Photo-Electric Effect

DISCLAIMER

Photo-Electric Effect

1. NATURE OF LIGHT

It was a topic of great interest for scientists to know that what exactly light is made up of and how light behaves. This is briefly described over here.

Newton's Corpuscular theory :

Newton was the first scientist who assumed that the light is made up of tiny elastic particles called "corpuscles" which travels with speed of light. So according to his theory, light is a particle.

Huygen's wave theory:

Huygen was a scientist who was working parallel to Newton who came with a thoroughly different idea for the nature of light & said that light is not a particle but a wave.

Maxwell's electromagnetic wave theory :

In the time of Huygen, his views concerning the nature of light were not accepted, as Newton was a very popular scientist of his time. But, when Maxwell stated that light is an electromagnetic wave, scientists started believing that light is a wave.

Concept Reminder

Light has wave character as well as particle character.

Max Planck's quantum theory of light :

Once again when scientists started believing that light is a wave. Max Plank came with different idea & stated that the light is not a wave rather it is a photon (i.e., a particle) which he manifested through black body radiation spectrum. At this point of time, there was a big confusion about the nature of light which was later solved by de-Broglie which lead to origin of theory of matter wave come into picture.

De-Broglie Hypothesis:

It supports dual nature of light (wave nature & particle nature). According to him, light consists of particles which are associated with definite amount of energy and momentum. These particles were later named as photons.

2. PROPERTIES OF PHOTONS

- Photon is a packet of energy emitted from a source of radiation. Photons are carrier particle of electromagnetic interaction.
- Photons travel in straight lines with speed of light $c = 3 \times 10^8$ m/s.
- The energy of photons is given as

$$E = h\upsilon = \frac{hc}{\lambda} = mc^2$$

Electron volt : It is the energy gained which an electron gains when it is accelerated through a potential difference of one volt.

$1 \text{ eV} = 1.6 \times 10^{-19}$ Joule.

Putting all values in above formula of energy

$$E = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{\lambda} \text{ in Joule}$$

$$E = \frac{6.62 \times 10^{-34} \times 3 \times 10^{-8}}{\lambda \times 1.6 \times 10^{-19}} \text{eV}$$

$$E = \frac{12400}{\lambda} \text{eV}$$

where λ is in Å.

Therefore, we get $E = \frac{hc}{\lambda} = \frac{12400}{\lambda} \text{eV} - \text{Å}$

[$\because$ hc = 12400 (Å-eV)]

where ν is frequency, λ is wavelength, h is Planck's constant.

- The effective or motional mass of photon is given as

$$m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{\lambda c}.$$

- The momentum of a photon is given by

$$p = mc = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}.$$

- The photon is a charge less particle of zero rest mass.
- Photons are electrically neutral. They are not deflected by the presence of electric and magnetic fields.

Concept Reminder

J.J. Thomson, using the mass spectrograph, experimentally determined the velocity and specific charge of cathode rays and predicted that the cathode rays are nothing abut the stream of fast-moving electrons.

KEY POINTS

- Photons
- Rest Mass
- Motional Mass

- If E is the energy of source in joule then number of photons emitted is

$$n = \frac{\text{total energy radiated}}{\text{energy of each photon}} = \frac{E}{h\nu} = \frac{E\lambda}{hc}$$

Intensity of photons is defined as amount of energy carried per unit area per unit time or power carried per unit area

$$\text{Intensity}(I_p) = \frac{\text{energy}}{\text{area} \times \text{time}} = \frac{\text{power}}{\text{area}}$$

$$I_p = nh\nu = \frac{N}{4\pi r^2}P$$

where n = number of photons per unit area per unit time

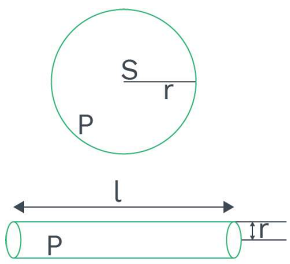
N = number of photons, P = power of source

e.g. (a) For a point source

$$I_p = nh\nu = \frac{N}{4\pi r^2}P$$

(b) For a line source

$$I_p = nh\nu = \frac{N}{2\pi rl}P$$



3. ELECTRON EMISSION

Electrons move randomly inside metals. They can't escape from the metal surface because as they attempt to escape, the metal surface gets positively charged and attracts these electrons back into the metal. Therefore, for escaping/ ejecting of electrons, some minimum additional energy is required. This minimum energy is called work function ϕ_0.

Definitions

- Work function: The minimum amount of energy required to pull out electron from surface of metal is known as work function.

METAL	WORK FUNCTION ϕ_0 (eV)	METAL	WORK FUNCTION ϕ_0 (eV)
Cs	2.14	Al	4.28
K	2.30	Hg	4.49
Na	2.75	Cu	4.65
Ca	3.20	Ag	4.70
MO	4.17	Ni	5.15
Pb	4.25	Pt	5.56

Types of Electron Emission:

(i) Thermionic emission: with the use of suitable heating, sufficient thermal energy can be imparted to the free electrons to enable them to come out of the metal.

(ii) Field emission: On applying a very strong electric field (of the order of 10^8 Vm^{-1}) to a metal, electrons can be pulled out of the metal, as in a spark plug.

(iii) Photo-electric emission: When a light of suitable frequency illuminates a metal surface, electrons are emitted from the metal surface.

4. PHOTOELECTRIC EFFECT

- **Hertz's observations**

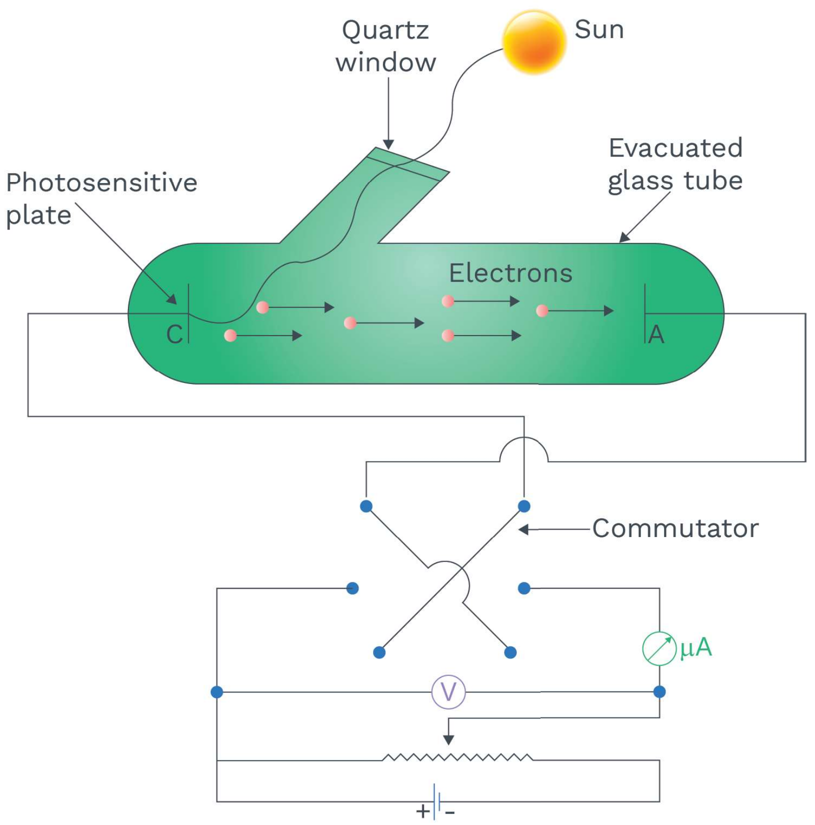
The phenomenon of photoelectric emission was first discovered in 1887 by the scientist Heinrich Hertz (1857–1894), during his experiment on electromagnetic wave. In his experimental investigation on the production of the electromagnetic waves by the means of a spark discharge, Hertz observed that the high voltage sparks across the detector loop were enhanced when the emitter plate was illuminated by an ultraviolet light from an arc lamp.

- **Hallwachs' and Lenard's observations**

During 1886–1902, Wilhelm Hallwachs and Philipp Lenard investigated the phenomenon of photoelectric emission in detail.

KEY POINTS

- Thermionic emission
- Field emission
- Photoelectric emission



Experimental arrangement for study of photoelectric effect

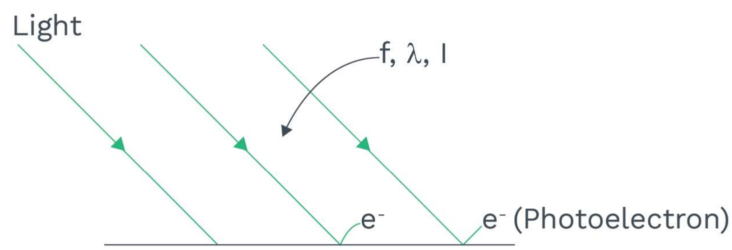
Lenard (1862–1947) observed that when the ultraviolet radiations were allowed to fall on an emitter plate of the evacuated glass tube enclosing two electrodes (metal plates), then the current flows in the circuit. As soon as ultraviolet radiations were stopped, the current flow in the circuit also stopped. These observations indicate that when ultraviolet radiations fall on the emitter plate C, electrons are ejected from it which are attracted towards the positive, collector plate A by the electric field. The electrons flow through the evacuated glass tube, resulting in the current flow. Thus, light falling on the surface of the emitter causes current in the external circuit. Hallwachs and Lenard studied how this photo

Concept Reminder

- The process of emission of free electrons when highly energetic electron beam is incident on a metal surface is called secondary emission.

current varied with the collector plate potential, and with frequency and intensity of incident light.

5. ELECTRON EMISSION PROCESS



When the light is incident on a metal surface it was observed that electrons are ejected from a metal surface even when very dim light such as that from stars and distant galaxies incident on it and sometimes electrons do not come out from the metal surface even when the high energetic or high intensity light is falling on the metal surface.

- This shows, that the electron emission from metal surface does not depends upon the intensity of incident light but basically it depends on the energy of the incident light.
- The number of photons falling are very less in a dim light, still the photoelectric effect can be observed.
- During this phenomenon of photoelectric effect, one incident photon on the metal surface can eject not more than only one electron.
- A photon is a packet of energy which is fully absorbed no partial absorption takes place. Thus one photon cannot be absorbed by more than one electron.

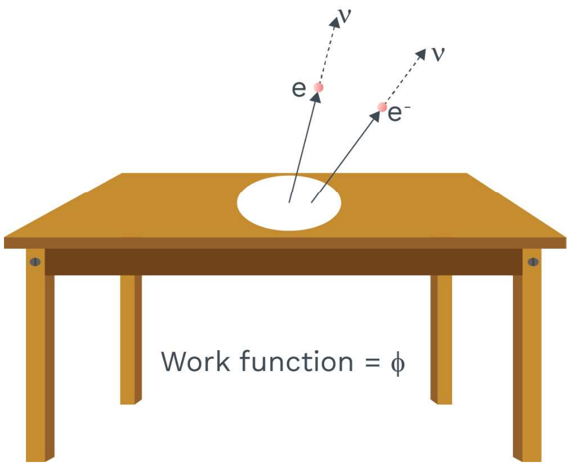
6. SOME DEFINITIONS

Work Function

- The minimum amount of energy or work, which is necessary to take a free electron out of a metal against the attractive forces of surrounding positive ions inside metals is called the work function of the metal.
  $W_0 = h\nu_0$, where ν_0 is the threshold frequency.
- It is also denoted by ϕ.
- Caesium has the lowest work function among all metals.
- The work function of metal surface depends on the nature of the metal.
- The electron emission from a metal depends on the work function or energy of one photon. But number of electrons comes out from the metal depends on intensity of the falling light.
- Energy of photon which is incident on the metal surface, will not necessarily

cause the emission of an electron even if the energy is more than that of work function. The electron after the absorption may be involved in many other processes like collision etc. in which it can lose energy. Hence, the ratio of number of electrons emitted to the number of photons which are incident on metal surface is less than unity.

- An electron can undergo collisions with other electrons, protons or macroscopically with the atom. In this process it will fritter away its energy. Therefore, electrons with Kinetic energy ranging from 0 to $K.E_{max}$ will be produced.



