Kinetic Theory of Gases

DISCLAIMER

Kinetic Theory of Gases

Pressure:

- *Pressure is the force per unit area.*
- $P = \frac{F}{A}$

$$= \frac{\text{Component of force perpendicular to surface}(F_{\perp})}{\text{Magnitude of area}(A)}$$

- Unit = N/m^2 or Pascal (Pa).
- Pressure is scalar quantity.
- 1 atm = 1.01325×10^5 Pa = 10^5 Pa
- Dimension = $[ML^{-1}T^{-2}]$

Volume:

- Volume is the quantity of three-dimensional space enclosed by a closed surface, for example the space that a substance (solid, liquid, gas or plasma)
- Unit = m^3
- 1 litre = 10^{-3} m^3
- 1000 lit = 1 m^3
- 1 cc = 10^{-6} m^3

Density (ρ):

- Density is measure of mass per unit of volume.
- $\rho = \frac{m}{V}$ Unit → kg/m^3 or gm/cc.
- Density of water = 1000 kg/m^3 = 1 gram/cc.

Some important points:

- m = mass of each molecule of gas.
- M = Total mass of gas sample.
- M_w = Molecular weight e.g.- H_2 = 2 gm/mol, He = 4 gm/mol, O_2 = 32 gm/mol.
- N = Total number of molecule in gas sample.
- n = Total number of mole in gas sample.
- Na = Avogadro number = 6.023×10^{23}.
- K = Boltzmann constant ($K = 1.38 \times 10^{-23}$ J/K).
- R = Universal gas constant (R = 8.314 J/mol K = $\frac{25}{3}$ J/mol K, R = 1.98 cal/mol K $\simeq$ 2 cal/mol K).

Concept Reminder

Units of pressure
1 atm = 1.013×10^5 Pa
1 bar = 10^5
1 torr = 133 Pa

Key Points

- Pascal
- Density
- Atmospheric pressure

Concept Reminder

Relative density = $\frac{\text{density of substance}}{\text{density of water at } 4°C}$

Rack your Brain

When a Vander Waal's gas undergoes free expansion then its temperature.

(1) Decreases
(2) Increases
(3) Does not change
(4) Depends upon the nature of the gas

$$\boxed{n = \frac{M}{M_w} = \frac{N}{N_a}}$$

$$\boxed{M = mN}$$

$$\boxed{M_w = mN_a}$$

$$\boxed{K = \frac{R}{N_a}}$$

Ideal Gas Equation:

$$\boxed{PV = nRT}$$

P = pressure, n = number of moles
V = volume, K = gas constant,
T = absolute temperature

Case-I:

$$PV = \frac{M}{M_w}RT \Rightarrow P = \frac{M}{V}\frac{RT}{M_W}$$

$$\Rightarrow \boxed{P = \frac{\rho RT}{M_w}}$$

Case-II:

$$PV = \frac{N}{N_a}RT \Rightarrow PV = \frac{NRT}{N_a}$$

$$\Rightarrow \boxed{PV = NKT}$$

If we move in upward direction in atmosphere then pressure is decrease in exponential form.

Assumption of ideal gas:

1. Assumption regarding the molecule:

- Every gas consists of extremely small particle known as molecules. The molecules of given gas are all identical but are different than those another gas.
- The molecules of an ideal gas are identical, spherical, rigid and perfectly elastic point masses.
- The size is negligible in comparison to inter molecular distance (10^{-9} m)

2. Assumption regarding volume:

- The volume of molecule is negligible in comparison to the volume of gas.

Key Points

- Universal gas constant
- Avogadro number
- Boltzmann constant

Concept Reminder

The temperature at which volume of an ideal gas becomes zero is called critical temperature.

Rack your Brain

Two vessels separately contain two ideal gases A and B at the same temperature the pressure of A being twice that of B. Under such conditions, the density of A is found to be 1.5 times the density of B. Then find the ratio of molecular weight of A and B.

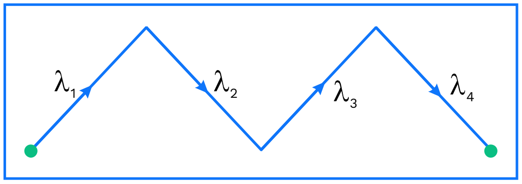
3. Assumption regarding motion:

- Molecules of ideal gas keep on moving randomly in all possible direction with all possible speed.
- The speed of gas molecules lie between zero and infinity (very high speed).

