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# **Newton's Law of Motion**





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# Newton's Law of Motion

## 1. NEWTON'S FIRST LAW:

- According to this law force is a factor which can change the state of object.  
or
- Force explained as the push or pull which changes or tends to change the state of rest or of uniform motion.

## FORCE

- Any push or pull which either changes or tends to change the state of rest or of uniform motion (constant motion) of a body is known as force.
- Unit: M.K.S = Newton (N), C.G.S. = dyne.
- $1 \text{ N} = 10^5 \text{ dyne}$ .
- Gravitational unit = Kg.wt.
- $1 \text{ Kg.wt} = 9.8 \text{ N}$ ,  $1 \text{ g-wt} = 980 \text{ dyne}$ .
- Value of force in this unit will depends on value of 'g'.

## Effect of Resultant force:

A non-zero resultant force may produce the following effects on a body:

- (i) It may change the speed of the body.
- (ii) It may change the direction of motion.
- (iii) It may change both the speed and direction of motion.
- (iv) It may change the size or/and the shape of the body.

## Inertia:

It is the virtue of a body due to which it opposes any change in its current state. Mass of a body is the measure of body's inertia of translational motion.

- Inertia is directly proportional to mass of the object.

$$\text{Inertia} \propto \text{mass}$$

- Mass of a body is quantitative or numerical measure of a body's inertia.
- **Inertia of rest:** It is the inability of a body to change its state of rest by itself.

## Definition

An object can be considered as a point object if during motion in a given time, it covers distance much greater than its own size.

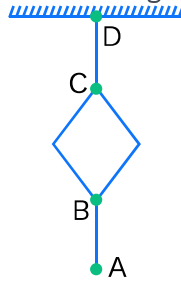


## Concept Reminder

Two bodies of equal mass, one in motion and another is at rest, possess same inertia because it is a factor of mass only and does not depend upon the velocity.

**Example:**

- (i) When a horse starts off suddenly, its rider falls backwards.
- (ii) When a bus starts suddenly, the passengers tend to fall backwards,
- (iii) When we shake a branch of a mango tree, the mangoes fall down.



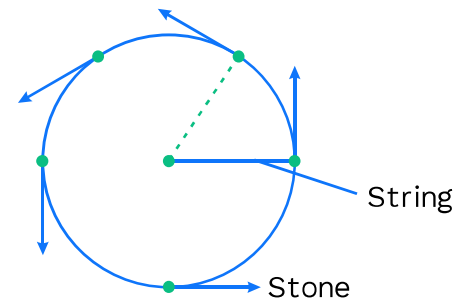
- When we apply sudden jerk then string (AB) will break from point 'B'.
- When we pull slowly of the string then tension transmits to the upper point 'D' then string (CD) will break from point 'D'.
- **Inertia of motion:** It is the inability of a body to change its state of uniform motion by itself.

**Example:**

- (i) A person who jumps out of a moving bus, train may fall in the forward.
- (ii) When a bus or train stops suddenly, the passengers sitting inside lean forward.
- (iii) A ball is thrown in the upward direction by a passenger sitting inside a moving train the ball will fall:
  - (a) Back to the hands of the passenger if the train is moving with constant velocity.
  - (b) Ahead of the passenger if the train is retarding (slowing down).
- **Inertia of direction:** It is the inability of a body to change its direction of motion by itself.

**Example:**

- (i) When a straight running car turns sharply, the person sitting inside feels a force radially outwards.
- (ii) Particle wants to move in tangential direction but due to centripetal force it will move in circular motion.

**Key Points**

- ◆ Force
- ◆ Inertia



**Momentum:** It tells us about amount and direction of motion at any instant. It can be calculated by multiplying the mass and velocity of a body. It is vector quantity.

$$\vec{p} = m\vec{v}$$

- SI unit = kg-m/s. Dimension =  $[MLT^{-1}]$
- Direction of  $\vec{p}$  is same as that of  $\vec{v}$ .
- Momentum of system of particle or body

$$\vec{p}_{\text{system}} = \vec{p}_1 + \vec{p}_2 + \dots + \vec{p}_n$$

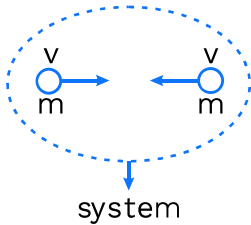
Relation between kinetic energy and magnitude of momentum.

$$K = \frac{p^2}{2m} \quad \text{Only for one body system.}$$

**Important points:**

1. If momentum of system is zero then kinetic energy of system may or may not be zero.

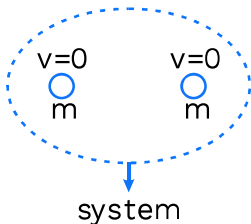
**Ex.**



$$p_{\text{system}} = mv + (-mv) = 0,$$

$$K_{\text{system}} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 \Rightarrow mv^2 \neq 0$$

**Ex.**



$$p_{\text{system}} = 0, K_{\text{system}} = 0$$

2. If kinetic energy of system is zero then momentum of system must be zero.

**% change in y**

$$= \frac{\Delta y}{y_1} \times 100 \quad \text{or}$$



**Concept Reminder**

If two bodies of different masses have same momentum, the lighter body possesses greater velocity.

$$P = m_1v_1 = m_2v_2$$

$$\frac{v_1}{v_2} = \frac{m_2}{m_1}$$

**Rack your Brain**



The mass of a hydrogen molecule is  $3.32 \times 10^{-27}$  kg. If  $10^{23}$  hydrogen molecules strike, per second, a fixed wall of area  $2 \text{ cm}^2$  at an angle of  $45^\circ$  to the normal and rebound elastically with a speed of  $10^3 \text{ m/s}$ , then calculate the pressure on the wall.



**Concept Reminder**

Slope of momentum - mass curve denote velocity.

$$= \frac{y_2 - y_1}{y_1} \times 100$$

$$= \left( \frac{y_2 - y_1}{y_1} \right) \times 100$$

3. If  $y \propto \frac{a^p b^q}{c^r}$  and % change  $\leq 5\%$ .

then % change in y:

$$= \frac{\Delta y}{y} \times 100 = p \frac{\Delta a}{a} \times 100 + q \frac{\Delta b}{b} \times 100 - r \frac{\Delta c}{c} \times 100$$

**Ex.** If momentum will increase by 0.3% and mass will decrease by 0.2%, then find out percentage change in kinetic energy.

**Sol.**  $K \propto \frac{p^2}{m}$

$$\frac{\Delta K}{K} \times 100 = \frac{2\Delta p}{p} \times 100 - \frac{\Delta m}{m} \times 100$$

$$\frac{\Delta K}{K} \times 100 = 2(0.3) - (-0.2) = 0.8\%$$

**4. Change in momentum:**

$$\Delta \vec{p} = \vec{p}_f - \vec{p}_i \quad (\text{mass constant})$$

$$m(\vec{v}_f - \vec{v}_i) = m(\Delta \vec{v})$$

**Ex.** If momentum will increase by 150% having constant mass, then find out percentage change in kinetic energy.

**Sol.**  $KE = \frac{p^2}{2m} \quad K \propto p^2$

$$K \propto (2.5)^2 = 6.25 \text{ times}$$

$$\% \text{ change in momentum} = \frac{p_f - p_i}{p_i} \times 100 =$$

$$= \frac{6.25 - 1}{1} \times 100 = 525\%$$

**Ex.** If kinetic energy will be 4 times of initial and mass will become half find out percentage change in momentum.

### Key Points

- ◆ Momentum
- ◆ Velocity
- ◆ Kinetic energy
- ◆ Change in Momentum



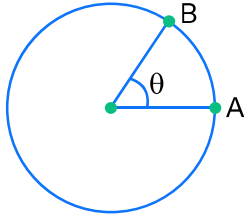
**Sol.**  $K = \frac{p^2}{2m}$ ,

$$p^2 = 2mk$$

$$p \propto \sqrt{mk} = \sqrt{4 \times \frac{1}{2}} = \sqrt{2} = 1.414 \text{ times}$$

$$\% \text{ change in } P = (1.414 - 1) \times 100 = 41.4\%$$

**Ex.** Particle of mass 'm' is moving with uniform speed 'v' then find out magnitude of change in momentum in AB interval shown in the diagram.



**Sol.**  $|\Delta \vec{p}| = m(\Delta \vec{v}) = mv \cdot \sin \frac{\theta}{2}$

**NEWTON'S SECOND LAW OF MOTION:**

- This law defines that, the rate of change of momentum of system is directly proportional to the applied external force and this change in momentum takes place in the direction of the applied force.

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m \cdot \frac{d\vec{v}}{dt} + \vec{v} \cdot \frac{dm}{dt}$$

**Case-I:** If  $m = \text{constant}$  then  $\frac{dm}{dt} = 0$

$$\Rightarrow \vec{F} = m \cdot \frac{d\vec{v}}{dt} = m\vec{a}$$

**Case-II:** If  $\vec{v}$  constant then  $\frac{d\vec{v}}{dt} = 0$

$$\Rightarrow \vec{F} = \vec{v} \cdot \frac{dm}{dt}$$

**Ex.** Conveyor belt, rocket propulsion.

- This is quantitative definition of force.
- This is fundamental law i.e. 1<sup>st</sup> and 3<sup>rd</sup> law derive/define by 2<sup>nd</sup> law.

**Rack your Brain**



A rigid ball of mass  $m$  strikes a rigid wall at  $60^\circ$  and gets reflected without loss of speed. Find the value of impulse imparted by the wall in the ball.



**Concept Reminder**

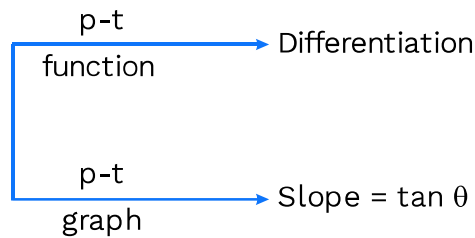
Newton's first and third law can be derived from second law therefore second law is the most fundamental law out of three.



- $$\vec{F}_{\text{avg}} = \frac{\Delta \vec{p}}{\Delta t}$$

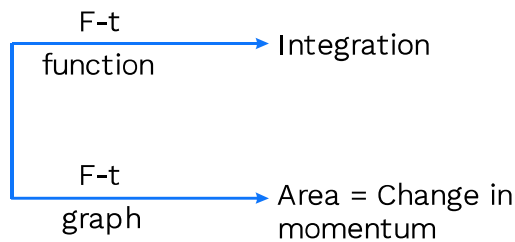
- Direction of force will be same as that of direction of change in momentum.

- $$F = \frac{dp}{dt}$$



- Slope of momentum-time graph is equal to the force on the particle at that instant.

- $$\int_{p_1}^{p_2} dp = \int_{t_1}^{t_2} F \cdot dt$$



**Important points:**

- Newton's first law of motion defines force and second law of motion measure force. It given the units, dimensions and magnitude of the force.  
Unit of force = (unit of mass) × (unit of acceleration)  
= 1 kg × 1 m/s<sup>2</sup> = 1 N  
1 N = 1 kg·m/s<sup>2</sup>; dyne = 1 g·cm/s<sup>2</sup>.  
Dimension: [MLT<sup>-2</sup>].
- If a particle move uniformly, means velocity = constant

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d(\text{constant})}{dt} = \vec{0}$$

so 
$$\vec{F} = m\vec{a} = m \times \vec{0} = \vec{0}$$

**Definition**

Force is a factor which try to displace or displace an object.

Force is a factor which try to accelerate or accelerate an object.



**Key Points**

- ◆ Acceleration
- ◆ Uniform motion



It means that in the absence of external force, a particle moves uniformly. This is Newton's first law of motion. It means that we can derive mathematically Newton's first law of motion with the help of Newton's second law of motion.

- Accelerated motion is always due to an external force.

Law of conservation of linear momentum is applicable in the direction in which external force is zero.

$$\text{If } F_x = 0 \Rightarrow \frac{dp_x}{dt} = 0; \text{ then } p_x = \text{constant}$$

$$\text{If } F_y = 0 \Rightarrow \frac{dp_y}{dt} = 0; \text{ then } p_y = \text{constant}$$

$$\text{If } F_z = 0 \Rightarrow \frac{dp_z}{dt} = 0; \text{ then } p_z = \text{constant}$$

### Impulse:

- This force will act for very very large force applied on a body for a very short time interval and in this interval it will change its value known as impulsive force.
- Net force on any system is directly proportional to rate of change of momentum.

$$F_{\text{net}} \propto \frac{dp}{dt}, \quad \vec{F}_{\text{net}} = \frac{K \cdot d\vec{p}}{dt},$$

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$$

$$\boxed{d\vec{p} = d\vec{I} = \vec{F} \cdot dt}$$

- Unit of impulse = N-s or kg-m/s.
- Dimension = [MLT<sup>-1</sup>].



### Concept Reminder

If there is change in velocity is very large then impulsive force is in a large amount.



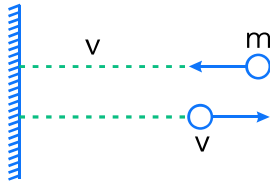
### Key Points

- ♦ Impulse
- ♦ Conveyor belt
- ♦ Rocket propulsion
- ♦ Equilibrium of forces



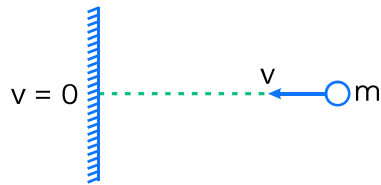
**Change in momentum in following conditions:**

1. Elastic collision



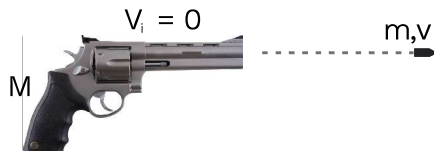
$$\Delta p = m(v - (-v)), \quad \boxed{\Delta p = 2mv}$$

2. Perfectly, absorbing surface (stick after collision)



$$\boxed{\Delta p = mv}$$

3. During firing of bullet,



$$p_i = p_f \quad \Rightarrow \quad 0 = mv + MV$$

$$V = -\frac{mv}{M} \quad (\text{-ve sign show that gun move}$$

opposite direction to bullet.)

**Ex.** Particle of mass 2 kg is moving according to displacement time function  $x = t^3/6$ . Find out force at time  $t = 2$  sec.

**Sol.**  $x = t^3/6$

$$\frac{dx}{dt} = v = \frac{t^2}{2} \quad \Rightarrow \quad \frac{dv}{dt} = a = t$$

$$F = ma = 2t$$

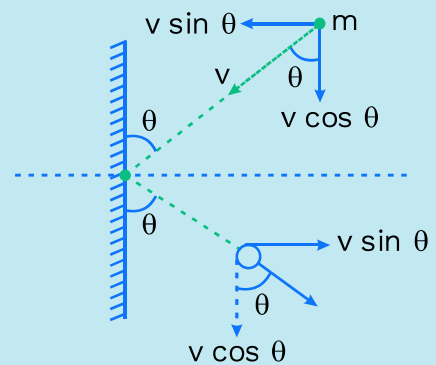
at  $t = 2$  sec force is

$$\therefore F = 2 \times 2 = 4 \text{ N}$$



**Concept Reminder**

If a ball is thrown with velocity 'v' at an angle  $\theta$  from vertical, then after collision from wall if it revert at same angle then:



$$\Delta P_y = 0, \Delta P_x = 2mv \sin \theta$$



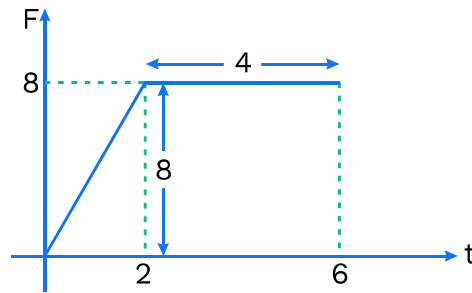
**Key Points**

- ◆ Elastic collision
- ◆ Perfect, absorbing surface



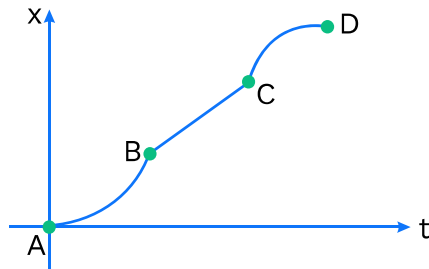


**Ex.** Particle having mass 2 kg is moved in from rest the variation of force with time is shown in the following diagram find out velocity at  $t = 6$  sec.



**Sol.**  $\Delta p = \text{Area of curve}$   
 $m(v_6 - v_0) = \left(\frac{1}{2} \times 2 \times 8\right) + (4 \times 8)$   
 $2v_6 = 40 \quad \Rightarrow \quad v_6 = 20$

**Ex.** In the following x-t graph write down the sign of force of AB, BC and CD interval.



**Sol.** AB =  $\tan \theta = +ve$  (increasing)  
 $\Rightarrow$  Force positive  
 BC =  $\tan \theta = \text{constant}$ ,  $a = 0$   
 CD =  $\tan \theta = -ve$  (decreasing)  
 $\Rightarrow$  Force negative

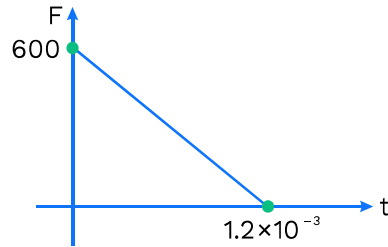
**Ex.** A particle having initial velocity 20 m/sec and mass 2 kg is moving under constant retardation if it will change the direction of velocity in opposite direction the time taken is 2 sec find out resistance force.

**Sol.**  $F = \frac{\Delta p}{\Delta t} = \frac{m(v_f - v_i)}{t} \Rightarrow F = \frac{2(-20 - 20)}{2}$   
 $\Rightarrow F = -40 \text{ N}$  (-ve sign shown resistive force)



**Ex.** A bullet will fire a force applied by gun is  $F = 600 - 5 \times 10^5 t$  then find out impulse provided by gun, when value of force will be zero.