Threshold energy

- The minimum energy of an incident light for which photoelectric effect is possible is called threshold energy.
- If $h\nu_0$ is the threshold energy and ϕ is work function of the material, then

  $h\nu_0 = \phi$

Threshold frequency (ν_0)

- The minimum frequency of photons of incident light for which photoelectric effect is possible is called threshold frequency.

$$\upsilon_0 = \frac{\phi_o}{h}$$

KEY POINTS

- Work function
- Threshold energy
- Threshold frequency
- Threshold wavelength

Threshold wavelength (λ_0)

- The maximum wavelength of an incident light for which photoelectric effect is possible is called the threshold wavelength of the metal.

$$\Rightarrow \lambda_0 = \frac{hc}{\phi} \qquad \Rightarrow \lambda_0 = \frac{12400\,\text{eV} - \text{Å}}{\phi \text{ in eV}}$$

$$\Rightarrow h\upsilon_0 = \phi \qquad \Rightarrow \frac{hc}{\lambda_0} = \phi$$

$$\text{or } \lambda_0 = \frac{c}{\upsilon_0}$$

Observations based on the experiments on Photo-Electric Effect:

(i) The emission of photoelectrons is instantaneous.

(ii) The number of photoelectrons emitted per second is proportional to the intensity of incident light.

(iii) The maximum velocity with which electrons emerge is dependent only on frequency and not on intensity of the incident light.

(iv) There is always a lower limit of frequency called threshold frequency below which no emission takes place, however high the intensity of the incident radiation may be.

7. EINSTEIN'S PHOTO-ELECTRIC EQUATION

- Radiations absorbed by the surface are in the form of quanta (photon). Energy of each photon depends on frequency. One photon can interact with one electron at a time. In the interaction between photon and electron incident photon transfers its whole energy to the electron.
  If energy is sufficient then electron comes out without any time delay. It means photo electric effect is an instantaneous process.
- If intensity of the given source is increased, then number of photons increases. So that, a greater number of electrons are emitted, and

Rack your Brain

Photoelectric emission occurs only when the incident light has more than a certain minimum.
(1) Power
(2) Wavelength
(3) Intensity
(4) Frequency

KEY POINTS

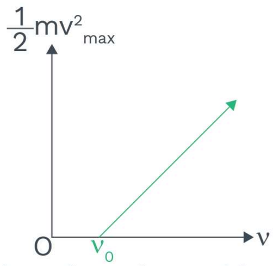
- Einstein's photoelectric equation
- Stopping potential

greater saturation current is obtained. It means saturation current depends upon intensity of the given source.

- At a time, only one photon can interact with one electron.
  Energy of photon used by the electron is
  $h\upsilon$ = Kinetic energy of electron + Energy required to make electron free from the metal surface (ϕ_0) + Energy lost in collision before emission (Q).
  If Q = 0, means there is no heat loss, then kinetic energy of electron is maximum.
  Now $h\upsilon = (K.E_{max}) + \phi_0$
  It is known as Einstein's equation of Photoelectric effect.

$$(K.E_{max}) = h\upsilon - \phi_0$$

Here ν_0 is threshold frequency.
It means maximum K.E. depends on frequency. It is independent of intensity of the given source.



Plot of maximum kinetic energy versus frequency of incident light

- Kinetic energy cannot be negative so that,
  $h\upsilon > \phi_0$

$$h\upsilon > h\nu_0 \left[\text{Here } \phi_0 = h\upsilon_0 = \frac{hc}{\lambda_0},\ \phi_0 = \frac{12400}{\lambda_0} eV - Å\right]$$

$\upsilon > \nu_0$
It means if frequency is less than 'ν_0', electrons will not come out.

- **Stopping Potential:**

Rack your Brain

A source of light is placed at a distance of 50 cm from a photocell and the stopping potential is found to be V_0. If the distance between the light source and photocell is made 25 cm, then what will be the new stopping potential?

This is the value of negative potential difference which just stops the electrons with maximum kinetic energy from reaching the anode. If V_o is the stopping potential, then

$$eV_o = \frac{1}{2}mv^2_{max}$$

7. INTENSITY OF LIGHT

We know $I = \frac{E}{At} = \frac{P}{A}$ ...(i)

SI unit: $\frac{\text{Joule}}{m^2 - s}$ or $\frac{\text{Watt}}{m^2}$

Here P = power of source, A = Area, t = time taken

E = energy incident in t time = Nhυ,

N = number of photon incident in t time

Intensity $I = \frac{N(h\upsilon)}{At} = \frac{n(h\upsilon)}{A}$ ...(ii)

$$\left[\because \; n = \frac{N}{t} = \text{no. of photons per sec.}\right]$$

from equation (i) and (ii),

$$\frac{P}{A} = \frac{n(h\upsilon)}{A}$$

$$\Rightarrow n = \frac{P}{h\upsilon} = \frac{P\lambda}{hc}$$

or $n = (5 \times 10^{24} J^{-1} m^{-1}) P \times \lambda$

Ex. A source S_1 is producing 10^{15} photon/s of wavelength 5000 Å. Another source S_2 is producing 1.02×10^{15} photon/s of wavelength 5100 Å. Calculate ratio of power of S_2 to power of S_1.

Sol. $n = \frac{P\lambda}{hc}$ or $P = \frac{nhc}{\lambda}$

$$\frac{P_2}{P_1} = \frac{n_2}{\lambda_2} \times \frac{\lambda_1}{n_1} = \frac{1}{1}$$

Ex. Intensity of U.V. radiations is 10 Watt/m^2 and its average wavelength is

6000 Å. It is incident on a surface. Calculate number of photons per 2 sec. per 2 m^2.

Sol. $n = \frac{P\lambda}{hc} = (5 \times 10^{24} J^{-1} m^{-1}) IA\lambda \qquad [P = IA]$

$= (5 \times 10^{24}) \times 10 \times 2 \times 6000 \times 10^{-10} = 6 \times 10^{19}$

$N = n \times t = 6 \times 10^{19} \times 2$

$= 1.2 \times 10^{20}$

Ex. In an photo-electric experiment frequency of incident light is doubled and its intensity is tripled. How much time the photon current changes?

Sol. Since , $I = nh\nu$

And given that ,n is $\frac{3}{2}$ times increases

As $\quad i \propto n$

So i will increases by $\frac{3}{2}$ times.

Also, Photoelectric current does not depend upon frequency of incident light. Therefore, doubling the incident frequency will not change it.

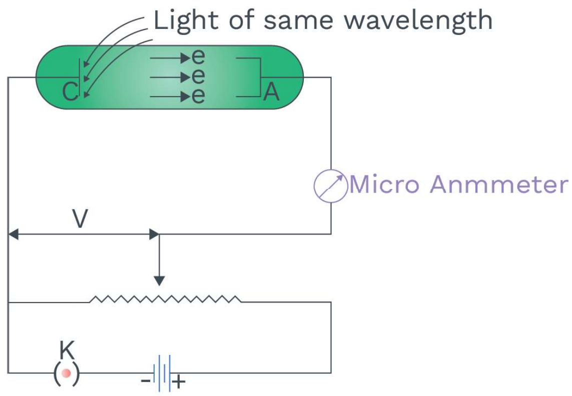
Ex. Energy flux of sunlight which is reaching the Earth's surface is 1.388 × 10^{-3} Wm^{-2}. How many photons (approximately) are incident on the earth per square meter per second? Assuming the photons in the sunlight have an average wavelength of 1550 nm.

Sol. Using , $n = \frac{IA}{E} = \frac{IA\lambda}{hc}$

Put all values to get answer.

8. LENARD'S EXPERIMENTAL STUDY OF PHOTOELECTRIC EFFECT

- The apparatus used for experimental study of photoelectric effect. A metal plate C called cathode (emitter) and a metal cup A called anode (collector) are sealed in a vacuum chamber.
- A beam of monochromatic light enters the window of a vacuum chamber and falls on cathode C. The photoelectrons emitted are collected by the anode A.



Concept Reminder

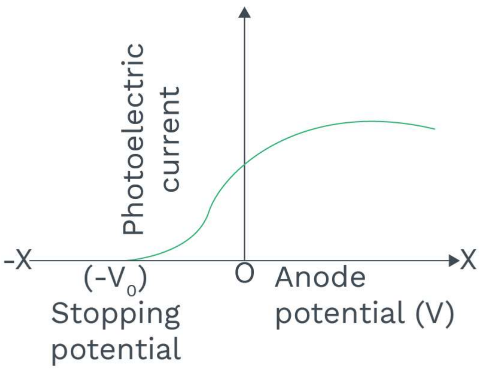
Non-metals also show photoelectric effect. Also liquids and gases show this effect but to a limited extend only.

- When key K is open and monochromatic light is made incident on the cathode, then current is measured by the ammeter. i.e., even though applied voltage is zero, current flows in the circuit.
  These photoelectrons emitted from the cathode C moves towards anode A. But less energetic electrons come to rest before reaching the anode.
  When anode is given positive potential w.r.t the cathode, electrons in the space charge are attracted towards the anode so photocurrent increases. If potential of the anode is increased gradually the effect of space charge becomes negligible at some potential and then every electron that is emitted from the cathode will be able to reach the anode. The current then becomes constant even though voltage is increased and this current is called saturation photocurrent.
- When anode is given negative potential w.r.t the cathode, the photo electrons will be repelled by the anode and some electrons will go back to cathode so current decreases. At some negative potential anode current becomes zero.
- The minimum negative potential (V_0) given to the collector with respect to the emitter for which 'photocurrent' becomes zero is called 'stopping potential'.
- Stopping potential is related to maximum kinetic energy of photoelectrons, because at this potential even the most energetic electron just fails to reach the anode.
  So work done by the stopping potential is equal to the maximum kinetic energy of the electrons.

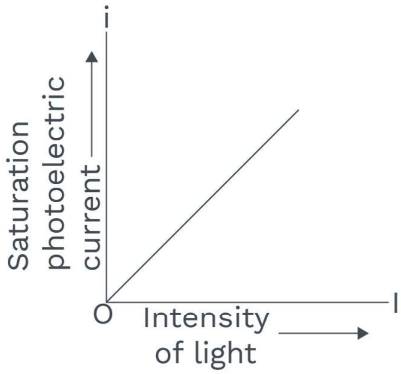
$$(-e)(-V_0) = \frac{1}{2}mv_{max}^2 ;$$

$$\therefore \quad eV_0 = \frac{1}{2}mv_{max}^2$$

A graph is plotted with current on y-axis and applied voltage on x-axis. It is as shown in below graph.



Variation of saturation photo current with Intensity of incident light: Keeping the frequency of incident light and nature of the cathode constant, for different intensities of incident light saturation photo current is measured. Graph is plotted with saturation photocurrent on y-axis and intensity of incident light on x-axis and is shown below.



Observation:

(i) It is observed that saturation photocurrent is proportional to the intensity (I) of incident light at given frequency.

Rack your Brain

When ultraviolet rays incident on metal plate then photoelectric effect does not occur, if will occur by incidence of

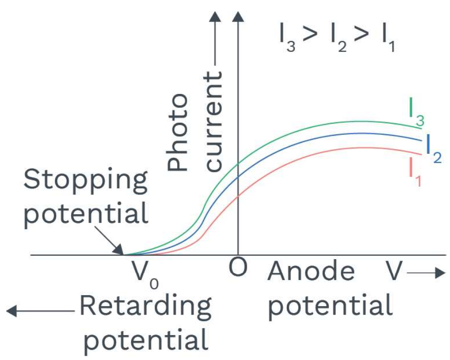
(1) Infrared rays
(2) X-rays
(3) Radio-waves
(4) Micro-waves

Rack your Brain

The number of photoelectrons emitted for light of frequency ν (higher than threshold frequency ν_0) is proportional to

(1) threshold frequency
(2) intensity of light
(3) frequency of light
(4) $\nu - \nu_0$

- **Variation of saturation photo current with Anode potential at different intensities:**
  Keeping the frequency of incident light and nature of the cathode constant, for different intensities of incident light photo current is measured. When a graph is plotted with photocurrent on y-axis and applied voltage on x-axis. It is as shown in figure.