4. Assumption regarding collision:

- The molecules of ideal gas keep on colliding among themselves as well as with the walls of containing vessels. These collisions are perfectly elastic (i.e. total kinetic energy before collision = total kinetic energy after the collision)
- Molecules of gas move in a straight line with constant speed between two successive collisions.
- The distance covered by the molecule between two successive collisions is known as free path and mean of all the path is known as mean free path.

Mean free path: The path transversed in a straight line by a molecule of a gas between two successive collision is called mean free path.



$$\lambda = \frac{\lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_4}{N}$$

$$\Rightarrow \quad \lambda = \frac{1}{\sqrt{2}\,\pi n' d^2} \left[n' = \frac{N}{V} = \frac{P}{KT} \right]$$

- The time taken in a collision between two molecules is negligible in comparison of time to travel distance between two consecutive collisions.
- In a gas the number of collisions per unit volume remains constant.

Concept Reminder

Unit of gas constant
= Joule mol^{-1} k^{-1}
Dimension
= $[M^1L^2T^{-2}mol^{-1}k^{-1}]$

Rack your Brain

The mean free path of molecules of a gas, (radius r) is inversely proportional to-
(1) r
(2) $\sqrt{r}$
(3) r^3
(4) r^2

5. Assumption regarding force:

- No attractive or repulsive force acts between gas molecules i.e. potential energy = 0.
- Gravitational attraction on the molecules is ineffective due to extremely small masses and very high speed of molecules.

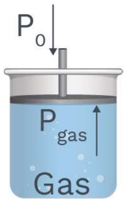
6. Assumption regarding pressure:

- Molecules in gas constantly collide with the walls of container due to which their momentum changes. This change in momentum is transferred to walls of the container consequently pressure is exerted by gas molecules on walls of container.

7. Assumption regarding density:

- The density of gas is constant at all points of the container.

Ex. In a vertical open vessel an ideal gas is filled and it is enclosed by a piston of mass 4 kg and cross-section area is 2 cm^2 then find out pressure applied by gas on piston in equilibrium.



Sol. F = mg = 40 N

$A = 2\ cm^2 = 2 \times 10^{-4}\ m^2$

$$P = \frac{F}{A} = \frac{40}{2 \times 10^{-4}} = 2 \times 10^5 N/m^2 = 2\,atm = 2P_0$$

$$P_{gas} = P_0 + 2P_0 = 3\ P_0$$

Ex. An ideal gas filled in closed vessel at 27°C temperature if gas is heated then its pressure is made 2 times. Then find out the final temperature of gas.

Sol. Closed vessel, V = constant

$$\frac{P_1}{P_2} = \frac{T_1}{T_2} \Rightarrow \frac{P}{2P} = \frac{300}{T_2} \Rightarrow T_2 = 600\,K$$

Ex. In a close vessel temperature of gas is increased by 4°C then find out the initial temperature of gas when pressure is increased by 2%.

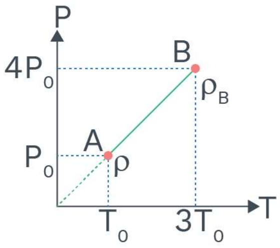
Sol. $PV = nRT \quad \Rightarrow P \propto T, \quad (V = \text{constant})$

$\Delta P\% = \Delta T\% \quad \Rightarrow \Delta T = 4°C = 4\ K$

$$2\% = \frac{T_f - T_i}{T_i} \times 100\% \Rightarrow 2 = \frac{4}{T_i} \times 100$$

$$T_i = 200\,K = -73°C$$

Ex. If density at point A is ρ, then find out density at point B for given curve

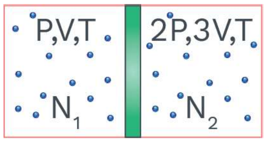


Sol. $P = \dfrac{\rho RT}{M_w} \quad \Rightarrow P \propto \rho T$

$$\frac{P_i}{P_f} = \frac{\rho_i T_i}{\rho_f T_f} \quad \Rightarrow \frac{P_0}{4P_0} = \frac{\rho_A}{\rho_B} \cdot \frac{T_0}{3T_0}$$

$$\rho_B = \frac{4}{3}P$$

Ex. An insulating closed vessel is divided in two parts by on movable piston as shown in figure.



