Magnetic Effect of Current

Magnetic Effect of Current

In earlier lessons we found it easy to define to describe the interaction between electrically charged bodies in terms of electric fields. Recall that 'An electric field surrounding an electric charge'. The region of space surrounding a moving electric charge includes a magnetic field in addition to the electric field. A magnetic field also surrounds a magnetic material.

In order to define any form of field, we must define its strength, or magnitude and its direction. Magnetic field is the area surrounding a moving charge in which its magnetic effects are perceptible on a moving charge (electric current). Intensity of magnetic field is a vector quantity and also known as 'magnetic induction vector'. It is represented by $\vec{B}$.

Lines of magnetic field may be drawn in the same way as lines of electric field. The field lines per unit area passing a small area normal to the direction of the field being equal to $\vec{B}$. The number of lines of magnetic field $\vec{B}$ crossing a given area is referred to as the magnetic flux linked with that area. For this reason, $\vec{B}$ is also called 'magnetic flux density'. There are 2 methods of calculating magnetic field at some point. One is 'Biot-Savart' law which gives the magnetic field due to an infinite long small current having wire at some point and the another is 'Ampere' law', which is useful to find out the magnetic field of a symmetrical configuration carrying a steady current. The unit of magnetic induction is Weber/m^2 and is known as 'Tesla' (T) in the SI system.

Magnet: It has always two poles. Monopole does not exist

m → pole strength

Unit → A.m.

$m \propto A$ Cross section area

KEY POINTS

- Magnetic effect of current
- Magnetic flux density
- Magnetic field
- Tesla
- Biot- Savart Law

Definitions

The number of lines of $\vec{B}$ crossing a given area is referred to as the magnetic flux linked with that area. For this reason, $\vec{B}$ is also called magnetic flux density.

Concept Reminder

Biot-savart's law and Ampere's law both are used to find magnetic field due to charge configuration.

Concept Reminder

Sources of magnetic fields are:

1. Magnet
2. Current carrying element
3. Motion of charge
4. Geomagnetism
5. Time varying E field

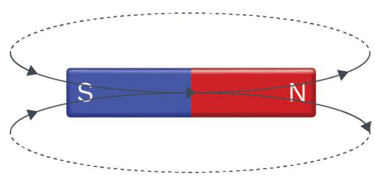
There is not physical existence of magnetic poles. It is used for mathematical calculation only.

Properties of Magnetic field Lines:

(1) Magnetic lines are imaginary lines which represents direction of magnetic field continuously.

(2) Direction of magnetic field lines is from N to S outside the Magnet & south to north inside the magnet.

(3) M.F. lines always form closed loop or curves.



Rack your Brain

Tesla is the unit of
(1) electric field
(2) magnetic field
(3) electric flux
(4) magnetic flux

Source of Magnetic Field

⟶ Magnet
⟶ Current carrying element
⟶ Motion of charge
⟶ Geomagnetism
⟶ Time varying E field

Biot-Savart Law:

'Biot-Savart law' gives the magnetic field due to an infinite long current element. Let AB be a conductor of an arbitrary shape carrying a current 'i', and P be a point in free space at which the field is to be determined. Let us divide the conductor into infinitesimal current elements. Let $\vec{r}$ be a displacement vector from the element to the point P.

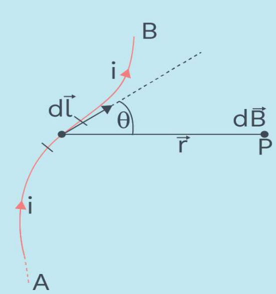
According to 'Biot-Savart Law', the magnetic field $d\vec{B}$ at P due to the current element $d\vec{l}$ is given by-

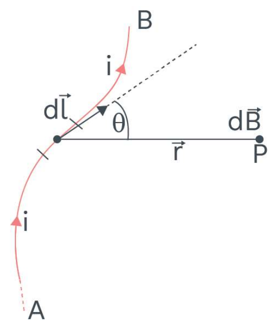
$$d\vec{B} \propto \frac{i(d\vec{l} \times \vec{r})}{r^3} \text{ or } d\vec{B} = k\frac{i(d\vec{l} \times \vec{r})}{r^3}$$

Concept Reminder

According to Biot savart's law

$$d\vec{B} = \frac{\mu_o i(d\vec{l} \times \vec{r})}{4\pi r^3}$$





Where k is a proportionality constant. Here length element $d\vec{l}$ is vector points in the direction of current i.

In S.I. system,

$$k = \frac{\mu_0}{4\pi} = 10^{-7} \frac{Wb}{amp \times metre} \quad [\mu_0 = 4\pi \times 10^{-7}]$$

$$\therefore d\vec{B} = \frac{\mu_0}{4\pi}\frac{i(d\vec{l} \times \vec{r})}{r^3} \quad \text{... (1)}$$

Equation (1) is the vector form of the 'Biot-Savart Law'. The magnitude of the field at P is given by

$$dB = \frac{\mu_0}{4\pi}\frac{idl \times \sin\theta}{r^2}$$

where θ is angle between $d\vec{l}$ and $\vec{r}$.

Now, if the medium is other than air or vacuum, the magnetic field is

$$d\vec{B} = \frac{\mu_0\mu_r}{4\pi}\frac{i(d\vec{l} \times \vec{r})}{r^3} \quad \text{... (2)}$$

where μ_r is relative permeability of the medium and is a dimensionless quantity.

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$$

$dl \rightarrow$ length of small current element.

$d\vec{l} \rightarrow$ direction of $d\vec{l}$ vector is same as current.

$r \rightarrow$ distance between current element and point.

$\vec{r} \rightarrow$ direction of vector is always from wire to point not from point to wire.

$\theta \rightarrow$ Angle between $d\vec{l}$ & $\vec{r}$.

$\mu_0 \rightarrow$ Magnetic permeability of free space

KEY POINTS

- Current element
- Permeability of free space
- Relative permeability

Rack your Brain

A long wire carrying a steady current is bent into a circular loop of one turn. The magnetic field at centre of loop is B. It is then bent into a circular coil of n turns. Find magnetic field at centre of this loop.

Concept Reminder

Relative permeability (μ_r) is defined as

$$\mu_r = \frac{\mu_m}{\mu_0} \text{(unitless)}$$

$\mu_m \rightarrow$ permeability of medium

$\mu_0 \rightarrow$ permeability of free space

B→ Magnetic field or magnetic induction or magnetic flux density.

UNIT		
SI	T → Tesla	
CGS	G → Gauss	$1T = 10^4 G$

Relative permeability (μ_0)

$$\mu_r = \frac{\mu_m}{\mu_0} \text{(unitless)}$$

μ_m → permeability of medium

μ_0 → permeability of free space

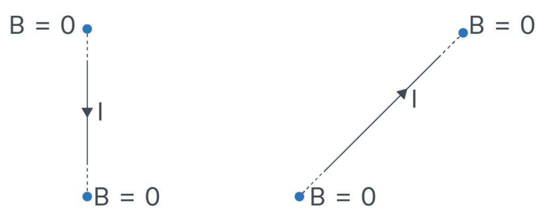
Magnetising field/Magnetic intensity:

$$B = \mu H$$

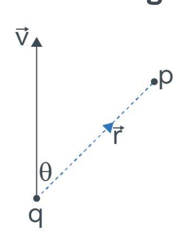
$$H = \frac{B}{\mu} \left(\text{unit} = \frac{A}{m}\right)$$

Note: H does not depend on medium

- **Important:** Magnetic field on the axis of current carrying wire is always zero.



Magnetic field Due to Moving charge:-



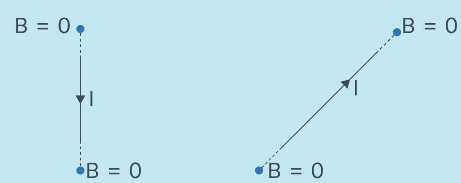
$\because qv \equiv Idl$

$$\Rightarrow B_p = \frac{\mu_0 Idl \sin\theta}{4\pi r^2}$$

$$\boxed{B_p = \frac{\mu_0 qv \sin\theta}{4\pi r^2}}$$

Concept Reminder

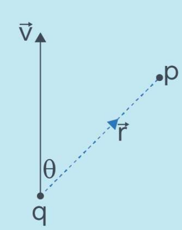
Magnetic field on the axis of current carrying wire is always zero.



Concept Reminder

Magnetic field Due to Moving Charge: -

$$\boxed{B_p = \frac{\mu_0 qv \sin\theta}{4\pi r^2}}$$



Method of Direction Finding :

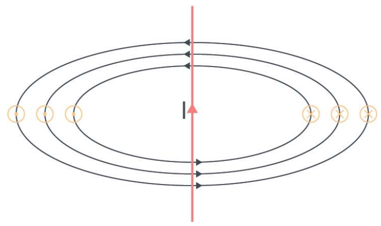
(1) Right Hand Thumb Rule

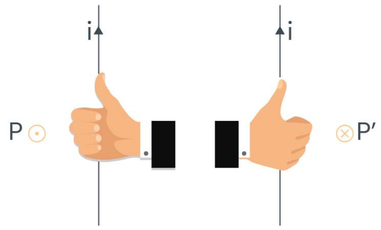
(2) Right Hand Palm Rule

(1) Right Hand Thumb Rule

Right hand thumb should be in direction of current then curling figures will show the pattern of magnetic field lines. If wire is in horizontal, then magnetic field lines will be in vertical plane & if wire is in vertical plane then magnetic field line will be in horizontal plane.

Imp. For a straight current carrying wire magnetic field lines are in form of concentric circles.



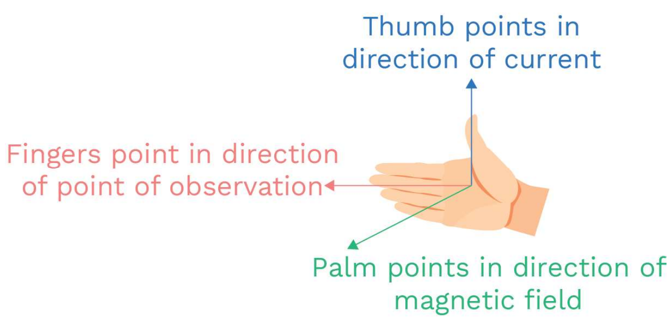


(2) Right Hand Palm Rule

I → Thumb

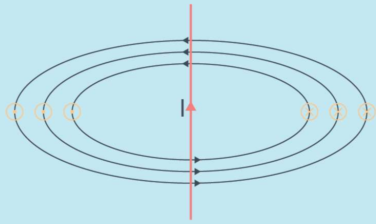
Point of observation → Fingers

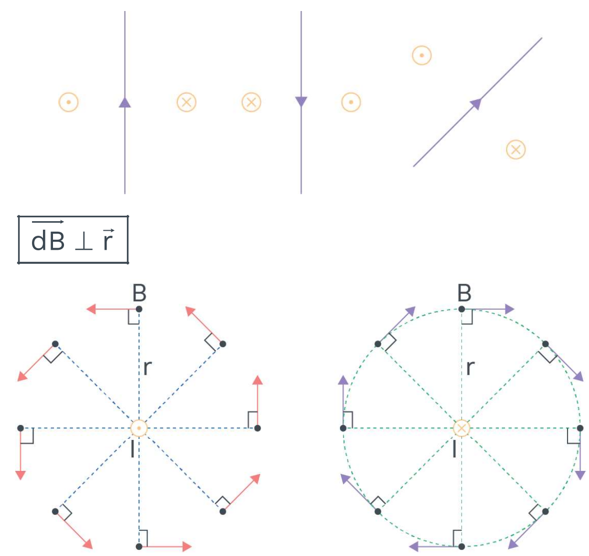
B → Palm



Concept Reminder

For a straight current carrying wire magnetic field lines are in form of concentric circles.





Rack your Brain

Two similar coil of radius R are lying concentrically with their planes at right angles to each other, the current flowing in them are I and 2I, respectively. The resultant magnetic field induction at center will be

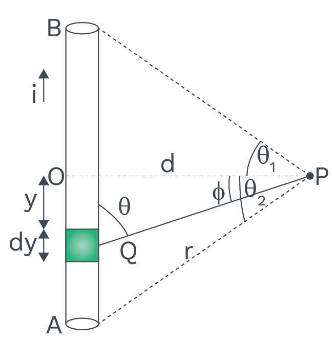
(1) $\frac{\sqrt{5}\mu_0 I}{2R}$ (2) $\frac{\sqrt{5}\mu_0 I}{R}$

(3) $\frac{\mu_0 I}{2R}$ (4) $\frac{\mu_0 I}{R}$

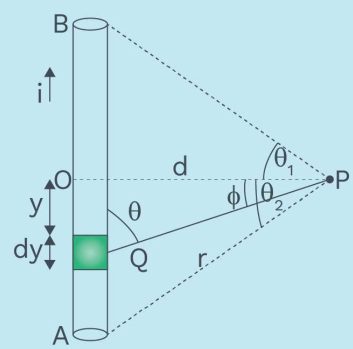
Field Due to A Straight Current Carrying Wire:
When the wire is of finite length:

Take a straight wire segment carrying a current i and there is a point P at which magnetic field to be calculated as given in the figure. This wire segment makes angle θ_1 and θ_2 at that point with normal OP. Assume a length element 'dy' at a distance y from O and distance of this element from 'P' is r and line joining P to Q makes an angle θ with the direction of current as given in figure. Using 'Biot-Savart Law' magnetic induction at point P due to small current element (idl) is given by-

$$dB = \frac{{}_0 i}{4\pi}\left(\frac{dy \sin\theta}{r^2}\right)$$



Concept Reminder



$$B = \frac{\mu_0 i}{4\pi d}[\sin\theta_1 + \sin\theta_2]$$

As every length element of the wire contributes to $\vec{B}$ in the same direction, then we have

$$B = \frac{\mu_0 i}{4\pi}\int_A^B \frac{dy \sin\theta}{r^2} \qquad \text{... (i)}$$

From the triangle OPQ as given in diagram, we have

$$y = d\tan\phi$$

or $\quad dy = d\sec^2\phi\, d\phi$

and in same triangle,

$$r = d\sec\phi \text{ and } \theta = (90° - \phi),$$

where ϕ is angle between line OP and PQ

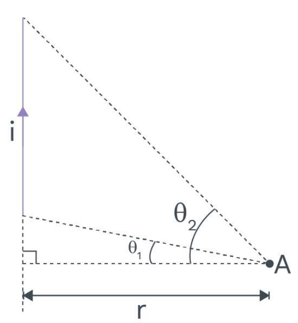
Now equation (i) can be expressed in this form

$$\therefore \quad B = \frac{\mu_0 i}{4\pi d}\int_{-\theta_2}^{\theta_1} \cos\phi\, d\phi$$

or $$\boxed{B = \frac{\mu_0 i}{4\pi d}[\sin\theta_1 + \sin\theta_2]} \qquad \text{.... (3)}$$

Note: θ_1 & θ_2 must be taken with sign

For the case given in figure

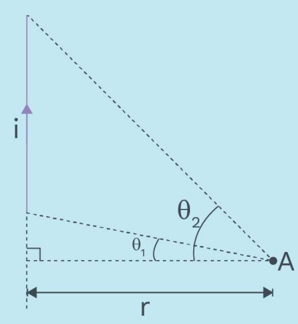


$$B \text{ at } A = \frac{\mu_0 i}{4\pi r}(\sin\theta_2 - \sin\theta_1) \otimes$$

Direction of $\vec{B}$:- The direction of magnetic field is obtained by the cross product of the current element $id\vec{l}$ with $\vec{r}$. So, at point P, the direction of the magnetic field due to the entire conductor will be normal to the plane of paper and going

Concept Reminder

θ_1 & θ_2 must be taken with sign



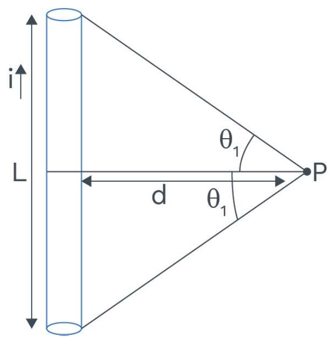
Magnetic field at A is

$$B = \frac{\mu_0 i}{4\pi r}(\sin\theta_2 - \sin\theta_1) \otimes$$

into the plane.

Case-1 : When the point P is on the perpendicular bi-sector:

In this case angle $\theta_1 = \theta_2$, by using result of equation (3), the magnetic field at point 'P' is

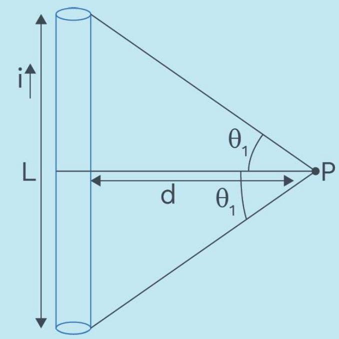


$$B = \frac{\mu_0 2i}{4\pi d}\sin\theta_1$$

where $\sin\theta_1 = \dfrac{L}{\sqrt{L^2 + 4d^2}}$

Concept Reminder

When the point P is on the perpendicular bisector

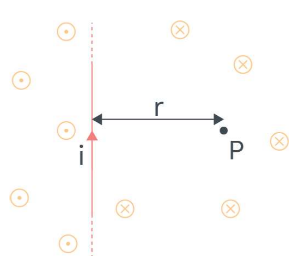


$$B = \frac{\mu_0 2i}{4\pi d}\sin\theta_1$$

Case-2: Magnetic field due to infinite wire:

If the wire is infinitely long then the magnetic field at 'P' (as given in the diagram) is given by (using $\theta_1 = \theta_2 = 90°$ and the formula 'B' due to straight wire)

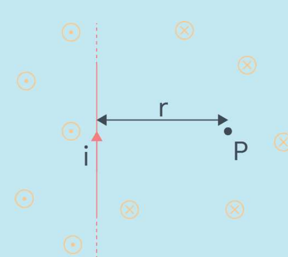
$$B = \frac{\mu_0 I}{2\pi r} \Rightarrow B \propto \frac{I}{r}$$



The direction of $\vec{B}$ at different point is as shown in the diagram. The magnetic lines will be

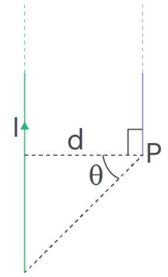
Concept Reminder

If the wire is infinitely long, then the magnetic field is $B = \dfrac{\mu_0 I}{2\pi r}$



concentric circles around the wire (as shown earlier).

Case-2: Magnetic field due to semi-infinite wire:



$$B = \frac{\mu_0 I}{4\pi d}(\sin\theta_1 + \sin\theta_2)$$

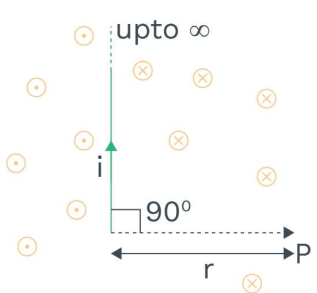
$$\theta_1 = 90°, \theta_2 = \theta$$

$$= \frac{\mu_0 I}{4\pi d}(\sin 90° + \sin\theta)$$

$$B = \frac{\mu_0 I}{4\pi d}(1 + \sin\theta)$$

- If the wire is infinitely long but point 'P' is as shown in the diagram then direction of magentic field $\vec{B}$ at various points is as shown in the diagram. At 'P'

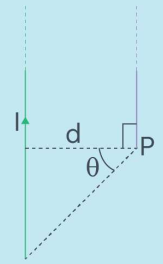
$$\theta_1 = 90°, \theta_2 = 0°$$



$$B = \frac{\mu_0 I}{4\pi d}(\sin 0° + \sin 90°)$$

Concept Reminder

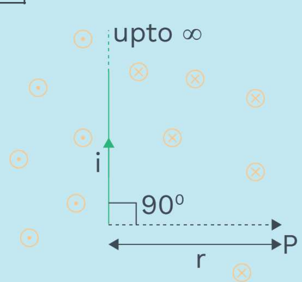
For given figure, Magnetic field at point P is $B = \frac{\mu_0 I}{4\pi d}(1 + \sin\theta)$



Concept Reminder

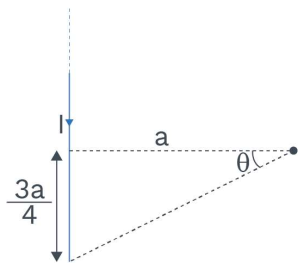
If the wire is semi infinitely long, then the magnetic field is

$$\boxed{B = \frac{\mu_0 I}{4\pi r}}$$



$$\boxed{B = \frac{\mu_0 I}{4\pi r}}$$

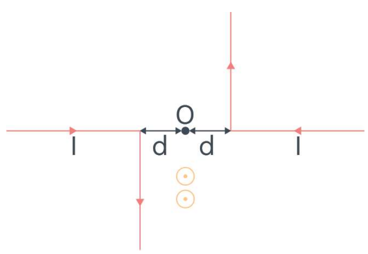
Ex. Find magnetic field for given figure.



Sol. $B = \frac{\mu_0 I}{4\pi a}(1+\sin\theta)$

$$\boxed{B = \frac{\mu_0 I}{4\pi a} \times \frac{8}{5}}$$

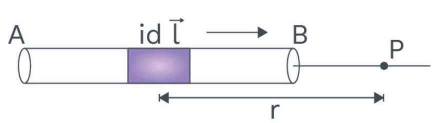
Ex. Find magnetic field at O.



Sol. $B = 2\frac{\mu_0 I}{4\pi d}$

$B = \frac{\mu_0 I}{2\pi d} \odot$

Case-3: When the point 'P' lies along the length of wire:



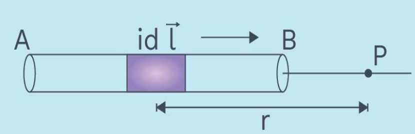
If the point 'P' is along the length of the wire (but not one it), then as $d\vec{l}$ & $\vec{r}$ will either be parallel or anti-parallel, i.e., $\theta = 0$ or π, so $i d\vec{l} \times \vec{r} = 0$ and hence using equation (1)

Rack your Brain

A charge q is uniformly spread on a thin ring of radius R. The ring rotates about its axis with uniform frequency f Hz. Find the magnitude of magnetic induction at center of the ring.

Concept Reminder

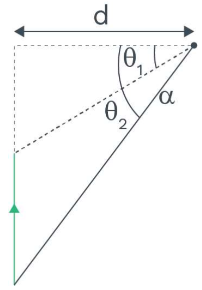
When the point lies along the length of wire (but not on it), then magnetic field will be zero.



