Nucleus

DISCLAIMER

Nucleus

It exists at the centre of an atom, containing the entire positive charge and almost the whole of the mass. The electrons revolve around the nucleus to form an atom. The nucleus consists of protons (+ve charge) & neutrons (no charge).

- A proton has positive charge which is equal in magnitude to that of an electron (1.6×10^{-19}C) and a mass equal to 1836 times that of an electron.
- A neutron has no charge, & its mass is approximately equal to that of the proton (1.6726×10^{-27} kg). (1837 times that of an electron)
- The number of protons in a nucleus of an atom is called as the atomic number (Z) of that atom. The number of protons and neutrons (together called Nucleons) in the nucleus of an atom is called the mass number (A) of the atom.
- A particular set of nucleons forming an atom is called a nuclide. It is represented as $_ZX^A$.
  Central core of atom that is nucleus, was discovered by Rutherford in α -scattering experiment. The order of the size of a nucleus = 10^{-15} m or fm while the order of atomic size = 10^{-10} m or Å.
- The atomic nucleus consists of the two types of elementary particles, viz. protons & neutrons. These particles are called nucleons.
- Z = atomic number = p (no. of protons)
- A = mass number = total no. of the nucleons = n + p

Atomic Mass Unit (A.M.U):

- The masses of atoms, nuclei and subatomic particles are very small. Hence, a small unit is used to express these masses. This unit is called as atomic mass unit (amu). **1 amu is equal to one twelfth part of the mass of carbon ($_6C^{12}$) isotope. Mass of $_6C^{12}$ is exactly 12 amu.**
- The mass of 1 gm -mole of carbon is 12 gm and according to Avogadro's Hypothesis it has N (Avogadro's Number) atoms. Thus, the mass of one atom of the carbon is (12/N) gm. According to the definition.

KEY POINTS

- Nucleus
- Proton
- Neutron
- Atomic number
- Mass number
- Atomic mass unit
- Sub-atomic particles

Concept Reminder

The atomic nucleus consists of two types of elementary particles, viz. protons and neutrons. These particles are called nucleons.

Definitions

- 1 amu is equal to one twelfth part of the mass of carbon ($_6C^{12}$) isotope. Mass of $_6C^{12}$ is exactly 12 amu.

$$1\text{amu} = 1\text{u} = \frac{1}{12} \times (\text{mass of one carbon atom})$$

$$= \frac{1}{12} \times \frac{12}{N} = \frac{1}{N}\text{gm} = \frac{1}{6.023 \times 10^{23}}\text{gm}$$

$$= 1.660565 \times 10^{-24}\text{ gm} = 1.660565 \times 10^{-27}\text{ Kg}$$

$m_p = 1.6726 \times 10^{-27}$ kg = 1.00727 u
$m_n = 1.6749 \times 10^{-27}$ kg = 1.00866 u
$m_e = 9.1 \times 10^{-31}$ kg = 0.00055 u

Properties of Nuclei

- **Size of Nucleus:** (Order is fermi)
  As the number of nucleons in nucleus increases its size also increases and relation between its radius and mass number is

  $R \propto A^{1/3}$ $\quad$ $\boxed{\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}}$

  $R = R_0 A^{1/3}$

  Here R_0 is a constant and its value $R_0 \approx 1.2$ fm.
- Nuclear sizes are very small & are measured in fermi (or) femtometer. 1 fermi = 10^{-15} m.
- Radius of the nucleus depends upon number of nucleons. $R = R_0 A^{1/3}$

  Above equation does not apply to the heavy nuclei Value of $R_0 = 1.25 \times 10^{-15}$ m
- Radius of the nucleus is of the order of 10^{-15} m.
- Size of an atom is of the order of 10^{-10} m.
- **Volume of Nucleus**

  Volume $= \frac{4\pi R^3}{3} = \frac{4\pi R_o^3 A}{3}$

  Volume $\propto R^3 \propto A$
- **Mass of Nucleus**
  It is measured in Atomic Mass Unit (amu)
  Mass of a nucleus of mass number A is $\simeq Am_p \simeq A$
  or mass of a nucleus $(m) \propto A$

Concept Reminder

Atomic nucleus is represented as ${}^{A}_{Z}X$ where Z = atomic number and A = mass number.

Definitions

- **Atomic Number (Z):** The number of protons in a nucleus is known as atomic number.
- **Mass Number (A):** The sum of number of proton and number of neutrons inside nucleus is known as mass number.

- **Density of Nucleus (ρ)**

$$\rho = \frac{\text{mass}}{\text{volume}} \cong \frac{Am_p}{\frac{4}{3}\pi R^3} = \frac{Am_p}{\frac{4}{3}\pi R_0^3 A} = \frac{3m_p}{4\pi R_0^3} \simeq 2.3 \times 10^{17} \text{kg} / \text{m}^3$$

Density of nucleus is independent of mass number of the atom.

- Density of the nucleus is 2.3 x 10^{17} Kgm^{-3}.
- The density is maximum at the centre and gradually falls to zero as we move radially outwards.
- Radius of the nucleus is taken as the distance between the centre and the point where the density falls to half of its value at the centre.
- Density of the nucleus is of the order of 10^{14} gm / cc $= 10^{17}$ kg / m^3

Ex. Nuclear radius of Fe^{56} is r. Calculate nuclear radius of Li^7.

Sol. $\frac{r}{r'} = \left(\frac{56}{7}\right)^{\frac{1}{3}}$ or $r' = \frac{r}{2}$

Forces acting inside the nucleus:

There are three forces which takes place between the nucleons, these are

(i) Gravitational force $\rightarrow$ weakest force of the nature.

(ii) Electrostatic repulsive (coulombian) force $\rightarrow$ it only works between proton and proton. This is stronger than gravitational force.

(iii) Nuclear force $\rightarrow$ strongest interaction that holds nucleons together to form nuclei and it is powerful enough to overcome the electrostatic repulsion of proton and proton.

Features of Nuclear Force (F_n):

1. The strongest force in the universe. $F_n : F_e : F_g \approx 1 : 10^{-2} : 10^{-36}$

2. Works only between the nucleons.

KEY POINTS

- Radius of nucleus
- Volume of nucleus
- Density of nucleus

Rack your Brain

The mass number of a nucleus is:

(1) Always less than its atomic number

(2) Always more than its atomic number

(3) Sometimes equal to its atomic number

(4) Sometimes less and sometimes more than atomic number

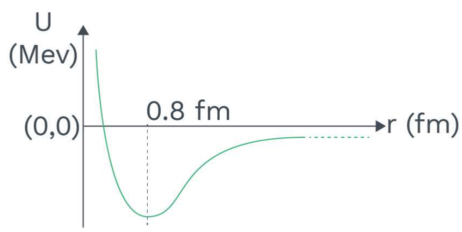
Concept Reminder

- Radius of nuclei $R = R_0 A^{\frac{1}{3}}$
- Volume of nuclei $V = \frac{4}{3}\pi R_0^3 A$
- Density of nuclei $\rho = \frac{3m_p}{4\pi R_0^3}$

3. **Very short range**: It's range is only upto the size of nucleus (3 or 4 Fermi). More than this distance, the nuclear force is almost zero.
4. **Very much depends upon distance**: – Small variation in distance may cause of large change in nuclear force while electrostatic force remains almost unaffected.
5. **Independent of charge: –** Same between n–n as well as between p–p and also between n–p.
6. **Spin dependent: –** It is stronger between nucleons having same sense of spin than between nucleons having opposite sense of spin.
7. **It is not a central force: –** Definition of central force (F_c): Whose line of action always passes through a fixed point and its magnitude depends only on distance, if medium is same.

   $\vec{F}_c = \frac{K}{r^n}(\pm \hat{r})$ is central force.

8. **Potential Energy for Nuclear Force**



Attractive – If distance is greater than 0.8 fm or above.

Repulsive – If distance is lesser than 0.8 fm.

TYPES OF THE NUCLEI:

- **Isotopes:** Atomic nuclei having the same atomic number, but different mass number are known as isotopes. They occupy the same position in the periodic table & possess identical chemical properties. They have same proton number.

Ex: (1) ${}_3Li^6, {}_3Li^7$ (2) ${}_1H^1, {}_1H^2, {}_1H^3$

Concept Reminder

Properties of nuclear force:
- Strongest force
- Short range
- Independent of charge
- Non-central
- Non-conservative

Definitions

Isotopes: Atomic nuclei having same atomic number, but different mass numbers are known as isotopes.

Isotones: Atomic nuclei having same number of neutrons are called isotones.

Isomers: Atomic nuclei having same mass number and same atomic number, but different nuclear properties are called isomers.

- **Isotones:** Atomic nuclei having the same number of neutrons are called isotones.
  Ex.: (1) ${}_8O^{18}, {}_9F^{19}$ (2) ${}_3Li^7, {}_4Be^8$
- **Isomers:** Atomic nuclei having the same mass number and same atomic number, but different nuclear properties are called isomers.
  Ex.: $m_{35}Br^{80}$ metastable Bromine and $g_{35}Br^{80}$ ground state Bromine are two isomers with different half-lives.

Ex. Compare the radii of the nuclei of mass numbers 27 and 64.

Sol. The ratio of radii of the nuclei is

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{3}} = \left(\frac{27}{64}\right)^{\frac{1}{3}} = \frac{3}{4} \quad (\because R = R_0 A^{1/3})$$

Ex. The radius of the oxygen nucleus ${}^{16}_{8}O$ is 2.8 x 10^{-15} m. Find the radius of lead nucleus ${}^{205}_{82}Pb$.

Sol. $R_0 = 2.8 \times 10^{-15} m$

$$\frac{R_0}{R_{Pb}} = \left(\frac{A_0}{A_{Pb}}\right)^{1/3} = \frac{2.8 \times 10^{-15}}{R_{Pb}} = \left(\frac{16}{205}\right)^{1/3}$$

$R_{Pb} = 6.55 \times 10^{-15} m$.

Einstein's Mass Energy Equivalence

According to the Einstein, mass can be converted into energy and energy can be converted into mass. This relation is given by -

$E = mc^2$

Here E = the total energy associated with mass m

c^2 = used as a conversion coefficient

1. Rest mass energy $= E_0 = m_0c^2$

2. Total energy of a moving particle

$= E = mc^2 = m_0c^2 + K.E.$

3. $\boxed{K.E. = mc^2 - m_0c^2}$ and at relatively low speed $KE \simeq \frac{1}{2}m_0v^2$

KEY POINTS

- Isotopes
- Isomer
- Isotones

Rack your Brain

If the nucleus of ${}^{27}_{13}Al$ has a nuclear radius of about 3.6 fm then find out radius of ${}^{125}_{32}Te$.

Concept Reminder

- 1 amu = 931 MeV/C^2
- m_e = 0.511 MeV/C^2
- m_p = 938.28 MeV/C^2
- m_n = 939.57 MeV/C^2

4. 1 amu is equivalent to Energy of = 931.5 MeV $\simeq$ 931 MeV

or $\boxed{1\text{amu} = 931\frac{\text{MeV}}{c^2}}$

5. Rest mass energy of proton = 938.28 MeV
6. Rest mass energy of neutron = 939.57 MeV
7. Rest mass energy of electron/positron = 0.511 MeV

Ex. Energy of a moving e^- is 0.55 MeV. Calculate its K.E.

Sol. K.E. = $mc^2 - m_0c^2$ = 0.04 MeV

Ex. An electron is accelerated by 500 KeV potential difference. Calculate % change in its mass.

Sol. K.E. = eV_{acc} = 500 KeV

% Change in mass $= \frac{m - m_0}{m_0} \times 100$

$= \frac{E - E_0}{E_0} \times 100 = \frac{\text{K.E.}}{E_0} \times 100$

$= \frac{500\,\text{KeV}}{511\text{ KeV}} \times 100 = 97.8\%$

KEY POINTS

- Energy-mass equivalence
- Mass defect

Mass Defect

(i) The rest mass of the nucleus is smaller than the sum of the rest masses of the nucleons constituting it. This is due to the fact that when the nucleons combine to form a nucleus, some energy (binding energy) is liberated.

(ii) If the observed mass of nucleus ${}_ZX^A$ be m_{nuc}, mass of proton is m_p and mass of neutron is m_n then

mass defect = $\Delta m = [Zm_p + (A - Z)m_n] - m_{nuc}$

(iii) If m_{atom} is taken as mass atom of ${}_ZX^A$, then

$\Delta m = [Z(m_p + m_e) + (A - Z)m_n] - m_{atom}$

Definitions

Mass defect: Difference between the mass of the constituent nucleons of nucleus in free state and the mass of the nucleus.

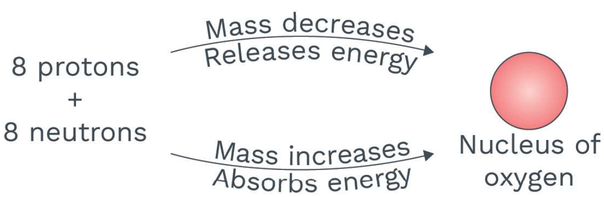
Binding Energy (E_b)

(i) The binding energy is equal to the work that must be done to split the nucleus into the particles constituting it.

(ii) $E_b = \Delta mc^2$, if the masses are taken in atomic mass unit, the binding energy is given by $E_b = \Delta m\,(931.5)$ MeV

(iii) It is always positive & numerically equal to the energy equivalent of mass defect (or equal to the energy liberated when it was formed)

(iv) Let us take example of oxygen nucleus. It contains 8 protons and 8 neutrons.
We can discuss concept of binding energy by following diagram.