(i) Find out ratio of number of molecule in both part.

(ii) If piston is freely moving with isothermal process then find out the volume of both part in equilibrium.

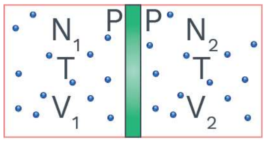
Sol. (i) $PV = NKT$

$$\frac{PV}{(2P)(3V)} = \frac{N_1KT}{N_2KT} \quad \Rightarrow \frac{N_1}{N_2} = \frac{1}{6}$$

Rack your Brain

If pressure of CO_2 (real gas) in a container is $P = \dfrac{RT}{2V - b} - \dfrac{a}{4b^2}$, then find mass of the gas in the container.

(ii)



PV = NKT

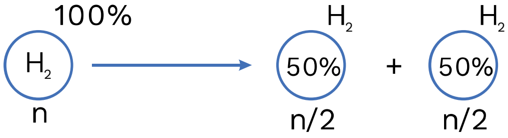
$$\frac{P_1V_1}{P_2V_2} = \frac{N_1KT}{N_2KT} \Rightarrow \frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{1}{6}$$

$$V_1 = \frac{1}{7} \times 4V = \frac{4V}{7}$$

$$V_2 = \frac{6}{7} \times 4V = \frac{24V}{7}$$

Ex. In a close vessel H_2 gas is filled at 'T' temperature. If its temperature is made 2 times then 50% H_2 gas is dissociates in atoms then find out final pressure of gas when initial pressure is P_0.

Sol.



$PV = nRT \Rightarrow P \propto nT$

$$\frac{P_0}{P_f} = \frac{nT}{\frac{3}{2}n(2T)} \Rightarrow P_f = 3P_0$$

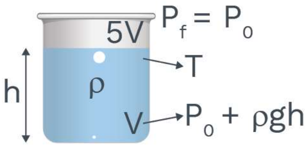
$H_2 \rightarrow H + H$

$$n = \frac{M}{M_w} \Rightarrow n' = \frac{M}{\frac{M_w}{2}} = \frac{2M}{M_w}$$

$\Rightarrow$ n' = 2n

Ex. A bubble start rising from the bottom of a lake and reach on surface then its volume is made 5 times then out the depth of lake when temperature of bottom and surface is same.

Sol.



PV = nRT,   PV = constant,

$P_iV_i = P_fV_f$

$\Rightarrow \quad (P_0 + \rho gh)V_0 = (P_0) \times 5V_0$

$\rho gh = 5P_0 - P_0 \qquad \Rightarrow \qquad \rho gh = 4P_0$

$1000 \times 10h = 4 \times 10^5$

$\Rightarrow \quad h = 40\text{m}$

Note: $\boxed{h = 10(n - 1)}$ (If temperature is constant then depth of length). Where n is times of volume.

$h = 10(5 - 1) = 40$ m

Ex. In the above question if radius of bubble is made 2 times then find out the depth of lake.

Sol. $V = \frac{4}{3}\pi R^3 \Rightarrow V \propto R^3, \quad V_f = 8V$

$h = 10(8 - 1) = 70$ m

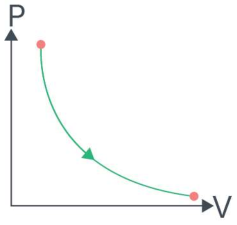
Laws of Gases:

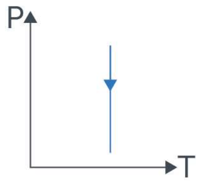
1. **Boyle's law:** At constant temperature for constant mass of the gas.

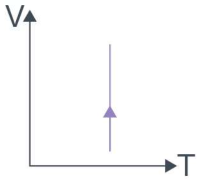
$\boxed{T = \text{constant}}$

$PV = nRT \Rightarrow PV = \text{constant}$

$\boxed{P \propto \frac{1}{V}}$







Concept Reminder

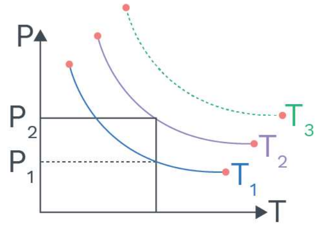
When T = Constant

$P \propto \frac{1}{V}$

PV = C

$\frac{P_1}{P_2} = \frac{V_2}{V_2}$

Ex. Compare the temperature in given isothermal curve.