Observations:

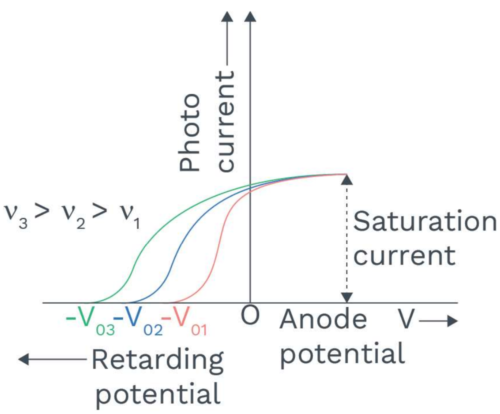
(i) The value of the stopping potential, is independent of the intensity of incident light, if frequency is constant.

(ii) The magnitude of saturation current depends on the intensity of light. Higher the intensity, larger the saturation current.

- **Variation of saturation photo current with Anode potential at different frequencies:**
  Keeping the intensity of incident light and nature of the cathode constant, for different frequencies of incident light, photo current is measured. When a graph is plotted with photocurrent on y-axis and applied voltage on x-axis. It is as shown in figure.

Concept Reminder

Below a certain frequency (threshold frequency) ν_0, characteristic of metal, no photoelectric emission takes place, no matter how large the intensity may be.



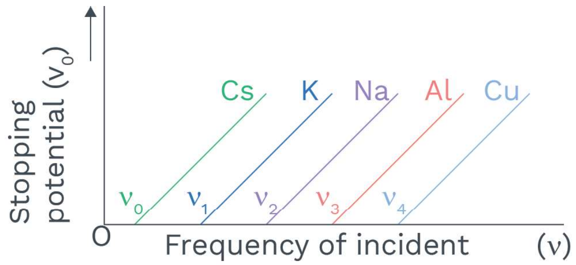
Observations:

(i) Larger the frequency of incident radiation, larger is the stopping potential. So the maximum kinetic energy of the emitted electrons depends on the frequency of incident light and nature of the metal plate. Maximum kinetic energy of photo electrons is independent of the intensity of incident light.

(ii) The saturation photocurrent is independent of the frequency of incident radiation.

- **Variation of the Stopping potential with frequency of incident light:**

When a graph is plotted with stopping potential on y-axis and frequency of incident radiation on x- axis, keeping the metal constant, then it is as shown in figure.



Observations:

(i) Threshold frequency (ν_0) is a characteristic of the metal plate and at this frequency,

KEY POINTS

- Stopping Potential

kinetic energy of the photo electrons is zero. Above threshold frequency, kinetic energy of photo electrons ranges from zero to a maximum value.

(ii) Maximum kinetic energy and Stopping potential increases linearly with increasing frequency as shown in the above figure.

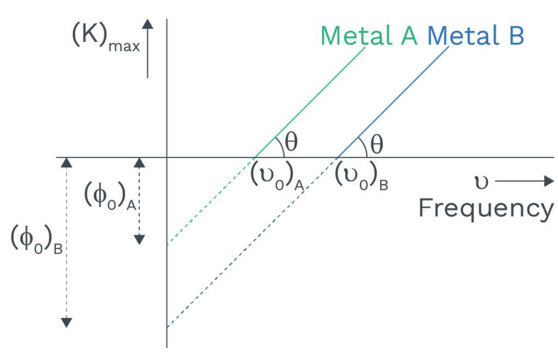
LAWS OF PHOTOELECTRIC EFFECT

- If the frequency of radiation which is being incident is less than a certain value called threshold frequency, electrons are not emitted from a given metal surface, whatever be the intensity of the incident radiation.
- The maximum kinetic energy of the incident photoelectrons depends on the frequency of the incident radiation, but it is independent of the intensity of the radiation. The maximum kinetic energy of photoelectrons is a linear function of the frequency of the incident radiation.
- The saturation photocurrent increases with intensity of incident radiation, but it is independent of the frequency of the incident radiation.
- There is no time lag inbetween the incidence of the incident radiation and the emission of photo electrons.
- **Graph between $(K.E.)_{max}$ and frequency:**

$$(K)_{max} = h\upsilon - \phi_0 \quad [\because \; Y = mx - c]$$

slope = m = tan θ = h (same for all metals)

$$(\phi_0)_B > (\phi_0)_A$$



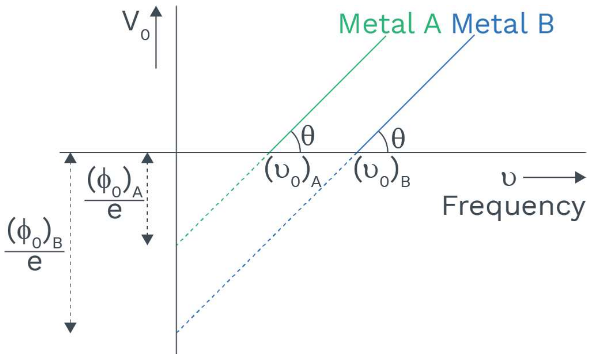
Concept Reminder

- The maximum velocity of an electron emitted by light of wavelength $\lambda(\lambda < \lambda_0)$ incident on surface of metal of work function ϕ_0 is $V = \left[\frac{2(hc - \lambda\phi_0)}{m\lambda}\right]^{1/2}$

- **Graph between stopping potential (V_0) and frequency (υ):**

$\because \quad eV_0 = h\upsilon - \phi_0$

$$V_0 = \left[\frac{h}{e}\right]\upsilon - \left[\frac{\phi_0}{e}\right]$$



$$\text{slope} = m = \tan\theta = \frac{h}{e} \text{ (same for all metals)}$$

Ex. The work function of the caesium metal is 2.14 eV. When the light of frequency 6×10^{14} Hz is incident on the metal surface, photo electrons are emitted. Determine:

a. Maximum kinetic energy of the emitted photo electrons.

b. Stopping potential and

c. Maximum speed of the emitted photoelectron?

Sol. $\phi_0 = 2.14$ eV

$h\upsilon = (4.14 \times 10^{-15} \text{ eVs}) \times 6 \times 10^{14} \text{Hz} = 2.48 \text{ eV}$

(a) $KE_{max} = h\upsilon - \phi = 2.48 - 2.14 = 0.34 \text{ eV}$

(b) $V_0 = \frac{KE_{max}}{e} = \frac{0.34 \text{ eV}}{e} = 0.34 \text{ V}$

(c) As $KE_{max} = \frac{1}{2}mv_{max}^2$

$$v_{max} = \sqrt{\frac{2\,KE_{max}}{m}} = 345.8 \text{ km/s}$$

Ex. In P.E.E. max K.E. of photoelectrons is K_1 and work function of emitter is ϕ. If frequency of incident light is doubled. Then calculate K_{max}.

Sol. $K_1 = h\upsilon - \phi$ ...(i)

$K_2 = 2h\upsilon - \phi$ ...(ii)

Put from eq. (1) into (2)

$$K_2 = 2(K_1 + \phi) - \phi$$

$$= 2K_1 + \phi$$

Note: If frequency is made N times then K_{max} and stopping potential both are greater than N times while maximum speed of e^- is greater than $\sqrt{N}$ times.

Ex. Power of source is 24 W and it is emitting 3×10^{19} ph/sec. This light incident on e^- emitter of work function 3 eV. Calculate stopping potential required?

Sol. $eV_0 = h\upsilon - \phi$

Given $P = nh\nu$

$$h\upsilon = \frac{P}{n} = \frac{24}{3 \times 10^{19}} \times \frac{e}{1.6 \times 10^{-19}} = 5\,eV$$

$$eV_0 = 5\,eV - 3\,eV$$

$\Rightarrow$ $V_0 = 2$ volts

Ex. In P.E.E. if incident wavelength is λ, then stopping potential is $5\,V_0$. If incident wavelength is 3λ then stopping potential is V_0. Then calculate threshold wavelength.

Sol. $5\,eV_0 = \frac{hc}{\lambda} - \phi$ ...(i)

$eV_0 = \frac{hc}{3\lambda} - \phi$ ...(ii)

$$\frac{5}{3}\frac{hc}{\lambda} - 5\phi = \frac{hc}{\lambda} - \phi$$

or $\frac{5}{3}\frac{hc}{\lambda} - \frac{hc}{\lambda} = 4\phi$

or $\frac{2}{3}\frac{hc}{\lambda} = 4\phi$

or $\frac{2}{3}\frac{hc}{\lambda} = 4\left(\frac{hc}{\lambda_0}\right)$

$\Rightarrow$ $\lambda_0 = 6\lambda$.

Ex. In P.E.E. if λ = 4000 Å, then Stopping potential = 1.2 volt. Calculate stopping potential if λ = 3000 Å.

Sol. $K_{max} = 1.2\,eV = \frac{hc}{4000} - \phi = \frac{12400}{4000} - \phi$

$1.2\,eV = 3.1 - \phi$

$\phi = 1.9\,eV$

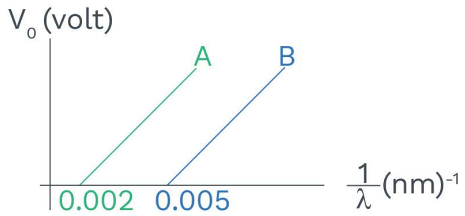
$eV_0' = \frac{12400}{3000} - 1.9$

$eV_0' = 4.13 - 1.9$

$eV_0' = 2.23\,eV$

$\Rightarrow \quad V_0' = 2.23\,V$

Ex. Find-



(a) λ_{0A} and λ_{0B}

(b) ϕ_A and ϕ_B

(c) Which metal can emit e^- with visible light.

(d) Slope of each line.

Sol. (a) $\frac{1}{\lambda_{0A}} = 0.002\,(nm^{-1})$

$\Rightarrow \quad \lambda_{0A} = \frac{1nm}{0.002} = 500\,nm = 5000\,Å$

and $\lambda_{0B} = 0.005 \frac{1000}{0.005} = 200\,nm = 2000\,Å$

(b) $\phi_A = \frac{hc}{\lambda_0} = \frac{12400}{5000}\,eV = 2.48\,eV$

$\phi_B = \frac{12400}{2000} = 6.2\,eV$

(c) Only metal A [4000 Å – 8000 Å]

Rack your Brain

Light of frequency 1.5 times the threshold frequency is incident on a photosensitive material. What will be the photoelectric current if the frequency is halved and intensity is doubled?

(d) $(V_0) = \left(\frac{hc}{e}\right)\frac{1}{\lambda} - \frac{\phi}{e}$

y = mx + c

$$\text{slope} = \tan\theta = \frac{hc}{e} = \frac{12400\,\text{eVÅ}}{e} = 12400\,\text{VÅ}$$

SOME SPECIAL POINTS:

- Intensity $\propto$ photon per second $\propto \frac{1}{d^2}$ (for a given point source)

$$\frac{i_2}{i_1} = \frac{d_1^2}{d_2^2}$$

i = current flowing in the circuit
d = distance between the source of light and electron emitter.

- **Quantum Efficiency:**

Quantum efficiency =

$$\frac{\text{Number of electrons emitted per second}}{\text{Total number of photons incident per second}} = \frac{n_e}{n_{ph}}$$

If quantum efficiency is x% then

$$n_e = \frac{x}{100}n_{ph} \quad [\text{Here } n_{ph} = (5\times10^{24}\,J^{-1}m^{-1})P\lambda]$$

KEY POINTS

- Quantum efficiency
- Photoelectric current

- **Photoelectric Current:**

Photoelectric current

$$I_e = \frac{\text{charge}}{\text{time}} = \frac{Q}{t} = n_e e = 1.6\times10^{-19} n_e$$

Ex. In P.E.E. distance between light source and e^- emitter is 5 m then obtained saturation current and stopping potential are 27 mA and 8 volt respectively. If distance is made 15 m; then calculate i_s and V_0.

Sol. $V_0' = V_0 = 8$ volt [Stopping potential is independent of intensity of light.]

and $i_s \propto \frac{1}{r^2}$

so, $\frac{i_{s_1}}{i_{s_2}} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{15}{5}\right)^2 = 9$

$\Rightarrow \quad \frac{27}{i_{s_2}} = 9 \quad \Rightarrow i_{s_2} = 3 \text{ mA}$

Ex. Wave equation of electric field in the light is given as $E = E_0 \sin [1.57 \times 10^7 \text{ m}^{-1} (x - Ct)]$. This light is incident on a metal of work function 1.9 eV. Calculate stopping potential.