**Sol.**  $F = 0 \Rightarrow 600 - 5 \times 10^5 t = 0$



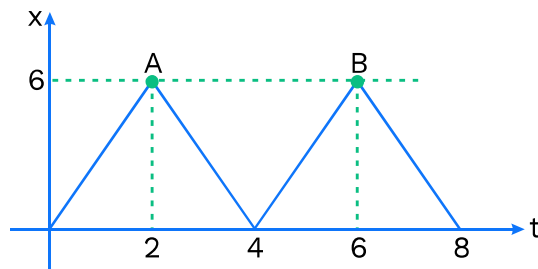
$$t = \frac{600}{5 \times 10^5} = 1.2 \times 10^{-3}$$

F-t curve area gives impulse

$$I = \text{Area} = \frac{1}{2} \times 600 \times 1.2 \times 10^{-3}$$

$$= 360 \times 10^{-3} = 3.6 \times 10^{-1}$$

**Ex.** According to following x-t graph find out impulse of 2 kg body at reflection point (only magnitude).



**Sol.**  $I = \Delta p = m(v_f - v_i) = 0.2 \left( \frac{1}{2} \times 4 \times 6 \right) = 1.2$

**Ex.** In the following diagram find out number of bullets per second are firing for that plate is in equilibrium in air effect of gravity is negligible on bullet.





**Sol.** F.B.D. of plate

$$\therefore F_{\text{net}} = 0 \Rightarrow F_{\text{bullet}} = Mg$$

$$2 mnv = Mg \Rightarrow n = \frac{Mg}{2mv}$$

**Impulse-momentum theorem:**

- Impulse of force is equal to change in momentum.

**Law of conservation of linear momentum:**

According to this law, the linear momentum of a system remains constant, if the total external force on the system is zero.

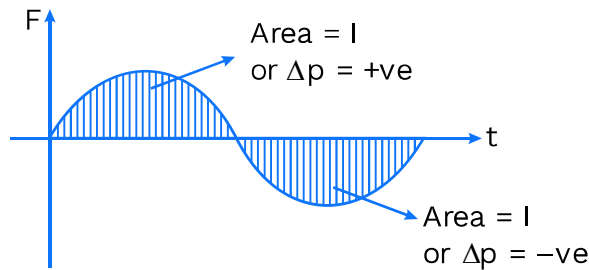
$$\text{If } \vec{F}_{\text{ext}} = \vec{0} \text{ then } \vec{F}_{\text{ext}} = \frac{d\vec{p}}{dt} = 0$$

$$\vec{p} = \text{constant} \quad \text{or} \quad \vec{p}_{\text{initial}} = \vec{p}_{\text{final}}$$

$$\Delta \vec{p}_1 + \Delta \vec{p}_2 + \Delta \vec{p}_3 + \dots + \Delta \vec{p}_n = 0$$

Conservation of linear momentum.

**Note:** Area under the force-time graph represents impulse or change in momentum.



$$I \text{ or } \Delta p = \int F \cdot dt = \text{Area under } F - t \text{ graph}$$

Since  $F_{\text{avg}} = \frac{\Delta p}{\Delta t}$

If time interval is increased for a certain momentum, then the average force exerted on body will decrease.

**Ex.:** Shockers are provided in vehicles to avoid jerks.

A man jumping on a hard cement floor receives more injuries than a man jumping on muddy or sandy road.

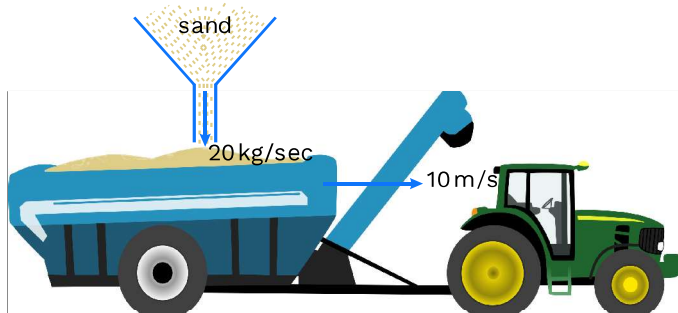
**Key Points**



- ♦ Law of conservation of linear momentum
- ♦ Impulsive force



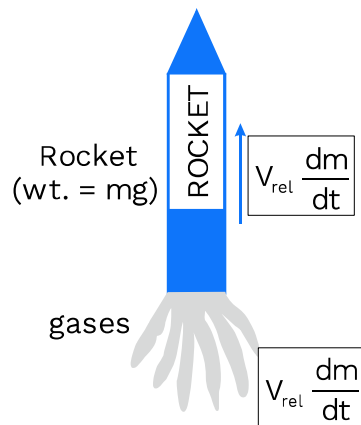
**Ex.** A trolley is moving with velocity 10 m/s and is dropping on it at the rate of 20 kg/sec. Find out additional horizontal force 'F' applied to maintain its velocity.



**Sol.**  $F = v \cdot \frac{dm}{dt} = 10 \times 20 = 200 \text{ N}$

**Rocket Propulsion:**

Ejected gas will apply reaction force on rocket in the opposite direction of ejection known thrust force.



**Concept Reminder**

Net upward force on rocket  
= ma

$$V_{rel} \frac{dm}{dt} - mg = ma$$

Where  $V_{rel}$  is the velocity of the ejected mass w.r.t. rocket.

**Case-I:** If rocket is accelerating upwards, then net upward force on rocket = ma

$$V_{rel} \frac{dm}{dt} - mg = ma$$

Where  $V_{rel}$  is the velocity of the ejected mass w.r.t. rocket.

**Case-II:** If rocket is moving with constant velocity, then a = 0

$$V_{rel} \frac{dm}{dt} = mg$$



**Ex.** A rocket of mass 2000 kg ejects the gases at the rate 50 kg/s with speed of 20 m/s in gravity free space. Find out the acceleration of rocket after 20 sec of firing.

**Sol.** Remaining mass of rocket after 20 sec  
 $= 2000 - 50 \times 20 = 1000 \text{ kg}$

$$\therefore v \frac{dm}{dt} = ma \Rightarrow 20 \times 50 = 1000 \times a$$

$$a = 1 \text{ m/s}^2$$

**Ex.** Mass of any rocket at any instant is 20 kg and its acceleration is 10 m/s. Find out thrust force on rocket.

**Sol.**  $a = \frac{F_{th}}{M} - g \Rightarrow 10 = \frac{F_{th}}{20} - 10$

$$F_{th} = 400 \text{ N}$$

### NEWTON'S 3<sup>RD</sup> LAW OF MOTION

- Each and every action have equal and opposite reaction.
- Physical contact is not necessary for action reaction pair.
- Action reaction pair will always exist on different object or system.

#### Important points:

- Force is always produced in action-reaction pair.
- There is not time gap in between action and reaction.
- Action-reaction law is applicable on both the states either at rest or in motion.
- Bodies which are not in physical contact then action-reaction is also applicable.
- Action reaction law is applicable to all the interaction force, tension, friction gravitational force etc.
- Action and reaction never cancel each other because they act on two different bodies.



### Key Points

- ♦ Thrust force
- ♦ Action-reaction pair
- ♦ Interaction force



### Concept Reminder

While walking, a person presses the ground in the backward direction (action) by his feet. The ground pushes the person in forward direction with an equal force (reaction). The component of reaction in horizontal direction makes the person move forward.

### Rack your Brain



A block is kept on a frictionless inclined surface with angle of inclination 'a'. The incline is given an acceleration 'a' to keep the block stationary. Then calculate the value of a.



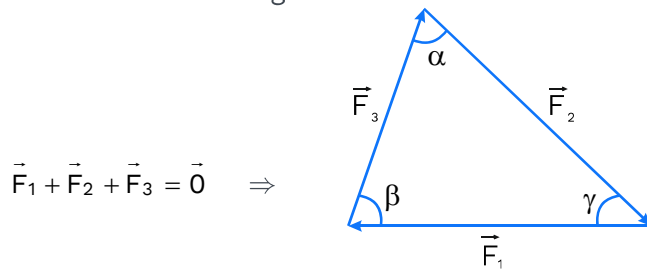
**Free body diagram (F.B.D.):**

**Step-I:** Applying forces on the appropriate system.

**Step-II:** Consider two appropriate mutually perpendicular axis and resolve forces along these axes.

**Step-III:** Frame equation along both axis acceleration to requirement as follows:

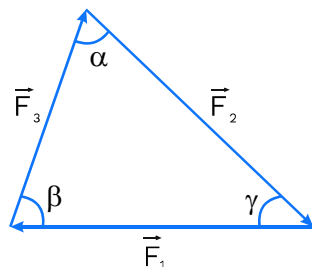
- If system is in equilibrium along then apply  $F_{net} = 0 \Rightarrow$  along that axis  
If an object is in state of equilibrium under the action of only two external forces, the forces must be equal and opposite.  
If an object is in state of equilibrium under the action of three forces, their resultant must be zero. Therefore, according to the triangle law of vector addition they must be coplanar and should form a closed triangle.



- If system is accelerated along the axis then apply  $F_{net} = ma$   
If an object is acted upon by a single external force, it cannot be in equilibrium.

**Lami's theorem:**

- Graphical method makes use of sine rule or Lami's theorem



**Rack your Brain**

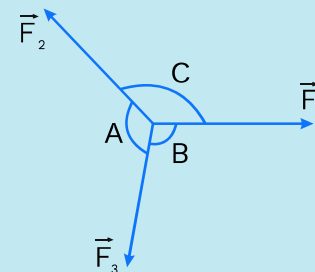


A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviated at an angle of  $45^\circ$  at the roof point. If the suspended mass is at equilibrium, then calculate the magnitude of the force applied. ( $g = 10 \text{ ms}^{-2}$ )



**Concept Reminder**

Lami's theorem



$$\frac{F_1}{\sin A} = \frac{F_2}{\sin B} = \frac{F_3}{\sin C}$$

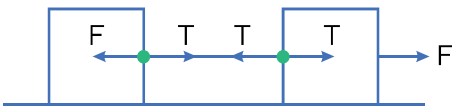


Sine rule:  $\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}$

Lami's theorem:  $\frac{F_1}{\sin A} = \frac{F_2}{\sin B} = \frac{F_3}{\sin C}$

### 1. Tension:

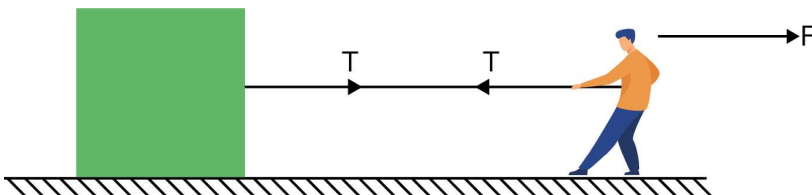
- It is a type of reaction force which will generate on applying an extensive force at its both ends. This force will generate due to opposition of intermolecular force.
  - It is a type of pulling force.
  - It will represent at any point on system along the length of the string away from that point.



- If string is massless, uniform and continuous then value of tension throughout the string remain, same.

### Tension in a string:

- It is the intermolecular forces between the molecules of a string, which become active when the string is stretched. Some important points about the tension in a string are as follows:
  - Force of tension act on an object in the direction away from the point of contact or tied end of the string.

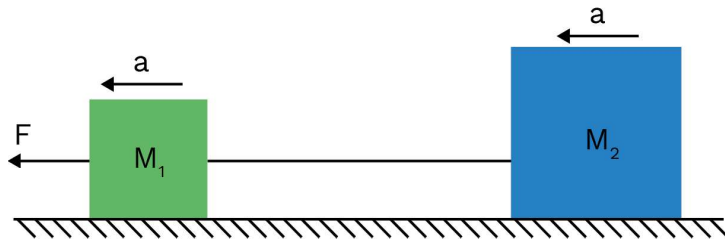


- String is supposed to be inextensible so that the magnitude of accelerations of the blocks tied to the strings are always same.

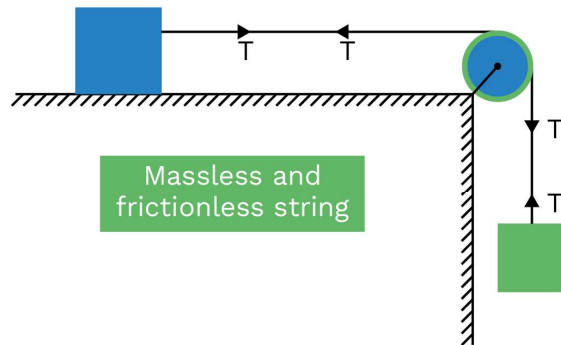
### Rack your Brain



A mass of 10 kg is suspended by a rope of length 4 m, from the ceiling. A force  $F$  is applied horizontally at the mid-point of the rope such that the top half of the rope makes an angle of  $45^\circ$  with the vertical. Then calculate the  $F$ . (Take  $g = 10 \text{ ms}^{-2}$  and the rope to be massless)



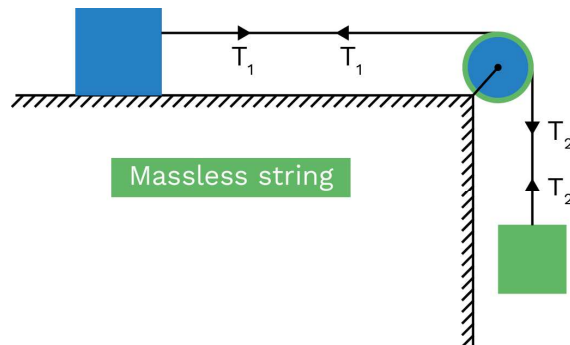
(c) (i) If the string is massless and frictionless, tension throughout the string remains same.



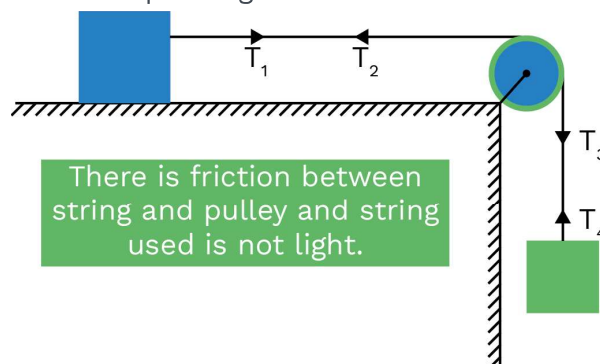
### Key Points

- ◆ Free body diagram
- ◆ Tension
- ◆ String

(ii) If a string is massless but not frictionless, then at every contact tension changes.



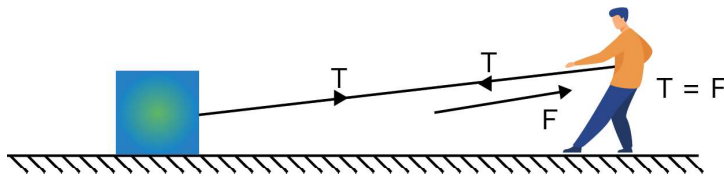
(iii) If the string is not light, tension at each point of the string will be different depending on the acceleration.



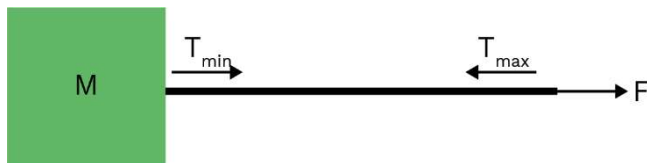




- (d) If a force is directly applied to a string, say a man is pulling a string from the other end with some force, then tension will be equal to the given force regardless of the pulling agent, irrespective of whether the box moves or not, man moves or not.



- (e) String is supposed to be massless unless stated, hence tension in it remains the same every where and equal to the applied force. However, if the string has a mass, tension at different points will be different being maximum (= applied force) at the end by which force is given and minimum at the other end connected to a body.



- (f) In order to produce tension in a string 2 equal and opposite stretching forces must be applied. The tension thus produced is equal in magnitude to either applied force (i.e.,  $T = F$ ) and is directed inwards opposite to  $F$ .



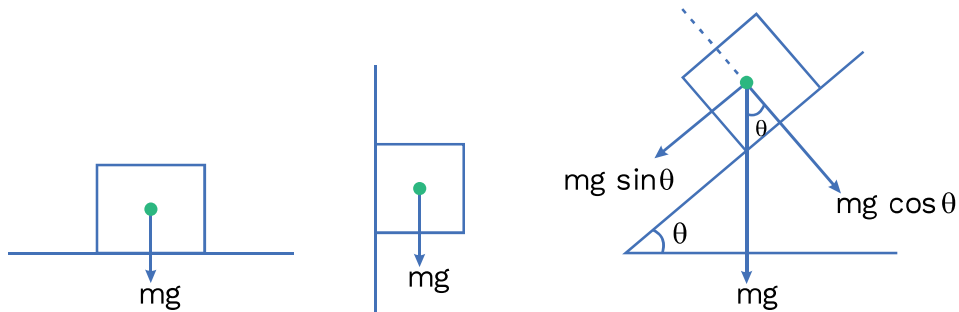
- (g) Every string can manage a maximum tension i.e. if the tension in a string is continuously increased it will break beyond a certain limit. The maximum tension which a string can bear without breaking is called its 'breaking strength'.

## 2. Weight:

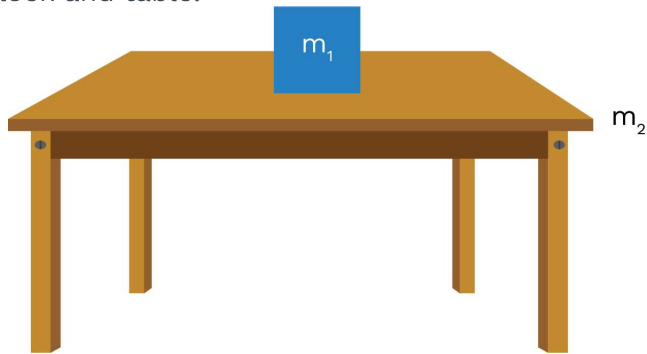
Applied at centre of gravity vertically downwards.

### Key Points

- ◆ Weight
- ◆ Breaking strength



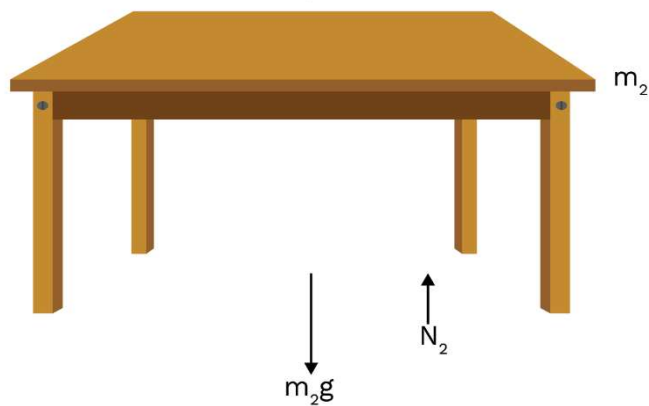
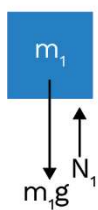
**Ex.** A block is placed on a table and the table is kept on earth. Assuming no other body in the universe exerts any force on the system, make the FBD of block and table.