Sol. PV = nRT $\Rightarrow$ $P \propto T$

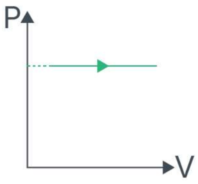
$P_2 > P_1$ $\Rightarrow$ $T_2 > T_1$

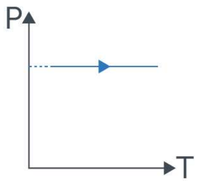
$\Rightarrow T_3 > T_2 > T_1$

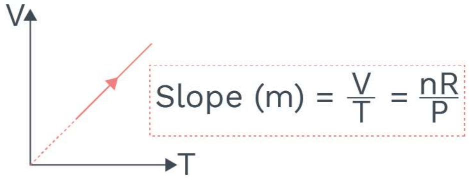
2. **Charle's Law:** At constant pressure for constant mass of the gas.

$\boxed{P = \text{constant}}$

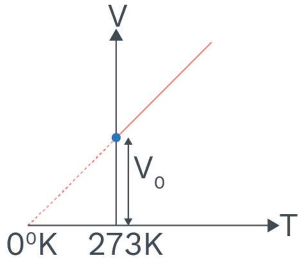
$$\Rightarrow \quad PV = nRT \quad \Rightarrow \quad V \propto T, \quad \frac{V_1}{V_2} = \frac{T_1}{T_2}$$







$$\text{Slope (m)} = \frac{V}{T} = \frac{nR}{P}$$



$V \propto T \Rightarrow V = AT$

$V = A(T + 273)$

$V = AT + A \times 273$

compare with equation

$y = mx + c$

Key Points

- Boyle's law
- Isothermal

Concept Reminder

When P = Constant

$V \propto T$

$\frac{V}{T} = C$

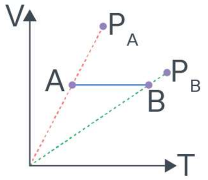
$\rho \times T = C$

$\rho_1 T_1 = \rho_2 T_2$

Rack your Brain

A gas at 27°C temperature and 30 atm pressure is allowed to expand to the atmospheric pressure. If the volume becomes 10 times its initial volume, then calculate the final temperature.

Ex. For given curve compare the pressure at point A and B?



Sol. $V \propto T = P =$ constant,

Slope (m) $\propto \frac{1}{P}$

$\theta_A > \theta_B$

$\Rightarrow$ Slope of $A(m_A) >$ Slope of $B(m_B)$

$\Rightarrow$ $P_B > P_A$

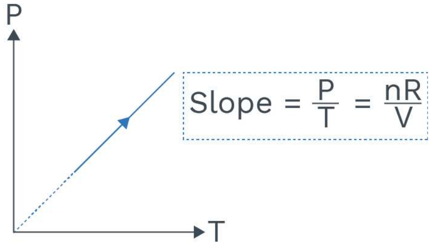
Key Points

- Charle's law
- Isobaric

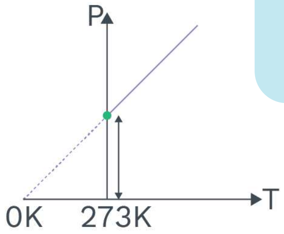
3. Gay-lussac Law:

At constant volume for constant mass of gas.

$\boxed{V = \text{constant}} \Rightarrow PV = nRT \Rightarrow P \propto T$



Slope = $\frac{P}{T} = \frac{nR}{V}$

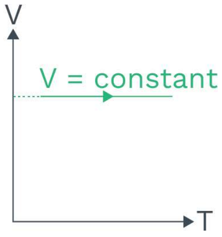


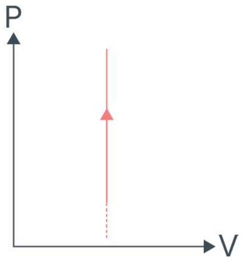
Concept Reminder

When V = Constant

$P \propto T$

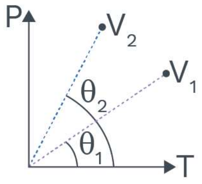
$\boxed{\frac{P_1}{T_1} = \frac{P_2}{T_2}}$





- Pressure and volume never be negative.
- Temperature in Kelvin is never be negative but temperature in °C may be negative.
- 0°K temperature is called absolute Kelvin temperature.