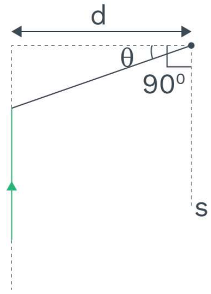
$$\vec{B} = \int_A^B d\vec{B} = 0$$

Special Case:

When point is not in front of wire



- $B = \frac{\mu_0 I}{4\pi d}(\sin\theta_2 - \sin\theta_1)$

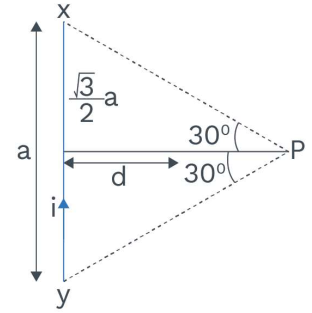


- $B = \frac{\mu_0 I}{4\pi d}(1 - \sin\theta)$

Ex. Find out the magnetic field at a point distance, $\frac{a\sqrt{3}}{2}$ metre from a straight wire of length 'a' metre carrying a current of i amp. The point is on the perpendicular bi-sector of the wire.

Sol. $B = \frac{\mu_0}{4\pi}\frac{i}{d}[\sin\theta_1 + \sin\theta_2]$

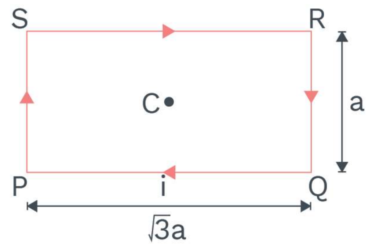
$$= 10^{-7}\left[\frac{i}{(a\sqrt{3}/2)}\left(\frac{1}{2} + \frac{1}{2}\right)\right]$$



$$= \frac{2i}{a\sqrt{3}} \times 10^{-7}\,T$$

Perpendicular to the plane of figure (inward)

Ex. Find out the resultant magnetic field at 'C' in the figure shown.



Sol. It is clear that magnetic field $\vec{B}$ at 'C' due all the wires is directed $\otimes$. Also that magnetic field at 'C' due PQ and SR is same. Also because of QR & PS is equal

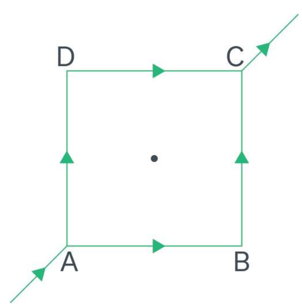
$$\therefore \quad B_{res} = 2(B_{PQ} + B_{SP})$$

$$B_{PQ} = \frac{\mu_0 i}{4\pi \frac{a}{2}}(\sin 60° + \sin 60°)$$

$$B_{SP} = \frac{\mu_0 i}{4\pi \frac{\sqrt{3}\,a}{2}}(\sin 30° + \sin 30°)$$

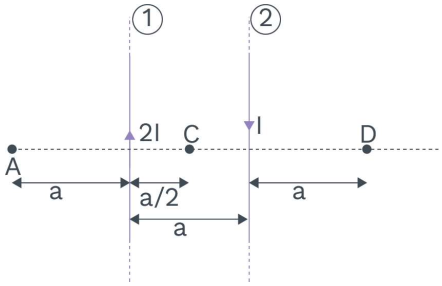
$$\Rightarrow \quad B_{res} = 2\left(\frac{\sqrt{3}\mu_0 i}{2\pi a} + \frac{\mu_0 i}{2\pi a\sqrt{3}}\right) = \frac{4\mu_0 i}{\sqrt{3}\pi a}$$

Ex. As shown in figure a square loop made from a uniform wire. Calculate the magnetic field at the centre of the square if a battery source is connected between the points 'A' & 'C'.



Sol. The current will be equally divided at 'A'. The fields at the mid-point due to the currents in the wires AB and DC will be same in magnitude & opposite in direction. The resultant of these '2' fields will be zero. Same as, the resultant of the fields due to the wires 'AD' & 'BC' will be zero. Hence, the net field at the centre will be zero.

Ex. In the diagram shown there are '2' parallel long wires (placed in the plane of paper) are carrying currents '2I' and 'I' consider points A, C, D on the line normal to both the wires and also in plane of the paper. The distances are mentioned.



Find: (i) $\vec{B}$ at A, C, D

(ii) position of point on line (A, C, D) where $\vec{B}$ is zero.

Sol. (i) Let's call $\vec{B}$ due to (1) and (2) as $\vec{B}_1$ and $\vec{B}_2$ respectively. Then

at A : $\vec{B}_1$ is $\odot$ and $\vec{B}_2$ is $\otimes$

$$B_1 = \frac{\mu_0 2I}{2\pi a} \text{ and } B_2 = \frac{\mu_0 I}{2\pi 2a}$$

$$\therefore \quad B_{res} = B_1 - B_2 = \frac{3}{4}\frac{\mu_0 I}{\pi a} \odot$$

at C : $\vec{B}_1$ is $\otimes$ and $\vec{B}_2$ also $\otimes$

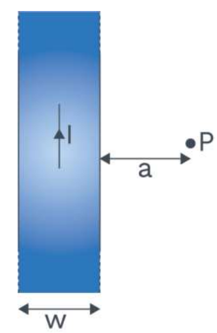
$$\therefore \quad B_{res} = B_1 + B_2 = \frac{\mu_0 2I}{2\pi \frac{a}{2}} + \frac{\mu_0 I}{2\pi \frac{a}{2}} = \frac{6\mu_0 I}{2\pi a} = \frac{3\mu_0 I}{\pi a} \otimes$$

at D : $\vec{B}_1$ is $\otimes$ and $\vec{B}_2$ is $\odot$ and both are equal in magnitude.

$$\therefore \quad B_{res} = 0$$

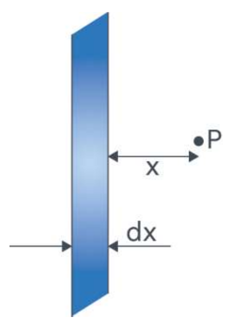
(ii) It is clear from the above result that magnetic field B = 0 at point 'D'.

Ex. In the diagram shown a large metal sheet of width 'w' carries a current I (uniformly distributed in its width 'w'. Find out the magnetic induction at point 'P' which lies in the plane of given sheet?



Sol. To find out 'B' at 'P' the sheet can be considered as set of large number of infinitely long wires. Take a long wire distance 'x' from point 'P' and of width 'dx'. Due to this the magnetic field at point 'P' is 'dB'.

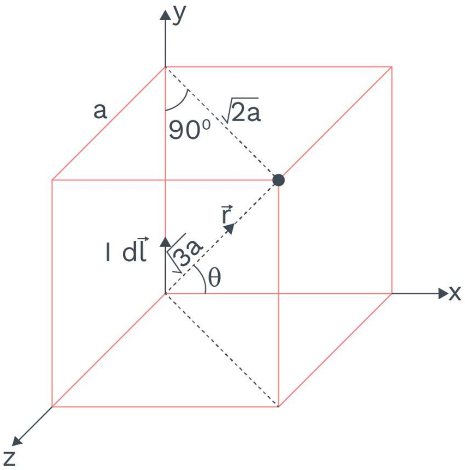
$$dB = \frac{\mu_0 \left(\frac{I}{w} dx \right)}{2\pi x} \otimes$$



Due to each such wire magnetic field $\vec{B}$ will be directed inwards

$$\therefore \quad B_{res} = \int dB = \frac{\mu_0 I}{2\pi w} \int_{x=a}^{a+w} \frac{dx}{x} = \frac{\mu_0 I}{2\pi w} . \ln \frac{a+w}{a}$$

Ex. A small current element $I\overrightarrow{dl}$ is placed at origin as shown in figure. Then find magnetic field $\overrightarrow{dB}$ at position vector .



Sol. $dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$

$r = \sqrt{3}\,a$

$\sin\theta = \frac{\sqrt{2}a}{\sqrt{3}a} = \sqrt{\frac{2}{3}}$

$dB = \frac{\mu_0}{4\pi}\frac{Idl}{3a^2}\sqrt{\frac{2}{3}}$

$dB = \frac{\mu_e}{4\pi}\frac{I\overrightarrow{dl}\times\vec{r}}{r^3}$

$\overrightarrow{dl} = dl\hat{j}$

$\vec{r} = a\hat{i} + a\hat{j} + a\hat{k}$

$= a(\hat{i} + \hat{j} + \hat{k})$

$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{Idl\,\hat{j}\times a(\hat{i}+\hat{j}+\hat{k})}{\left(\sqrt{3}a\right)^3}$

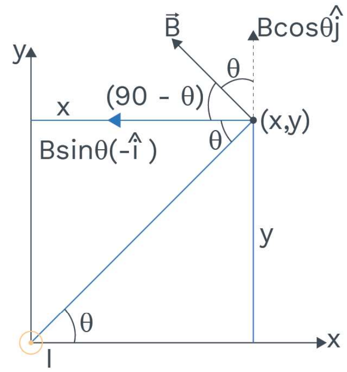
$= \frac{\mu_0}{4\pi}\frac{Idl\,\hat{j}\times a(\hat{i}+\hat{j}+\hat{k})}{3\sqrt{3}\,a^3}$

$= \frac{\mu_0}{4\pi}\frac{Idl\,\hat{j}\times a(\hat{i}+\hat{j}+\hat{k})}{3\sqrt{3}\,a^2}$

$$\boxed{\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{Idl\,a}{3\sqrt{3}\,a^3}\left(-\hat{k}+\hat{i}\right)}$$

Ex. Current I is flowing in a long straight wire in positive z-direction and this wire is passing through origin than find magnetic field at point (x, y).

Sol.



$$B = \frac{\mu_0 I}{2\pi\sqrt{x^2 + y^2}}$$

$$\cos\theta = \frac{x}{\sqrt{x^2 + y^2}} \text{ and } \sin\theta = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\vec{B} = B\cos\theta\hat{j} - B\sin\theta\hat{i}$$

$$= B\left(\cos\theta\hat{j} - \sin\theta\hat{i}\right)$$

$$= \frac{\mu_0 I}{2\pi\sqrt{x^2 + y^2}}\left(\frac{x}{\sqrt{x^2 + y^2}}\hat{j} - \frac{y}{\sqrt{x^2 + y^2}}\hat{i}\right)$$

$$\vec{B} = \frac{\mu_0 I}{2\pi\left(x^2 + y^2\right)}(x\hat{j} - y\hat{i})$$

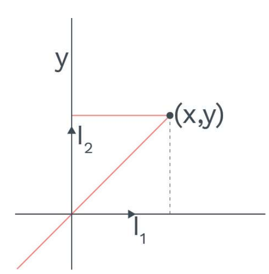
Ex. Current I_1 and I_2 are flowing in two long wires along +x and +y direction. These wires are placed on x-axis and y axis respectively. Then find out locus where magnitude of magnetic field is equal.

Sol. $|\vec{B}_1| = |\vec{B}_2|$

$$\frac{\mu_0 I_1}{2\pi y} = \frac{\mu_0 I_2}{2\pi x}$$

$$\frac{I_1}{I_2} = \frac{y}{x}$$

$$y = \frac{I_1}{I_2}x$$

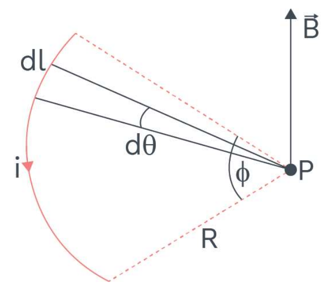


$$\boxed{y = mx}$$

Note: If magnetic field of both wires cancel each other then their direction must be opposite and magnitude must be same.

Magnetic Field at the centre of an arc:

Assume an arc of radius R carrying current i and subtending an angle ϕ at the centre. According to 'Biot-Savart Law', the magnetic field at the point P is given by



$$B = \frac{\mu_0}{4\pi}\int_0^{\phi}\frac{idl}{R^2}$$

Here, $dl = Rd\theta$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\int_0^{\phi}\frac{iRd\theta}{R^2}$$

or $$B = \frac{\mu_0}{4\pi}\frac{i\phi}{R} \qquad \text{... (5)}$$

If 'l' is the length of the circular arc, we have

$$B = \frac{\mu_0}{4\pi}\frac{il}{R^2} \qquad \text{... (6)}$$

Take some special cases involving the application of equation (5).

CASE I: If the loop is semi-circular:

In this case $\phi = \pi$, so

Concept Reminder

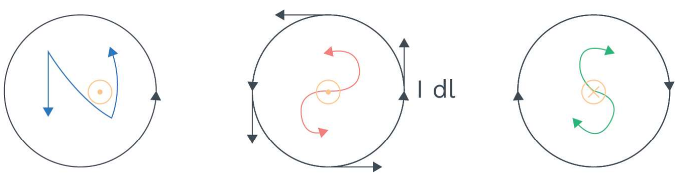
(1) When $\theta = 2\pi$, $B = \frac{\mu_0 i}{2R}$ (field at centre of complete circular loop)

(2) When $\theta = \pi$, $B = \frac{\mu_0 i}{4R}$ (field at centre of a semi wire)

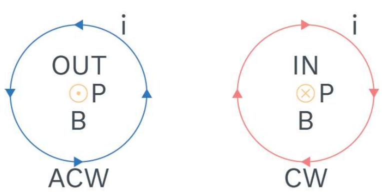
$$B = \frac{\mu_0}{4\pi}\frac{\pi i}{R}$$

and it will be out of the page for anti-clockwise current while into the page for clockwise current as shown in the figure.

CASE II. Magnetic Field of a Current Carrying Circular Loop :



In this case $\phi = 2\pi$, So, magnitude of magnetic field at the centre $\boxed{B = \frac{\mu_0 I}{2r}}$ and it will be out of the paper for anti-clockwise current while into the page for clockwise current as shown in the diagram.



Current carrying loop behaves as a magnetic dipole.

- **If the loop is a full complete with N turns:**

$$B = \frac{\mu_0}{4\pi}\frac{2\pi NI}{R}$$

- **Note:**

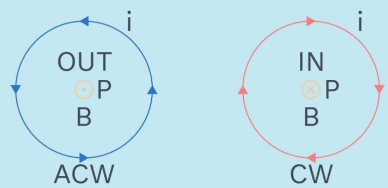
$$2\pi \longrightarrow \frac{\mu_0 I}{2r} \qquad I \longrightarrow \frac{\mu_0 I}{2r}\left(\frac{1}{2\pi}\right)$$

$$\alpha \longrightarrow \frac{\mu_0 I}{4\pi r}\alpha \qquad \boxed{B_{Arc.} = \frac{\mu_0 I}{4\pi r}\alpha}$$ α is in radian

- **Below are some examples**

Concept Reminder

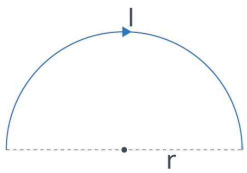
Direction of magnetic field at centre of circular loop.



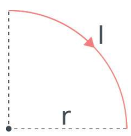
Rack your Brain

A circular loop consists of 2 identical semi-circular parts each of radius R one lying in x-y plane and the other in x-z plane. If the current in loop is I. Find out resultant magnetic field due to two semi-circular parts at their common centre.

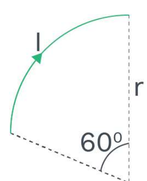
1. $B = \frac{\mu_0 I}{4\pi r} \times \pi = \frac{\mu_0 I}{4r}$



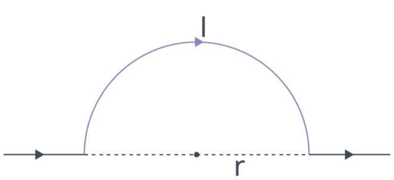
2. $B = \frac{\mu_0 I}{4\pi r} \times \frac{\pi}{2} = \frac{\mu_0 I}{8r}$



3. $B = \frac{\mu_0 I}{4\pi r} \frac{\pi}{3} = \frac{\mu_0 I}{12r}$

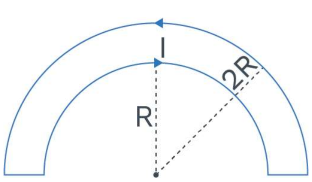


4. $B = \frac{\mu_0 I}{4r} \otimes$



5. $B = \frac{\mu_0 I}{4}\left[\frac{1}{R} - \frac{1}{2R}\right]$

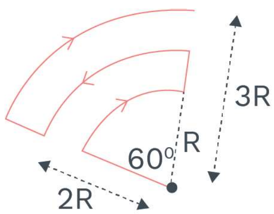
$B = \frac{\mu_0 I}{8R} \otimes$



6. $B = \frac{\mu_0 I}{12R} - \frac{\mu_0 I}{12(2R)} + \frac{\mu_0 I}{12(3R)}$

$B = \frac{\mu_0 I}{12R} \times \left(1 - \frac{1}{2} + \frac{1}{3}\right)$

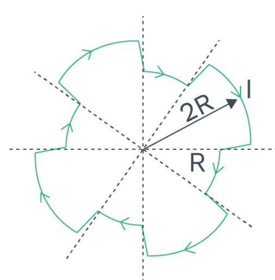
$B = \left(\frac{\mu_0 I}{12R} \times \frac{5}{6}\right) \otimes$



7. $B = 4 \times \frac{\mu_0 I}{16R} + 4 \frac{\mu_0 I}{16(2R)}$

$B = \frac{\mu_0 I}{4R} + \left(1 + \frac{1}{2}\right)$

$B = \frac{3}{2} \times \frac{\mu_0 I}{4R} \otimes$

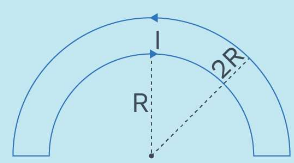


8. $B_e = B_l + B_{arc}$

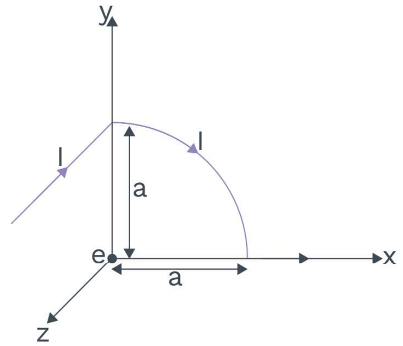
$= \frac{\mu_0 I}{4\pi a} + \frac{\mu_0 I}{8a} \otimes$

Concept Reminder

$B = \frac{\mu_0 I}{8R} \otimes$



$$= \frac{\mu_0 I}{4a}\left(\frac{1}{\pi} + \frac{1}{2}\right)$$



$$B_l = \frac{\mu_0 I}{4\pi a}\left(-\hat{i}\right)$$

$$B_{arc} = \frac{\mu_0 I}{8a}\left(-\hat{k}\right)$$

$$B = \sqrt{B_1^2 + B_{arc}^2}$$

$$B = \sqrt{\left(\frac{\mu_0 I}{4\pi a}\right)^2 + \left(\frac{\mu_0 I}{8a}\right)^2} = \frac{\mu_0 I}{a}\sqrt{\frac{1}{16\pi^2} + \frac{1}{64}}$$

Ex. Circular loop is made by a wire of length 12.5m. If current I is flowing in the loop then find magnetic field at centre.

Sol. $B = \frac{\mu_0 I}{2r} \times \frac{\pi}{\pi}$

$$= \frac{4\pi \times 10^{-7} \times I\pi}{12.5}$$

$$= \frac{4\pi^2 \times 10^{-7} \times I}{12.5}$$

$$= \frac{4\pi^2 \times 10^{-7} \times I}{4\pi} = \pi \times 10^{-7} I \text{ Tesla}$$

Ex. Two concentric circular coils of radius 10 cm and 20 cm are placed in same plane. If current I is flowing in both coils in opposite direction then find magnetic field at the centre.

Sol. $B = \frac{\mu_0 I}{2 \times 0.1} = \frac{\mu_0 I}{0.2}$

$B_2 = \frac{\mu_0 I}{2 \times 0.2} = \frac{\mu_0 I}{0.4}$

$B = \frac{\mu_0 I}{0.2} - \frac{\mu_0 I}{0.4}$

$= \mu_0 I \left[\frac{10}{2} - \frac{10}{4}\right]$

$= \mu_0 I \, 10 \times \frac{2}{8} = \frac{5}{2}\mu_0 I$

Ex. For a current carrying loop number of turns are 50 and radius is 10 cm at this place magnitude of earth's magnetic field $5\pi \times 10^{-4}\,T$. If magnetic field at the centre of loop is zero, then find I in coil.

Sol. $5\pi \times 10^{-4} = \frac{50 \times 4\pi \times 10^{-7} \times I}{2 \times 0.1}$

$I = \frac{5 \times 0.2 \times 10^{-4+7}}{5 \times 40} = \frac{2}{400} \times 1000$

I = 5 Amp.

If magnetic field of coil and earth cancel each other than their direction must be opposite and magnitude must be same.

Ex. A circular ring of 1 turns is made by a conducting wire. The magnetic field at the center is B, now. This same wire is bent to form circular ring of 3 turns. Find magnetic field at centre (current is same).

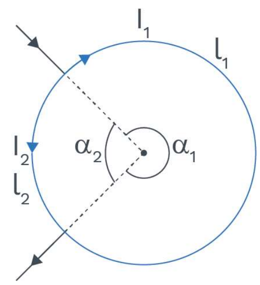
Sol. $B = \frac{N\mu_0 I}{2r}$

$l = N(2\pi r)$, $\quad r \propto \frac{1}{N}$

$B \propto N^2$, $\quad B_2 = 9B_1$

When current divides in any symmetrical loop.

(Uniform wire)



At centre

$$B_1 = \frac{\mu_0 I_1 \alpha_1}{4\pi r} \otimes$$

$$B_2 = \frac{\mu_0 I_2 \alpha_2}{4\pi r} \odot$$

$$B = B_1 - B_2$$

$$B = \frac{\mu_0}{4\pi r}(I_1\alpha_1 - I_2\alpha_2)$$

$$\frac{B_1}{B_2} = \frac{I_1\alpha_1}{I_2\alpha_2}$$

$$R = \rho\frac{l}{A} \Rightarrow \boxed{R \propto l}$$

$$R \propto l \Rightarrow I \propto \frac{1}{l}$$

$$I_1 \longrightarrow \frac{1}{l_1} \qquad I_2 \longrightarrow \frac{1}{l_2}$$

$$\text{Angle} = \frac{\text{Arc}}{\text{Radius}}$$

$$\alpha_1 = \frac{l_1}{r}; \qquad \alpha_2 = \frac{l_2}{r}$$

$$\boxed{\alpha_1 \propto l_1}; \qquad \boxed{\alpha_2 \propto l_2}$$

$$\frac{B_1}{B_2} = \frac{I_1\alpha_1}{I_2\alpha_2} = \frac{\frac{1}{l_1}l_1}{\frac{1}{l_2}l_2}$$

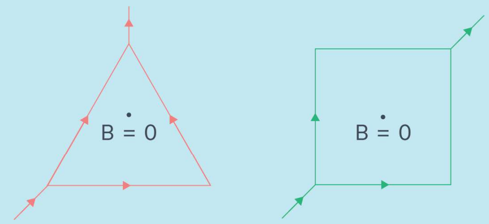
$$\boxed{\frac{B_1}{B_2} = 1}$$

Concept Reminder

Magnetic field at the centre of a square of a side a carrying current I is $B = \frac{2\sqrt{2}\ {}_0 i}{\pi a}$

Concept Reminder

When current divides in any symmetrical loop made with uniform wire then magnetic field at the centre is zero.