**Sol.** FBD of block,  
FBD of table,

$$N_1 = m_1 g$$

$$N_2 = N_1 + m_2 g = m_1 g + m_2 g = (m_1 + m_2) g$$

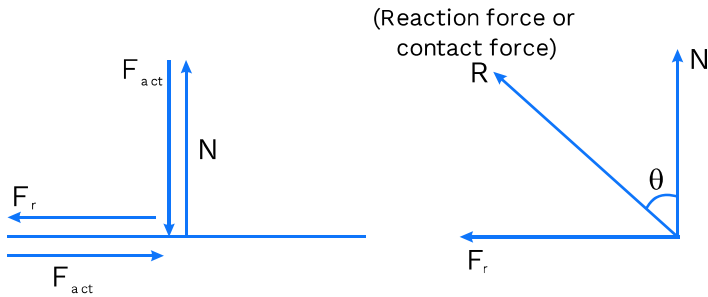


### 3. Contact force:

- This is also a reaction force which will generate on applying an external force towards perpendicular to contact surface or parallel to contact surface.



- Thus, following two types of contact force will generate.

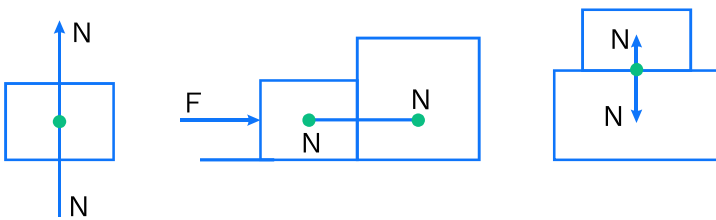


**Concept Reminder**

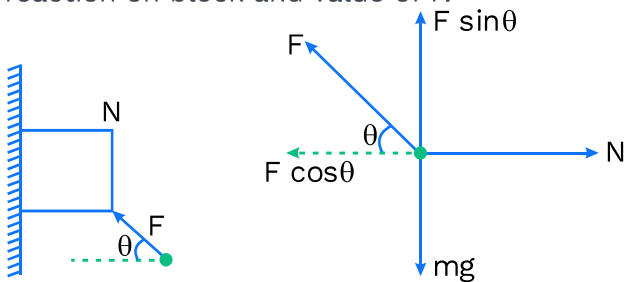
- ◆  $R = \sqrt{N^2 + F_r^2}$
- ◆  $\tan \theta = \frac{F_r}{N}$

**Normal Reaction (N):**

- When an external force applied perpendicular towards contact surface then this reaction force will generate.
- This force is of pushing nature.
- This will represent perpendicular to contact surface passing through the body (on which it is applied).



**Ex.** In the following diagram if block is in equilibrium then find out normal reaction on block and value of  $F$ .

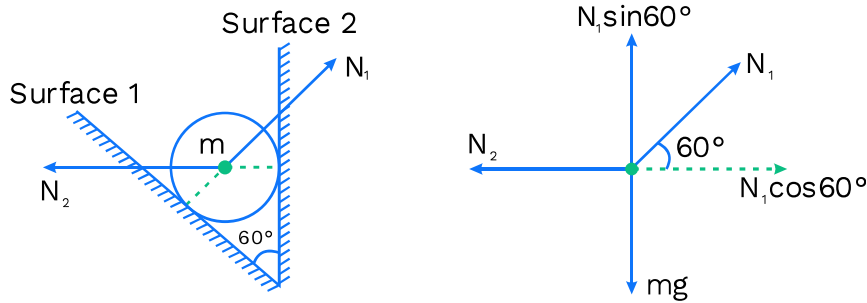


**Sol.**  $F \sin \theta = mg$ ,  $F \cos \theta = N \Rightarrow F = \frac{mg}{\sin \theta}$

$$N = F \cos \theta = \frac{mg}{\sin \theta} \times \cos \theta = \frac{mg}{\tan \theta}$$



**Ex.** Find out normal reaction  $N_1$  and  $N_2$ ?



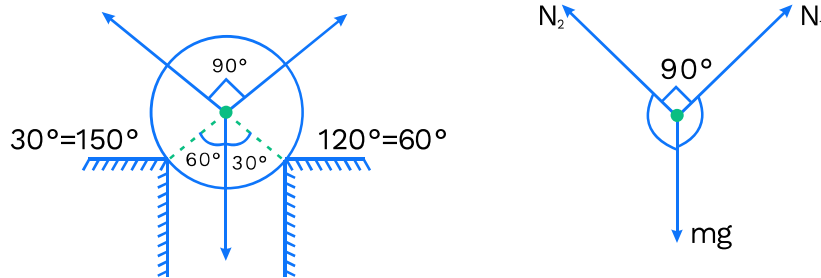
**Sol.** (1)  $N_1 \sin 60^\circ = mg \Rightarrow N_1 = \frac{2mg}{\sqrt{3}}$

(2)  $N_2 = N_1 \cos 60^\circ \Rightarrow N_2 = \frac{2mg}{\sqrt{3}} \times \frac{1}{2} = \frac{mg}{\sqrt{3}}$

**Key Points**

- ◆ Contact force
- ◆ Normal reaction

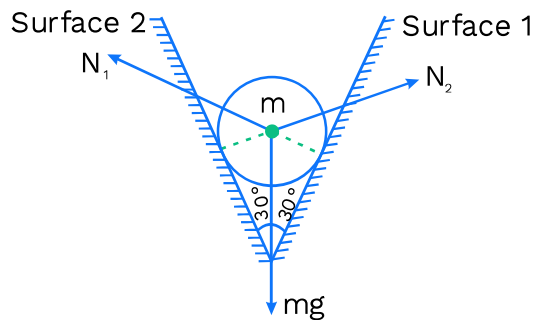
**Ex.**



**Sol.**  $\frac{N_1}{\sin 30} = \frac{N_2}{\sin 60} = \frac{mg}{\sin 90} \Rightarrow \frac{N_1}{\sin 30} = \frac{mg}{\sin 90}$

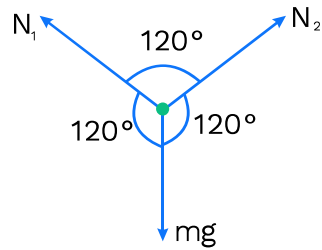
$N_1 = \frac{mg \sin 30}{\sin 90} = \frac{mg}{2} \Rightarrow N_2 = \frac{mg \sin 60}{\sin 90} = \frac{mg\sqrt{3}}{2}$

**Ex.** In the following if spherical body of mass  $m$  is placed between two rigid surface, then find out normal reaction on ball.

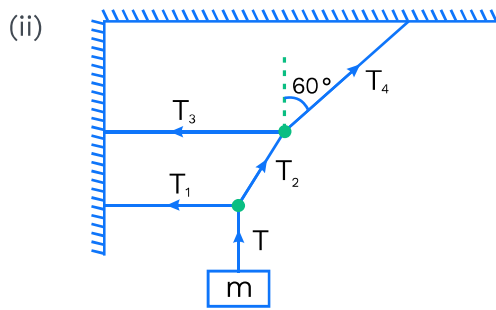
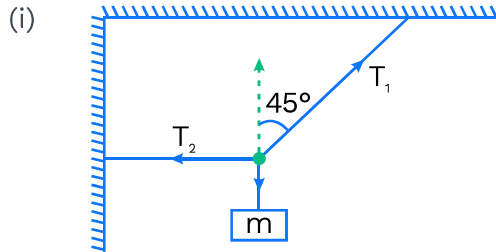




**Sol.**  $N_1 = N_2 = mg$

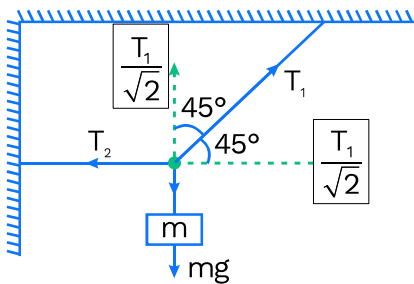


**Ex.** In following figure find the value of tension in all string



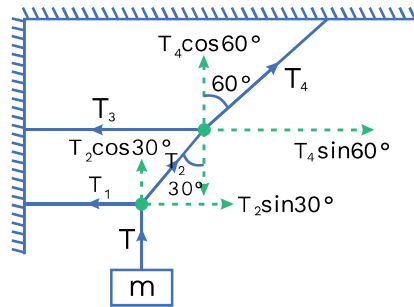
**Sol.** (i)  $T_1 \cos 45^\circ = mg \Rightarrow T_1 = mg\sqrt{2}$

$T_1 \sin 45^\circ = T_2 \Rightarrow \boxed{T_2 = mg}$





(ii)  $T = mg \Rightarrow T_2 \cos 30^\circ = T$



$$T_2 \times \frac{\sqrt{3}}{2} = mg \Rightarrow T_2 = \frac{2mg}{\sqrt{3}}$$

$$T_1 = T_2 \sin 30^\circ \Rightarrow T_1 = \frac{2mg}{\sqrt{3}} \times \frac{1}{2}$$

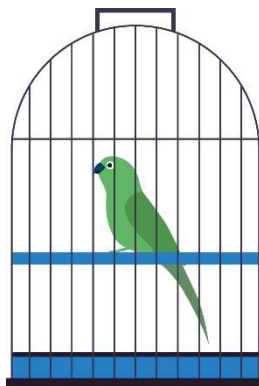
$$T_1 = \frac{mg}{\sqrt{3}}$$

$$T_4 \cos 60^\circ = T_2 \cos 30^\circ$$

$$T_4 \times \frac{1}{2} = \frac{mg}{\sqrt{3}} \times \frac{\sqrt{3}}{2} \Rightarrow T_4 = mg$$

$$T_3 = T_4 \sin 60^\circ \Rightarrow T_3 = mg \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}mg}{2}$$

### Bird-cage problem:



- A parrot is sitting on the base in an airtight cage. Now, if the bird starts flying, then
  - (i) Weight of system will not change, if the bird flies with constant velocity.
  - (ii) Weight of the system will increase, if the bird flies with upward acceleration.

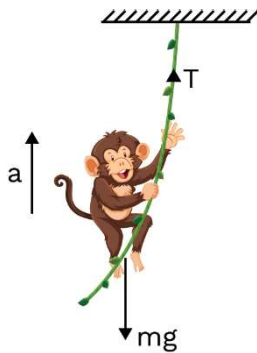


(iii) Weight of system will decrease, if the bird flies with downward acceleration

- A parrot is sitting on the base in a wire cage. Now if the parrot flies upward its weight of surface will decrease in all the cases.

### Monkey problem:

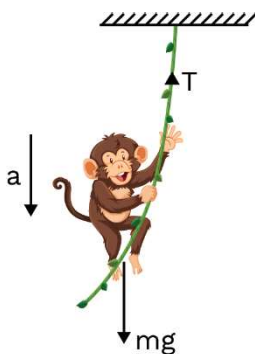
**Ex.** A monkey of 20 kg is climbing on a rope. If breaking strength of rope is 250 N. Find maximum acceleration with which monkey can climb on rope.



**Sol.**  $T - mg = ma \Rightarrow T = m(g + a)$

$$250 = 20(10 + a) \Rightarrow a = 2.5 \text{ m/s}^2$$

**Ex.** A monkey is moving down on string with constant acceleration. If breaking strength of a string is half of the weight of monkey. Find minimum acceleration with which monkey can move down on a string.



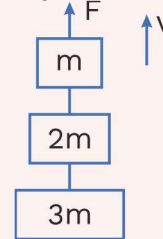
**Sol.**  $mg - T = ma \Rightarrow mg - \frac{mg}{2} = ma$

$$\Rightarrow a = 5 \text{ m/s}^2$$

### Rack your Brain



Three blocks with masses  $m$ ,  $2m$  and  $3m$  are connected by strings, as shown in figure. After an upward force  $F$  is applied on block  $m$ , the masses move upward at constant speed  $v$ . What is the net force on the block of mass  $2m$  ( $g$  is the acceleration due to gravity).



### Rack your Brain



Three blocks A, B and C of masses 4 kg, 2 kg and 1 kg respectively, are in contact on a frictionless surface, as shown. If a force of 14 N is applied on the 4 kg block then calculate the contact force between A and B.





**Motion of bodies in contact:**

**Case-I:**

$F - N = m_1 a$

$N = m_2 a$

$F = (m_1 + m_2) a$

$a = \frac{F}{m_1 + m_2}$  or  $a = \frac{F_{\text{net}}}{m_{\text{total}}}$

$N = \frac{m_2 F}{m_1 + m_2} \Rightarrow N = \text{contact force}$

System of masses tied by string

**Case-II:**

- Motion of connected bodies:

$T = m_1 a$

$F - T = m_2 a$  ... (ii)

... (i)

$a = \frac{F}{m_1 + m_2} = \frac{F_{\text{net}}}{m_{\text{total}}}$

$T = \frac{m_2 F}{m_1 + m_2}$

**Case-III:**

Tension in a massive string.

- If mass is distributed uniformly along the length of the string.

$T$

$m$

$x$

$F$

$a$

$L$

$M$

$F$

$L$





$$F - T = ma \quad \dots(i)$$

$$F = Ma \quad \dots(ii)$$

$$T = F - ma$$

$$\boxed{F = Ma}$$

$$\boxed{a = F / M}$$

$$T = F - m \cdot \frac{F}{M}$$

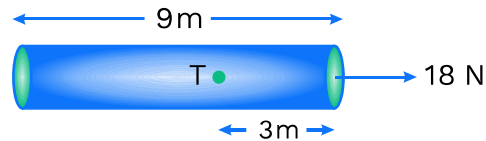
L length mass of rod = M

$$1 \text{ length mass of rod} = \frac{M}{L}$$

$$x \text{ length mass of rod} = \frac{M}{L} \cdot x = m$$

$$T = F - \left(\frac{M}{L} \cdot x\right) \cdot \frac{F}{M} \Rightarrow \boxed{T = F \left(1 - \frac{x}{L}\right)}$$

**Ex.** Find out the value of tension (T).



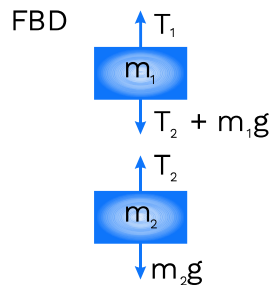
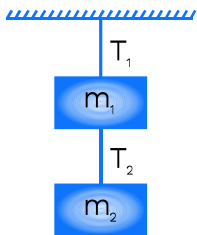
**Sol.**  $T = F \left(1 - \frac{x}{L}\right) = 18 \left(1 - \frac{3}{9}\right) \Rightarrow T = 18 \times \frac{2}{3} = 12 \text{ N}$

**Case-IV:**

- If all the bodies are in equilibrium therefore net force on each body is zero.

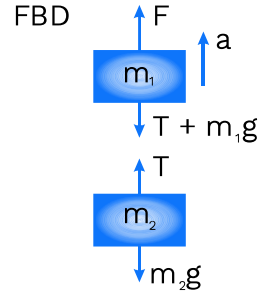
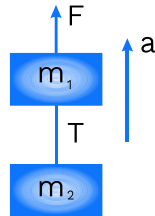
$$T_1 = T_2 + m_1g, \quad T_2 = m_2g$$

$$T_1 = (m_1 + m_2)g$$





**Case-V:**



$$F - (T + m_1g) = m_1a$$

$$T - m_2g = m_2a$$

$$F - (m_1 + m_2)g = (m_1 + m_2)a$$

$$a = \frac{F - (m_1 + m_2)g}{m_1 + m_2}$$

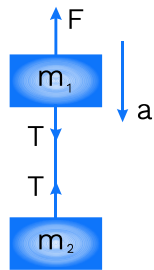
$$T = m_2(g + a) \quad \Rightarrow \quad F = (m_1 + m_2)(g + a)$$

...(i)

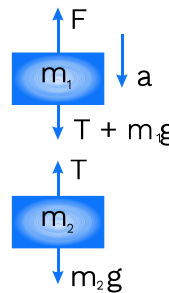
...(ii)

**Case-VI:**

- Bodies accelerating vertically downwards.



FBD



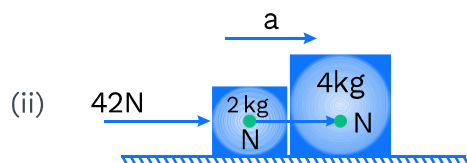
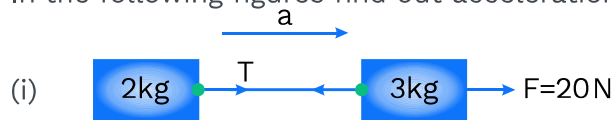
$$(T + m_1g) - F = m_1a$$

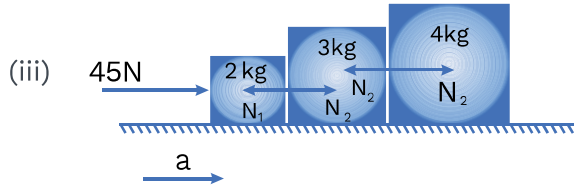
$$m_2g - T = m_2a$$

$$a = \frac{(m_1 + m_2)g - F}{m_1 + m_2} \quad \Rightarrow \quad F = (m_1 + m_2)(g - a)$$

$$T = m_2(g - a)$$

**Ex.** In the following figures find out acceleration and tension/normal force.





Sol. (i)

$$T = 2a \quad \dots(i)$$

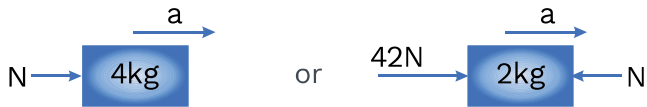
$$20 - T = 3a \quad \dots(ii)$$

By (i) and (ii)

$$20 = 2a + 3a \Rightarrow a = 4 \text{ m/s}^2$$

$$T = 2a \Rightarrow T = 2(4) \Rightarrow T = 8 \text{ N}$$

(ii)  $a = \frac{42}{6} \Rightarrow 7 \text{ m/s}^2$

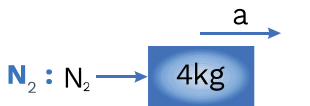


$$N = 4a \Rightarrow N = 4 \times 7$$

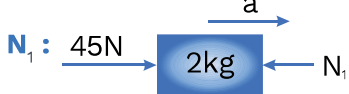
$$42 - N = 2a \Rightarrow 42 - N = 14$$

$$N = 28 \text{ N}$$

(iii)  $a = \frac{45}{2+3+4} = \frac{45}{9} = 5 \text{ m/s}^2$



$$N_2 = 4a = 4 \times 5 = 20 \text{ N}$$



$$45 - N_1 = 2a \Rightarrow N_1 = 35 \text{ N}$$

**Effective or apparent weight of a man in lift:**

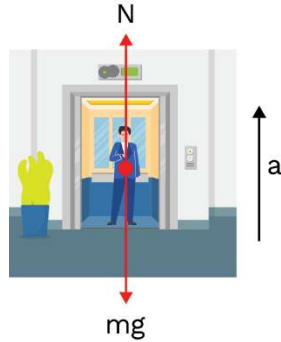
**Case-I:** If the lift is at rest or moving uniformly ( $a = 0$ ), then





$$\boxed{N = mg} \Rightarrow W_{\text{app}} = W_{\text{actual}}$$

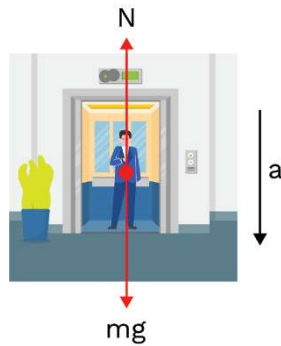
**Case-II:** If the lift is accelerating upwards, then



$$N - mg = ma \Rightarrow \boxed{N = m(g + a) = W_{\text{app}}}$$

$$W_{\text{app}} > W_{\text{actual}}$$

**Case-III:** If the lift is acceleration downwards, then



$$mg - N = ma \Rightarrow \boxed{N = m(g - a) = W_{\text{app}}}$$

$$W_{\text{app}} < W_{\text{actual}}$$

- If the lift is falling freely  $a = g$  then  $W_{\text{app}} = m(g - g) = 0$ . It means the man will feel weightless.
- If  $a > g$  then man will move up with respect to the lift.

