \times 135}{150}$$

t = 9 sec

Q39 **Q.39 A body starting from rest moves with constant acceleration. The ratio of distance covered by the body during the 5th sec to that covered in 5 sec is.**

(1) $\frac{9}{25}$ **(2)** $\frac{3}{5}$

(3) $\frac{25}{9}$ **(4)** $\frac{1}{25}$

A39

$$S_{n^{th}} = u + \frac{a}{2}[2n - 1] \quad ...(i)$$

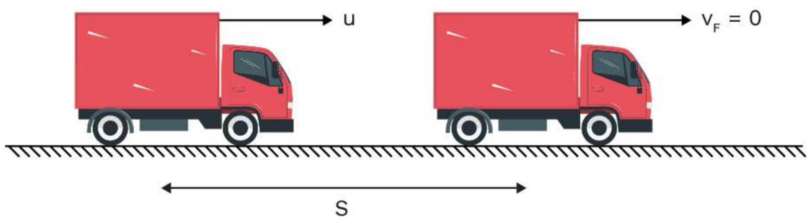
$$S_n = ut + \frac{1}{2}at^2 \quad ...(ii)$$

u = 0, a = constant.

$$\frac{S_{n^{th}}}{S_n} = \frac{(2 \times 5 - 1)}{(5)^2}$$

$$\frac{S_{n^{th}}}{S_n} = \frac{9}{25}$$

Uniform Brake [stopping distance]:



$v^2 = u^2 + 2as$

$0 = u^2 + 2(-a)S$

$$S = \frac{u^2}{2a}$$

$S \propto u^2$

$$\boxed{\frac{S_2}{S_1} = \frac{u_2^2}{u_1^2}}$$

If 'u' becomes 'n' times then 's' becomes n^2 times.

Stopping time

$v = u - at, \quad 0 = u - at, \quad t = \frac{u}{a}$

$$\boxed{t \propto u}$$

If 'u' becomes 'n' times then 't' becomes n times.

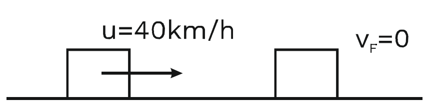
> **Concept Reminder:**
> **Stopping distance:**
>
> - = —
> - $S \propto u^2$
> - — = —
>
> **Stopping time:**
>
> - = —
> - $\propto$

Q40 **A car moving with speed of 40 km/h can be stopped by applying brakes after atleast 2m. If the same car is moving with speed of 80 km/h, what is the minimum stopping distance?**

(1) 8 m **(2) 2 m**

(3) 4 m **(4) 6 m**

A40



stopped by applying brake

$s \propto u^2$

$$\frac{S_2}{S_1} = \frac{u_2^2}{u_1^2} = \frac{(80)^2}{(40)^2}$$

$S_2 = 4S_1$

$S_2 = 4 \times 2$

$S_2 = 8m$

Keywords

- Stopping distance
- Stopping time

Q41 **Average velocity of a particle moving in a straight line, with constant acceleration a and initial velocity u in first t second is?**

(1) $u+\frac{1}{2}at$ **(2)** **u + at**

(3) $\frac{u+at}{2}$ **(4)** $\frac{u}{2}$

A41 $\vec{v}_{avg} = \frac{\text{Total displacement}}{\text{Total time}}$

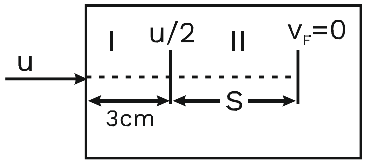
$S = \frac{ut+\frac{1}{2}at^2}{t} \Rightarrow \vec{v}_{avg} = u+\frac{1}{2}at$

Q42 **A bullet fired into a fixed target. Which is loses half of it's velocity after penetrating 3 cm. Find out how much further it will penetrate before coming to rest assuming that it faces constant resistance to motion?**

(1) **1.5 cm** **(2)** **1.0 cm**

(3) **3.0 cm** **(4)** **2.0 cm**

A42



F = constant
a = constant

$v^2 = u^2 + 2as$

in second condition when final velocity is zero.

$0 = \left(\frac{u}{2}\right)^2 - 2as \quad ...(i)$

in first condition when velocity is half of initial velocity

$\left(\frac{u}{2}\right)^2 = u^2 + 2(-a)(3)$

$a = \frac{u^2}{8}$

From equation (i)

$0 = \left(\frac{u}{2}\right)^2 + 2\times\left(-\frac{u^2}{8}\right)S \Rightarrow S = 1\text{ cm}$

Q43 **Average velocity of a body moving with uniform acceleration travelling a distance of 3.06 m is 0.34 m/s. If change in velocity of the body is 0.18 m/s during this time, then its uniform acceleration is.**

(1) 0.01 m/s² **(2) 0.02 m/s²**

(3) 0.03 m/s² **(4) 0.04 m/s²**

A43 $\vec{a}_{avg} = \frac{\text{Change in velocity}}{\text{Time}}$

average velocity = $\frac{\text{Total displacement}}{\text{Time}}$

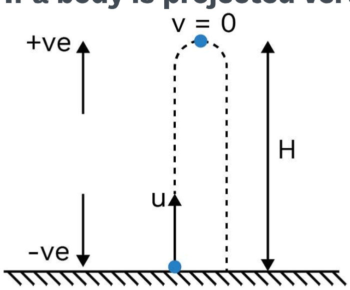
time = $\frac{3.06}{0.34}$ $\Rightarrow$ time = 9 sec

$\vec{a}_{avg} = \frac{0.18}{9}$ $\Rightarrow \vec{a}_{avg} = 0.02 \text{ m/s}^2$

Motion Under Gravity:

- Acceleration produced by force of gravity.
- It is denoted by 'g'.
- $g = 9.8 \text{ m/s}^2 = 32 \text{ ft/s}^2 = 980 \text{ cm/s}^2$ (applicable from a small altitude).

(i) If a body is projected vertically upward



[positive/negative direction are a matter of choice]

Equation of motion:

If a particle is projected with velocity u and after time t it reaches a height H then

$v = u - gt, H = ut - \frac{1}{2}gt^2$

$v^2 = u^2 - 2gh$

$H_{n^{th}} = u - \frac{g}{2}(2n - 1)$

Concept Reminder:

When a body thrown upward with velocity:

- $v = u - gt$,
- $H = ut - \frac{1}{2}gt^2$
- $v^2 = u^2 - 2gh$
- $H_{n^{th}} = u - \frac{g}{2}(2n - 1)$

For maximum height, v = 0

$v = u - gt \Rightarrow \quad u = gt$

$t_1 = \frac{u}{g}$; t_1 is called time of ascent.

In motion under gravity, time taken to fall down is equal to the time taken to go up through the same distance.

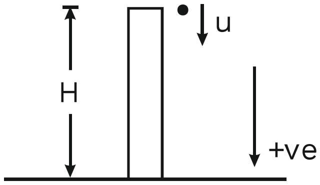
time of ascent (t_1) = time of descent (t_2) = $\frac{u}{g}$

time of flight $\boxed{T = t_1 + t_2 = \frac{2u}{g}}$

$u^2 = 2gH$

$\boxed{H = \frac{u^2}{2g}}$

(ii) If a body is projected vertically downward with some initial velocity (u) from some height (H)



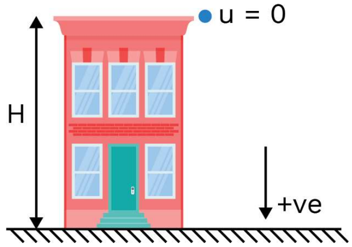
when we choose downward direction as a positive,

$v = u + gt, H = ut + \frac{1}{2}gt^2$

$v^2 = u^2 + 2gh$

$H_{n^{th}} = u + \frac{g}{2}(2n - 1)$

(iii) When a body is dropped (u = 0) from some height (H):



Taking downward direction as a positive, u = 0 [as

Rack your Brain

A ball is dropped from a high rise platform at t = 0 starting from rest. After 6 seconds another ball is thrown downwards from the same platform with a speed v. The two balls meet at t = 18 s. What is the value of v? (Take g = 10 m/s²) a body starts from rest]