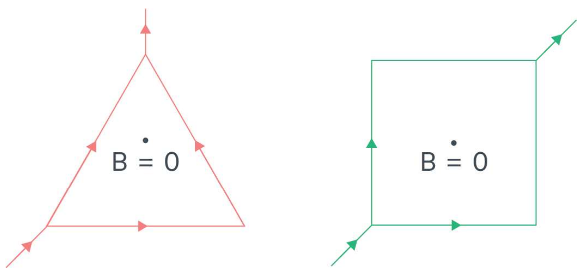


$B_1 = B_2$

$B = B_1 - B_2$

$B = 0$

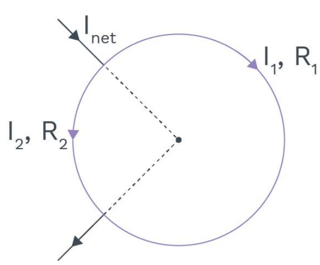
- When current divides in any symmetrical loop made with uniform wire then magnetic field at the centre is zero.



- **If wire is not uniform**

$I_1 = \frac{R_2}{R_1 + R_2} \times I_{net}$ $\quad R_2 \rightarrow$ Resistance of other part

$I_2 = \frac{R_1}{R_1 + R_2} \times I_{net}$



If cross-section area of wire are different

$R = \rho \frac{l}{A}$ $\boxed{R \propto \frac{l}{A}}$

$\frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{A_2}{A_1}$

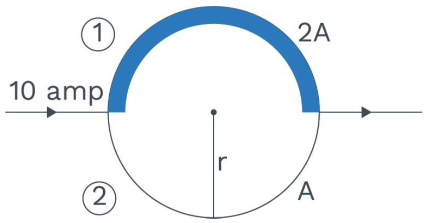
Below are some examples:

1.

Rack your Brain

A straight wire of diameter 0.5 mm carrying a current of 1 A is replaced by another wire of 1 mm diameter carrying the same current. The strength of magnetic field far away is

(1) one-quarter of the earlier value
(2) one-half of the earlier value
(3) twice the earlier value
(4) Same as the earlier value.



$$\frac{R_1}{R_2} = \frac{A_2}{A_1} = \frac{A}{2A} = \frac{1}{2}$$

$$I_1 = \frac{2}{3} \times I_{net} = \frac{20}{3}$$

$$I_2 = \frac{1}{3} \times I = \frac{10}{3}$$

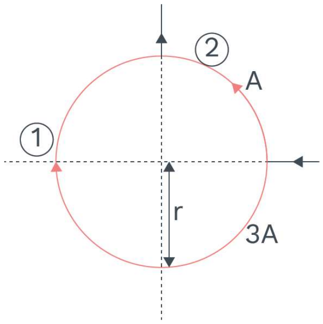
$$B = B_2 - B_1$$

$$B = \frac{\mu_0 I_1}{4r} - \frac{\mu_0 I_2}{4r}$$

$$B = \frac{5\mu_0}{6r} \otimes$$

2. $R = \rho \dfrac{l}{A}$

$$\frac{R_1}{R_2} = \frac{l_1}{l_2}\frac{A_2}{A_1} = \frac{3}{1} \times \frac{A}{3A} = \frac{1}{1}$$



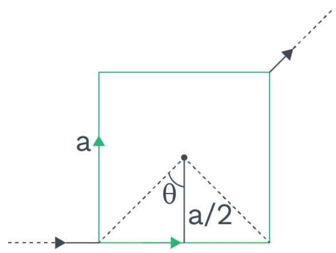
$$I_1 = I_2 = \frac{I}{2}$$

$$B = B_1 - B_2$$

$$= \frac{3\mu_0 I_1}{8r} - \frac{\mu_0 I_2}{8r}\left(I_1 = I_2 = \frac{I}{2}\right)$$

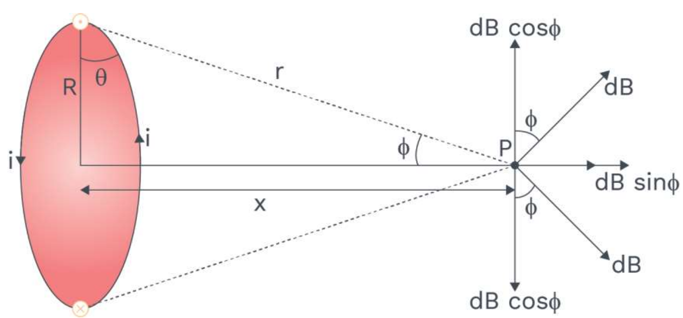
$$B = \frac{\mu_0 I}{8r} \otimes$$

3. $B = \frac{\mu_0 I_1}{4\pi a}(1 - \sin\theta) = \frac{\mu_0 I}{2\pi a}\left(1 - \frac{1}{\sqrt{2}}\right) \odot$



Magnetic Field At A Point On Axis Of A Circular Coil:-

Take a circular loop of radius 'R' and carrying current i. We have to find magnetic field at the axial point P, which is at distance 'x' from the centre of the loop.



Consider an element $id\vec{l}$ of the loop as given in diagram, and the distance of point 'P' from current element is r. The magnetic field at Point 'P' due to this current element (idl) from the equation (1) can be given by-

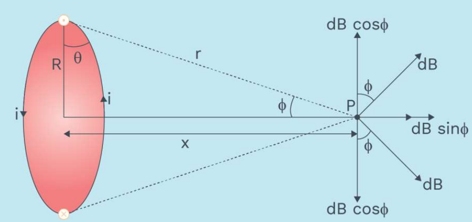
$$\overline{dB} = \frac{\mu_0}{4\pi}\frac{id\vec{l} \times \vec{r}}{r^3}$$

In case of point on axis of a circular coil, as for every current element there is asymmetrically situated opposite element, the component of the field normal to the axis cancel each other while along the axis add up.

$$\therefore \quad B = \int dB\sin\phi = \frac{\mu_0}{4\pi}\int \frac{idl\sin\theta}{r^2}\sin\phi$$

Concept Reminder

Magnetic field of an axial point of circular coil is $B = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}$



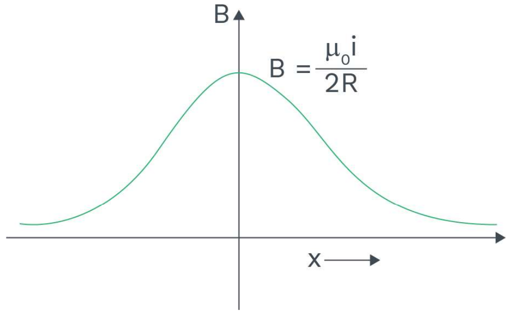
Concept Reminder

(1) If x << R $\quad B = \frac{\mu_0 i}{2R}\left[1 - \frac{3x^2}{2R^2}\right]$

(2) If x >> R $\quad B = \frac{\mu_0 i R^2}{2x^3}$

Here, θ is angle between the current element $id\vec{l}$ and $\vec{r}$, which is $\frac{\pi}{2}$ everywhere and

$$\phi = \frac{R}{r} = \frac{R}{\sqrt{R^2 + x^2}}$$



$$\therefore \quad B = \frac{\mu_0}{4\pi} \frac{iR}{(R^2 + x^2)^{3/2}} \int_0^{2\pi R} dL$$

or $$B = \frac{\mu_0}{4\pi} \frac{iR}{(R^2 + x^2)^{3/2}} (2\pi R)$$

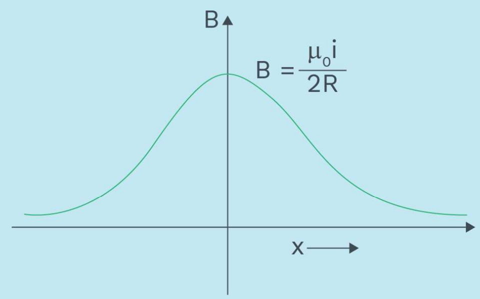
or $$\boxed{B = \frac{\mu_0}{4\pi} \frac{2\pi i R^2}{(R^2 + x^2)^{3/2}}}$$

If the coil has N turns, then

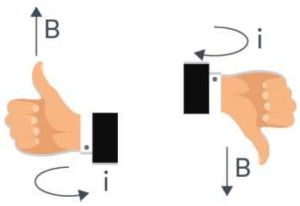
$$\boxed{B = \frac{\mu_0}{4\pi} \frac{2\pi N i R^2}{(R^2 + x^2)^{3/2}}}$$

Concept Reminder

Variation of B with x



Direction of $\vec{B}$: The direction of magnetic field at a point on the axis of a circular loop is along the axis and its orientation can be obtained by using the "right-hand thumb rule". If the fingers are bent along the current, the stretched thumb will point towards magnetic field induction.

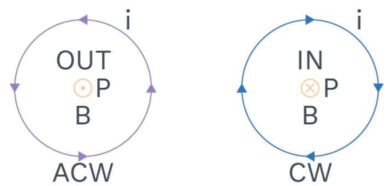


Magnetic field will be out of the paper for anticlockwise current while into the paper for clockwise current as shown in the diagram given. Now consider some special conditions/ cases involving the application of equation (4)

CASE I : Magnetic field at the centre of the coil

In this situation distance of the point P from the centre (x) = 0, the magnetic field is

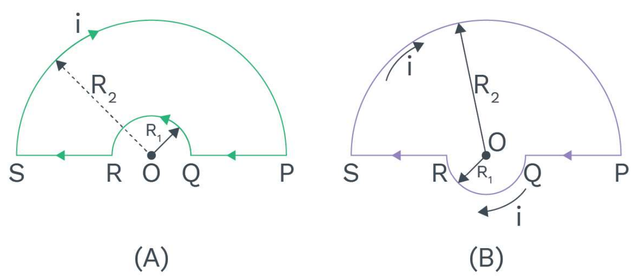
$$B = \frac{\mu_0}{4\pi}\frac{2\pi i}{R} = \frac{\mu_0}{2}\frac{i}{R}$$



CASE II : Magnetic field at a point far away from the centre-

It means x >> R, $B = \frac{\mu_0}{4\pi}\frac{2\pi R^2 i}{x^3} = \frac{\mu_0 i R^2}{2x^3}$

Ex. Two wire loop PQRS P formed by joining two semicircular wires of radii 'R_1' and 'R_2' carries a current i as given in the figure given shown below. Calculate the magnetic field at the centre point O in cases (A) and (B) ?



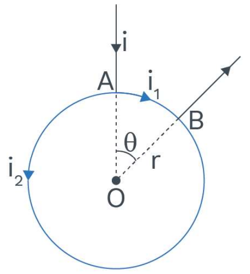
Sol. (a) As the point 'O' is along the length of the straight wires, so the magnetic field at point O due to them will be zero and thus magnetic field induction is only due to semicircular portions

$$\therefore \quad |\vec{B}| = \frac{\mu_0}{4\pi}\left[\frac{\pi i}{R_2}\otimes + \frac{\pi i}{R_1}\odot\right]$$

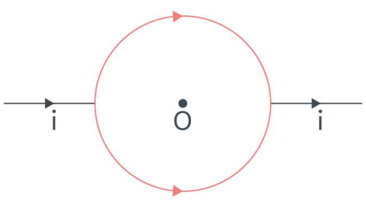
or $$|\vec{B}| = \frac{\mu_0}{4\pi}\pi i\left[\frac{1}{R_2} - \frac{1}{R_1}\right]$$ out of the page

(b) $$|\vec{B}| = \frac{\mu_0}{4\pi} i\pi\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$ into the page

Ex. A battery source is connected between 2 points A and B on the perimeter of a uniform circular ring of radius r and resistance R as given in the figure shown below. One of the arcs 'AB' of the ring subtends an angle θ at the middle of ring. Find out the value of magnetic field at the centre due to the current in the circular ring?



(A)



(B)

Sol. (a) As the field due to arc at the centre is given by $B = \frac{\mu_0}{4\pi}\frac{i\phi}{r}$

$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{i_1\theta}{r} \otimes + \frac{\mu_0}{4\pi}\frac{i_2(2\pi-\theta)}{r} \odot$

But $(V_A - V_B) = i_1R_1 = i_2R_2$

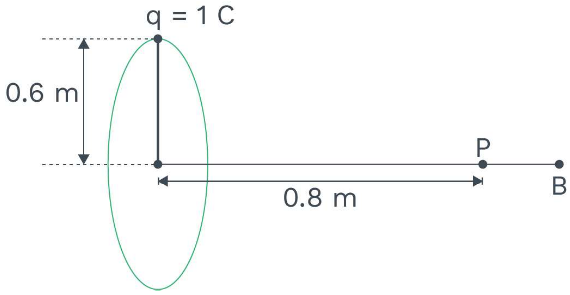
or $\quad i_2 = i_1\frac{R_1}{R_2} = i_1\frac{L_1}{L_2}$ $\quad [\therefore R \propto L]$

$\therefore \quad B_R = \frac{\mu_0}{4\pi}\frac{i_1\theta}{r} \otimes + \frac{\mu_0}{4\pi}\frac{i_1\theta}{r} \odot = 0$

i.e., the magnetic field at the centre of the coil is zero and is independent of θ.

Ex. 1 Coulomb charge of one coulomb is placed at one end of a non-conducting rod of length 0.6 metre. Rod is rotated in the vertical plane about a horizontal axis passing through another end of the rod with angular velocity $10^4\pi$ rad/second. Find the magnetic field at the point on axis of rotation at a distance of 0.8 metre from the centre of the path. Now, half of the total charge is removed from one end and placed on the another end. Now, rod is rotated in a vertical plane about the horizontal axis passing through the mid-point of rod with the equal angular frequency. Find out the magnetic field induction at a point on the axis at a distance of 0.4 metre from the mid point of the rod.

Sol. As the revolving charge 'q' is equivalent to a current

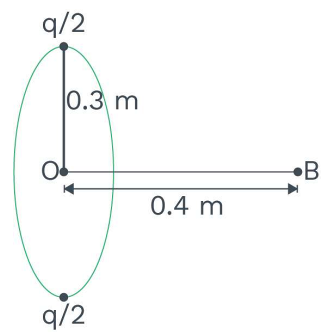


$$i = qf = q \times \frac{\omega}{2\pi} = 1 \times \frac{10^4 \pi}{2\pi} = 5 \times 10^3 \text{ A}$$

Now $B = \frac{\mu_0}{4\pi} \frac{2\pi i R^2}{(R^2 + x^2)^{3/2}}$

$$\therefore \quad B = 10^{-7} \times \frac{2\pi \times 5 \times 10^3 (0.6)^2}{[(0.6)^2 + (0.8)^2]^{3/2}} = 1.13 \times 10^{-3} \text{ T}$$

If half of the total charge is placed at the other end and the rod is rotated at the same frequency with the same current.



$$i' = \left(\frac{q}{2}\right) f + \left(\frac{q}{2}\right) f = qf = i = 5 \times 10^3 \text{ A}$$

In this condition, R' = 0.3 m and x' = 0.4 m

$\therefore$

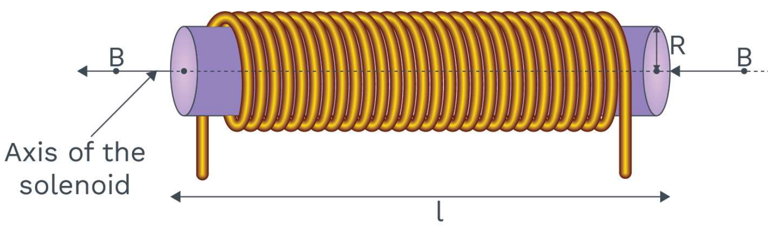
$$B' = 10^{-7} \times \frac{2\pi \times 5 \times 10^3 (0.3)^2}{[(0.3)^2 + (0.4)^2]^{3/2}} = 2.3 \times 10^{-3} \text{ T}$$

Solenoid :

(i) Solenoid has large number of circular loops wrapped around a non-conducting cylinder.

Definitions

A solenoid is a tight wound helical coil of wire which contains large number of circular loops wrapped around a non-conducting cylinder.

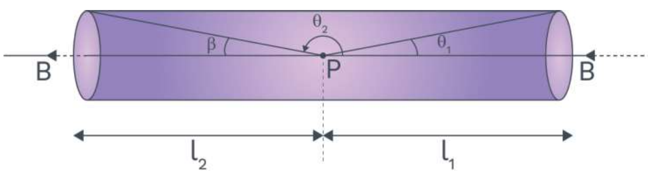


(ii) The winding of wire is uniform direction of the magnetic field is same at all points of the axis.

(iii) $\vec{B}$ on axis (turns should be very near to each other).

$$B = \frac{\mu_0 ni}{2}(\cos\theta_1 - \cos\theta_2) \quad \text{... (7)}$$

where n = number of turns per unit length.



$$\cos\theta_1 = \frac{l_1}{\sqrt{l_1^2 + R^2}}; \cos\beta = \frac{l_1}{\sqrt{l_2^2 + R^2}} = -\cos\theta_2$$

$$B = \frac{\mu_0 ni}{2}\left[\frac{l_1}{\sqrt{l_1^2 + R^2}} + \frac{l_2}{\sqrt{l_2^2 + R^2}}\right]$$

$$= \frac{\mu_0 ni}{2}(\cos\theta_1 + \cos\beta)$$

Derivation :-

Take an element dx at a distance 'x' from point 'P'. [point P is the point on axis at which we are going to find out magnetic field. Number of turns in the element is dn = ndx,

here n = number of turns per unit length]

Concept Reminder

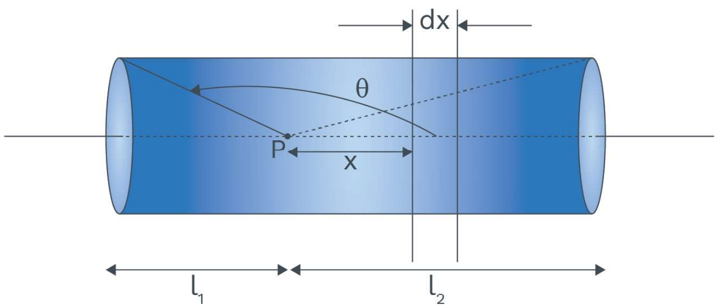
For an ideal solenoid, length should be very large compared to its radius.

Rack your Brain

A long solenoid carrying a current produces a magnetic field B along its axis. If the current is doubled and number of turns per cm is halved, then new value of magnetic field is

(1) B/2 (2) B

(3) 2B (4) 4B



$$dB = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}(ndx)$$

$$B = \int dB = \int_{-l_1}^{l_2} \frac{\mu_0 i R^2 ndx}{2(R^2 + x^2)^{3/2}}$$

$$= \frac{\mu_0 ni}{2}\left[\frac{l_1}{\sqrt{l_1^2 + R^2}} + \frac{l_2}{\sqrt{l_2^2 + R^2}}\right]$$

$$= \frac{\mu_0 ni}{2}[\cos\theta_1 - \cos\theta_2]$$

Concept Reminder

For an ideal solenoid, a reasonably uniform magnetic field is present in its central region. Magnitude of magnetic field is $\boxed{B = \mu_0 ni}$ in vacuum.

(iv) For 'Ideal Solenoid' :

*** Inside** (at the mid point)

$l >> R$ or length is infinite

$\theta_1 \to 0$

$\theta_2 \to \pi$

$$B = \frac{\mu_0 ni}{2}[1 - (-1)]$$

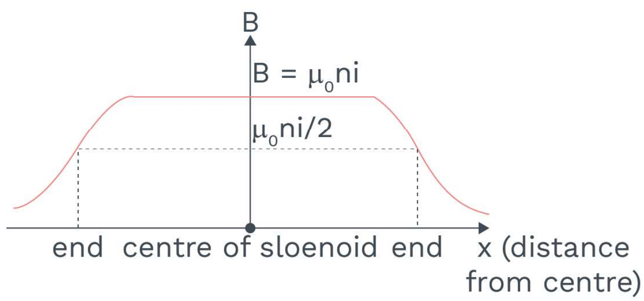
$$\boxed{B = \mu_0 ni}$$

If material of solid cylinder has relative permeability, 'μ_r' then $B = \mu_0 \mu_r ni$

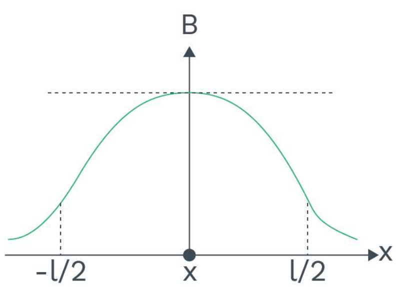
At the ends $\boxed{B = \frac{\mu_0 ni}{2}}$

(v) Comparison between ideal and real solenoid:

Ideal Solenoid



Real Solenoid



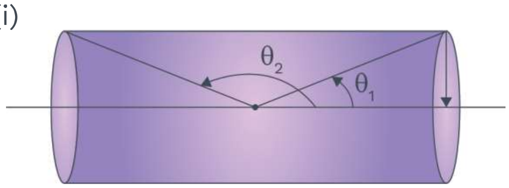
Ex. A solenoid of length 0.4 metre and diameter 0.6m consists of a single layer of 10^3 turns of fine wire carrying a current of 5×10^{-3} ampere. Find out the magnetic field on the axis at the middle and at the ends of the solenoid. (Gives $\mu_0 = 4\pi \times 10^{-7} \frac{V-s}{A-m}$)

Sol. $B = \frac{1}{2}\mu_0 ni[\cos\theta_1 - \cos\theta_2]$

$\Rightarrow \quad n = \frac{1000}{0.4} = 2500 \text{ per meter}$

$i = 5 \times 10^{-3} A$

(i)

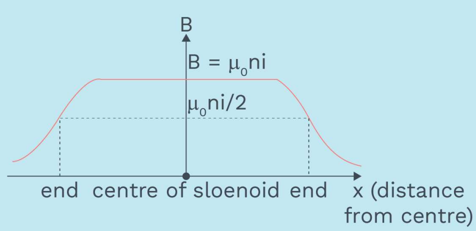


$$\cos\theta_1 = \frac{0.2}{\sqrt{(0.3)^2 + (0.2)^2}} = \frac{0.2}{\sqrt{0.13}}$$

Concept Reminder

Variation of magnetic field in an ideal solenoid

Ideal Solenoid



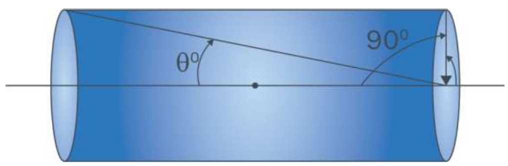
$$\cos\theta_2 = \frac{0.2}{\sqrt{0.13}}$$

$\Rightarrow$

$$B = \frac{1}{2} \times (4 \times \pi \times 10^{-7}) \times 2500 \times 5 \times 10^{-3} \times \frac{2 \times 0.2}{\sqrt{0.13}}$$

$$= \frac{\pi \times 10^{-5}}{\sqrt{13}} T$$

(ii)



At the end

$$\cos\theta_1 = \frac{0.4}{\sqrt{(0.3)^2 + (0.4)^2}} = 0.8$$

$$\cos\theta_2 = \cos 90° = 0$$

$$B = \frac{1}{2} \times (4 \times \pi \times 10^{-7}) \times 2500 \times 5 \times 10^{-3} \times 0.8$$

$$\Rightarrow B = 2\pi \times 10^{-6} Wb / m^2$$

Ampere's Law

"Ampere's circular law" is useful in finding the magnetic field induction due to electric currents under certain conditions of symmetry. Take a closed plane curve enclosing some current-carrying conductors. The line integral $\oint \vec{B}.\vec{dl}$ taken along this closed loop is equal to μ_0 times the total current crossing the area bounded by the close curve.

i.e., $\oint \vec{B}.\vec{dl} = \mu_0 i$ ... (8)

where i = algebraic sum of total current passing the area.