$8m_p + 8m_n >$ mass of nucleus of oxygen
For nucleus we apply mass energy conservation,
$8m_p + 8m_n = \text{mass of nucleus} + \dfrac{\text{B.E.}}{c^2}$

Binding Energy per Nucleon $\left[\dfrac{E_b}{A}\right]$

(i) The value of the binding energy per nucleon decides the stability of a nucleus. It is obtained by dividing the binding energy by the mass number of the given nucleus.

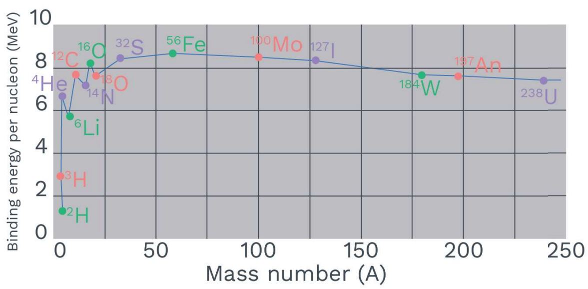
(ii) The following figure shows the graph between binding energy per nucleon plotted against the mass number of the various atoms nuclei.
Greater the binding energy per nucleon, more is the stability the nucleus.

Concept Reminder

- Total binding energy
$= [[Zm_p + (A - Z)m_n] - M_N]C^2$
- Average BE/nucleon
$= \dfrac{\text{Total BE}}{\text{Mass number}}$

KEY POINTS

- Binding energy
- Binding energy per nucleon
- Binding energy curve



The binding energy per nucleon as a function of mass number.

(iii) It is the maximum for isotope of iron $^{56}_{26}Fe$ and equal to 8.8 MeV/nucleon. It is the most stable nucleus.

(iv) For Uranium $_{92}U^{238}$, the binding energy per nucleon is about 7.7 MeV/nucleon and it is unstable.

(v) Exception in graph.

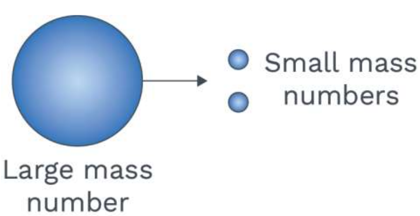
$He^4 > Li^7$

$C^{12} > N^{14}$

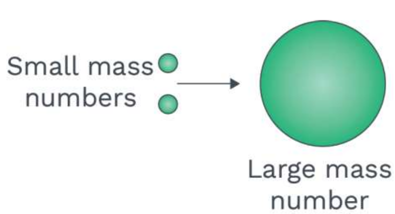
$O^{16} > F^{19}$

(vi) The medium size nuclei are more stable than light or heavy nuclei.

(vii) Heavy nuclei achieve stability by breaking into two smaller nuclei and this reaction is called fission reaction.



(viii) Light nuclei achieve stability by combining and resulting into heavy nucleus and this reaction is called fusion reaction.



Rack your Brain

The energy equivalent of 0.5 g of a substance is:
(1) 4.5×10^{16} J
(2) 4.5×10^{13} J
(3) 1.5×10^{13} J
(4) 0.5×10^{13} J

Concept Reminder

B.E./A is maximum for $^{56}_{26}Fe$ and equal to 8.8 MeV/nucleon. For A = 30 to 170 it is nearly about 8 MeV/nucleon.

Rack your Brain

The mass of proton is 1.0073 u and that of neutron is 1.0087 u. Find the binding energy of $^{4}_{2}He$.

Note:

- When stability of system increases then energy is released by it.
- Released energy = Final B.E. – Initial B.E. = $(m_i - m_f)c^2$

Ex. Calculate binding energy of helium nuclei

Given $m_p = 1.0073$ u

$m_n = 1.0087$ u

Nuclear mass of $He^4 = 4.0015$ u

Sol. $\Delta m = (2m_p + 2m_n) - M_{He} = 0.0305\,u$

B.E. $= 0.0305 \times 931\,MeV = 28.4\,MeV$

Ex. The radio nuclide ^{11}C decays according to:

$$^{11}_{6}C \longrightarrow {}^{11}_{5}B + {}^{+}e + \nu;\ T_{1/2} = 20.3\,min$$

The value of the maximum energy of the emitted positron is 0.962 MeV. Given the mass values: $m\left({}^{11}_{6}C\right) =$ 11.011434 u and $m\left({}^{11}_{5}B\right) = 11.009305\,u$. Calculate Q and compare it with the maximum energy of the positron emitted.

Sol. $^{11}_{6}C \longrightarrow {}^{11}_{5}B + {}^{0}_{+1}e + \nu + Q$

Mass defect $\Delta m = m_i - m_f$

$\Delta m = [m_N({}^{11}_{6}C) - m_N({}^{11}_{5}B) - m_e]$

$\{m_N \to \text{mass of nucleus}\}$

$\Delta m = [m({}^{11}_{6}C) - 6m_e] - [m({}^{11}_{5}B) - 5m_e] - m_e$

$\{m \to \text{mass of atom}\}$

$\Delta m = m({}^{11}_{6}C) - m({}^{11}_{5}B) - 2m_e$

$\Delta m = [11.011434 - 11.009305 - 2 \times 0.000548]u$

$\Delta m = 0.001033\,u$

$Q = \Delta mc^2 = 0.001033 \times 931.5\,MeV$

$= 0.962\,MeV$

Clearly, $Q = E_d + E_e + E_\nu = 0.962\,MeV$, since

Rack your Brain

How does the binding energy per nucleon vary with the increase in the number of nucleons?

(1) Decreases continuously

(2) First decreases then increases

(3) First increases then decreases

(4) Increases continuously

daughter nucleus is too heavy as compared to ${}_{+1}^{0}e$(positron) and ν(neutrino), positron carries practically all the energy (in this case the energy carried by neutrino is minimum i.e. zero).

Ex. The nucleus ${}_{10}^{23}Ne$ decays by β^{-} emission write down the β-decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

$$m\left({}_{10}^{23}Ne\right) = 22.994466\,u$$

$$m\left({}_{11}^{23}Na\right) = 22.989770\,u$$

Sol. ${}_{10}^{23}Ne \longrightarrow {}_{11}^{23}Na + {}_{-1}e^{0} + \upsilon + Q$

Mass defect $\Delta m = m_i - m_f$

$$\Delta m = [m_N({}_{10}^{23}Ne)] - [m_N({}_{11}^{23}Na) + m_e]$$

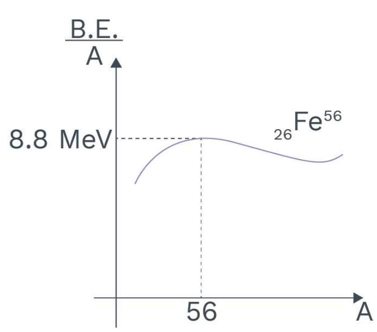
$$\Delta m = [m({}_{10}^{23}Ne) - 10m_e] - [\{m({}_{11}^{23}Na) - 11m_e\} + m_e]$$

$$\Delta m = m({}_{10}^{23}Ne) - m({}_{11}^{23}Na)$$

$$Q = \Delta m.c^2 = 4.374 \text{ MeV}$$

Variation Of The Binding Energy Per Nucleon With Mass Number:

The binding energy per nucleon first increases on an average and reaches a maximum of about 8.7 MeV for A (atomic mass) $50 \rightarrow 80$. For still the heavier nuclei, the binding energy per nucleon slowly decreases as A increases. Binding energy per nucleon is maximum for ${}_{26}Fe^{56}$, which is equal to 8.8 MeV.



Definitions

Binding energy: The total energy required to disintegrate the nucleus into its constituent particles (i.e., nucleons) is called binding energy of the nucleus.

The binding energy per nucleon is more for medium nuclei than that for heavy nuclei. Hence, the medium nuclei are highly stable.

- The heavier nuclei being unstable have tendency to split into medium nuclei. This process is called Fission.
- The Lighter nuclei being unstable, have the tendency to fuse into a medium nucleus. This process is called Fusion.

Radioactivity:

It was discovered by the scientist Henry Becquerel. The spontaneous emission of radiations (α, β, γ) from unstable nucleus is called radioactivity. Substances which show radioactivity are known as the radioactive substances. Radioactivity was studied in detail by the scientist Rutherford. In the radioactive decay, an unstable nucleus emits α particle or β particle. After the emission of α or β, the remaining nucleus may emit γ -particle & converts it into more stable nucleus.

α-particle:

It is a doubly charged helium nucleus. It contains two protons & two neutrons.

Mass of α -particle = Mass of $_2He^4$ atom – $2m_e$ = $4\ m_p$

Charge of α -particle = +2e

β-particle

(a) **β^-(electron):**

Mass = m_e; Charge = –e

(b) **β^+(positron):**

Mass = m_e; Charge = +e

Positron is an antiparticle of the electron.

Antiparticle:

A particle is called antiparticle of the other if on collision with the particle of equal mass and opposite charge, both can annihilate (destroy completely) & converts into energy. For example:

Concept Reminder

Energies associated with nuclear processes are about a million times larger than chemical process.

KEY POINTS

- Radioactivity
- Radioactive substance

Definitions

Radioactivity: The phenomenon of spontaneous emission of radiation by heavy elements is called radioactivity.

Antiparticle: A particle is called antiparticle of other if on collision both can annihilate (destroy completely) and converts into energy.

(i) Electron ($-e$, m_e) and positron ($+e$, m_e) are anti particles.

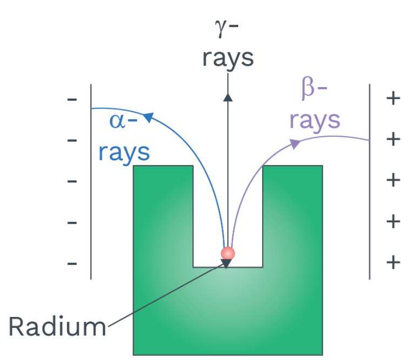
(ii) Neutrino (ν) & anti-neutrino ($\bar{\nu}$) are anti particles.

γ-**particle**:

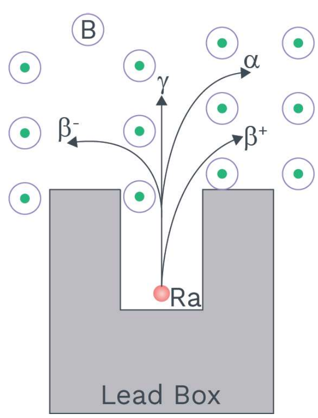
They are the energetic photons of energy of the order of MeV and having rest mass zero.

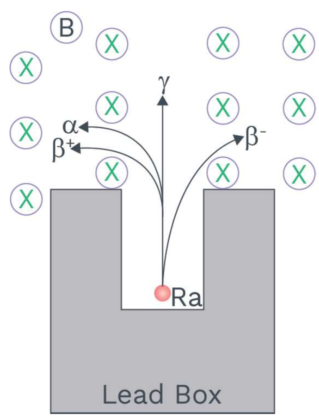
Nature Of Radioactive Radiations:

- **Rutherford's Experiment: -**



Pb box (best absorber of nuclear radiations)





He put a sample of radioactive substance in a lead box and allow the emission of radiations through a small hole only. When the radiation enters into an external electric field, they split it into three parts.

Concept Reminder

There are 3 types of radioactivity decay-

(a) α-decay

(b) β-decay

(c) γ-decay

KEY POINTS

- Antiparticles
- α-particle
- β-particle
- γ-particle

Radiations which deflect towards negative plate are called α – rays.

Radiations which deflect towards positive plate are called β – rays.

Radiations which are undeflected, called γ – rays.

(i) **Alpha rays: –** These are stream of positive charged particles i.e., particle nature.

(ii) **Beta rays: –** These are stream of negative charged particles i.e., particle nature.

(iii) **Gamma rays: –** These are electromagnetic waves.

Rack your Brain

α-particle consists of:
(1) 2 protons only
(2) 2 protons and 2 neutrons only
(3) 2 electrons, 2 protons and 2 neutrons
(4) 2 electrons and 4 protons only

- **Properties of Alpha, Beta & Gamma rays: -**

PROPERTIES OF α, β AND γ RAYS :–

S.No.	Features	α-particles	β-particles	γ-particles
1.	Identity	Helium nucleus or doubly ionised helium ion ($_2He^4$)	Fast moving electrons ($_{-}\beta^0$ or β^-)	Electromagnetic wave (photons)
2.	Charge	Twice of proton (+ 2e)	Electronic (– e)	Neutral
3.	Mass	$\approx 4m_p$ m_p - mass of proton	(rest mass of β) = (rest mass of ele.)	rest mass = 0
4.	Speed	$\approx 10^7$ m/s Their speed depends on nature of the nucleus. So, it is a characteristic speed.	$\approx 10^7$ m/s β-particles come out with different speeds from the same type of nucleus. Therefore can not be a characteristic speed.	Only c = 3×10^8m/s γ-photons come out with same speed from all types of nucleus. So, can not be a characteristic speed.
5.	K.E.	$\approx$ MeV	$\approx$ MeV	$\approx$ MeV
6.	Energy spectrum	Line and discrete	Continuous	Line and discrete
7.	Ionization power ($\alpha > \beta > \gamma$)	10,000 times of γ-rays	100 times of γ-rays (or $\frac{1}{100}$ times of α)	1 (or $\frac{1}{100}$ times of β)
8.	Penetration power ($\gamma > \beta > \alpha$)	$\frac{1}{10000}$ times of γ-rays	$\frac{1}{100}$ times of γ-rays (100 times of α)	1 (100 times of β)
9.	Effect of electric or magnetic field	Deflection	Deflection (More than α)	No Deflection

Type Of Radioactive Decay:

(1) α-Emission

$$_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_2He^4 + Q$$

$$ {}_{Z}^{A}X \longrightarrow {}_{Z-2}^{A-4}Y + {}_{2}He^{4} + Q $$

(i) In each and every nuclear reaction, charge and mass no. remains conserved, but conservation of mass is not essential.