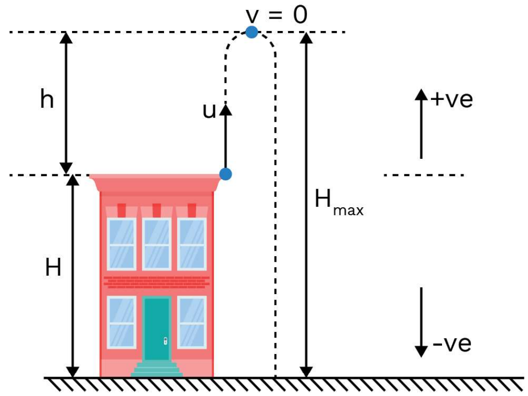
$$v = gt$$

$$H = \frac{1}{2}gt^2$$

$$v^2 = 2gh$$

$$H_{n^{th}} = \frac{g}{2}(2n-1)$$

(iv) If a body is projected vertically upward with some initial velocity (u) from a certain height (H):



Equation of motion (taking upward direction is positive and downward direction is negative)

$$v = u - gt$$

$$h = ut - \frac{1}{2}gt^2, \quad -H = ut - \frac{1}{2}gt^2,$$

$$v^2 = u^2 - 2gh$$

$$h_{n^{th}} = u - \frac{g}{2}(2n-1), \quad H_{max} = H + h = H + \frac{u^2}{2g}$$

total distance travelled by the body

$$H + 2h = H + \frac{u^2}{g}$$

Note:

(I) Body is released or dropped from stationary frame then its initial velocity will be zero.

(II) If body is projected or thrown or fired then its initial velocity is non-zero.

Rack your Brain

Two bodies A (of mass 1 kg) and B(of mass 3 kg) are dropped from heights of 16m and 25m respectively. Calculate the ratio of the time taken by them to reach the ground.

(III) In case of rocket and hydrogen balloon initial velocity is zero.

Effect of air resistance:

a = retardation due to air friction for ascending motion.

(i) $v^2 = v_1^2 - 2a_{net} \cdot h,$

$0 = v_1^2 - 2(g+a) \cdot h$ $\boxed{v_1 = \sqrt{2(g+a)h}}$

(ii) $\boxed{t_1 = \frac{v_1}{a_{net}} = \sqrt{\frac{2h}{(g+a)}}}$

For descending motion

(i) $v_2^2 = v^2 + 2a_{net} \cdot h$

$v_2^2 = 0 + 2(g-a) \cdot h$

$\boxed{v_2 = \sqrt{2(g-a) \cdot h}}$

(ii) $\boxed{t_2 = \sqrt{\frac{2h}{(g-a)}}}$

result $\boxed{\begin{array}{l} v_1 > v_2 \\ t_1 < t_2 \end{array}}$

Concept Reminder:

Effect of air resistance:

During upward:

- $\boxed{t_1 = \frac{v_1}{a_{net}} = \sqrt{\frac{2h}{(g+a)}}}$

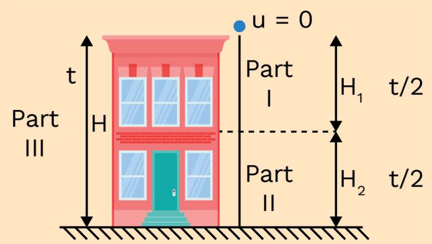
During downward:

- $\boxed{t_2 = \sqrt{\frac{2h}{(g-a)}}}$
- $\boxed{\begin{array}{l} v_1 > v_2 \\ t_1 < t_2 \end{array}}$

Key Points

- Motion under gravity
- Air resistance
- Ascent
- Decent

Q44 **If a ball drop height H from ground and H_1 distance travelled in half time and H_2 distance travelled in next half time than find out relation between H_1 and H_2 = ?**



A44 $\vec{S} = \vec{u}t + \frac{1}{2}at^2$

I^st Part

$$-H_1 = 0 - \frac{1}{2}g\left(\frac{t}{2}\right)^2$$

$$H_1 = \frac{1}{2}g\frac{t^2}{4} \qquad ...(i)$$

III[rd] **Part**

$$-(H_1 + H_2) = 0 - \frac{1}{2}gt^2$$

$$H_1 + H_2 = \frac{1}{2}gt^2 \qquad \text{...(ii)}$$

Equation (i) divided by equation (ii)

$$\frac{H_1}{H_1 + H_2} = \frac{1}{4}$$

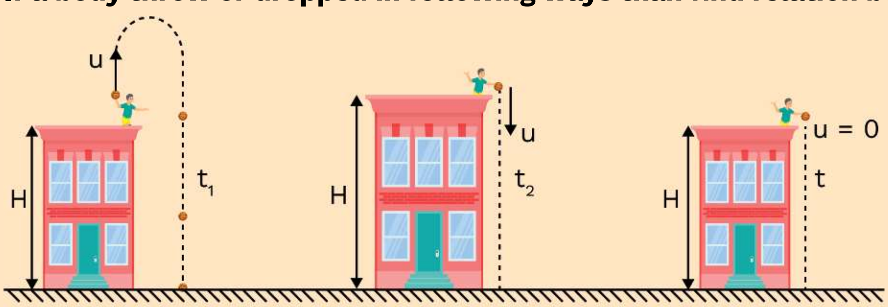
$$4H_1 = H_1 + H_2$$

$$H_2 = 3H_1$$

Rack your Brain

A body starts from rest, what is the ratio of the distance travelled by the body during the 4[th] and 3[rd] second?

Q45 **If a body throw or dropped in following ways than find relation between t, t_1, t_2?**



Note: Height of man is negligible compare to height of building

A45 (i) $-H = ut_1 - \frac{1}{2}gt_1^2$ . ..(i)

(ii) $-H = -ut_2 - \frac{1}{2}gt_2^2$ ...(ii)

(iii) $-H = 0 - \frac{1}{2}gt^2$ ...(iii)

$$T_{up} = \frac{u + \sqrt{u^2 + 2gH}}{g} = t_1$$

$$T_{down} = \frac{-u + \sqrt{u^2 + 2gH}}{g} = t_2$$

$$T_{drop} = \sqrt{\frac{2H}{g}} = t$$

Using [(A + B) (A – B) = $A^2 - B^2$]

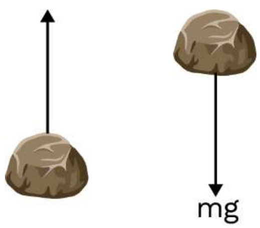
$$t_1t_2 = \frac{(u^2 + 2gH) - u^2}{g.g} = \frac{2H}{g}$$

$$t_1t_2 = t^2 \quad \Rightarrow \quad t = \sqrt{t_1t_2}$$

Q46 **A stone of mass 0.05 kg is thrown vertically upward. What is the magnitude and direction of net force on the stone during its upward motion.**

(1) 0.49 N vertically downward
(2) 9.8 N vertically downward
(3) 0.49 N vertically upward
(4) 0.98 N vertically downward

A46 $\vec{F} = mg = 0.05 \times 9.8 = 0.49\,N$ vertically downward



Q47 **A ball is thrown up under gravity (g = 10 m/s²). Find its velocity after 1.0 sec at a height of 10 m.**

(1) 5 m/s² **(2) 5 m/s**
(3) 10 m/s **(4) 15 m/s**

A47 $S = ut + \frac{1}{2}at^2$

$S = 10,$ $u = +u,$ $a = -g$

$h = ut - \frac{1}{2}gt^2$ $\Rightarrow 10 = u \times 1 - \frac{1}{2} \times 10(1)^2$

$u = 5\ m/s$ $\Rightarrow v = u - gt$

$v = 15 - 10 \times 1 = 5\ m/s$

Q48 **A body is released from a great height and falls freely toward the earth. Another body is released from same height exactly 1 sec later. Find out separation between the two bodies, 2 sec after the release of the second body is.**