As a simple application of Ampere's law, we can obtain magnetic induction due to a long straight wire carrying current i.

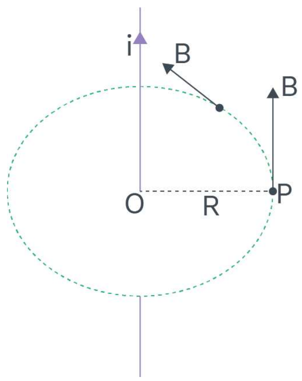
Definitions

Ampere's Law

The line integral $\oint \vec{B}.\vec{dl}$ taken along this closed curve is equal to μ_0 times the total current crossing the area bounded by the curve.

i.e., $\oint \vec{B}.\vec{dl} = \mu_0 i$

Suppose the magnetic magnetic field at point P at distance R from the wire is required.



Draw the circular loop through P with centre O and radius R as given in

The magnetic field $|\vec{B}|$ at all points of loop along this loop will be same and will be along to tangent on the circle, which is also the direction of the length element $d\vec{l}$.

So, $\oint \vec{B}.d\vec{l} = \oint Bdl = B\oint dl = B \times 2\pi R$

The current passing the circular area is i.

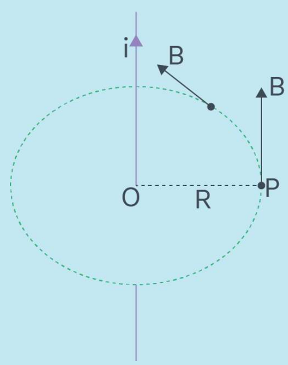
Thus, by Ampere's law, $B \times 2\pi R = \mu_0 i$

$$\Rightarrow \quad B = \frac{\mu_0 i}{2\pi R}$$

Note:-

- Line integral is independent of shape of path and position of wire with in it.
- The statement $\oint \vec{B}.\overrightarrow{dl} = 0$ does not necessarily mean that $\vec{B} = 0$ everywhere along the path but only that no net current is crossing through the path.
- **Sign of Current :-** The current due to which magnetic field $\vec{B}$ is produced in the same sense as (i.e. $\vec{B}.d\vec{l}$) positive will be taken +ve and the current which produces $\vec{B}$ in the sense opposite to $d\vec{l}$ will be –ve.

Concept Reminder



$$B = \frac{\mu_0 i}{2\pi R}$$

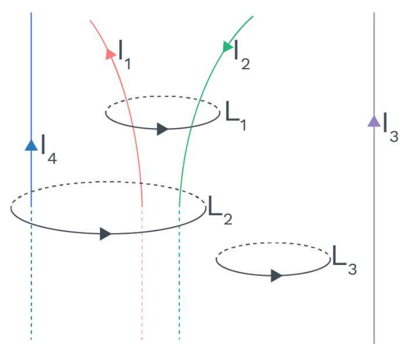
Concept Reminder

The statement $\oint \vec{B}.\overrightarrow{dl} = 0$ does not necessarily mean that $\vec{B} = 0$ everywhere along the path but it can also mean that no net current is passing through the path.

Ex. Find out the value of $\oint \vec{B}.\overrightarrow{dl}$ for the loops L_1, L_2, L_3 in the figure shown. The sense of $\overrightarrow{dl}$ is mentioned in the diagram.

Sol. For $L_1 \oint \vec{B}.\overrightarrow{dl} = \mu_0(I_1 - I_2)$

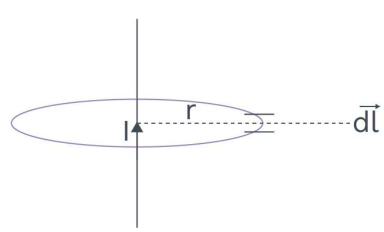
here I_1 is taken +ve because magnetic lines of force produced by I_1 is anti-clockwise as seen from top. I_2 produces lines of $\vec{B}$ in clockwise direction sense as seen from top. The sense of $\overrightarrow{dl}$ is anti-clockwise as seen from top.



$L_2 : \oint \vec{B}.\overrightarrow{dl} = \mu_0(I_1 - I_2 + I_4)$

for $L_3 : \oint \vec{B}.\overrightarrow{dl} = 0$

Magnetic field induction due to a infinite long straight current carrying wire:-



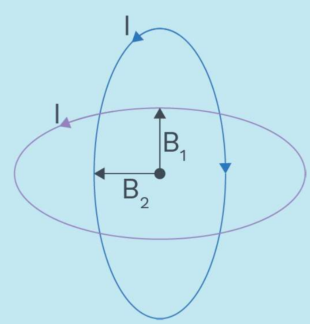
$\oint \vec{B}.\overrightarrow{dl} = \mu_0 I$

$\oint B.dl\cos\theta = \mu_0 I$

KEY POINTS

- Gravitation
- Newton's Law of gravitational

Concept Reminder



If $B_1 = B_2 = B$

$B_{Net} = \sqrt{B_1^2 + B_2^2} = \sqrt{2}\,B$

$\theta = 0^\circ \Rightarrow \cos\theta = 1$

$\oint B.dl = \mu_0 I$

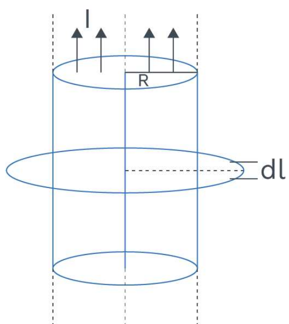
$\oint B dl = \mu_0 I$

$B(2\pi r) = \mu_0 I$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

Magnetic field due to a long current solid cylinder:-

(1) r >R(outside)



$\oint B dl = \mu_0 I$

$B.2\pi r = \mu_0 I$

$B = \frac{\mu_0 I}{2\pi r}$

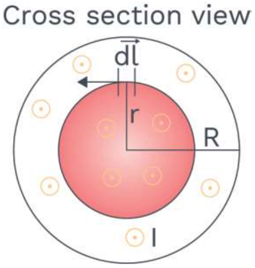
(2) r =R (surface)

$B = \frac{\mu_0 I}{2\pi r}$

(3) r < R (inside)

$\oint B.dl = \mu_0 I$

Cross section view



Rack your Brain

Magnetic field intensity at the centre of the coil of 50 turns, radius 0.5 m and carrying current of 2A, is

(1) 3×10^{-5} T

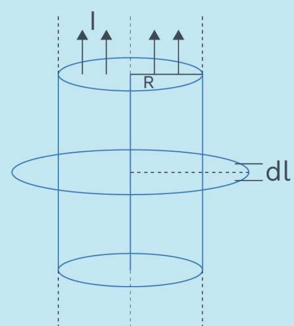
(2) 1.25×10^{-4} T

(3) 0.5×10^{-5} T

(4) 4×10^{-6} T

Concept Reminder

Magnetic field of a long solid cylinder



1. r > R(outside)

   $B = \frac{\mu_0 I}{2\pi r}$

2. r = R (surface)

   $B = \frac{\mu_0 I}{2\pi r}$

3. r < R (inside)

   $B_{in} = \frac{\mu_0 I r}{2\pi R^2}$

(4)

distance of point from axis of cylinder.

Radius of cylinder

Current density

- Magnetic field is maximum at surface of cylinder.

Concept Reminder

Variation of Magnetic field of a long solid cylinder with distance

Magnetic Field of a Long Hollow Cylinder

(1) r > b (outside)

$$B = \frac{\mu_0 I}{2\pi r}$$

(2) r = b (surface)

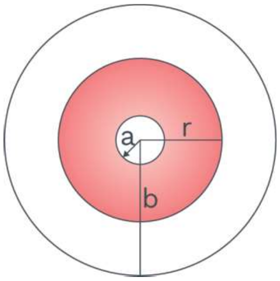
$$B = \frac{\mu_0 I}{2\pi b}$$

(3) $r \leq a$

I enclosed = 0

B = 0

(4) a < r < b



$$\pi b^2 - \pi a^2 \longrightarrow I$$

$$\longrightarrow \frac{I}{\pi b^2 - \pi a^2}$$

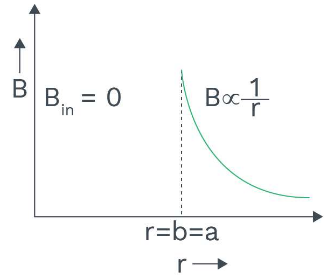
$$\pi r^2 - \pi a^2 \longrightarrow \frac{I}{b^2 - a^2}\left(r^2 - a^2\right) = I'$$

$$\oint B.dl = \mu_0 I'$$

$$B.2\pi r = \mu_0 \frac{I\left(r^2 - a^2\right)}{\left(b^2 - a^2\right)}$$

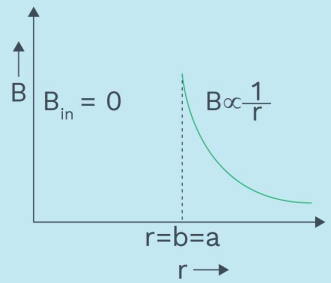
$$B = \frac{\mu_0 I}{2\pi r}\frac{I\left(r^2 - a^2\right)}{\left(b^2 - a^2\right)}$$

If inner radius is equal to outer radius (a = b)

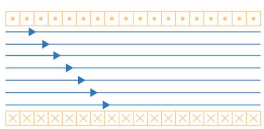


Concept Reminder

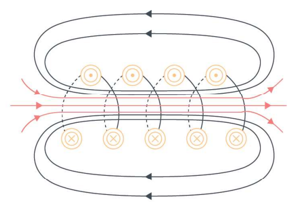
Variation of Magnetic field of a LONG HOLLOW cylinder with distance



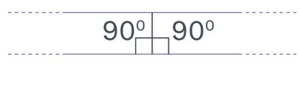
Magnetic field line of a long solenoid (Infinite solenoid):-



(Ideal)



(Real)



- [at centre of long solenoid]

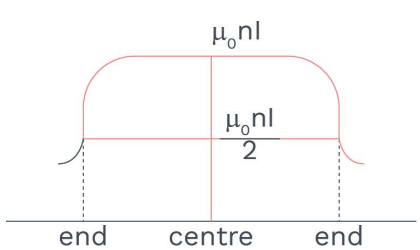
$$B = \frac{\mu_0 nI}{2}[\sin\theta_1 + \sin\theta_2]$$

$$= \frac{\mu_0 nI}{2}[\sin 90^\circ + \sin 90^\circ]$$

$$= \mu_0 nI$$

- [at end of long solenoid]

$$B = \frac{\mu_0 nI}{2}$$



- **Imp.:** For a long solenoid, Magnetic field at the end point of solenoid is ½ times of M.F at any internal point. And for all ideal long solenoid, no mutual effect considered.

Toroid

Toroid is an endless solenoid M.F is produce along the Axis of toroid outside the toroid MF is zero.

$$\left(n = \frac{N}{2\pi r}\right)$$

KEY POINTS

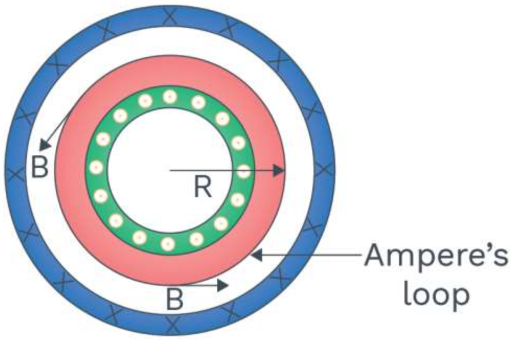
- Solenoid
- Toroid

Concept Reminder

For an ideal solenoid, Magnetic field at the end point of solenoid is ½ times of Magnetic field at any internal point

Definitions

A toroid is a solenoid bent to form a ring. Magnetic field outside the toroid is zero.



$$r \rightarrow \text{Mean Radius} = \frac{R_1 + R_2}{2}$$

Assume, we have to find the field at a point P inside the toroid.

Let the distance of P from the centre be r. Draw a circular loop through the point P and concentric with the toroid. By symmetry, the field will have same magnitude at all points of this circle. Also, the magnetic field is every where tangential to the circular loop. So,

$$\oint \vec{B}.d\vec{l} = \int B\,dl = B\int dl = 2\pi rB$$

If the total number of turns in toroid is N, the current passing the area enclosed by the circle is Ni where i is the current in the toroid. Using Ampere's law on this circle,

$$\oint \vec{B}.d\vec{l} = \mu_0 Ni$$

or $2\pi rB = \mu_0 Ni$

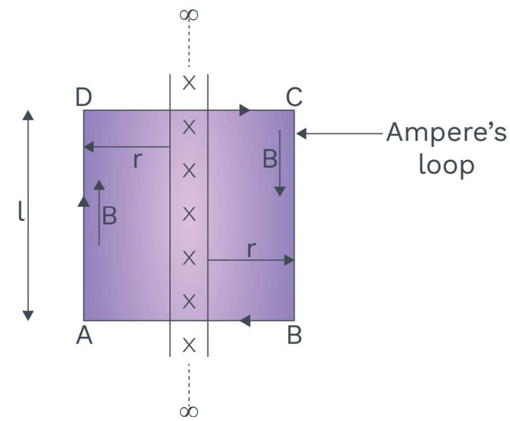
or $B = \frac{\mu_0 Ni}{2\pi r}$

Magnetic moment of Toroid is zero. It is not a Magnetic Dipole. It produces Non-Uniform Magnetic field.

Concept Reminder

Toroidal solenoid is a case in which magnetic field lines exist without polarity x. No apparent north or south pole exist.

Infinite Current Carrying sheet:-



Now from Ampere's loop $\oint B.dl = \mu_0 \lambda l$

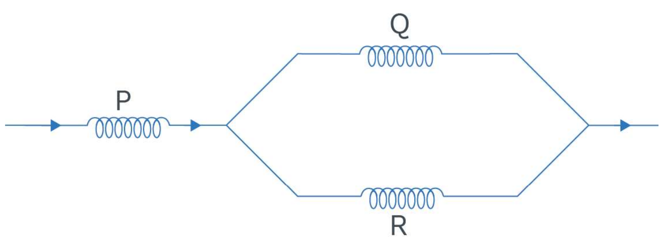
$$\int_{AB} B.dl + \int_{BC} B.dl + \int_{CD} B.dl + \int_{DA} B.dl = \mu_0 \lambda l$$

$$B\int dl + B\int dl\cos 90^\circ + B\int dl + B\int dl\cos 90^\circ = \mu_0 \lambda l$$

$$B = \frac{\mu_0 \lambda}{2}$$

Ex. 3 identical long solenoids P, Q and R are connected to each other as shown in figure. if the magnetic field at centre of P is 2 T, what would be the magnetic field at the centre of solenoid Q? Suppose that the magnetic field due to any solenoid is confined within the volume of that solenoid only.

Sol.



As solenoids are identical, the currents in Q and R will be the same and will be half of

KEY POINTS

- Atomic magnetism
- Magnetic moment
- Bohr's magneton

the current in P. The magnetic field induction in a solenoid is given by $B = \mu_0 ni$. Hence the

magnetic field in Q will be equal to the magnetic field in R and will be half of the field in P i.e., will be 1.0 T

Current And Magnetic Field Due To Circular Motion Of A Charge:-

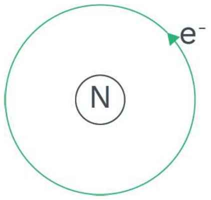
(i) According to the theory of atomic structure every atom is made of electrons, protons and neutrons. protons and neutrons (nucleon) are in the nucleus of each atom and electrons are assumed to be moving indifferent orbits around the nucleus.

(ii) An electron and a proton present in the atom constitute an electric dipole at every moment but the direction of this dipole changes continuously and hence at any time the average dipole moment is zero. As a result, static electric field is not observed.

(iii) Moving charge produces magnetic field and the average value of this field in the atom is not zero.

(iv) **Atomic Magnetism**

When electron moves around nucleus in any orbit then a current is induced in this orbit.

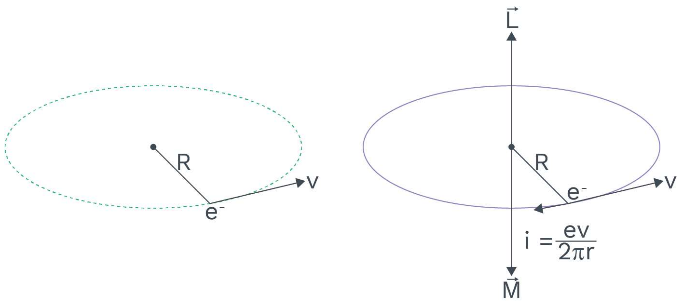


In an atom an electron moving in a circular path around the nucleus. Due to this motion current appears to be flowing in the electronic orbit and the orbit behaves like a current carrying coil. If e is the electron

Concept Reminder

The magnetic dipole moment of a current loop is independent of its shape. It depends on area of the loop. The loop can be circular, square or rectangle.

charge, R is the radius of the orbit and f is the frequency of motion of electron in the orbit, then



(1) $I = \frac{e}{T} = ef = \frac{ev}{2\pi r}$

(2) $B_{in} = \frac{\mu_0 I_{in}}{2r} = \frac{\mu_0 ev}{4\pi r^2}$

(3) $M = IA$

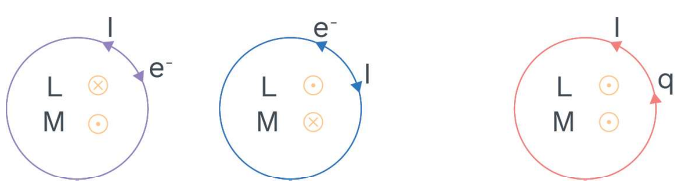
$M = \frac{ev}{2\pi r}\pi r^2$

$M = \frac{evr}{2}$

(4) Relation between L & M

$M = \frac{evr}{2} \times \frac{m}{m}$ $(L = mvr)$

$M = \frac{eL}{2m}$ $\left(\frac{m}{L} = \frac{q}{2m} \rightarrow \text{Gyromagnetic ratio}\right)$



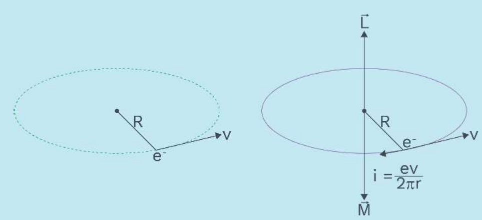
$\vec{M} = \frac{q\vec{L}}{2m} \rightarrow$ for all charge

For a negative charge which is moving in circular path L and M are anti-parallel and for a positive charge L and M are parallel.

(5) Acc. Bohr's Theory

Concept Reminder

In case of e⁻ orbiting around nucleus, angular momentum and magnetic moment are in opposite direction.



Concept Reminder

Magnetic moment

$= \frac{q}{2m} \times (\text{angular momentum})$

$L = \frac{nh}{2\pi}$ $\quad\quad x = 1, 2, 3, ...$

$M = \frac{eL}{2m} = \frac{e}{2m}\frac{nh}{2\pi}$

$M = \frac{neh}{4\pi m}$

$n = 1$

Bohr magneto $M = \mu_B = 0.923 \times 10^{-23}\, Am^2$

Magnetic moment of e^- which is moving in first orbit of H-atom

(v) If a charge q (or a charged ring of charge q) is moving in a circular path of radius 'R' with a frequency f or angular velocity ω, then

(a) current due to moving charge

$i = qf = q\omega / 2\pi$

(b) magnetic field at the centre of ring

$B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 qf}{2R}$ or $B_0 = \frac{\mu_0 q\omega}{4\pi R}$

(c) magnetic moment

$M = i(\pi R^2) = qf\pi R^2 = \frac{1}{2}q\omega R^2$

(vi) If a charge q is distributed uniformly over the surface of plastic disc of radius R and it is rotated about its axis with an angular velocity ω, then

(a) the magnetic field produced at its centre will be

$B_0 = \frac{\mu_0 q\omega}{2\pi R}$

(b) the magnetic moment of the disc will be

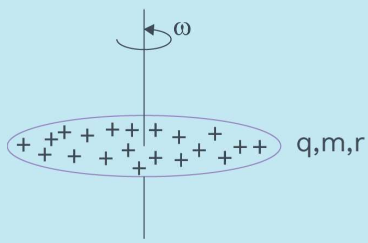
$dM = (di)\,\pi x^2$

$= \frac{\omega}{2\pi} dq\, \pi x^2 = \frac{\omega q}{R^2} x^3 dx$

$\Rightarrow \quad M = \int dM = \frac{\omega q}{R^2}\int_0^R x^3 dx$

Concept Reminder

For rotating charged disc, magnetic moment is equal to $M = \frac{qr^2\omega}{4}$



Concept Reminder

$\mu_B = \frac{eh}{4\pi m}$ is known as Bohr's magneton.

$$\Rightarrow \quad M = \frac{q\omega R^2}{4}$$

$$\Rightarrow \quad M = \frac{q\omega R^2}{4}$$

Note: For rigid bodies

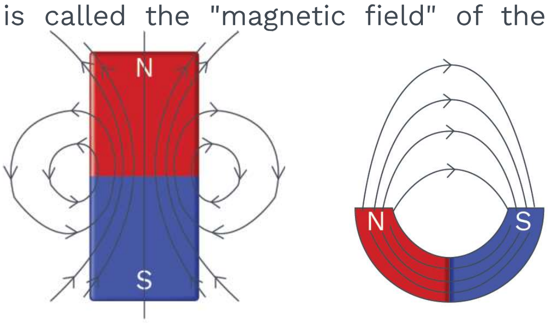
$$L = I\omega$$

which I = moment of inertia

So, $M = \frac{eL}{2m} \Rightarrow M = \frac{qI\omega}{2m}$

Magnetic Lines And Their Characteristics:-

The area surrounded by a magnet or magnetic configuration in which its effects are perceptible is called the "magnetic field" of the magnet.