**Case-IV:** If the lift is retarding upwards, then

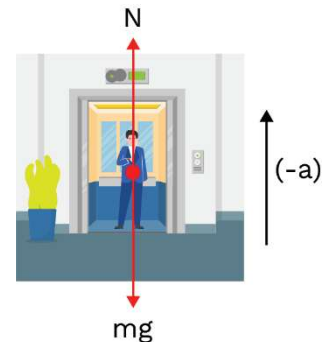
$$N - mg = m(-a) \Rightarrow \boxed{N = m(g - a) = W_{\text{app}}}$$

$$W_{\text{app}} < W_{\text{actual}}$$

### Rack your Brain

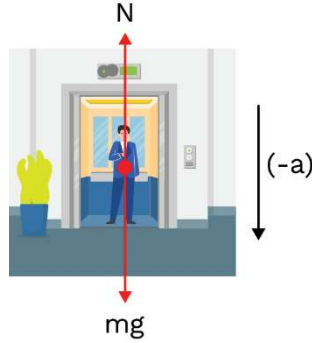


A person of mass 60 kg is inside a lift of mass 940 kg and presses the button on a control panel. The lift starts moving upwards with an acceleration  $1.0 \text{ m/s}^2$ . If  $g = 10 \text{ ms}^{-2}$ , then find the tension in the supporting cable.





**Case-V:** If the lift is retarding downwards, then



$$mg - N = m(-a) \Rightarrow \boxed{N = m(g + a) = W_{app}}$$

$$W_{app} > W_{actual}$$

### Key Points



- ◆ Apparent weight
- ◆ Weightless
- ◆ Stationary

**Ex.** A lift having mass 50 kg is connected with cable having breaking strength 100 N then find out minimum acceleration for lift for rate rise.

**Sol.**  $T_{max} < W_{act}$  (means lift downward accelerated)

$$\therefore T_{max} = m(g - a_{min})$$

$$100 = 50(10 - a_{min})$$

$$\Rightarrow 2 = 10 - a_{min} \Rightarrow a_{min} = 8 \text{ m/sec}^2$$

**Ex.** If lift is stationary the tension in cable is 50 N. If it is moving under acceleration, then tension in the cable is 80 N. Find out acceleration in this case.

**Sol.** When lift is stationary

$$T = Mg \Rightarrow 50 = Mg \Rightarrow M = 5 \text{ kg}$$

When lift is moving

$$T_2 = M(g + a) \Rightarrow 80 = M(g + a)$$

$$\Rightarrow 80 = 5(g + a)$$

$$30 = 5a \Rightarrow a = 6 \text{ m/s}^2$$

**Ex.** A lift is moving under acceleration and the tension in the string is 75% of weight of lift find out acceleration?

**Sol.**  $T = 0.75 mg$

$$\therefore T = m(g - a) = 0.75 Mg$$

$$g - a = \frac{3}{4}g \Rightarrow a = g - \frac{3}{4}g$$

$$a = \frac{g}{4}$$

### Case of pulley system:

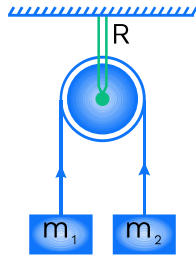
- Ideal pulley is considered massless and frictionless.
- Ideal string is massless and inextensible.



- A pulley may change the direction of force in the string but not the tension.

The only function of pulley (which has no friction on its axle to retard rotation) is to change the direction of force through the cord that joins the two blocks.

**Case-I:**



$m_1 = m_2 = m$   
 Tension in the string  
 $T = mg$   
 acceleration 'a' = zero  
 Reaction at point of suspension of the pulley or thrust on pulley.  
 $R = 2T = 2 mg$

**Case-II:**

$m_1 > m_2$   
 at mass  $m_1$ ,  
 $m_1g - T = m_1a$  ... (i)  
 at mass  $m_2$ ,  
 $T - m_2g = m_2a$  ... (ii)  
 $(m_1 - m_2)g = (m_1 + m_2)a$

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)}$$

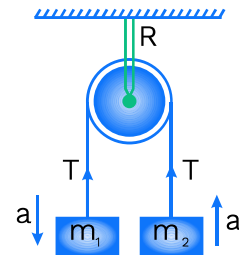
$$T = \frac{2m_1m_2}{(m_1 + m_2)} \cdot g$$

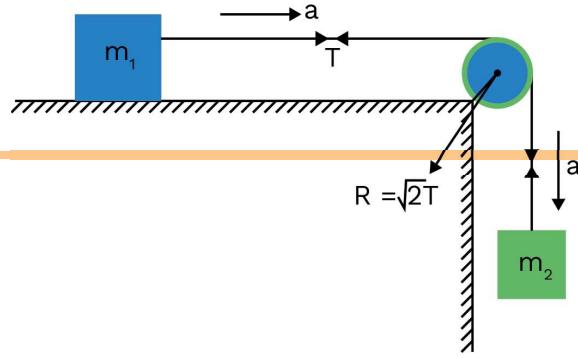
Reaction at the suspension point of pulley (thrust on pulley)

$$R = 2T = \frac{4m_1m_2}{(m_1 + m_2)} \cdot g$$

**Key Points**

- ◆ Ideal pulley





**Case-III:**

For mass  $m_1$ ,  $T = m_1 a$  ... (i)

For mass  $m_2$ ,  $m_2 g - T = m_2 a$  ... (ii)

Equation (i) + equation (ii),

$$a = \frac{m_2 g}{m_1 + m_2} \quad T = \frac{m_1 m_2}{(m_1 + m_2)} \cdot g$$

Reaction at suspension point of pulley

$$R = \sqrt{2} T$$

**Case-IV:**

For mass  $m_1$ ,  $m_1 g - T_1 = m_1 a$  ... (i)

For mass  $m_2$ ,  $T_1 - T_2 = M a$  ... (ii)

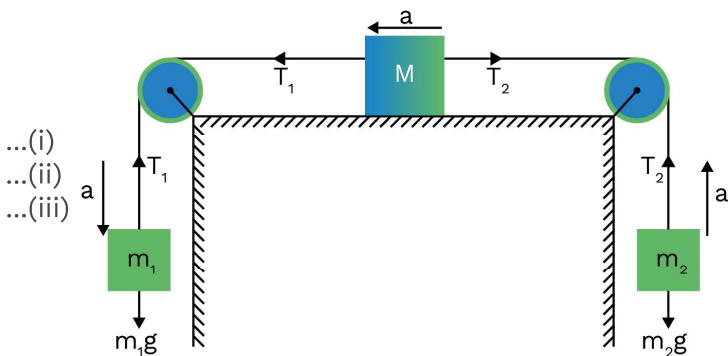
For mass  $m_3$ ,  $T_2 - m_2 g = m_2 a$  ... (iii)

Add equation (i) + (ii) + (iii),

$$a = \frac{(m_1 - m_2)g}{m_1 + M + m_2}$$

$$T_1 = m_1 g - \frac{m_1 \cdot (m_1 - m_2)g}{(m_1 + M + m_2)}$$

$$T_2 = \frac{m_2(m_1 - m_2)g}{(m_1 + M + m_2)} + m_2 g$$



**Case-V:**  $m_1 > m_2$

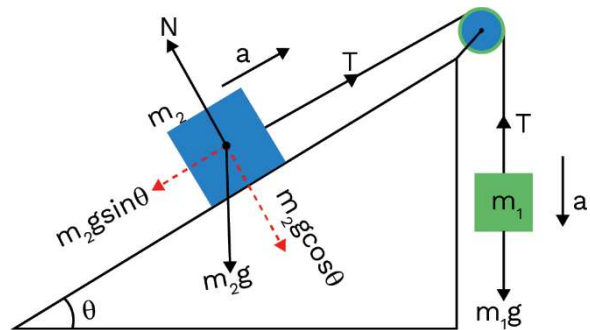
For mass  $m_1$ ,  $m_1 a = m_1 g - T$  ... (i)

For mass  $m_2$ ,  $T - m_2 g \sin \theta = m_2 a$  ... (ii)

Add equation (i) + (ii),

$$a = \frac{(m_1 - m_2 \sin \theta)g}{(m_1 + m_2)}$$

$$T = \frac{m_1 m_2 \cdot (1 + \sin \theta)g}{m_1 + m_2}$$



**Case-VI:**

If  $m_1 g \sin \alpha > m_2 g \sin \beta$

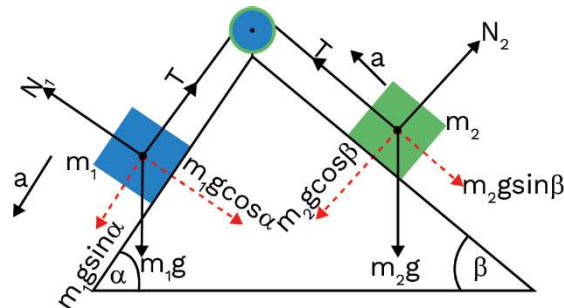
For mass  $m_1$ ,  $m_1 g \sin \alpha - T = m_1 a$  ... (i)

$T - m_2 g \sin \beta = m_2 a$  ... (ii)

Adding equation (i) and (ii)

$$a = \frac{(m_1 \sin \alpha - m_2 \sin \beta)g}{m_1 + m_2}$$

$$T = \frac{m_1 m_2 (\sin \alpha + \sin \beta)}{(m_1 + m_2)} \cdot g$$





**Case-VII:**

For mass  $m_1$ ,

$$T_1 - m_1g = m_1a \quad \dots(i)$$

For mass  $m_2$ ,

$$T_2 + m_2g - T_1 = m_2a \quad \dots(ii)$$

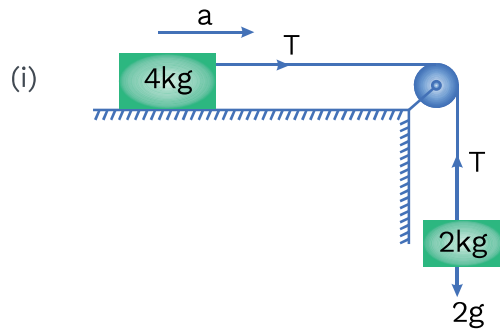
For mass  $m_3$ ,

$$m_3g - T_2 = m_3a \quad \dots(iii)$$

Solving equation (i), (ii) and (iii)

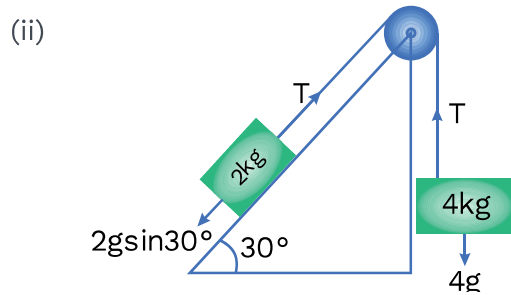
$$a = \frac{(m_2 + m_3 - m_1)}{(m_1 + m_2 + m_3)} \cdot g$$

**Ex.** Find out the acceleration and tension of block in following cases.



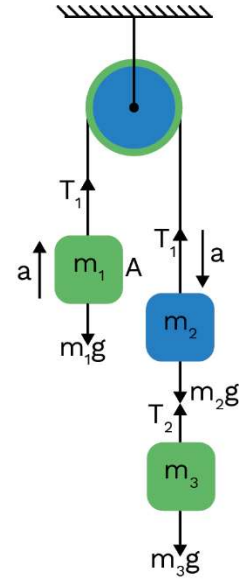
**Sol.** (a)  $a = \frac{2g}{4 + 2} = \frac{g}{3}$

(b)  $T = ma = 4 \times \frac{g}{3} \Rightarrow T = \frac{4g}{3}$



**Sol.** (a)  $a = \frac{4g - 2g \sin 30^\circ}{6} = \frac{4g - g}{6} = \frac{g}{2}$

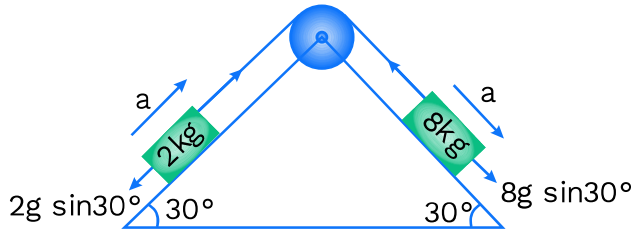
(b)  $4g - T = 4a \Rightarrow T = 4(g - a) = 4\left(g - \frac{g}{2}\right) \Rightarrow T = 2g$







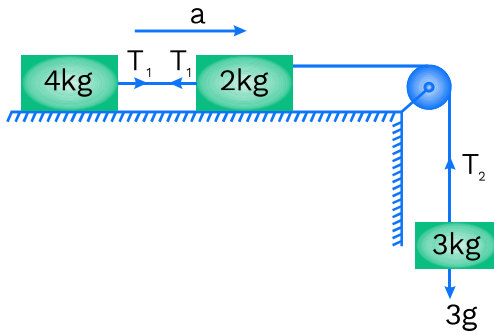
(iii)



**Sol.** (a) 
$$a = \frac{8g \sin 30^\circ - 2g \sin 30^\circ}{8 + 2} = \frac{4g - g}{10} = \frac{3g}{10}$$

(b) 
$$8g \sin 30^\circ - T = 8a \Rightarrow 4g - T = \frac{8 \times 3g}{10} \Rightarrow T = \frac{8g}{5}$$

(iv)

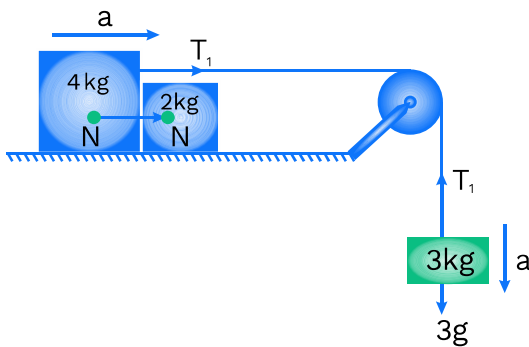


**Sol.** (a) 
$$a = \frac{3g}{4 + 2 + 3} = \frac{g}{3}$$

(b) 
$$3g - T_2 = 3a \Rightarrow T_2 = 3(g - a) = \frac{6g}{3} = 2g$$

(c) 
$$T_1 = 4a \Rightarrow T_1 = 4a = \frac{4g}{3}$$

(v)



**Sol.** (a) 
$$a = \frac{3g}{4 + 2 + 3} = \frac{g}{3}$$

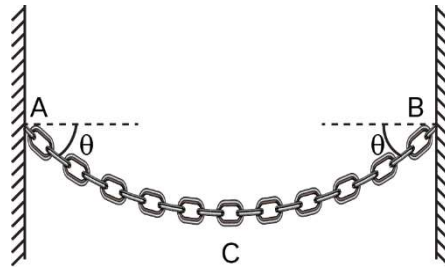


$$(b) \quad 3g - T_1 = 3a \quad \Rightarrow \quad T_1 = 3(g - a)$$

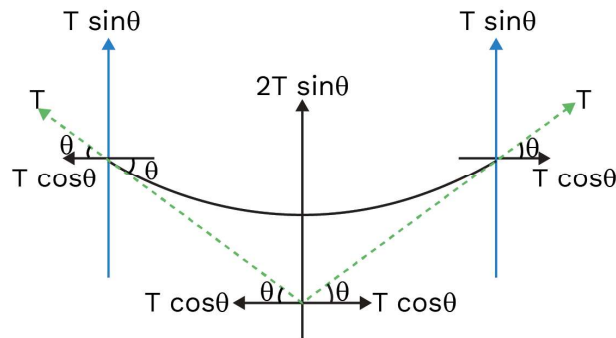
$$T_1 = 3\left(g - \frac{g}{3}\right) \quad \Rightarrow \quad T_1 = 2g$$

$$(c) \quad N = 2a \quad \Rightarrow \quad N = 2 \times \frac{g}{3} = \frac{2g}{3}$$

**Ex.** A chain of mass 'm' is attached at 2 points A & B of two fixed walls as shown in the figure. Find the tension in the chain near the walls at point A and at the midpoint C.



**Sol.** (i) At point A



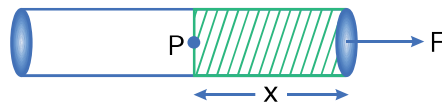
$$2T \sin \theta = mg \quad \Rightarrow \quad T = \frac{1}{2} mg \operatorname{cosec} \theta$$

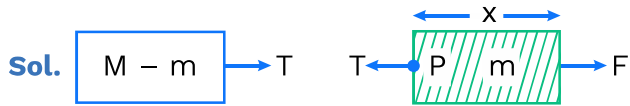
(ii) Tension along horizontal direction is same everywhere.  
 $\therefore$  no external force is acting on it in horizontal direction  
 At point C

$$T' = T \cos \theta = \frac{mg \cos \theta}{2 \sin \theta} = \frac{mg \cot \theta}{2}$$

**Ex.** Find out tension at point p if mass and length of string is M and L.

(i) In this case tension is same through the length in massive string.