(ii) Mass-Energy remains conserved.

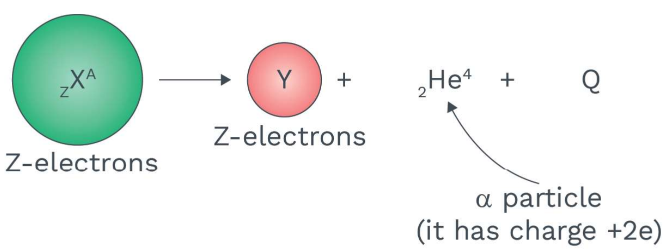
(iii) In a spontaneous nuclear reaction momentum is also conserved.

(iv) Daughter nucleus is isodiaphere of parent nucleus, because value of (n – p) remain same.

Q value: It is defined as the energy released during the decay process.

Q value = rest mass energy of the reactants – rest mass energy of the products.

This energy is available in the form of the increase in K.E. of products



Let, M_x = mass of atom ${}_{Z}X^{A}$

M_y = mass of atom ${}_{Z-2}Y^{A-4}$

M_{He} = mass of atom ${}_{2}He^{4}$

Q value = $[(M_x - Zm_e) - \{(M_y - (Z-2)m_e) + (M_{He} - 2m_e)\}]\, c^2$

$= [M_x - M_y - M_{He}]\, c^2$

Considering actual number of electrons in α - decay

Q value = $[M_x - (M_y + 2m_e) - (M_{He} - 2m_e)]\, c^2$

$= [M_x - M_y - M_{He}]c^2$

Calculation of Kinetic energy of the final products:

As atom 'X' was initially at rest and no external forces are acting, so final momentum also has to be zero.

Definitions

Q value: It is defined as energy released during the decay process. Q value = rest mass energy of reactants – rest mass energy of products.

Concept Reminder

Q value in α-decay process is:

$Q = [M_x - M_y - M_{He}]c^2$ (M_x, M_y and M_{He} are atomic mass)

Concept Reminder

In α-decay, the daughter nucleus is different from parent nucleus. This change of one element into another is called transmutation.

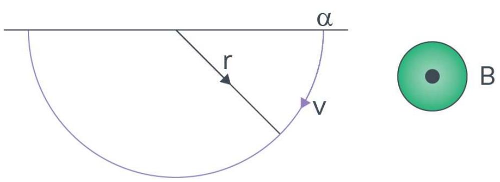
Hence both Y and α - particle will have same momentum in magnitude but in opposite direction.

(Here we are representing 'T' for kinetic energy)

$$Q = T_y + T_\alpha \qquad m_\alpha T_\alpha = m_Y T_y$$

$$T_\alpha = \frac{m_Y}{m_\alpha + m_Y} Q \; ; \qquad T_Y = \frac{m_\alpha}{m_\alpha + m_Y} Q$$

$$T_\alpha = \frac{A-4}{A} Q \; ; \qquad T_Y = \frac{4}{A} Q$$

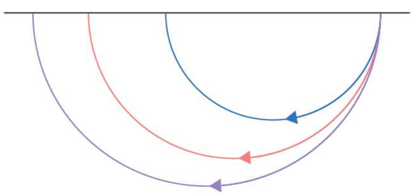


From the above calculation, one can observe that all the α - particles emitted should have same value of kinetic energy. Hence, if they are made to pass through a region having uniform magnetic field having direction perpendicular to that of velocity, they should move in a circle of same radius.

$$r = \frac{mv}{qB} = \frac{mv}{2eB} = \frac{\sqrt{2Km}}{2eB}$$

Experimental Observation:

Experimentally it has been seen that all the α -particles do not move in the circle of the same radius, but they move in the circles having different radii.



Concept Reminder

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He + Q$$

Kinetic energies are-

$$T_\alpha = \left(\frac{A-4}{A}\right) Q$$

$$T_y = \frac{4}{A} Q$$

Rack your Brain

A radioactive nucleus of mass M emits a photon of frequency υ and nucleus recoils. The recoil energy will be:

(1) $Mc^2 - h\upsilon$ (2) $\frac{h^2\upsilon^2}{2Mc^2}$

(3) Zero (4) $h\upsilon$

This shows that they have different value of kinetic energies. But it is also observed that they follow circular paths of some fixed values of radius i.e. yet the energy of emitted α -particles is not same but it is quantized. The reason behind this is that all the daughter nuclei produced are not in their ground state but some of the daughter nuclei may be produced in their excited states and they emit photon to acquire their ground state.

$$X \longrightarrow Y^*(\text{excited state}) + \alpha + Q$$
$$\downarrow$$
$$Y + \text{photon } (\gamma \text{ particle})$$

The only difference between Y and Y* is that Y* is in excited state and Y is in ground state.
Let, the energy of emitted γ -particles be E

$$\therefore \quad Q = T_\alpha + T_Y + E$$

where $Q = [M_x - M_Y - M_{He}]c^2$

$$T_\alpha + T_Y = Q - E$$

$$T_\alpha = \frac{m_Y}{m_\alpha + m_Y}(Q - E) \;;\; T_Y = \frac{m_\alpha}{m_\alpha + m_Y}(Q - E)$$

Ex. $\;{}^{A}_{Z}X \longrightarrow {}_{Z-2}Y^{A-4} + {}_2He^4 + Q$

Calculate K.E. of α-particle.

Sol. $\; Y \xleftarrow{(p)} \underset{\text{Rest}}{\otimes} \xrightarrow{(p)} \alpha \qquad K.E. = \dfrac{p^2}{2m}$

{Momentum of both Y and α is same and equal to P}

$$\frac{K_\alpha}{K_y} = \frac{m_y}{m_\alpha} = \frac{A-4}{4} \Rightarrow K_\alpha = \frac{A-4}{A}Q$$

and $K_y = \dfrac{4}{A}Q$

Therefore $K_\alpha >>> K_y$

Rack your Brain

Alpha particles are:
(1) Neutrally charged
(2) Positron
(3) Protons
(4) Ionized helium atom

Ex. Find the Q-value & the kinetic energy of the emitted α-particle in the α-decay of ${}^{226}_{88}Ra$

Given:

$m({}^{226}_{88}Ra) = 226.02540\,u, m({}^{220}_{86}Rn) = 222.0175\,u$

Sol. ${}^{226}_{88}Ra \longrightarrow {}^{222}_{86}Rn + {}^{4}_{2}He + Q$

$Q = [m({}^{226}_{88}Ra) - m({}^{222}_{86}Rn) - m({}^{4}_{2}He)]C^2$

K.E. of the α-particle

$$= \left(\frac{A-4}{A}\right)A = \left(\frac{226-4}{226}\right)Q = \frac{222}{226}Q$$

(2) β-emission

(a) Negative β-emission: - If a nucleus has (n >p) then a neutron disintegrate itself into proton and electron. These electrons come out of nucleus in the form of β particle. Nuclear reaction of β^- emission is given as

$$\boxed{{}_{Z}X^{A} \longrightarrow {}^{A}_{Z+1}Y + \beta^- + \bar{\nu}}$$

Here X and Y are isobars.
Also, decay equation of neutron: -

$n \longrightarrow p + e^- + \bar{\nu}$

β^-particle comes out of nucleus along with $\bar{\nu}$

After $_{-}\beta^{\circ}$ decay, n/p ratio decreases.

During β^- - decay, inside the nucleus a neutron is converted to a proton with emission of an electron and antineutrino.

$$n \rightarrow p + {}_{-1}e^{0} + \bar{\nu}$$

Let, M_x = mass of atom ${}_{z}X^{A}$
M_Y = mass of atom ${}_{z+1}Y^{A}$
m_e = mass of electron
Q value = $[(M_x - Zm_e) - \{(M_Y - (z+1)m_e) + m_e\}]c^2$
$= [M_x - M_Y]\ c^2$
Considering actual number of electrons.

Q value $= [M_x - \{(M_Y - m_e) + m_e\}]c^2 = [M_x - M_y]c^2$

(b) Positive β^+ -emission: - If a nucleus has

Concept Reminder

In beta decay, either a neutron is converted into proton or proton is converted into neutron.

Concept Reminder

β^--decay: To achieve stability, it increases Z by conversion of neutron into proton

${}_{Z}X^{A} \rightarrow {}_{Z+1}Y^{A} + e^{-1} + \bar{\upsilon}$

Q-value = $[M_x - M_y]c^2$

(p > n) then a proton disintegrates itself into neutron and positron. This positron comes out of nucleus in the form of positive β rays. Nuclear reaction of β^+ emission is given as

$$\boxed{{}_{Z}X^{A} \longrightarrow {}^{\;\;A}_{Z-1}Y + \beta^{+} + \nu}$$

Here X and Y are isobars.

Also, decay equation of proton:

$$p \longrightarrow n + e^{+} + \nu$$

After $_{+}\beta^{0}$ decay, n/p ratio increases.

$_{+}\beta^{0}$ always comes out from the nucleus along with neutrino.

As mass increases during conversion of proton to a neutron, hence it requires energy for β^+ decay to take place,

$\therefore$ β^+ decay is rare process. It can take place in the nucleus where a proton can take energy from the nucleus itself.

Q value = $[(M_X - Zm_e) - \{(M_y - (Z-1)\,m_e) + m_e\}]\,c^2$

$= [M_X - M_Y - 2m_e]\,c^2$

Considering actual number of electrons.

Q value = $[M_x - \{(M_y + m_e) + m_e\}]c^2$

$= [M_X - M_Y - 2m_e]\,c^2$

- **This reaction is possible only inside the nucleus. Free proton can never disintegrate. So, free proton is a stable particle whereas a free neutron disintegrates spontaneously. So, free neutron is an unstable particle.**
- **Properties of Neutrino & antineutrino**

(i) Both are chargeless

(ii) Have almost zero rest mass (very light particles)

(iii) Have spin quantum number ± 1/2 and spin angular momentum $\pm\; h/2\pi$ similar to electron.

(iv) These are suggested by Pauli to explain the problems of energy conservation, linear momentum conservation, spin conservation

Concept Reminder

β^+-decay: To achieve stability, it decreases Z by the converting a proton into neutron

$${}_{Z}X^{A} \rightarrow {}_{Z-1}Y^{A} + e^{+} + \upsilon$$

Q-value = $[M_x - M_y - 2m_e]c^2$

Concept Reminder

Besides β^- and β^+ emission, there is a third related process. This is electron capture and occurs when a nucleus absorbs one of its orbiting electrons.

and spin angular momentum conservation in β–decay.

(v) The electrons and positrons emitted in β-decay have a continuous spectrum of energies from zero to a limit $[Q = (\Delta m)c^2]$ as emitted particles share the available disintegration energy in random proportion.

(3) γ-decay: -

Similar to an atom, nucleus also have certain energy levels and nucleons occupy them. After α - decay (or β decay), daughter nucleus may be in excited state and return to ground state by emitting photons of high
energy (MeV order) called γ - photons.

Equation of γ-decay: –

$({}_ZX^A)^* \rightarrow {}_ZX^A + \gamma - \text{photons}\,(\text{or } h\nu)$

*Shows excited nucleus

(i) γ emission don't change the structure of nucleus

(ii) No change in Z and A

Ex. Consider the beta decay

$_{198}Au \rightarrow {}^{198}Hg^* + \beta^- + \overline{\nu}$

where ${}^{198}Hg^*$ represents nucleus of mercury in an excited state at energy 1.088 MeV above the ground state. What can be the maximum value of kinetic energy of the electron emitted? The atomic mass ${}^{198}Au$ is 197.968233 u & that of ${}^{198}Hg$ is 197.966760 u.

Sol. If the product nucleus ${}^{198}Hg$ is being formed in its ground state, then the kinetic energy available to the electron and the antineutrino is

$Q = [m({}^{198}Au) - m({}^{198}Hg)]c^2.$

As ${}^{198}Hg^*$ has energy 1.088 MeV more than ${}^{198}Hg$ in ground state, value of kinetic energy actually available is

Rack your Brain

A nucleus ${}^m_nX$ emits one α-particle and two β^- particles. The resulting nucleus is:

(1) ${}^{m-6}_{n-4}Z$ (2) ${}^{m-6}_{n}Z$

(3) ${}^{m-4}_{n}X$ (4) ${}^{m-4}_{n-2}Y$

$Q = [m(^{198}Au) - m(^{198}Hg)]c^2 - 1.088\,MeV$

= 1.3686 MeV – 1.088 MeV = 0.2806 MeV.

This is also the maximum possible kinetic energy of the electron emitted.

Ex. Calculate the Q-value in the following decays:

(a) $^{19}O \rightarrow {}^{19}F + e^- + \overline{\nu}$

(b) $^{25}Al \rightarrow {}^{25}Mg + e^+ + \nu$

The atomic masses needed are as follows:

^{19}O	^{19}F
19.003576 u	18.998403 u
^{25}Al	^{25}Mg
24.990432 u	24.985839 u

Sol. **(a)** The Q-value of β^--decay is

= [19.003576 u – 18.998403 u] (931 MeV/u)

= 4.816 MeV

(b) The Q-value of β^+-decay is

$Q = [m(^{25}Al) - m(^{25}Mg) - 2m_e]c^2$

$$= \left[24.99032\,u - 24.985839\,u - 2 \times 0.511\frac{MeV}{c^2}\right]c^2$$

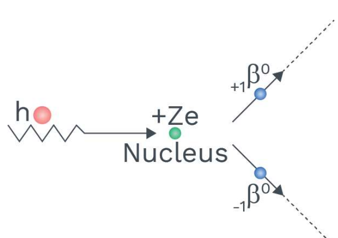
= (0.004593 u) (931 MeV/u) – 1.022 MeV

= 4.276 MeV – 1.022 MeV = 3.254 MeV.