Bar magnet (A) U-Shape magnet (B)

In order to visualise a magnetic field graphically, "Michael Faraday" introduced the idea of lines. According to "Michael Faraday" a line in magnetic field on which drawn tangent represent direction of magnetic field at that point.

For magnetic field it is worth noting that :

(i) Outside a magnet, field lines are from north to to south pole while inside from south to north, it means that magnetic lines are closed curves i.e., they appear to converge or diverge at poles.

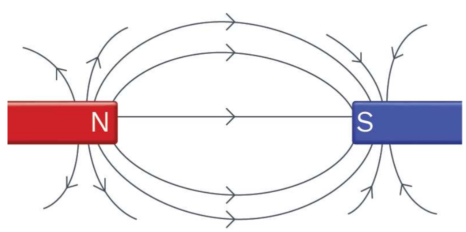
(ii) The number of magnetic field lines of field originating or terminating on a pole is

Definitions

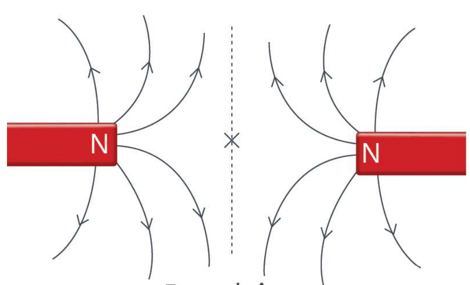
The space surrounding a magnet or magnetic configuration in which its effects are perceptible is called the magnetic field of the given magnet or magnetic configuration proportional to its strength. μ_0 lines are assumed to be associated with a unit magnetic pole. So, if an object encloses a pole of strength strength 'm', total lines linked with the body (called magnetic flux) will be μ_0 (m).

(iii) Magnetic field lines of field can never intersect each other because if they intersect at a point, field intensity at that point will have 2 directions which is absurd.

(iv) Magnetic field lines of field have a tendency to contract longitudinally like a stretched elastic
string (producing attraction force between opposite poles) and repel each other laterally (resulting in repulsion between similar poles)



Attraction
(A)



Repulsion
(B)

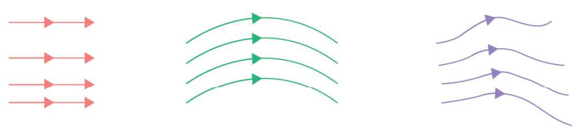
(v) Number of field lines of field per unit area, perpendicular to the area at a point, represents the magnitude of field at that

Concept Reminder

Magnetic lines of field can never intersect each other because if they intersect at a point, intensity at that point will have two directions which is absurd.

point. so crowded lines represent a strong magnetic field while distant lines represent weak field. Further, if the field lines of force are equidistant and straight the field is uniform otherwise not.

Non uniform fields

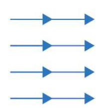


Magnitude is not constant (A)

Direction is not constant (B)

Both magnitude and direction are not constant (C)

Uniform fields



Both magnitude and direction are constant (D)

(vi) In the region of space where there is no field, there will be no field lines. That's why, at a neutral point, where resultant magnetic field is zero there cannot be any line of field.

(vii) Magnetic lines of field can originate from or enter on the surface of the magnetic material at any angle.

(viii) Magnetic field lines are exist inside every magnetised material

(ix) As monopoles do not exist, so the total magnetic flux linked with a closed surface is always zero, i.e.,

$$\oint \vec{B}.\vec{ds} = \mu_0(0) = 0$$

This law is called "Gauss's law for magnetism".

KEY POINTS

- Magnetic field lines

Concept Reminder

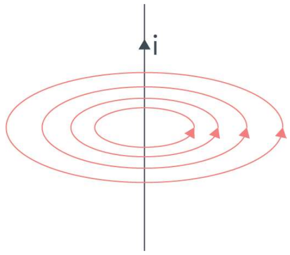
As monopoles do not exist, the total magnetic flux linked with a closed surface is always zero, i.e.,

$$\oint \vec{B}.\vec{ds} = \mu_0(0) = 0$$

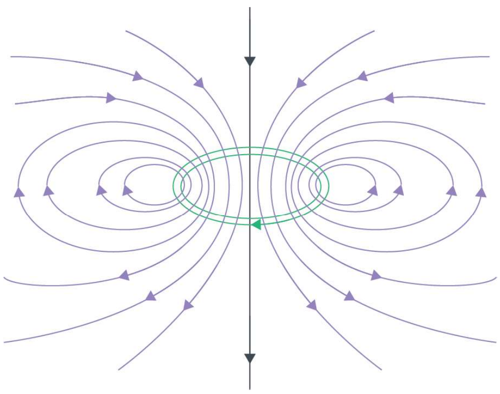
This law is called Gauss's law for magnetism.

Magnetic Field Line Due To Some Important Structure:-

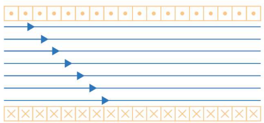
1. Straight Current Carrying Wire



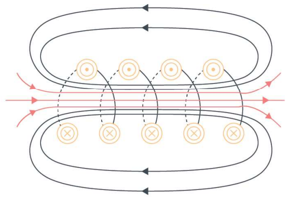
2. Circular coil



3. Solenoid



(Ideal)



(Real)

Magnetic Force On Moving Charge:-

When an electric charge q moves with velocity $\vec{v}$, in a magnetic field $\vec{B}$, then magnetic force experienced by the moving charge is given by following formula:

$\vec{F} = q(\vec{v} \times \vec{B})$ (Put q with sign). ... (9)

$\vec{v}$ = Instantaneous velocity

$\vec{B}$ = Magnetic field

Rack your Brain

Ionized hydrogen atoms and α-particles with same moment a enters perpendicular to a constant magnetic field B. The ratio of their radii of their path $r_H : r_\alpha$ will be-

(1) 1 : 4 (2) 1 : 2

(3) 2 : 1 (4) 4 : 1

DIFFERENCE BETWEEN MAGNETIC FORCE AND ELECTRIC FORCE:-

(1) Magnetic force is always normal to the field while electric force is collinear with the magnetic field.

(2) Magnetic force is dependent on velocity, i.e., acts only when the charged particle is in the motion while electric force (qE) is independent of the state of rest or motion of charge particle.

(3) Work done by Magnetic force is zero, when

Note :

- $\vec{F} \perp \vec{v}$ and also $\vec{F} \perp \vec{B}$
- $\therefore \vec{F} \perp \vec{v} \therefore$ power due to magnetic force, power on a charged particle is zero. (Use the formula of power $P = \vec{F}.\vec{v}$ for its proof)
- Since the $\vec{F} \perp \vec{B}$ so work done by the magnetic force is zero in every part of the motion. The magnetic force cannot decrease or increase kinetic energy of the charged particle. It will only change the direction of velocity.
- On a stationary charged particle, the value of magnetic force is '0'.
- If $\vec{v} \parallel \vec{B}$, then also magnetic force on the charged particle is zero. It moves along the straight line if magnetic field is acting only.

Ex. The charged particle of mass 5 mg and charge $q = +2\mu C$ has velocity $\vec{v} = 2\hat{i} - 3\hat{j} + 4\hat{k}$.

Find out the magnetic force on charged particle and its acceleration at this instant due to the magnetic field $\vec{B} = 3\hat{j} - 2\hat{k}$. Where $\vec{v}$ and $\vec{B}$ are in m/s and Wb/m^2 respectively.

Sol. $\vec{F} = q\vec{v} \times \vec{B} = 2 \times 10^{-6}(2\hat{i} - 3\hat{j} + 4\hat{k}) \times (3\hat{j} - 2\hat{k})$

$= 2 \times 10^{-6}[-6\hat{i} + 4\hat{j} + 6\hat{k}]N$

By Newton's Law

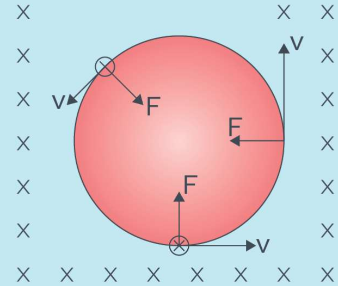
$$\vec{a} = \frac{\vec{F}}{m} = \frac{2 \times 10^{-6}}{5 \times 10^{-6}}(-6\hat{i} + 4\hat{j} + 6\hat{k})$$

Concept Reminder

When $\theta = 0°$ or 180° i.e., charged particle moves parallel or anti-parallel with field then magnetic force is zero. In this case charged particle moves in a straight line.

Concept Reminder

When $\theta = 90°$



(i) Radius of path $r = \frac{mv}{qB}$

(ii) Time period $T = \frac{2\pi m}{qB}$

$= 0.8(-3\hat{i} + 2\hat{j} + 3\hat{k})\, m/s^2$

Ex. A charged particle has acceleration $\vec{a} = 2\hat{i} + x\hat{j}$ in the magnetic field $\vec{B} = -3\hat{i} + 2\hat{j} - 4\hat{k}$. Find the value of x.

Sol. $\because \quad \vec{F} \perp \vec{B}$

$\therefore \quad \vec{a} \perp \vec{B}$

$\therefore \quad \vec{a}.\vec{B} = 0$

$\therefore \quad (2\hat{i} + x\hat{j}).(-3\hat{i} + 2\hat{j} - 4\hat{k}) = 0$

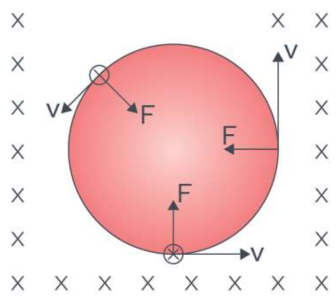
$\Rightarrow \quad -6 + 2x = 0 \qquad \Rightarrow x = 3$.

Motion Of the Charged Particle In the Uniform Magnetic Field.

Case.1 WHEN THE VELOCITY OF CHARGED PARTICLE IS PERPENDICULAR TO THE MAGNETIC FIELD

Let a particle of charged q and mass 'm' is moving with the velocity 'v' and enters at right angles to the uniform magnetic field '$\vec{B}$' as shown in the figure.

The force on the particle is 'qvB' and this force will always act in the direction perpendicular to 'v'. Hence, the particle will move on the circular path. If the radius of the path is 'r' then



$$\frac{mv^2}{r} = Bqv \text{ or } r = \frac{mv}{qB} \qquad \ldots (10)$$

Thus, radius of path is proportional to the momentum mv of the particle and inversely proportional to the magnitude of the magnetic field.

Time period : Time period is the time taken by the particle to complete one rotation of circular path which is given by,

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB} \qquad \ldots (11)$$

The time period is independent of speed 'v'.

The frequency is the number of revolution of charged particle in one second, which is given by,

$$\nu = \frac{1}{T} = \frac{qB}{2\pi m} \quad \text{... (12)}$$

and angular frequency = $\omega = 2\pi\nu$

Ex. A proton (p), α -particle and deuteron (D) are moving in the circular paths with same kinetic energies in the same magnetic field. Find ratio of their radii and time periods. (Neglect interaction between particles).

Sol. $R = \frac{\sqrt{2mk}}{qB}$

$$\therefore \quad R_p : R_\alpha : R_D = \frac{\sqrt{2mk}}{qB} : \frac{\sqrt{2.4mk}}{2qB} : \frac{\sqrt{2.2mk}}{qB}$$

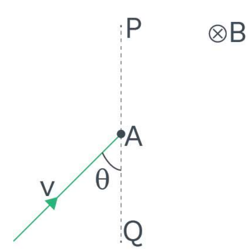
$$= 1 : 1 : \sqrt{2}$$

$$\therefore \quad T = 2\pi m / qB$$

$$\therefore \quad T_p : T_\alpha : T_D = \frac{2\pi m}{qB} : \frac{2\pi 4m}{2qB} : \frac{2\pi 2m}{qB}$$

$$= 1 : 2 : 2$$

Ex. A positive charge particle of charge 'q', mass 'm' enters into a uniform magnetic field with velocity 'v' as shown in figure. There is no magnetic field to the left of 'PQ'.



Find

(i) time spent,

(ii) distance travelled in magnetic field

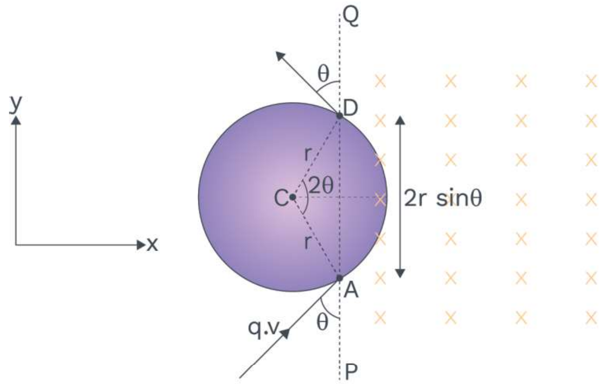
(iii) impulse of the magnetic force.

Sol. Particle will move in the field as shown. Angle subtended by the arc at the centre = 2θ

(i) Time spent by charge in magnetic field

$$\omega t = \theta \Rightarrow \frac{qB}{m} t = \theta \Rightarrow t = \frac{m\theta}{qB}$$

(ii) Distance travelled by charge particle in magnetic field:



$= r(2\theta) = \frac{mv}{qB}.2\theta$

(iii) Impulse = change in momentum of the charge

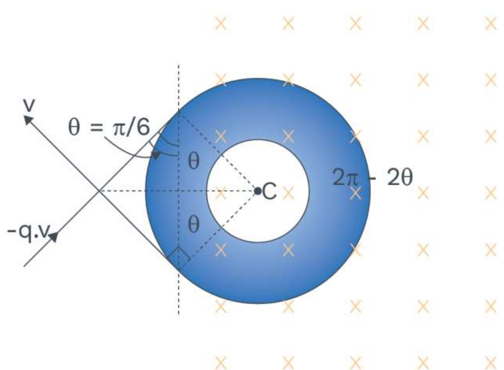
$= (-mv\sin\theta\hat{i} + mv\cos\theta\hat{j})$

$-(mv\sin\theta\hat{i} + mv\cos\theta\hat{j}) = -2mv\sin\theta\hat{i}$

Ex. Repeat above question if the charge is –ve and the angle made by the boundary with the velocity is $\frac{\pi}{6}$.

Sol. **(i)** $2\pi - 2\theta = 2\pi - 2.\frac{\pi}{6} = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$= \omega t = \frac{qBt}{m} \Rightarrow t = \frac{5\pi m}{3qB}$



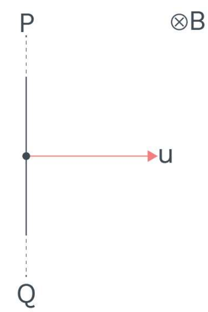
(ii) Distance travelled $s = r(2\pi - 2\theta) = \frac{5\pi r}{3}$

(iii) Impulse = charge in linear momentum

$$= m(-v\sin\theta\hat{i} + v\cos\theta\hat{j}) - m(v\sin\theta\hat{i} + v\cos\theta\hat{j})$$

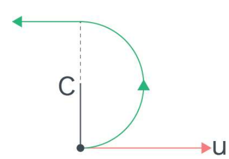
$$= -2mv\sin\theta\,\hat{i} = -2mv\sin\frac{\pi}{6}\hat{i} = -mv\,\hat{i}$$

Ex. In the figure shown the magnetic field on the left on 'PQ' is zero and on the right of 'PQ' it is uniform.

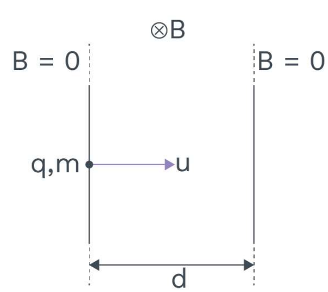


Find the time spent in the magnetic field.

Sol. The path will be semi-circular time spent $= T/2 = \pi m / qB$

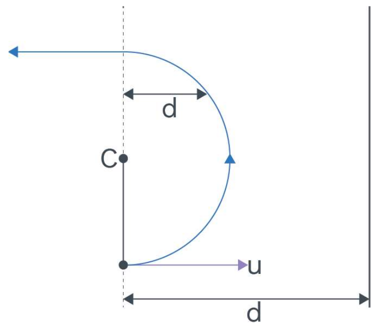


Ex. A uniform magnetic field of strength 'B' exists in a region of width 'd'. A particle of charge 'q' and mass 'm' is shot perpendicularly (as shown in the figure)into the magnetic field. Find the time spend by the particle in the magnetic field if



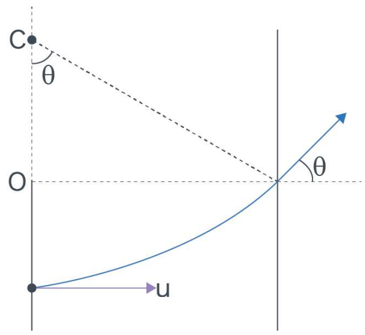
(i) $d = \frac{mu}{qB}$ **(ii)** $d < \frac{mu}{qB}$

Sol.



(i) $d > \frac{mu}{qB}$ means $d > R \therefore\ t = \frac{T}{2} = \frac{\pi m}{qB}$

(ii) $\sin\theta = \frac{d}{R}$

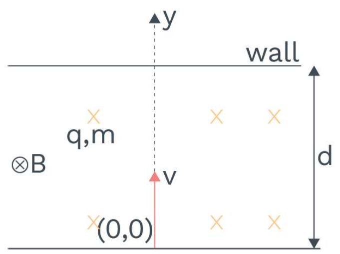


$$\theta = \sin^{-1}\left(\frac{d}{R}\right)$$

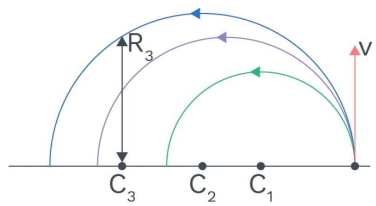
$$\omega t = \theta$$

$$\Rightarrow \quad t = \frac{m}{qB}\sin^{-1}\left(\frac{d}{R}\right)$$

Ex. What should be the speed of charged particle so that it can't collide with the upper wall? Also find the coordinate of the point where the particle strikes the lower plate in the limiting case of velocity.



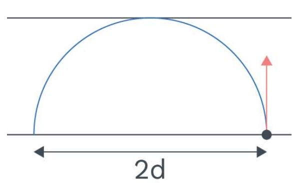
Sol. (i) The path of the particle will be circular larger the velocity, larger will be the radius. For particle not to s strike R < d



$\therefore \quad \frac{mv}{qB} < d$

$\Rightarrow \quad v < \frac{qBd}{m}$

(ii) for limiting case $v = \frac{qBd}{m}$



$R = d$

$\therefore$ coordinate = (–2d, 0, 0)

Case.2 WHEN THE CHARGED PARTICLE IS MOVING AT AN ANGLE TO THE FIELD

In this case the charged particle having charge q and mass m is moving with velocity v and it enter the magnetic field B at angle θ as shown in figure. Velocity can be resolved in two components, one along magnetic field and the other perpendicular to it. Let these components are $v_{||}$ and $v_{\perp}$

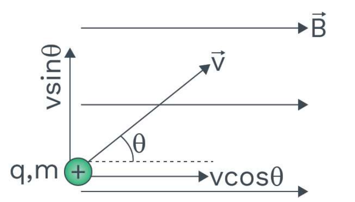
Concept Reminder

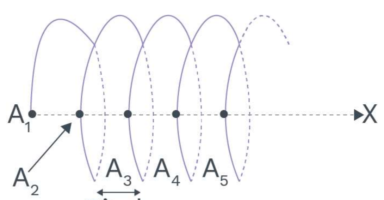
Path of charged particle in a magnetic field can be straight line, circular or helical depending upon angle between velocity and magnetic field.

$v_{||} = v\cos\theta$

and $v_{\perp} = v\sin\theta$

The parallel component $v_{||}$ of velocity remains unchanged as it is parallel to $\vec{B}$. Due to the $v_{\perp}$ particle will move on a circular path. So the resultant path will be combination of straight line motion and circular motion, which will be helical as shown in figure.





The radius of path is (r) $\dfrac{mv_{\perp}}{qB} = \dfrac{mv\sin\theta}{qB}$ ... (13)

Time period (T) = $\dfrac{2\pi r}{v_{\perp}} = \dfrac{2\pi mv\sin\theta}{v\sin\theta qB} = \dfrac{2\pi m}{qB}$ ... (14)

Frequency (f) = $\dfrac{Bq}{2\pi m}$ ... (15)

Pitch : Pitch of helix described by charged particle is defined as the distance moved by the centre of circular path in the time in which particle completes one revolution.

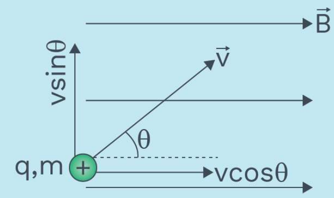
Pitch = distance $A_1A_2 = A_3A_4 = \ldots\ldots\ldots\ldots = v\cos\theta.T$

$v_{||}.T = v\cos\theta\dfrac{2\pi m}{Bq} = \dfrac{2\pi mv\cos\theta}{qB}$ .... (16)

If the velocity of the charge is not perpendicular to the magnetic field, we can break the velocity in two components $v_{||}$, parallel to the field and

Concept Reminder

In helical motion



(i) Radius of path $r = \dfrac{mv\sin\theta}{qB}$

(ii) Time period $T = \dfrac{2\pi m}{qB}$

(iii) Pitch $P = (v\cos\theta) = \dfrac{2\pi m}{qB}$

$v_{\perp}$, perpendicular to the field. The components $v_{||}$ remains unchanged as the force $q\vec{v} \times \vec{B}$ is perpendicular to it. In the plane perpendicular to the field, the particle traces a circle of radius $r = \frac{mv_{\perp}}{qB}$ as given by equation. The resultant path is helix.

