Magnetism

DISCLAIMER

Magnetism

MAGNET

- A body which attracts the substances like Iron, Cobalt, Nickel and which exhibits directive property is called magnet.

Types of Magnet:

1. Natural magnets:

(a) The magnet which is found in nature is called a natural magnet.
e.g., magnetite (Fe_3O_4).

(b) In generally, they are the weak magnets.

2. Artificial magnets:

The magnets which are artificially prepared are known as artificial magnets. These are generally made of iron, steel and nickel.

Properties of Magnets:

1. Attractive property: The property of attracting pieces of iron, steel, cobalt, nickel etc. by a magnet is called attractive property.

- It was found that when a magnet is dipped into iron fillings the concentrations of iron fillings is maximum at ends and minimum at centre.
- The places in a magnet where the attracting power is maximum are called poles.

2. Directive property:

- If a magnet is suspended freely, its length becomes parallel to N-S direction. This is called directive property.
- The pole at the end pointing north is called north pole while the other pointing south is called south pole.
- The magnetic poles always exist in pairs if a magnet is broken into number of pieces, each piece becomes a magnet with two equal and opposite poles. This implies that monopole does not exist.
- The two poles of a magnet are found to be equal in strength and opposite in nature.

Key Points

- Magnet
- Natural magnet
- Artificial magnet
- Poles

Definition

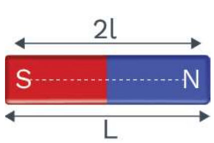
A body which attracts iron, cobalt, nickel, like substances and exhibits directive property is called magnet.

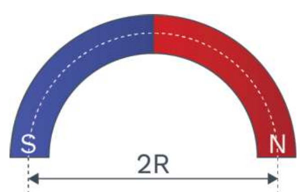
Concept Reminder

Due to their odd shapes and weak attracting powers natural magnets are rarely used.

- Unlike poles attract each other and like poles repel each other.
- There can be magnets with no poles. e.g., Solenoid and toroid have properties of magnet but no poles.
- **Magnetic axis and magnetic meridian:** The line which is joining the poles of any magnet is called magnetic axis and the vertical plane passing through the axis of a magnet which is freely suspended is called magnetic meridian.
- **Geometrical length (L):** The actual length of any magnet is called geometric length.
- **Magnetic length $(2\vec{l})$:** The shortest distance inbetween two poles of a magnet along the axis is called magnetic length or effective length. As the two poles are not exactly at the ends. The magnetic length is always lesser than geometric length of a magnet. Effective length depends only on the positions of the poles but not on the magnet.

Examples:





Magnetic length = 2 l
Geometrical length = L

Magnetic length = 2 R
Geometrical length = πR

Magnetic length is a vector quantity. Its direction is given from south pole to north pole along is axis.

Magnetic length = $\frac{5}{6}$ geometrical length.

- **Pole Strength (m):** The ability of a magnetic pole, to attract or to repel another pole of a magnet is called pole strength. SI unit: ampere-meter. The pole strength is a scalar quantity. It depends

Concept Reminder

The idea of magnetic monopoles so far has frustrated all its investigator as experimenters has fainted to find any sign of the particle while theorists on the other hand have failed to find any good reason for its non-existence.

Definitions

- The property of attracting pieces of iron, steel, cobalt, nickel etc. by a magnet is called attractive property.
- If a magnet is suspended freely then its length becomes parallel to N-S direction. This is called directive property.

Key Points

- Geometric length
- Magnetic length
- Magnetic meridian
- Inductive property

upon the area of cross section of the pole. Its dimensional formula is $[M^0LT^0A^1]$.

Inductive property: When a magnetic substance such as iron bar is kept very close to a magnet an opposite pole is induced at the nearer end and a similar pole is induced at the farther end of the magnetic substance. This property is known as inductive property.

- A magnet attracts certain other magnetic substance through the phenomenon of magnetic induction. Induction precedes attraction.
- Repulsion is a sure test of magnetism.
- The pole of a magnet attracts the opposite pole while it repels similar pole.
- However, a sure test of magnetism is repulsion but not attraction.
- Because attraction can take place between opposite poles or between a pole and a piece of unmagnetized material due to induction.

Concept Reminder

A pole of magnet attracts the opposite pole while repels similar pole. However, a sure test of polarity is repulsion and not attraction, as attraction can take place between opposite poles or a pole and a piece of unmagnetized magnetic material due to induction effect.

Definition

A configuration of two magnetic poles of opposite nature and equal strength separated by a finite distance is called as magnetic dipole.

MAGNETIC MOMENT

Magnetic dipole and magnetic dipole moment(M):

- A configuration of the two magnetic poles having opposite nature & equal strength which is separated by a finite distance is called as magnetic dipole.
- The product of pole strength (either pole) and the magnetic length of a magnet is called magnetic dipole moment or simply called as magnetic moment.
- If we assume that 'm' be the pole strength of each pole and '2l' be the magnetic length, then magnetic moment M is given by

$$M = m \times 2\,l.$$

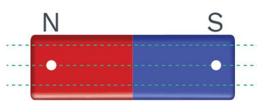
In vector form, $\vec{M} = 2\vec{l}m$

- The magnetic moment is a vector quantity, whose direction is along the axis of the magnet from south to north pole. The S.I. unit of the magnetic moment is ampere-meter^2 (A-m^2) its dimensional formula $[M^0L^2T^0A^1]$.

Variation of the magnetic moment due to cutting of magnets:

Let us take a bar magnet having length '2*l*', pole strength **'m'** and magnetic moment **'M'**.

- When a bar magnet is being cut into 'n' equal parts parallel to its length, then



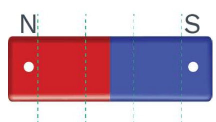
Pole strength of each part of the magnet = m/n
(∴ the area of cross section becomes (1/n) times of original magnet)
Length of each part = 2*l* (remains same)
∴ Magnetic moment of each part,

$$M' = 2l \times \frac{m}{n} = \frac{M}{n}$$

Note: If it is cut 'n' times, parallel to its length then magnetic moment of each part is

$$M' = 2l \times \frac{m}{n+1} = \frac{M}{n+1}$$

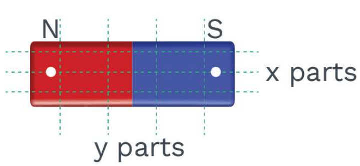
- When the magnet is being cut into 'n' equal parts perpendicular to its length then,



- Pole strength of each part of magnet = m(area of cross section remains same)
  Length of each part = 2l/n
- Magnetic moment of each part,

$$M' = \frac{2l}{n} \times m = \frac{M}{n}$$

- When the magnet is being cut into 'x' equal parts, which are parallel to its length and 'y' equal parts perpendicular to its length, then



Rack your Brain

A thin bar magnet is cut into two equal parts perpendicular to its length. Find out ratio of moment of inertia to the magnetic moment of each part will become as compare to original magnet.

Concept Reminder

Remember that no substance is non-magnetic. Though, magnetic effect may be very weak, all substance exhibit some kind of magnetism.

Key Points

- Pole strength
- Magnetic moment

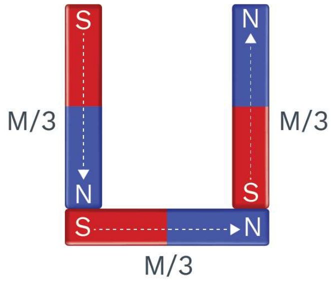
Pole strength of each part = m/x

Length of each part = 2l / y

Magnetic moment of each part,

$$M' = \frac{2l}{y} \times \frac{m}{x} = \frac{M}{xy}$$

- Variation of the magnetic moment due to bending of magnets.
- When a bar magnet is being bent, its pole strength remains same but magnetic length decreases. Therefore, magnetic moment decreases.
- When a thin bar magnet of the magnetic moment M is bent in the form of U-shape with the arms of equal length as shown in figure, then



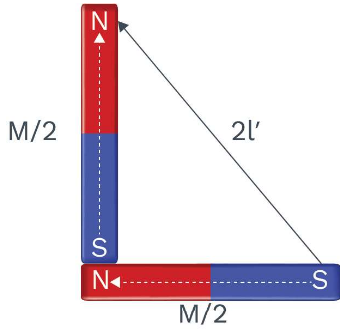
Magnetic moment of each part = M / 3

Net magnetic moment of the combination,

$$\vec{M}' = \frac{M}{3}(-\hat{j}) + \frac{M}{3}(\hat{i}) + \frac{M}{3}(\hat{j}) = \frac{M}{3}(\hat{i})$$

$$\therefore \quad M' = \frac{M}{3}$$

- When a thin magnetic needle of the magnetic moment M is being bent at the middle, so that the two equal parts are perpendicular as shown in figure, then



Concept Reminder

- If a magnet is cut into two equal parts perpendicular to length. Then,

$$m' = m \text{ and } M' = \frac{M}{2}$$

- If a magnet is cut into two equal parts parallel to its length. Then,

$$m' = \frac{m}{2} \text{ and } M' = \frac{M}{2}$$

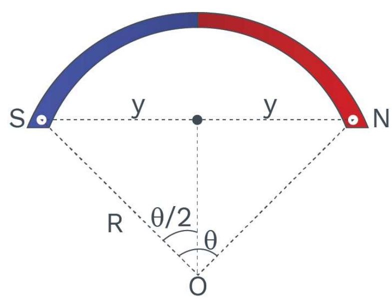
Magnetic moment of each part = $\frac{M}{2}$

Net magnetic moment of the combination,

$$\vec{M}' = \frac{M}{2}(-\hat{i}) + \frac{M}{2}(\hat{j})$$

$$\therefore \quad M' = \sqrt{2} \times \frac{M}{2} = \frac{M}{\sqrt{2}}$$

When a thin bar magnet of the magnetic moment M is bent into an arc of a circle subtending an angle ‘θ’ radians at the centre of the circle, then its new magnetic moment is given by,



$$M' = \frac{2M\sin\left(\frac{\theta}{2}\right)}{\theta} \quad (\theta \text{ must be in radians})$$

$$\text{i.e.} \quad \theta = \frac{2l}{R} \quad \Rightarrow R = \frac{2l}{\theta}$$

from the figure, effective length

$$2y = 2R\sin\frac{\theta}{2}$$

$$\because \quad \sin\left(\frac{\theta}{2}\right) = \frac{y}{R} \Rightarrow y = R\sin\left(\frac{\theta}{2}\right)$$

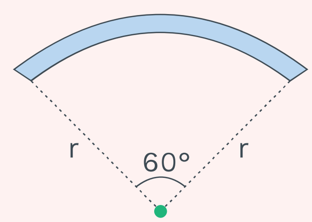
$\therefore$ New magnetic moment,

$$M' = m \times 2y = m \times 2\left(\frac{2l}{\theta}\right)\sin\left(\frac{\theta}{2}\right)$$

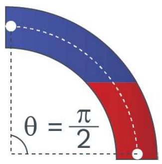
$$\Rightarrow \quad M' = \frac{2M\sin\left(\frac{\theta}{2}\right)}{\theta} \quad (\because M = 2l \times m)$$

Rack your Brain

A bar magnet of length ‘l’ and magnetic dipole moment ‘M’ is bent in the form of an arc as shown in figure. Find out new magnetic moment.



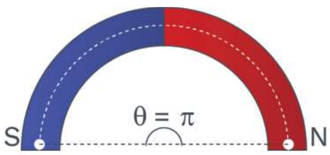
- If $\theta = \frac{\pi}{2}$ radians,



i.e., if the magnet is bent in the form of quadrant of a circle, then

$$M' = \frac{2M\sin\frac{\pi}{4}}{\left(\frac{\pi}{2}\right)} = \frac{2\sqrt{2}M}{\pi}$$

- If θ = π radians, i.e., if the magnet is bent in the form of a semi circle, then



$$M' = \frac{2M\sin\frac{\pi}{2}}{\pi} = \frac{2M}{\pi}$$

- If θ = 2π radians, i.e., if the magnet is bent in the form of a circle, then

$$M' = \frac{2M\sin\pi}{2\pi} = 0$$

- When a magnet in the form of an arc of a circle making an angle θ at the centre having magnetic moment M is straightened, then effective length of the magnet increases. Hence magnetic moment increases.

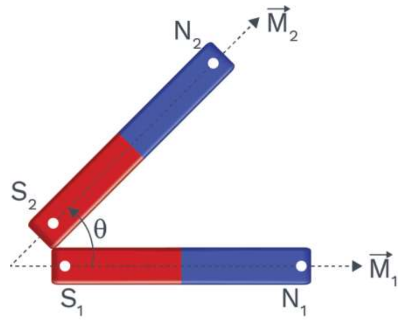
  New magnetic moment is given by

$$M' = \frac{M\theta}{2\sin\left(\frac{\theta}{2}\right)}$$ (θ must be in radians)

Concept Reminder

- When a magnet is brought near an unmagnetized iron piece, the dipoles in different domains in iron align parallel to the field, and so the iron piece get magnetized and gets attracted towards magnet.

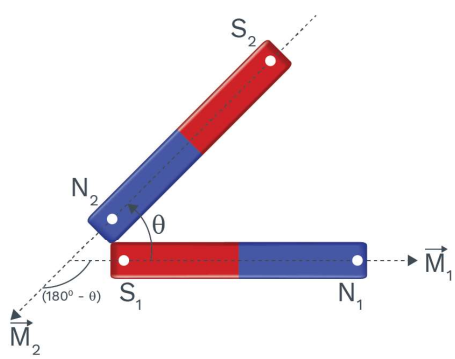
Resultant magnetic moment due to this combination of magnets:



- When the two bar magnets of moments M_1 and M_2 are joined so that their like poles touch each other and their axes are inclined at an angle 'θ', then the resultant magnetic moment of the combination M' is given by

$$M' = \sqrt{M_1^2 + M_2^2 + 2M_1M_2\cos\theta}$$

(θ = angle between the directions of magnetic moments)



- When the two bar magnets of moments M_1 and M_2 are joined so that their unlike poles touch each other and their axes are inclined at an angle 'θ', then the resultant magnetic moment

$$M' = \sqrt{M_1^2 + M_2^2 + 2M_1M_2\cos(180^\circ - \theta)}$$

[∵ the angle between directions of magnetic moments is (180° – θ)]

$$\therefore \quad M' = \sqrt{M_1^2 + M_2^2 - 2M_1M_2\cos\theta}$$

Concept Reminder

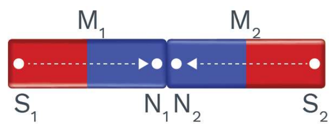
- Resultant magnetic moment due to combination of magnets is

$$\vec{M} = \vec{M}_1 + \vec{M}_2$$

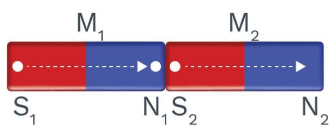
$$M = \sqrt{M_1^2 + M_2^2 + 2M_1M_2\cos\theta}$$

Concept Reminder

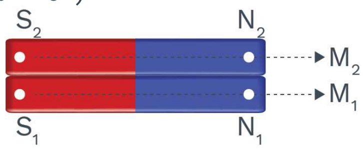
- A magnet gets demagnetized, i.e., loses its power of attraction if it is heated, hammered or ac is passed through a wire wound over it.