Ex. For given curve, compare the volume.



Sol. $P \propto T \Rightarrow V = \text{constant}$

Slope (m) $\propto \frac{1}{V}$

$m = \tan\theta \quad \Rightarrow \quad \theta_2 > \theta_1$

$\Rightarrow m_2 > m_1$

$V_1 > V_2$

4. Avogadro Law:

- According to this law at same pressure, same volume and same temperature number of molecules are same.
- At NTP number of molecules in 1 mol is $N_A = 6.023 \times 10^{23}$.

Key Points

- Isochoric
- Absolute

5. Grahm's Law of Diffusion:-

- When 2 gases at the same temperature and pressure are allowed to diffuse into each other, then rate of diffusion of each gas is inversely proportional to the square root of the density of the gas, i.e. $r \propto \frac{1}{\sqrt{\rho}} \propto \frac{1}{\sqrt{M}}$ (M is the molecular weight of the gas) $\Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{M_2}{M_1}}$.

If V is the volume of gas diffused to t sec then

$$r = \frac{V}{t} \Rightarrow \frac{r_1}{r_2} = \frac{V_1}{V_2} \times \frac{t_2}{t_1}$$

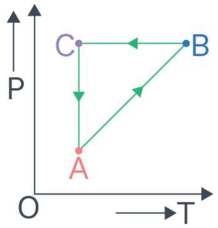
6. Daltons Law:

- According to this law total pressure of gases mixture is equal to sum of partial pressure of gases.

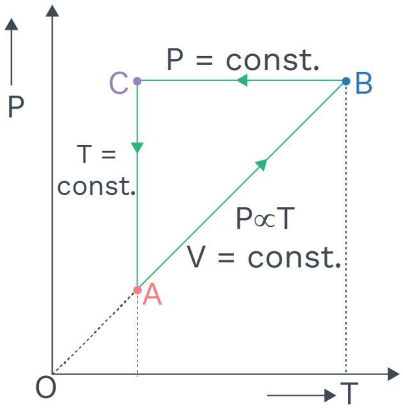
$$\boxed{P_{mix} \propto n}$$

$$\boxed{P_{total} = P_1 + P_2 + P_3 + ...}$$

Ex. Plot P-V, T-V graph corresponding to the P-T graph for an ideal gas shown in figure.



Sol.



(i) A → B

$PV = nRT,$ V = constant

$P \propto T(k)$

T ↑, P ↑, V = constant

(ii) B → C

$PV = nRT,$ P = constant

$V \propto T(k)$

T ↓, V ↓, P = constant

(iii) C → A

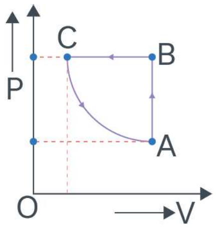
$PV = nRT,$ T = constant

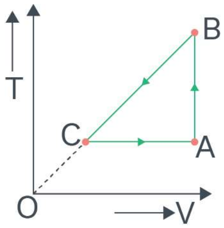
$$P \propto \frac{1}{V}$$

P ↓, V ↑, T = constant

By graph:

(i) P and V (ii) T and V





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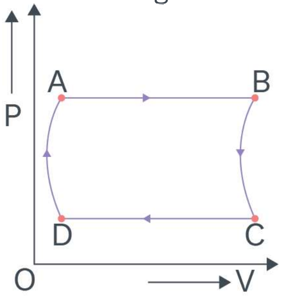

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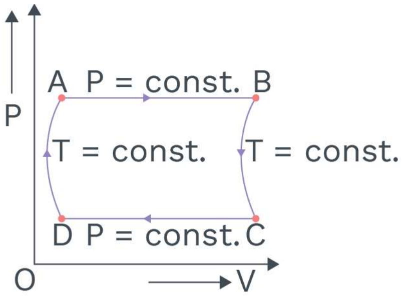