Ex. A beam of protons with a velocity 4 × 10^5 m/s enters a uniform magnetic field of 0.3 Tat an angle 60° to the magnetic field. Find the radius of the helical path taken by the proton beam. Also find the pitch of the helix m_p = 1.67 × 10^{-27} kg

Sol. Radius of the helical path taken by the proton beam $r = \frac{m(v\sin\theta)}{qB} = 1.2\,\text{cm}$

Time period $T = \frac{2\pi r}{v\sin\theta} = 2.175 \times 10^{-7}\,\text{s}$

$\therefore$ pitch of the helix $p = v\cos\theta.T$

$\Rightarrow$ $p = 4 \times 10^5 \times \frac{1}{2} \times 2.175 \times 10^{-7} = 4.35\,\text{cm}$

Motion of A charged Particle In Combined Electric And Magnetic Field

When the moving charged particle is subjected simultaneously to both electric field $\vec{E}$ and magnetic field $\vec{B}$, the moving charged particle will experience electric force $\overrightarrow{F_e} = q\vec{E}$ and magnetic force $\overrightarrow{F_m} = q(\vec{v} \times \vec{B})$

$$\vec{F} = q[\vec{E} + \vec{v} \times \vec{B}] \qquad \text{... (17)}$$

which is **'Lorentz force equation'**. Now consider two special cases involving the application of above equation

Note :

- Magnetic force is frame dependent, Electric force is frame dependent but Lorentz force is frame

Concept Reminder

Lorentz force

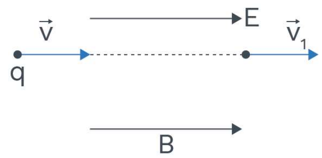
$\vec{F} = q[\vec{E} + \vec{v} \times \vec{B}]$

independent.

Case-I: When $\vec{v}, \vec{E}$ and $\vec{B}$ all the three are collinear

In this situation as the particle is moving parallel or anti-parallel to the field, the magnetic force on it will be zero and only electric force will act, so

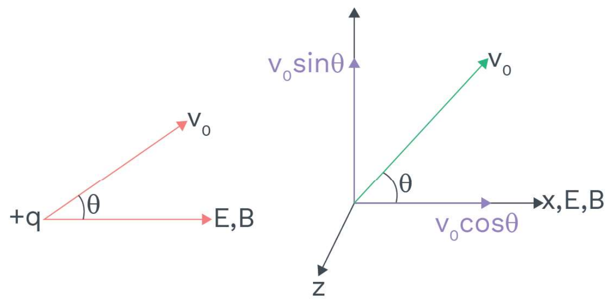
$$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$$

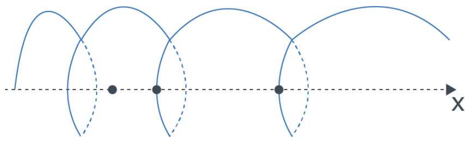


Hence the particle will pass through the field following a straight-line path (parallel to the field) with change in its speed. So, in this situation speed, velocity, momentum and kinetic energy all will change without change in direction of motion as shown in the figure.

$\vec{v}, \vec{E}$ and $\vec{B}$ are collinear.

Case-II: $\vec{E} \parallel \vec{B}$ and uniform $\theta \neq 0$ ($\vec{E}$ and $\vec{B}$ are constant and uniform)





along X axis: $F_x = qE, a_x = \frac{qE}{m}, v_x = v_0 \cos\theta + a_x t$,

$$x = v_0 \cos\theta t + \frac{1}{2} a_x t^2$$

Rack your Brain

Find the magnitude and direction of magnetic force acting on a charged particle of charge −2μC in a magnetic field of 2T acting in y-direction, when the particle velocity is $\left(2\hat{i} + 3\hat{j}\right) \times 10^6 ms^{-1}$.

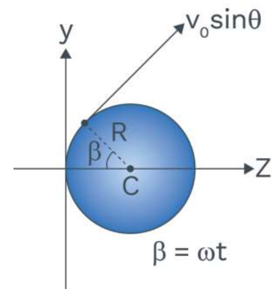
in y-z plane:

$$qv_0 \sin\theta B = m(v_0 \sin\theta)^2 / R$$

$$\Rightarrow R = \frac{mv_0 \sin\theta}{qB}$$

$$\omega = \frac{v_0 \sin\theta}{R} = \frac{qB}{m}$$

$$= \frac{2\pi}{T} = 2\pi f$$



$$\vec{v} = \vec{r}\{(v_0 \cos\theta)t + \frac{1}{2}\frac{qE}{m}t^2\}\hat{i} + \sin\omega t\hat{j} + (R - R\cos\omega t)(-\hat{k})$$

$$\vec{v} = \left(v_0 \cos\theta + \frac{qE}{m}t\right)\hat{i} + (v_0 \sin\theta)\cos\omega t\hat{j}$$

$$+v_0 \sin\theta \sin\omega t - \hat{k})$$

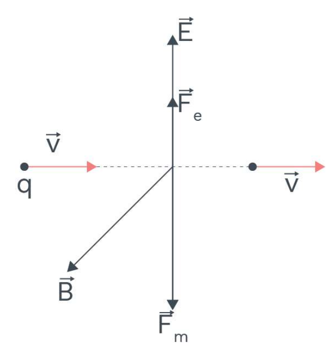
$$\vec{a} = \frac{qE}{m}\hat{i} + \omega^2 R[-\sin\beta\,\hat{j} - \cos\beta\,\hat{k}]$$

Case-III: $\vec{v}, \vec{E}$ **and** $\vec{B}$ are mutually perpendicular

$\vec{v}, \vec{E}$ and $\vec{B}$ are mutually perpendicular. In case situation of $\vec{E}$ and $\vec{B}$ are such that

$$\vec{F} = \vec{F}_e + \vec{F}_m = 0$$

or $\vec{a} = \left(\frac{\vec{F}}{m}\right) = 0$, then the particle will pass through the field with the same velocity. In this situation,



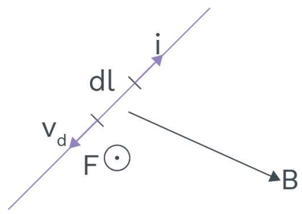
$$F_e = F_m \text{ or } qE = qvB$$

Or $v = \frac{E}{B}$

This principle is used in velocity-selector to get a charged beam having a specific velocity.

Magnetic Force On A Current Carrying Wire:

Suppose a conducting wire, carrying a current i, is placed in a magnetic field $\vec{B}$. Consider a small element dl of the wire (figure). The free electrons drift



with a speed v_d opposite to the direction of the current. The relation between the current i and the drift speed v_d is

$$i = jA = nev_dA$$

Here A is the area of cross-section of the wire and n is the number of free electrons per unit volume. Each electron experiences an average (why average?) magnetic force

$$\vec{f} = -e\vec{v}_d \times \vec{B}$$

The number of free electrons in the small element considered in nAdl . Thus, the magnetic force on the wire of length dl is

$$d\vec{F} = (nAdl)(-e\vec{v}_d \times \vec{B})$$

If we denote the length dl along the direction of the current by $d\vec{l}$, the above equation becomes

$$d\vec{F} = nAev_d d\vec{l} \times \vec{B}$$

Using (i), $d\vec{F} = id\vec{l} \times \vec{B}$

The quantity $id\vec{l}$ is called a current element

$$\vec{F}_{res} = \int d\vec{F} = \int id\vec{l} \times \vec{B} = i\int d\vec{l} \times \vec{B}$$

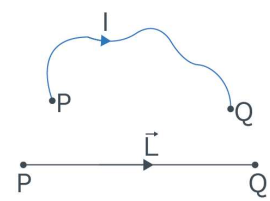
($\because$ i is same at all points of the wire)

If $\vec{B}$ is uniform, then $\vec{F}_{res} = i\left(\int d\vec{l}\right) \times \vec{B}$

$$\vec{F}_{res} = i\vec{L} \times \vec{B} \qquad \text{... (18)}$$

Concept Reminder

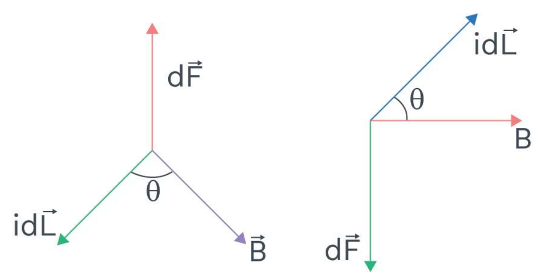
- Force on current carrying wire is $F = iBdl$
- This formula holds for straight conductor of length l placed in a uniform magnetic field $\vec{B}$.



Here $\vec{L} = \int d\vec{l}$ = vector length of the wire = vector connecting the end points of the wire.

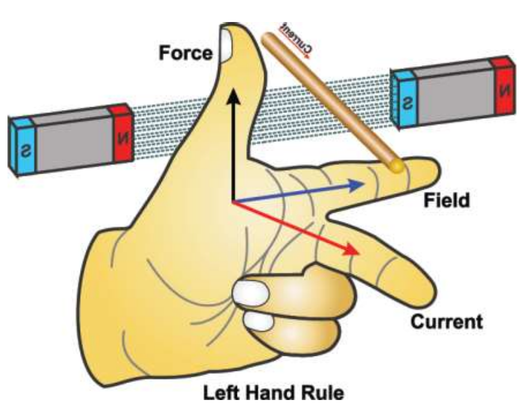
Note:-If a current loop of any shape is placed in a uniform $\vec{B}$ then $\left(\vec{F}_{res}\right)_{magnetic}$ on it = 0 $(\because \vec{L} = 0)$

Direction of force: The direction of force is always perpendicular to the plane containing $i d\vec{L}$ and $\vec{B}$ is same as that of cross-product of two vectors $(\vec{a} \times \vec{b})$ with $\vec{a} = i d\vec{L}$ and $\vec{b} = \vec{B}$



The direction of force when current element $i d\vec{L}$ and $\vec{B}$ are perpendicular to each other can also be determined by applying either of the following rules.

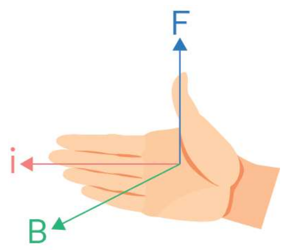
(a) Fleming's Left-hand Rule : Stretch forefinger, central finger and thumb of left hand mutually perpendicular. If the fore-finger points in direction of the field '$\vec{B}$' and central in direction of current i, the thumb will point in direction of the force.



Concept Reminder

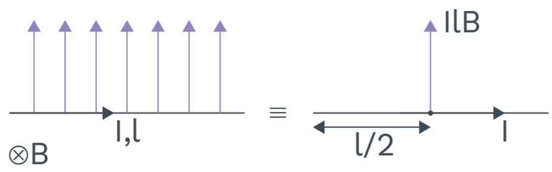
Force on a closed loop (of any shape) placed in a uniform magnetic field is zero because $l_{eff} = 0$

(b) Right-hand Palm rule : Stretch your fingers and thumb of right-hand at the right angles to each other. If the fingers point in direction of current i, and the palm in the direction of field $\vec{B}$ then thumb will point in direction of 'Force'.



Point Of Application of Magnetic Force

On a straight current carrying wire magnetic force in a uniform magnetic field can be assumed to be acting at its mid point.



This can be used for the calculation of torque.

Force Between The Two Long Straight Parallel Current Carrying Conductors

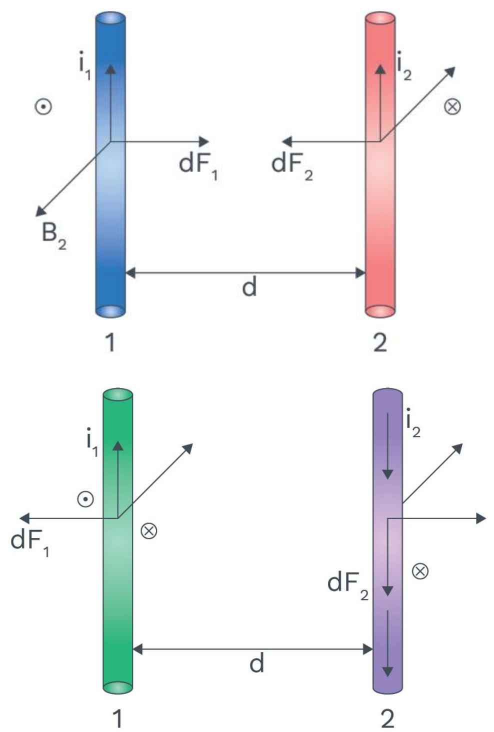
Let us consider Two very long parallel straight wires carrying the currents i_1 and i_2. Each wire is placed in the region of the magnetic induction of other and hence will experience the force. The net force on the current-carrying conductor due to its own field is '0'. So, if there are two long parallel current-carrying wires '1' and '2' (as shown below), the wire '1' will be in the field of wire '2' and vice versa.

Concept Reminder

Parallel current carrying conductors attract each other while anti-parallel current carrying conductors repel each other.

Concept Reminder

Force per unit length acting on both conductor is $\frac{\mu_0 i_1 i_2}{2\pi d}$



> **Concept Reminder**
>
> If wires were able to move freely, they would gain kinetic energy at cost of electric field not by magnetic field. Work done by magnetic field is still zero.

The force on dl_1 length of wire-2 due to field of wire-1, $dF_2 = i_2\, dL_2\, B_1$

$$dF_2 = \frac{\mu_0}{4\pi}\frac{2i_1i_2}{d} \qquad \left[\because B_1 = \frac{\mu_0}{4\pi}\frac{2i_1}{d}\right]$$

or $$\frac{dF_2}{dL_2} = \frac{\mu_0}{4\pi}\frac{2i_1i_2}{d} \qquad \ldots (19)$$

It will be true for wire '1' in the field of wire '2'. The direction of the force in accordance with right-hand screw rule will be as shown above. So, force per unit length in the case of two parallel current-carrying wire separated by a distance 'd' is

$$\boxed{\frac{dF}{dL} = \frac{\mu_0}{4\pi}\frac{2i_1i_2}{d}}$$

If 'i_1' and 'i_2' are along the same direction, forces between the wires is attractive in nature and if i_1 and i_2 are oppositely directed, force is repulsive. The direction of the forces is given by Fleming's left-hand rule.

Definition of 'ampere'

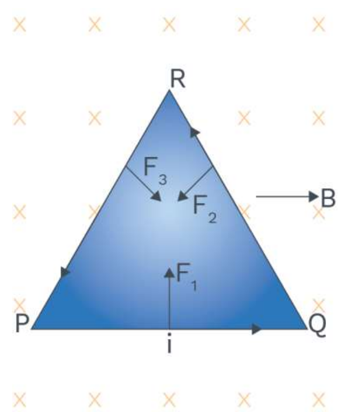
We have $\frac{dF}{dL} = \frac{\mu_0}{4\pi}\frac{2i_1i_2}{d}$

If $i_1 = i_2 = 1A; d = 1m; dL = 1m;$ then

$dF = 2 \times 10^{-7}$ N

Hence, 'ampere' is defined as the current passing through each of two parallel infinitely long straight conductor placed in the free space at the distance of 1 metre from each other produces between them force of 2×10^{-7} N for 1 metre of their length.

Ex. A wire is bent in form of the equilateral triangle PQR of side 10 cm and carries a current of 5 Ampere. It is placed in magnetic field 'B' of magnitude 2 T direction perpendicular to the plane of the loop as shown in figure. Find the forces on the three sides of the triangle.



Sol. The field and the current have directions as shown in figure. The force on PQ is

$\vec{F}_1 = i\vec{l} \times \vec{B}$

or $F_1 = 5.0\,A \times 10\,cm \times 2.0\,T = 1.0\,N$

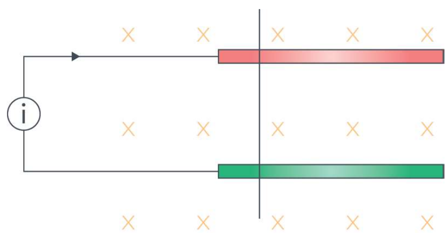
The rule of vector product shows that force F_1 is perpendicular to PQ and is directed towards inside of the triangle.

The forces $\vec{F}_2$ and $\vec{F}_3$ on QR and RP can also be obtained similarly. Both the forces are 1 Newton directed perpendicularly to the respective side and towards the inside of triangle. The three forces $\vec{F}_1, \vec{F}_2$, and $\vec{F}_3$ will have '0' resultant, so that there is no net magnetic force on triangle. This result can be generalised for any closed current loop, placed in a homogeneous magnetic field, does not experience a net magnetic force.

Ex. As shown in the figure, two long metal rails placed horizontally and parallel to each other at a separation l. A uniform magnetic field B exists in vertically downward direction. A wire of mass m can slide on the rails. The rails are connected to the constant current source which drives a current i in the circuit. The friction coefficient between the rail sand the wire is μ.

(a) What should the minimum value of 'μ' which can prevent the wire from the sliding on rails?

(b) Describe motion of the wire if value of 'μ' is half the value found in the previous part



Sol. (a) Force on the wire due to magnetic field is

$\vec{F} = i\vec{l} \times \vec{B}$

or $F = ilB$

It acts towards right. If wire does not slide on the rails, the force of friction by rails should be equal to 'F'. If 'μ_0' be the minimum coefficient of friction which can prevent sliding, this force is also equal to μ_0 mg. Thus,

$\mu_0 mg = ilB$

or $\mu_0 = \dfrac{ilB}{mg}$

(b) If the friction coefficient is $\mu = \dfrac{\mu_0}{2} = \dfrac{ilB}{2mg}$, the wire will slide towards the right.

The frictional force by the rails is

$f = \mu mg = \dfrac{ilB}{2}$ towards left.

The resultant force is $ilB - \dfrac{ilB}{2} = \dfrac{ilB}{2}$ towards right. The acceleration will be $a = \dfrac{ilB}{2m}$. The wire will slide towards the right with this acceleration

Ex. As shown in the figure a semicircular wire is placed in a uniform $\vec{B}$ directed toward right. Find the resultant magnetic force and torque on it.

Sol. The wire is equivalent to

forces on individual parts are marked in the figure with and . Though symmetry their will be pair of forces forming couples.

Ex. Find resultant magnetic force and torque on loop.

Sol.

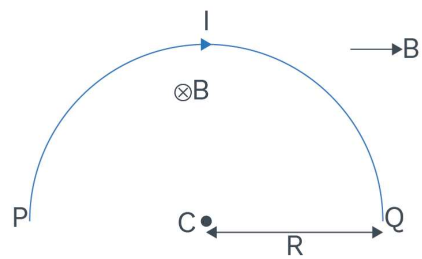
,

and

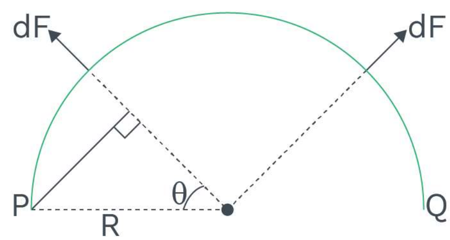
using the above method

Ex. Find the resultant magnetic force and torque about 'C', and 'P' as shown in the figure.

Sol.



∵ wire is equivalent to



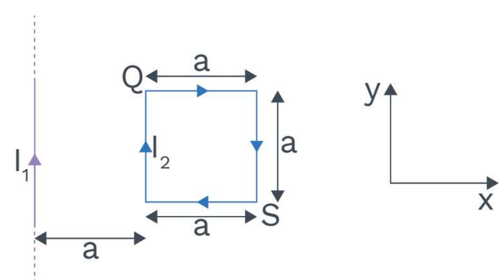
Force on each element is radically outward: $\tau_c = 0$

about point P

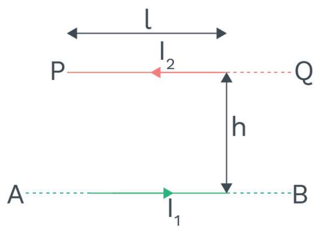
$$P = \int_0^{\pi} [i / (Rd\theta)B\sin 90°]\ R\sin\theta = 2IBR^2$$

Ex. Find the magnetic force on loop 'PQRS' due to straight wire.

Sol. $F_{res} = \frac{\mu_0 I_1 I_2}{2\pi a} a(-\hat{i}) + \frac{\mu_0 I_1 I_2}{2\pi(2a)} a(\hat{i}) = \frac{\mu_0 I_1 I_2}{4\pi a}(-\hat{i})$



Ex. As shown in the figure wires AB and PQ carry constant currents 'I_1' and 'I_2' respectively. PQ is of uniformly distributed mass 'm' and length 'l' AB and PQ both are horizontal and kept in the same vertical plane. The PQ is in equilibrium at the height 'h'. Find



(i) 'h' is terms of I_1I_2, l, m, g and other standard constants.

(ii) If the wire PQ is displaced vertically by small distance, then prove that it performs SHM. Find the time period in terms of 'h' and g.

Sol. (i) Magnetic repulsive force balances the weight.

$$\frac{\mu_0 I_1 I_2}{2\pi h} l = mg \Rightarrow h = \frac{\mu_0 I_1 I_2}{2\pi mg}$$

(ii) Let the wire is displaced downward by the distance x (<< h). Magnetic force will increase, so it goes back towards the equilibrium position. Hence it performs oscillations.

$$F_{res} = \frac{\mu_0 I_1 I_2}{2\pi(h-x)} l - mg$$

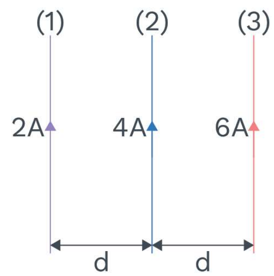
$$= \frac{mgh}{h-x} - mg = \frac{mg(h-h+x)}{h-x}$$

$$= \frac{mg}{h-x} x \cong \frac{mg}{h} x \text{ for } x << h$$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{mg/h}} = 2\pi\sqrt{\frac{h}{g}}$$

Imp. If direction of current is same in parallel wires they will attract each other end if direction of current is opposite then they will repel each other.

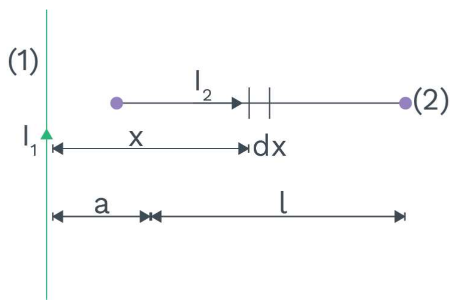
Ex. Find force per unit length on wire 2.