> **Concept Reminder**
>
> The neutrino doesn't contribute to either the mass or charge balance since it has Q = 0 and m = 0.

Pair Production & Pair Annihilation

Collision of γ-ray photon by a nucleus & production of electron positron pair is known as pair production.



The rest mass of each of the electron & the positron is 9.1×10^{-31} kg. so, the rest mass energy

> **Definitions**
>
> - Collision of γ-ray photon by a nucleus & production of electron positron pair is known as pair production.

of each of than is

$E_0 = m_0 c^2 = (9.1 \times 10^{-31})(3 \times 10^8)^2$

$= 8.2 \times 10^{-14}$ joule

$= 0.51$ MeV

Hence for pair-production, it is essential that the energy of γ-photon must be at least $2 \times 0.51 = 1.02$ MeV.

$_{+1}\beta^0$ $_{-1}\beta^0$	γ-proton $\leftarrow$ $h\nu$ γ-proton $\rightarrow$ $h\nu$
(Before combining)	(After combining)

$$\underset{\text{(positron)}}{_{+1}\beta^0} + \underset{\text{(electron)}}{_{-1}\beta^0} = \underset{(\gamma\text{-proton})}{h\nu} + \underset{(\gamma\text{-proton})}{h\nu}$$

Concept Reminder

For pair-production, it is essential that the energy of γ-photon must be at least $2 \times 0.51 = 1.02$ MeV.

Pair Production And Pair Annihilation

Pair Production	**Pair Annihilation**
A γ-photon of energy more than $\geq$ 1.02 MeV, when interact with a nucleus produces pair of rest mass of $e^-\left(\text{or } e^+\right) = 0.51\text{MeV}$.	When electron and positron combine, they annihilates to each other and only energy is released in the form of two gamma photons.

The energy equivalent of rest of pair

$$\left(e^- + e^+\right) = 1.02\,\text{MeV}.$$

For pair production Energy of photon 1.02 MeV. If energy of photon is more than 1.02 MeV, the extra energy (E – 1.02) MeV divides approximately in equal amount to each particle as the kinetic energy.

$$(\text{K.E.})_{e^- \text{or } e^+} = \left[\frac{E_{Ph} - 1.02}{2}\right]\text{MeV}$$

If E < 1.02 MeV, pair will not produce.

KEY POINTS

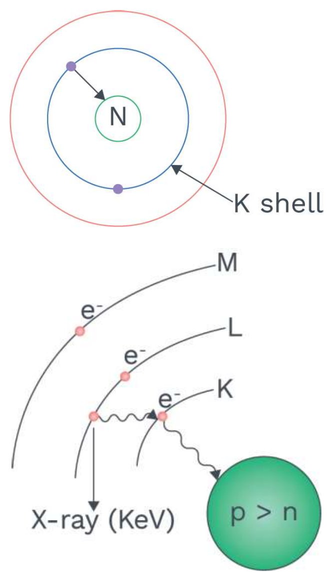
- Pair production
- Pair annihilation
- K capture

K Capture:

It is a rare process which is found only in few nuclei. In this process the nucleus captures one of the atomic electrons from the K shell. A proton in the nucleus combines with this electron and converts itself into a neutron. A neutrino is also emitted in the process and is emitted from the nucleus.

$$p + {}_{-1}e^{0} \rightarrow n + \nu$$

If X and Y are atoms, then reaction is written as:



$${}_{Z}X^{A} \rightarrow {}_{Z-1}Y^{A} + \nu + Q + \text{ characteristic x-rays of Y.}$$

If X and Y are taken as nucleus, then reaction is written as:

$${}_{Z}X^{A} + {}_{-1}e^{0} \rightarrow {}_{Z-1}Y^{A} + \nu$$

Note:

(1) Nuclei having atomic numbers from Z = 84 to 112 shows radioactivity.

(2) Nuclei having Z = 1 to 83 are stable (only few exceptions are there)

(3) Whenever a neutron is produced, a neutrino is also produced.

Concept Reminder

In K-capture, electron disappears in the process and a proton in the nucleus becomes a neutron, a neutrino is emitted as a result

$${}^{A}_{Z}N + e^{-} \rightarrow {}^{A}_{Z-1}N' + \upsilon$$

Rack your Brain

α-particle, β-particle and γ-rays are all having same energy. Their penetrating power in a given medium in increasing order will be:

(1) γ, α, β (2) α, β, γ

(3) β, α, γ (4) β, γ, α

(4) Whenever a neutron is converted into a proton, an antineutrino is produced.
(5) It is usually accompanied by x-ray emission.

Uses Of Radioactive Isotopes

1. In Medicine
- Co^{60} for treatment of cancer
- Na^{24} for circulation of blood
- I^{131} for thyroid
- Sr^{90} for treatment of skin & eye
- Fe^{59} for location of brain tumour
- Radiographs of castings and teeth

2. In Industries
- For detecting leakage in water and oil pipelines.
- For the investigation of wear & tear, study of plastics & alloys, thickness measurement.

3. In Agriculture
- C^{14} to study kinetics of plant photosynthesis.
- P^{32} to find nature of phosphate which is best for given soil & crop.
- Co^{60} for protecting potato crop from earth worm and sterilization of insects for pest control.

4. In Scientific research
- K^{40} to find age of meteorites
- S^{35} in factories

5. Carbon dating
- It is used to find the age of earth and fossils
- The age of earth is found by the disintegration of Uranium and fossil age by disintegration of C^{14}.
- The estimated age of the earth is about 5×10^9 years.
- The half-life of C^{14} is 7500 years.

6. As Tracers
- A very small quantity of the radio isotope present in any specimen is called tracer.
- This technique is used to study complex biochemical reactions, in detection of

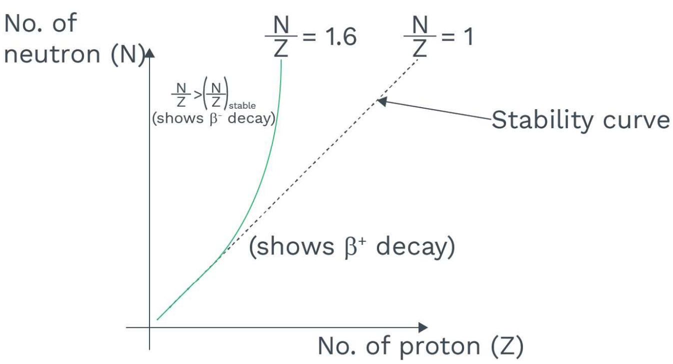
Concept Reminder

In β-decay, it is the weak nuclear force that plays the crucial role. The neutrino is unique as it interacts with matter only via the weak force.

cracks, blockage etc., tracing sewage or silt in sea

9. Nuclear Stability:

Figure shows a plot of neutron number N versus proton number Z for the nuclide found in nature. In the figure the solid line represents the stable nuclide. For light stable nuclide, the neutron number is equal to the proton number so that ratio N/Z is equal to 1. The ratio N/Z increases for the heavier nuclide and becomes about 1.6 for the heaviest stable nuclide. The points (Z, N) for stable nuclide fall in a rather well-defined narrow region. There are nuclide to the left of the stability belt as well as to the right of it. The nuclide to the left of the stability region have excess neutrons, whereas those to the right of the stability belt have excess protons. According to the laws of radioactive disintegration, these nuclide are unstable and decay with time. Nuclide with excess neutrons (lying above stability belt) show β^- decay while nuclide with excess protons (lying below stability belt) show β^+ decay and K - capture.



Ex. The nucleus $^{23}_{10}Ne$ decays by β^- emission. Write down the β-decay equation and determine the maximum kinetic energy of the electrons emitted. Give that:

$$m(^{23}_{10}Ne) = 22.994466\,u;\, m(^{23}_{11}Na) = 22.989770\,u$$

Rack your Brain

A nuclear reaction given by-

$${}_Z X^A \rightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + \overline{\upsilon}$$

represents:

(1) β-decay (2) γ-decay
(3) Fusion (4) Fission

KEY POINTS

- Nuclear stability
- Radioactive decay law

Sol. $^{23}_{10}Ne \rightarrow ^{23}_{11}Na + \bar{e} + \bar{\nu} + Q$

For β^- - decay, Q = [M(x)-M(y)]C²

$= [22.994466 - 22.989770]931.5$

$= 0.004696 \times 931.5 = 4.37\,MeV$

Ex. Calculate the binding energy of an α - particle. Given that mass of proton = 1.0073 u, mass of neutron = 1.0087 u. and mass of α -particle = 4.0015 u.

Sol. m_P = 1.0073 u, m_N = 1.0087 u, M = 4.0015 u

$N = A - Z = 4 - 2 = 2 \left(\because {}_2He^4 = {}_ZX^A\right)$

B.E = $\Delta m \times 931.5\,MeV$

$= \left\{\left[Zm_p + (A - Z)m_n\right] - M\right\} \times 931.5$

$\left[\left[(2 \times 1.0073) + (2 \times 1.0087) - 4.0015\right]\right] \times 931.5\,MeV$

$= 0.0305 \times 931.5\,MeV; B.E = 28.4\,MeV$

Ex. How many α and β -particles are emitted when uranium nucleus $({}_{92}U^{238})$ decay to ${}_{82}Pb^{214}$?

Sol. Let n be the number of α-particles and m be the number of β-particles emitted.

${}_{92}U^{238} \rightarrow {}_{82}Pb^{214} + n\ {}_2He^4 + m\ {}_{-1}e^0.$

As mass number is conserved, 238= 214 + 4n

4n = 24

n = 6

As charge is conserved, $92 = 82 + 2n + m(-1)$

$10 = 2(6) - m \ (\because n = 6)$

m = 2.

$\therefore$ 6α -particles and 2 β -particles are emitted

Radioactive Decay Law:

Based on their experimental observations and analysis of certain radioactive materials Rutherford and Soddy formulated a theory of radioactive decay. According to them After decay of a nucleus the new product (daughter) of nucleus has totally different physical as well as chemical properties.

The rate of radioactive decay (or) the number of nuclei decaying per unit time at any instant is directly proportional to the number of nuclei (N) present at that instant and is independent of the external physical conditions like temperature, pressure etc.

Let 'N' be the number of radioactive atoms present at a time 't' and N_0 is the initial number of radioactive nuclei. Let dN atoms disintegrate in time 'dt'. According to the law of radioactive decay

$$\frac{-dN}{dt} \propto N$$

$$\frac{-dN}{dt} = \lambda N \text{ (N = No. of active nuclei)}$$

where, λ is decay constant

Note: λ is characteristic property of nucleus, so, it remains unaffected with any physical and chemical changes.

Now, $\frac{-dN}{dt} = \lambda N$

$$\frac{dN}{N} = -\lambda dt$$

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt \Rightarrow [\ln N]_{N_0}^{N} = -\lambda [t]_0^t$$

$$\ln N - \ln N_0 = -\lambda t$$

$$\ln \frac{N}{N_0} = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t} \Rightarrow \boxed{N = N_0 e^{-\lambda t}}$$

No. of decayed nuclei after time t.

$$N' = N_0 - N$$

$$N' = N_0(1 - e^{-\lambda t})$$

It means No. of active nuclei decreases exponentially w.r.t. time.

Concept Reminder

In all three types of decay of any radioactive sample. It is found that the number of nuclei undergoing the decay per unit time is proportional to total number of undecayed nuclei in sample.

Concept Reminder

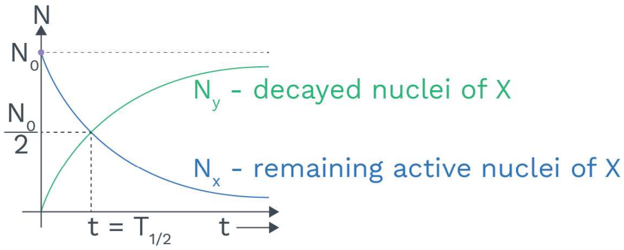
- $\frac{dN}{dt} = -\lambda N$
- $N = N_0 e^{-\lambda t}$
- When an α or β-decay takes place, the daughter nucleus is usually in higher energy state. Such a nucleus comes to ground state by emitting a photon or protons.

Rack your Brain

For a radioactive nucleus, half-life is 10 minutes. If there are 600 number of nuclei, the time taken for disintegration of 450 nuclei is?

EX.- X $\longrightarrow$ y

	Unstable	stable
$t = 0$	N_0	0
$t = T_{1/2}$	$\frac{N_0}{2}$	$\frac{N_0}{2}$
$t \to \infty$	0	N_0

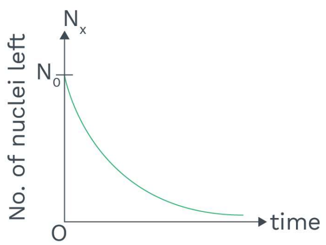


This shows that the number of radioactive nuclei decreases exponentially with time.

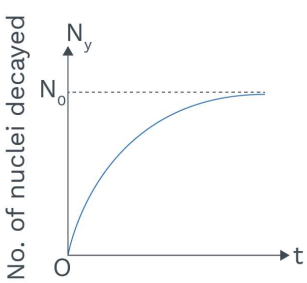
Above equation is known as the decay law (or) the law of radio-active decay. It is an exponential law.