Sol. $E = E_0 \sin [1.57 \times 10^7 \text{ m}^{-1} (x - Ct)]$

$E = E_0 \sin(kx - \omega t)$

$= E_0 \sin\left[k\left(x - \frac{\omega}{k}\right)t\right]; \quad \frac{\omega}{k} = \text{velocity} = C$

$E = E_0 \sin[k(x - Ct)]$

$k = \frac{2\pi}{\lambda} = 1.57 \times 10^7$

$\Rightarrow \quad \lambda = 4 \times 10^{-7} \text{m} = 4000 \text{ Å}$

$eV_0 = (3.1 - 1.9) \text{ eV}$

$eV_0 = 1.2 \text{ eV}$

$\Rightarrow \quad V_0 = 1.2$ volt.

Three Major Features of the Photoelectric effect that cannot be explained by classical wave theory of light:

(a) **The intensity problem:** The wave theory requires, that the oscillating electric field vector (E) of the light wave increases in amplitude as the intensity of the light beam is increased. As the force applied to the electron is eE, this suggests that the kinetic energy of the photo-electrons should also be increased when the light beam is made up more intense. However, the observation shows that the maximum kinetic energy is independent of the light intensity.

(b) **The frequency problem:** According to the wave theory, the photoelectric effect can take place for any frequency of the light, provided only that the light should be intense enough to supply the energy which is required to eject the photoelectrons. However, the observations shows that there exists, for each metal surface, a characteristic cut-off frequency ν_0 such that for frequency less than ν_0 the photoelectric effect does not occur no matter how intense light beam is.

(c) **The time delay problem:** If the energy attained by a photoelectron is absorbed directly from the wave which is incident on the metal plate, then the "effective target area" for an electron in the metal is limited and is probably not much more than that of diameter of a circle roughly equal to that of an atom. In the classical theory, light energy is distributed uniformly over the wave front. Thus, if the light is weak enough, then there will definitely be some measurable time lag, inbetween the impinging of the light on the surface and the ejection of the photoelectron from the surface. Meanwhile, in the interval the electron should be absorbing the energy from the beam until it had accumulated enough energy to escape.

However, there has no detectable time lag ever been measured.

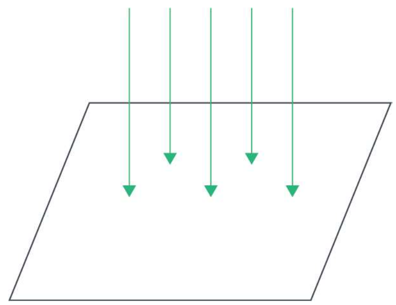
Remember that quantum theory solves all these problems by providing the correct interpretation of the photoelectric effect as discussed earlier in this chapter.

FORCE DUE TO RADIATION (PHOTON)

Every photon has a definite linear momentum and a definite energy. All the photons of light having a particular wavelength λ have the same energy $E = \frac{hc}{\lambda}$ and the same momentum $p = \frac{h}{\lambda}$.

When light of intensity I falls on a surface, it exerts force on that surface. Assume that absorption and reflection coefficient of surface be 'a' and 'r' and assuming that there is no transmission.

Assuming that light beam falls on the surface of surface area 'A' perpendicularly as shown in figure.

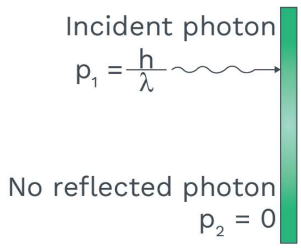


In order to calculate the force exerted by beam on surface, we consider following cases.

Rack your Brain

Monochromatic light of wavelength 667 nm is produced by a helium neon laser. The power emitted is 9 mW. Find the number of photons arriving per second on the average at target irradiated by this beam.

Case-I:



$a = 1, r = 0$

Initial momentum of the photon $= \frac{h}{\lambda}$

Final momentum of photon = 0

Change in momentum of photon $= \Delta P = \frac{h}{\lambda}$

Energy incident per unit time = IA

Number of photons incident per unit time

$$= \frac{IA}{h\upsilon} = \frac{IA\lambda}{hc}$$

∴ Total change in momentum per unit time

$$= n\Delta P = \frac{IA\lambda}{hc} \times \frac{h}{\lambda} = \frac{IA}{c}$$

Force on photons = the total change in momentum per unit time

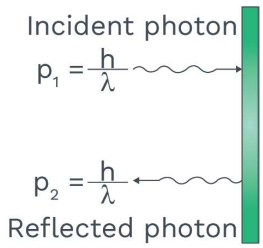
$$= \frac{IA}{c}$$

∴ Force on plate due to photon

$$F = \frac{IA}{c}$$

$$\text{pressure} = \frac{F}{A} = \frac{IA}{cA} = \frac{I}{c}$$

Case-II:



When r = 1, a = 0

Initial momentum of the photon = $\frac{h}{\lambda}$

Final momentum of photon = $\frac{h}{\lambda}$

Change in momentum = $\frac{h}{\lambda} + \frac{h}{\lambda} = \frac{2h}{\lambda}$

∴ Energy incident per unit time = IA

Number of photons incident per unit time = $\frac{IA\lambda}{hc}$

∴ Total change in momentum per unit time

$$= n \cdot \Delta P = \frac{IA\lambda}{hc} \cdot \frac{2h}{\lambda} = \frac{2IA}{c}$$

Force = total change in momentum per unit time

$$F = \frac{2IA}{c}$$

Pressure

$$P = \frac{F}{A} = \frac{2IA}{cA} = \frac{2I}{c}$$

Case-III:

When, 0 < r < 1, a + r = 1

Change in the momentum of photon when it is reflected = $\frac{2h}{\lambda}$

Change in the momentum of photon when it is absorbed = $\frac{h}{\lambda}$

No. of photons incident per unit time = $\frac{IA\lambda}{hc}$

No. of photons reflected per unit time = $\frac{IA\lambda}{hc} \cdot r$

No. of photon absorbed per unit time = $\frac{IA\lambda}{hc}(1-r)$

Force due to absorbed photon

$$(F_a) = \frac{IA\lambda}{hc}(1-r) \cdot \frac{h}{\lambda} = \frac{IA}{c}(1-r)$$

Force due to reflected photon

$$(F_r) = \frac{IA\lambda}{hc} \cdot r \frac{2h}{\lambda} = \frac{2IAr}{c}$$

Total force = $F_a + F_r$

$$= \frac{IA}{c}(1-r) + \frac{2IAr}{c} = \frac{IA}{c}(1+r)$$

Now pressure $P = \frac{IA}{c}(1+r) \times \frac{1}{A} = \frac{I}{c}(1+r)$

PHOTO CELL

A photo cell is a practical application of the phenomenon of photo electric effect. It converts light energy into electrical energy.

Construction:

A photo cell consists of an evacuated sealed glass tube containing anode and a concave cathode of suitable emitting material such as Caesium (Cs).

Working:

When the light of frequency greater than that of threshold frequency of cathode material falls on the cathode, emitted photoelectrons are then collected by the anode and an electric current starts to flow in the external circuit. With the increase in the intensity of light, the current also increases. The current would thus stop, if the light does not fall on the cathode surface.

Application:

(i) In television camera.

(ii) In automatic door

(iii) Burglar's alarm

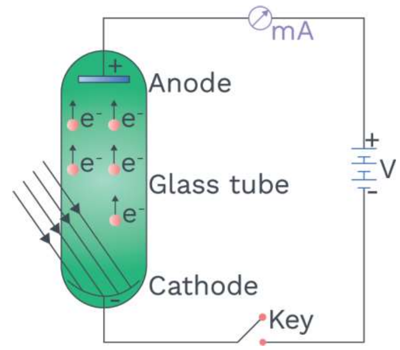
(iv) Automatic switching of street light and traffic signals.

DUAL NATURE OF MATTER-(De-BROGLIE HYPOTHESIS)

- Photoelectric effect and Compton effect proves that radiation behaves like particles (photons), where as Interference and Diffraction proves that radiation behaves like waves.

Concept Reminder

- The current produced in vacuum type photocell is very small (of the order of A). To increase the current, the cell is filled with suitable inert gas.



KEY POINTS

- Photocell
- de-Broglie hypothesis

So 'radiation has dual nature' i.e., radiation behaves like particles when interacting with matter and radiation behaves like waves when propagating in a medium.

De-Broglie hypothesis:

(a) The universe consists of matter and radiation only.

(b) Nature loves symmetry

(c) If radiation has dual nature then matter should also have dual nature.

- According to de Broglie particles like electron, proton & neutron, shows the property of both wave and particle properties. The waves which are associated with the moving particle are called matter waves and the wavelength is called the De-Broglie wavelength of a particle.

  For a photon energy, $E = \frac{hc}{\lambda} = mc^2$

  Where m = effective mass

  then wavelength

  $$\lambda = \frac{h}{mc} = \frac{h}{p}$$

  Where p = momentum of the photon.

- De-Broglie extended this for particles also. So if a particle of mass 'm' is moving with velocity 'v' then its momentum

  p = mv, hence de Broglie wavelength of the matter wave associated with is given by

  $$\lambda = \frac{h}{p} = \frac{h}{mv}$$

- Davisson and Germer studied the scattering of electrons by a nickel target. The wavelength of diffracted electrons was determined by both the scientist Davisson & Germer. The experimental values of wavelength λ were found in order to agree with the theoretical value $\lambda = \frac{h}{mv}$.

  Hence it is concluded that electrons behaves like waves and undergo diffraction.

- For definite sized objects like a car the corresponding wavelength is very small to detect the wave properties. But the de-Broglie wavelength of the electron is large enough to be observed. Because of their small mass, electrons have a small momentum and hence large wavelength

  $$\lambda = \frac{h}{p}.$$

Formula for de-Broglie wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mK}}$$

Where:

λ = de-Broglie wavelength associated with the moving particles.

h = Planck's constant

= 6.63×10^{-34} J-s = 4.132×10^{-15} eV-s

p = Linear momentum = mv

K = kinetic energy = $\frac{1}{2}mv^2 = \frac{p^2}{2m}$

$\Rightarrow\ p^2 = 2mK$

$\Rightarrow\ p = \sqrt{2mK}$

de-Broglie wavelength for a charged particle:

Consider a charged particle of mass 'm' and charge 'q' accelerated from rest through a potential V, then its

(a) Kinetic energy K = qV

(b) Momentum p = $\sqrt{2mK} = \sqrt{2mqV}$

(c) Velocity $v = \sqrt{\frac{2qV}{m}}$ $\left[\because K = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}}\right]$

(d) de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

Examples of different charged particles:

1. Electron ($_{-1}e^0$)

$m_e = 9.1 \times 10^{-31}$ kg

$q_e = e = 1.6 \times 10^{-19}$ C

$$\boxed{\lambda_e = \frac{12.27}{\sqrt{V}} Å\sqrt{volt} = \sqrt{\frac{150}{V}} Å\sqrt{volt}}$$

2. Proton ($_1p^1$)

$m_p = 1.67 \times 10^{-27}$ kg

Concept Reminder

- If the energy of photon is $\frac{2\lambda_p mc}{h}$ times the kinetic energy of the electron then the wavelength of photon λ_p and the de-Broglie wavelength of an electron will have same value.