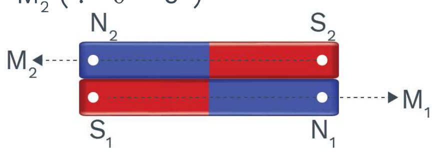
- When the two bar magnets of moments M_1 and M_2 ($M_1 > M_2$) are placed coaxially with like poles in contact then resultant magnetic moment,
  $M' = M_1 - M_2$
  (angle between directions of magnetic moments, $\theta = 180°$)



When the two bar magnets of moments M_1 and M_2 ($M_1 > M_2$) are placed coaxially with unlike poles are in contact then resultant magnetic moment,
$M' = M_1 + M_2$
(angle between directions of magnetic moments, $\theta = 0°$)



- When two bar magnets of magnetic moments M_1 and M_2 are placed one over the other with like poles on the same side, then resultant magnetic moment,
  $M' = M_1 + M_2$ ($\because$ $\theta = 0°$)



- When two bar magnets of magnetic moments M_1 and M_2 are placed one over the other with unlike poles on the same side, then resultant magnetic moment,

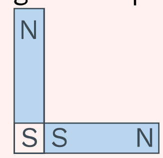
$$M' = M_1 - M_2 \quad (\because \theta = 180°)$$

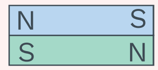
Concept Reminder

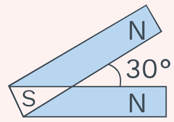
- There can be magnets with no poles e.g., toroid. Sometimes due to faulty magnetization we can temporarily have magnets with three poles but remember they are just temporary and shouldn't be confused with magnetic monopoles.

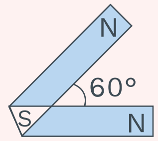
Rack your Brain

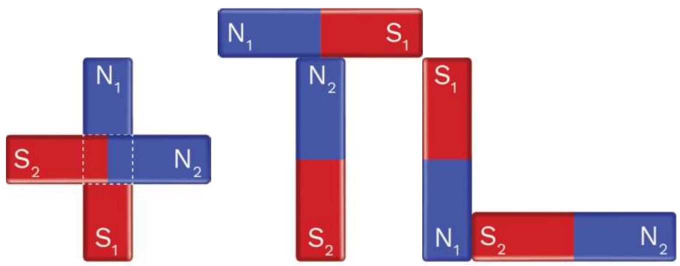
Which of the following configuration has highest net magnetic dipole moment?

(1) 

(2) 

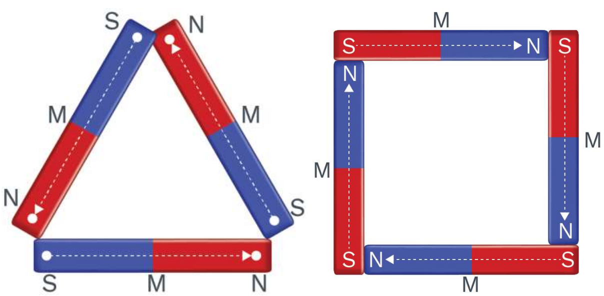
(3) 

(4) 



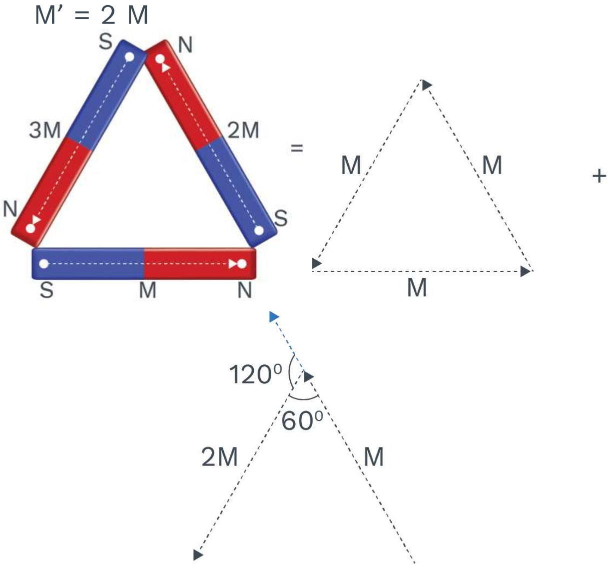
- When two bar magnets of magnetic moments M_1 and M_2 are placed at right angles to each other than resultant magnetic moment,

$$M' = \sqrt{M_1^2 + M_2^2} \quad (\because \theta = 90°)$$



- When the identical magnets each of magnetic moment M are arranged to form a closed polygon like a triangle (or) square with unlike poles at each corner, then resultant magnetic moment, M′ = 0.
- In the above point, if one of the magnets is reversed pole to pole then resultant magnetic moment,

$$M' = 2\ M$$



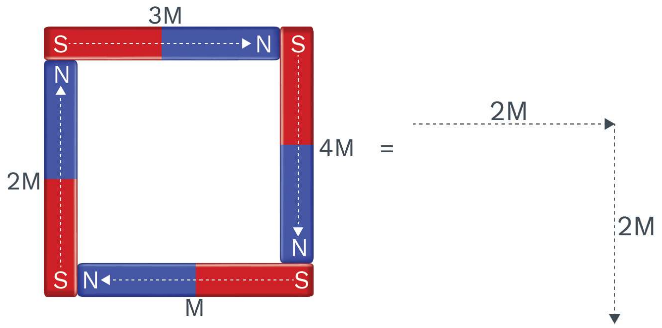
Concept Reminder

- Magnetic keepers are pieces of soft iron put at the end of magnets to neutralize their pole and hence reduce the demagnetizing effects.

- When three bar magnets of equal length but moments M, 2 M and 3 M are arranged to form an equilateral triangle with unlike poles at each corner, resultant magnetic moment is given by

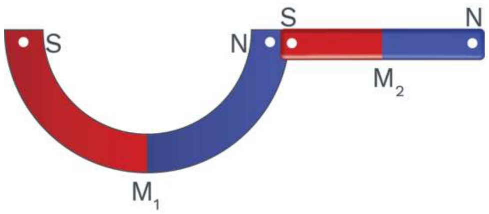
$$M' = \sqrt{(2M)^2 + M^2 + 2(2M)(M)\cos 120^\circ} = \sqrt{3}\,M$$

- When four bar magnets of moments M, 2M, 3M and 4M are arranged to form a square with unlike poles at each corner, then resultant magnetic moment is given by



$$M' = \sqrt{(2M)^2 + (2M)^2 + 2(2M)(2M)\cos 90^\circ} = 2\sqrt{2}\,M$$

- When half of the length of a thin bar magnet of magnetic moment M is bent into a semi circle as shown in figure, then



Resultant magnetic moment,

$M' = M_1 + M_2$

$$= \frac{2\left(\frac{M}{2}\right)}{\pi} + \frac{M}{2} = \frac{M}{\pi} + \frac{M}{2} = M\left(\frac{2+\pi}{2\pi}\right)$$

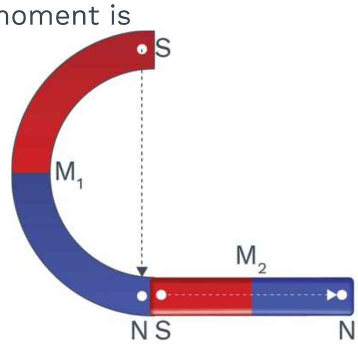
Concept Reminder

- Permanent magnets show their magnetic effect for long time and are made of alcino, ticonal or alcomax, steel.
- Temporary magnets are made of soft iron, stalloy or mumetal.

Rack your Brain

A bar magnet of magnetic moment M is bend in the fom of quadrant of circular arc. Find the new magnetic moment.

- In the above case if the two parts are arranged perpendicular to each other, then resultant magnetic moment is



$$M' = \sqrt{M_1^2 + M_2^2} = \sqrt{\left(\frac{M}{\pi}\right)^2 + \left(\frac{M}{2}\right)^2}$$

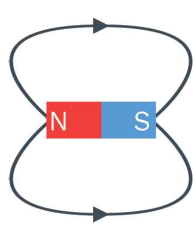
$$= \frac{M}{2\pi}\sqrt{(4+\pi^2)}$$

MAGNETIC FIELD

- There exist a region around a pole called magnetic field in which the influence of the pole is felt.
- The field associated with the space around the magnet is known as magnetic field, if another magnet is brought into the space, it is acted upon by a force due to this energy.
- Magnetic induction is the measure of the magnetic field both in direction and magnitude.

Magnetic Field Lines:

- The magnetic line of force or simply magnetic "field line" is the imaginary path in which a free unit north pole would tend to movein a magnetic field.



Characteristics of lines of force :

(i) Magnetic lines of force are closed curves. Inside the magnet they are directed from

Concept Reminder

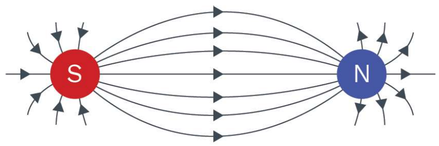
- Micheal Faraday introduced the concept of lines of force.
- His concept is that a line of force is an imaginary curve the tangent to which at a point gives the direction of the field at that point.

south to north pole while outside the magnet, their direction is from north to south pole, while. Hence, they have neither origin nor end.

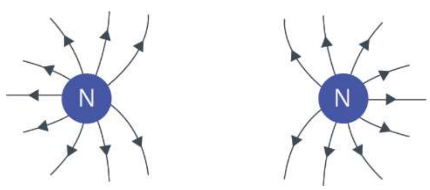
(ii) Tangent drawn at any point to the line of force gives the direction of magnetic field at that point.

(iii) Two lines of the force never intersect each other. If these two lines of force intersect, at the intersecting point the field should have two directions, which is not possible.

(iv) These lines of force tend to contract longitudinally or length wise. Because of this property the two unlike poles attract each other.



(v) The lines of force tend to repel each other laterally. Due to this property the two similar poles repel each other.



(vi) If at any point, in the combined field due to two magnets, there are no lines of force, it follows that the resultant field at that point is zero. Such points are called null or neutral points.

(vii) Lines of force in a field represent the strength of the field at a point in the field. Lines of force are crowded themselves in regions where the field is strong and they spread themselves apart at places where the field is weak.

(viii) Lines of force have a tendency to pass through magnetic substances. They show maximum tendency to pass through ferromagnetic materials.

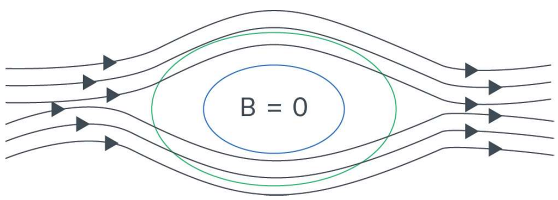
Key Points

- Magnetic field lines
- North pole
- South pole

Concept Reminder

- Magnetic lines of force are closed curves. Outside a magnet, lines of force are from north to south pole, while inside from south of north.

(ix) When a soft iron ring is placed in magnetic field, then most of lines of force pass through the ring and no lines of force pass through the space inside the ring as shown in figure. The phenomenon is known as magnetic screening or shielding.



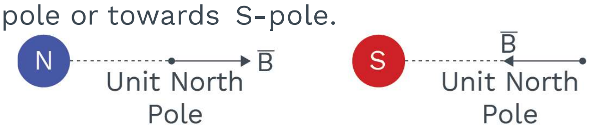
(x) If the magnetic lines of force are straight and parallel, and equally spaced the magnetic field is said to be uniform.

Magnetic Induction (or) Induction Field Strength (B):

- The magnetic induction field strength at a point in the magnetic field is defined as the force experienced by unit north pole placed at that point. It is denoted by B'.
- If a pole of strength 'm' is placed at a point in a magnetic field experiences a force 'F', the magnetic induction (B) at that point is given by

$$\vec{B} = \frac{\vec{F}}{m} \quad i.e., \vec{F} = m\vec{B}$$

- B is a vector quantity which is directed away from N-pole or towards S-pole.



S.I. unit of B:

$$\frac{N}{A-m} \text{ (or) } \frac{J}{A-m^2} \text{ (or) } \frac{V-s}{m^2} \text{ (or) tesla(T)}$$

CGS unit of B:

Gauss (G) 1 G = 10^{-4} T

Dimensions of B:

$$B = \frac{F}{m} = \frac{[MLT^{-2}]}{[AL]} = [MT^{-2}A^{-1}]$$

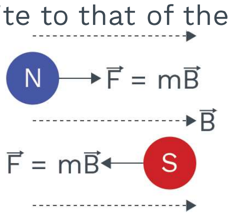
Key Points

- Magnetic induction
- Gauss
- Magnetic screening

Concept Reminder

- When a soft ring is placed in a magnetic field, no line passes through the space inside the ring. This phenomenon is called "magnetic screening" or "shielding".

- When these poles are placed in an external magnetic field, all the N-poles experience a force (F = mB) in the direction of the field and all the S-poles experience the same force in the direction opposite to that of the field.



Magnetic induction at a point due to isolated magnetic pole:

- A magnetic pole of strength 'm' kept at the point 'O'. Take a point 'P' which is at a distance 'r' from 'O'. To find the magnetic induction at point 'P', imagine a unit north pole at 'P'.

Force on unit north pole at P $= \frac{\mu_0}{4\pi}\frac{m \times 1}{r^2}$N

Force on unit north pole at 'P' gives the magnetic induction at that point.

$\therefore$ Magnetic induction at P is

$$B = \frac{\mu_0}{4\pi}\frac{m}{r^2} \text{ newton/amp-metre (r) or tesla(T)}$$

Magnetic Potential:

- The amount of the work done in bringing a unit north pole from infinity to a point in magnetic field is known as magnetic potential at the point.
- It is a scalar.
- SI unit: Joule/amp-m.
- For a pole strength m, the filed at a distance 'r' is $B = \frac{\mu_0}{4\pi}\frac{m}{r^2}$, and radially away from the pole. The potential at a distance 'r' is given by

$$V = -\int_{\infty}^{r} \vec{B}\,\overrightarrow{dr} = -\int_{\infty}^{r} \frac{\mu_0}{4\pi}\frac{m}{r^2} \times dr = \frac{\mu_0}{4\pi}\frac{m}{r}$$

Rack your Brain

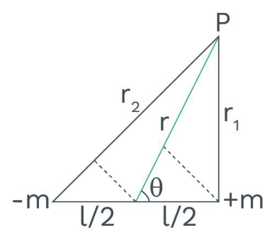
Two magnetic field lines:
(1) Never intersect at all
(2) Intersect at the neutral points
(3) Intersect on the equatorial axis of magnet
(4) Intersect near the poles

Note: $B = -\frac{\partial V}{\partial r}$

Magnetic potential due to a dipole:

- Consider a magnetic dipole of moment M. If 'm' is the pole strength and l is the distance between the poles, then M = ml. If r_1 and r_2 are the distances of point P from the poles, then

$$r_1 = r - \frac{l}{2}\cos\theta \text{ and } r_2 = r + \frac{l}{2}\cos\theta$$



Magnetic potential at P,

$$V = V_N + V_S = \frac{\mu_0}{4\pi}\left[\frac{m}{r_1} - \frac{m}{r_2}\right] = \frac{\mu_0 m}{4\pi}\left[\frac{l\cos\theta}{r^2 - \frac{l^2}{4}\cos^2\theta}\right]$$

Types of Magnetic Field:

Uniform magnetic field

- The magnetic field in which magnetic induction field strength is same both in magnitude and direction at all points, is known as uniform magnetic field.
- In this type of magnetic field the magnetic lines of force are parallel straight lines and equidistant.

Ex : The horizontal component of earth's magnetic field in a limited region.

Non-uniform magnetic field:

- The magnetic field, in which magnetic induction or field strength differs either in magnitude, in direction or both is known as non uniform magnetic field.
- It is represented by the non-parallel lines of force. Ex: The magnetic field lines near the pole of any magnet.

Concept Reminder

- $\oint \vec{B}\cdot d\vec{S} = 0$ i.e., the net magnetic flux through the guassian surface for magnetic fields is equal to zero. It means magnetic monopoles do not exist.

Concept Reminder

- μ_0 number of magnetic lines are associated with unit pole. Also remember that magnetic flux numerically means number of magnetic lines of force.