(1) 4.9 m **(2) 9.8 m**
(3) 19.6 m **(4) 24.5 m**

A48 $h_1 - h_2 = ?$

$$\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

For 1st body

$$-h_1 = 0 - \frac{1}{2}g(3)^2$$

$$h_1 = \frac{9g}{2} \quad ...(i)$$

For 2nd body

$$-h_2 = 0 - \frac{1}{2}g(2)^2$$

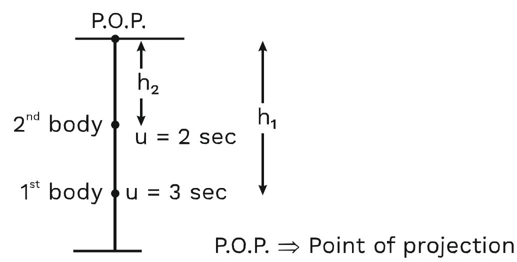
$$h_2 = \frac{4g}{2} \quad ...(ii)$$

$$h_1 - h_2 = \frac{9g}{2} - \frac{4g}{2}$$

$$h_1 - h_2 = \frac{5g}{2}$$

$$h_1 - h_2 = \frac{5}{2} \times 9.8$$

$$h_1 - h_2 = 24.5\,m$$



Q49 **A particle is free fall from the top of a tower of height h. It takes t sec to reach the ground. Where will be the particle after time t/2 sec.**

(1) **At $\frac{h}{2}$ from ground**

(2) **At $\frac{h}{4}$ from the ground**

(3) **Depends upon mass of particle**

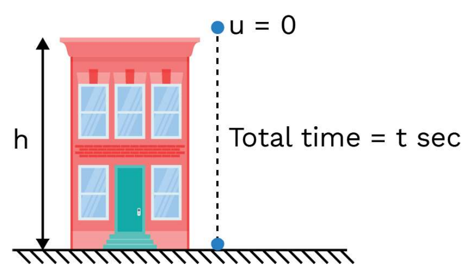
(4) **At $\frac{3h}{4}$ from the ground**

A49 $s = ut + \frac{1}{2}gt^2$

$$-h = 0 - \frac{1}{2}gt^2 \quad \Rightarrow h = \frac{1}{2}gt^2$$

After t/2 sec

$$h' = ut + \frac{1}{2}gt^2 \quad \Rightarrow -h' = 0 + \frac{1}{2}(-g)\left(\frac{t}{2}\right)^2$$



$$h' = \frac{1}{2}g \times \frac{t^2}{4}$$

$$h' = \frac{h}{4} \text{ from top}$$

Ball will be at $\frac{3h}{4}$ from the ground.

Q50 **A particle is projected up with an initial velocity of 80 ft/sec. The ball will be at a height of 96 ft from the ground after**

(1) 2.0 and 3.0 sec

(2) Only at 3.0 sec

(3) Only at 12.0 sec

(4) After 12 and 2 sec

A50 $g = 9.8 \text{ m/sec}^2$ or 32 ft/sec^2

$$S = ut + \frac{1}{2}at^2$$

$$96 = 80t - \frac{1}{2} \times 32(t)^2 \Rightarrow 16t^2 - 80t + 96 = 0$$

$$-5t + t^2 + 6 = 0 \Rightarrow (t-3)(t-2) = 0$$

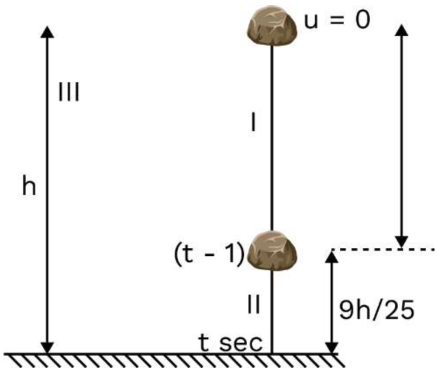
$t = 2, 3$ sec

Q51 **A stone falls from rest from a height h and it travels a distance 9h/25 in the last second. The value of h is**

(1) 145 m **(2) 100 m**

(3) 122.5 m **(4) 200 m**

A51



$$S = ut + \frac{1}{2}at^2$$

After t sec

$$-h = 0 - \frac{1}{2}at^2$$

$$h = \frac{1}{2}gt^2 \quad \text{...(i)}$$

After (t − 1) sec

$$-\frac{16h}{25} = 0 - \frac{1}{2}g(t-1)^2$$

$$\Rightarrow \frac{16h}{25} = \frac{1}{2}g(t-1)^2 \quad \text{...(ii)}$$

From equation (i) and (ii)

$$\frac{16}{25} = \frac{(t-1)^2}{t^2}$$

$\Rightarrow$ t = 5 sec

From equation (i)

$$h = \frac{1}{2}gt^2$$

$$\Rightarrow \quad h = \frac{1}{2}9.8 \times (5)^2 \qquad \Rightarrow \quad h = 122.5 \text{ m}$$

Q52 **A particle starts from rest with constant acceleration. Then find out ratio of space average velocity to the time average velocity.**

A52 a = constant

$v = at$ ...(i)

$s = \frac{1}{2}at^2$ ...(ii)

$v^2 = 2as$ ...(iii)

time average velocity

$$< v >_t = \frac{\int_0^t v \cdot dt}{\int_0^t dt} = \frac{\int_0^t at \cdot dt}{\int_0^t dt} = \frac{\frac{at^2}{2}}{t} \Rightarrow < v >_t = \frac{at}{2}$$

space average velocity

$$< v >_s = \frac{\int v \cdot ds}{\int ds}$$

from equation (ii) ds = at . dt

$$< v >_s = \frac{\int_0^t (at)^2 dt}{\int_0^t at \cdot dt} \Rightarrow < v >_s = \frac{\frac{a^2t^3}{3}}{\frac{at^2}{2}} = \frac{2at}{3} \Rightarrow < v >_s = \frac{2at}{3}$$

$$\text{ratio} = \frac{< v >_s}{< v >_t} = \frac{\frac{2at}{3}}{\frac{at}{2}} = \frac{4}{3}$$

Q53 **A stone thrown upward with a speed u from the top of the tower reaches the ground with a velocity 3u. The height of the tower is**

(1) $\frac{3u^2}{g}$ **(2)** $\frac{4u^2}{g}$

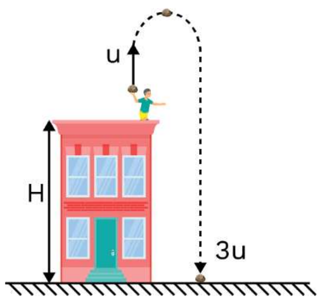
(3) $\frac{6u^2}{g}$ **(4)** $\frac{9u^2}{g}$

A53 $v = \sqrt{u^2 + 2gh}$

$v^2 = u^2 + 2gh$

$(3u)^2 = u^2 + 2gh$

$h = \frac{4u^2}{g}$



Q54 **If a freely falling body travels in the last second a distance equal to the distance travelled by it in the 1st three second, the time of the travel is.**

(1) **6 sec** **(2)** **5 sec**

(3) **4 sec** **(4)** **3 sec**

A54 Distance travelled in first 3 sec = distance travelled in last t^{th} sec

$ut + \frac{1}{2}g(t)^2 = u + \frac{g}{2}(2n-1)$

$0 - \frac{1}{2}g(3)^2 = 0 - \frac{g}{2}(2t-1)$

$9 = 2t - 1$

$t = 5$ sec

Q55 **A packet is dropped from a balloon which is going upward with the velocity 12 m/s, the velocity of the packet after 2 second will be.**

(1) **– 12 m/s** **(2)** **12 m/s**

(3) **– 7.6 m/s** **(4)** **7.6 m/s**

A55 Balloon velocity is transferred to packet initial velocity will be 12 m/s.

v = u + at

v = 12 – 9.8 × 2 = –7.6 m/s

Q56 **A body is thrown vertically upwards. It air resistance is to be taken into account, then the time during which the body rises is.**

(1) Equal to the time of fall

(2) Less than the time of fall

(3) Greater than the time of fall

(4) Twice the time of fall

A56
$$t_{top} = \frac{u}{g+a}$$

$$H = \frac{u^2}{2(g+a)}$$

$$H = (g-a)t_{down}^2$$

$$t_{down} = \sqrt{\frac{2H}{g-a}} = \sqrt{\frac{2u^2}{2(g+a)(g-a)}}$$

$$t_{down} = \frac{u}{\sqrt{g^2 - a^2}}$$

$$t_{down} > t_{top}$$

Q57 **A ball is dropped from a high rise platform at t = 0 starting from rest. After 6 sec another ball is thrown downward from the same platform with a speed v. The two balls meet at t = 18 sec. What is the value of v ? (take g = 10 m/s²)**