Sol. $F_2 = \frac{\mu_0 24\hat{i}}{2\pi d} - \frac{\mu_0 8\hat{i}}{2\pi d}$

$$= \frac{16\mu_0}{2\pi d} = \frac{8\mu_0}{\pi d}\hat{i}\,\frac{N}{m}$$

Ex. Find force on wire number (2)



Sol.

$$B_1 = \frac{\mu_0 I_1}{2\pi x}$$

$$F = I_2 l B_1 \sin 90^\circ$$

$$dF = I_2 dx B_1$$

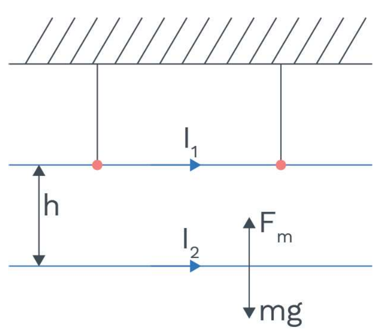
$$dF = I(d\vec{x} \times \vec{B})$$

$$= I_2 \int_a^{a+l} \frac{\mu_0 I_1 I_2}{2\pi x} dx$$

$$= \frac{\mu_0}{2\pi} I_1 I_2 \log_e (x)_a^{a+l}$$

$$= \frac{\mu_0 I_1 I_2}{2\pi} \log_e \left(\frac{a+l}{a}\right)$$

Free-wire Balancing

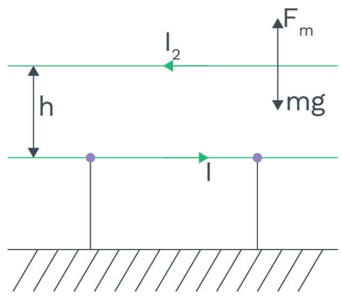


$$F_m = mg$$

$$\frac{F_m}{l} = \frac{mg}{l}$$

$$\frac{\mu_0 I_1 I_2}{2\pi h} = \lambda g \left(\lambda = \frac{m}{L}\right)$$

$$\frac{\mu_0 I_1 I_2}{2\pi h} = \lambda g$$



$$F = \frac{\mu_0 I_1 I_2 l}{2\pi a} - \frac{\mu_0 I_1 I_2 l}{2\pi(a+l)} = \frac{\mu_0 I_1 I_2 l}{2\pi}\left[\frac{1}{a} - \frac{1}{a+l}\right]$$

$$= \frac{\mu_0 I_1 I_2 l}{2\pi}\left[\frac{a+l-a}{a(a+L)}\right] = \frac{\mu_0 I_1 I_2 l^2}{2\pi(a)(a+L)}$$

$$F = \frac{\mu_0 I_1 I_2 l^2}{2\pi(a)(a+L)}$$

Ex. Two wires are in stable equilibrium. If direction of current is reversed in one wire then find instantaneous acceleration of wire.

Sol. a = 2g

$F_m = F_g$

After reversing the direction

$F_{net} = F_m + mg$

$= 2mg$

$a = 2g$

Current Loop In A Uniform Magnetic Field

- **Magnetic Dipole Moment:-**

According to magnetic effects of current, in case of current-carrying coil for axial point,

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\pi NiR^2}{(R^2 + x^2)^{3/2}}$$

when $x >> R$, $x >> R, \vec{B} = \frac{\mu_0}{4\pi}\frac{2\pi NiR^2}{x^3}$

If we compare this conclusion with the field due to a small bar magnet for a axial point, i.e.,

Concept Reminder

If a charged particle projected in a gravity free room deflects, then both fields cannot be zero, both fields can be non-zero.

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{M}}{x^3}$$

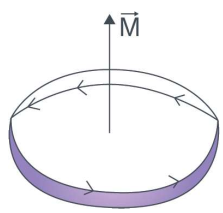
Where 'M' is magnetic dipole moment of bar magnet.

We find that a current-carrying coil for a distant point behaves as a magnetic dipole moment

$$\vec{M} = Ni\pi R^2 = NiA \qquad \text{... (20)}$$

where A is area of the loop. So the magnetic moment of the current carrying coil is defined as product of current in the coil with the area of coil in vector form.

Magnetic moment of the current carrying loop is the vector quantity and direction is perpendicular to plane of the loop. its dimensions is $[L^2A]$ and units is $A\text{-}m^2$.



Magnetic dipole moment in case of the charged particle having charge 'q' and moving in a circle of radius 'R' with speed 'v' is given by $\frac{1}{2}qvR$

As we know, the equivalent current $i = qf = q\frac{v}{2\pi R}$

and $|A| = \pi R^2$

$\therefore \quad M = i|\vec{A}| = \frac{1}{2}qvR$

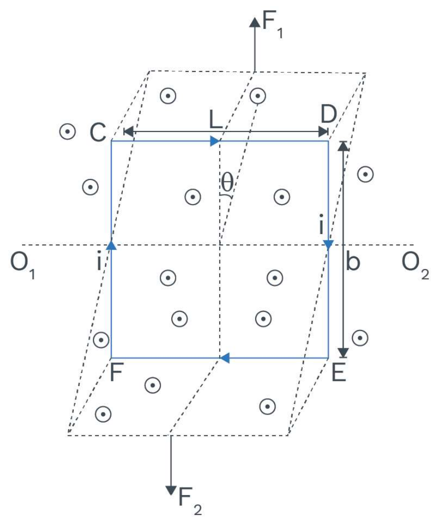
- **Torque on A Current Loop:-**

Take a rectangular coil CDEF of length L and width b is placed vertically, while a uniform magnetic field B passes normally through it as given. The coil is capable rotate about an axis O_1O_2.

If the loop is placed in the magnetic field induction such that the perpendicular to plane of the coil makes an angle 'θ' with the direction of $\vec{B}$, then the torque experienced by the loop is

Rack your Brain

A coil in the shape of an equilateral triangle of side l is suspended between the pole pieces of permanent magnet such that $\vec{B}$ is in plane of the coil. If due to a current i in the triangle a torque τ acts on it then find slide l of the triangle.



$$\tau = \frac{b}{2}(iLB)\sin\theta + \frac{b}{2}(iLB)\sin\theta$$

i.e., $\tau = iLbB\sin\theta = iAB\sin\theta$

where A = Lb is the area of loop.

Maximum torque experienced is $\tau = iAB$, when $\theta = 90°$

and for a coil of N turns

$$\tau = NiAB$$

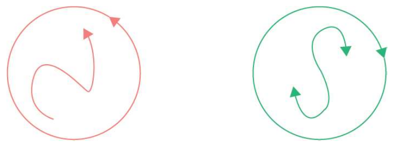
Here NiA = M = Magnetic moment of the loop.

In vector rotation $\vec{\tau} = \vec{M} \times \vec{B}$ ... (21)

This result holds good for plane loops of all shapes rectangular, circular or otherwise.

Magnetic dipole moment of a current carrying coil:-

- Current carrying coil behaves as a magnetic dipole



$$\vec{M} = I\vec{A}$$

$$\vec{M} = NI\vec{A}$$

Concept Reminder

Current carrying coil behaviour as a magnetic dipole

Concept Reminder

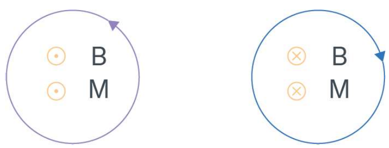
If two symmetrical current carrying rings are placed perpendicular to each other with common centre and magnetic moment of one ring is M, then net magnet moment is

$M_{net} = \sqrt{2}M$

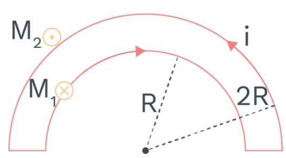
It is an Axial vector and its direction is $\perp$ to the plane

Imp.

For a current carrying coil. (unidirectional current) dir. of magnetic moment is same as direction of magnet field at the centre.



Ex. Find magnetic moment at center for given figure.



Sol. $M = M_2 - M_1$

$$M = -i \times \frac{\pi R^2}{2} + i\frac{\pi 4R^2}{2}$$

$$= \frac{3}{2} i\pi R^2 \odot$$

Ex. An equilateral Δ, square and a circular loop are made by wire of same length l if current is same then compare their magnet moment.

Sol. $M_\Delta = i \times A = i\left(\frac{\sqrt{3}}{4}\left(\frac{l}{3}\right)^2\right)$

$$M_s = i\left(\frac{l}{4}\right)^2 = \frac{l^2}{16} i$$

$$M_{cl} = \pi \times \left(\frac{l}{2\pi}\right)^2 i = \frac{\pi l^2}{4\pi^2} i = \frac{l^2}{4\pi} i$$

10. Three circular rings A, B, C having turns 2, 4 & 6 respectively if current is same then

Concept Reminder

If different polygons are made by same length of wires, then magnetic moment will increase with no. of sides.

Magnetic Effect of Current

compare their magnet moments these rings are made by wires of same length.

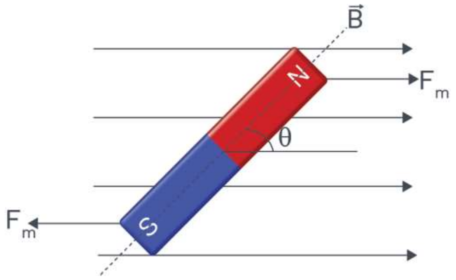
$$M_A = ni \times \pi r^2 = 2i\pi r^2 = \frac{2i\pi l^2}{4\pi^2} \quad \frac{2(2\pi r) = nl}{4\pi r = 2l}$$

$$M_B = 4\pi r^2 = 4i\pi\left(\frac{l}{8\pi}\right)^2 \quad r = \frac{l}{\eta}$$

$$\boxed{M_A > M_B > M_C} \qquad 4(2\pi r) = l$$

 # Torque on a Bar Magnetic/Magnetic dipole in uniform Magnetic field:-

$\tau = \text{force} \times \perp \text{distance}$



$\tau = F_M l \sin\theta$ $\qquad [F_m = mB]$

$\tau = mBl\sin\theta \; [M = ml]$

$\tau = mB\sin\theta$

$\vec{\tau} = \vec{M} \times \vec{B}$

Coil

$\vec{\tau} = \vec{M} \times \vec{B}$

$\tau = MB\sin\theta$

$\tau = NIAB\sin\theta$

> **Concept Reminder**
>
> **Torque on a Magnetic dipole in uniform Magnetic field :**
>
> $\vec{\tau} = \vec{M} \times \vec{B}$
>
> For coil of N turns
>
> $\tau = NIA\sin\theta$

$\vec{M} \rightarrow$ Magnetic moment of bar magnet.

$\theta \rightarrow$ Angle between area vector ($\vec{M}$) & magnetic field.

Note: Force on any magnetic dipole in uniform magnetic field is always zero. Torque may or may not be zero.

Ex. A circular ring of 100 turns & radius 2 cm is placed in uniform magnet field B. 2T making an angle 60° with field if current is I, then find torque experienced by ring.

Sol. $\tau = 100 \times I \times 2(2 \times 10^{-2})^2 \pi \sin 30°$

$= I \; 100\pi \times 4 \times 10^{-4}$

$= 4I\pi \times 10^{-2}$

$= (4\pi \times 10^{-2})I$

Ex. Magnetic-moment of a current carrying coil is M and this coil is placed in uniform magnetic field B. If torque experienced by coil is $\frac{MB}{2}$ then find angle between coil & magnet field

Sol. $\frac{MB}{2} = MB\sin\theta$

$\theta = 30°$

This is the angle between coil & magnet field.

Work done in rotating a dipole from θ_1 to θ_2 angle:-

$$W = \int_{\theta_1}^{\theta_2} \tau \, d\theta$$

$$W = \int_{\theta_1}^{\theta_2} MB\sin\theta . d\theta$$

$$W = MB[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$W = -MB[\cos\theta_2 - \cos\theta_1]$$

$$W = MB[\cos\theta_1 - \cos\theta_2]$$

Potential Energy

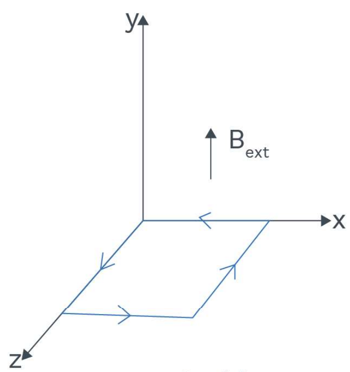
$$W = MB(\cos\theta_1 - \cos\theta_2)$$

Work done in rotating a dipole in uniform magnetic field from $\theta_1 = 90°$ to $\theta_2 = \theta$

$$U = -MB\cos\theta$$

- $\theta = 0°$

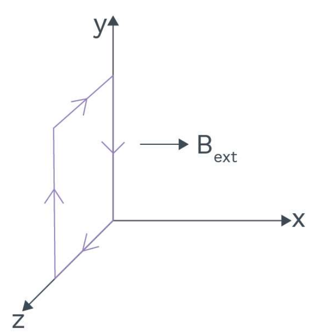
$U = -MB_{(min.)}$ [stable equilibrium]



Stable

- $\theta = 180°$

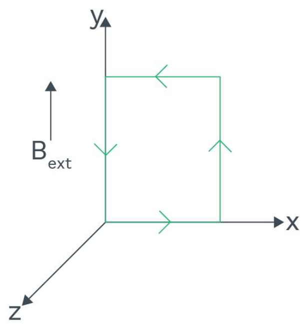
$U = MB_{(mex.)}$ [unstable equilibrium]



Unstable

- $\theta = 90°$

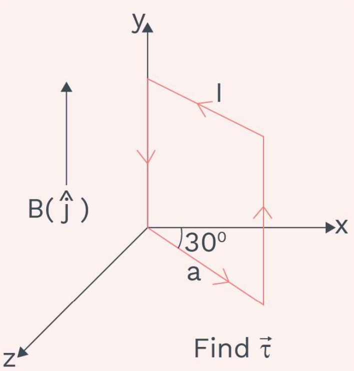
U = 0 [zero potential energy and no equilibrium]



No equilibrium

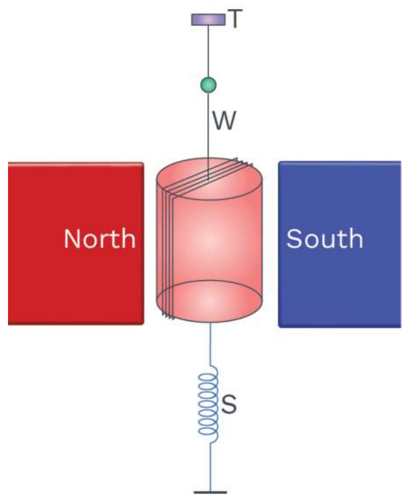
Rack your Brain

Find torque acting on coil for following case.

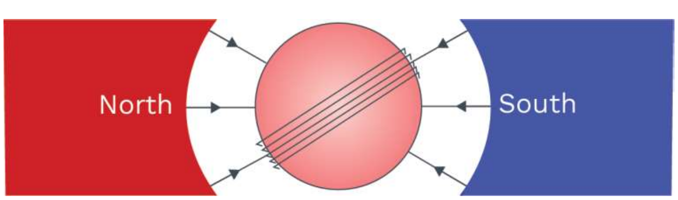


Find $\vec{\tau}$

Moving-Coil Galvanometer:-



The main parts of a 'moving-coil galvanometer' are shown in figure. A rectangular coil of several turns is rolled over a soft-iron core. The wire of the coil is coated with an non-conducting material so that each turn is insulated from each other and from iron core. The coil is suspended between the 2 pole pieces of a strong stable magnet. The current to be determine is passed through the moving coil-galvanometer. As the coil is in magnetic field $\vec{B}$ of the permanent magnet, a torque $\vec{\Gamma} = ni\vec{A} \times \vec{B}$ acts on the coil. Where n = number of turns in the coil, i = current, $\vec{A}$ = area-vector of the coil and $\vec{B}$ = magnetic field at the site of the coil. This torque deflects the coil from its balance position.



The magnetic pole pieces are made cylindrical in shape. So, the magnetic field induction at arms of the coil remains parallel to the plane of the

Concept Reminder

The neutral position of the pointer (when no current is flowing through the galvanometer) is in the middle of scale and not at the left end.

coil everywhere even as the coil rotates. The deflecting torque is $\Gamma = niAB$. Upper end of the suspension strip 'W' is fixed, so strip gets twisted when the coil rotates. This produces a 'restoring torque' acting on the coil. If the deflection of the coil is 'θ' and the tensional constant of the suspension strip is k, the restoring torque is $k\theta$. The coil will remain at a deflection θ where

$$niAB = k\theta \text{ or } i = \frac{k}{nAB}\theta$$

So, the current is directly proportional to the deflection. Constant $\frac{k}{nAB}$ is non the galvanometer constant, and may be found by passing a known current, measuring the deflection θ and putting these values in above equation.

Sensitivity

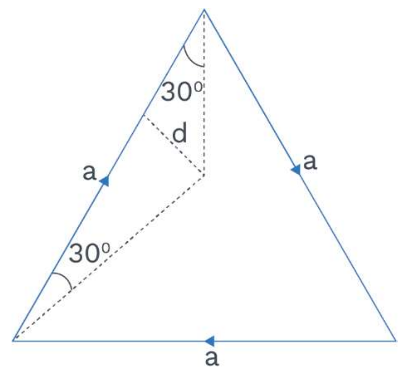
The sensitivity of a "moving-coil galvanometer" is defined as

θ/i. From equation, the sensitivity is $\frac{nAB}{k}$. For large sensitivity, the magnetic field B should be high. The presence of soft iron core increases the magnetic field.

EXAMPLE

Q1 **A current of 1 amp is flowing in the sides of an equilateral triangle of side 4.5 x 10^{-2} m. Find out the magnetic field at the centroid of triangle.**

Sol:

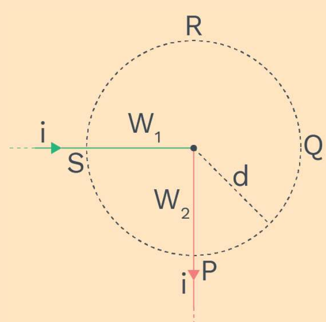


$$\vec{B}_{Net} = 3\left(\vec{B}_{\text{due to one side}}\right)$$

$$\vec{B}_{Net} = 3 \times \frac{\mu_0 i}{4\pi d}\left[\cos 30° + \cos 30°\right]$$

$$\vec{B}_{Net} = 3 \times 10^{-7} \times \frac{1}{\frac{a}{2}\tan 30°} 2\cos 30° = 4 \times 10^{-5}\ \text{Wb/m}^2.$$

Q2 **Diagram shows a long wire bent at the middle to form a right angle. Show that the intensity of the magnetic fields at the points Q and R are unequal and find these magnitudes. The wire W_1 and the circumference of circle are coplanar and W_2 is perpendicular to plane of paper. Also find the ratio of field at Q and R**

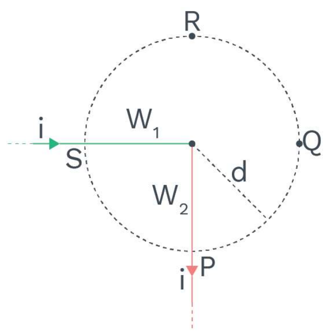


Sol: Magnetic field at Q is

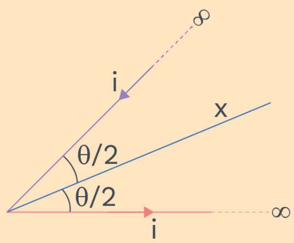
$$B_Q = \frac{\mu_0 i}{4\pi d}$$

Now magnetic field at R is

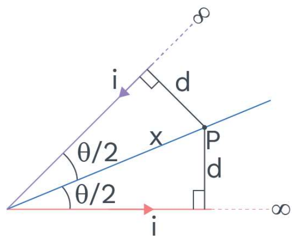
$$B_R = \frac{\mu_0 i}{4\pi d}\sqrt{2}$$



Q3 **A long wire carrying a current i is bent to form a plane angle θ. If the magnetic field B at a point on the bisector of this angle situated at a distance x from the vertex is written in the form of $K\cot\frac{\theta}{4}$ Tesla. Then, find the value of K.**



Sol:



$$B = 2\frac{\mu_0 i}{4\pi d}\left[\cos\frac{\theta}{2} + \cos 0^\circ\right] \quad \text{Out of the plane of paper}$$

$$= 2\frac{\mu_0 i}{4\pi x \sin\frac{\theta}{2}}\left[1 + \cos\frac{\theta}{2}\right]$$

$$= \frac{\mu_0 i}{2\pi x}\cot\frac{\theta}{4}$$

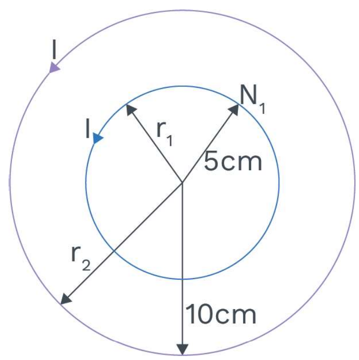
$$\therefore \quad K = \frac{\mu_0 i}{2\pi x}.$$

Q4 **(i) Two circular coils of radii 5.0 cm and 10 cm carry equal currents of 1 A. The circular coils have 50 and 100 turns respectively and are placed in such a way that their planes as well as centre coincide. Find out the magnitude of the magnetic field B at the common centre when the currents in the coils are**

(a) in same sense **(b) in opposite sense**

(ii) If the outer coil of the given problem is rotated through 90° about a diameter, then what would be the magnitude of the magnetic field B at the centre ?