Magnetic flux (ϕ):

- It is equivalent to the total number of magnetic lines of the force passing normal through a given area. Its S.I. unit is weber and CGS unit is maxwell

  1 weber = 10^8 maxwell

  $\phi = \vec{B} \cdot \vec{A} = BA\cos\theta$
- Where θ is the angle made by magnetic field $(\vec{B})$ with the area $(\hat{n})$

  $\vec{A} = A\hat{n}$ (A = area of the coil)
- It is a scalar. Dimensional formula is $[ML^2T^{-2}A^{-1}]$.

Magnetic Flux Density (B):

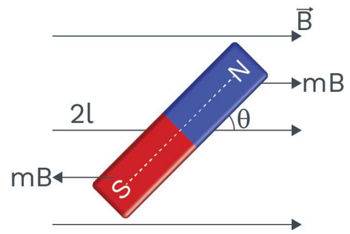
- The number of magnetic flux lines passing per unit area of cross section normal to the cross section is called magnetic flux density.

  $B = \phi_B / A$
- SI unit is weber metre^{-2} or tesla or $NA^{-1}m^{-1}$.
- Its CGS unit is gauss

  1 gauss = 10^{-4} tesla.
- Its dimensional formula is $[M^1L^0T^{-2}A^{-1}]$
- It is also called magnetic induction and magnetic field.
- The relation between B and H is $B_0 = \mu_0 H$ in vacuum and $B = \mu H$ in a material medium Where μ is the absolute permeability of the medium.
- The force experienced by a pole of strength 'm' ampere meter in a field of induction B is F = mB.

Couple acting on the bar Magnet / Torque on a magnetic dipole:

- When a bar magnet of the moment M & length $2l$ is placed in the uniform field of induction B, then each pole experiences a force = mB in opposite directions.



Concept Reminder

- Magnetic field $\vec{B}$ inside superconductors is always zero, if kept in external $\vec{B}$ field. This effect is known as "Meissner effect".

Key Points

- Magnetic flux density
- Magnetic flux
- Uniform magnetic field
- Non-uniform magnetic field

- As a result of this, the bar magnet experiences a couple & moment of couple is developed.
- Moment of couple which is acting on the bar magnet is C = Torque x perpendicular distance between two forces.

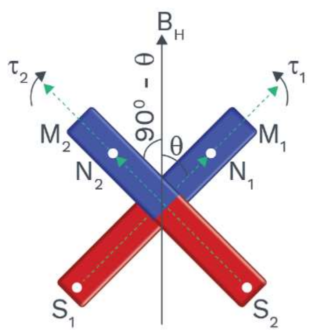
$$C = (m)\ (2l)\ B \sin\theta \Rightarrow C = M\ B \sin\theta$$

- Where θ is the angle between the magnetic moment and magnetic field.
  In vector notation $\vec{C} = \vec{M} \times \vec{B}$
- When bar magnet is either along or is opposite to the direction of magnetic field then moment of couple = 0.
- When bar magnet is perpendicular to the direction of the applied magnetic field, then the moment of couple is maximum. i.e., $C_{max} = MB$.
- In a uniform magnetic field, the bar magnet experiences only a couple but no net force. Therefore, it undergoes only rotatory motion.
- In a non-uniform the magnetic field, a bar magnet experiences a couple and also a net force. So, it under goes both rotational and translational motion.
- Two magnets of magnetic moments M_1 and M_2 are joined in the form of a (+) and this arrangement is pivoted so that it is free to rotate in a horizontal plane under the influence of earth's horizontal magnetic field. If 'θ' is the angle made by the magnetic meridian with M_1 in equilibrium position, then

$$\tau_1 = \tau_2$$

i.e., $M_1B_H \sin\theta = M_2B_H \sin(90 - \theta)$;

$$\therefore \quad \tan\theta = \frac{M_2}{M_1}$$



Rack your Brain

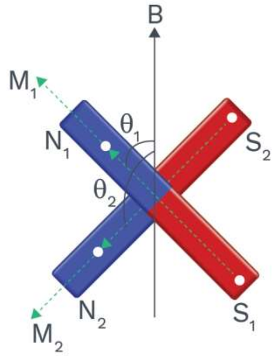
A bar magnet is hung by a thin cotton thread in a uniform horizontal magnetic field and is in equilibrium state. The energy required to rotate it by 60° is W. Find the torque required to keep the magnet in this new position.

Concept Reminder

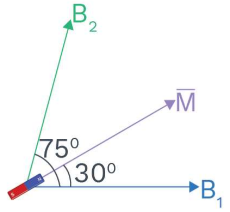
- When a magnetic dipole is kept in a uniform external magnetic field $\vec{B}$.
  (a) The net force on dipole is zero.
  (b) The net torque on it is $\vec{\tau} = \vec{M} \times \vec{B}$.

- Two magnets of moments M_1 and M_2 are joined as shown in figure and the arrangement is pivoted so that it is free to rotate in a horizontal plane under the influence of magnetic field B. Then net torque acting on the system is given by

$$\tau = \tau_1 + \tau_2$$
$$= M_1 B \sin\theta_1 + M_2 B \sin\theta_2$$
$$= B(M_1 \sin\theta_1 + M_2 \sin\theta_2)$$



- Two uniform magnetic fields of strengths B_1 and B_2 acting at an angle 75° with each other in horizontal plane are applied on a magnetic needle of moment M, which is free to move in the horizontal plane. If the needle gets aligned at an angle 30° with B_1, then the ratio B_1/B_2 is
- In equilibrium position,



$$\tau_1 = \tau_2;$$

i.e., $MB_1 \sin 30°$

$$= MB_2 \sin(75° - 30°);$$

$$\therefore \quad \frac{B_1}{B_2} = \frac{\sin 45°}{\sin 30°} = \frac{\sqrt{2}}{1}$$

- A pivoted magnetic needle of length $2l$ and pole strength 'm' is at rest in magnetic meridian. It is

Concept Reminder

- If magnetic dipole is slightly disturbed from equilibrium position and released, then it will oscillate with time period

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

Concept Reminder

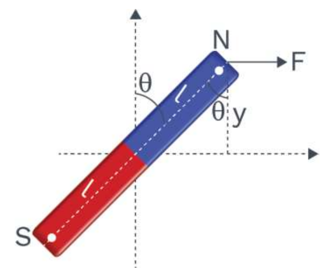
- Potential energy stored in system of dipole $\vec{M}$ and magnetic field $\vec{B}$ making an angle θ with $\vec{M}$ is $-\vec{M}\cdot\vec{B}$. So work done in rotating a dipole from angular position θ_1 to θ_2 with respect to field is

$$W = MB(\cos\theta_1 - \cos\theta_2)$$

held in the equilibrium at an angle 'θ' with B_H by pulling its north pole towards east by a string. Then tension in the string is from the figure,

$$\cos\theta = \frac{y}{l} \Rightarrow y = l\cos\theta$$

In equilibrium,

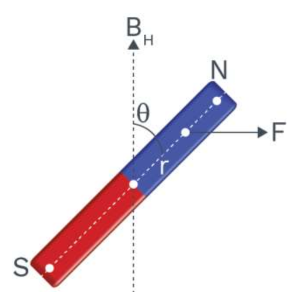


$$\tau_{tension} = \tau_{B_H}; \quad \text{i.e., } Fl\cos\theta = MB_H\sin\theta$$

or $\quad Fl\cos\theta = 2lmB_H\sin\theta$

$\therefore \quad F = 2mB_H\tan\theta$

- In the above case, if the magnetic needle is held in equilibrium at an angle 'θ' to a uniform magnetic induction field B_H by applying a force F at a distance 'r' from the pivot along a direction perpendicular to the field, then



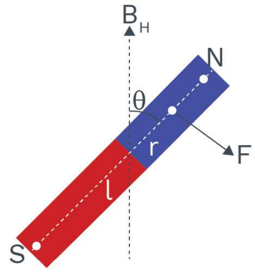
$$Fr\cos\theta = MB_H\sin\theta;$$

$$\therefore \quad F = \frac{MB_H\tan\theta}{r} = \frac{(2lm)B_H\tan\theta}{2}$$

Rack your Brain

A magnetic needle suspended parallel to a magnetic field requires $\sqrt{3}$ J of work to turn it through 60°. Find torque needed to maintain the needle in this position.

- In the above case, if the force is applied at one end which is always perpendicular to length of the magnetic needle, then

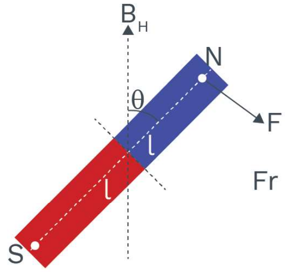


$\tau_{tension} = \tau_{B_H}$

i.e., $Fl = (2lm)B_H \sin\theta$;

$\therefore \quad F = 2mB_H \sin\theta$

- In the above case, if the force applied is always perpendicular to length of the magnetic needle but at a distance 'r' from the pivot, then

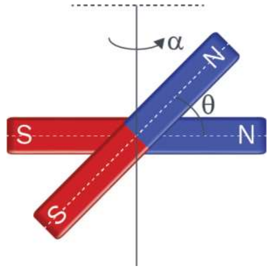


$Fr \sin 90^o = MB_H \sin\theta$

$Fr \sin 90° = MB_H \sin\theta$

$\therefore \quad F = \frac{MB_H \sin\theta}{r} = \frac{2lmB_H \sin\theta}{r}$

- A magnet of moment 'M' is suspended in the magnetic meridian with untwisted wire. The upper end of the wire is then rotated through an angle 'α' to deflect the magnet by an angle 'θ' from magnetic meridian. Then deflecting couple acting on the magnet = $MB_H \sin\theta$



Concept Reminder

- A magnetic dipole is in stable equilibrium when $\vec{M}$ is pointing towards $\vec{B}$ because $\theta = 0°$ and energy is minimum here. The equilibrium is unstable when dipole moment $\vec{M}$ is exactly opposite to $\vec{B}$ i.e., $\theta = 180°$ because energy is maximum here.

- Restoring couple developed in suspension wire = $C(\alpha - \theta)$ where C is couple per unit twist of suspension wire.
In equilibrium position,

$$B_H \sin\theta = C(\alpha - \theta)$$

Ex. When a bar magnet is placed at 90° to a uniform magnetic field, then it is acted upon by a couple which is maximum. For the couple to be half of its maximum value, at what angle should the magnet be inclined to the magnetic field (B)?

Sol. We know that,

$\tau = MB\sin\theta$

If $\theta = 90°$ then

$\tau_{max} = MB$ ...(i)

$\frac{\tau_{max}}{2} = MB\sin\theta$ ...(ii)

From equations (i) and (ii)

$$2 = \frac{1}{\sin\theta} \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$$

Ex. A bar magnet of the magnetic moment M_1 is suspended by a wire in a magnetic field. The upper end of the wire is being rotated through 180°, then the magnet rotated through 45°. Under similar conditions another magnet of magnetic moment M_2 is rotated through 30°. Then find the ratio of M_1 & M_2.

Sol. $C(\alpha - \theta) = MB\sin\theta$

For first magnet,

$C(180 - 45) = M_1B\sin 45°$ ...(i)

For second magnet,

$C(180 - 30) = M_2B\sin 30°$ ...(ii)

Dividing equations (i) by (ii)

$$\frac{135}{150} = \frac{M_1}{M_2} \times \sqrt{2}$$

$$\Rightarrow \frac{M_1}{M_2} = \frac{9}{10\sqrt{2}}$$

Ex. Magnetic dipole is under the influence of the two magnetic fields. Angle between the direction of two field is 60° and one of the fields has a magnitude of 1.2×10^{-2}T. If the dipole comes to the stable equilibrium at an angle of 15° with this field, what is the magnitude of the other field?

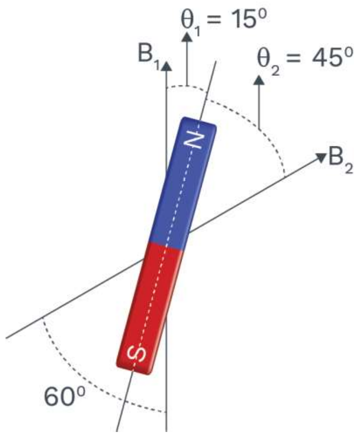
Sol. Here $B_1 = 1.2 \times 10^{-2}$ T

Inclination of dipole with B_1 is $\theta_1 = 15°$

Therefore, inclination of dipole with

B_2 is $\theta_2 = 60° - 15° = 45°$

As the dipole is in the equilibrium, therefore the torque on the dipole is due to the two fields are equal and opposite. If M is magnetic dipole moment of the dipole, then



$$MB_1 \sin\theta_1 = MB_2 \sin\theta_2$$

$$\text{or } B_2 = \frac{B_1 \sin\theta_1}{\sin\theta_2} = \frac{1.2 \times 10^{-2} \times \sin 15°}{\sin 45°}$$

$$= \frac{1.2 \times 10^{-2} \times 0.2588}{0.707} = 4.39 \times 10^{-3}\,T$$

Ex. A compass needle of magnetic moment 60 A-m^2, pointing towards geographical north at a certain place where the horizontal component of earth's magnetic field is 40 μwb/m^2 experiences a torque of 1.2×10^{-3} Nm. Find the declination at that place.

Sol. If θ is the declination of the place, then the torque acting on the needle is $\tau = MB_H \sin\theta$

$$\Rightarrow \quad \sin\theta = \frac{\tau}{MB_H} = \frac{1.2 \times 10^{-3}}{60 \times 40 \times 10^{-6}} = \frac{1}{2}$$

$\therefore \quad \theta = 30°$

Work done in the rotating of a magnetic dipole in the magnetic field

- The work done in the deflecting a magnet from angular position θ_1 to an angular position θ_2 with the field is change in PE given as

$$W = MB(\cos\theta_1 - \cos\theta_2)$$

- The work done in the deflecting of a bar magnet through an angle θ from its state of equilibrium position in a uniform magnetic field is given by

$$W = MB(1 - \cos\theta) \quad [\text{here } \theta_1 = 0°, \theta_2 = \theta]$$

When it is released, then this work done converts into rotational kinetic energy

$$MB(1 - \cos\theta) = \frac{1}{2}I\omega^2$$

- When a bar magnet is at an angle θ with the magnetic field, then the potential energy possessed by the magnet is $U = -MB \cos\theta$.
- When the bar magnet is held parallel to the applied field, then $\theta = 0°$ and potential energy is $(-MB)$.It is said to be stable equilibrium.
- When the bar magnet is held perpendicular to the applied field, then $\theta = 90°$ and potential energy is zero
- When the bar magnet is held anti-parallel to the applied field, then $\theta = 180°$ and potential energy is maximum. i.e., $U = +MB$. It is said to be unstable equilibrium.

Rack your Brain

A 250 turn rectangular coil of length 2.1 cm and width of 1.25 cm carries a current of 85 μA and subjected to a magnetic field of strength 0.85 T. Find the work done for rotating the coil by 180° against the torque.

Ex. A magnet is being suspended at an angle 60° in an external magnetic field of 5×10^{-4} T. What is the work done by the magnetic field in bringing it in its direction? [The magnetic moment = 20 A-m^2]

Sol. Work done by the magnetic field,

$$W = MB(\cos\theta_1 - \cos\theta_2)$$

Here $\theta_1 = 60°$ and $\theta_2 = 0°$

$$\therefore \quad W = 20 \times 5 \times 10^{-4}[\cos 60° - \cos 0°]$$

$$= 10^{-2}\left[\frac{1}{2} - 1\right] = -5 \times 10^{-3} \text{ J}$$

Ex. A magnetic needle which is lying parallel to a magnetic field requires W units of the work to turn it through an angle of 60°. What is amount of torque needed to maintain the needle in this position?