Taking logarithm on both sides for the above equation. $\log_e N = \log_e N_0 - \lambda t \Rightarrow \lambda t = \log_e \frac{N_0}{N}$

$$\therefore \boxed{t = \frac{1}{\lambda}\ln\left(\frac{N_0}{N}\right)}$$



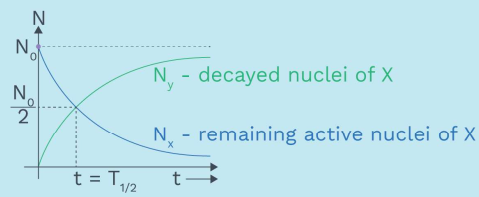
(a)



(b)

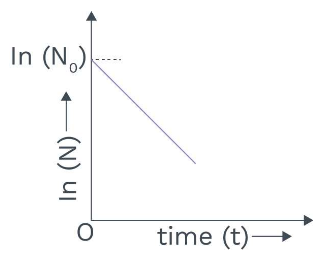
Concept Reminder

The number of radioactive nuclei decreases exponentially with time.



Rack your Brain

Half-lives of two radioactive substances A and B are respectively 20 minutes and 40 minutes. Initially the samples of A and B have equal number of nuclei. After 80 minutes find ratio of remaining numbers of A and B.

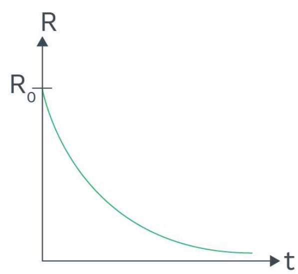


(c)

Activity (R):
The number of decays per unit time (or) decay rate is called activity (R).

$$|R| = \left|\frac{dN}{dt}\right| = \frac{d}{dt}\left(N_0 e^{-\lambda t}\right) \text{ (or) } R = \lambda N = \lambda N_0 e^{-\lambda t} \text{ (or)}$$

$R = R_0 e^{-\lambda t}$, where $R_0 = \lambda N_0$ is the decay rate at $t = 0$, called initial activity.



If a nucleus can decay simultaneously by n processes, which have activities R_1, R_2, ... and R_n. Then the resultant activity $R = R_1 + R_2 + \ldots\ldots\ldots\ldots + R_n$. If nucleus decays simultaneously more than one process is called parallel decay. The S.I unit of activity is Becquerel (Bq) and other units are curie (Ci) and Rutherford (Rd).

1 Bq = 1 decay per second,

1 Rd = 10^6 decays per second = 10^6 Bq

1 Ci = 3.7×10^{10} decays per second = 10^{10} Bq

Note: Curie is approximately equal to the activity of one gram of pure radium.

Definitions

- The number of decays per unit time (or) decay rate is called activity (R).
- $R = R_0 e^{-\lambda t}$, where $R_0 = \lambda N_0$ is the decay rate at $t = 0$, called initial activity.

Concept Reminder

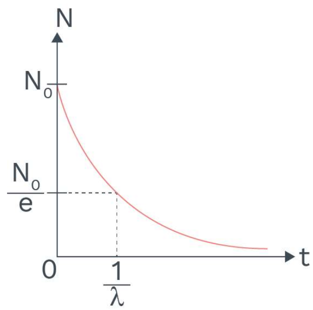
The S.I unit of activity is Becquerel (Bq) and other units are curie (Ci) and Rutherford (Rd).
1 Bq = 1 decay per second,
1 Rd = 10^6 decays per second.
1 Ci = 3.7×10^{10} decays per second.

Definitions

The decay constant λ for a given radioactive sample is defined as the reciprocal of the time during which the number of nuclei decreases to $\frac{1}{e}$ times their original value.

Decay Constant (λ)

It gives the ability of a nucleus to decay. The decay constant λ for a given radioactive sample is defined as the reciprocal of the time during which the number of nuclei decreases to $\frac{1}{e}$ times their original value.



(1) Larger value of λ corresponding to decay in smaller time and vice versa.

(2) λ = 0 for stable nuclei.

(3) Decay constant is the characteristic of the sample taken and does not vary with time.

(4) If a nucleus can decay simultaneously by more than one process (say n), which have decay constants λ_1, λ_2 and λ_n, then the effective decay constant is $\lambda = \lambda_1 + \lambda_2 + ... + \lambda_n$. This is called parallel decay.

Half Life (T) :

As the name tells, the half-life of a radioactive sample is defined as "The time interval during which the activity of a radioactive sample falls to half of its value, (or) The time interval during which the number of radioactive nuclei of a sample disintegrate to half of its original number of nuclei" Half-lives vary from isotope to isotope. While T may be as small as 10^{-16} s, its largest value may be as big as 10^9 years.

Concept Reminder

If a nucleus can decay simultaneously by more than one process (say n), which have decay constants λ_1, λ_2 and λ_n, then the effective decay constant is $\lambda = \lambda_1 + \lambda_2 + ... + \lambda_n$. This is called parallel decay.

Definitions

Half Life: The time interval during which the number of radioactive nuclei of a sample disintegrate to half of its original number of nuclei"

E.g., Half-life of uranium $\left({}^{238}_{92}U\right)$ is 4.47×10^9 years and half-life of krypton $\left({}^{89}_{36}Kr\right)$ is 3.16 minutes.

Relation between decay constant (λ) and half-life period (T).

From law of Radioactive decay $\frac{N}{N_0} = e^{-\lambda t}$

when $N = \frac{N_0}{2}, t = T$

$\therefore \frac{1}{2} = e^{-\lambda T}$ or $2 = e^{\lambda T}$

(or) $\log_e 2 = \lambda T \quad \therefore T = \frac{2.303 \log_{10} 2}{\lambda} = \frac{0.693}{\lambda}$

$$\therefore \boxed{T = \frac{\ln 2}{\lambda} = \frac{2.303 \log 2}{\lambda} = \frac{0.693}{\lambda}}$$

The above relation establishes that the half - life (T) depends upon the decay constant λ of the radioactive substance. The value of λ is different for different radioactive substances.

Note:

(i) Half-life is the characteristic property of the sample and T cannot be changed by any known method.

(ii) At any given instant whatever be the amount of the undecayed sample, it will be reduced to exactly half its value after a time equal to the half-life of the sample.

(ii) In parallel decay $\lambda = \lambda_1 + \lambda_2 + \ldots\ldots \lambda_n$ hence $\frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2} + \ldots\ldots \frac{1}{T_n}$, where T is the equivalent half-life and $T_1, T_2 \ldots\ldots T_n$ are the half-lives in individual decay.

Application:

In a radioactive sample the number of nuclides undecayed after n-half-lives (i.e., t = nT) is

$$t = nT = \frac{1}{\lambda} \ln\left(\frac{N_0}{N}\right) \text{ or } \frac{n(\ln 2)}{\lambda} = \frac{1}{\lambda} \ln\left(\frac{N_0}{N}\right)$$

KEY POINTS

- Half life
- Decay constant

Concept Reminder

- $T = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$
- $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

Where $n = \frac{t}{T_1 / 2}$

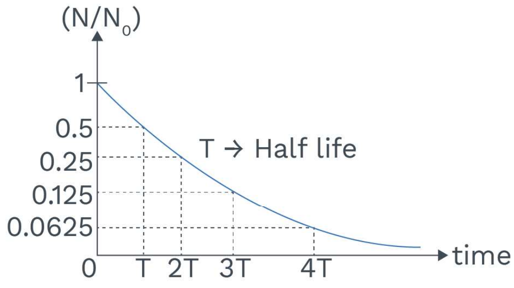
or $2^n = \frac{N_0}{N}$; or $\boxed{N = N_0\left(\frac{1}{2}\right)^n}$

Note: The number of nuclei remain in the sample after half of half-life period (t = T/2) is given by

$$N = N_0\left(\frac{1}{2}\right)^n \text{ here } n = \frac{1}{2}\text{, then } N = N_0\left(\frac{1}{2}\right)^{\frac{1}{2}}$$

$$\therefore \boxed{N = \frac{N_0}{\sqrt{2}}} \text{ taking } N_0 = 100,\ N = 50\sqrt{2} = 70.7$$

70.7% of nuclei remain and 29.3% of nuclei decayed.



Average Life (Or) Mean Life:

The phenomenon of radioactivity is random because we just can't predict which of the atoms in a given sample will decay first and when. Hence radioactivity process totally depends on chance. In decay process some of the atoms of the given sample may have very short life span, and others may not decay even after a very large span of time. So, to determine the ability of the nucleus to decay it would be useful to calculate the average life. Hence average life is defined **as the total lifetime of all the nuclei divided by the total number of original nuclei.**

$$\text{i.e } T = \frac{\sum \text{life span of individual nucleus}}{\text{Total number of original nuclei}} = \frac{\sum t}{N_0}$$

Let N_0 be the radioactive nuclei that are present at t = 0 in the radioactive sample. The number of nuclei which decay between t and (t + dt) is dN

Definitions

Average life: Hence average life is defined as the total lifetime of all the nuclei divided by the total number of original nuclei.

Rack your Brain

Half-life of a radioactive element is 12.5 hours, and its quantity is 256 g. After how much time its quantity will remain 1 g ?

i.e., the lifetime of these nuclei is 't'.

The total lifetime of these dN nuclei is (t dN)

$\therefore$ The total lifetime of all the nuclei presents initially in the sample $\int_{t=0}^{t=\infty} t\,dN$ [$\because$ N = 0 at infinity]

Average lifetime $\tau = \dfrac{\int t\,dN}{N_0}$ But $\dfrac{-dN}{dt} = \lambda N$

$dN = -\lambda N dt = -\lambda N_0 e^{-\lambda t} dt \qquad (\because N = N_0 e^{-\lambda t})$

$$\tau = \int_0^\infty t \frac{\lambda N_0 e^{-\lambda t}}{N_0} dt; \quad \boxed{\tau = \frac{1}{\lambda}}$$

The mean life (or) average life of a radioactive sample is reciprocal to decay constant.

We know that $N = N_0 e^{-\lambda t}$;

When $t = T$,

$$N = N_0 e^{-\frac{1}{T} \times T} = \frac{N_0}{e} = 0.37 N_0 = 37\% \text{ of } N_0$$

Hence average life period of a radioactive sample can also be defined as **"The time interval during which 63% of sample decays or sample reduces to 37% of its original amount".**

Relation Between Half Life Period And Average Life Period

We know that $T = \dfrac{0.693}{\lambda}$ & $\tau = \dfrac{1}{\lambda}$

Hence $T = 0.693\tau$

(or) $\boxed{\tau = \dfrac{T}{0.693} = 1.443T}$

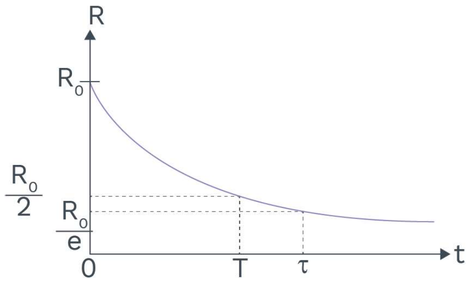
From the above given equation it is clear that average life period is 44.3% greater than half-life period.

Definitions

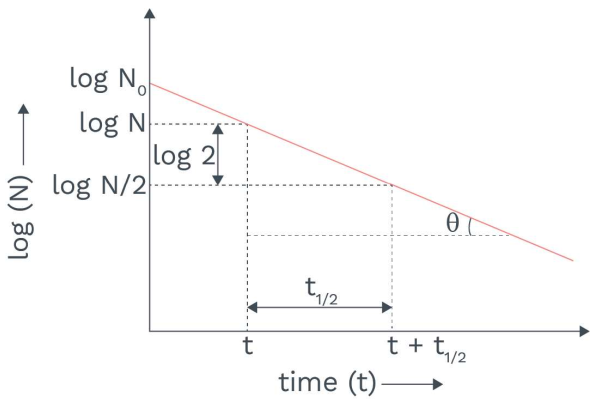
Average life period of a radioactive sample can also be defined as "The time interval during which 63% of sample decays or sample reduces to 37% of its original amount".

Concept Reminder

- $\tau = \dfrac{1}{\lambda}$
- $\tau = \dfrac{T}{0.693}$



Determination of decay constant (λ) and half-life period (T) of a radioactive sample graphically



If N_0 & N be the number of atoms present undecayed initially and after a time t, then

We know that $N = N_0 e^{-\lambda t}$ taking log on both sides

$$\log_e N = \log N_0 - \lambda t \Rightarrow \log N = \log N_0 - \frac{\lambda t}{2.303}$$

$$\log N = \left(\frac{-\lambda}{2.303}\right)t + \log N_0$$

Slope of the graph $m = -\tan\theta = \frac{-\lambda}{2.303}$

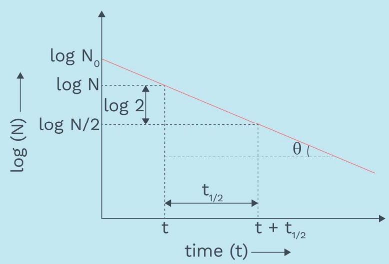
$\Rightarrow$ $\boxed{\lambda = 2.303 \tan\theta}$

Half-life period $T = \frac{2.03 \log 2}{\lambda}$

$T = \frac{2.03 \log 2}{2.303 \tan\theta}$ $\therefore$ $\boxed{T = (\log 2)\cot\theta}$

Note: In radioactive sample decay

Concept Reminder



- $\log N = \log N_0 - \frac{\lambda t}{2.303}$

(1) The probability survival of nucleus after time $P_S =$

(2) The probability of nucleus to disintegrate in time i

Concept Reminder

- **Active Fraction (A.F.)** $= \frac{1}{2^n}$ **and Decayed fraction (D.F.)** $= 1 - \frac{1}{2^n}$.
- $\boxed{m = m_0 e^{-\lambda t}}$

Remember for solving problems

- $N_0 \xrightarrow{t=T_{1/2}} \frac{N_0}{2} \xrightarrow{t=2T_{1/2}} \frac{N_0}{4}$

  $= \frac{N_0}{2^2} \xrightarrow{t=3T_{1/2}} \frac{N_0}{8} = \frac{N_0}{2^3}$

- If $t = nT_{1/2}$, then $N = \frac{N_0}{2^n} = N_0 e^{-\lambda t}$ where n = no. of $T_{1/2}$

- t and $T_{1/2}$ (given) $\longrightarrow n = \frac{t}{T_{1/2}} \longrightarrow$ **Active Fraction (A.F.)**

  $= \frac{1}{2^n}$ **and Decayed fraction (D.F.)** $= 1 - \frac{1}{2^n}$.