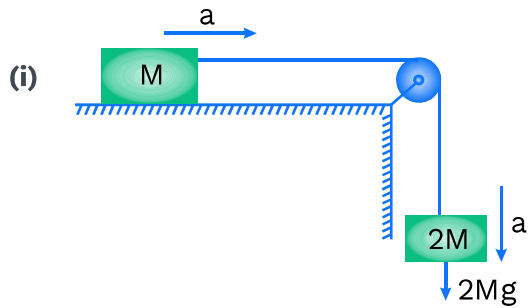


$$F = Ma \quad \Rightarrow \quad a = \frac{F}{M}$$

$$F - T = m \times a \quad \left( m = \frac{M}{L} \cdot x \right)$$

$$F - T = \frac{M}{L} \cdot x \cdot \frac{F}{M} \quad \Rightarrow \quad T = F \left( 1 - \frac{x}{L} \right)$$

**Ex.** In the following diagram find out acceleration of system.

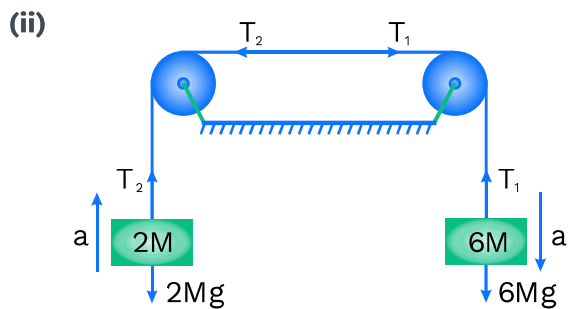


**Sol.**  $2Mg - T = 2Ma$  ... (i)

$T = Ma$  ... (ii)

Add equation (i) and (ii)

$$2Mg = 3Ma \quad \Rightarrow \quad a = \frac{2g}{3}$$



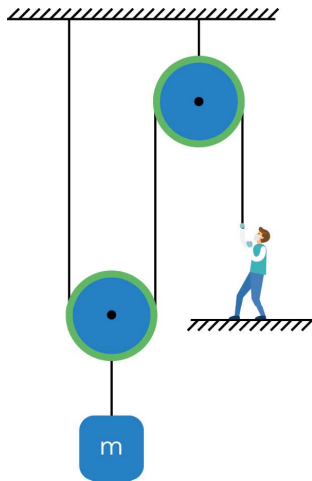
**Sol.**  $a = \frac{6Mg - 2Mg}{8M} = \frac{4g}{8} = \frac{g}{2}$



**Sol.**  $a = \frac{F}{M_{\text{total}}} = \frac{F}{2M + M} = \frac{F}{3M}$



**Ex.** Find out force applied by man (F) to hold the block in air.



### Rack your Brain

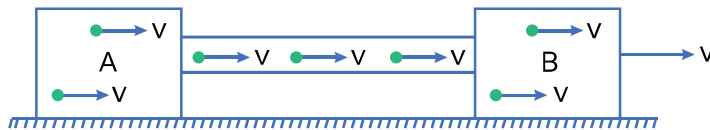


Two masses of 5 kg and 10kg are connected by string over a pulley then calculate acceleration of the system.

**Sol.**  $2T = mg \Rightarrow T = \frac{mg}{2}$

### CONSTRAINT MOTION:

- When the motion of one body is affected by the motion of another body, then motion is said to be constraint motion.



- The object is said to be in constrained environment.

### Constraint motion in string:

#### Properties of string:

- String is massless.
- String is inextensible.
- Remains taut.

**Note:** The velocity of each and every point of the string is same.

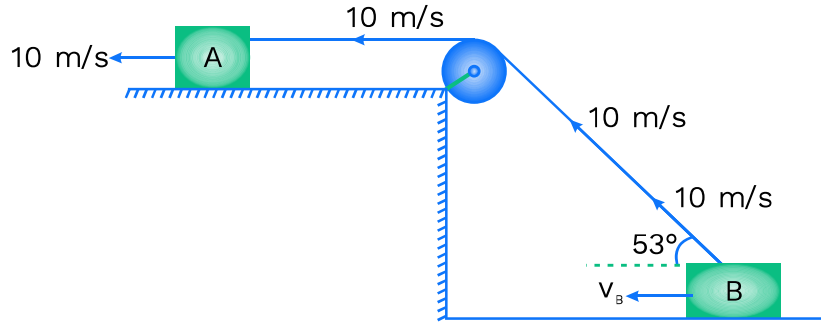
#### Constraint motion in string study about 3 formats:

- Fixed pulley format
- Moving pulley format
- Point method.



### 1. Fixed pulley format:

**Ex.** Find out the velocity of B when block A move with 10 m/s.

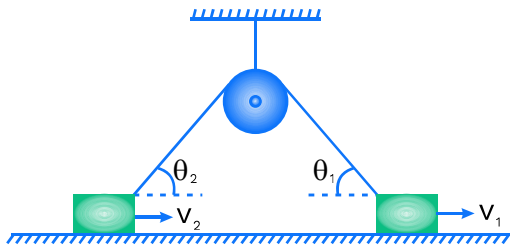


**Sol.**  $v_B \cos 53^\circ = 10 \Rightarrow v_B \times \frac{3}{5} = 10$

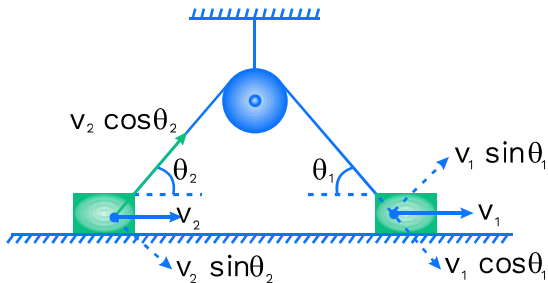
$$v_B = \frac{50}{3} \text{ m/s}$$

**Note:** The component of velocity must always be done along the string.  
Never may components of strings velocity along the direction of blocks motion.

**Ex.** Find out relation between  $v_1$  and  $v_2$ .



**Sol.**



$$v_1 \cos \theta_1 = v_2 \cos \theta_2$$

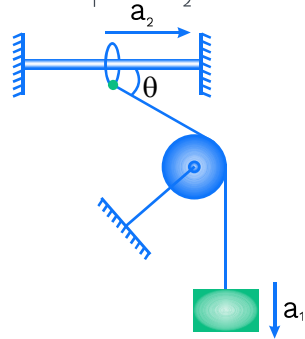
### Key Points



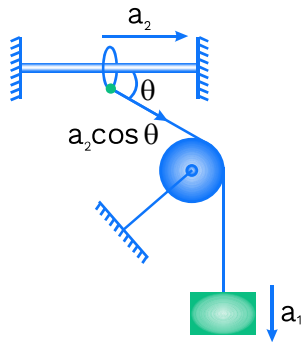
- ◆ Constraint motion
- ◆ Fixed pulley format
- ◆ Moving pulley format
- ◆ Inextensible string



**Ex.** Find out relation between  $a_1$  and  $a_2$ .

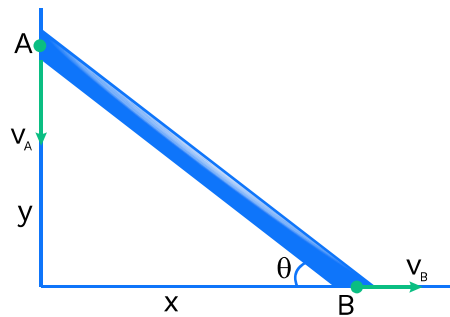


**Sol.**

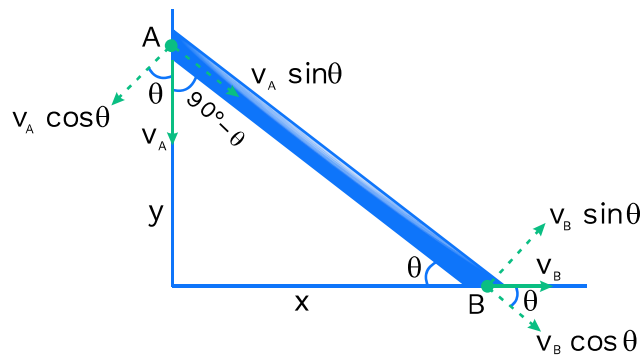


$$a_1 = a_2 \cos \theta$$

**Ex.** Find the relation between  $v_A$  and  $v_B$ .



**Sol.**  $v_B \cos \theta = v_A \sin \theta \Rightarrow v_B = v_A \tan \theta$  (by constraint motion)





Solving by calculus method

$$l^2 = x^2 + y^2$$

differentiate w.r.t. time

$$2l \cdot \frac{dl}{dt} = 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt}$$

( $l = \text{constant}$  then  $\frac{dl}{dt} = 0$ )

$$0 = x \cdot \frac{dx}{dt} + y \cdot \frac{dy}{dt} \quad \left( \frac{dx}{dt} = v_B \text{ and } \frac{dy}{dt} = v_A \right)$$

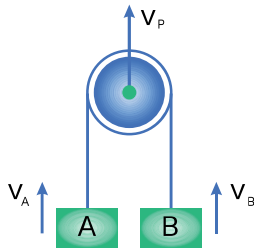
$$x \cdot v_B = -y \cdot v_A$$

(negative sign indicate y decreases w.r.t. time)

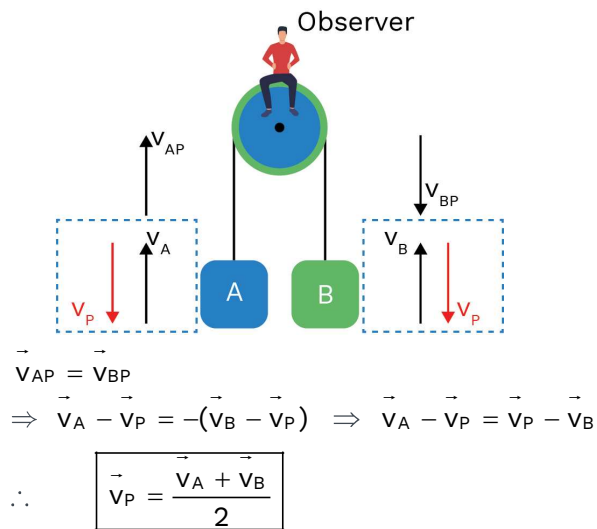
$$v_B = \frac{y}{x} \times v_A \quad (\text{by diagram } \tan \theta = \frac{y}{x})$$

$$v_B = v_A \tan \theta$$

## 2. Moving pulley format:



When observer sit on pulley and observe block A and B w.r.t. to pulley.



### Concept Reminder

$$\vec{v}_P = \frac{\vec{v}_A + \vec{v}_B}{2}$$

$$\vec{a}_P = \frac{\vec{a}_A + \vec{a}_B}{2}$$

$$\vec{x}_P = \frac{\vec{x}_A + \vec{x}_B}{2}$$



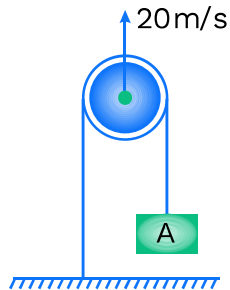
(direction has to be considered)  
differentiating w.r.t. time

$$\vec{a}_P = \frac{\vec{a}_A + \vec{a}_B}{2}$$

when integrate w.r.t. time

$$\vec{x}_P = \frac{\vec{x}_A + \vec{x}_B}{2}$$

**Ex.** Find out the velocity of A block.

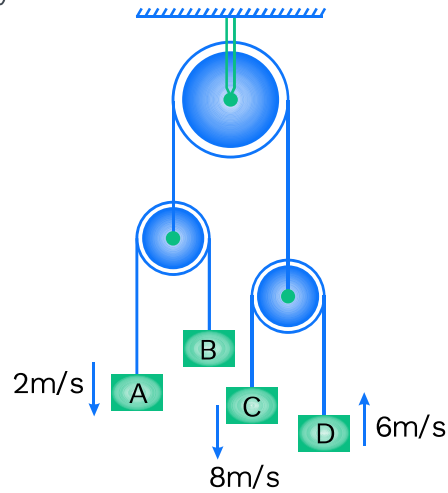


**Sol.** Fixed end velocity is zero.

$$\vec{v}_P = \frac{\vec{v}_1 + \vec{v}_2}{2} \Rightarrow 20 = \frac{0 + \vec{v}_2}{2}$$

$$v_2 = 40 \text{ m/s (upward)}$$

**Ex.** Find out velocity of block B?



$$\text{Sol. } \vec{v}_1 = \frac{8 - 6}{2} = \frac{2}{2} = 1 \text{ m/s (downward)}$$

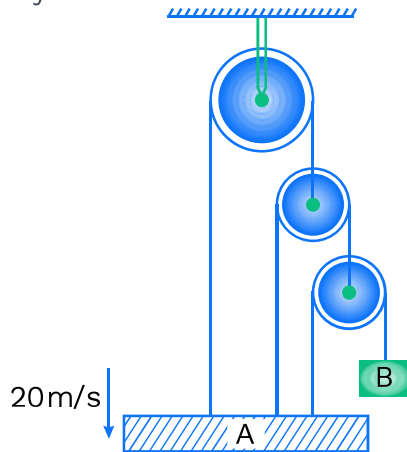
$$\vec{v}_1 = \frac{-v_A + v_B}{2} \Rightarrow 1 = \frac{-2 + v_B}{2}$$

$$v_B = 4 \text{ m/s (upwards)}$$

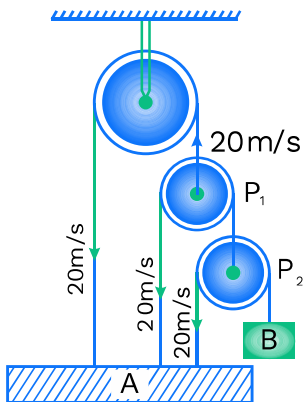




**Ex.** Find out the velocity of B?



**Sol.**



$$\vec{v}_{p_1} = \frac{\vec{v}_A + \vec{v}_{p_2}}{2}$$

$$20 = \frac{-20 + v_{p_2}}{2}$$

$$v_{p_2} = 60 \text{ m/s (upwards)}$$

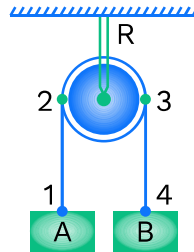
$$\vec{v}_{p_2} = \frac{\vec{v}_A + \vec{v}_B}{2}$$

$$60 = \frac{-20 + v_B}{2}$$

$$v_B = 140 \text{ m/s (upwards)}$$

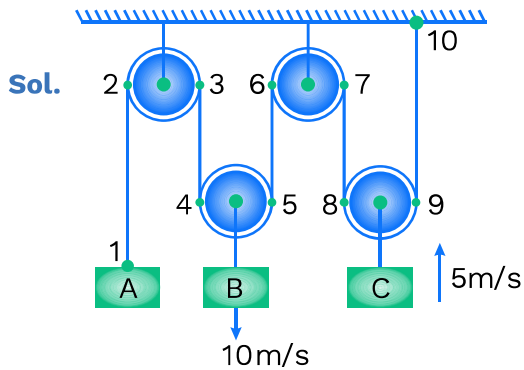
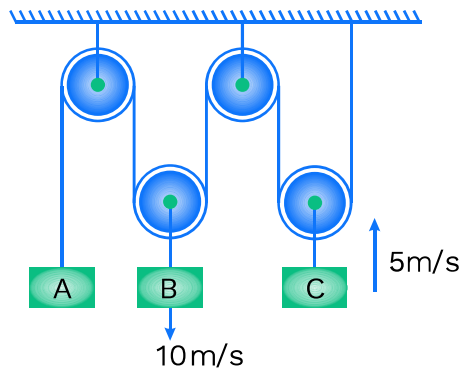


### 3. Point method:



- Search for the longest string along with the point whose velocity has to be found.
  - Number and mark the points where string touches or losses contact with rigid bodies.
  - make the sum of velocities of all points equal to zero. (because length is constant)
- $$\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \vec{v}_4 = 0$$
- [sign convection] : +ve  $\rightarrow$  Length of string increases and -ve  $\rightarrow$  length of string decreases.

**Ex.** Find out the velocity of A block?





By point method

$$v_1 + v_2 + v_3 + \dots + v_{10} = 0$$

$$v_4 = v_5 = v_B = 10 \text{ m/s (length of string increases)}$$

$$v_8 = v_9 = v_C = -5 \text{ m/s (length of string decreases)}$$

$$v_2 = v_3 = v_6 = v_7 = v_{10} = 0$$

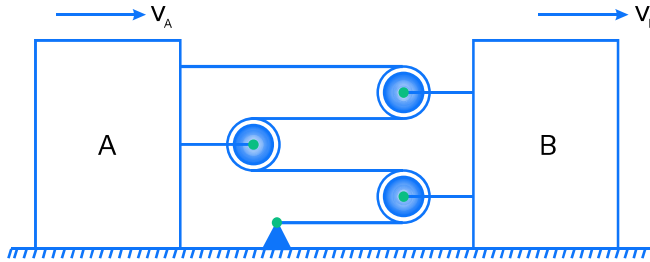
$$v_1 + 0 + 0 + 10 + 10 + 0 + 0 - 5 - 5 + 0 = 0$$

$$v_1 + 20 - 10 = 0$$

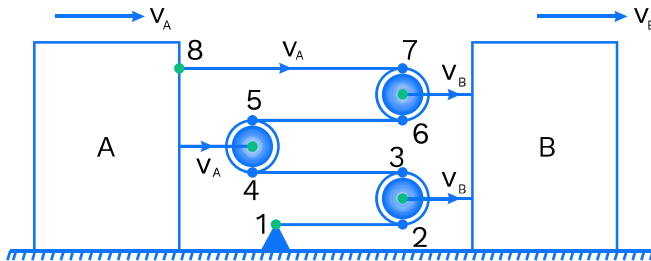
$$v_1 = -10 \text{ m/s}$$

Block A moves upward with 10 m/s.