Sol. In case of a dipole in a magnetic field,

$$W = MB(\cos\theta_1 - \cos\theta_2) \text{ and } C = MB\sin\theta$$

Here, $\theta_1 = 0°$ and $\theta_2 = 60°$

So, $W = MB(1 - \cos\theta) = 2MB\sin^2\frac{\theta}{2}$

and, $C = MB\sin\theta = 2MB\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

So, $\frac{C}{W} = \cot\left(\frac{\theta}{2}\right)$

i.e., $C = W\cot 30° = \sqrt{3}\,W$

Ex. A bar magnet has a magnetic moment is 2.5 JT^{-1} and is placed in a magnetic field of 0.2 T. Calculate the work done in turning the magnet from parallel to antiparallel position relative to field direction.

Sol. Work done in changing the orientation of the dipole of moment in a field from position θ_1 to θ_2 is given by

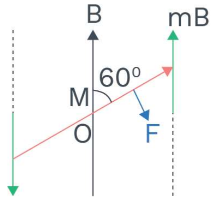
$W = MB(\cos\theta_1 - \cos\theta_2)$

Here, $\theta_1 = 0°$ and $\theta_2 = 180°$

So, $W = 2MB = 22.5 \times 0.2 = 1J$

Ex. A bar magnet with poles are 25 cm apart and pole strength 14.4 A-m rests with its centre on a frictionless pivot. It is held in the equilibrium at 60° to a uniform magnetic field of induction 0.25 T by applying a force F at right angles to its axis, 10 cm from its pivot. Calculate F. What will happen if the force is removed?

Sol. The situation is shown in figure. In equilibrium the torque on M due to B is balanced by torque due to F,



i.e., $\vec{M} \times \vec{B} = \vec{r} \times \vec{F}$

$MB\sin\theta = Fr\sin 90°$

or $F = \frac{(m \times 2l)B\sin\theta}{r}$ (as M = m × 2l);

So substituting the given data,

$$F = \frac{14.4 \times (25 \times 10^{-2}) \times 0.25(\sqrt{3}/2)}{10 \times 10^{-2}} = 7.8\,N$$

- If the force $\vec{F}$ is removed, the torque $\vec{M} \times \vec{B}$ will become unbalanced and under its action the magnet will execute oscillatory motion about the direction of B on its pivot O which will not be simple harmonic as angular displacement θ is not small.

Field of a Bar-Magnet:

- **Axial Line:** Magnetic induction at a point on the axial line is

$$B_a = \left(\frac{\mu_0}{4\pi}\right)\frac{2Md}{(d^2 - l^2)^2}$$

For a short bar magnet
i.e., $l << d$ then

$$B_a = \left(\frac{\mu_0}{4\pi}\right)\frac{2M}{d^3}$$

- The direction of the magnetic induction on the axial line is along the direction of the magnetic moment.

Equatorial Line:

- Magnetic induction at a point on an equatorial line at a distance d from the centre is

$$B_e = \frac{\mu_0}{4\pi}\frac{M}{(d^2 + l^2)^{3/2}}$$

For the short bar magnet i.e., $l << d$ then

$$B_e = \left(\frac{\mu_0}{4\pi}\right)\frac{M}{d^3}$$

- The direction of the magnetic induction on the equatorial line is in the direction which is opposite to the magnetic moment.

At any point in the plane of axial & equatorial lines:

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{M\sqrt{(3\cos^2\theta + 1)}}{d^3}$$

$\theta = 0°$ for axial line;
$\theta = 90°$ for equatorial line.

- For a short bar magnet, at two equidistant points, one on the axial and the other on the equatorial line
$B_a = 2B_e$

Force between the two magnets:
When one magnet is being placed in the field of another magnet it experiences a couple or force

Concept Reminder

Magnetic field of a short bar-magnet:

- Axial line $B_a = \left(\frac{\mu_0}{4\pi}\right)\frac{2M}{d^3}$
- Equatorial line $B_e = \left(\frac{\mu_0}{4\pi}\right)\frac{M}{d^3}$
- At any point

$$B = \frac{\mu_0 M}{4\pi}\frac{\sqrt{3\cos^2\theta + 1}}{d^3}$$

or both and has potential energy. Depending upon the orientation of the magnets relative to each other, the following situations are discussed below.

When the magnets are along the line joining their centres:

If the opposite poles of the two magnets face each other as shown in figure (A), then the field due to $\vec{M}_1$ at the position of $\vec{M}_2$, i.e., at O_2, will be:

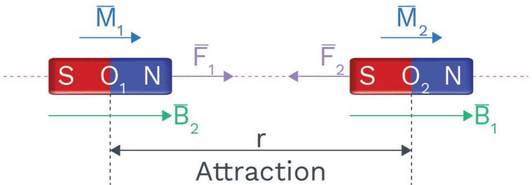
$\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{2M_1}{r^3}$ with $\theta = 0°$ [as O_2 lies on the axis of $\vec{M}_1$]

So couple on $\vec{M}_2$ due to $\vec{M}_1$,

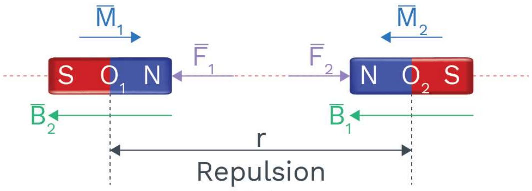
i.e., $\vec{B}_1$ is

$\vec{C}_2 = \vec{M}_2 \times \vec{B}_1 = 0$ [as $\vec{M}_2$ is parallel to $\vec{B}_1$]

i.e., $\theta = 0$ ...(i)



Attraction
(A)



Repulsion
(B)

Similarly,

$\vec{C}_1 = \vec{M}_1 \times \vec{B}_2 = 0$ [as $\vec{M}_1$ is parallel to $\vec{B}_2$]

i.e., $\theta = 0$ ...(ii)

Now, the magnets will not exert any couple on each other.

Concept Reminder

- Force between two identical short bar magnets whose centres are r metre apart is proportional to $\frac{1}{r^4}$.

Key Points

- Axial line
- Equatorial line
- Force between two magnet

- And as $U = -\vec{M}\cdot\vec{B}$, the interaction energy of the system (i.e., P.E. of $\vec{M}_2$ in the field of $\vec{M}_1$ or P.E. of $\vec{M}_1$ in the field of $\vec{M}_2$) will be-

$U = -\vec{M}_2 \cdot \vec{B}_1 = -\vec{M}_1 \cdot \vec{B}_2$

$$= -\frac{\mu_0}{4\pi}\frac{2M_1M_2}{r^3} \quad [\text{as } \vec{M}_2 \text{ is parallel to } \vec{B}_1]$$

i.e., $\theta = 0°$ ...(iii)

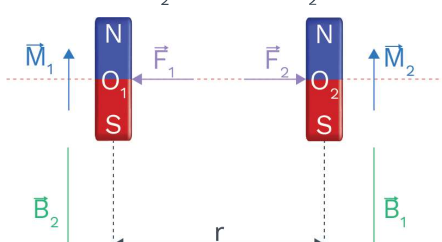
- Now as $F = -\left(\frac{dU}{dr}\right)$ so force on $\vec{M}_1$ due to $\vec{M}_2$ or force on $\vec{M}_2$ due to $\vec{M}_1$ will be-

$$F_1 = F_2 = -\frac{d}{dr}\left[-\frac{\mu_0}{4\pi}\frac{2M_1M_2}{r^3}\right]$$

$$= -\frac{\mu_0}{4\pi}\frac{6M_1M_2}{r^4} \quad ...(iv)$$

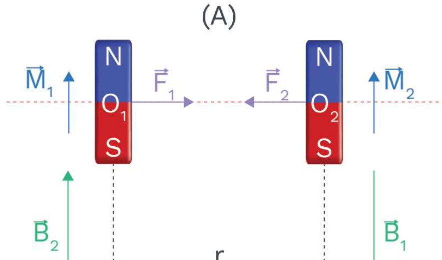
(From equation (iv) it is clear that there is an interaction).

- When the magnets are perpendicular to the line joining their centers, then force between the magnets varies as $(1/r^4)$.
- If the similar poles of the two magnets face each other as shown in figure (A), the field due to $\vec{M}_1$ at the position of $\vec{M}_2$, i.e., at O_2 will be-



Repulsion

(A)

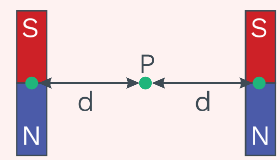


Attraction

(B)

Rack your Brain

Two identical bar magnets, each of magnetic moment M, are placed as shown in figure. Find magnetic field at point P.



Concept Reminder

- When one dipole is placed in the field of another dipole it usually experienced a couple or force or both and has potential energy. This is known as dipole-dipole interaction.

$$\overrightarrow{B_1} = \frac{\mu_0}{4\pi}\frac{M_1}{r^3} \text{ with } \phi = 90°$$

[as O_2 lies on the equatorial line of $\overrightarrow{M_1}$]

Now as $\overrightarrow{B_1}$ is antiparallel to $\overrightarrow{M_2}$ and $\overrightarrow{B_2}$ to $\overrightarrow{M_1}$,

i.e., $\theta = 180°$

So $\overrightarrow{C_2} = \overrightarrow{M_2} \times \overrightarrow{B_1} = 0$ and $\overrightarrow{C_1} = \overrightarrow{M_1} \times \overrightarrow{B_2} = 0$ ...(v)

i.e., magnets will not exert any couple on each other.

And as $U = -\overrightarrow{M_2} \cdot \overrightarrow{B_1} = -\overrightarrow{M_1} \cdot \overrightarrow{B_2}$

$$= \frac{\mu_0}{4\pi}\frac{M_1M_2}{r^3} \quad \text{...(vi)}$$

[as $\overrightarrow{}$ is anti parallel to $\overrightarrow{B_1}$, i.e., $\theta = 180°$]

- Now as $F = -\left(\frac{dU}{dr}\right)$, so force on $\overrightarrow{M_1}$ due to $\overrightarrow{M_2}$ or on $\overrightarrow{M_2}$ due to $\overrightarrow{M_1}$ will be-

$$F_1 = F_2 = -\frac{d}{dr}\left(\frac{\mu_0}{4\pi}\frac{M_1M_2}{r^3}\right)$$

$$= \frac{\mu_0}{4\pi}\frac{3M_1M_2}{r^4} \quad \text{...(vii)}$$

From equation (vii) it is clear that interaction force varies as $\left(\frac{1}{r^4}\right)$.

Superposition of magnetic fields

Neutral points and their location:

- It is the combined field due to bar magnet and horizontal component of earth's magnetic field (B_H).
- Earth's magnetic field is present everywhere and its horizontal component extends from south to north. When a magnet is placed anywhere, its field gets superimposed over the earth's field, which gives rise to the resultant magnetic field. In this, the resultant magnetic field, there are certain points where the resultant magnetic induction field becomes zero.

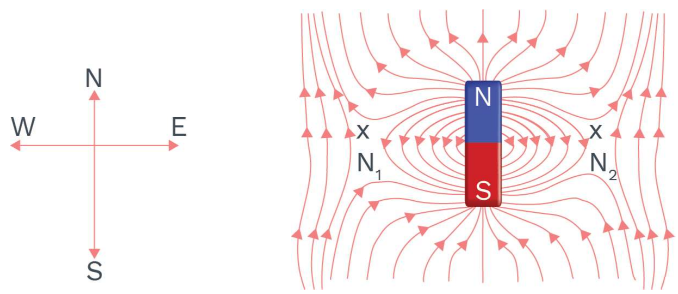
Concept Reminder

- The most likely cause of earth's magnetism is thought to be the motion and distribution of negatively charged material in and outside the earth, however, no definite theory has been accepted so far.

- At these points, the horizontal component of the earth's magnetic field exactly balances the field due to the magnet. These points are called null points or the neutral points.
  "The points in magnetic field where the resultant magnetic induction field becomes zero is called null points".

North pole of magnet pointing towards the geographical north:

- A magnet is being placed in the magnetic meridian, with its north pole facing towards geographic north, then the combined magnetic field lines due to the earth and the bar magnet are as shown in the figure below:



Magnetic lines of force when the north pole of the magnet pointing towards geographic north Results:

- Along the axial line, on both the sides, the two fields have the same direction. The magnitude of the resultant magnetic field is sum of the magnitudes of the two fields.
- As we gets deviated from the axial line, the two fields differ in the direction.
- On an equatorial line, the direction of two fields are exactly opposite to each other.
- At N_1 & N_2 on the equatorial line, the magnetic induction field due to the magnet is exactly the same as that of earth's horizontal component. These points are called null points.
- If the average distance of N_1 & N_2 from the centre of the magnet is 'd' then

$$B_{magnet} = B_H$$

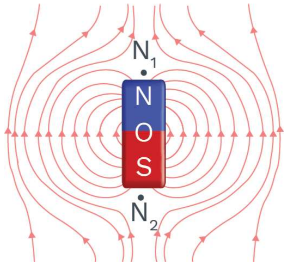
(horizontal component of earth's magnetic field)

$$\therefore \quad \frac{\mu_0}{4\pi}\frac{M}{(d^2 + l^2)^{3/2}} = B_H \text{ F}$$

For short magnet $\frac{\mu_0}{4\pi}\frac{M}{d^3} = B_H$

The North pole of the magnet pointing towards geographic south:

- When a magnet is placed in magnetic meridian with its north pole facing geographic south, then the field lines of the resultant magnetic field as shown in the figure.



Magnetic lines of force when the north pole of the magnet is pointing towards geographic south Results:

- The directions of the two fields (the horizontal component of earth's magnetic field & the field due to the magnet) are exactly opposite to each other, on the axial line.
- As we deviate from axial line, then the two fields differ in their direction.
- The directions of two fields at all the points on the equatorial line is same.
- Along the axial line, the magnetic field is due to the magnet decreases in magnitude on moving away from the centre of the magnet. There will be points N_1 & N_2 situated at equal distances from the centre of the magnet where the fields are exactly balanced by the earth's horizontal component field. These points are called null points.

  At null points,

$$B = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2 - l^2)^2} = B_H$$

(where B_H is the earth's horizontal magnetic induction field)

For short magnet,

$$\frac{\mu_0}{4\pi}\frac{2M}{d^3} = B_H$$

- If the horizontal component of the earth's magnetic field B_H at a given place is known, then the magnetic moment (M) of the magnet can be then determined by locating the neutral points.

Magnet is placed perpendicular to the magnetic meridian:

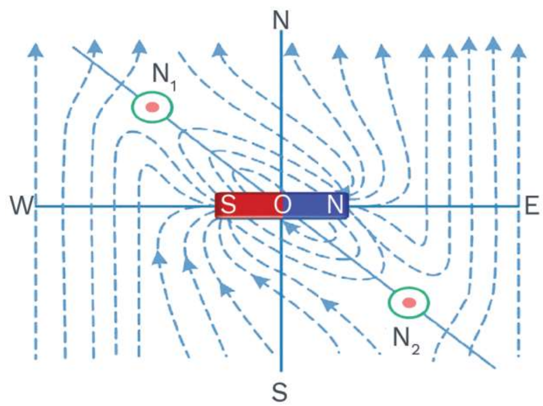
- When a bar magnet is being placed with its axial line perpendicular to the magnetic meridian with its north pole facing east of earth, the resultant magnetic field is shown in the figure. Along a line making an angle of

$\tan^{-1}(\sqrt{2})$ with east west line, there are two points (N_1 & N_2) where the resultant magnetic induction field is zero. Thus N_1 (on the N-W line) & N_2 (on the S-E line) are the null points.