(1) 60 m/s **(2) 7 m/s**

(3) 55 m/s **(4) 40 m/s**

A57 $\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2 \quad \Rightarrow -h_1 = -\frac{1}{2}g(18)^2$

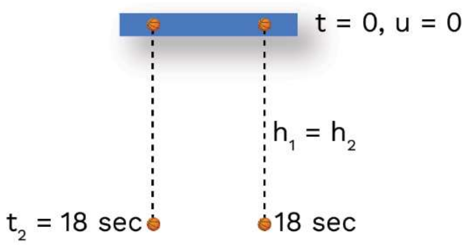
$h_1 = \frac{1}{2}g(18)^2$ ...(i)

$-h_2 = -v(12) - \frac{1}{2}g(12)^2$

$h_1 = h_2 = 12v + \frac{1}{2}g(12)^2$ ...(ii)

$\frac{1}{2}g(18)^2 = 12v + \frac{1}{2}g(12)^2$

v = 75 m/s

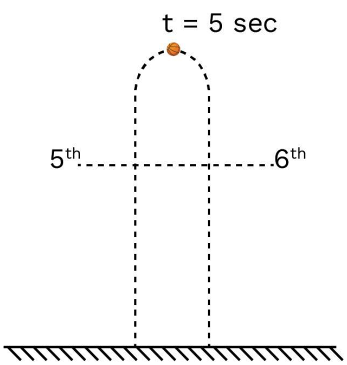


Q58 **A body is thrown vertically upward with speed u. The distance travelled by it in the 5th and 6th second are equal. The speed u is given by (g = 9.8 m/s²)**

(1) 24.5 m/s **(2) 49.0 m/s**

(3) 73.5 m/s **(4) 98.0 m/s**

A58



$t = \frac{u}{g}$

u = gt = 9.8 × 5 = 49 m/s

Q59 **When a body is thrown up vertically with velocity v_0, it reaches a maximum height of 'h'. If one wishes to 3 times the maximum height then the body should be thrown with velocity.**

(1) $\sqrt{3}\, v_0$ **(2)** $3\, v_0$

(3) $9\, v_0$ **(4)** $\frac{3}{2} v_0$

A59 $h = \frac{v^2}{2g} \Rightarrow h \propto v^2$

$\sqrt{\frac{h_1}{h_2}} = \frac{v_0}{v_2}$

$v_2 = \sqrt{3}\, v_0$

Q60 **From the top of a tower two stones whose masses are in the ratio 1 : 2 are thrown one straight up with an initial speed u and the second straight down with the same speed u. Then neglecting air resistance.**

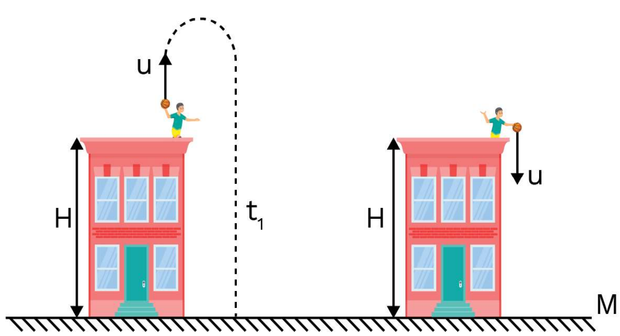
(1) **The heavier stone hits the ground with a higher speed**

(2) **The lighter stone hits the ground with a higher speed**

(3) **Both the stone will have the same speed when they hit the ground**

(4) **The speed can't be determined with the given data**

A60



$v_1 = \sqrt{u^2 + 2gh} = v_2$

$T_{up} > T_{down}$

$\therefore$ First stone which is lighter will take more time to hit the ground.

Q61 **A body falling from a high building travels 40 meters in the last 2 sec of its fall to ground. Height of building is (g = 10 m/s²)**

(1) 60 **(2) 45**

(3) 80 **(4) 50**

A61

$-H = 0 - \frac{1}{2}gt^2$

$H = \frac{1}{2}gt^2$ ...(i)

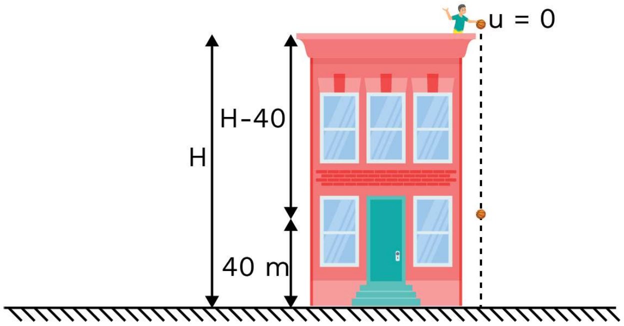
$-(H - 40) = 0 - \frac{1}{2}g(t-2)^2$

$H - 40 = \frac{1}{2}g(t-2)^2$ ...(ii)

From equation (i) and (ii)

t = 3 sec

H = 45 m



Q62 **A very large number of balls are thrown vertically upward in quick succession hence that the next ball is thrown when the previous one is at the maximum height. If the ball is maximum height 5 m, the number of ball thrown per minute is. (g = 10 m/s²)**

(1) 120 **(2) 80**

(3) 60 **(4) 40**

A62

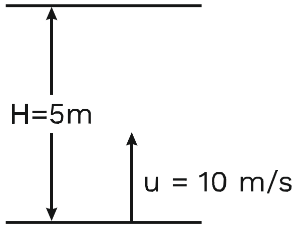
$H = \frac{u^2}{2g}$

$u = \sqrt{2gH} = \sqrt{100} = 10 \text{ m/s}$

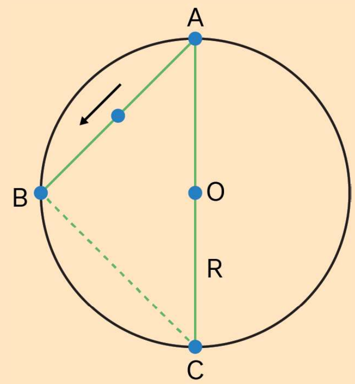
$t = \frac{u}{g} = \frac{10}{10} = 1\text{sec}$

1 min = 60 sec

$\Rightarrow$ 60 ball.



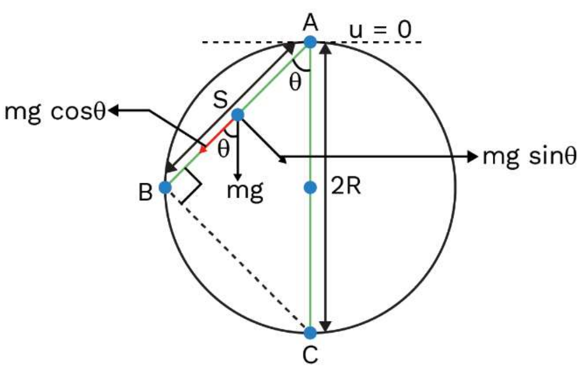
Q63 **A frictionless wire AB is fixed on a sphere of radius R. A very small spherical particle slips on this wire. The time taken by this particle to slip from A to B is.**



(1) $\frac{2\sqrt{gR}}{g\cos\theta}$ **(2)** $2\sqrt{gR}\cdot\frac{\cos\theta}{g}$

(3) $2\sqrt{\frac{R}{g}}$ **(4)** $\frac{gR}{\sqrt{g\cos\theta}}$

A63



$$\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

F = mg cos θ = ma

$$-AB = 0 - \frac{1}{2}(g\cos\theta)t^2$$

$$a = g\cos\theta$$

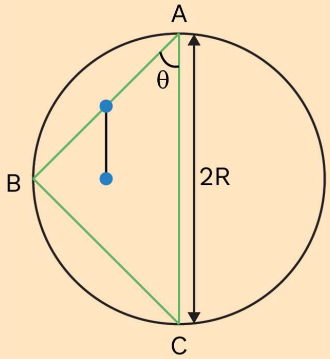
$$t_{AB} = \sqrt{\frac{2AB}{g\cos\theta}}$$

$$t_{AB} = \sqrt{\frac{2\cdot 2R\cos\theta}{g\cos\theta}}$$

$$\Rightarrow \quad t_{AB} = 2\sqrt{\frac{R}{g}}$$

Time does not depend on angle.