$q_p = e = 1.6 \times 10^{-19} C$

$$\boxed{\lambda_p = \frac{0.2863}{\sqrt{V}} Å\sqrt{volt}}$$

3. Deutron ($_1H^2$ or d)

$m_d = m_p + m_n \approx 2m_p$

$q_d = q_p + q_n = e + 0 = e$

$$\boxed{\lambda_d = \frac{0.2021}{\sqrt{V}} Å\sqrt{volt}}$$

4. Alpha particle (α or $_2He^4$)

$m_\alpha = 2m_p + 2m_n \approx 4m_p$

$q_\alpha = 2q_p + 2q_n = 2e + 0 = 2e$

$$\boxed{\lambda_\alpha = \frac{0.1012}{\sqrt{V}} Å\sqrt{volt}}$$

de-Broglie wavelength of neutral particles:

1. Neutron ($_0n^1$)

$$\boxed{\lambda_n = \frac{h}{p_n} = \frac{h}{m_n v_n} = \frac{h}{\sqrt{2m_n K}}}$$

Here K must be in joule (J)

$$\boxed{\lambda_n = \frac{0.2863}{\sqrt{K}} Å\sqrt{eV}}$$

Here K must be in in eV

Special Note: For thermal neutrons, average kinetic energy is given by $\frac{3}{2}kT$

$$\therefore \lambda_n = \frac{h}{\sqrt{2m_n\left(\frac{3}{2}kT\right)}}$$

$$= \frac{h}{\sqrt{3m_n kT}} \approx \frac{25.6}{\sqrt{T}} Å\sqrt{K}$$ (temperature is in Kelvin)

Rack your Brain

An electron of mass m with an initial velocity $\vec{v} = v_0\hat{i}\,(v_0 > 0)$ enters an electric field $\vec{E} = -E_0\hat{i}$ $(E_0 = \text{constant} > 0)$ at t = 0. If λ_0 is its de-Broglie wavelength initially, then find out its de-Broglie wavelength at time t.

2. **Gas molecule:** Total kinetic energy of any gas molecule is $\frac{f}{2}kT$ where; f = degree of freedom.

But total translatory kinetic energy of any gas molecule is $\frac{3}{2}kT$. Because de-Broglie wavelength is associated with translational motion only. Therefore, for any gas molecule

$$\lambda = \frac{h}{\sqrt{2m\left(\frac{3}{2}kT\right)}}$$

$$= \frac{h}{\sqrt{3mkT}} \text{(here T is in Kelvin scale)}$$

Ex. Find the ratio of de-Broglie wavelengths for a proton and an alpha particle, if both have same speed.

Sol. $\lambda = \frac{h}{mv} \Rightarrow \lambda \propto \frac{1}{m}$

$$\Rightarrow \frac{\lambda_p}{\lambda_\alpha} = \frac{m_\alpha}{m_p} = \frac{4m_p}{m_p} = 4 : 1$$

Ex. Calculate the ratio of de-Broglie wavelengths for a proton and an alpha particle, accelerated with same potentials from rest.

Sol. $\lambda = \frac{h}{\sqrt{2mqV}} \Rightarrow \lambda \propto \frac{1}{\sqrt{mq}}$

$$\Rightarrow \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{4m_p}{m_p} \times \frac{2e}{e}} = 2\sqrt{2} : 1$$

Ex. If de-Broglie wavelength of a moving electron is decreased from 1 Å to 0.5 Å, then calculate change in its K.E. in eV.

Sol. $\lambda_1 = \sqrt{\frac{150}{V_1}}\text{ Å}\sqrt{\text{volt}} = 1\text{Å}$

$$\Rightarrow V_1 = 150 \text{ volt}$$

Rack your Brain

An electron is accelerated from rest through a potential difference of V volts. If the de-Broglie wavelength of electron is 1.227×10^{-2} nm, then find out value of V.

$\Rightarrow$ $K_1 = 150$ eV

$$\lambda_2 = \sqrt{\frac{150}{V}}\,\text{Å}\sqrt{\text{volt}} = 0.5\,\text{Å}$$

$\Rightarrow$ $V_1 = 600$ volt

$\Rightarrow$ $K_2 = 600$ eV

$\therefore$ Change in kinetic energy

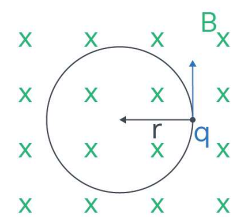
$$\Delta K = K_2 - K_1 = 600 - 150 = 450\,\text{eV}$$

Ex. A particle having charge q is revolving in a circle of radius r in an uniform transverse magnetic field of strength B. Calculate its de-Broglie wavelength.

Sol. For circular motion,

$F_c = F_m$

$$\frac{mv^2}{r} = qvB\sin 90°$$



$\Rightarrow$ $mv = rqB$

$$\therefore \quad \lambda = \frac{h}{mv} = \frac{h}{rqB}$$

Ex. Compute the typical de-Broglie wavelength of an electron in a metal at 27°C and also compare it with the mean separation between two electrons in a metal, which is given to be about 2×10^{-10} m.

Sol. $$\lambda = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{3(9.1\times 10^{-31}) \times (1.38 \times 10^{-23}) \times (273 + 27)}}$$

$= 6.2 \times 10^{-9}$ m

As mean separation between two electrons

$r = 2 \times 10^{-10}$ m

$$\therefore \quad \frac{r}{\lambda} = \frac{2 \times 10^{-10}\,\text{m}}{6.2 \times 10^{-9}\,\text{m}} \approx 0.03$$

Ex. The strength of magnetic field required to bend photoelectrons of maximum energy in a circle of radius 50 cm when light of wavelength 3300 Å is incident on a barium emitter is 6.7×10^{-6} T. What value of charge on the photoelectrons is obtained from this data? (Given: Work function of barium = 2.5 eV; mass of the electron = 9×10^{-31} kg)

Sol. Maximum KE of photoelectron $\frac{1}{2}mv_{max}^2 = \frac{hc}{\lambda} - \phi$

$$\Rightarrow \quad v_{max} = \sqrt{\frac{2}{m}\left(\frac{hc}{\lambda} - \phi\right)}$$

$$= \sqrt{\frac{2}{9 \times 10^{-31}}\left(\frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{3300 \times 10^{-10}} - 2.5 \times 1.6 \times 10^{-19}\right)}$$

$$= \sqrt{\frac{4}{9} \times 10^{12}} = \frac{2}{3} \times 10^{6}\, ms^{-1}$$

Now $Bev_{max} = \frac{Mv_{max}^2}{R_{max}}$

$$\Rightarrow \quad e = \frac{Mv_{max}}{BR_{max}} = \frac{9 \times 10^{-31} \times \frac{2}{3} \times 10^{6}}{6.7 \times 10^{-6} \times 0.5} = 1.8 \times 10^{-19}\, C$$

SPECIAL POINTS:

1. In de-Broglie wavelength $\lambda = \frac{h}{p}$.

   λ is a scalar while p is vector, therefore it is concluded that: For two different particles

   (i) Momentum same then λ must be same.

   (ii) λ same then momentum may be same.

2. de-Broglie wavelength of microscopic particles can be observed in specific experiments but de-Broglie wavelength of macroscopic objects can never be observed due to extremely low λ. (order 10^{-34} m)

3. de-Broglie wavelength of the particle moving with very high speed (v in the order of C)

   $$\lambda \neq \frac{h}{m_0 v}$$

   it is $\lambda = \frac{h}{mv} = \frac{h}{m_0 v}\sqrt{1 - \frac{v^2}{c^2}}$

   Where m_0 = rest mass and

   m = moving or effective mass of the particle.

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}.$$

4. Total energy of a particle moving relativistically is given as $E = \sqrt{m_0^2c^4 + p^2c^2}$

Ex. (a) Evaluate the speed with which electrons emitted from a heated cathode of an evacuated tube impinge on the anode maintained at a potential difference of 500 V with respect to the cathode. Ignore the small speed of the electrons. The specific charge of electron, i.e., its e/m ratio is given to be 1.76×10^{11} C/kg.

(b) Use the same formula you implemented in (a) to obtain electron speed for an anode potential 10 MV. Do you see what is wrong? In what ways is the formula to be modified?

Sol. (a) $\frac{1}{2}mv^2 = eV$

$$v = \sqrt{2\left(\frac{e}{m}\right)V} = \sqrt{2 \times 1.76 \times 10^{11} \times 500}$$

$= 1.33 \times 10^7\ ms^{-1}$

(b) If V = 10 MV = 10×10^6 V

$$v = \sqrt{2 \times 1.76 \times 10^{11} \times 10^7}$$

$v = 1.88 \times 10^9 ms^{-1} > c$ (speed of light)

This is not possible $\Rightarrow$ above method is not applicable

When v is comparable to c

$$E_K = mc^2 - m_0c^2$$

$$eV = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2$$

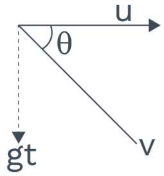
or $$\frac{eV}{m_0c^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1$$

$$\frac{1.76 \times 10^{11} \times 10^7}{(3 \times 10^8)^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1$$

$\Rightarrow$ $v \approx 0.99c$

Ex. A particle of mass 'm' projected horizontally with velocity u. If it makes an angle, with the horizontal after some time, then at that instant, its de Broglie wavelength is.

Sol. For a projectile, the horizontal component of velocity is constant



$\therefore \quad v_x = u_x; \quad v\cos\theta = u$

$\therefore$ de-Broglie wavelength

$$= \frac{h}{mv} = \frac{h\cos\theta}{mu}$$

Ex. Electrons are accelerated through a potential difference of 150 V. Calculate the de Broglie wavelength.

Sol. V = 150 V; h = 6.62×10^{-34} Js,

m = 9.1×10^{-31} kg, e = 1.6×10^{-19}C

$$\therefore \quad \frac{h}{\sqrt{2\,Vem}} = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 150}} = 1\text{Å}$$

Ex. Find the ratio of de Broglie wavelength of molecules of hydrogen and helium which are at temperatures 27°C and 127°C respectively.

Sol. Since,

$$\lambda = \frac{h}{m\upsilon} = \frac{h}{\sqrt{3\,mkT}}$$

$$\frac{\lambda_H}{\lambda_{He}} = \sqrt{\frac{m_{He}T_{He}}{m_H T_H}} = \sqrt{\frac{8}{3}}$$

Ex. With what velocity must an electron travel so that its momentum is equal to that of a photon with a wavelength of 5000 Å.

(h = 6.6×10^{-34} Js, 9.1×10^{-31} kg)

Sol. $mv = \frac{h}{\lambda}$

$$\Rightarrow \quad v = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 5000 \times 10^{-10}} = 1450 \text{ m/s}$$

HEISENBERG UNCERTAINITY PRINCIPLE

- The matter-wave picture effectively incorporated the Heisenberg's uncertainty principle. According to the principle, it is not possible to measure both, the position and momentum of an electron (or any particle) at exactly same time. There is always some uncertainty in the specification of position (Δx) and some uncertainity in the specification of momentum

(Δp). The product of Δx & Δp is of the order of $\hbar$

$$\left(\hbar = \frac{h}{2\pi}\right)$$

i.e., $\Delta x \Delta p = \hbar$.

- Equation allows the possibility that Δx is zero, but then Δp must be infinite in order that the product is nonzero. Likewise, if Δp is zero, Δx have to be infinite. Generally, both Δx and Δp are non-zero such that their product is of the order of $\hbar$.
- Now, if an electron has a momentum p, (i.e., Δp = 0), by the de Broglie relation, it has a wavelength 'λ'. A wave of definite (single) wavelength extends all over the space. By the Born's probability interpretation this means that the electron is not localized in any finite region of space. That is, its position uncertainty is infinite $(\Delta x \to \infty)$, which is compatible with the uncertainty principle.
- In general, the matter wave which is associated with the electron is not extended all over the space. It is a wave packet extends over some finite region of the space. In that case, Δx is not infinite rather it has some finite value depending upon the extension of the wave packet. Also, one must appreciate that a wave packet of finite extension does not have a single wavelength. It is built up wavelengths which are spread around the central wavelength.
- By de Broglie's relation, the momentum of the electron will also have a spread with uncertainty Δp. This is an expected from the uncertainty principle. It can be convey that the wave packet description together with de Broglie relation and Born's probability interpretation reproduce the Heisenberg's uncertainty principle exactly.
- The de Broglie relation will be seen to justifying Bohr's postulate on quantization of angular momentum of electron in an atom.

KEY POINTS

- Matter wave
- Heisenberg's uncertainty principle

Figure shows a schematic diagram of (a) a localised wave packet, and (b) an extended wave with fixed wavelength.