At the null point,

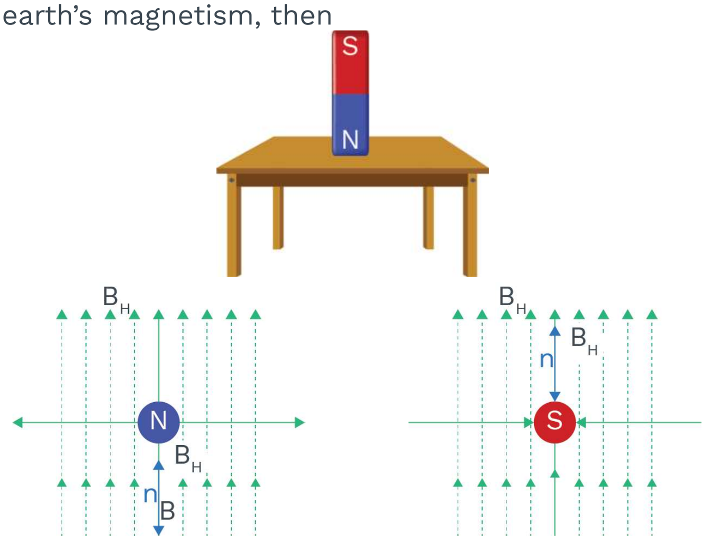
$$B_H = \frac{\mu_0}{4\pi}\frac{M}{d^3}\sqrt{1+3\cos^2\theta}$$

Where $\tan\theta = \sqrt{2}$



$$\therefore \quad \cos\theta = \frac{1}{\sqrt{3}} \quad \Rightarrow B_H = \sqrt{2}\frac{\mu_0}{4\pi}\frac{M}{d^3}$$

- If a very long magnet is being placed vertically with its one pole on the wooden horizontal table (or) if an isolated magnetic pole is being kept in the field of earth's magnetism, then

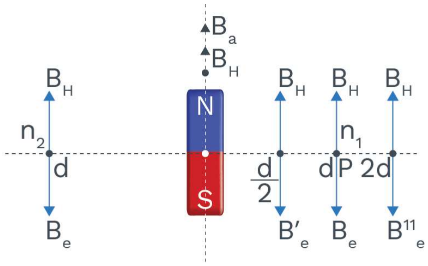


- A single neutral point will be then formed in the combined field on the horizontal table.
- If 'm' is the pole strength and 'd' is the distance from the pole of the magnet where the neutral point is formed, then

$$B_H = \frac{\mu_0}{4\pi}\frac{m}{d^2}$$

- If north pole of the magnet is on the table, then the neutral point is formed towards the geographic south side of the pole.
- If south pole of the magnet is on the table, then the neutral point is formed towards the geographic north side of the pole.

Note: A short bar magnet is kept along magnetic meridian with its north pole pointing north. A neutral point is formed at point 'P' at distance 'd' from the centre of the magnet then



- At a distance 'd' on equatorial line, net magnetic induction $B_{net} = 0$

i.e., $B_e = B_H \Rightarrow \frac{\mu_0}{4\pi}\cdot\frac{M}{d^3} = B_H$

- At a distance $\frac{d}{2}$ from the centre of magnet on equatorial line, the net magnetic induction is given by

$$B_{net} = B'_e - B_H = \frac{\mu_0}{4\pi}\frac{M}{\left(\frac{d}{2}\right)^3} - B_H = 7B_H$$

- At a distance '2d' on equatorial line, the magnetic induction is given by

$$B_{net} = B_H - B''_e = B_H - \frac{\mu_0}{4\pi}\frac{M}{(2d)^3}$$

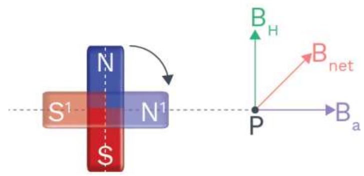
$$= B_H - \frac{B_H}{8} = \frac{7B_H}{8}$$

- At a distance 'd' on axial line of the bar magnet, the net magnetic induction is given by

$$B_{net} = B_a + B_H = 2B_e + B_H = 2B_H + B_H = 3B_H$$

- If the axis of the bar magnet is rotated through 90° clockwise at the same position then the net magnetic induction at the same point 'P' is

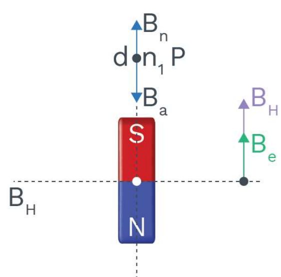
$$B_{net} = \sqrt{B_a^2 + B_H^2} = \sqrt{5}\,B_H \quad (\because B_a = 2B_e = 2B_H)$$



- If the axis of the magnet is rotated through 180° at the same position, then net magnetic induction at the same point 'P' is

$$B_{net} = B_e + B_H = 2\,B_H$$

Note: A short bar magnet is kept along magnetic meridian with its south pole pointing north. A neutral point is formed at a point 'P' at a distance 'd' from the centre of the magnet then at a distance 'd' on axial line of the bar magnet net magnetic induction,



$$B_{net} = 0$$

i.e., $B_a = B_H \Rightarrow \frac{\mu_0}{4\pi} \cdot \frac{2M}{d^3} = B_H$

- At a distance $\frac{d}{2}$ on axial line of bar magnet, net magnetic induction is given by

$$B_{net} = B_a - B_H = \frac{\mu_0}{4\pi} \cdot \frac{2M}{\left(\frac{d}{2}\right)^3} - B_H = 7\,B_H$$

- At a distance '2d' on axial line of the bar magnet, net magnetic induction is given by

$$B_{net} = B_H - B_a'' = B_H - \frac{\mu_0}{4\pi}\frac{2M}{(2d)^3}$$

$$= B_H - \frac{B_H}{8} = \frac{7B_H}{8}$$

- At a distance 'd' on equatorial line of the bar magnet, net magnetic induction is

$$B_{net} = B_e + B_H = \frac{B_a}{2} + B_H = \frac{B_H}{2} + B_H = \frac{3}{2}B_H$$

- If axis of the magnet is rotated through 90° clockwise at the same position, then net magnetic induction at the same point 'P' is

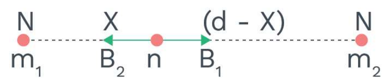
$$B_{net} = \sqrt{B_e^2 + B_H^2} = \frac{\sqrt{5}}{2}B_H \quad \left(\because B_e = \frac{B_a}{2} = \frac{B_H}{2}\right)$$

- If axis of the magnet is rotated through 180°, then magnetic induction at the point 'P' is

$$B_{net} = B_a + B_H = B_H + B_H = 2B_H$$

Neutral points in combined field due to an isolated magnetic poles:

- When the two like magnetic poles of the pole strengths m_1 and $m_2 (m_1 < m_2)$ are separated by a distance 'd', then neutral point is formed in between the poles and on the line joining them.
- Let 'x' be the distance of the neutral point from weaker pole of strength m_1.



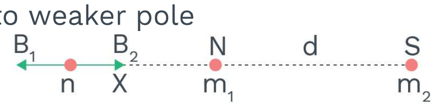
At the neutral point, $B_1 = B_2$

$$\Rightarrow \quad \frac{\mu_0}{4\pi}\cdot\frac{m_1}{x^2} = \frac{\mu_0}{4\pi}\cdot\frac{m_2}{(d-x)^2}$$

On solving, we get $x = \dfrac{d}{\sqrt{\dfrac{m_2}{m_1}} + 1}$

- When the two unlike magnetic poles of strengths m_1 and m_2 ($m_1 < m_2$) are separated by a distance 'd', then the neutral point is formed outside and on the line passing through the poles.

  It always lies closer to weaker pole



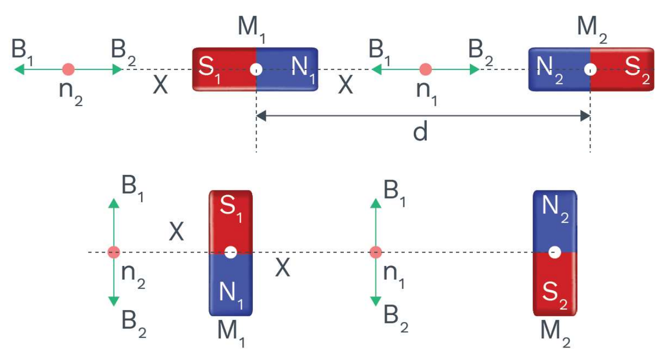
At the neutral point, $B_1 = B_2$

$$\Rightarrow \quad \frac{\mu_0}{4\pi} \cdot \frac{m_1}{x^2} = \frac{\mu_0}{4\pi} \cdot \frac{m_2}{(d+x)^2}$$

On solving, we get $x = \dfrac{d}{\sqrt{\dfrac{m_2}{m_1}} - 1}$

Neutral points are in the combined field due to the short bar magnets:

- The two short bar magnets of magnetic moments M_1 and M_2 (where $M_1 < M_2$) are placed at a distance 'd' between their centers with their magnetic axes oriented as shown in the figure. Then two neutral points are formed
  (i) In between and
  (ii) Outside and on the line passing through centers of the magnets. In both the case, null point is always closer to magnet of weaker moment.



Case-I: If the neutral point is formed in between the magnets, then $B_1 = B_2$

$$\Rightarrow \quad \frac{\mu_0}{4\pi} \cdot \frac{2M_1}{x^3} = \frac{\mu_0}{4\pi} \cdot \frac{2M_2}{(d-x)^3}$$

On solving, we get

$$x = \frac{d}{\left(\dfrac{M_2}{M_1}\right)^{1/3} + 1}$$

Case-II: If the neutral point is formed outside the combination, then

$$\Rightarrow \quad \frac{\mu_0}{4\pi} \cdot \frac{2M_1}{x^3} = \frac{\mu_0}{4\pi} \cdot \frac{2M_2}{(d+x)^3}$$

On solving, we get

$$x = \frac{d}{\left(\dfrac{M_2}{M_1}\right)^{1/3} - 1}$$

Note: No null points are obtained when unlike poles of the magnets are placed closer to each other.

- When two or more magnetic fields are superimposed in the same region, according to the resultant magnetic field the space in the region gets modified.
- The magnetic field of induction at any point is the resultant of all the fields superimposed at that point.

Null Point (or) Neutral Point:

- That point at which the resultant magnetic field is zero is called null point.
- If two poles having pole strengths m_1 and m_2 ($m_1 < m_2$) are separated by a distance d, then the distance of the neutral point from the first pole m_1 is

$$x = \frac{d}{\sqrt{\frac{m_2}{m_1}} \pm 1} \begin{pmatrix} + \text{ for like poles} \\ - \text{ for unlike poles} \end{pmatrix}$$

(a) For the like poles the neutral point is situated in between the poles.
(b) For the unlike poles the neutral point is situated on line joining the poles. But not in between them.
(c) In either of the case null point is always closer to the weaker pole.

- If the two short bar magnets of magnetic moments M_1 and M_2 ($M_1 < M_2$) are placed along the same line with like poles facing each other and 'd' is the distance between their centres, the distance of null point from M_1 is

$$x = \frac{d}{\left(\frac{M_2}{M_1}\right)^{1/3} \pm 1}$$

(a) + for null point formed between the magnets.
(b) – for the null point formed outside the magnets.
(c) When the unlike poles face each other, null point.

Time period of Suspended Magnet in the Uniform Magnetic Field
Principle:

- A bar magnet is being suspended freely in a uniform magnetic field and displaced from its equilibrium, it starts executing angular SHM.
- Time period of oscillation and frequency of magnet is

$$T = 2\pi\sqrt{\frac{I}{MB_H}} \text{ and } n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{MB_H}{I}}$$

- Where M magnetic moment, B_H horizontal component of earth magnetic induction and I moment of inertia,

$$I = m\frac{(l^2 + b^2)}{12}$$

For a thin bar magnet, $I = \frac{ml^2}{12}$

Where m is mass, l is length and b is breadth of the magnet.

- For small percentage changes in moment of inertia

$$\frac{\Delta T}{T} \times 100 = \frac{1}{2}\frac{\Delta I}{I} \times 100$$

As M increases, T increases

- For small percentage changes in magnetic moment

$$\frac{\Delta T}{T} \times 100 = \frac{-1}{2}\frac{\Delta M}{M} \times 100$$

As M increases, T decreases

Comparison of magnetic moments:

- If two magnets of moment M_1 and M_2 of same dimensions and same mass are oscillating in the same field separately, then

$$\frac{T_1}{T_2} = \sqrt{\frac{M_2}{M_1}} \text{ (bar magnets of equal size)}$$

$$\left(\because T \propto \frac{1}{\sqrt{M}}\right)$$

A magnet is oscillating in a magnetic field and its time period is T sec. If another identical magnet is placed over that magnet with similar poles together, then time period remain unchanged.

$\because$ I' = 2 I and M' = 2M,

$$T' = 2\pi\sqrt{\frac{I'}{M'B}} = 2\pi\sqrt{\frac{2I}{2MB}} = 2\pi\sqrt{\frac{I}{MB}} = T$$

- A magnet is oscillating in a magnetic field B and its time period is T sec. If another identical magnet

Rack your Brain

The combination of 2 bar magnets makes 10 oscillations per second in an oscillation when like poles are tied together and 2 oscillations per second when unlike poles are tied together. Find the ratio of magnetic moments of magnets.

is placed over that magnet with unlike poles together, then time period becomes infinite. i.e., it does not oscillate.

$$\left[M' = M - M = 0;\ T = 2\pi\sqrt{\frac{I}{0 \times B}} = \infty\right]$$

- Time period of a thin bar magnet is T. It is cut into 'n' equal parts by cutting it normal to its length. The time period of each piece when oscillating in the same magnetic field will be-

$$T' = \frac{T}{n} \left(\because I' = \frac{\left(\frac{m}{n}\right)\left(\frac{l}{n}\right)^2}{12} = \frac{I}{n^3}\ \&\ M' = \frac{M}{n}\right)$$

$$\therefore \quad T' = 2\pi\sqrt{\frac{I'}{M'B}} = \frac{T}{n}$$

- Time period of a thin bar magnet is T. It is cut into 'n' equal parts by cutting it along its length. The time period of each piece remains unchanged, when oscillating in the same field.

$$\therefore \quad M' = \frac{M}{n} \text{ and } I' = \frac{\left(\frac{m}{n}\right)l^2}{12} = \frac{I}{n}$$

$$\Rightarrow \quad T' = 2\pi\sqrt{\frac{I'}{M'B}} = 2\pi\sqrt{\frac{\frac{I}{n}}{\frac{M}{n}B}} = T$$

- The two magnets of magnetic moments M_1 and M_2 ($M_1 > M_2$) are placed one over the other. If T_1 is the time period when like poles touch each other and T_2 is the time period when unlike poles touch each other, then

$$\frac{T_2^2}{T_1^2} = \frac{M_1 + M_2}{M_1 - M_2} \quad \left(\because T \propto \frac{1}{\sqrt{M}}\right)$$

$$\Rightarrow \quad \frac{M_1}{M_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2}$$

Concept Reminder

- If 2 bar magnets of dipole moments M_1 and M_2 are tied with similar poles together such that its oscillations time period is T_1 and if opposite poles are together then time period is T_2 then $\frac{M_1}{M_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2}$.

If n_1 and n_2 are the corresponding frequencies, then

$$\frac{M_1}{M_2} = \frac{n_1^2 + n_2^2}{n_1^2 - n_2^2}$$

- When same bar magnet used in the vibration magnetometer at two different places 1 and 2, then

$$\frac{B_{H_1}}{B_{H_2}} = \frac{T_2^2}{T_1^2} \quad \left(\because T \propto \frac{1}{\sqrt{B_H}} \right)$$

- When two bar magnets of moments and are placed one over the other in such a way (i) like poles together (ii) unlike poles together and (iii) their axes are perpendicular to each other. When vibrated in the same magnetic field, the ratio of their time periods respectively is

$$T = 2\pi\sqrt{\frac{I}{MB}} \quad \Rightarrow T \propto \frac{1}{\sqrt{M}}$$

$$\therefore \quad T_1 : T_2 : T_3 = \frac{1}{\sqrt{M_1 + M_2}} : \frac{1}{\sqrt{M_1 - M_2}} : \frac{1}{\left(\sqrt{M_1^2 + M_2^2}\right)}$$

- If T_0 is the time period of oscillation of the experimental magnet oscillating in $B_{H.}$ An external field B is applied due to a bar magnet in addition to B_H at the point where the first magnet is oscillating. Then its new time period is T.