Q64 **When reach to bottom then find out velocity?**



A64 $a = g\cos\theta, \quad v^2 = u^2 + 2\vec{a}\cdot\vec{s}$

$v^2 = 2\cdot(-g\cos\theta)(-AB)$

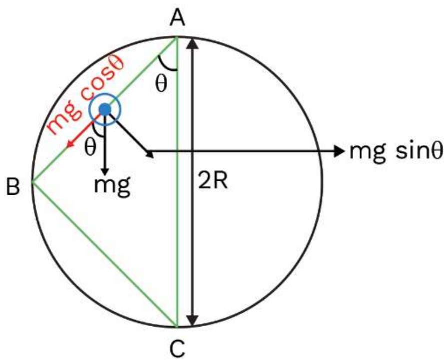
$v = \sqrt{2g\cos\theta \cdot 2R\cos\theta}$

$v = 2\cos\theta\sqrt{Rg}$

Speed depend on angle

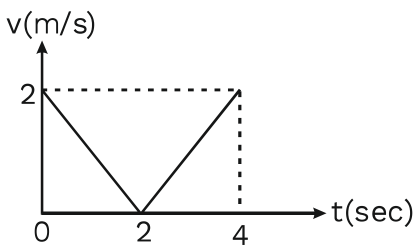
$v \propto \cos\theta$

$\theta\uparrow, \quad \cos\theta\downarrow, \quad v\downarrow$



Q65 **Particle moves along a straight line with speed v = |t – 2| m/s. Find out distance travelled by it in first 4 sec.**

A65



By graphical method

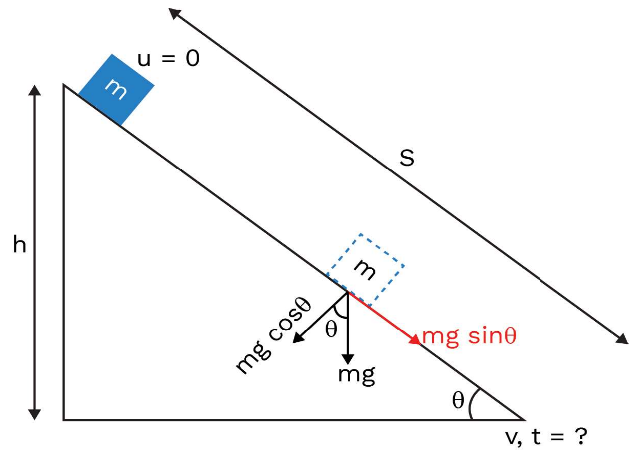
At $t = 0$, $v = 2$ m/s,

$t = 2$s, $v = 0$

$t = 4$ s, $v = 2$ m/s

$\text{distance} = 2\left[\frac{1}{2}\times 2\times 2\right] = 4\text{m}$

Motion along smooth inclined plane:



$a = g \sin \theta$

$v^2 = u^2 + 2\vec{a} \cdot \vec{s}$

$v = \sqrt{2(-g \sin\theta)(-S)}$

$\boxed{v = \sqrt{2gS\sin\theta}}$

$\frac{h}{S} = \sin\theta$

$\boxed{v = \sqrt{2gh}}$

$\boxed{h = S\sin\theta}$

(ii) $\vec{} = \vec{} + -\vec{}$ $\Rightarrow -S = -\frac{1}{2} g \sin\theta t^2$

$t = \sqrt{\frac{2S}{g\sin\theta}}$ $\Rightarrow t = \sqrt{\frac{2h}{g\sin^2\theta}}$

$\boxed{t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}}$

$\theta \uparrow, \quad \sin\theta \uparrow, \quad t \downarrow$

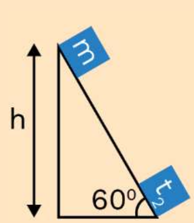
Concept Reminder:

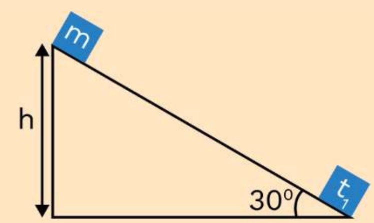
- $\boxed{h = S\sin\theta}$
- $\boxed{v = \sqrt{2gS\sin\theta}}$
- $\boxed{t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}}$

Q66 **Find out the ratio of $\frac{t_1}{t_2}$?**

(1) $\frac{1}{\sqrt{3}}$ **(2)** $\frac{1}{2}$

(3) $\frac{2}{1}$ **(4)** $\sqrt{3}$



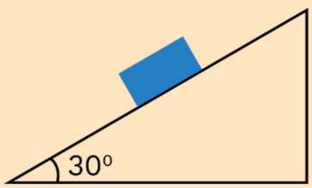


A66 $t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$

$\frac{t_1}{t_2} = \frac{\sin 60°}{\sin 30°}$

$\frac{t_1}{t_2} = \frac{\sqrt{3}}{1}$

Q67 **The time taken a block of wood (initially at rest) to slide down a smooth inclined plane 9.8 long (angle of inclination is 30°) is.**



(1) $\frac{1}{2}$ **sec** **(2)** **2 sec**

(3) **4 sec** **(4)** **1 sec**

A67 a = g sin 30°

$\vec{} = \vec{} + -\vec{} \Rightarrow -S = -\frac{1}{2} g \sin\theta\, t^2$

$t = \sqrt{\frac{2S}{g \sin 30^\circ}}$

t = 2 sec

Q68 **A body freely falling has a speed 'v' after it falls through a height 'h'. Find out the distance it has to fall down for its speed to become double is.**

(1) **2 h** **(2)** **4 h**

(3) **6 h** **(4)** **8 h**

A68 $v^2 = u^2 + 2gh$

$v^2 = 2gh$

$h \propto v^2$

$h_2 = 4h$

Non-uniform acceleration:

If $a \propto t^n$ (when $n \neq 0$)

then acceleration is known as non-uniform acceleration.

Q69 **If a = bt where b is positive constant. Particle is at rest when t = 0. Find**

(i) **v = ?** **(ii)** **S = ?**

A69 (i) $\int_u^v \frac{dv}{dt} = \int_0^t bt \Rightarrow v - u = b\left(\frac{t^2}{2}\right)_0^t$

$v - u = \frac{bt^2}{2} \Rightarrow v = u + \frac{bt^2}{2}$

(ii) $\frac{}{} = \Rightarrow \frac{ds}{dt} = u + \frac{bt^2}{2}$

$$\int ds = \int \left(u + \frac{bt^2}{2} \right) dt$$

$$s = ut + \frac{bt^3}{6}$$

Q70 **If** $f = f_0 \left[1 - \frac{t}{T} \right]$**, where f = acceleration at time t then find value of v at t = T.**

when t = 0, u = 0

A70 $\frac{dv}{dt} = f_0 - \frac{f_0 t}{T}$

$$\int \quad = \int \quad - \int \; -$$

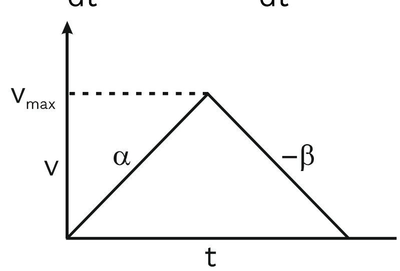
$$v = f_0 t - \frac{f_0}{T} \frac{t^2}{2} \quad \Rightarrow \quad v = f_0 t - \frac{f_0 t^2}{2T}$$

At $t = T$ $\Rightarrow$ $v = f_0 T - \frac{f_0 T^2}{2T}$

$$v = \frac{f_0 T}{2}$$

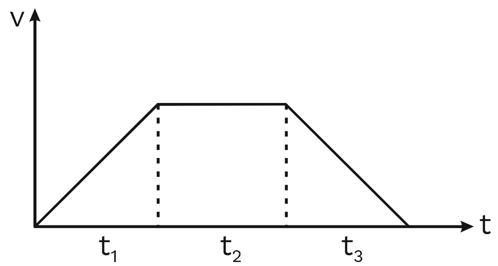
Q71 **Particle start from rest and uniformly acceleration α and attain maximum speed and in same uniform retardation β and finally come to rest (straight line). Draw the graph between v and t**