- Decayed Fraction (given at time t) $\longrightarrow$ A.F. = 1 – D.F. = $\frac{1}{2^n} \Rightarrow t = nT_{1/2}$

- Active mass also decreases exponentially w.r.t. time

  $\boxed{m = m_0 e^{-\lambda t}}$

  Here m_0 = initial active mass, m = remains active mass after time t

Ex. $\frac{15}{16}$ fraction of a radioactive substance disintegrates in t time. How long will it take in decay of $\frac{31}{32}$ fraction?

Sol. In time t D.F. $= \frac{15}{16}$

$$\text{A.F.} = \frac{1}{16} = \frac{1}{2^4} \Rightarrow n = 4 \quad \Rightarrow T_{1/2} = \frac{t}{4}$$

In $\frac{31}{2^5}$ decay, time is

$$t' = 5 \times \frac{t}{4} = \frac{5t}{4}$$

Ex. For a radioactive 'material' $T_{1/2} = 2$ hrs how much fraction will decay in 9 hrs?

Sol. $t = nT_{1/2} \Rightarrow n = 4.5$

$$A.F. = \frac{1}{2^n} = \frac{1}{2^{4.5}} = \frac{1}{2^4 \times 2^{0.5}} = \frac{1}{16\sqrt{2}}$$

$$D.F. = 1 - \frac{1}{16\sqrt{2}}$$

Ex. For a radioactive substance $T_{1/2}$ is 100 days. How much fraction will decay in 50 days?

Sol. $n = \frac{50}{100} = \frac{1}{2}$

$$A.F. = \frac{1}{2^{0.5}} = \frac{1}{\sqrt{2}} = 0.707$$

$$D.F. = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}} = 0.3$$

Ex. $T_{1/2} = 10\,\text{hr}$. How much time is required in decay of $\frac{19}{20}$ fraction?

Sol. $\frac{19}{20} \Rightarrow \frac{N}{N_0} = \frac{1}{20} \Rightarrow e^{-\lambda t} = 1/20$

or $t = \frac{1}{\lambda}\ln(20) = T_{1/2}\frac{\ln(20)}{\ln(2)} = 43.16\,\text{hrs}$

Ex. 10% of radioactive substance disintegrates in one hr. How much percentage of substance will decay in 2 hrs.?

Sol. **I[st] method:**

10% 1 hrs

$$100(A.F.) \xrightarrow{1hr} 90 \xrightarrow{1hr} 90 \times \frac{90}{100} = 81$$

d.F. = 100 – 81 = 19%

II[nd] method

$$\frac{N}{N_0} = A.F. = e^{-\lambda t}$$

$$0.9 = e^{-\lambda t}(t = 1hr)$$

At $t = 2$ hr

$$A.F. = x = e^{-\lambda(2t)} = \left(e^{-\lambda t}\right)^2 = (0.9)^2 = 0.81$$

d.F. = 0.19 or 19%

Ex. A and B are two radioactive samples with half period 2 hrs and 3 hrs respectively. Ratio of their active nuclei is 1 : 3 calculate ratio of active nuclei at the end of 6 hrs.

Sol. **I method:**

A	B
N	$3N$
2 hrs. $\downarrow$	3 hrs. $\downarrow$
$\boxed{\frac{N}{2}}$	$\boxed{\frac{3N}{2}}$
2 hrs. $\downarrow$	3 hrs. $\downarrow$
$\boxed{\frac{N}{4}}$	$\boxed{\frac{3N}{4}}$
2 hrs. $\downarrow$	
$\boxed{\frac{N}{8}} \Rightarrow N_A : N_B = N/8 : 3N/4 = 1 : 6$	

II method:

$$n_A = \frac{t}{T_A} = \frac{6}{2} = 3$$

$$n_B = \frac{6}{3} = 2$$

$$N = \frac{N_0}{2^n} \Rightarrow \frac{N_A}{N_B} = \frac{N_{0A}}{N_{0B}} \times \frac{2^{n_B}}{2^{n_A}} = \frac{1}{3} \times \frac{2^2}{2^3} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

Ex. Obtain the amount of ${}^{60}_{27}Co$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of ${}^{60}_{27}Co$ is 5.3 years.

Sol. $R = \left(\frac{N_A}{M_w} m\right)\frac{0.693}{T_{1/2}} = 7.106 \times 10^{-6}\, g$

Ex. A source contains two phosphorous radio nuclides ${}^{32}_{15}P(T_{1/2} = 14.3\text{days})$ and ${}^{33}_{15}P(T_{1/2} = 25.3\text{ days})$. Initially, 10% of the decays come from ${}^{33}_{15}P$. For how long one must wait until 90% do so?

Sol. Let the source initially contains 90% ${}^{33}_{15}P$ (say P_1) and 10% ${}^{32}_{15}P$ (say P_2) [i.e., 9x grams of P_1 and x gram of P_2]. If the source contains 90% P_2 & 10% P_2 (i.e., 9y grams of P_2 and y grams of P_1) after t days, we are to calculate t,

$$\text{As} \frac{N_0}{N} = 2^{t/T_{1/2}}$$

for $P_1 = \frac{9x}{y} = 2^{t/14.3}$

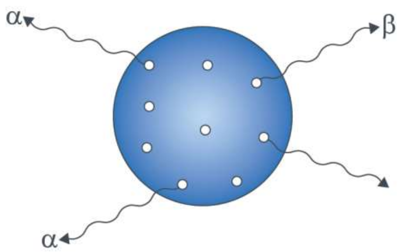
and for $P_2 = \frac{x}{9y} = 2^{t/25.3}$

clearly $\frac{9x/4}{x/9y} = \frac{2^{t/14.3}}{2^{t/25.3}}$

$81 = 2^{0.0304t} \Rightarrow t = 209\,d$

Decay by two simultaneous processes: -

$_{85}Ra^{226}$ is an exception which can emit α as well as β particle.



λ_α and λ_β

left $\lambda_{eff} = \lambda_\alpha + \lambda_\beta$

$$\frac{1}{(T_{1/2})\,eff} = \frac{1}{T_\alpha} + \frac{1}{T_\beta}$$

$$(T_{1/2})\,eff = \frac{T_\alpha T_\beta}{T_\alpha + T_\beta}$$

Concept Reminder

If α and β-decay occur simultaneously then:

- $\lambda_{eff} = \lambda_\alpha + \lambda_\beta$
- $T_{eff} = \frac{T_\alpha T_\beta}{T_\alpha + T_\beta}$

Carbon Dating:

- Half period of C^{14} is 5730 yrs. In our atmosphere ratio of C^{12} and C^{14} is almost constant and same ratio exist in all living things.
- After death of living things its ^{14}C decreases. To find age of fossil current ratio of C^{12} and C^{14} is measured and compared with initial ratio of C^{12} and C^{14}.
- Carbon dating is best suitable upto 10^4 yrs older fossils.

Concept Reminder

- Half-lives of radioactive elements vary over a very wide range. They can be short as 10^{-15} s and as long as 10^{10} years.

- For older objects uranium dating can be used, because half-life of uranium is of the order of 10^9 yrs.

Ex. A radioactive substance initiative has 6.0 × 10^{18} active nuclei. What is the required time for the active nuclei of the same substance to become 1.0 × 10^{18} if its half-life is 40 s.

Sol. The number of active nuclei at any instant of time t,

$$\frac{N_0}{N} = e^{\lambda t}; \log_e\left(\frac{N_0}{N}\right) = \lambda t$$

$$\therefore t = \frac{\log_e\left(\frac{N_0}{N}\right)}{\lambda} = \frac{2.303\log_{10}\left(\frac{N_0}{N}\right)}{\lambda}$$

In this problem, the initial number of active nuclei,

$$N_0 = 6.0\times10^{18}; N = 1.0\times10^{18}, T = 40\,s$$

$$\lambda = \frac{0.693}{T} = \frac{0.693}{40} = 1.733\times10^{-2}\,s^{-1}$$

$$t = \frac{2.303\log_{10}\left(\frac{6.0\times10^{18}}{1.0\times10^{18}}\right)}{1.733\times10^{-2}} = 1.034\times10^{2}\ s$$

Ex. Plutonium decays with a half-life of 24,000 years. If plutonium is stored for 72,000 years, what fraction of it remains?

Sol. $T_{1/2}$ = 24,000 years

Duration of time (t) = 72,000 years

Number of half-lives (n) = $\frac{t}{T_{1/2}} = \frac{72000}{24000} = 3$

$$\therefore 1g \xrightarrow{1} \frac{1}{2}g \xrightarrow{2} \frac{1}{4}g \xrightarrow{3} \frac{1}{8}g$$

$\therefore$ Fraction of plutonium remains = $\frac{1}{8}g$

Ex. A certain substance decays to 1/32 of its initial activity in 25 days. Calculate its half

Rack your Brain

The half-life of a radioactive isotopes X is 50 years. If decays to another element Y which is table. The two elements X and Y were found to be in the ratio of 1 : 15 in a sample of rock. Find the age of rock.

-life.

Sol. $1g \xrightarrow{1} \frac{1}{2}g \xrightarrow{2} \frac{1}{4}g \xrightarrow{3} \frac{1}{8}g \xrightarrow{4}$

$\frac{1}{16}g \xrightarrow{5} \frac{1}{t_{1/2}} \Rightarrow t_{1/2} = \frac{t}{n} = \frac{25}{5}; t_{1/2} = 5\,\text{days}$

Ex. The half -life period of a radioactive substance is 20 days. What is the time taken for $(7/8)^{th}$ of its original mass to disintegrate?

Sol. Let the initial mass be one unit.

Mass remaining = $= 1 - \frac{7}{8} = \frac{1}{8}$

A mass of 1 unit becomes $\frac{1}{2}$ unit in 2^{nd} half-life $\frac{1}{4}$ unit becomes $\frac{1}{8}$ unit in 3^{rd} half life

∴ Time taken = 3 half-lives = 3 × 20 = 60 days

Ex. How many disintegrations per second will occur in one gram of ${}^{238}_{92}U$, if its half-life against α -decay is 1.42×10^{14} s?

Sol. Given Half -life period

$(T) = \frac{0.693}{\lambda} = 1.42 \times 10^{17}\,s$

$\lambda = \frac{0.693}{1.42 \times 10^{17}} = 4.88 \times 10^{-18}$

Avogadro number (N) = 6.023×10^{23} atoms

n = Number of the atoms present in 1 g of

${}^{238}_{92}U = \frac{N}{A} = \frac{6.023 \times 10^{23}}{238} = 25.30 \times 10^{20}$

Number of disintegrations = $\frac{dN}{dt} = \lambda n$

$= 4.88 \times 10^{-18} \times 25.30 \times 10^{20}$

$= 1.2346 \times 10^{4}$ disintegrates/sec

Ex. The half-life of a radioactive substance is 5000 years. In how many years, its activity will decay to 0.2 times of its initial value?

Concept Reminder

In a magnetic field, γ-rays are un-deviated, and β-particles are the most deviated.

Given $\log_{10} 5 = 0.6990$.

Sol. T = 5000 years,

$$\frac{N}{N_0} = 0.2 = \frac{2}{10} = \frac{1}{5}$$

$$\lambda = \frac{0.693}{T} = \frac{0.693}{5000}$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\frac{1}{5} = \frac{1}{e^{\lambda t}} \Rightarrow 5 = e^{\lambda t}$$

$$\log_e 5 = \lambda t$$

$$t = \frac{2.303 \times 0.6990 \times 5000}{0.693}$$

$t = 11614.6$ years $= 1.1615 \times 10^4$ years

Concept Reminder

We can represent a nuclear collision or reaction by the following notation, which means X (a, b) Y

$$a + \underset{(\text{rest})}{X} \rightarrow Y + b$$

Nuclear Fission

The splitting of a heavy nucleus (A > 230) into the two or more lighter nuclei when struck by a thermal neutron.

In this process a certain mass disappears which is obtained in the form of energy (enormous amount)

A + n $\rightarrow$ excited nucleus $\rightarrow$ B + C + Q.

Hahn and Strass-man did the first fission experiment (fission of the nucleus of U^{235}).

When U^{235} is bombarded with a neutron it splits into two fragments & 2 - 3 secondary neutrons & releases about 200 MeV energy per fission (or from single nucleus)

Fragments are uncertain but each time the energy released is almost same.

Possible reactions are –

$$U^{235} + {}_0n^1 \rightarrow Ba + Kr + 3\,{}_0n^1 + 200\,\text{MeV}$$

or $$U^{235} + {}_0n^1 \rightarrow Xe + Sr + 2\,{}_0n^1 + 200\,\text{MeV}$$

and many other reactions are also possible.