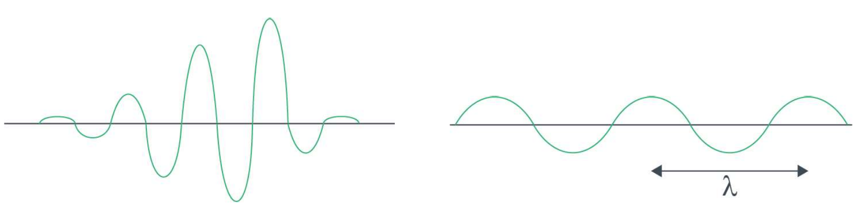


Figure (a) the wave packet description of an electron. The wave packet corresponds to the spread of wavelength around some central wavelength (and hence by de Broglie relation, a spread in momentum). Consequently, it is associated with an uncertainty in position (Δx) and an uncertainty in momentum (Δp). (b) the matter wave corresponding to a definite momentum of an electron extends all over space. In this case, $\Delta p = 0$ and $\Delta x \to \infty$.

Ex. If the uncertainty in the position of the proton is 6×10^8 m , then the minimum uncertainty in its speed is.

Sol. $\Delta p = m\Delta v = \dfrac{\hbar}{\Delta x}$

or $\Delta v = \dfrac{\hbar}{m\Delta x} = \dfrac{1.034 \times 10^{-34}}{1.67 \times 10^{-27} \times 6 \times 10^{-8}} = 1\ ms^{-1}$

Ex. The correctness of velocity of an electron which is moving with velocity 50 ms^{-1} is 0.005%. The accuracy with which its position can be measured will be.

Sol. Here,

$$\Delta v = \frac{0.005 \times 50}{100} = 0.0025\ ms^{-1}$$

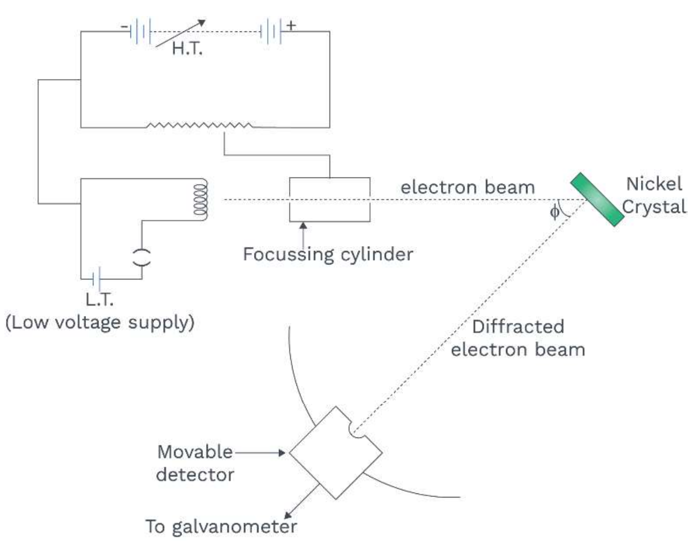
$$\Delta x = \frac{\hbar}{m\Delta v} = \frac{1.034 \times 10^{-34}}{9.1 \times 10^{-31} \times 0.0025} = 4634 \times 10^{-5}\ m$$

Davisson Germer Experiment

1. This experiment verifies the concept of matter waves.

2. In this experiment a sharp accelerated electron beam is diffracted by the nickel crystal.

3. Experimental set-up



Rack your Brain

In the Davisson and Germer experiment, the velocity of electrons emitted from electron gun can be increased by

(1) Increasing the potential difference between the anode and filament.

(2) Increasing the filament current

(3) Decreasing the filament current.

(4) Decreasing the potential difference between anode and filament.

4. Basic elements used in the experimental set-up:-

(i) Electron gun: It provides a sharp beam of e^-s of different K.E., which is given by

$$KE = eV_{acc}$$

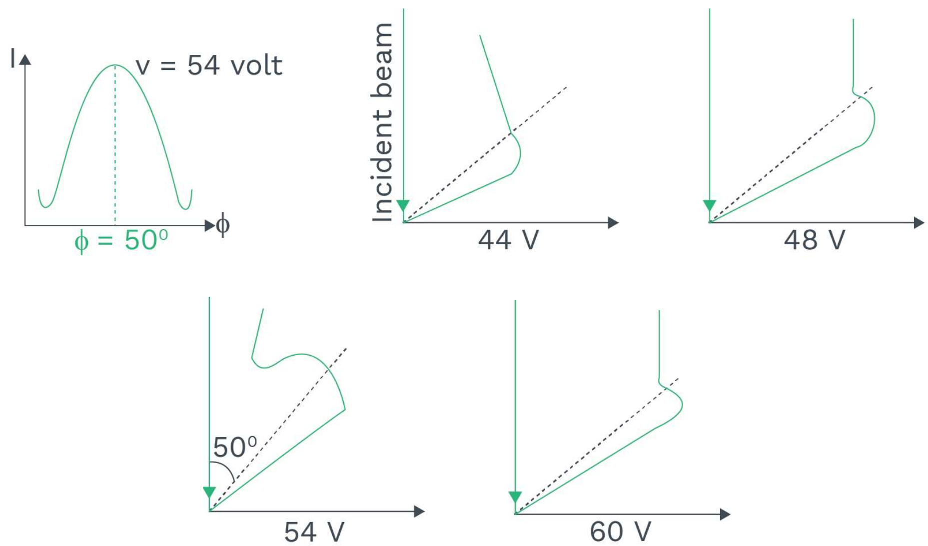
Electron gun consists a tungsten filament F, coated with barium oxide. By thermionic emission, electrons are emitted by filament and accelerated to a desired velocity by applying high voltage power supply. They are made to pass through a cylinder with the fine holes along its axis, producing a fine collimated beam.

(ii) Ni crystal: It is used to diffract/scatter electrons in all direction.

(iii) Detector: It is used to measure intensity of scattered electron beam in given direction.

The detector can be freely moved on a circular scale & it is connected to a sensitive galvanometer whose deflection is proportional to the intensity of the electron beam entering the collector.

5. Observations: On varying the accelerating potential from 44 V to 68 V, it was noticed, that a strong peak appeared in the intensity (I) of the scattered electron for an accelerating voltage of 54 V at a scattering angle $\phi = 50°$.

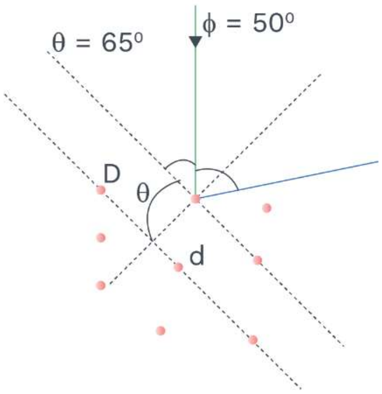


6. Calculations and result: The appearance of the peak in a particular direction is due to the constructive interference and it is called diffraction maxima. This indicates that moving electrons are behaving like wave. Hence wavelength of these can be calculated by following two methods:

Considering moving e⁻s as particle or considering moving e⁻s as wave
Using de-Broglie formula (particle nature)

$$\lambda_e = \frac{12.27}{\sqrt{V}} = \frac{12.27}{\sqrt{54}}$$

$$\lambda_e = 1.67\,Å$$



Using Bragg's equation (wave nature)

$$2d\sin\theta = n\lambda$$

Where,

$$\theta = \frac{180 - \phi}{2} = 65°$$

and d = 0.91 Å for Ni

$n = 1$

and $\lambda_e = 1.65\,Å$

Both calculated and experimental wavelengths are equal hence verifies the concept of matter waves (moving particles shows wave character).

Ex. Calculate de-Broglie wavelength of an e^- revolving in first excited state in hydrogen atom.

Sol. Given n = 2 (first excited state)

Z = 1 (for hydrogen)

$$2\pi r_2 = 2\lambda$$

$$\Rightarrow \quad \lambda = \pi r_2 = \pi(0.53\,Å)\frac{(2)^2}{(1)}$$

$$\lambda = 2.116\,Å$$

Ex. In third orbit of hydrogen atom wavelength of revolving electron is λ. Calculate the circumference of third Bohr orbit.

Sol. $2\pi r_3 = 3\lambda$

Circumference of 3rd Bohr orbit = 3λ

Ex. Estimating the following two numbers will be interesting. The first number tells why radio engineers don't need to worry much about the photons. The second number tells you that, why our eye cannot 'count photons' even in the barely detectable light.

(a) The number of photons which are emitted per second by a 10 kW power transmitter emitting radio waves of the wavelength 500m.

(b) The number of photons which are entering the pupil of eye per second corresponding to that of the minimum intensity of white light that we humans can perceive ($\sim 10^{-10}\ Wm^{-2}$). Take the area of the pupil to be about 0.4 cm^2 and the average frequency of white light to be about 6×10^{14} Hz.

Sol. (a) $n = \frac{P}{E} = \frac{P\lambda}{hc}$

$n = 2.51 \times 10^{31}$

and energy of each photon

$$E = \frac{P}{n} = \frac{10^4}{2.51 \times 10^{31}} = 3.98 \times 10^{-28} J$$

Because the number of photons emitted by the transmitter is very very large and the energy of each photon is very small, thus radio engineers make no error, when they overlook the existence of the photon (due to its very small energy) and instead consider the radio wave as continuous (due to large no. of photons)

(b) $n = \frac{IA\lambda}{hc} \simeq 10^4$ photon per second

Thus in one second 10,000 photons enter the eye. Since the persistence of vision is $\left(\frac{1}{16}\right)^{th}$ of a second, these photons cannot be counted by the eye.

Ex. While working with light and X-rays, there is a useful relation between the energy of a photon in electron volts (eV) and the wavelength of the photon in angstrom (Å). Suppose the wavelength of a photon is λ Å. Then energy of the photon is.

Sol. $E = h\upsilon = \frac{hc}{\lambda}$

Here wavelength =

$\lambda \times 10^{-10} m$; $h = 6.62 \times 10^{-34}$ Js,

$c = 3 \times 10^8\ ms^{-1}$

$$\therefore \quad E = \frac{(6.62 \times 10^{-34}) \times (3 \times 10^8)}{\lambda \times 10^{-10}}$$

$$= \frac{(6.62 \times 10^{-34}) \times (3 \times 10^8)}{(\lambda \times 10^{-10}) \times (1.6 \times 10^{-19})} eV = \frac{12400}{\lambda} eV$$

$$\therefore \quad E = \frac{12400}{\lambda} eV$$

Note: (λ is taken in Å and 12400 in Å eV)

Ex. If wavelength of radiation is 4000 Å , then the energy of the photon is.

Sol. $E = \frac{hc}{\lambda} = \frac{12400 \, eV \, Å}{4000 \, Å}$

$$= \frac{1240 \, eV \, nm}{400 \, nm} = 3.1 \text{ eV}$$

Ex. A monochromatic source of light which is operating at 200 W emits 4 × 10^{20} photons per second. Find the wavelength of the light.

Sol. Power (P) = $\frac{N}{t} h\upsilon$

Energy of photon

$$E = \frac{P}{\left(\frac{N}{t}\right)} = \frac{200}{4 \times 10^{20}} = 5 \times 10^{-19}$$

$$\lambda = \frac{(6.62 \times 10^{-34}) \times (3 \times 10^{8})}{5 \times 10^{-19}} m = 3.972 \, Å$$

Ex. The work function of a metal is 3.0 eV. It is illuminated by a light of wavelength 3 x 10^{7} m. Calculate (i) threshold frequency, (ii) the maximum energy of photoelectrons, (iii) the stopping potential. (h = 6.63 x 10^{-34} Js and c = 3 x 10^{8} ms^{-1}).