A71 $a = \frac{dv}{dt}$, $\quad v = \frac{ds}{dt}$

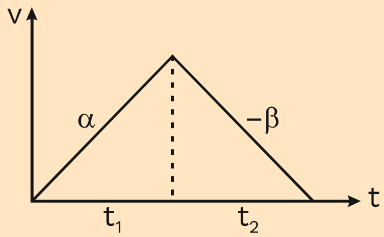


Q72 **Particle start from rest and in same uniform acceleration and maximum speed with this speed moving for some time and finally in some time uniformly retarded and come to rest. Then draw the graph between v and t?**

A72



Q73 **v-t graph are shown in figure. Find total time and d total distance?**



A73

$t = t_1 + t_2$ $\qquad t_1 = \frac{v_{max}}{\alpha}$

$\alpha = \frac{v_f - v_i}{t_1} = \frac{v_{max}}{t_1}$ $\qquad \Rightarrow t_1 = \frac{v_{max}}{\alpha}$

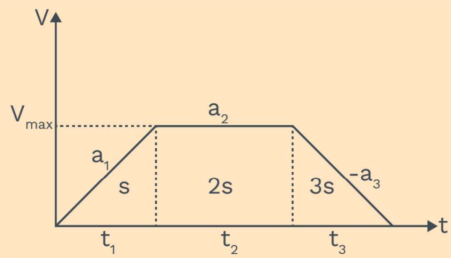
$-\beta = \frac{0 - v_{max}}{t_2}$ $\qquad \Rightarrow t_2 = \frac{v_{max}}{\beta}$

$\Rightarrow \quad t = t_1 + t_2$ $\qquad \Rightarrow t = \frac{v_{max}}{\alpha} + \frac{v_{max}}{\beta}$

$t = \frac{v_{max}(\alpha + \beta)}{\alpha\beta}$, $\qquad \Rightarrow v_{max} = \frac{\alpha\beta t}{\alpha + \beta}$

$s = \frac{1}{2} \times t \times v_{max}$ $\qquad \Rightarrow s = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha + \beta}\right)t^2$

Q74 **Particle start from rest moving along straight line. Find $\frac{v_{avg}}{v_{max}}$ = ?**



A74 $$v_{avg} = \frac{\text{total distance}}{\text{total time}}$$

$$v_{avg} = \frac{s + 2s + 3s}{t_1 + t_2 + t_3} \quad \text{...(i)}$$

$$s = \frac{1}{2} v_{max} t_1 \quad \Rightarrow \quad t_1 = \frac{2s}{v_{max}}$$

$$2s = v_{max} \cdot t_2 \quad \Rightarrow \quad t_2 = \frac{2s}{v_{max}}$$

$$3s = \frac{1}{2} \times v_{max} \cdot t_3 \quad \Rightarrow \quad t_3 = \frac{6s}{v_{max}}$$

From equation (i)

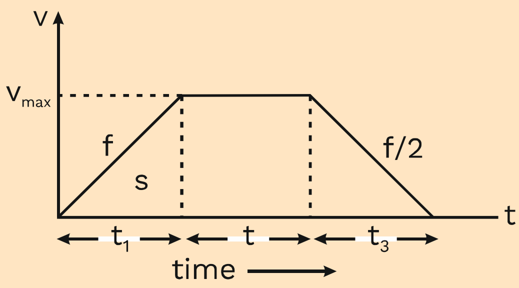
$$v_{avg} = \frac{6s}{\frac{2s}{v_{max}} + \frac{2s}{v_{max}} + \frac{6s}{v_{max}}} \quad \Rightarrow \quad v_{avg} = \frac{v_{max} \times 3}{5}$$

$$\frac{v_{avg}}{v_{max}} = \frac{3}{5}$$

Q75 **Total distance = 15 s then:**

(i) $s = \frac{f t_1^2}{72}$ **(2)** $s = \frac{f t^2}{72}$

(3) $s = \frac{f t_1^2}{2}$ **(4)** $s = \frac{f t^2}{24}$



A75 $v = \frac{ds}{dt} \Rightarrow ds = vdt$

$\Rightarrow s = \int vdt$

$s = \frac{1}{2} \times v_{max} \times t_1$ ...(i)

$\frac{v_{max} - 0}{t_1} = f \Rightarrow v_{max} = f t_1$

$s = \frac{1}{2}(f t_1)t_1 \Rightarrow s = \frac{f t_1^2}{2}$

$s_3 = \frac{1}{2} v_{max} \cdot t_3 \Rightarrow s_3 = \frac{1}{2} f t_1 \cdot t_3$

$-\frac{f}{2} = \frac{0 - v_{max}}{t_3} \Rightarrow t_3 = \frac{2v_{max}}{f}$

$\Rightarrow t_3 = 2t_1$

$s_3 = \frac{1}{2} f t_1 \cdot 2t_1 \Rightarrow s_3 = ft_1^2$

$s_3 = 2s$

$s_1 + s_2 + s_3 = 15\ s$

$s + s_2 + 2s = 15\ s$

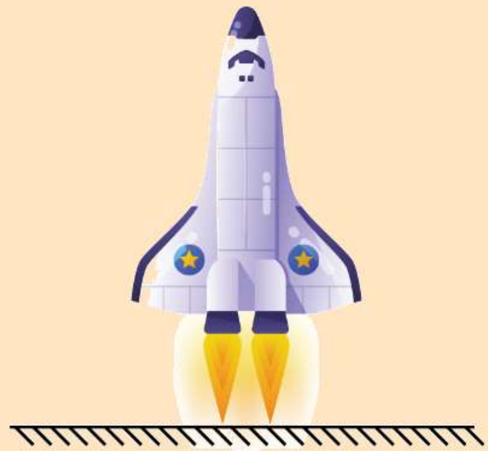
$s_2 = 12s$

$12s = v_{max} \times t \Rightarrow \frac{12 \times ft_1^2}{2} = ft_1 \times t$

$t_1 = \frac{t}{6}$

$\Rightarrow s = \frac{ft^2}{72}$

Q76 **Start from rest, moving with constant acceleration $10 m/s^2$. After t = 30 sec, fuel finished. Then find maximum height from ground?**



A76

$(v_f)_I = (v_i)_I + a_1 t$

$(v_f)_I = 0 + 10 \times 30 = 300 \text{ m/s}$

$$s = ut + \frac{1}{2}at^2$$

$$H_1 = 0 + \frac{1}{2} \times 10(30)^2$$

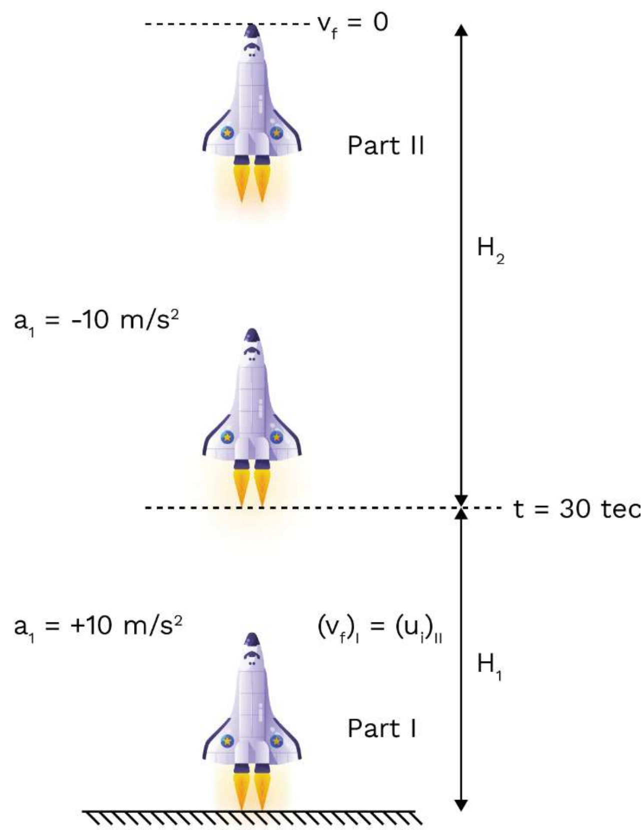
$H_1 = 4500 \text{ m}$

$$H_2 = \frac{u_{II}^2}{2g} = \frac{300 \times 300}{2 \times 10} = 4500 \text{m}$$