Sol: **(i)** (a)

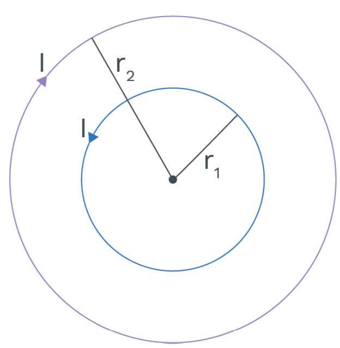


$$B_{net} = \frac{\mu_0 N_1 I}{2r_1} + \frac{\mu_0 N_2 I}{2r_2} = \frac{\mu_0 I}{2}\left[\frac{N_1}{r_1} + \frac{N_2}{r_2}\right]$$

$$= \frac{4\pi \times 10^{-7} \times 1}{2}\left[\frac{20}{10^{-2}}\right]$$

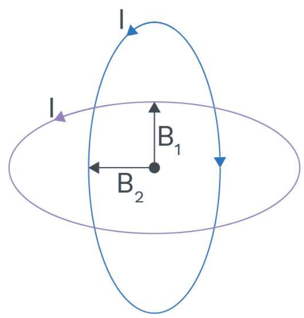
$$= 8\pi \times 10^{-4} Wb/m^2.$$

(b)



$$B_{Net} = \frac{\mu_0 N_1 I}{2r_1} - \frac{\mu_0 N_2 I}{2r_2} = 0$$

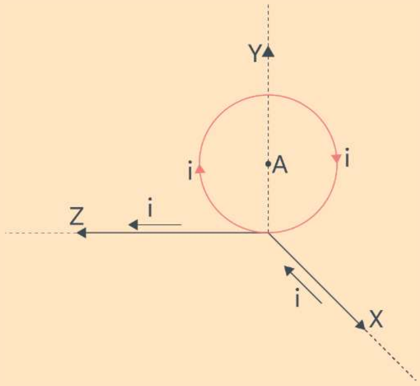
(ii)



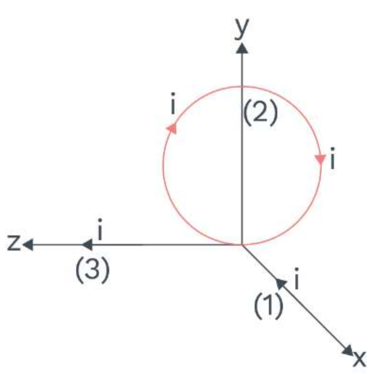
$$B_{Net} = \sqrt{B_1^2 + B_2^2} = \sqrt{2}\, B_1$$

$$= \sqrt{2}\, \frac{\mu_0 N_1 I}{2r_1} = 4\sqrt{2}\pi \times 10^{-4}\, T.$$

Q5 **Find out the intensity of the magnetic induction B generated by the system of thin conductors (along which a current i is flowing) at the point A (0, R, 0), which is at centre of the circular conductor of radius R. The circular part is in yz-plane.**



Sol:



$$\vec{B}_{\text{due to}(1)} = \frac{\mu_0 i}{4\pi R}\left(-\hat{k}\right)$$

$$\vec{B}_{\text{due to}(2)} = \frac{\mu_0 i}{2R}\left(-\hat{i}\right)$$

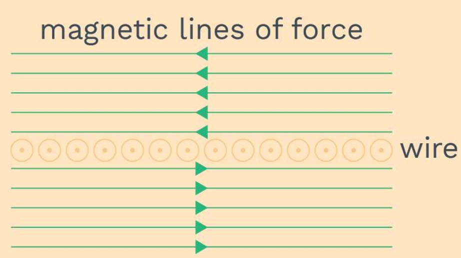
$$\vec{B}_{\text{due to}(3)} = \frac{\mu_0 i}{4\pi R}\left(-\hat{i}\right)$$

$$\vec{B}_{\text{Net}} = \frac{\mu_0 i}{4\pi R}\left[\left(2\pi - 1\right)\hat{i} - 1\hat{k}\right]$$

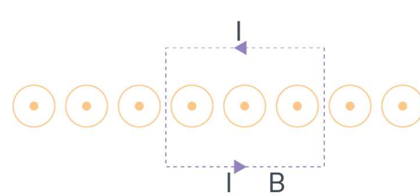
$$\left|\vec{B}\right| = \frac{\mu_0 i}{4\pi R}\left[\sqrt{4\pi^2 + 1 - 4\pi + 1}\right]$$

$$= \frac{\mu_0 i}{4\pi R}\left[\sqrt{4\pi^2 - 4\pi + 2}\right].$$

Q6 **A conductor contains of an infinite number of adjacent wires, each infinitely long and carrying a current i. Prove the lines of B will be as represented in figure & that magnetic field B for all points in front of the infinite current sheet will be given by, B = (1/2)$\mu_0$ni, where n is the number of conducting wires per unit length.**



Sol:



$$\oint \vec{B}.dl = \mu_0 I$$

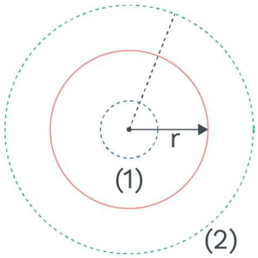
$$Bl + Bl = \mu_0\left[n\,l\,i\right]$$

$$B = \frac{\mu_0 ni}{2}.$$

Q7 **A thin but long, hollow, cylindrical tube of radius r carries a current i along its length. Find out the magnitude of the magnetic field at a distance r/4 from the surface**

(a) inside the tube **(b) outside the tube.**

Sol:



(a) loop (1)

$$B\left(2\pi\frac{r}{2}\right) = \mu_0(0)$$

$$B = 0$$

(b) loop (2)

$$B\left(2\pi\frac{5r}{4}\right) = \mu_0 i$$

$$B = \frac{2\mu_0 i}{5\pi r}.$$

Q8 **A closely-wound, long solenoid is kept with its axis parallel to a large conducting sheet carrying a surface current. The surface current through a width 'dl' of the sheet is 'Kdl' and number of turns per unit length of the solenoid is 'n'. The magnetic field close to the centre of the solenoid is found to be zero.**

(a) Obtain the current in the long solenoid.

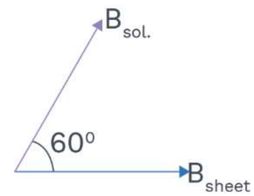
(b) If solenoid is rotated to make its axis 60° to the plane of conducting sheet, then what would be the magnitude of the magnetic field near its centre ?

Sol: (a) $B_{Net} = 0$

$$\Rightarrow \mu_0 ni = \frac{\mu_0 k}{2}$$

$$i = \frac{k}{2n}$$

(b) $B_{Net} = \sqrt{\mu_0^2 n^2 i^2 + \frac{\mu_0^2 k^2}{4} + 2.\mu_0 ni.\frac{\mu_0 k}{2}\cos 60°}$

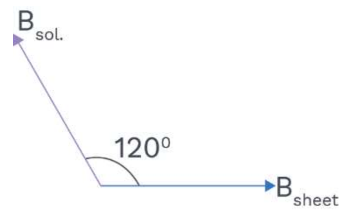


$= \frac{\mu_0 k}{2}\sqrt{3}$

and one more at a angle of 120°

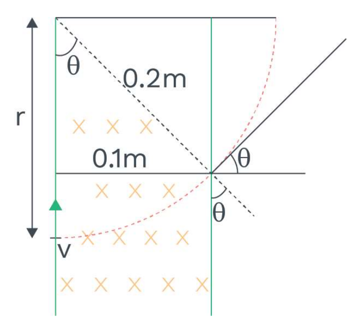
$B_{Net} = \sqrt{\mu_0^2 n^2 i^2 + \frac{\mu_0^2 k^2}{4} + 2.\mu_0 ni.\frac{\mu_0 k}{2}\cos 120°}$

$= \frac{\mu_0 K}{2}$.



Q9 **An α–particle is accelerated by a potential difference of 10^4V. Find out the change in its direction of motion, if it enters normally in a region of thickness 0.1 metre having transverse magnetic induction of 0.1 Tesla. (Given: mass of α-particle is equal to 6.4×10^{-27} kg)**

Sol:



$r = \frac{mv}{qB}$ and $\frac{1}{2}mv^2 = qV$ $= \frac{m}{qB}\sqrt{\frac{2qV}{m}}$

$$r = \sqrt{\frac{2mV}{q}}\frac{1}{B} = 0.2$$

$$\sin\theta = \frac{d}{r} = \frac{1}{2} \quad \Rightarrow \quad \theta = 30° \ = 0.2\ \text{m.}$$

Q10 **A magnetic field of $8\hat{k}$ mT exerts a force of $\left(4.0\,\hat{i} + 3.0\,\hat{j}\right)\times 10^{-10}$ N on a particle having a charge of 5 × 10^{-10} C and going in the X – Y plane. Find the velocity of the particle.**

Sol: $\vec{F} = q\left(\vec{v}\times\vec{B}\right)$

Let $\vec{V} = V_x\hat{i} + V_y\hat{j}$

$$= \left(4.0\hat{i} + 3.0\hat{j}\right)\times 10^{-10} = \left(0.5\times 10^{-9}\right)$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ V_x & V_y & 0 \\ 0 & 0 & 8\times 10^{-3} \end{vmatrix}$$

$$\left(4\hat{i} + 3\hat{j}\right)\times 10^{-1} = \hat{i}\left[8\times 10^{-3}V_y\right] - \hat{j}\left[8\times 10^{-3}V_x\right]0.5$$

So V_y = 100 m/s and V_x = – 75 m/s.

Q11 **An observer's diary reads as follows; "a charged particle is projected in a magnetic field of and particle's acceleration is found to be $\left(x\hat{i} + 7.0\hat{j}\right)\times 10^{-6}$ m/s². Find the value of x.**

Sol: $\vec{F} = q\left(\vec{v}\times\vec{B}\right)$

$\vec{F}\perp\vec{B} \quad \Rightarrow \quad \vec{a}$ is $\perp$ to $\vec{B}$

So, $\vec{a}.\vec{B} = 0$

7x – 21 = 0

x = 3.0

Q12 **An electron having a kinetic energy of 400 eV circulates in a path of radius 20 cm in a magnetic field. Find the magnetic field and the number of revolutions per second made by the electron.**

Sol: $r = \frac{mv}{qB} = \frac{\sqrt{2km}}{qB}$

$B = \frac{\sqrt{2km}}{qr} = \frac{\sqrt{182}}{4} \times 10^{-4}\,T$

$f = \frac{\omega}{2\pi}$

$= \frac{V}{2\pi r} = \frac{\sqrt{\frac{2k}{m}}}{2\pi r}$

$= \frac{\sqrt{8}}{\pi\sqrt{91}} \times 10^{8}$ rps.

Q13 **(a) An e^- moves along a circle of radius 1 metre in a normal magnetic field of strength 0.50 T. What would be its speed ? Is it reasonable ?**
(b) If a proton moves along a circle of the same radius in the same magnetic field, what would be its speed? mass of proton = $\frac{5}{3} \times 10^{-27}$ kg.

Sol: (a) $r = \frac{mv}{qB}$

$v = \frac{qBr}{m}$

$= \frac{1.6 \times 10^{-19} \times 0.5 \times 1}{9.1 \times 10^{-31}}$

$= \frac{8}{91} \times 10^{12}$ m/s.

As $V > C$ $\rightarrow$ It is Not reasonable.

(b) $v = \frac{qBr}{m}$

$= 48 \times 10^{6}$ m/s.

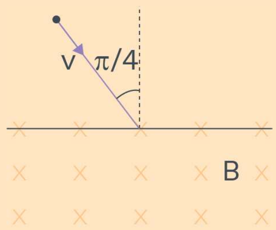
Q14 **A particle of mass m and +ve charge q, moving with a uniform velocity v, enters a magnetic field B as given in diagram.**

(a) Find out the radius of circular arc it describes in the magnetic field.

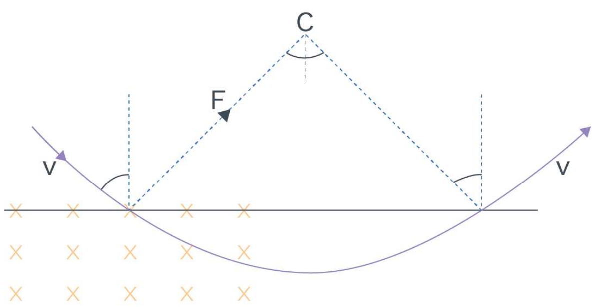
(b) Find out the angle subtended by the arc at centre of circle.

(c) How long does particle stay inside magnetic field ?

(d) Solve the 3 parts of the above problem if the charge q on the particle is negative.



Sol:



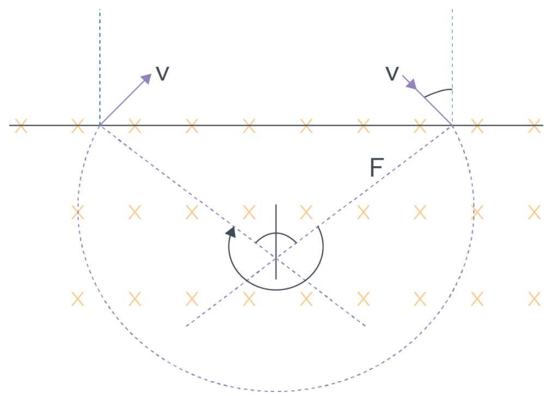
(a) $r = \dfrac{mv}{qB}$

(b) Angle Subtended by the arc $= \dfrac{\pi}{2}$.

(c) $\theta = \omega t$

$$\frac{\pi}{2} = \frac{qBt}{m} \Rightarrow t = \frac{\pi m}{2qB}$$

(d)



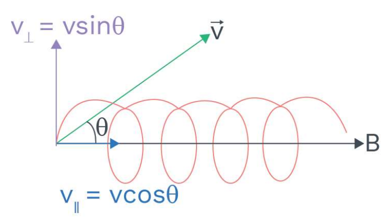
$$r = \frac{mv}{qB}$$

$$\theta = \frac{3\pi}{2}$$

$$t = \frac{\theta}{\omega} = \frac{3\pi m}{2qB}$$

Q15 **A particle having a charge of 5 μC and a mass of 5.0 × 10⁻¹² kilogram is projected with a speed of 1.0 km/s in a magnetic field of magnitude 5.0 mT & angle between the velocity vector and the magnetic field vector is $\sin^{-1}$ (0.90). Prove that the path of the particle will be a helix. Find the diameter of the helix and its pitch.**

Sol:



$$r = \frac{mV_\perp}{qB}$$

$$= \frac{5 \times 10^{-12} \times 1 \times 10^{3} \times 0.9}{5 \times 10^{-6} \times 5 \times 10^{-3}} = 18\,\text{cm}\,.$$

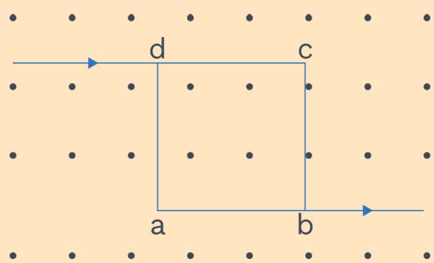
Diameter = 36 cm

$$T = \frac{2\pi m}{qB}$$

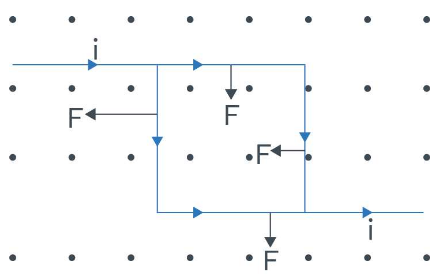
$$P = v\cos\theta \times \frac{2\pi m}{qB}$$

$$= 4\pi\sqrt{19} \text{ cm.}$$

Q16 **A current of 2 A enters at the corner d of a square frame abcd of side 10 centimetre and leaves at the opposite corner b. A magnetic field B = 0.1 T exists in the free space in direction perpendicular to the plane of the frame as shown in figure. Find out the magnitude and direction of magnetic forces on the four sides of the frame.**



Sol:



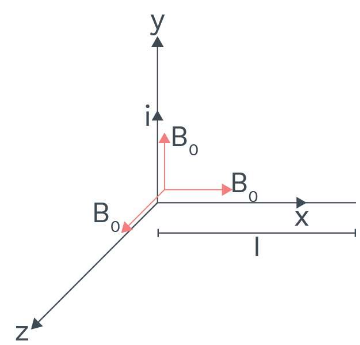
$$F = B\frac{i}{2}l$$

$$= 0.1 \times 1 \times 0.1$$

$$= 1 \times 10^{-2} \text{ N.}$$

Q17 **A wire of length ℓ carries a current i along the y-axis. A magnetic field exists which is given by $\vec{B} = B_0(\hat{i} + \hat{j} + \hat{k})$T. Find the magnitude of the magnetic force acting on the wire.**

Sol:



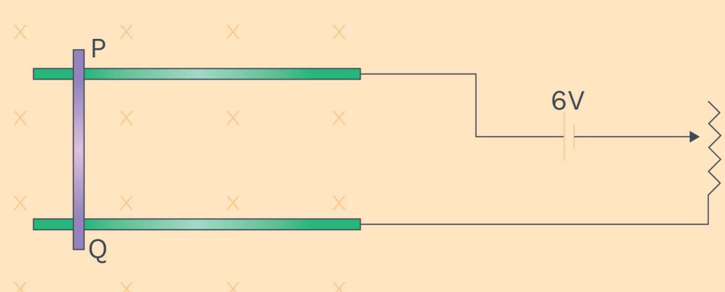
$$\vec{F} = i\left[\vec{l} \times \vec{B}\right]$$

$$= B_0 il\left[\hat{i} + \hat{j} + \hat{k}\right]$$

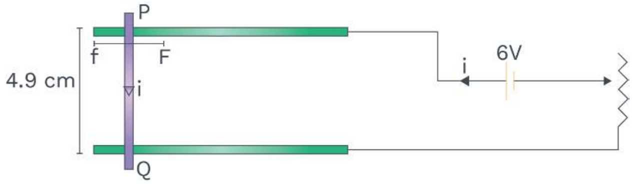
$$= B_0 il\left[-\hat{k} + \hat{i}\right]$$

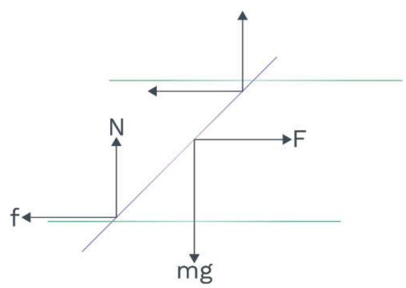
$$F = \sqrt{2}\, B_0 il\,.$$

Q18 **A metal wire PQ of mass 10 gm lies at rest on two horizontal metal rails separated by 4.90 cm (figure). A vertically downward magnetic field of value 0.800 Tesla exists in the free space. The resistance of the circuit is slowly reduced and it is found that when the resistance goes below 20 Ω, the wire PQ starts sliding on the rails. Find the coefficient of friction. Neglect magnetic force acting on wire PQ due to metal rails ($g = 9.8\ m/s^2$)**



Sol:





$F = f_L$

$N = mg$

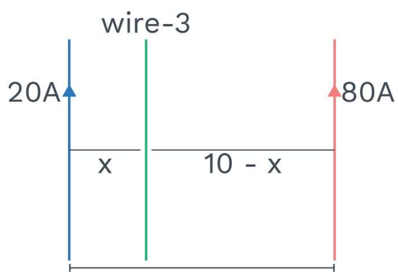
$BiL = \mu\, mg$

$$\mu = \frac{BIL}{mg}$$

$$= \frac{0.8 \times \frac{6}{20} \times 4.9 \times 10^{-2}}{10 \times 10^{-3} \times 9.8} = 0.12\,.$$

Q19 **Two parallel wires separated by a distance of 10 cm carry currents of 20 A and 80 A along the same direction. Where should a third current carrying wire be placed so that it experiences no magnetic force ?**

Sol:



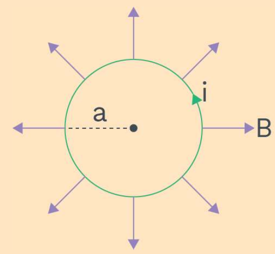
$$\frac{\mu_0 i_1}{2\pi x} = \frac{f}{\ell} = \frac{\mu_0 i_2}{2\pi(10 - x)}$$

$$\frac{20}{x} = \frac{80}{10 - x}$$

$10 - x = 4x$

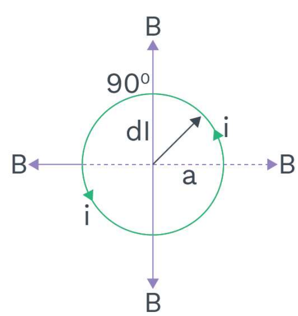
$x = 2$ cm.

Q20 **(a) A circle of radius a, carrying a current i, is placed in a 2-dimensional magnetic field. The centre of the circular loop coincides with the centre of the field (in diagram). The density of the magnetic field at the boundary of the loop is B. Find the magnetic force on the wire.**



(b) A hypothetical magnetic field existing in a region is given by $\vec{B} = B_0\vec{e}_r$, where $\vec{e}_r$ denotes the unit vector along the radial direction of a point relative to the origin and B_0 = constant. A circle of radius a, carrying a current i, is placed in X-Y plane and the centre of circle is at (0, 0, a). Find out the magnitude of magnetic force acting on loop.

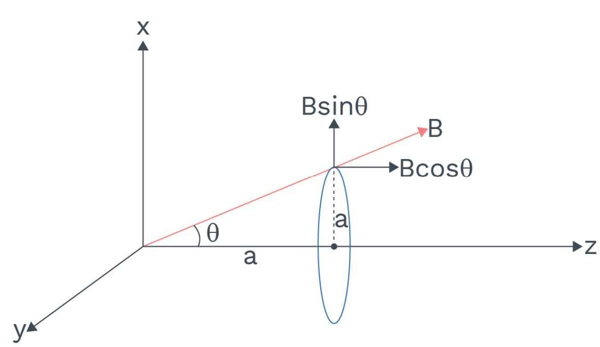
Sol:



(a) df = BidL

F = Bi (2πa) into the plane of paper.

(b)



$\vec{B} = B_0\vec{e}_r$

All force elements due to $B_0\cos\theta$ will be added up and due to $B_0 \sin\theta$ will get cancelled

$dF = B_0 \cos\theta \, I \, dl$

$$dF = B_0 \frac{1}{\sqrt{2}} \times i\,dl$$

$$F = \frac{B_0 i}{\sqrt{2}}(2\pi a)$$

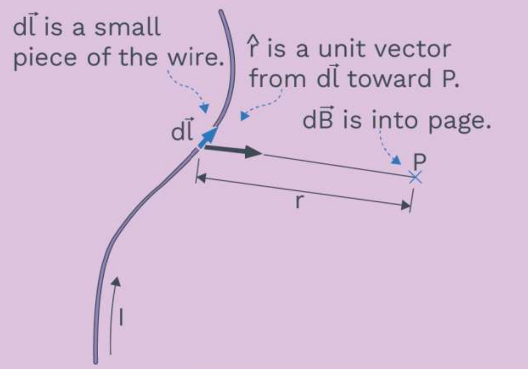
$$= \sqrt{2} i B_0 \pi a.$$

Mind Map

MAGNETIC EFFECT OF CURRENT

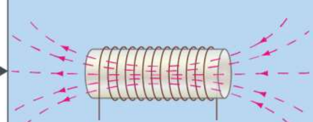
Biot Savart's Law

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{l} \times \vec{r}}{r^3}$

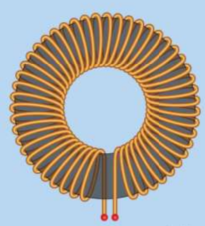


Ampere's Circuital Law

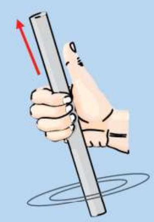
$\oint \vec{B}.d\vec{l} = \mu_0 I_{enclosed}$



Magnetic field inside a long solenoid.



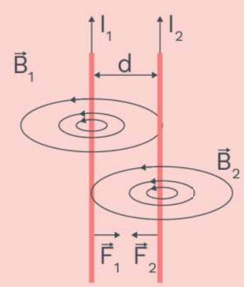
Magnetic field inside a toroidal coil.

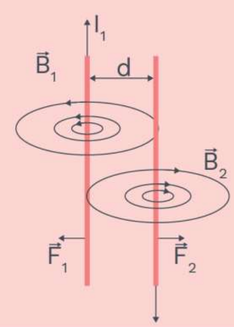


Magnetic field from a long straight wire.

Force on current Carring Conductor

$\vec{F} = i\,(\vec{l} \times \vec{B})$





Torque on Current Carrying Coil

$\vec{\tau} = \vec{M} \times \vec{B}$