Sol. (i) ϕ_o = 3.0 eV = 3 x 1.6 x 10^{-19} J

Threshold frequency

$$\upsilon_0 = \frac{W}{h} = \frac{3 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = 0.72 \times 10^{15} Hz$$

(ii) Maximum kinetic energy (K_{max}) =

$$h(\upsilon - \upsilon_o) = 3 \times 10^{-7} m$$

$$\upsilon = \frac{c}{h} = \frac{3 \times 10^{8}}{3 \times 10^{-7}} = 1 \times 10^{15} \text{ Hz}$$

$$K_{max} = h(\upsilon - \upsilon_o) = 6.63 \times 10^{-34} (1 - 0.72) \times 10^{15}$$

$$K_{max} = 1.86 \times 10^{-19} J$$

(iii) $K_{max} = eV_0$

where V_0 is stopping potential in volt and e is the charge of electron

$$V_0 = \frac{K_{max}}{e}$$

Here $K_{max} = 1.86 \times 10^{-19}\,J$ and

$$e = 1.6 \times 10^{-19}\,C$$

$$V_0 = \frac{1.86 \times 10^{-19}\,J}{1.6 \times 10^{-19}\,C} = 1.16\,V$$

Ex. The work function of a photosensitive element is 2 eV. Calculate the velocity of a photoelectron when the element is exposed to a light of wavelength 4×10^3 Å.

Sol. Einstein's photoelectric equation is

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - W_0$$

$$\frac{1}{2}mv^2 = \frac{6.62 \times 3}{4 \times 10^3 \times 10^{-10}} \times 10^{-26} - 2 \times 1.6 \times 10^{19}$$

$$v^2 = \frac{1.765 \times 2}{9.1} \times 10^{12}$$

$$v = \sqrt{\frac{1.765 \times 2}{9.1}} \times 10^6 = 6.228 \times 10^5\ ms^{-1}$$

Ex. A metal of work function 4 eV is exposed to a radiation of wavelength 140×10^{-9} m. Find the stopping potential.

Sol. $E = \frac{hc}{\lambda}$

$$E = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{140 \times 10^{-9} \times 1.6 \times 10^{-19}}\,eV = 8.86\,eV$$

Work function $W_0 = 4$ eV

$eV_0 = E - W_0 = 8.86 - 4 = 4.86$ eV

∴ Stopping potential $V_o = 4.86$ V

Ex. Radiations of the wavelength 200 nm propagating in the form of a parallel beam, falling normally on the plane metallic surface. The intensity of the beam used is 5 mW and its cross sectional area is 1.0 mm^2. Find out the pressure exerted by the radiation on the metallic surface, if the radiation is completely reflected.

Sol. $E = \frac{12400}{\lambda} = \frac{12400}{200} = 6.2\,eV \approx 10^{-18}\,J$

The number of photons passing a point per second is

$$n = \frac{P}{E} = \frac{5 \times 10^{-9}}{10^{-18}} = 5 \times 10^{9}$$

Momentum of each photon

$$p = \frac{E}{C} = 3.3 \times 10^{-27}\ J/s.$$

Change in momentum after each strike

$= 2p = 6.6 \times 10^{-27}$ J/s

Total momentum change per second is

$$F = \frac{dp}{dt} = \frac{n \times 2p}{t} = 5 \times 10^{9} \times 6.6 \times 10^{-27}$$

$= 33 \times 10^{-18}$ N

$\therefore$ Pressure $\frac{F}{A} = 33 \times 10^{-12}\ N/m^2$

Ex. The photoelectric threshold of the photo electric effect of a certain metal is 2750 Å. Find-

(i) The work function of the emission of an electron from this metal

(ii) Maximum kinetic energy of these electrons

(iii) The maximum velocity of the electrons which are ejected from the metal by using wavelength of light as 1800 Å.

Sol. (i) Given that the threshold wavelength of a metal is $\lambda_{th} = 2750\,Å$. Thus the work function of metal can be stated as shown

$$\phi = \frac{hc}{\lambda_{th}} = \frac{12431}{2750}\,eV = 4.52\,eV$$

(ii) The energy of the incident photon of given wavelength 1800 Å on metal surface in eV is

$$E = \frac{12431}{1800}\,eV = 6.9\,eV$$

Thus, the maximum kinetic energy of ejected electrons is

$KE_{max} = E - \phi = 6.9 - 4.52$ eV $= 2.38$ eV

(iii) If the maximum speed of the ejected electrons is v_{max} then we have

$$\frac{1}{2}mv_{max}^2 = 2.38\,\text{eV}$$

or $$v_{max} = \sqrt{\frac{2 \times 2.38 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}}$$

$$= 9.15 \times 10^5 \text{ m/s}$$

Ex. In a experiment, the tungsten cathode which has a threshold 2300 Å is irradiated by ultra-violet light of given wavelength 1800 Å. Calculate

(i) Maximum energy of an emitted photoelectron and

(ii) Work function for tungsten (Mention both the results in electron-volts)

(Given: Plank's constant h = 6.6×10^{-34} joule-sec, 1 eV = 1.6×10^{-19} joule and velocity of light c = 3×10^8 m/sec)

Sol. The work function of tungsten cathode is

$$\phi = \frac{hc}{\lambda_{th}} = \frac{12431}{2300}\text{eV} = 5.4\,\text{eV}$$

The energy of incident photons is in eV

$$E = \frac{hc}{\lambda} = \frac{12431}{1800}\text{eV}$$

Therefore, maximum kinetic energy for the ejected electrons can be given as

$$KE_{max} = E - \phi = 6.9 - 5.4\,\text{eV} = 1.5\,\text{eV}$$

Ex. Determine the frequency of light which ejects electrons from a metal surface fully stopped by a retarding potential of 3 V. The photo electric effect begins in this metal at frequency of 6×10^{14} sec^{-1}. Find out the work function for this metal.

Sol. The threshold frequency for a given metal surface is

$$\upsilon_{th} = 6 \times 10^{14} \text{ Hz}$$

Thus the work function for metal surface is

$$\phi = h\upsilon_{th} = 6.63 \times 10^{-34} \times 6 \times 10^{-19}$$

$$= 3.978 \times 10^{-19} \text{ J}$$

As the stopping potential for the ejected electrons is 3V, the maximum kinetic energy of ejected electrons will be

$$KE_{max} = 3\,\text{eV} = 3 \times 1.6 \times 10^{-19}\,\text{J}$$

$= 4.8 \times 10^{-19}$ J

According to photo electric effect equation, we have

$$h\upsilon = h\upsilon_{th} + KE_{max}$$

or frequency of incident light is

$$v = \frac{\phi + KE_{max}}{h} = \frac{3.978 \times 10^{-19} + 4.8 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$= 1.32 \times 10^{15}$ Hz

Ex. Evaluate the velocity of a photo-electron, if the work function of the target material is 1.24 eV and the wavelength of the incident light is 4360 Å. What retarding potential is necessary in order to stop the emission of the electrons?

Sol. The energy of incident photons in eV on metal surface is

$$E = \frac{12431}{4360} eV = 2.85\, eV$$

According to the photo electric effect equation,

$$E = \phi + \frac{1}{2} mv_{max}^2$$

or $$\frac{1}{2} mv_{max}^2 = E - \phi$$

$= 2.85 - 1.24$ eV $= 1.61$ eV

The value of the stopping potential for these ejected electrons can be given as

$$V_0 = \frac{1/2\, mv_{max}^2}{e} = \frac{1.61 \text{ eV}}{e} = 1.61 \text{volts}$$

Ex. Find the number of photons in 6.62 joule of radiation energy of frequency 10^{12} Hz?

Sol. No. of photons

$$n = \frac{E}{h\upsilon} = \frac{6.62}{6.62 \times 10^{-34} \times 10^{12}} = 10^{22}$$

Ex. Calculate the energy and momentum of a photon of wavelength 6600 Å.

Sol. Energy of photon

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{6600 \times 10^{-10}} = 3 \times 10^{-19} J$$

Momentum of photon

$$p = \frac{h}{\lambda}$$

$$= \frac{6.6 \times 10^{-34}}{6600 \times 10^{-10}} = 10^{-27} \text{ kg m / sec}$$

Ex. In an experiment on photo electric emission, following observations were made;

(i) Wavelength of the incident light = 1.98×10^{-7} m;

(ii) Stopping potential = 2.5 volt.

Find :

(a) Kinetic energy of photoelectrons with maximum speed.

(b) Work function and

(c) Threshold frequency

Sol. (a) Since

$V_s = 2.5$ V, $K_{max} = eV_s$

so, $K_{max} = 2.5$ eV

(b) Energy of incident photon

$$E = \frac{12400}{1980} eV = 6.26 \, eV$$

$W = E - K_{max} = 3.76$ eV

(c) $h\upsilon_{th} = W = 3.76 \times 1.6 \times 10^{-19} J$

$$\therefore \quad \upsilon_{th} = \frac{3.76 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} \approx 9.1 \times 10^{14} Hz$$

EXAMPLES

Q1 **When a light of wavelength 400 nm falls on a metal of work function 2.5 eV, what will be the maximum magnitude of linear momentum of emitted photoelectron?**

Sol: $\frac{P^2}{2m} = \left(\frac{1.24 \times 10^4}{4000} - 2.5\right) eV = 0.6\, eV$;

$P = \sqrt{2 \times 9.1 \times 10^{-31} \times 0.6 \times 1.6 \times 10^{-19}}$

$= 4.2 \times 10^{-25}$ kg m/s

Q2 **The electric field associated with a monochromatic light to given $E = E_0 \sin (1.2 \times 10^{15} \pi t - kx)$. Find the maximum kinetic energy of the photoelectrons when this light falls on a metal surface whose work function is 2.0 eV**

Sol: frequency of the light f

$= \frac{1}{2} \times 1.2 \times 10^{15} = 0.6 \times 10^{15}$ Hz

$KE = hf - \phi = 0.48$ eV

Q3 **The magnetic field at a point associated with a light wave is $B = 2 \times 10^{-6}$ Tesla $\sin[(3.0 \times 10^{15} s^{-1})t] \sin[(6.0 \times 10^{15} s^{-1})t]$. If this light falls on a metal surface having a work function of 2.0 eV , what will be the maximum kinetic energy of the photoelectrons?**

Sol: **Sol.** $\left(\frac{9 \times 10^{15}}{2\pi e} h - 2\right) eV = 3.93$ eV

Q4 **In an experiment on photoelectric effect, light of wavelength 800 nm (less than threshold wavelength) is incident on a caesium plate at the rate of 5.0 W. The potential of the collector plate is made sufficiently positive with respect to that of the emitter, such that the current reaches its saturation value. Assume that on the average one of every 10^6 photons is able to eject a photo-electron, find the photo current in the circuit.**

Sol: No. of Photons $= \frac{P}{E_\lambda} = \frac{P\lambda}{hc}.s^{-1}$

No. of photo electron $= \frac{P\lambda}{hc}.\frac{1}{10^6}$

$\therefore$ Photo current $= \frac{P\lambda}{hc \times 10^6}.\ e = \frac{5 \times 800 \times 10^{-9} \times (1.6 \times 10^{-19})}{6.63 \times 10^{-34} \times 3 \times 10^8 \times 10^6}$ A = 3.2 μA.

Q5 **Intensity of sunlight falling normally on the earth surface is 1.4×10^3W/m^2. Assume that the light is monochromatic with average wavelength 5000Å and that no light is absorbed in between the sun and the earth's surface. The distance between the sun and the earth is 1.5×10^{11}m.**

(a) Calculate the number of the photons falling per second on each square meter of earth's surface directly below the sun.

(b) How many photons are there in each cubic meter near the earth's surface at any instant?

(c) How many photons does the sun emits per second?