(i) The average number of the secondary neutrons is 2.5.

Definitions

- Nuclear fission is a process of splitting a heaving nucleus into two lighter nuclei along the conversion of mass defect into energy.

(ii) The nuclear fission can be explained by using " liquid drop model" also.

(iii) The mass defect Δ m is about 0.1% of mass of fission nucleus

(iv) About 93% of the released energy (Q) is appear in the form of kinetic energies of products and about 7% part in the form of γ -rays.

Natural Uranium

It is mixture of U^{235} (0.7%) and U^{238} (99.3%)

U^{235} is easily fissionable, by slow neutron (or thermal neutrons) having K.E. of the order of 0.03 eV.

To improve the quality, percentage of U^{235} is increased to 3%. The improved uranium is called 'Enriched Uranium' (97% U^{238} and 3% U^{235})

Losses of Secondary Neutrons

Leakage of neutrons from the system

Due to their high K.E. some neutrons escape from the system.

Absorption of neutrons by U^{238}

U^{238} is not fissionable by these secondary fast neutrons. But U^{238} absorbs some fast neutrons.

Critical Size (or mass)

In order to sustain chain reaction in a sample of enriched uranium, it is required that the number of lost neutrons should be much smaller than the number of neutrons produced in a fission process. For it the size of uranium block should be equal or greater than a certain size called critical size.

Reproduction factor-

$$(K) = \frac{\text{rate of production of neutrons}}{\text{rate of loss of neutrons}}$$

Concept Reminder

U^{235} is easily fissionable, by slow neutron (or thermal neutrons) having K.E. of the order of 0.03 eV.

KEY POINTS

- Nuclear reaction
- Nuclear fission
- Secondary neutron
- Reproduction factor
- Nuclear reactor

Concept Reminder

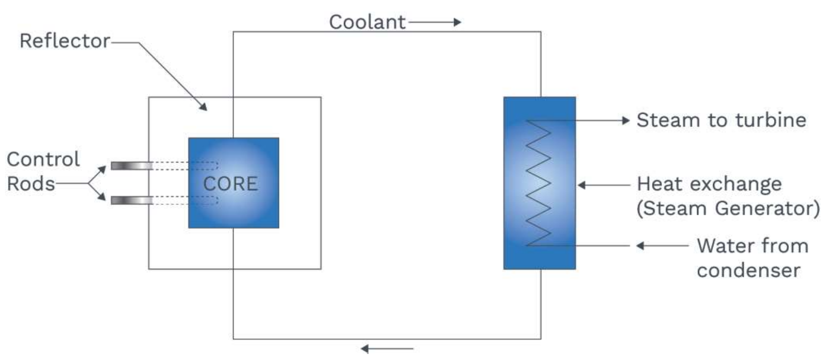
Reproduction factor:

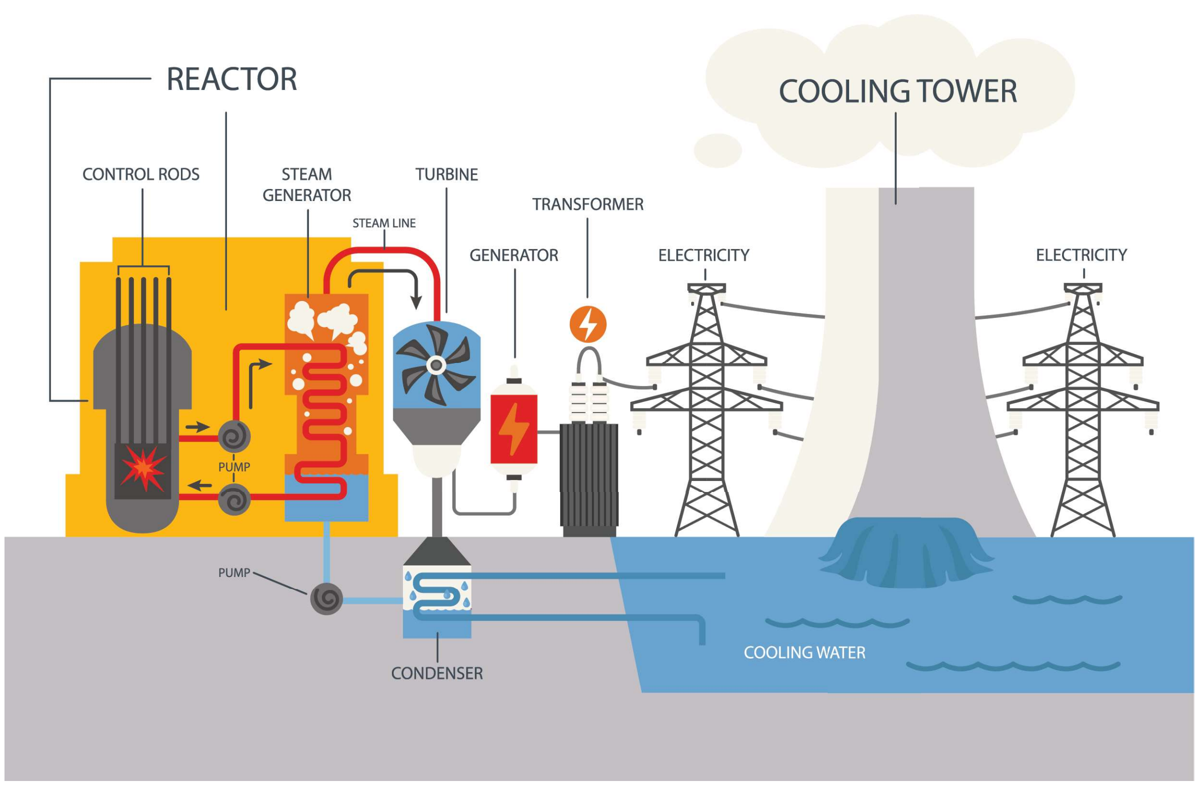
$$(K) = \frac{\text{rate of production of neutrons}}{\text{rate of loss of neutrons}}$$

(i) If size of Uranium used is 'Critical' then K = 1 and the chain reaction will be steady or sustained (As in nuclear reaction).
(ii) If size of Uranium used is 'Super critical' then K > 1 and chain reaction will accelerate resulting in a explosion (As in atom bomb).
(iii) If size of Uranium used is 'Sub Critical' then K < 1 and chain reaction will retard and ultimately stop.

Nuclear Reactor (K = 1)

Its main constituents are -





Moderator

Its function is to slow down the fast secondary neutrons. Because only slow neutrons are capable for the fission of U^{235}. The moderator should be light and it should not absorb neutrons. Commonly, Heavy water (D_2O, molecular weight 20 gm.) are used.

Control rods:

They have the ability to capture the slow neutrons and can control the rate of chain reaction at any stage.

Boron and Cadmium are best absorber of neutrons.

Coolant:

A substance which absorbs the produced heat and transfers it to water for further use. Generally, coolant is water at high pressure

Fast Breeder Reactors

The atomic reactor in which fresh fissionable fuel (Pu^{239}) is produced along with energy.

Fuel: Natural Uranium.

During fission of U^{235}, energy and secondary neutrons are produced. These secondary neutrons are absorbed by U^{238} and U^{239} is formed. This U^{239} converts into Pu^{239} after two beta decays. This Pu^{239} can be separated, its half-life is 2400 years.

$$_{92}U^{238} + {}_0n^1 \rightarrow {}_{92}U^{239} \xrightarrow{2\beta^-} {}_{94}Pu^{239}$$

(Best fuel of fission)

This Pu^{239} can be used in nuclear weapons because of its small critical size than U^{235}.

Ex. In a nuclear reactor, the fission is produced in 1 g for U^{235} (235.0439) in 24 hours by slow neutrons (1.0087 u). Assume that $_{35}Kr^{92}$ (91.8973 u) and $_{56}Ba^{141}$ (140.9139 amu) are produced in all reactions and no energy is lost.

KEY POINTS

- Moderator
- Control rods
- Coolants
- Fast breeder reactors
- Fuel

Concept Reminder

The fact that more neutrons are produced in fission than are consumed gives the possibility of chain reaction.

Rack your Brain

The power obtained in a reactor using U^{235} disintegration is 1000 kw. The mass decay of U^{235} per hour, is:

(1) 10 μg (2) 20 μg
(3) 40 μg (4) 1 μg

(i) Write the complete reaction (ii) Calculate the total energy produced in kilowatt hour. Given 1u = 931 MeV.

Sol. The nuclear fission reaction is

$$_{92}U^{235} + {}_0n^1 \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3{}_0n^1$$

Mass defect

$$\Delta m = \left[(m_u + m_n) - (m_{Ba} + m_{kr} + 3m_n)\right]$$

$$= 256.0526 - 235.8373 = 0.2153\,u$$

Energy released Q = 0.2153 × 931 = 200 MeV.

Number of atoms in

$$1g = \frac{6.02 \times 10^{23}}{235} = 2.56 \times 10^{21}$$

Energy released in the process of fission of 1 g of U^{235} is E

$= 200 \times 2.56 \times 10^{21} = 5.12 \times 10^{23}$ MeV

$= 5.12 \times 10^{23} \times 1.6 \times 10^{-13} = 8.2 \times 10^{10}$ J

$$= \frac{8.2 \times 10^{10}}{3.6 \times 10^6} kWh = 2.28 \times 10^4\ kWh$$

Ex. How much mass of U^{235} is required per sec to operate the nuclear reactor generating power 100 KW.

Sol. $\frac{\text{energy}}{\text{time}} = 10^5\ J$

$$1\text{sec} \longrightarrow 10^5\ \text{Joule energy}$$

$$1\text{gm} \rightarrow 8 \times 10^{10}\ J$$

'm' gm $8m \times 10^{10}$J = E

$$10^5 = \frac{8m \times 10^{10}}{1}$$

$$m = \frac{10^5}{8 \times 10^{10}} = \frac{1}{8} \times 10^{-5} = 0.12 \times 10^{-5} = 1.2 \times 10^{-6}$$

Ex. Calculate no. of fissions of U^{235} required per sec to generate power of 10 KW.

Sol. $1\text{fission} \rightarrow 200\,\text{MeV}$

1 fission = $200 \times 10^6 \times 1.6 \times 10^{-19}$ Joule

Rack your Brain

The binding energy per nucleon in deuterium and helium nuclei are 1.1 MeV and 7.0 MeV, respectively. When two deuterium nuclei, fuse to form a helium nucleus then find energy released in fusion.

10^4 Joule $\rightarrow$? fission

N fission = 3.2×10^{-11} N J

$$P = \frac{E}{t} \Rightarrow 10^4 = \frac{3.2 \times 10^{-11} N}{1} \Rightarrow N = 3.12 \times 10^{14}$$

Nuclear Fusion

It is the phenomenon of fusion two or more lighter nuclei to form a single heavy nucleus.

$A + B \rightarrow C + Q$ (Fusion)

The product (C) is more stable than reactants (A and B).

and $m_C < (m_A + m_B)$

and mass defect $\Delta m = [(m_A + m_B) - m_C]$ amu

Energy released is E = $\Delta m \times 931$ MeV / amu

The total binding energy and the binding energy per nucleon C both are more than of A and B.

$$\Delta E = E_C - (E_A + E_B)$$

Fusion of four hydrogen nuclei into helium nucleus–

$$4(_1H^1) \rightarrow {}_2He^4 + 2\,{}_{+}\beta^0 + 2\nu + 26.7\,\text{MeV}$$

(i) Energy released per fission >> Energy released per fusion

(ii) Energy per nucleon in fission $\left[= \frac{200}{235} \simeq 0.85\,\text{MeV} \right]$ << energy per nucleon in fusion $\left[= \frac{26}{4} \simeq 6.5\,\text{MeV} \right]$

Required Condition for Nuclear fusion

(a) **High Temperature**

Which provide kinetic energy to nuclei to overcome the repulsive electrostatic force between them.

KEY POINTS

- Nuclear fusion

Definitions

It is the phenomenon of fusing two or more lighter nuclei to form a single heavy nucleus.

Concept Reminder

Required Condition for Nuclear fusion:

(a) High Temperature

(b) High Pressure

(b) High Pressure (or density)

Which ensure frequent collision and increases the probability of fusion.

These conditions exist in the sun and in many other stars. The source of energy in the sun is nuclear fusion, where hydrogen is in plasma state and protons fuse to form helium nuclei.

S.No.	Nuclear Fission	Nuclear Fusion
1.	Neutrons are required for it	Protons are required for it
2.	It is possible at normal pressure and temperature	It is possible at high pressure and temperature
3.	Energy released per nucleon $\cong 0.9$ MeV	Energy released per nucleon $\cong 6$ MeV
4.	% of mass getting converted into energy = 0.1%	% of mass getting converted into energy = 0.7%
5.	Fissionable materials are expensive	Fusion materials are cheap
6.	Harmful reactions are produced	Harmful reactions are not produced

Examples

Q1 **Assuming the radius of a nucleus to be equal to** $R = 1.3\,A^{1/3} \times 10^{-15}\,m$, **where A is its mass number, evaluate the density of nuclei and the number of nucleons per unit volume of the nucleus. Take mass of one nucleon =** 1.67×10^{-27} kg **.**

Sol: The radius of nucleus is $R = R_0 A^{1/3}$

Where A = mass number $R_0 = 1.3 \times 10^{-15} m = 1.3\,fm$

The volume of nucleus is $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(R_0 A^{1/3}\right)^3 = \frac{4}{3}\pi R_0^3 A$

A = mass number = number of nucleons

$\therefore$ The number of nucleons per unit volume is = $\frac{A}{V}$

$= \frac{1}{\frac{4}{3}\pi R_0^3} = 1.9 \times 10^{38}$ nucleons/cc

Density is $\rho = \frac{\text{mass}}{\text{volume}}$

= 1.09×10^{38} × mass of nucleon per cc

= 1.82×10^{11} kg/cc.