**Ex.** Find out the relation between  $v_A$  and  $v_B$ .



**Sol.**



$$v_1 = 0$$

$$v_2 = v_3 = v_6 = v_7 = v_B$$

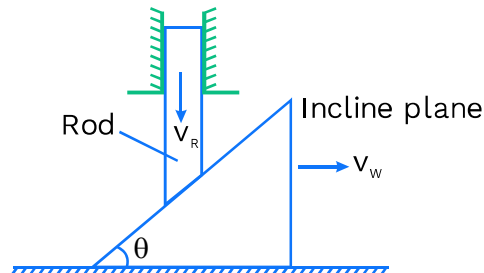
$$v_4 = v_5 = v_8 = -v_A$$

$$4v_B - 3v_A = 0$$

$$v_A = \frac{4}{3}v_B$$

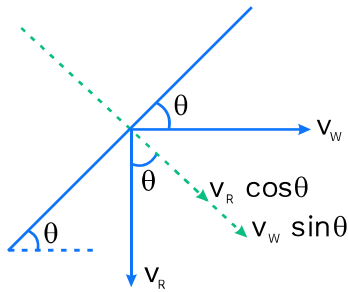
**Wedge Constraint:**

**Ex.** Find out the relation between  $v_R$  and  $v_W$ ?





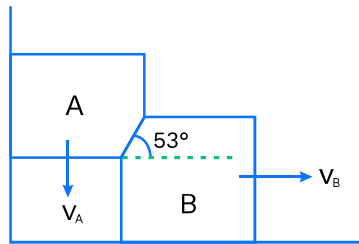
**Sol.**



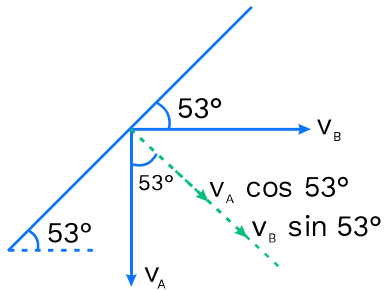
$$v_R \cos \theta = v_W \sin \theta$$

$$\Rightarrow v_R = v_W \tan \theta$$

**Ex.** Find out the relation between  $v_A$  and  $v_B$ .



**Sol.**

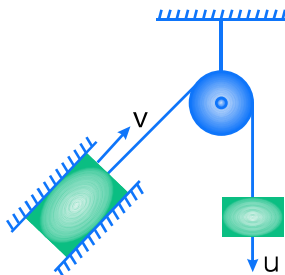


$$v_A \cos 53^\circ = v_B \sin 53^\circ$$

$$v_A = v_B \cdot \tan 53^\circ \quad \Rightarrow \quad v_A = \frac{4}{3} v_B$$

**Note:** Component of velocity perpendicular to the contact surface remains same as the blocks do not lose contact with each other.

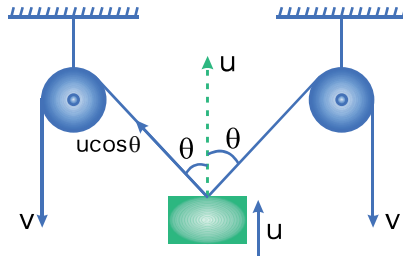
**Ex.** Find the relation between  $v$  and  $u$ .



**Sol.**  $v = u$



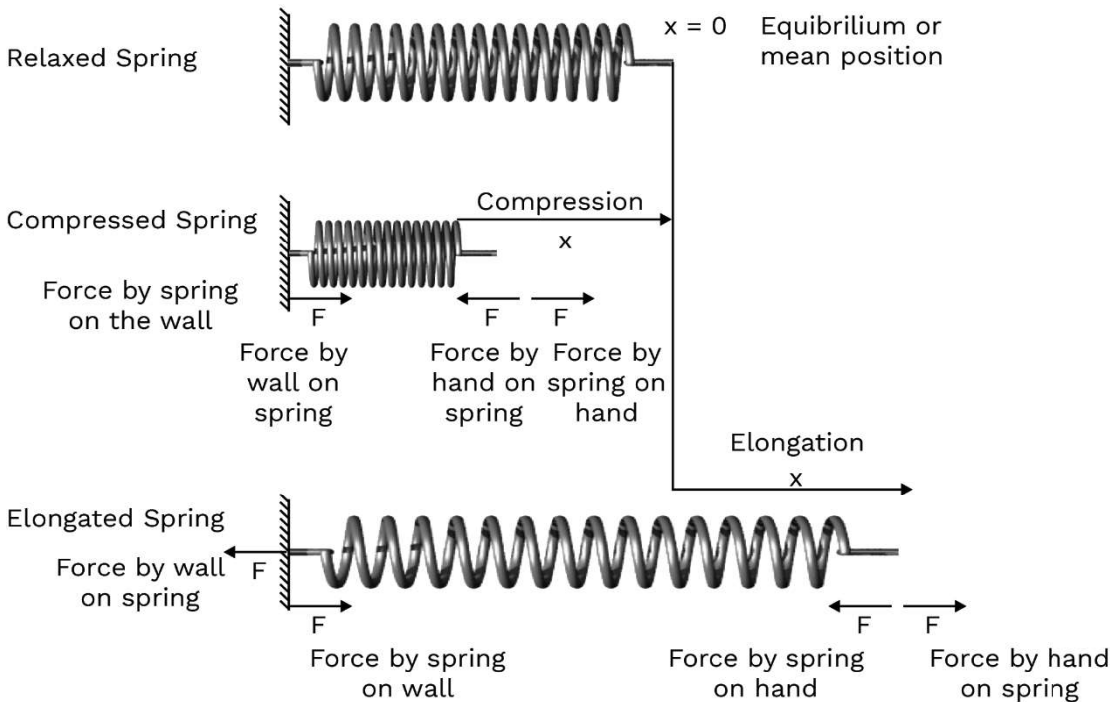
**Ex.** Find the relation between  $v$  and  $u$ .



**Sol.**  $v = u \cos \theta$

**SPRING FORCE:**

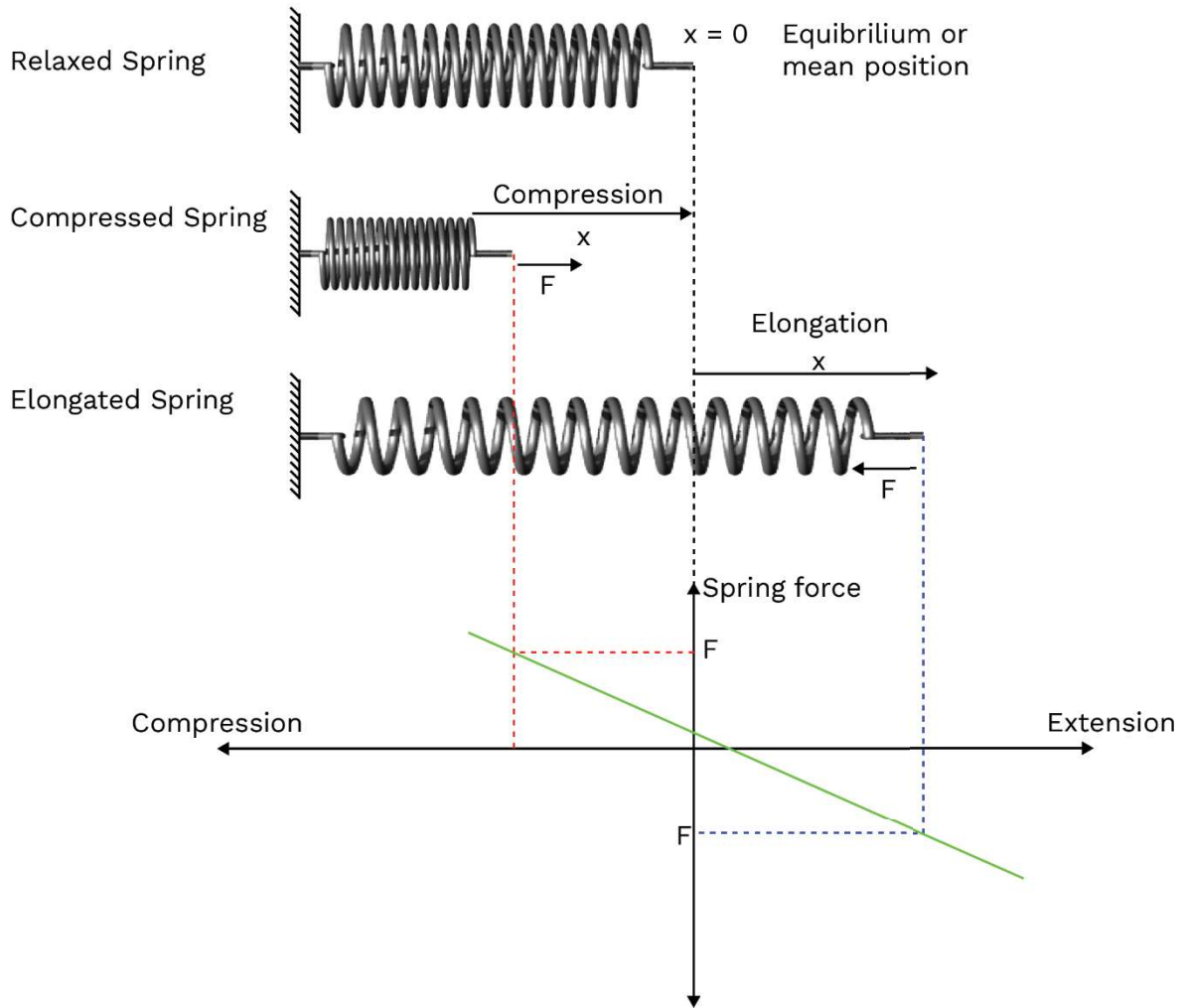
If there is force working on a spring, then spring will be in normal condition means the spring is neither compressed nor elongated. Assume a spring attached to a fixed support at one of its ends and the other end being free. If we neglect gravity, it remains in a relaxed state. When it is pushed by a force  $F$ , it is compressed and displacement  $x$  of its free end is called compression. When the spring is pulled by a force  $F$ , it is elongated and displacement  $x$  of its free end is called elongation. Various forces developed in these situations are shown in the following figure. The force applied by the spring on the wall and the force applied by the wall on the spring form an action-reaction pair according to Newton's 3<sup>rd</sup> law.





### Hooke's Law:

- Variation in spring force with deformation in length  $x$  of the spring is also shown in the following figure.



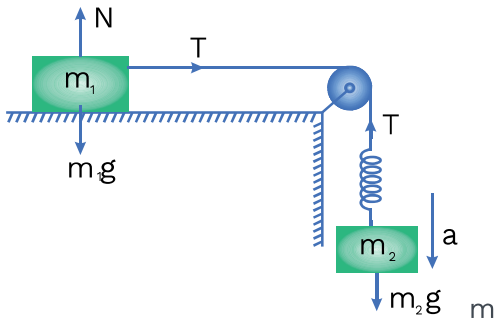
- Force  $F$  changes linearly with  $x$  and acts in a direction opposite to  $x$ .

$$F = -kx$$

- $-ve$  sign shows that  $F$  is always opposite  $x$ .
- $K$  is known as force constant or spring constant.
- Its SI unit is  $N/m$ .
- Dimension:  $[MT^{-2}]$ .



### Pulley-spring system (at steady state):

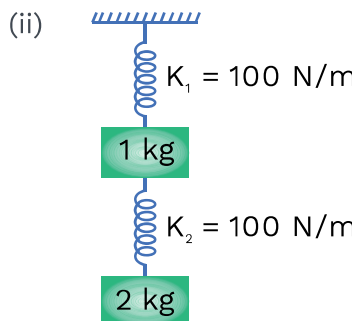
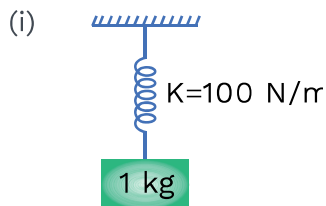


For mass  $m_1$ ,  $T = m_1 a$  ... (i)  
 For mass  $m_2$ ,  $m_2 g - T = m_2 a$  ... (ii)  
 By equation (i) and (ii)

$$a = \frac{m_2 g}{m_1 + m_2}; T = \frac{m_1 m_2}{(m_1 + m_2)} \cdot g$$

$$x = \frac{T}{K} = \frac{m_1 m_2}{K(m_1 + m_2)} \cdot g$$

**Ex.** If given system is in equilibrium, then find out elongation in spring.



**Sol.** (i)  $1 g = K \cdot x \Rightarrow 1 \times 10 = 100 \cdot x$   
 $\Rightarrow x = 0.1 \text{ m}$

(ii)  $2 g = K_2 x_2 \Rightarrow x_2 = \frac{2 \times 10}{100} = 0.2 \text{ m}$

$K_1 x_1 = 1 g + 2 g \Rightarrow x_1 = \frac{3 \times 10}{100} = 0.3 \text{ m}$

### Key Points

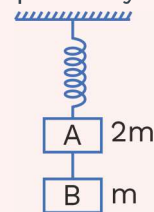


- ◆ Spring force
- ◆ Force constant
- ◆ Spring constant

### Rack your Brain

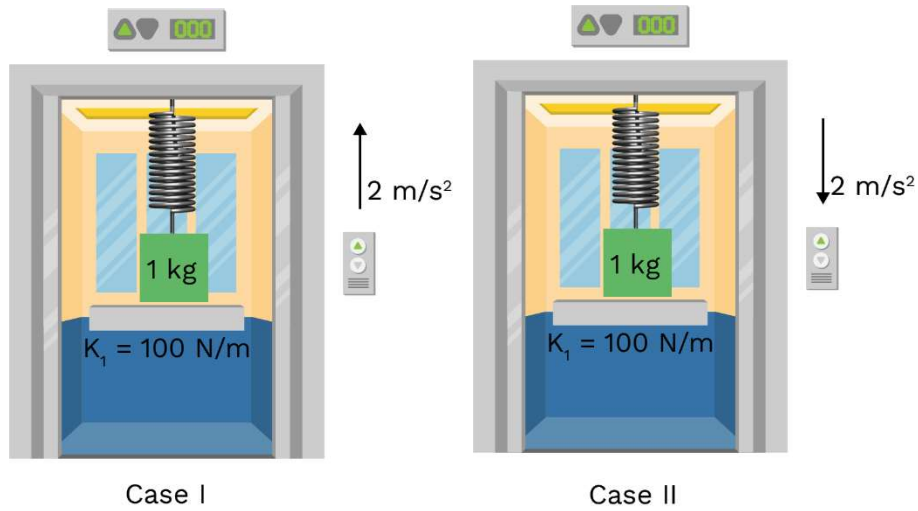


Two blocks A and B of masses  $2m$  and  $m$ , respectively, are connected by a massless and inextensible string. The whole system is suspended by a massless spring as shown in the figure. Calculate the magnitudes of acceleration of A and B, immediately after the string is out, are respectively.





**Ex.** Find elongation in following case.



**Sol.** (i)  $Kx = mg + ma \Rightarrow kx = m(g + a)$

$$x = \frac{1(10 + 2)}{100} = 0.12 \text{ m}$$

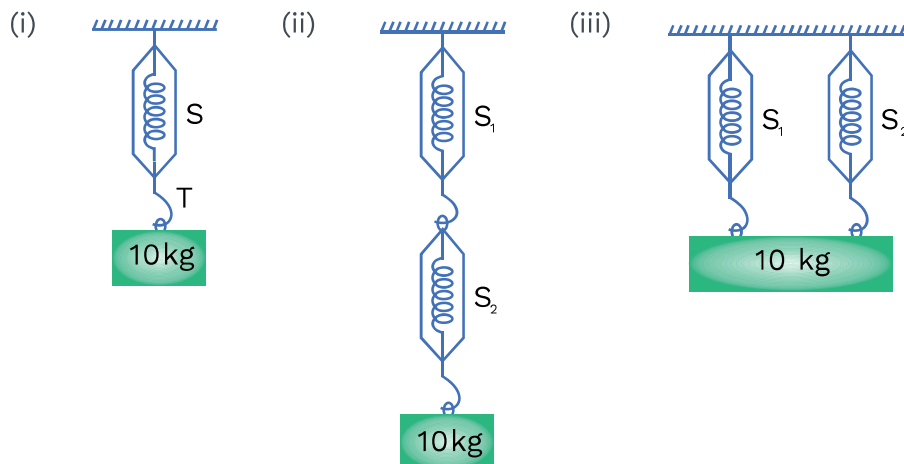
(ii)  $kx = m(g - a) \Rightarrow x = \frac{1(10 - 2)}{100} = 0.08 \text{ m}$

**Ex.** Find reading of dynamometer (spring balance) at steady state.

### Rack your Brain



A balloon with mass  $m$  is descending down with an acceleration  $a$  (where  $a < g$ ). How much mass should be removed from it so that it starts moving up with an acceleration  $a$ .