Sol: (a) $n = \frac{P}{E_\lambda} = \frac{1.4 \times 10^3}{\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10}}} = 3.5 \times 10^{21}$

(b) If N be the no. of photons then these photons will fall on the surface in time

$$\Delta t = \frac{1}{C}$$

$\therefore \quad n = \frac{N}{\Delta t} = N.\,C$

$\therefore \quad n = \frac{n}{C} = \frac{3.5 \times 10^{21}}{3 \times 10^8} = 1.2 \times 10^{13}$

(c) No of photon emitted per second by the sun

$= n.\ 4\pi r^2 = 3.5 \times 10^{21} \times 4\pi\ (1.5 \times 10^{11})^2$

$= 9.9 \times 10^{44}$

Q6 **A parallel beam of monochromatic light of wavelength 663 nm is incident on a totally reflecting plane mirror. The angle of incidence is 60° and the number of photons striking the mirror per second is 5×10^{19}. Calculate the force exerted by the light beam on the mirror. ($h = 6.63 \times 10^{-34}$ J.s.)**

Sol: $F = \frac{\Delta P}{\Delta t}$ = change of momentum of one photon x no. of photons per second =

$$\left(\frac{h}{\lambda} \times 2 \times \cos 60^{\circ}\right) \times \frac{dn}{dt}$$

$$= \frac{6.63 \times 10^{-34}}{663 \times 10^{-9}} \times 2 \times \frac{1}{2} \times (5 \times 10^{19}) = 5 \times 10^{-8}\ N$$

Q7 **A sodium lamp of power 10 W is emitting photons of wavelength 590 nm. Assuming that 60% of the consumed energy is converted into light, find the number of photons emitted per second by the lamp.**

Sol: Power consumed = 10 W

Power emitted = 0.6 × 10W = 6W

$$\text{Number of photons emitted per second} = \frac{\text{Power emitted}}{\text{energy of each photon}}$$

$$= \frac{6}{(6.63 \times 10^{-34}) \times \frac{(3 \times 10^{8})}{590 \times 10^{-9}}} = 1.77 \times 10^{19}$$

Q8 **Photo electrons are liberated by ultraviolet light of wavelength 3000 Å from a metallic surface for which the photoelectric threshold wavelength is 4000 Å. Calculate the de Broglie wavelength of electrons emitted with maximum kinetic energy.**

Sol: Maximum K.E. of the electron $e^{-} = E_{\lambda} - \phi$

$$E = \frac{hc}{\lambda} - \frac{hc}{\lambda_{th}}$$

λ_d = de–Broglie wavelength,

$$\frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2m \cdot hc \cdot \left(\frac{\lambda_{th} - \lambda}{\lambda \cdot \lambda_{th}}\right)}}$$

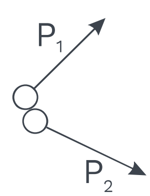
$$\lambda_d = \sqrt{\frac{h.\lambda.\lambda_{th}}{2m.c.(\lambda_{th} - \lambda)}}$$

= 12.08 Å.

Q9 **Two identical nonrelativistic particles move at right angles to each other, possessing de-Broglie wavelengths, λ_1 & λ_2. Find the De Broglie wavelength of each particle in the frame of their centre of mass.**

Sol: Let $\vec{P}_1$ and $\vec{P}_2$ be the momenta of the two particles.

The velocity of the centre of mass



$$\vec{V}_{cm} = \frac{\vec{P}_1 + \vec{P}_2}{2m}$$

Hence, momentum of particle 1 in COM frame

$$\vec{P}_1' = m(\vec{v}_1 - \vec{v}_{cm})$$

$$= m\vec{v}_1 - m\frac{\vec{P}_1 + \vec{P}_2}{2m}$$

$$= \frac{\vec{P}_1 + \vec{P}_2}{2}$$

$$|\vec{P}_1'| = \frac{1}{2}\sqrt{P_1^2 + P_2^2 - 2P_1P_2 \cos 90º}$$

$$\frac{h}{\lambda} = \frac{1}{2}\sqrt{\frac{\lambda_1^2h^2 + \lambda_2^2h^2}{\lambda_1^2 \times \lambda_2^2}}$$

$$\Rightarrow \qquad \lambda = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}.$$

Q10 **The work function of caesium is 2.14 eV. Find the threshold frequency for caesium.**

Sol: For the cut-off or threshold frequency, the energy $h\nu_0$ of the incident radiation must be equal to work function ϕ_0, so that,

$$\nu_0 = \frac{\phi_0}{h} = \frac{2.14\,\text{eV}}{6.63 \times 10^{-34}\,\text{J s}}$$

$$= \frac{2.14 \times 1.6 \times 10^{-19}\,\text{J}}{6.63 \times 10^{-34}\,\text{J s}} = 5.16 \times 10^{14}\,\text{Hz}$$

Thus, for frequencies less than this threshold frequency, photoelectrons are ejected.

Q11 **Monochromatic light of frequency 6.0 × 10^{14} Hz is produced by a laser. The power emitted is 2.0 × 10^{-3} W. (a) What is the energy of a photon in the light beam? (b) How many photons per second, on an average, are emitted by the source?**

Sol: (a) Each photon has an energy

$$E = h\nu = (6.63 \times 10^{-34}\,\text{J s})(6.0 \times 10^{14}\,\text{Hz})$$

$$= 3.98 \times 10^{-19}\,\text{J}$$

(b) If N is the number of photons emitted by the source per second, the power P transmitted in the beam equals N times the energy per photon E, so that P = N E. Then

$$N = \frac{P}{E} = \frac{2.0 \times 10^{3}\,\text{W}}{3.98 \times 10^{-19}\,\text{J}}$$

= 5.0 × 10^{15} photons per second.

Q12 **The wavelength of light in the visible region is about 390 nm for violet colour, about 550 nm (average wavelength) for yellow-green colour and about 760 nm for red colour.**
What are the energies of photons in (eV) at the (i) violet end, (ii) average wavelength, yellow-green colour, and (iii) red end of the visible spectrum? (Take $h = 6.63 \times 10^{-34}$ J s and 1 eV = 1.6×10^{-19}J.)

Sol: Energy of the incident photon, $E = h\nu = \dfrac{hc}{\lambda}$

$$= \frac{1.989 \times 10^{-25}\,\text{Jm}}{\lambda}$$

(i) For violet light, $\lambda_1 = 390\,\text{nm}$ (lower wavelength end)

Incident photon energy, $E_1 = \dfrac{1.989 \times 10^{-25}\,\text{Jm}}{390 \times 10^{-9}\,\text{m}}$

$$= 5.10 \times 10^{-19}\,\text{J}$$

$$= \frac{5.10 \times 10^{-19}\,\text{J}}{1.6 \times 10^{-19}\,\text{J / eV}}$$

$$= 3.19\,\text{eV}$$

(ii) For yellow-green light, $\lambda_2 = 550\,\text{nm}$ (average wavelength)

Incident photon energy, $E_2 = \dfrac{1.989 \times 10^{-25}\,\text{Jm}}{550 \times 10^{-9}\,\text{m}}$

$$= 3.62 \times 10^{-19}\,\text{J} = 2.26\,\text{eV}$$

(iii) For red light, $\lambda_3 = 760\,\text{nm}$ (higher wavelength end)

Incident photon energy, $E_3 = \dfrac{1.989 \times 10^{-25}\,\text{Jm}}{760 \times 10^{-9}\,\text{m}}$

$$= 2.62 \times 10^{-19}\,\text{J} = 1.64\,\text{eV}$$

Q13 **What is the de Broglie wavelength associated with (a) an electron moving with a speed of 5.4×10^6 m/s, and (b) a ball of mass 150 g travelling at 30.0 m/s?**

Sol: (a) For the electron:

Mass $m = 9.11 \times 10^{-31}$ kg, speed $\upsilon = 5.4 \times 10^6$ m/s. Then, momentum

$p = m\upsilon = 9.11 \times 10^{-31}\ (kg) \times 5.4 \times 10^6\ (m/s)$

$p = 4.92 \times 10^{-24}$ kg m/s

de Broglie wavelength, $\lambda = h/p$

$$= \frac{6.63 \times 10^{-34}\,Js}{4.92 \times 10^{-24}\,kg\,m/s}$$

$\lambda = 0.135\,nm$

(b) For the ball:

Mass $m' = 0.150$ kg, speed $\upsilon' = 30.0$ m/s.

Then momentum $p' = m'\ \upsilon' = 0.150\ (kg) \times 30.0\ (m/s)$

$p' = 4.50$ kg m/s

de-Broglie wavelength $\lambda' = \frac{h}{p'}$.

$$= \frac{6.63 \times 10^{\pm 34}\ Js}{4.50 \times kg\,m/s}$$

$\lambda' = 1.47 \times 10^{-34}$ m

The de Broglie wavelength of electron is comparable with X-ray wavelengths. However, for the ball it is about 10^{-19} times the size of the proton, quite beyond experimental measurement.

Q14 **An electron, an α-particle, and a proton have the same kinetic energy. Which of these particles has the shortest de Broglie wavelength?**

Sol: For a particle, de-Broglie wavelength, $\lambda = \frac{h}{p}$

Kinetic energy, $K = \frac{p^2}{2m}$

Then, $\lambda = h / \sqrt{2mK}$

For the same kinetic energy K, the de Broglie wavelength associated with the particle is inversely proportional to the square root of their masses. A proton $\left({}_1^1H\right)$ is 1836 times massive than an electron and an α -particle $\left({}_2^4He\right)$ four times that of a proton.

Hence, α -particle has the shortest de-Broglie wavelength.

Q15 **A particle is moving three times as fast as an electron. The ratio of the de Broglie wavelength of the particle to that of the electron is 1.813 × 10^{-4}. Calculate the particle's mass and identify the particle.**

Sol: de-Broglie wavelength of a moving particle, having mass m and velocity υ:

$$\lambda = \frac{h}{p} - \frac{h}{m\upsilon}$$

Mass, $m = \frac{h}{\lambda\upsilon}$

For an electron, mass $m_e = h / \lambda_e \upsilon_e$

Now, we have $\upsilon / \upsilon_e = 3$ and $\lambda / \lambda_e = 1.813 \times 10^{-4}$

Then, mass of the particle, $m = m_e \frac{\lambda_e}{\lambda} \frac{\upsilon_e}{\upsilon}$

$m = (9.11 \times 10^{-31}\ kg) \times (1/3) \times (1/1.813 \times 10^{-4})$

$m = 1.675 \times 10^{-27}$ kg.

Thus, the particle, with this mass could be a proton or a neutron.

Q16 **What is the de-Broglie wavelength associated with an electron, accelerated through a potential difference of 100 volts?**

Sol: Accelerating potential V = 100 V. The de-Broglie wavelength λ is.

$$\lambda = h / p = \frac{1.227}{\sqrt{V}} \text{nm}$$

$$\lambda = \frac{1.227}{\sqrt{100}} \text{nm} = 0.123 \, \text{nm}$$

The de-Broglie wavelength associated with an electron in this case is of the order of X-ray wavelengths.

MIND MAP

PHOTOELECTRIC EFFECT

Photon

- Particle of light is called as 'photon'. It is packet of energy.
- $E = \frac{hc}{\lambda} = h\nu = \frac{1242}{\lambda(\text{in nm})}$ in eV
- Momentum of photon $P = \frac{h}{\lambda} = \frac{E}{c}$

  Planck's constant 'h' = 6.626×10^{-34} Js

  $1\text{eV} = 1.6 \times 10^{-19}$J

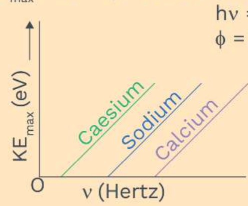
Einstein's Equation

$0 \le KE \le h\nu - \phi$

$(KE)_{max} = h\nu - \phi$ Where, $(KE)_{max}$ = Value between 0 to KE_{max}

$h\nu = E_{photon}$

ϕ = Work function



Work Function - Minimum energy required to exit an electron from surface of the substance is called 'Work Function (ϕ)' of the substance.

Threshold Wavelength - Maximum wavelength for which e^- just comes out is called threshold wavelength (λ_0)

$\frac{hc}{\lambda_0} = \phi$

Photoelectric effect will take place for $\lambda \le \lambda_0$

$KE_{max} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$

Threshold Frequency - Minimum frequency for which electron just comes out is called threshold frequency (ν_0)

$h\nu = \phi$

Photoelectric effect takes place for $\nu > \nu_0$

$KE_{max} = h\nu - \phi = h\nu - h\nu_0$

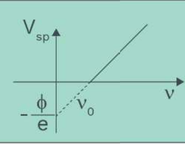
$KE_{max} = h(\nu - \nu_0)$

Photoelectric Experiment

$eV_{sp} = h\nu - \phi$

$V_{sp} = \left[\frac{h}{e}\right]\nu - \left[\frac{\phi}{e}\right]$

Slope = $\frac{h}{e}$ is independent of metals



Saturation Current i_s - The maximum value of photoelectric currect is called saturation current.

Stopping Potential V_{sp} - The value of $V_C - V_A$ at which photo current just stops is called Stopping Potential (V_{sp}).

$eV_{sp} = KE_{max}$