$H = H_1 + H_2$

$H = 4500 + 4500$

$H = 9000 \text{ m}$



Q77 **A body starts from rest, with uniform acceleration a. The acceleration of body as function of time t is given by the equation a = pt, where p is constant then the displacement of the particle in the time interval t = 0 to t = t_1 will be.**

(1) $\frac{1}{2}pt_1^3$

(2) $\frac{1}{3}pt_1^2$

(3) $\frac{1}{2}pt_1^2$

(4) $\frac{1}{6}pt_1^3$

A77 $a = \frac{dv}{dt}$

$\int dv = \int a\,dt \quad \Rightarrow \quad v = \int pt\,dt$

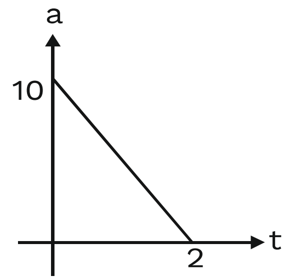
$v = \frac{pt^2}{2} \quad \Rightarrow \quad v = \frac{ds}{dt} = \frac{pt^2}{2}$

$\int \quad = \int — \quad \Rightarrow \quad = -\left(— \right)$

displacement $s = \frac{pt_1^3}{6}$.

EXAMPLES

1. 1-D motion a particle starting with 5 m/s. Find out maximum velocity according to following a-t curve.



Sol. u = +5 m/s

In interval $0 \le +t \le 2 \Rightarrow a = +ve$

So velocity will increase.

So at t = 2 second velocity will be maximum.

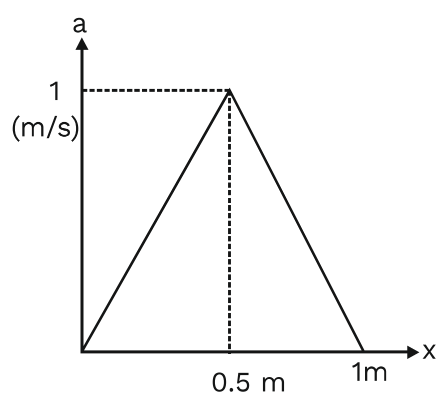
Change in velocity = Area of a-t curve

$$\Delta v = \frac{1}{2} \times 10 \times 2$$

$$v_{max} - 5 = 10$$

$$v_{max} = 15 m/s.$$

2. In the following a-x graph find out maximum velocity if initial velocity is zero.



Sol. $a = \frac{vdx}{dx}$

$$\int_{x_1}^{x_2} a\,dx = \int_{v_1}^{v_2} v\,dv$$

Area of a–x curve = $\frac{v_2^2}{2} - \frac{v_1^2}{2}$

at x = 1, velocity is maximum

Area = $\frac{v_{max}^2}{2} - \frac{v_0^2}{2}$ $\quad \{v_0 = 0 m/s$

$- = ——$, v_{max} = 1 m/s

3. If relation between velocity and time is given as $v^2 = t^2 + 1$, then find out acceleration at t = 0 second.

Sol. Give that $\quad v^2 = t^2 + 1$

$$\frac{d}{dt}(v^2) = \frac{d}{dt}(t^2 + 1)$$

$$2v\frac{dv}{dt} = 2t$$

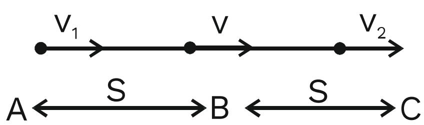
$$\frac{vdv}{dt} = t$$

$$\frac{dv}{dt} = \frac{t}{v}$$

$$\frac{dv}{dt} = \frac{t}{v}$$

4. A particle is moving under constant acceleration. If will change its velocity from v_1 to v_2 in an interval. Find out velocity at mid-point of interval.

Sol.



Let acceleration during motion is a.
Let velocity at mid-point = v
For distance

AB $v^2 = v_1^2 + 2as$ ---(i)

for distance BC $v_2^2 = v^2 + 2as$ --- (ii)

from equation (i) & (ii)

$$v^2 = v_1^2 + 2as$$

$$v_2^2 = v^2 + 2as$$

$$v^2 - v_2^2 = v_1^2 - v^2$$

$$2v^2 = v_1^2 + v_2^2$$

$$v = \sqrt{\frac{v_1^2 + v_2^2}{2}}$$

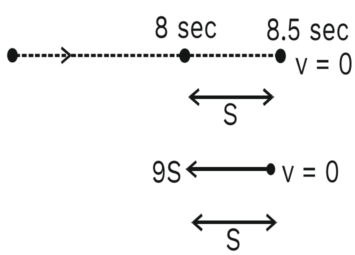
5. During one-dimensional motion a particle having initial velocity 17 m/s under constant retardation 2 m/s^2. Find out distance travelled by it in 9^{th} second.

Sol. Initial velocity = 17 m/s
retardation = 2 m/s^2
According to equation of motion

$v = u + at$

Take v = 0, $0 = 17 - 2t$

$t = 8.5$ second



velocity is at t = 8.50. After it particle return to its path.
Distance in 8.5s to 9s is

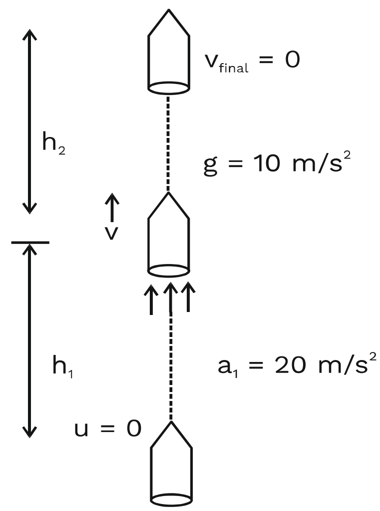
$$S = \frac{1}{2}at^2 = \frac{1}{2} \times 2 \times (0.5)^2 = 0.25$$

Distance in 9^{th} second

= 0.25 + 0.25 = 0.5 m

6. A rocket start rising under constant acceleration 20 m/s^2. At t = 2 second, it's engine will shut down, then find out maximum height achieved by it from ground.

Sol.



$h_1 = ut_1 + \frac{1}{2}at_1^2$, at t = 2 s,

$= 0 \times 2 + \frac{1}{2} \times 20 \times 2^2$

$h_1 = 40$ m

Velocity of rocket at t = 2 second

v = u + at

= 0 + 20 × 2 = 40 m/s

$v_f = 0$

so $v_f^2 = v^2 + 2 \times g \times h_2$, $0^2 = v^2 - 2 \times 10 \times h_2$, $h_2 = \frac{40^2}{20} = 80\text{m}$

Total light $h = h_1 + h_2$

= 40 + 80 = 120 m

7. A ball is released from any height. It take 10 s in 1/3rd distance. Find out time in rest journey.

Sol. Ratio of time to cover same distance during motion under gravity is

$t_1 : t_2 : t_3 ..... = (\sqrt{1} - \sqrt{0}) : (\sqrt{2} - \sqrt{1}) : (\sqrt{3} - \sqrt{2})....$

$\frac{\quad}{\quad + \quad} = \frac{\sqrt{\quad} - \quad}{\sqrt{\quad} - \sqrt{\quad} + \sqrt{\quad} - \sqrt{\quad}}$

$\frac{10\,s}{t} = \frac{1}{\sqrt{3} - 1}$, $t = 10(\sqrt{3} - 1)$ seconds

8. If an object travel 1/3rd of total distance with speed v, next 1/3rd distance with v_2 and remaining distance with speed v_3 then calculate average speed during motion.