Then $\frac{T_0}{T} = \sqrt{\frac{B_r}{B_H}}$ where $\overrightarrow{B_r} = \vec{B} + \overrightarrow{B_H}$

(a) If $\vec{B}$ and $\overrightarrow{B_H}$ are along the same direction,

$B_r = B + B_H \quad \Rightarrow T < T_0$

(b) If $\vec{B}$ and $\overrightarrow{B_H}$ are in opposite directions,

$B_r = B - B_H \quad \Rightarrow T_0 < T$

(c) If $\vec{B}$ and $\overrightarrow{B_H}$ are in opposite directions and also

if $|\vec{B}| = |\overrightarrow{B_H}|$

$B_r = B - B_H = 0 \quad \Rightarrow T = \infty$

(i.e., it does not oscillate)

(d) If $\vec{B}$ and $\overrightarrow{B_H}$ are perpendicular to each other,

$B_r = \sqrt{B^2 + B_H^2} \quad \Rightarrow T < T_0$

Here $B = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$ (if the point is on the axial line)

$B = \frac{\mu_0}{4\pi}\frac{M}{d^3}$ (if the point is on the equatorial line)

(e) If a straight wire carries current vertically up or down placed on the east or west or north or south side, then

$B = \frac{\mu_0}{2\pi}\frac{i}{r}$

(From ampere's law in electro magnetism)

- If n_1 and n_2 are frequencies of oscillation of the bar magnet in uniform magnetic field when B supports B_H and when B opposes B_H, then

$$\frac{n_1}{n_2} = \sqrt{\frac{B + B_H}{B - B_H}} \text{ (let } B > B_H\text{)}$$

$$\Rightarrow \frac{B}{B_H} = \frac{n_1^2 + n_2^2}{n_1^2 - n_2^2}$$

For a bar magnet,

$$T \propto \frac{1}{\sqrt{B_H}} \text{ or } n \propto \sqrt{B_H}$$

- If B_1 and B_2 be the earth's magnetic induction at two different places having angles of dip θ_1 and θ_2 then

$$\frac{T_1}{T_2} = \sqrt{\frac{B_{H_2}}{B_{H_1}}} = \sqrt{\frac{B_2 \cos\theta_2}{B_1 \cos\theta_1}}$$

or $$\frac{n_1}{n_2} = \sqrt{\frac{B_{H_1}}{B_{H_2}}} = \sqrt{\frac{B_1 \cos\theta_1}{B_2 \cos\theta_2}}$$

Ex. The two bar magnets placed together in a vibration magnetometer take 3 seconds for 1 vibration. If one magnet is reversed, the combination takes 4 seconds for 1 vibration. Find the ratio of their magnetic moments.

Sol. Given that,

T_1 = 3 s and T_2 = 4 s

$$\frac{M_1}{M_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2} = \frac{4^2 + 3^2}{4^2 - 3^2}$$

$$= \frac{16+9}{16-9} = \frac{25}{7} \Rightarrow \frac{M_1}{M_2} = 3.57$$

Ex. A bar magnet is taken, which makes 40 oscillations per minute in a vibration magnetometer. An identical magnet is being demagnetised completely and is placed over the magnet in the magnetometer. Determine the time taken for 40 oscillations by this combination. Ignore induced magnetism.

Sol. In the first case, frequency of oscillation,

$$n = \frac{1}{2\pi}\sqrt{\frac{MB}{I}}$$

In the second case, frequency of oscillation,

$$n' = \frac{1}{2\pi}\sqrt{\frac{MB}{2I}}$$

$$\Rightarrow \quad \frac{n'}{n} = \frac{1}{\sqrt{2}} \quad \Rightarrow \frac{T'}{T} = \sqrt{2}$$

or $\quad T' = \sqrt{2}\,T \quad \Rightarrow 40\,T' = \sqrt{2} \times 40\,T$

or $\quad t' = \sqrt{2}\,t = \sqrt{2}\,\text{minute} = 1.414\,\text{minute}$

Ex. A short magnet oscillates in a vibration magneto-meter with a time period of 0.1 s where the horizontal component of earth's magnetic field is 24 T. An upward current of 18 A is established in the vertical wire placed 20 cm east of the magnet. Find the new time period?

Sol. $\frac{T_2}{T_1} = \sqrt{\frac{B_1}{B_2}}$

Where $B_1 = B_H = 24 \times 10^{-6}\,T$

and $B_2 = B_H - B = B_H - \frac{\mu_0 i}{2\pi r}$

$$= 24 \times 10^{-6} - \frac{4\pi \times 10^{-7} \times 18}{2\pi \times 0.2} = 6 \times 10^{-6}\,T$$

$$\therefore \quad \frac{T_2}{0.1} = \sqrt{\frac{24 \times 10^{-6}}{6 \times 10^{-6}}} = 2 \quad \Rightarrow T_2 = 0.2\,s$$

Ex. A magnet is being suspended so as to swing horizontally makes 50 vibrations/min at a place where dip is 30° and 40 vibrations/ min where dip is 45°. Compare the earth's total fields these two places.

Sol. $n \propto \sqrt{B_H}$

$$\Rightarrow \quad \frac{n_1}{n_2} = \sqrt{\frac{B_1 \cos\theta_1}{B_2 \cos\theta_2}}$$

i.e., $\frac{50}{40} = \sqrt{\frac{B_1}{B_2} \times \frac{\cos 30°}{\cos 45°}}$

$\Rightarrow \quad \frac{25}{16} = \frac{B_1}{B_2} \times \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \frac{B_1}{B_2} = \frac{25}{8\sqrt{6}}$

Ex. When a short bar magnet is being kept in tan A position on a deflection magnetometer, the magnetic needle then oscillates with a frequency 'f' & the deflection produced is 45°. If the bar magnet is then removed find the frequency of oscillation of that needle?

Sol. $n \propto \sqrt{B}$

$\Rightarrow \quad \frac{n_1}{n_2} = \sqrt{\frac{B_1}{B_2}}$

Where $B_1 = \sqrt{B^2 + B_H^2} = \sqrt{(B_H \tan 45°)^2 + B_H^2}$

$= \sqrt{2}\, B_H$ and $B_2 = B_H$

$\therefore \quad \frac{n_1}{n_2} = \sqrt{\frac{\sqrt{2}\, B_H}{B_H}} = 2^{1/4}$

$\Rightarrow \quad n_2 = \frac{n_1}{2^{1/4}} = \frac{f}{2^{1/4}}$

Ex. Two bar magnets of the same length and breadth but having magnetic moments M and 2M are joined with like poles together and suspended by a string. The time of oscillation of this assembly in a magnetic field of strength is 3 sec. What will be the period of oscillation, if the polarity of one of the magnets is changed and the combination is again made to oscillate in the same field?

Sol. As magnetic moment is a vector, so when magnets are joined with like poles together

$M_1 = M + 2M = 3M$

So, $T = 2\pi\sqrt{\frac{(I_1 + I_2)}{3MB}}$ ...(i)

When the polarity of one of the magnets is reversed,

$M_2 = 2M - M = M$

So, $T' = 2\pi\sqrt{\frac{(I_1 + I_2)}{MB}}$ ...(ii)

Dividing equation (ii) by (i),

$$\frac{T'}{T} = \sqrt{3}$$

i.e., $T' = (\sqrt{3})T = 3\sqrt{3}\,\text{sec}$

Magnetic Materials:

- Curie & Faraday discovered that all the materials present in the universe are magnetic to some extent. These magnetic substances are categorized mainly into the two groups.
- Weak magnetic materials comes under diamagnetic and paramagnetic materials.
- Strong magnetic materials are called Ferro-magnetic materials.
- According to the modern electron theory of the magnetism, the magnetic response of any material is due to circulating electrons in the atoms.
- Each circulating charge constitutes of a magnetic moment in a direction which is perpendicular to the plane of circulation.
- In the magnetic material, all these magnetic moments due to the orbital & spin motion of the electrons present in the atoms of the material, add up vectorially in order to give resultant magnetic moment.
- The magnitude & direction of this resultant magnetic moment is responsible for the magnetic behaviour of various material.
- Magnetic material are studied in terms of the following physical parameters.

Intensity of Magnetizing Field $(\vec{H})$:

- Any magnetic field in which the magnetic material is being placed for its magnetization is called magnetizing field.
- In a magnetizing field the ratio of magnetizing field $\vec{B_0}$ to the permeability of free space is called intensity of magnetizing field

In air, $\vec{H} = \frac{\vec{B_0}}{\mu_0}$ or $\vec{B_0} = \mu_0 \vec{H}$

In a medium, $H = \frac{B}{\mu}$

The value of (H) is independent of the medium.

- Intensity of magnetizing field is a vector in the direction of magnetic field and has unit

$$\frac{Wb/m^2}{H/m} = \frac{V \times s}{\Omega \times s \times m} = \frac{A}{m} \text{ dimensions } AL^{-1}$$

Intensity of magnetization $\vec{I}$:

- When a magnetic material is magnetized by placing it in a magnetizing field, the induced dipole-moment per unit volume in the specimen is called intensity of magnetization.

i.e., $\vec{I} = \frac{\vec{M}}{V}$

but as $\vec{M} = mL\vec{n}$ and $V = SL$

$$\vec{I} = \frac{m}{S}\vec{n}$$

i.e., intensity of magnetization is numerically equal to the induced pole-strength per unit area of cross-section. It is a vector quantity having direction of magnetizing field or opposite to it. Its unit is (A/m) and the dimensions are $[AL^{-1}]$.

Magnetic Susceptibility (χ_m):

- The ratio of magnitude of intensity of magnetization to that of magnetizing field strength is called magnetic susceptibility.

$$\chi_m = \frac{I}{H}$$

It is a scalar quantity with no units and dimensions. It physically represents the ease with which a magnetic material can be magnetized. i.e., large value of χ_m implies that the material is more susceptible to the field and hence can be easily magnetized.

Magnetic permeability (μ):

- When a magnetic material is placed in a magnetizing field, the ratio of magnitude of total field inside the material to that of intensity of magnetizing field is called magnetic permeability; i.e.,

$$\mu = \frac{B}{H}, \quad \text{i.e., } B = \mu H$$

Concept Reminder

- The magnetic susceptibility χ_m is small and negative and $-1 < \chi_m < 0$ and is independent of temperature. Superconductors are most diamagnetic and for them $\chi = -1$.

Concept Reminder

- $\mu_r = 1 + \chi_m$.

- It measures the degree to which a magnetic material can be penetrated by the magnetizing field or ability of the material to allow magnetic lines of force. It is a scalar having unit and dimensions $[MLT^{-2}A^{-2}]$.

Relative permeability (μ_r):

- It is the ratio of magnitudes of total field inside the material to that of magnetizing field or it is the ratio of permeability of a medium to that of free space.

$$\mu_r = \frac{B}{B_0} = \frac{\mu H}{\mu_0 H} = \frac{\mu}{\mu_0}$$

It has no units and dimensions

Relation between relative permeability and susceptibility:

We know $B = \mu_0(H + I)$ or $\frac{B}{H} = \mu_0\left(1 + \frac{I}{H}\right)$

or, $\mu = \mu_0(1 + \chi)$ $\left[\text{as } \frac{B}{H} = \mu \text{ and } \frac{I}{H} = \chi\right]$

or $\frac{\mu}{\mu_0} = 1 + \chi$

$\therefore$ $\mu_r = 1 + \chi$

This is the desired result.

Ex. A magnetising field of 1600 Am^{-1} produces a magnetic flux of 2.4×10^{-5} weber in a bar of iron of cross section 0.2 cm^2. Calculate permeability and susceptibility of the bar.

Sol. Magnetic induction,

$$B = \frac{\phi}{A} = \frac{2.4 \times 10^{-5}}{0.2 \times 10^{-4}} = 1.2 \text{ Wb/m}^2$$

Permeability,

$$\mu = \frac{B}{H} = \frac{1.2}{1600} = 7.5 \times 10^{-4}\, TA^{-1}m$$

As $\mu = \mu_0(1 + \chi)$ then

Susceptibility,

$$\chi = \frac{\mu}{\mu_0} - 1 = \frac{7.5 \times 10^{-4}}{4\pi \times 10^{-7}} - 1 = 596.1$$

Ex. The permeability of substance is 6.28×10^{-4} wb/A-m. Find its relative permeability and susceptibility?

Sol. $$\mu_r = \frac{\mu}{\mu_0} = \frac{6.28 \times 10^{-4}}{4\pi \times 10^{-7}} = 500$$

$$\mu_r = 1 + \chi$$

$$\therefore \quad \chi = \mu_r - 1 = 500 - 1 = 499$$

Ex. The magnetic moment of a magnet of mass 75 gm is 9×10^{-7} A-m^2. If the density of the material of magnet is 7.5×10^3 kg m^{-3}, then find intensity of magnetization is.

Sol. $$I = \frac{M}{V}$$

Volume,

$$V = \frac{\text{mass}(m)}{\text{density}(\rho)}$$

$$= \frac{M \times \rho}{m} = \frac{9 \times 10^{-7} \times 7.5 \times 10^3}{75 \times 10^{-3}} = 0.09\,A/m$$

Ex. A magnetic field strength (H) 3×10^3 Am^{-1} produces a magnetic field of induction (B) of 12 πT in an iron rod. Find the relative permeability of iron?

Sol. $$\mu = \frac{B}{H} = \frac{12\pi}{3 \times 10^3} = 4\pi \times 10^{-3}$$

$$\therefore \quad \mu_r = \frac{\mu}{\mu_0} = \frac{4\pi \times 10^{-3}}{4\pi \times 10^{-7}} = 10^4$$

Ex. An iron bar having length 10 cm and diameter 2 cm is placed in a magnetic field of intensity 1000 Am^{-1} with its length parallel to the direction of the field. Determine the magnetic moment produced in the bar if permeability of its material is 6.3×10^{-4} TmA^{-1}.

Sol. We know that,

$$\mu = \mu_0(1 + \chi)$$

$$\Rightarrow \quad \chi = \frac{\mu}{\mu_0} - 1 = \frac{6.3 \times 10^{-4}}{4\pi \times 10^{-7}} - 1 = 500.6$$

Intensity of magnetisation,

$$I = \chi H = 500.6 \times 1000 = 5 \times 10^5 \text{ Am}^{-1}$$

∴ Magnetic moment,

$M = I \times V = I \times \pi r^2 l$

$= 5 \times 10^5 \times 3.14 \times (10^{-2})^2 \times (10 \times 10^{-2})$

$= 17.70 \text{ A-m}^2$

Electron Theory of Magnetism:

- Molecular theory of magnetism was first given by Weber and was later developed by Ewing.
- Electron theory of magnetism was proposed by Langevin.
- The main reason for the magnetic property of a magnet is spin motion of electron. Most of the magnetic moment is produced due to electron spin.
- The contribution of the orbital revolution is very small.

Explanation of diamagnetism:

- Since diamagnetic substance have paired electrons, magnetic moments cancel each other and there is no net magnetic moment.
- When a diamagnetic substance is placed in an external magnetic field each electron experiences radial force F = Bev either inwards or outwards. Due to this the angular velocity, current, and magnetic moment of one electron increases and of the other decreases. This results in a non-zero magnetic moment in the substances in a direction opposite to the field.
- Since the orbital motion of electrons in atoms is an universal phenomenon, diamagnetism is present in all materials. Hence diamagnetism is a universal property.

Properties of Dia-magnetic substances:

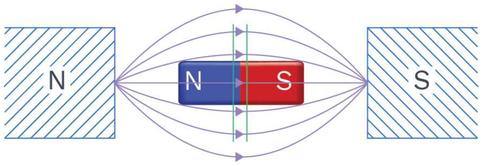
- The substances which when placed in a external magnetic field acquire feeble magnetism opposite to the direction of the magnetising field are known as dia-magnetic substances.