**Sol.** (i)  $T = 10 \text{ g} \Rightarrow T = 10 \text{ kg-wt}$   
Reading of S = 10 kg-wt





- (ii)  $T = 10 \text{ g} = 10 \text{ kg-wt}$   
Reading of  $S_1 = S_2 = 10 \text{ kg-wt}$
- (iii)  $2T = 10 \text{ g} \Rightarrow T = 5 \text{ g}$   
Reading of  $S_1 = 5 \text{ kg-wt}$  and  $S_2 = 5 \text{ kg-wt}$

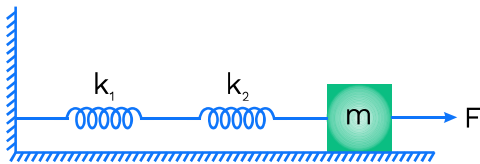
**Note:**

- When spring balance are connected in series, then reading of each balance is same and equal to applied load.
- When spring balance are connected in parallel, then reading of each balance is same and given by:

$$\text{Reading} = \frac{\text{Applied load}}{\text{Number of spring balance}}$$

**Spring combination:**

**(i) Series combination:**

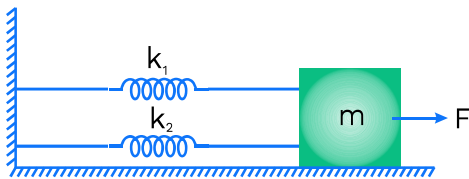


$$x = x_1 + x_2, \quad (F_1 = F_2, F_1 = k_1 x_1)$$

$$\frac{F}{k_{eq}} = \frac{F}{k_1} + \frac{F}{k_2} \quad \left( x_1 = \frac{F_1}{k_1} \right)$$

$$\boxed{\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}}$$

**(ii) Parallel combination:**



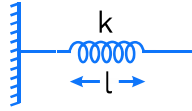
$$F = F_1 + F_2, \quad (x_1 = x_2)$$

$$k_{eq} \cdot x = k_1 x + k_2 x$$

$$\boxed{k_{eq} = k_1 + k_2}$$



### Cutting of spring:

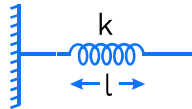


$$k \propto \frac{1}{l}, \quad l = \text{length of spring}$$

$$kl = \text{constant}$$

$$k_1 l_1 = k_2 l_2$$

**Ex.** If spring is cut by  $\frac{l}{3}$  distance, then find out relation between  $k_1$  and  $k_2$ .

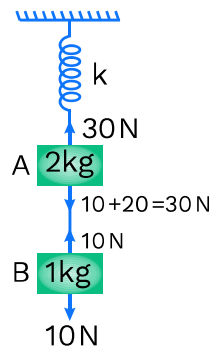


$$\text{Sol. } kl = k_1 \times \frac{l}{3}, \quad kl = k_2 \times \frac{2l}{3}$$

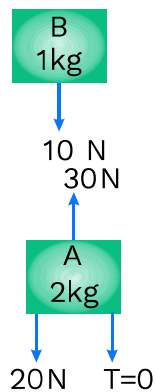
$$k_1 = 3k, \quad k_2 = \frac{3k}{2}$$

### Laziness of spring:

**Ex.** If system is in equilibrium, then string cutting, just after cutting find out acceleration of block A and B.



**Sol.**



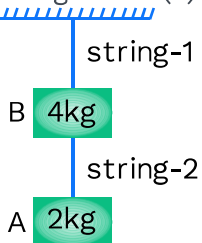


for 1 kg,  $10 = 1 a \Rightarrow a = 10\text{m/s}^2$   
 for 2 kg,  $30 - 20 = 2a \Rightarrow 2a = 10$

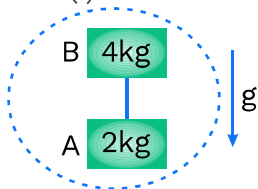
$$a = 5 \text{ m/s}^2, a = \frac{g}{2} \text{ m / s}^2 \text{ (upward)}$$

**Note:** A spring is in a stretched position when it is observed under equilibrium and thus it takes some time to again achieve its new mean position and thus the tension in a spring does not change instantaneously when changes are made to the system (Ex.-cutting etc.)

**Ex.** Find out the acceleration of block A and block B just after cutting (i) string-1 and (ii) string-2.



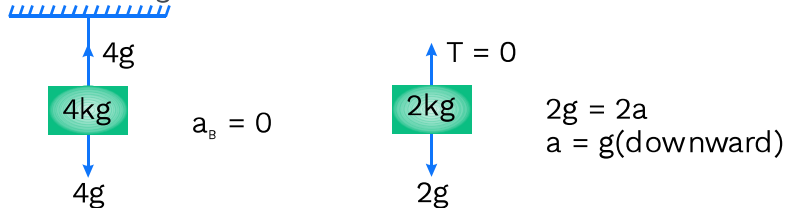
**Sol.** (i) When string-1 cut



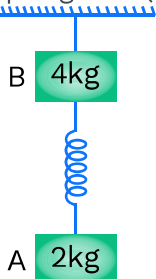
Fall motion under gravity

$$a_A = a_B = g$$

(ii) When string-2 is cut

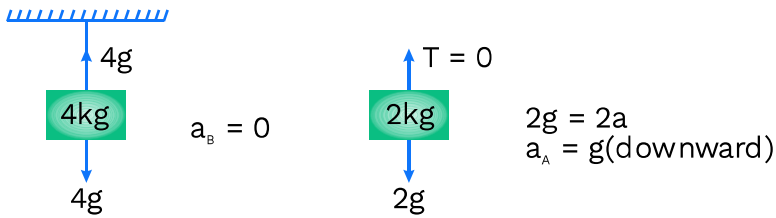


**Ex.** Find out the acceleration of block A and block B just after cutting (i) spring and (ii) string.

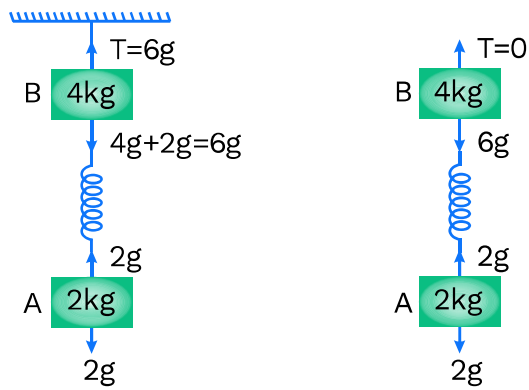




**Sol.** (i) When spring cutting



(ii) When string is cutting at equilibrium when string is cutting



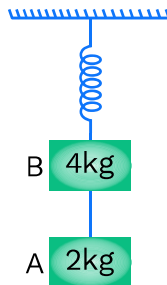
For block B                      For block A

$$6g - 0 = 4a_B \Rightarrow a_B = \frac{6g}{4}$$

$$2g - 2g = 2a_A$$

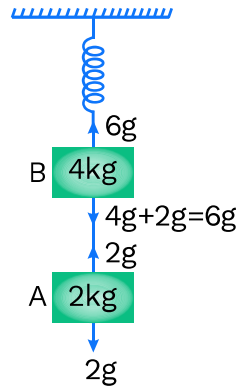
$$a_B = \frac{3g}{2} \text{ (downward), } a_A = 0$$

**Ex.** Find out the acceleration of block A and block B just after cutting (i) string (ii) spring.

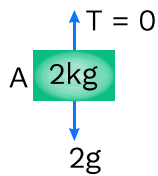




**Sol.** At equilibrium

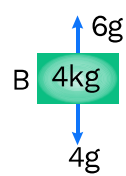


(i) When string is cut



$$2g = 2a_A$$

$$a_A = g \text{ (downward)}$$

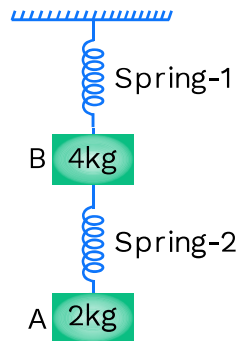


$$6g - 4g = 4a_B$$

$$2g = 4a_B$$

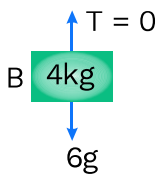
$$a_B = \frac{g}{2} \text{ (upward)}$$

**Ex.** Find out the acceleration of block A and block B just after cutting (i) spring-1 and (ii) spring-2.



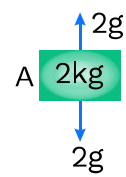
**Sol.** At equilibrium condition

(i) When spring-1 is cut

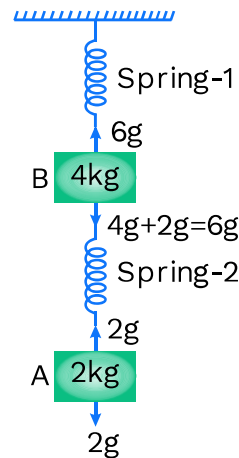


$$6g = 4a_B$$

$$\Rightarrow a_B = \frac{3g}{2} \text{ (downward)}$$

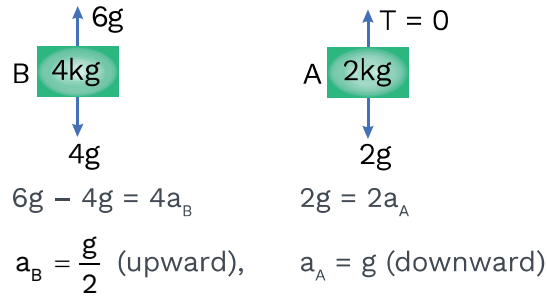


$$a_A = 0$$





(ii) When spring-2 is cut

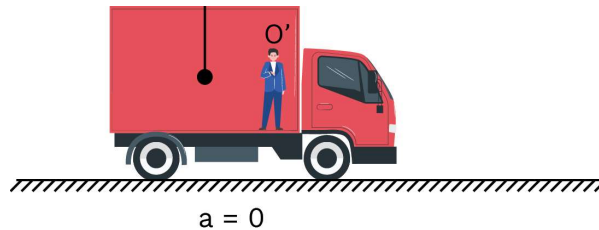


**Frame of Reference:**

- A system with respect to which the position or motion of a particle is described is known as a frame of reference.

**(i) Inertial frame of reference:**

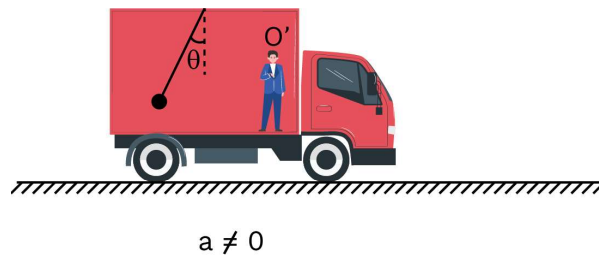
- The frame for which law of inertia is applicable is known as inertial frame of reference.



- All the frames which are at rest or moving uniformly with respect to an inertial frame, are inertial frame.

**(ii) Non-inertial frame of reference:**

- The frame for which law of inertia is not applicable is known as non-inertial frame of reference.
- All the frames which are accelerating or rotating with respect to an inertial frame will be non-inertial frames.



**Pseudo force:**

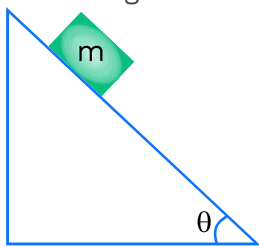
- It is a apparent force which is used to explain the motion of object in non-inertial reference frames.



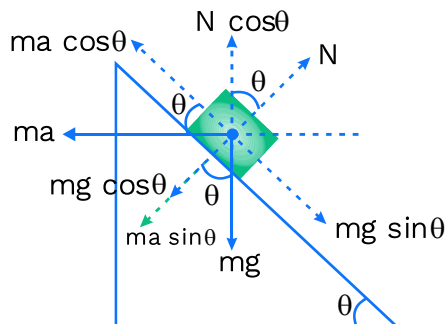
- This force always works in the direction opposite to that of acceleration of frame and it is equal to the product of mass of the body and the acceleration of the non-inertial reference frame.

$$\vec{F} = -m\vec{a}$$

**Ex.** Find out normal reaction between block and wedge and horizontal acceleration of wedge for which block remains at rest with respect to wedge.



**Sol.** Block is rest w.r.t. wedge, means net force on block is zero  
 $mg \sin \theta = ma \cos \theta$



$$\Rightarrow a = g \tan \theta$$

$$N = mg \cos \theta + ma \sin \theta$$

$$\Rightarrow N = mg \cos \theta + m \sin \theta \cdot g \cdot \frac{\sin \theta}{\cos \theta}$$

$$N = \frac{mg}{\cos \theta}$$

or  $N \cos \theta = mg$

$$N = \frac{mg}{\cos \theta}$$

**Ex.** A bob of mass 'm' is hanged from ceiling of horizontally accelerated car. Then in equilibrium find out tension in string or angle made by string from vertical.

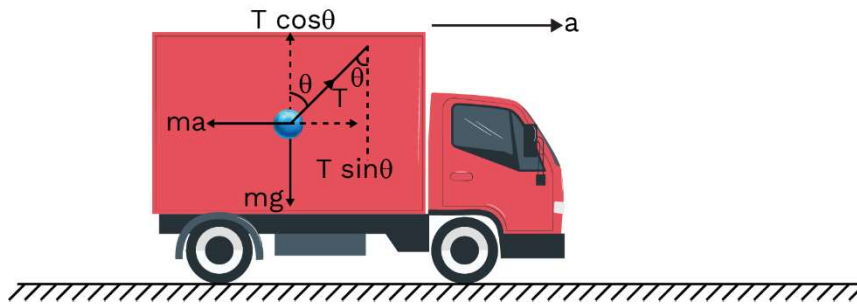
### Key Points



- ◆ Inertial frame
- ◆ Non-inertial frame
- ◆ Pseudo force



Sol.



$$T \sin \theta = ma$$

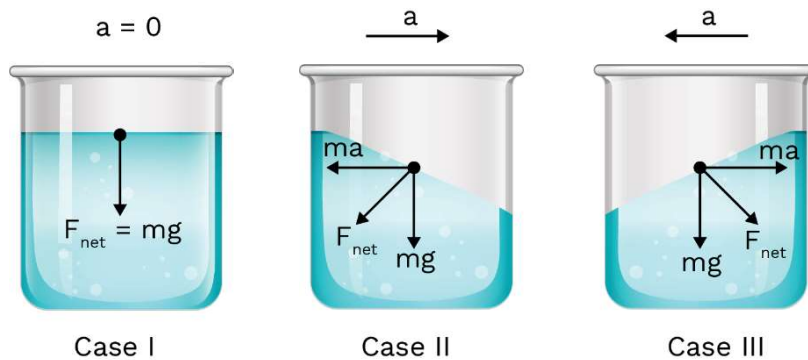
$$T \cos \theta = mg$$

$$\tan \theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1} \left( \frac{a}{g} \right)$$

$$T \sin \theta = ma \Rightarrow T \cdot \frac{a}{\sqrt{g^2 + a^2}} = ma$$

**Important Points:**

- Pseudo force does not follow the action-reaction law.
- Level of liquid in container

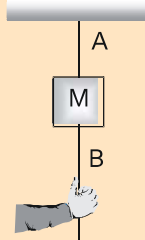






### EXAMPLE

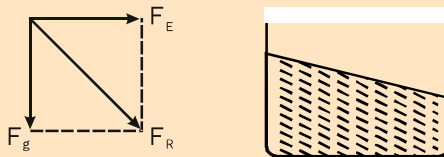
**Q1** In the arrangement shown in the figure, what will happen-



- (A) If the string 'B' is pulled with a sudden jerk?
- (B) If the string 'B' is pulled steadily?

**Sol.** (A) If the string 'B' is pulled with a sudden jerk, then it will experience tension while due to inertia of rest of mass 'M' this force will not be transmitted to the string 'A' and so the string 'B' will break.  
 (B) If the string 'B' is pulled steadily the force applied to it will be transmitted from string 'B' to 'A' through the mass 'M' and as tension in 'A' will be greater than in 'B' by  $Mg$  (weight of mass M) so the string 'A' will break.

**Q2** Drums of oil are carried in a truck. If a constant acceleration is applied on the truck, then predict the inclination of surface of oil in drum.



**Sol.** When a constant acceleration is applied on the truck, a constant force acts on it. Due to this constant force, a pseudo force will act on oil in backward direction. The resultant of this force and the force due to gravity acts along the surface of the oil. Hence, the surface of oil rises towards backward direction.

**Q3** Bullet of mass  $0.04 \text{ kg}$  moving with a velocity of  $90 \text{ ms}^{-1}$  penetrates a heavy wooden block and comes at after a  $60 \text{ cm}$ . Find the average resistive force exerted by the block on the bullet?

**Sol.** The retardation 'a' of the bullet  
 By using Newton's third equation of motion



$$v^2 = u^2 + 2as$$

$$\Rightarrow a = -\frac{u^2}{2s}$$

(assumed constant) is given by

$$a = \frac{-u^2}{2s} = \frac{-90 \times 90}{2 \times 0.6} \text{ms}^{-2} = -6750 \text{ms}^{-2}$$

The retarding force, by the 2<sup>nd</sup> Law of motion  $F = ma$

$$F = 0.04 \text{ kg} \times 6750 \text{ ms}^{-2} = 270 \text{ N}$$

The actual resistive force and therefore, de-acceleration of the bullet may not be constant. The answer therefore, only shows the average resistive force.

**Q4** The motion of a object of mass  $m$  is described by  $y = ut + \frac{1}{2}gt^2$ . Find the force acting on the object.

**Sol.** We know  $y = ut + \frac{1}{2}gt^2$

Now  $v = \frac{dy}{dt} = u + gt$

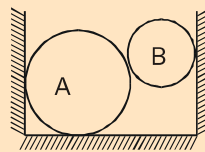
acceleration,  $a = \frac{dv}{dt} = g$

Then the force is given by

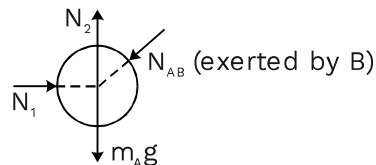
$$F = mg$$

So, the given equation defines the motion of an object under acceleration due to gravity and 'y' is the position coordinate in the direction of g.

**Q5** Two sphere A and B are placed between two vertical walls as shown in figure. Draw the free body diagrams of both the spheres.

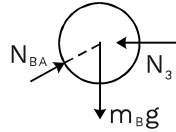


**Sol.** F.B.D. of sphere 'A':

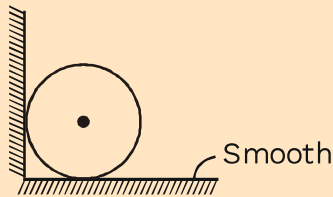




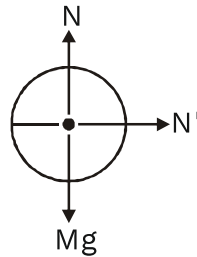
F.B.D. of sphere 'B': (exerted by A)



**Q6** Draw free body diagram of the sphere of mass 'M' placed between a vertical wall & ground as shown. (All surfaces are smooth)



**Sol.**

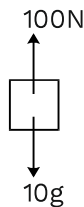


Vertical wall does not exerts force on sphere ( $N' = 0$ ).

**Q7** One end of string which passes over the pulley and connected to 10 kg mass at other end of string is pulled by 100 N force. Calculate the acceleration of 10 kg mass. ( $g = 9.8 \text{ m/s}^2$ )

**Sol.**

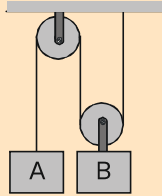
String is pulled by 100 N force. Therefore, tension in the string is 100 N.  
Free body diagram of 10 kg block



By using Newton's law of motion  
net force =  $Ma$   
 $100 - 10g = 10a$   
 $100 - 10 \times 9.8 = 10a$   
 $a = 0.2 \text{ m/s}^2$ .