Sol. $S_{average} = \frac{\text{total distance}}{\text{total time}}$

$= \frac{x}{\frac{x}{3v_1} + \frac{x}{3v_2} + \frac{x}{3v_3}}$, $S_{average} = \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$

9. A particle is released from a height and falls freely towards the earth. Exactly 1 sec later another particle is released. Find out the distance between the two bodies 2 sec after the release of the second body, if $g = 9.8\ m/s^2$?

Sol. According to given problem 2nd body falls for 2 s, so that

$h_2 = \frac{1}{2}g(2)^2$ ... (1)

while 1st has fallen for 2 + 1 = 3 s; so

$h_1 = \frac{1}{2}g(3)^2$ ... (2)

$\therefore$ Separation between two bodies 2 s after the release of 2nd body,

$$d = h_1 - h_2 = \frac{1}{2}g(3^2 - 2^2) = 4.9 \times 5 = 24.5\,m$$

10. A passenger is standing 'd' m away from a car. The car start to move with constant acceleration a. To catch the car the passenger runs at a constant speed u towards the car. Find out the minimum speed of the passenger so that he may catch the car?

Sol. Let the passenger catch the bust after time t. From 2nd equation of motion, the distance travelled by the bus,

$$s_1 = 0 + \frac{1}{2}at^2 \quad \text{... (i)}$$

and the distance travelled by the passenger

$$s_2 = ut + 0 \quad \text{... (ii)}$$

Now the passenger will catch the car if

$$d + s_1 = s_2 \quad \text{... (iii)}$$

Substituting the values of s_1 and s_2 from eqns. (i) and (ii) in (iii), we get

$$d + \frac{1}{2}at^2 = ut$$

i.e., $$\frac{1}{2}at^2 - ut + d = 0$$

or $$t = \frac{[u + \sqrt{u^2 - 2ad}]}{a}$$

So, the passenger will catch the car if t is real, i.e.,

$u^2 \ge 2ad$ or $\ge \sqrt{\quad}$

So, the minimum speed of passenger for catching the bus is $\sqrt{2ad}$.

11. At the instant tiem the traffic light turns green a bus starts with a constant acceleration 2 m/s^2. At the same instant time a truck, travelling with a speed (constant) of 10 m/s, overtakes and passes the bus. (a) How far (distance) beyond the starting point will the bus overtake the truck? (b) How fast will the bus be travelling at that instant?

Sol. Let the two vehicles meet after time 't'. Then from 2nd motion equation of motion, i.e., $s = ut + \frac{1}{2}at^2$, the distance travelled by bus

$$s_C = \frac{1}{2} \times 2t^2 \quad \text{[as u = 0] .... (1)}$$

And distance travelled by truck

$$s_T = 10 \times t \quad \text{[as a = 0] .... (2)}$$

But according to given problem

$s_C = s_T$, i.e., $t^2 = 10t$

i.e., $t = 0$ or 10 s

So, (a) The distance travelled by the bus in overtaking the truck,

$$s_C = 10^2 = 100\text{m}$$

(b) The speed of bus at t = 10 s, from equation v = u + at, will be

$$v = 0 + 2 \times 10 = 20 \text{ m/s}$$

12. If the initial velocity of a object is u and collinear acceleration at any time is 't', calculate the velocity of the particle after time 't'.

Sol. By definition acceleration = $\left(\frac{dv}{dt}\right)$

So, $\frac{dv}{dt} = at$ (given)

or $\int \quad = \int$

or $v - u = \frac{1}{2}at^2 \Rightarrow v = u + \frac{1}{2}at^2$

13. A object moves along a straight line such that its displacement at any time 't' is given by equation $s = (t^3 - 6t^2 + 3t + 4)$m. What is the velocity of the object when its acceleration is zero?

Sol. As according to given problem,

$s = t^3 - 6t^2 + 3t + 4$

instantaneous velocity

$$v = \frac{ds}{dt} = 3t^2 - 12t + 3 \qquad \text{... (1)}$$

and acceleration

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 6t - 12 \qquad \text{... (2)}$$

So, acceleration will be zero when $6t - 12 = 0$, i.e., t = 2 sec.

And so the velocity when acceleration is zero, i.e., at t = 2 sec from eqn. (1) will be

$v = 3 \times 2^2 - 12 \times 2 + 3 = -9$ m/s

[Negative velocity means that body is moving towards the origin, i.e., as time increases displacement decreases.]

14. A particle starts moving from the position of rest under a constant acceleration. If it travels a distance x in t sec, what distance will it travel in next t sec?

Sol. As acceleration is constant, from 2nd equation of motion, i.e., $s = ut + \frac{1}{2}at^2$ we have

$$x = \frac{1}{2}at^2 \qquad [\text{as } u = 0] \qquad \text{...(1)}$$

Now if it travels a distance y in next t sec., the total distance travelled in (t + t = 2t) sec will be x + y; so

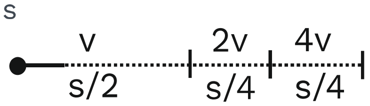
$$x + y = \frac{1}{2}a(2t)^2$$

Dividing Eqn. (2) by (1),

$$\frac{x+y}{x} = 4 \text{ or } y = 3x$$

15. A particle covers half its journey with a constant speed v, half the remaining part of journey with a constant speed of '2v' and the rest of journey with a constant speed of '4v'. Its average speed during the entire journey is.

Sol. Let total distance be s



Average speed $= \dfrac{\text{Total distance}}{\text{Time taken}}$

$$= \frac{s}{\frac{s/2}{v} + \frac{s/4}{2v} + \frac{s/4}{4v}} = \frac{16v}{11}$$

16. A particle moving with velocity equal to 0.4 m/s is subjected to an acceleration of 0.15 m/s^2 for 2 sec in a direction at right angle to its direction of motion. What is the magnitude of resultant velocity?

Sol. In vector form, 1st equation of motion is

$$\vec{v} = \vec{u} + \vec{a}t$$

So, $v = \sqrt{(u^2 + (at)^2 + 2u(at)\cos\theta}$

Here u = 0.4 m/s, a = 0.15 m/s^2, t = 2s and $\theta = 90°$

So, $v = \sqrt{[(0.4)^2 + (0.15 \times 2)^2 + 0]}$

= 0.5 m/s

Mind Map

Motion in a Straight Line

Instantaneous Speed & Velocity

$v = \frac{dx}{dt}$

$\Delta x = \int v\,dt$

Equation of Motion (Constant acceleration)

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s_n = u + \frac{a}{2}(2n-1)$

Acceleration

- Average Acceleration

  $\langle a \rangle = \frac{v - u}{t}$

- Instantaneous Acceleration

  $v = \frac{dv}{dt}$

Average Speed & Velocity

Average Speed $\langle v \rangle = \frac{\text{Total distance}}{\text{Time}}$

If particle travels with speed v_1 and v_2 for

Equal time interval $\langle v \rangle = \frac{v_1 + v_2}{2}$

Equal distance $\langle v \rangle = \frac{2v_1v_2}{v_1 + v_2}$

Average Velocity $= \frac{\text{Total displacement}}{\text{Time}}$

Graphs

Graphs	x - t	v - t	a - t
Slope	Velocity	Acceleration	
Area under curve		Displacement	Change in velocity

Distance & Displacement

- Distance:

  Total path length travel by object

- Displacement ($\Delta \vec{x}$):

  Vector joining initial to final position

  $\Delta x = x_f - x_i$

Motion under Gravity

Projected up with velocity u	Dropped (u=0) from height h
Time of flight $T = \frac{2u}{g}$	Time of flight $T = \sqrt{\frac{2h}{g}}$
Max. height $h = \frac{u^2}{2g}$	Final velocity =
Final velocity = -u	$v = \sqrt{2gh}$

Relative Motion

$\vec{r}_{B,A} = \vec{r}_B - \vec{r}_A$

$\vec{v}_{B,A} = \vec{v}_B - \vec{v}_A$

$\vec{a}_{B,A} = \vec{a}_B - \vec{a}_A$

Non-uniform Acceleration Variable Acceleration

$a = \frac{dv}{dt}$	$a = \frac{v\,dv}{dx}$
↓	$\int_u^v v\,dv = \int_{x_i}^{x_f} a\,dx$
$v - u = \int_{t_i}^{t_f} a\,dt$	
When acceleration is function of t	When a is function of position x