Concept Reminder

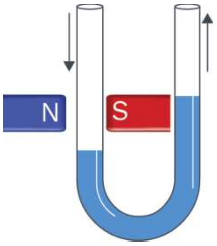
- All substances are diamagnetic because whenever a substance is kept in an external $\vec{B}$ field the rotational speed of electrons in the substance undergo a change and an opposite direction dipole moment is induced.

Ex: Bismuth (Bi), Zinc (Zn), Copper (Cu), Silver (Ag), Gold (Au). Salt (Nacl), Water (H_2O), Mercury (Hg), Hydrogen (H_2) etc.

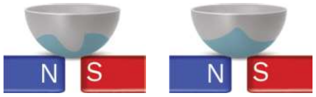
- When a bar of dia-magnetic substance is suspended freely between two magnetic poles [see figure], then the axis of the bar becomes perpendicular to magnetic field.



- When a dia-magnetic material is placed inside a magnetic field, the magnetic field lines become less dense in the material.
- If one limb of a narrow U-tube containing a dia-magnetic liquid is placed between the poles of an electromagnet, then on switching the field, the liquid shows a depression. This is shown in figure.



- When a dia-magnetic substance is placed in a non-uniform field, then it tends to move towards the weaker part from the stronger part of the field as shown in figure.

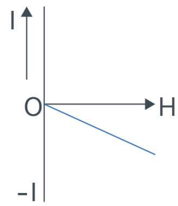


- Dia magnetic substances acquire feeble magnetism in a direction opposite to magnetizing field. The intensity of magnetization I is very small, negative and is directly proportional to magnetizing field as shown in figure.

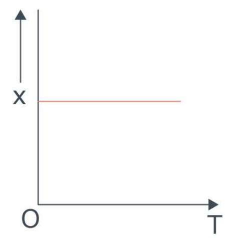
Rack your Brain

The magnetic susceptibility is negative for-

(1) Paramagnetic
(2) Diamagnetic
(3) Ferromagnetic
(4) All of above



- The magnetic susceptibility χ(I/H) is small and negative (Because I is small and opposite in direction to H). This is independent of temperature as shown in figure.



- The relative permeability is less than unity because $\mu_r = (1+\chi)$ and χ is negative.
- The origin of diamagnetism is the induced dipole moment due to change in orbital motion of electrons in atoms by the applied field. Diamagnetism is shown only by those substances which do not have any permanent magnetic moment.

Explanation of Paramagnetism:

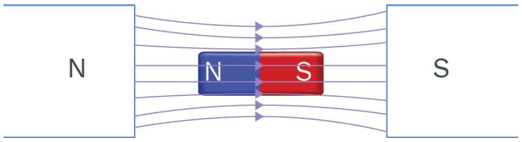
- Paramagnetic materials have a permanent magnetic moment in them. The moments arise from both orbital motion of electrons and the spinning of electrons in certain axis.
- In atoms whose inner shells are not completely filled, there is a net moment in them since more number of electrons spin in the same direction. This permanent magnet behaves like a tiny bar magnet called atomic magnet.
- In absence of external magnetic field atomic magnets are randomly oriented due to the thermal agitation and the net magnetic moment of the substance is zero.
- When it is placed in an external magnetic field the atomic magnets align in the direction of the field and thermal agitation oppose them to do so.
- At low fields the total magnetic moment would be directly proportional to the magnetic field B and inversely proportional to temperature T.

Properties of Para-magnetic substances:

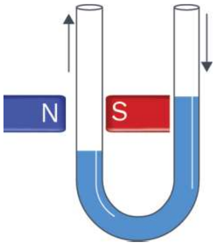
- The substances which when placed in a magnetic field, acquire feeble magnetism in the direction of magnetizing field are known as paramagnetic substances.

Ex: Aluminum (Al), Platinum (Pt), Manganese (Mn), Copper chloride ($CuCl_2$), Oxygen (O_2), solutions of salts of iron etc. are examples of paramagnetic substances.

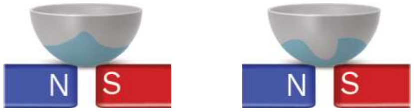
- When a bar of paramagnetic substance is placed in a magnetic field, it tries to concentrate the lines of force into it as shown in figure.



- This shows that the magnetic induction B in it is numerically slightly greater than the applied field H. So the permeability μ is greater than one because μ = (B/H).
- When the bar of paramagnetic material is suspended freely between two magnetic poles, its axis becomes parallel to magnetic field. Moveover, the poles produced at the ends of the bar are opposite to nearer magnetic poles.
- If a paramagnetic solution is poured in a U-tube and if one limb is placed between the poles of an electromagnet in such a way that liquid level is parallel to field, then on switching the field, the liquid rises. This is shown in figure.

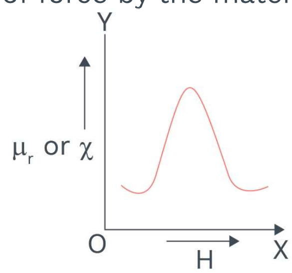


- In a non-uniform magnetic field, the paramagnetic substances are attracted towards the stronger parts of the magnetic field from the weaker parts of the field. The situation is shown in figure. In figure (a), the field is stronger in the middle as the poles are near to each other. In figure (b), the distance between the poles is increased.i.e., the field is stronger near the poles.



(a) b)

- The intensity of magnetization is very small and compared to one. It follows from the relation $\mu_r = 1 + \chi_m$. The variation of the values μ_r or χ with H is shown in figure below. As it is clear from the figure, that the variation is non-linear. The large value of 'μ_r' is due to the fact that the field 'B' inside the material is much stronger than the magnetizing field due to ‘pulling in’ of a large number of lines of force by the material.



Curie’s Law:

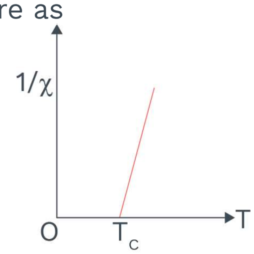
- The Curie law states that far away from saturation, the susceptibility χ(I/H) of paramagnetic substance is inversely proportional to absolute temperature, i.e.,

$$\chi \propto \frac{1}{T} \quad \Rightarrow \quad \chi = \frac{C}{T}$$

where C is constant & is called as Curie constant.

- When a ferromagnetic material is being heated, then it becomes paramagnetic at a certain temperature. This temperature is called Curie temperature and is denoted by T_C. After this temperature, the susceptibility of material varies with temperature as

$$\chi = \frac{C'}{(T - T_C)}$$



where C’ is another constant.

For iron,

T_C = 1043 K = 770°C

Ferromagnetic Substances:

- These substances possess a large resultant magnetic moment. According to Frenkel and Heisenberg domain theory.
- The spin magnetic moment of the electrons are responsible for the magnetic properties of ferromagnetic.
- Under certain definite forces, these spin magnetic moments are lined up parallel to the other. This results in setting up of the region of spontaneous magnetization which are called domains.

- A domain contains from 10^{21} to 10^{17} atoms and has a dimensions of the order of 10^{-8} to 10^{-12} m^3.
- The magnetization of the domains always tend to align in the direction of the field and the piece to matter becomes a magnet.

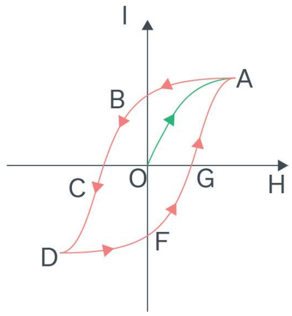
Hysteresis:

- It is defined as the tendency of demagnetization to lag behind the change in magnetic field applied to a ferromagnetic material.
- The process of taking a ferromagnet through a cycle of magnetization results in loss of energy.
- This is called hysteresis loss and it appears in the form of heat. Area of hysteresis loop is equal to the energy loss per cycle per unit volume. Area of hysteresis loop is equal to the energy loss per cycle per unit volume.

Definition

It is defined as the tendency of demagnetization to lag behind the change in magnetic field applied to a ferromagnetic material.

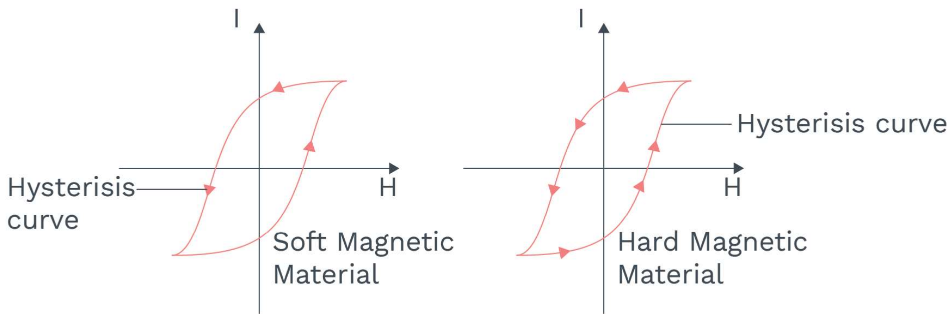
- When a bar of ferromagnetic material is being magnetized by a varying magnetic field H & the intensity of magnetization I induced is measured. The graph of I vs H is as shown is figure.



- When the magnetizing field is increased from O the intensity of magnetization I increases & becomes maximum i.e., at point (A). This maximum value is known as saturation value.
- When H is being reduced, I reduces but is not zero when H = 0. The remainder value OB of magnetization when H = 0 is called the residual magnetism or retentivity. OB is the retentivity.
- When the magnetic field H is reversed, I reduces & becomes zero i.e., for H = OC, I = 0. This value of H is called coercivity.
- When field is further increased in the reverse direction, the intensity of magnetisation attains saturation value in reverse direction (i.e., point D).
- When H is decreased to zero & changed direction in steps, we get the part DFGA.

Properties of soft iron and steel:

- For the soft iron, the susceptibility, permeability and retentivity are greater while coercivity and hysteresis loss per cycle are smaller than those of steel.



- Permanent magnets are made of steel & cobalt while electromagnets are made of soft iron.
- Diamagnetism is universal. It is present in all the materials. But it is weak & hard to detect if substance is para or ferromagnetic.

Shielding from magnetic fields:

- For shielding a certain region of the space from magnetic field, we surround the region by the soft iron rings. Magnetic field lines will be then drawn into the rings & the space enclosed will be free of magnetic field.

Elements of Earth's Magnetism (Terrestrial Magnetism):

There are three elements of earth's magnetism

(i) Angle of declination

(ii) Angle of dip

(iii) Horizontal component of earth's field.

Geographical Meridian: A vertical plane passing through the axis of rotation of the earth is called the geographic meridian.

A Magnetic Meridian: A vertical plane passing through the axis of a freely suspended magnet is called the magnetic meridian.

Angle of Declination (α): The acute angle between the magnetic meridian and the geographical meridian is called the 'angle of declination' at any place. The value of declination at equator is 17°.

Earth's Magnetic Field: The earth's magnetic field B_e in the magnetic meridian may be resolved into a horizontal component B_H and vertical component B_V at any place.

Definitions

- **Geographical Meridian:** A vertical plane passing through the axis of rotation of the earth is called the geographic meridian.
- **Magnetic Meridian:** A vertical plane passing through the axis of a freely suspended magnet is called the magnetic meridian.

$$B_r = \frac{\mu_0}{4\pi}\frac{2M\cos\theta}{r^3} \text{ and } B_\theta = \frac{\mu_0}{4\pi}\frac{M\sin\theta}{r^3}$$

and as $\tan\phi = \frac{B_V}{B_H} = -\frac{B_r}{B_\theta}$,

so in the light of equation (i)

$$\tan\phi = -2\cot\theta$$

But from figure $\theta = 90° + \lambda$

so, $\tan\phi = -2\cot(90° + \lambda)$

i.e., $\tan\phi = 2\tan\lambda$

Apparent Dip:

- If the dip circle is not kept in the magnetic meridian, the needle will not show the correct direction of earth's magnetic field. The angle made by needle with the horizontal is called the apparent dip for this plane. If the dip circle is at an angle θ to the meridian, the effective horizontal component in this place is $B_H' = B_H \cos\theta$. The vertical component is still B_V. If d_1 is the apparent dip and δ is the true dip, we have

$$\tan\delta_1 = \frac{B_V}{B'_H} = \frac{B_V}{B_H\cos\theta}$$

or $$\tan\delta_1 = \frac{\tan\delta}{\cos\theta} \left(\because \tan\delta = \frac{B_V}{B_H}\right) \quad ...(i)$$

- Now suppose, that the dip circle is then rotated through an angle of 90° from this position. Now, it will now make an angle (90° – θ) with the meridian. The effective horizontal component in this plane is $B''_H = B_H \sin\theta$. If d_2 be the apparent dip, we shall have

$$\tan\delta_2 = \frac{B_V}{B''_H} = \frac{B_V}{B_H\sin\theta}$$

or $$\tan\delta_2 = \frac{\tan\delta}{\sin\theta} \quad ...(ii)$$

From (i) and (ii),

$$\cot^2\delta_1 + \cot^2\delta_2 = \cot^2\delta$$

Key Points

- Geographic meridian
- Magnetic meridian
- Angle of declination
- Apparant dip

Concept Reminder

- $\tan\delta_1 = \frac{\tan\delta}{\cos\theta}$
- $\cot^2\delta_1 + \cot^2\delta_2 = \cot^2\delta$

Thus, one can get that the true dip δ without locating the magnetic meridian.

More about angle of dip (δ):

(i) At a point on the poles, earth's magnetic field is perpendicular to the surface of earth, i.e., $\delta = 90°$

$\therefore \quad B_V = B \sin 90° = B$

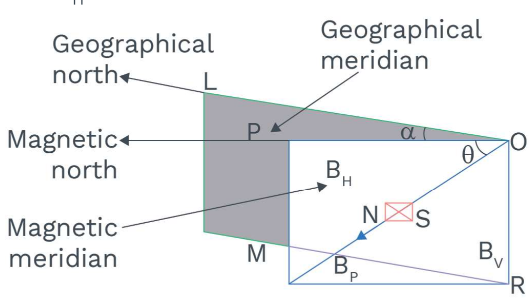
Further,

$B_H = B \cos 90° = 0$

So, except at the poles, the earth has a horizontal component of magnetic induction field.

(ii) At a place on equator, earth's magnetic field is parallel to the surface of earth, i.e., $\delta = 0°$

$\therefore \quad B_H = B \cos 0° = B$



θ = dip or inclination, α = declination

Horizontal component of earth's magnetic field

$B_H = B_e \cos\theta \qquad \text{...(i)}$

Vertical component of earth's magnetic field

$B_V = B_e \sin\theta \qquad \text{...(ii)}$

$$B_e = \sqrt{(B_H^2 + B_V^2)}$$

Dividing equation (ii) by equation (i), we have

$$\frac{B_V}{B_H} = \frac{B_e \sin\theta}{B_e \cos\theta} = \tan\theta$$

Rack your Brain

A compass needle which is allowed to move in a horizontal plane is taken to a geomagnetic pole. It-

(1) Will stay in N-S direction
(2) Will stay is E-W direction
(3) Will stay in any position
(4) All of above

Magnetic Maps:

- Usually lines are drawn joining all places having same value of an element.
- Such maps are called magnetic maps. The values of all the three magnetic elements
  - (a) Declination
  - (b) dip
  - (c) Horizontal component are found to be different at different places on the surface of earth.

Concept Reminder

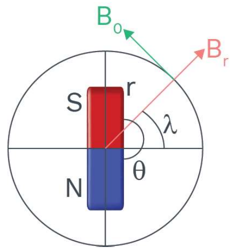
- Isoclinic lines are lines drawn through different places having the same dipole. The line joining places of zero dipole is called aclinic lines.