Q2 **In the decay $^{64}Cu \rightarrow {}^{64}Ni + e^+ + \nu$, the maximum kinetic energy carried by the positron is found to be 0.680 MeV**

(a) Find the energy of the neutrino which was emitted together with a positron of the energy 0.180 MeV

(b) What is the momentum of this neutrino in kg–m/s? Use the formula which is applicable to photon.

Sol: **(a)** KE of υ + KE of e^+ = Maximum energy of e^+

$\Rightarrow$ KE of υ = (0.680 – 0.180) MeV.

= 500 KeV.

(b) $p = \frac{E}{C} = \frac{500 \times 10^3 \times 1.6 \times 10^{-19}}{3 \times 10^8}$ Kg-m/s.

= 2.67×10^{-22} Kg-m/s.

Q3 **Beta decay of a free neutron takes place with a half-life of 14 minutes. Then find**
(a) decay constant
(b) energy liberated in the process.

Sol: **(a)** $\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{0.693}{14 \times 60} S^{-1} = 8.25 \times 10^{-4} S^{-1}$

(b) $n = p + e^{-} + \bar{\upsilon} + Q$

$$Q = \left(m_n - m_p - m_{e^-}\right).\ C^2$$

$= (1.008665 - 1.007276 - 0.0005486) \times 931$ MeV. $= 782$ KeV.

Q4 **The kinetic energy of an α-particle which flies out of the nucleus of a Ra^{226} atom in radioactive disintegration is 4.78 MeV. Find the total energy evolved during the escape of the α-particle.**

Sol: When α-particle will escape, the daughter nucleus will recoil back with same momentum. Applying momentum conservation $p_\alpha = p_d$

Total energy released

TE = KE of α + KE of daughter nucleus

$$= \frac{p_\alpha^2}{2m_\alpha} + \frac{p_d^2}{2m_d}$$

$$= \frac{p_\alpha^2}{2}\left(\frac{1}{m_\alpha} + \frac{1}{m_d}\right)$$

$$= m_\alpha . E_\alpha \left(\frac{1}{m_\alpha} + \frac{1}{m_d}\right)$$

$$= E_\alpha \left(1 + \frac{m_\alpha}{m_d}\right)$$

$$= 4.78\left(1 + \frac{4}{222}\right) = 4.87 \text{ MeV.}$$

Q5 **How many β-particles are emitted during one hour by 1.0 μg of Na^{24} radionuclide whose half–life is 15 hours? [Take $e^{(-0.693/15)}$ = 0.955, and Avogadro number = 6×10^{23}]**

Sol: No. of particle emitted in time t is

$$N = N_0\left(1 - e^{-\lambda t}\right) = N_0\left(1 - e^{-t\frac{\ln 2}{T_{1/2}}}\right)$$

N_0 = No. of nuclei in 1 μg of Na^{24}

$$N_0 = \frac{6 \times 10^{23} \times 10^{-6}}{24}$$

$$\therefore \quad N = \frac{6 \times 10^{23} \times 10^{-6}}{24} \times \left(1 - e^{-\frac{0.693}{15}}\right) = 1.128 \times 10^{15}.$$

Q6 **Calculate the specific activities of Na^{24} and U^{235} nuclides whose half-lives are 15 hours and 7.1×10^8 years respectively.**

Sol: Specific activity = No. of particles emitted per second by 1 g of the substance

For Na^{24}

$$\text{specific} = \frac{N_A}{24} \times \frac{0.693}{T_{1/2}} = 3.2 \times 10^{17} \text{ dps/g.}$$

For U^{235}

$$\text{Specific activity} = \frac{N_A}{235} \times \frac{0.693}{T_{1/2}} = 0.8 \times 10^5 \text{ dps/g.}$$

Q7 **For the D–T fusion reaction, find the rate at which deuterium and tritium are consumed to produce 1 MW. The Q–value of D–T reaction is 17.6 MeV and assume all the energy from the fusion reaction is available.**

Sol: Each deuterium nucleus produces 17.6 MeV.

1 kg of deuterium = $\frac{1\times10^3}{2}N_A$ no. of deuterium = $17.6\times\frac{10^3}{2}N_A$ MeV energy produced.

$\therefore$ To produce 1 MW, amount of deuterium in kg required per second.

$$= \frac{1\times10^6}{17.6\times\frac{10^3}{2}\times N_A\times e\times10^6} \text{ kg/s} = 1.179\times10^{-9} \text{ kg/s.}$$

Similarly for tritium $= \frac{1\times10^6}{17.6\times\frac{10^3}{2}\times N_A\times e\times10^6}$ kg/s = 1.769×10^{-9} kg/s.

Q8 **Consider a point source emitting α-particles and receptor of area 1 cm^2 placed 1 m away from source. Receptor records any α-particle falling on it. If the source contains $N_0 = 3.0\times10^{16}$ active nuclei and the receptor records a rate of A = 50000 counts/second, find the decay constant. Assume that the source emits an alpha particles uniformly in all directions and the alpha particles fall nearly normally on the window.**

Sol: Let λ = decay constant

N_0 = number of active nuclei = 3.0×10^{16}

number of α-particle falling on the window = $\frac{\lambda N_0 a}{4\pi R^2}$

$$\lambda = \frac{4\pi R^2 A}{N_0 a} = \frac{4\pi\times5\times10^4}{3.0\times10^{16}\times10^{-4}} = 2.1\times10^{-7}\,s^{-1}.$$

Q9 **The half-life of ^{40}K is T = 1.30 × 10^9 y. A sample of m = 1.00 g of pure KCl gives c = 480 counts/s. Calculate the relative percentage abundance of ^{40}K (fraction of ^{40}K present in term of number of atoms) in natural potassium. Molecular weight of KCl is M = 74.5, Avogadro number N_A = 6.02 × 10^{23}, 1y = 3.15 × 10^7 s.**

Sol: Number of KCl in the sample = $\frac{m}{M}N_A$

If N is the number of ^{40}K in the sample

$$C = \lambda.N = \frac{\ell n2}{T_{1/2}}N \quad \Rightarrow \quad N = \frac{CT_{1/2}}{\ell n2}$$

$$\text{The relative abundance} = \frac{N}{\frac{m}{M}N_A} \times 100 = \frac{CT}{\ln 2} \times \frac{M}{mN_A} \times 100$$

$$= \frac{480 \times \left(1.3 \times 10^9 \times 3.15 \times 10^7\right) \times 74.5 \times 100}{0.693 \times 1 \times 6.02 \times 10^{23}} = 0.36\%.$$

Q10 **A radioactive isotope is being produced at a constant rate dN/dt = R in an experiment. The isotope has a half-life $t_{1/2}$. Show that after a time t >> $t_{1/2}$, the number of active nuclei will become constant. Find the value of this constant. Suppose the production of the radioactive isotope starts at t = 0. Find the number of active nuclei at time t.**

Sol: Let N = no. of isotope at any instant

$$\frac{dN}{dt} = R - \lambda N \quad \Rightarrow \quad \frac{dN}{R - \lambda N} = dt$$

On integrating with initial condition t = 0, N = 0, we get

$$\int_0^N \frac{dN}{R - \lambda N} = \int_0^t dt \quad \Rightarrow \quad \frac{1}{-\lambda} \ln \frac{R - \lambda N}{R} = t$$

$$\Rightarrow \quad N = \frac{R}{\lambda}\left(l - e^{-\lambda t}\right)$$

When $t >> t_{1/2}$

$$N \approx \frac{R}{\lambda} = \frac{Rt_{1/2}}{\ell n2}.$$

Q11 **Consider a fusion reaction**

$^4He + {}^4He = {}^8Be$.

For the reaction find

(1) mass defect

(2) Q-value

(3) Is such a fusion energetically favorable? Atomic mass of ^8Be is 8.0053 u and that of ^4He is 4.0026 u.

Sol: $^4He + {}^4He = {}^8Be + Q$

(1) $\Delta m = 2 \times 4.0026 - 8.0053$

$= 8.0052 - 8.0053 = -0.0001$ amu

(2) $Q = \left(2m_{He} - m_{Be}\right).C^2$

$= (2 \times 4.0026 - 8.0053) \times 931$ MeV

$= -93.1$ KeV

Since Q is negative the fusion is not energetically favourable.

Q12 **In an ore containing uranium, the ratio (by number) of U-238 to Pb-206 is 3. Calculate the age of the ore, assuming that all the lead present in the ore is the final stable product of U-238. Take the half-life of U-238 to be 4.5×10^9 years. (ln 4/3 = 0.2876).**

Sol: $$\frac{\text{number of atoms of } {}^{238}\text{U initially}}{\text{number of radioactive } {}^{238}\text{U finally}} = \frac{a}{(a-x)}$$

[∵ One part lead (present to three parts Uranium) has initially Uranium]

$$\therefore \quad t = \frac{2.303}{\lambda}\log\frac{a}{(a-x)} = \frac{2.303 \times 4.5 \times 10^9}{0.693}\log\frac{4}{3} = 1.868 \times 10^9 \text{ yrs.}$$

Q13 **Nuclei of radioactive element A are being produced at a constant rate α. The element has a decay constant λ. At time t = 0, there are N_0 nuclei of the element.**

(a) Calculate the number N of nuclei of A at time t.

(b) If $\alpha = 2N_0\lambda$, calculate the number of nuclei of A after one half-life of A and also the limiting value of N as $t \to \infty$.

Sol: **(a)** Let at time t, number of radioactive nuclei are N.

Net rate of formation of nuclei of A

$$\frac{dN}{dt} = \alpha - \lambda N$$

or $$\frac{dN}{\alpha - \lambda N} = dt.$$

Q14 **A radioactive material decays by β-particle emission. During the first 2 seconds of a measurement, n β-particles are emitted and the next 2 seconds 0.75 n β-particles are emitted. Calculate the mean-life of this material in seconds to the nearest whole number. (ln 3 = 1.0986 and ln 2 = 0.6931).**

Sol: Let n_0 be the number of radioactive nuclei at time t = 0. Number of nuclei decayed in time t are given $n_0\,(1 - e^{-2\lambda})$, which is also equal to the number of beta particles emitted the same interval of time. For the given condition,

$$n = n_0\left(1 - e^{-2\lambda}\right) \quad \text{...(i)}$$

$$\left(n + 0.75n\right) = n_0\left(1 - e^{-4\lambda}\right) \quad \text{...(ii)}$$

Dividing (ii) by (i) we get

$$1.75 = \frac{1 - e^{-4\lambda}}{1 - e^{-2\lambda}}$$

or $$1.75 - 1.75e^{-2\lambda} = 1 - e^{-4\lambda}$$

$$\therefore \quad 1.75e^{-2\lambda} - e^{-4\lambda} = \frac{3}{4} \quad \text{...(iii)}$$

Let us take $e^{-2\lambda} = x$

Then the above equation is,

$$x^2 - 1.75x + 0.75 = 0$$

or $x = \frac{1.75 \pm \sqrt{(1.75)^2 - (4)(0.75)}}{2}$

or $x = 1$ and $\frac{3}{4}$

$\therefore$ From equation (iii) either $e^{-2\lambda} = 1$ or $e^{-2\lambda} = \frac{3}{4}$

But $e^{-2\lambda} = 1$ is not accepted because which means $\lambda = 0$. Hence

$e^{-2\lambda} = \frac{3}{4}$

or $-2\lambda \ell n(e) = \ell n(3) - \ell n(4) = \ell n(3) - 2\,\ell n(2)$

$\therefore$ $\lambda = \ell n(2) - \frac{1}{2}\ell n(3)$

Substituting the given values, $\lambda = 0.6931 - \frac{1}{2} \times (1.0986) = 0.14395\,s^{-1}$

$\therefore$ Mean life $t_{means} = \frac{1}{\lambda} = 6.947$ sec.

Q15 **Knowing the decay constant λ of a substance, find the probability of decay of a nucleus during the time from 0 to t.**

Sol: The number of nuclei decaying in time dt is

$$dN = N\lambda dt = N_0 \lambda e^{-\lambda t} dt$$

The probability of decaying of a nucleus in time 0 to t is

$$\int_0^t \frac{dN}{N_0} = \int_0^t \lambda e^{-\lambda t} dt = 1 - e^{-\lambda t}.$$

Mind Map

- **NUCLEUS**
  - Representation of Nucleus ${}_{Z}^{A}X$
    - **Atomic Number (Z)**
      Number of proton present inside nucleus
    - **Mass Number (A)**
      Total number of protons and neutrons present inside nucleus
  - Composition of Nucleus
    - **Protons (Z)**
      - Positively charged
      - Magnitude of charge = 1.6×10^{-19} C
      - Mass of $M_p = 1.67 \times 10^{-27}$ kg
    - **Neutron (N)**
      - Neutral particle
      - Mass roughly equal to mass of protons
  - Size of Nucleus
    - **Radius of Nucleus**
      $R = R_0 A^{1/3}$, where $R_0 = 1.2$ fm
    - **Volume of Nucleus**
      $V = \frac{4}{3}\pi R_0^3 A$
    - **Density of Nucleus**
      $\rho = \frac{3m}{4\pi R_0^3}$ = Constant
      Order of density = 10^{17} kg/m^3
  - Types of Nucleus
    - **Isotopes**
      Same Z but different A
    - **Isobars**
      Same A but different Z
    - **Isotones**
      Same N but different Z