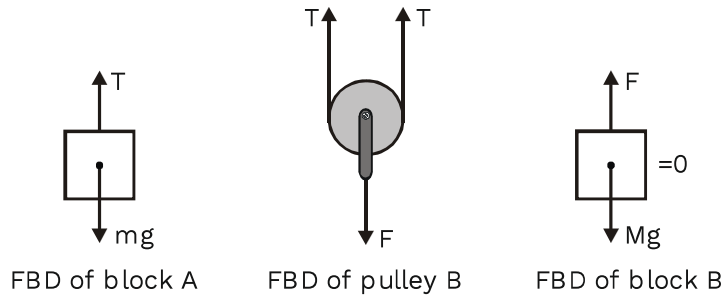


**Q8** Two boxes A and B of masses  $m$  and  $M$  are suspended by a system of pulleys are in equilibrium as shown. Express  $M$  in terms of  $m$ .



**Sol.**

Since tension on both sides of a pulley are equal and string is massless therefore tension everywhere on the string must have same magnitude.



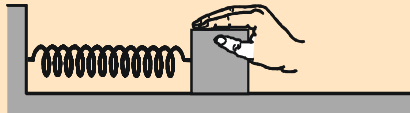
For equilibrium of block A  $\sum \vec{F} = \vec{0}$   
 $\Rightarrow T = mg$  ... (i)

For equilibrium of pulley attached to block B  $\sum \vec{F} = \vec{0}$   
 $\Rightarrow F = 2T$  ... (ii)

For equilibrium of block B  
 $\Rightarrow F = Mg$  ... (iii)

From equation (i), (ii) and (iii),  
 We have  $M = 2m$

**Q9** Consider a spring attached at one of its ends to a fixed support and at other end to a box, which rests on a smooth floor as shown in the figure. Denote mass of the box by  $m$ , force constant of the spring by  $k$  and acceleration due to gravity by  $g$ .

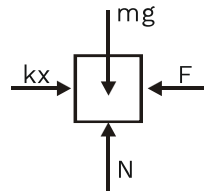


The box is pushed horizontally displacing it by distance  $x$  towards the fixed support and held at rest.

- (A) Draw free body diagram of the box.
- (B) Find force exerted by hand on the box.
- (C) Write all the third law action-reaction pairs.



**Sol.** (A)  $F$  is push by hand.



(B) Since the block is in equilibrium  $\Sigma F_x = 0 \Rightarrow F = kx$

(C) (i) Force by hand on box and force by box on hand.

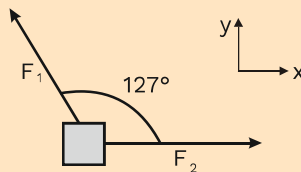
(ii) Force by spring on box and force by box on spring.

(iii) Normal reaction by box on floor and normal reaction by floor on box.

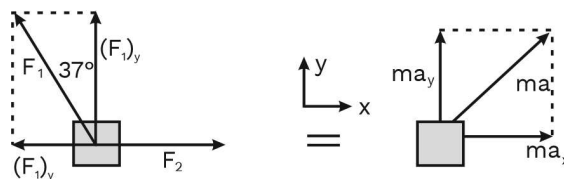
(iv) Weight of the box and the gravitational force by which box pulls the earth.

(v) Force by spring on support and force by support on spring.

**Q10** Two forces  $F_1$  and  $F_2$  of magnitudes 50 N and 60 N act on a free body of mass  $m = 5$  kg in directions shown in the figure. What is acceleration of object with respect to the free space?



**Sol.** In an inertial frame of reference with its  $x$ -axis along the force  $F_2$ , the forces are expressed in Cartesian components.



$$\vec{F}_1 = (-30\hat{i} + 40\hat{j}) \text{ and } \vec{F}_2 = 60\hat{i} \text{ N}$$

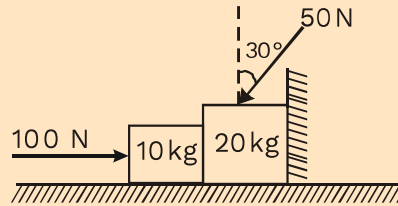
$$\Sigma F_x = ma_x \Rightarrow a_x = 6 \text{ m/s}^2$$

$$\Sigma F_y = ma_y \Rightarrow a_y = 8 \text{ m/s}^2$$

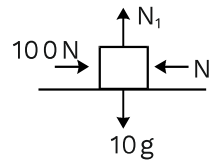
$$\vec{a} = (6\hat{i} + 8\hat{j}) \text{ m/s}^2$$



**Q11** Two blocks are kept in contact as shown in figure. Find  
**(A) forces exerted by surfaces (floor and wall) on blocks.**  
**(B) contact force between two blocks.**



**Sol.** F.B.D. of 10 kg block



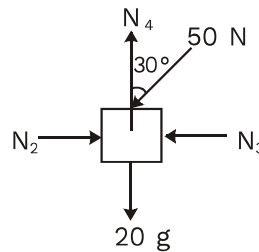
$$N_1 = 10g = 100 \text{ N}$$

...(i)

$$N_2 = 100 \text{ N}$$

...(ii)

F.B.D. of 20 kg block



$$N_2 = 50 \sin 30^\circ + N_3$$

$$\therefore N_3 = 100 - 25 = 75 \text{ N}$$

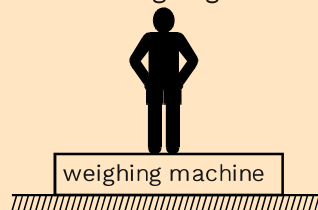
...(iii)

$$\text{and } N_4 = 50 \cos 30^\circ + 20g$$

$$N_4 = 243.30 \text{ N}$$

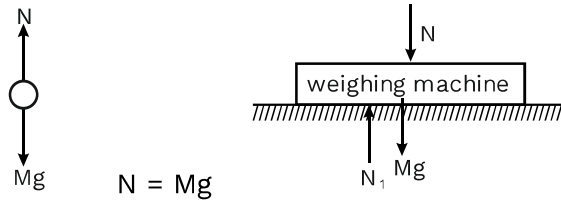
**Q12** A man of mass 60 Kg is standing on a weighing machine placed on ground.  
**Calculate the reading of machine ( $g = 10 \text{ m/s}^2$ )**

F.B.D. of weighing machine





**Sol.** For calculating the reading of weighing machine, we draw F.B.D. of man and machine separately.  
F.B.D. of man

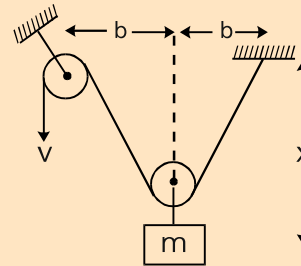


Here force exerted by object on upper surface is  $N$   
Reading of weighing machine

$$N = Mg = 60 \times 10$$

$$N = 600 \text{ N.}$$

**Q13** The figure shows one end of a string being pulled down at constant velocity  $v$ . Find the velocity of mass ' $m$ ' as a function of ' $x$ '.

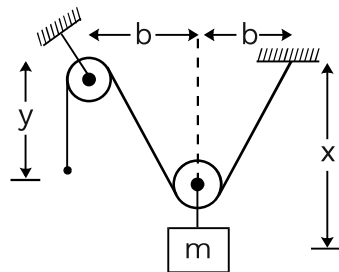


**Sol.** Using constraint equation  
 $2\sqrt{x^2 + b^2} + y = \text{length of string} = \text{constant}$   
Differentiating w.r.t. time

$$\frac{2}{2\sqrt{x^2 + b^2}} \cdot 2x \left( \frac{dx}{dt} \right) + \left( \frac{dy}{dt} \right) = 0$$

$$\Rightarrow \left( \frac{dy}{dt} \right) = v$$

$$\left( \frac{dx}{dt} \right) = -\frac{v}{2x} \sqrt{x^2 + b^2}$$



**Q14** A wagon of mass 1000 kg is moving with a speed 50 km/h on frictionless horizontal rails. A 250 kg mass is dropped into it. Then find the speed of it by which it will move now?

**Sol.** Initially the wagon of mass 1000 kg is moving with speed of 50 km/h  
So, its momentum =  $1000 \times 50 \frac{\text{kg} \times \text{km}}{\text{h}}$

When a mass 250 kg is dropped into it. New mass of the system  
 $= 1000 + 250 = 1250 \text{ kg}$

Let 'v' is the velocity of the system.

Using the conservation of linear momentum: Initial momentum = Final momentum

$$\Rightarrow 1000 \times 50 = 1250 \times v$$

$$\therefore v = \frac{50,000}{1250} = 40 \text{ km/h}$$

**Q15** The kinetic energy of two masses  $m_1$  and  $m_2$  are equal. Find the ratio of their linear momentum.

**Sol.** Relation between linear momentum (P), mass (m) and kinetic energy (E)

$$p = \sqrt{2mE} \Rightarrow p \propto \sqrt{m}$$

[as E is constant]

$$\therefore \frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}}$$

**Q16** The velocity of a particle of mass 2 kg is given by  $\vec{v} = at\hat{i} + bt^2\hat{j}$ . Find the force acting on the particle.

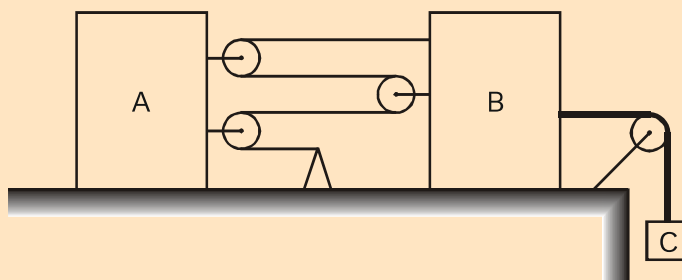
**Sol.** From second law of motion

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v})$$

$$= 2 \frac{d}{dt}(at\hat{i} + bt^2\hat{j}).$$

$$\Rightarrow \vec{F} = 2a\hat{i} + 4bt\hat{j}$$

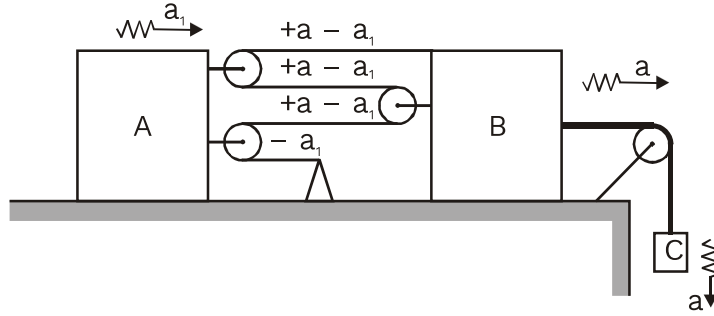
**Q17** Block C shown in figure is going down at the acceleration  $2 \text{ m/s}^2$ . Calculate the acceleration of the blocks A and B.







**Sol.** The assumed accelerations of the blocks A and B and corresponding changes in the different parts of the string between the blocks A and B are shown in figure.



It is obvious that the block B will have the same acceleration along horizontal direction as the block C is having along the downward direction. Constraint equation for the string between the blocks A and B can be written as

$$\begin{aligned}
 (+a - a_1) + (+a - a_1) + (+a - a_1) + (-a_1) &= 0 \\
 \Rightarrow 3a - 4a_1 &= 0 \\
 \Rightarrow a_1 &= \frac{3}{4}a = \frac{3}{4} \times 2\text{m/s}^2 = \frac{3}{2}\text{m/s}^2
 \end{aligned}$$

**Q18** If the force on a rocket moving with a constant velocity of 300 m/s is 210 N, then find the rate of combustion of the fuel.

**Sol.** Force on the rocket =  $\frac{u dm}{dt}$

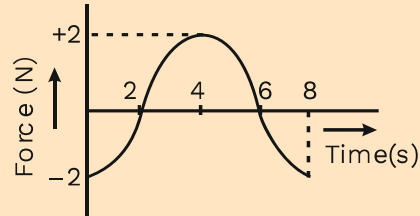
$$\therefore \text{Rate of combustion of fuel } \left( \frac{dm}{dt} \right) = \frac{F}{u} = \frac{210}{300} = 0.7 \text{ kg/s.}$$

**Q19** A batsman hits ball back straight in the direction of the bowler without changing its initial speed of 12 m s<sup>-1</sup>. If the mass of the ball is 0.15 kg, calculate the impulse imparted to the ball. (Assume linear motion of the ball)

**Sol.** Change in momentum = 0.15 × 12 - (- 0.15 × 12) = 3.6 N,  
Impulse = 3.6 N,  
In the direction from the batsman to the bowler.



**Q20** The force-time ( $F - t$ ) curve of a particle executing linear motion is as shown in the figure. Find the momentum acquired by the particle in time interval from zero to 8 second.



**Sol.** Momentum acquired by the particle is numerically equal to the area enclosed between the  $F-t$  curve and time Axis. For the given diagram area in a upper half is positive and in lower half is negative (and equal to the upper half). So net area is zero. Hence the momentum acquired by the particle will be zero.

# MIND MAP

**Analysis of Translational Motion using NLM**  
In translation motion of a body, velocity of every other point of the body.

In translation motion system can be treated as a particle.

### Steps To Follow

- (1) Define a System
- (2) Define the Environment of the System
- (3) Draw Free Body Diagram (FBD) of the system. Take only forces ON the system (not by the system).
- (4) Select appropriate axis and apply Newton's 2nd Law along each axis.

$\Sigma \vec{F} = m\vec{a}$  where  $\Sigma \vec{F}$  is net force acting on the system along the chosen axis.

### Note:

'ma' is not a force therefore, during the listing of forces in FBD in inertial frame, 'ma' should not be included.

### Translational Equilibrium

A system is said to be in Translational Equilibrium, when net force on the system is zero.

$$\Sigma \vec{F} = 0$$

## NLM

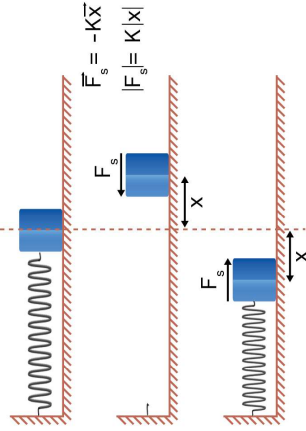
### Note:

If magnitude of acceleration of each particle connected with a string is same then

$$a = \frac{\text{Net pulling force on string}}{\text{total mass}}$$

### Spring Force:

NPL(Natural length position)



Variation of 'k' with natural length

$$k\ell = \text{Constant} \quad k\ell = 2k \frac{\ell}{2}$$

If both ends of a spring are attached with inertial mass, then sudden change in length of spring is not possible.

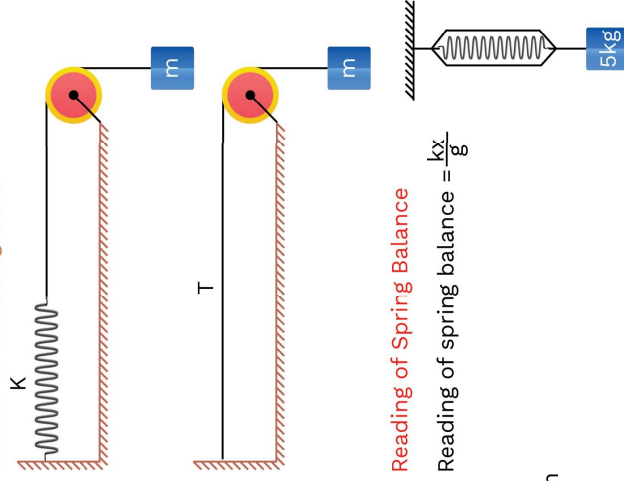
### Reading of Weighing Machine



If an object is put on a weighing machine and 'N' is normal contact force b/w object & machine, then reading of weighing machine will be given by

### Key Point

If a spring is connected with a string and then replace spring with string and find tension (T) in string and equate T with Kx to find elongation.



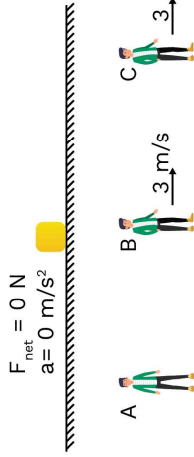
### Reading of Spring Balance

$$\text{Reading of spring balance} = \frac{Kx}{g}$$

**Newton's First Law of Motion (Law of Inertia)**

Every body preserves its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by external forces impressed on it.

Inertial and Non-Inertial Reference Frame



Inertial Frame    Inertial Frame    Non-Inertial Frame

If the net force acting on a body is zero, it is possible to find a reference frame in which that body has zero acceleration. Such reference frame is called Inertial Reference Frame.

**Newton's Second Law of Motion**

In Inertial Frame,  $\Sigma \vec{F} = m\vec{a}$  i.e.  $\vec{F}_{net} = m\vec{a}$

2nd Law for x-axis

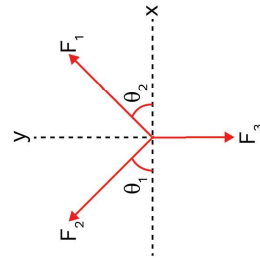
$$\Sigma \vec{F}_x = m\vec{a}_x$$

$$\Rightarrow F_1 \cos \theta_1 - F_2 \cos \theta_2 = ma_x$$

2nd Law for y-axis

$$\Sigma \vec{F}_y = m\vec{a}_y$$

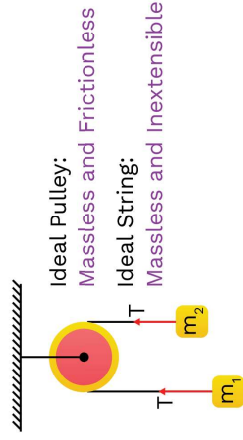
$$\Rightarrow F_1 \sin \theta_1 - F_2 \sin \theta_2 - F_3 = ma_y$$



**Tension Force**

- It is an electromagnetic type of force.
- This is a force applied by a string on a object or Force applied by one part of string on the remaining part of the string.
- It acts along the string and away from the system on which it acts.

Tension in a massless string remains constant throughout the string if no tangential force acts along the string.



**Newton's Third Law of Motion**

Every action has an equal and opposite reaction.

$$\vec{F}_1 = -\vec{F}_2$$

Action and Reaction act on different bodies and not on the same body.

Action and Reaction forces are of same type.

**NLM**

**Normal Contact Force**

- It is an electromagnetic type of force.
- It always acts among the common normal of the two surfaces in contact i.e. perpendicular to the surfaces.
- It is always directed towards to the system.



Linear Momentum is the product of Mass and Velocity

$$\vec{p} = m\vec{v} \text{ (kg m/s) or (N-s)}$$

$$F_{net} = \frac{d\vec{p}}{dt}$$

The rate of change of Linear Momentum of a body = net Force on a body.