Isogonic lines: Lines passing through different places having the same declination are called isogonic lines.
The line which passes through the places having zero declination is called agonic line.

Isoclinic lines: These are lines passing through place of equal dip.
The line which is joining places of zero dip is called aclinic line.

Ex. Considering earth as a short magnet with its centre coinciding with the centre of earth. Show that angle of dip ϕ is related to the magnetic latitude λ through the relation $\tan\phi = 2\tan\lambda$.

Sol. Considering the situation for dipole, at position (r, θ) we have



Further, $B_V = B_H \sin 0° = 0$

So, except at the equator, the earth has a vertical component of magnetic induction field.

(iii) In vertical plane at an angle θ to magnetic meridian

$$B'_H = B_H \cos\theta \text{ and } B'_V = B_V$$

So, the angle of dip δ' in a vertical plane making an angle θ to magnetic meridian is given by

$$\tan\delta' = \frac{B'_V}{B'_H} = \frac{B_V}{B_H \cos\theta}$$

or $\quad \tan\delta' = \dfrac{\tan\delta}{\cos\theta} \quad \left(\because \dfrac{B_V}{B_H} = \tan\delta\right)$

For a vertical plane which is other than magnetic meridian,
$\theta > 0^\circ$ and $\cos\theta < 1$, i.e., $\delta' > \delta$ (angle of dip increases)

For a plane perpendicular to magnetic meridian, $\theta = 90°$

$\therefore \quad \tan\delta' = \dfrac{\tan\delta}{\cos 90} = \infty \quad \Rightarrow \delta' = 90^\circ$

This shows that in a plane perpendicular to magnetic meridian, the dip needle will become vertical.

EXAMPLES

1. **When a bar magnet is being placed at 90° to a uniform magnetic field, it is acted upon by a couple which is maximum. For the couple to be half of the maximum value, at what angle should the magnet be inclined to the magnetic field (B) ?**

Sol. We know that, $\tau = MB\sin\theta$

If $\theta = 90°$ then $\tau_{max} = MB$ ...(i)

$$\frac{\tau_{max}}{2} = MB\sin\theta \quad ...(ii)$$

From equations (i) and (ii)

$$2 = \frac{1}{\sin\theta} \text{ or } \sin\theta = \frac{1}{2} \text{ or } \theta = 30°$$

2. **A bar magnet of magnetic moment M_1 is suspended by a wire in a magnetic field. The upper end of the wire is being rotated through 180°, then the magnet rotated through 45°. Under similar conditions another magnet of magnetic moment M_2 is rotated through 30°. Then find the ratio of M_1 & M_2.**

Sol. $C(\alpha - \theta) = MB\sin\theta$

For first magnet,

$$C(180 - 45) = M_1B\sin 45° \quad ...(i)$$

For second magnet,

$$C(180 - 30) = M_2B\sin 30° \quad ...(ii)$$

Diving equation (i) by equation (ii)

$$\frac{135}{150} = \frac{M_1}{M_2} \times \sqrt{2} \Rightarrow \frac{M_1}{M_2} = \frac{9}{10\sqrt{2}}$$

3. **A compass needle of magnetic moment 60 A-m², pointing towards geographical north at a certain place where the horizontal component of earth's magnetic field is 40 μwb/m² experiences a torque of 1.2 × 10⁻³ Nm. Find the declination at that place.**

Sol. If θ is the declination of the place, then the torque acting on the needle is
$\tau = MB_H \sin\theta$

$$\Rightarrow \quad \sin\theta = \frac{\tau}{MB_H} = \frac{1.2\times10^{-3}}{60\times40\times10^{-6}} = \frac{1}{2} \quad \therefore\ \theta = 30^\circ$$

4. **A magnet is suspended at an angle 60° in an external magnetic field of 5 × 10⁻⁴ T. What is the work done by the magnetic field in bringing it in its direction ? [The magnetic moment = 20 A-m²].**

Sol. Work done by the magnetic field,
$W = MB\left(\cos\theta_1 - \cos\theta_2\right)$ Here $\theta_1 = 60^\circ$ and $\theta_2 = 0^\circ$

$$\therefore \quad W = 20\times5\times10^{-4}\left[\cos60^\circ - \cos0^\circ\right]$$

$$= 10^{-2}\left[\frac{1}{2} - 1\right] = -5\times10^{-3}\,\text{J}.$$

5. **A bar magnet has a magnetic moment 2.5 J T⁻¹ and is placed in a magnetic field of 0.2 T. Calculate the work done in turning the magnet from parallel to antiparallel position relative to field direction.**

Sol. Work done in changing the orientation of a dipole of moment M in a field B from position θ_1 to θ_2 is

Given by $\quad W = MB\left(\cos\theta_1 - \cos\theta_2\right)$

Here, $\quad \theta_1 = 0^\circ$ and $\theta_2 = 180^\circ$

So, $\quad W = 2MB = 2\times2.5\times0.2 = 1\text{J}$

6. **Two bar magnets placed together in a vibration magnetometer take 3 seconds for 1 vibration. If one magnet is reversed, the combination takes 4 seconds for 1 vibration. Find the ratio of their magnetic moments.**

Sol. Given that, $T_1 = 3s$ and $T_2 = 4s$

$$\frac{M_1}{M_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2} = \frac{4^2 + 3^2}{4^2 - 3^2} = \frac{16+9}{16-9} = \frac{25}{7} \text{ or } \frac{M_1}{M_2} = 3.57$$

7. **A bar magnet makes 40 oscillations per minute in a vibrating magnetometer. Now, an identical magnet is being demagnetised completely and is then placed over the magnet in the magnetometer. Calculate the time taken for the 40 oscillations by this combination. Ignore the induced magnetism.**

Sol. In the first case, frequency of oscillation,

$$n = \frac{1}{2\pi}\sqrt{\frac{MB}{I}}$$

In the second case, frequency of oscillation,

$$n^1 = \frac{1}{2\pi}\sqrt{\frac{MB}{2I}} \Rightarrow \frac{T}{n} = \frac{1}{\sqrt{2}} \Rightarrow \frac{n'}{T} = \sqrt{2}$$

or $T' = \sqrt{2}T$ (or) $40\,T' = \sqrt{2} \times 40T$

or $T' = \sqrt{2}t = \sqrt{2}$ minutes = 1.414 minutes

8. **A short magnet oscillates in a vibration magnetometer with a time period of 0.1s where the horizontal component of earth's magnetic field is 24μT. An upward current of 18A is established in the vertical wire placed 20 cm east of the magnet. Find the new time period ?**

Sol. $\frac{T_2}{T_1} = \sqrt{\frac{B_1}{B_2}}$ Where $B_1 = B_H = 24 \times 10^{-6}\,T$

and $B_2 = B_H - B = B_H - \frac{\mu_0 i}{2\pi r}$

$$= 24 \times 10^{-6} \sim \sqrt{\frac{4\pi \times 10^{-7} \times 6}{6 \times 10^{-6}}} = 2 \quad \Rightarrow T_2 = 0.2s$$

9. **A magnet is suspended so as to swing horizontally makes 50 vibrations/min at a place where dip is 30°, and 40 vibrations/min where dip is 45°. Compare the earth's total field at the two places.**

Sol. $n \propto \sqrt{B_H}$

$$\Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{B_1 \cos\theta_1}{B_2 \cos\theta_2}} \Rightarrow \frac{50}{40} = \sqrt{\frac{B_1}{B_2} \times \frac{\cos 30^\circ}{\cos 45^\circ}}$$

$$\Rightarrow \frac{25}{16} = \frac{B_1}{B_2} \times \frac{\sqrt{3}}{\sqrt{2}} \Rightarrow \frac{B_1}{B_2} = \frac{25}{8\sqrt{6}}$$

10. **The two bar magnets of the same length and breadth but having magnetic moments M and 2M are joined with the like poles together and are suspended by a string. The time of oscillation for this assembly in a magnetic field of strength B is 3 sec. What will be the period of the oscillation, if the polarity of one of the magnets is changed and the combination is again made to oscillate in the same field ?**

Sol. As magnetic moment is a vector, so when magnets are joined with like poles together $M_1 = M + 2M = 3M$,

So, $T = 2\pi\sqrt{\frac{(I_1 + I_2)}{3MB}}$ ...(i)

Now, when the polarity of one of the magnets is being reversed, $M_2 = 2M - M = M$;

So $T' = 2\pi\sqrt{\frac{(I_1 + I_2)}{MB}}$ ...(ii)

Dividing Equation (ii) by (i),

$$\frac{T'}{T} = \sqrt{3}, \text{ i.e., } T' = \left(\sqrt{3}\right)T = 3\sqrt{3} \text{ sec}$$

11. **A magnetising field of 1600 Am^{-1} produces a magnetic flux of 2.4×10^{-5} weber in a bar of iron of cross section 0.2 cm^2. Calculate permeability and susceptibility of the bar.**

Sol. Magnetic induction, $B = \frac{\phi}{A} = \frac{2.4 \times 10^{-5}}{0.2 \times 10^{-4}} = 1.2 \ Wb / m^2$

(i) Permeability, $\mu = \frac{B}{H} = \frac{1.2}{1600} = 7.5 \times 10^{-4} TA^{-1}m$

(ii) As $\mu = \mu_0 (1 + \chi)$ then

Susceptibility, $\chi = \frac{\mu}{\mu_0} - 1 = \frac{7.5 \times 10^{-4}}{4\pi \times 10^{-7}} - 1 = 596.1$

12. **The permeability of substance is 6.28×10^{-4} wb/A-m. Find its relative permeability and susceptibility ?**

Sol. $\mu_r = \frac{\mu}{\mu_0} = \frac{6.28 \times 10^{-4}}{4\pi \times 10^{-7}} = 500$

$\mu_r = 1 + \chi \ \therefore \chi = \mu_r - 1 = 500 - 1 = 499$

13. **The magnetic moment of a magnet of mass 75 gm is 9×10^{-7} A-m^2. If the density of the material of magnet is 7.5×10^3 kg m^{-3}, then find intensity of magnetisation is.**

Sol. $I = \frac{M}{V}$ Where volume, $V = \frac{\text{mass}(m)}{\text{density}(\rho)}$

$= \frac{M \times \rho}{m} = \frac{9 \times 10^{-7} \times 7.5 \times 10^3}{75 \times 10^{-3}} = 0.094 A / m$

14. **A magnetic field strength (H) 3×10^3 Am^{-1} produces a magnetic field of induction (B) of $12\pi T$ in an iron rod. Find the relative permeability of iron ?**

Sol. $\mu = \frac{B}{H} = \frac{12\pi}{3 \times 10^3} = 4\pi \times 10^{-3}$

$\therefore \ \mu_r = \frac{\mu}{\mu_0} = \frac{4\pi \times 10^{-3}}{4\pi \times 10^{-7}} = 10^4$

15. **An iron bar having length 10 cm & diameter 2 cm is being placed in a magnetic field of intensity 1000 Am^{-1} with its length parallel to the direction of the field. Evaluate the magnetic moment produced in the bar if permeability of its material is 6.3×10^{-4} TmA^{-1}.**

Sol. We know that, $\mu = \mu_0 (1 + \chi)$

$$\Rightarrow \quad \chi = \frac{\mu}{\mu_0} - 1 = \frac{6.3 \times 10^{-4}}{4\pi \times 10^{-7}} - 1 = 500.6$$

Intensity of magnetization,

$$I = \chi H = 500.6 \times 1000 = 5 \times 10^5 \text{ Am}^{-1}$$

$\therefore$ magnetic moment, $M = I \times V = I \times \pi r^2 \ell$

$$= 5 \times 10^5 \times 3.14 \times (10^{-2})^2 \times (10 \times 10^{-2}) = 15.70 \text{ A-m}^2.$$

■ ■ ■

Mind Map

MAGNETISM AND MATTER

Magnetisation

$$\vec{M} = \frac{\vec{m}_{net}}{V} = \frac{\text{Net Magnetic Moment}}{\text{Volume}}$$

$\vec{B} = \vec{B}_0 + \vec{B}_m$ (where $\vec{B}_0 \to \mu_0\vec{H}$, $\vec{B}_m \to \mu_0\vec{M}$)

$\vec{M} = \vec{H}\chi$

$\vec{B} = \mu_0(\vec{H} + \vec{M})$

$\vec{B} = \mu_0(1 + \chi)\vec{H}$

$\vec{B} = \mu_0\mu_r\vec{H}$

$\vec{B} = \mu\vec{H}$

ELECTROSTATIC ANALOG

$\vec{B}$	$\vec{E}$
$\vec{m}$	$\vec{p}$
μ_0	$\frac{1}{\varepsilon_0}$
$\vec{\tau} = \vec{m} \times \vec{B}$	$\vec{\tau} = \vec{p} \times \vec{E}$
$U = -\vec{m} \cdot \vec{B}$	$U = -\vec{p} \cdot \vec{E}$
$\vec{B}_{axial} = \frac{\mu_0}{4\pi}\frac{2\vec{m}}{r^3}$	$\vec{E}_{axial} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}$
$\vec{B}_{eq} = -\frac{\mu_0}{4\pi}\frac{\vec{m}}{r^3}$	$\vec{E}_{eq} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3}$

Magnetism and Gauss's Law

Net magnetic flux through any closed surface ($\oint \vec{B}.d\vec{A}$) is always zero.

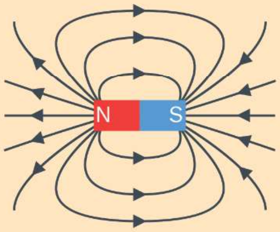
Dipole in Uniform Magnetic Field

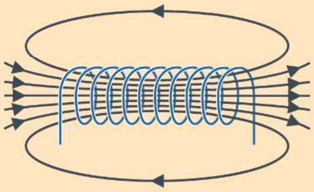
$$T = 2\pi\sqrt{\frac{I}{mB}}$$

Magnets

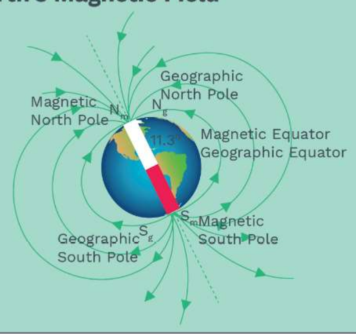
Same poles repel
Opposite poles attract each other

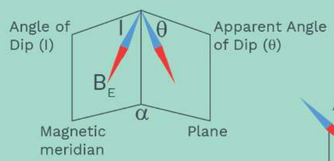
The magnetic field lines of a magnet and Solenoid form continuous closed loops.





Earth's Magnetic Field

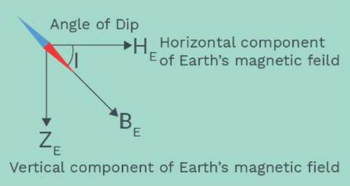




$B_E \cos I = H_E$

$B_E \sin I = Z_E$

$\tan I = \frac{Z_E}{H_E}$



PROPERTIES	DIAMAGNETIC	PARAMAGNETIC	FERROMAGNETIC
χ	$-1 \leq \chi < 0$	$0 < \chi < k$	$\chi >> 1$
μ_r	$0 \leq \mu_r < 1$	$1 < \mu_r < 1 + k$	$\mu_r >> 1$
μ	$\mu < \mu_0$	$\mu > \mu_0$	$\mu >> \mu_0$
Magnetisation	Weak Magnetization in opposite direction	Weak Magnetization in Same direction	Strong Magnetization in Same direction
Movement in magnetic field	(Weak tendency) From strong to weak magnetic field	(Weak tendency) From weak to strong magnetic field	(Strong tendency) From weak to strong magnetic field
Magnet	Weak Repulsion	Weak Attraction	Strong Attraction
Examples	Bi, Au, Pb, Si, H_2O, NaCl, N_2 (STP)	Al, Na, Ca, O_2 (STP)	Fe, Co, Ni, Gd
Magnetic Field Lines