(i) $A \rightarrow B$

$PV = nRT$,

$P = \text{constant}$

$V \propto T$

$V \uparrow, T \uparrow$

(ii) $B \rightarrow C$

$PV = nRT$, $T = \text{constant}$

$P \propto \dfrac{1}{V}$

$V \uparrow, P \downarrow$

(iii) $C \rightarrow D$

$PV = nRT$

$P = \text{constant}$

$V \propto T$

$V \downarrow, T \downarrow$

(iv) $D \rightarrow A$

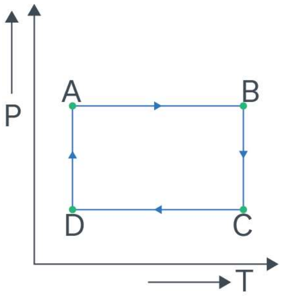
$PV = nRT$

$T = \text{constant}$

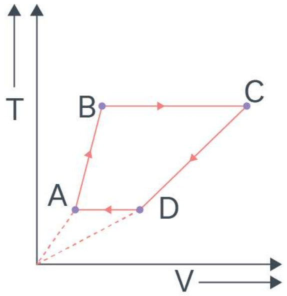
$P \propto \dfrac{1}{V}$

$V \downarrow, P \uparrow$

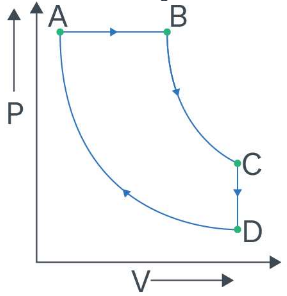
P and T



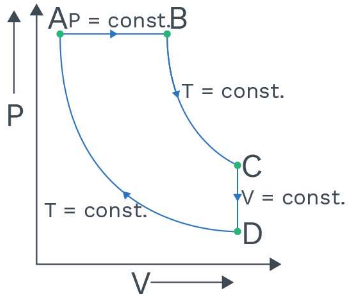
T and V



Ex. Plot P-T, T-V graph corresponding to the P-V graph for an ideal gas shown in figure.

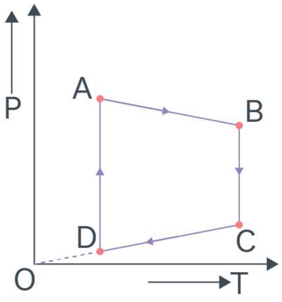


Sol.

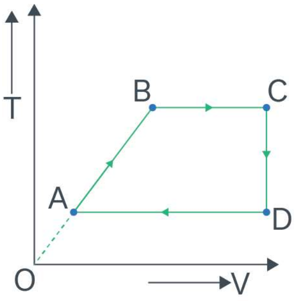


(i)	$A \rightarrow B$ $PV = nRT$, P = constant $V \propto T$ $V \uparrow, T \uparrow$	(ii)	$B \rightarrow C$ $PV = nRT$, T = constant $P \propto \frac{1}{V}$ $V \uparrow, P \downarrow$
(iii)	$C \rightarrow D$ $PV = nRT$ V = constant $P \propto T$ $P \downarrow, T \downarrow$	(iv)	$D \rightarrow A$ $PV = nRT$ T = constant $P \propto \frac{1}{V}$ $V \downarrow, P \uparrow$

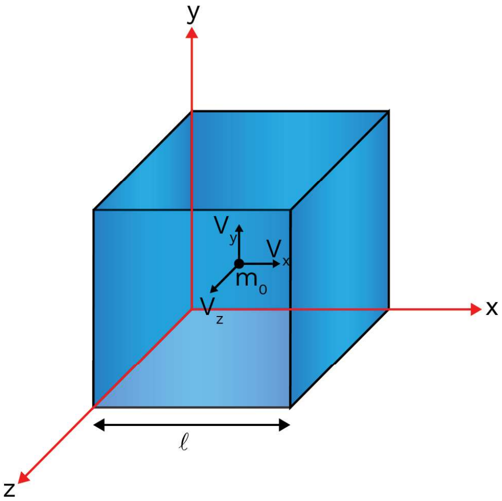
P and T



T and V



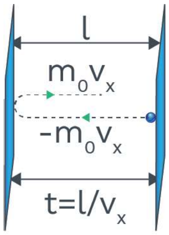
Pressure exerted by gas molecules on the wall of container:



$$\text{Pressure (P)} = \frac{F_\perp}{A}, \quad \vec{F} = \frac{d\vec{P}}{dt}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2} \Rightarrow v^2 = v_x^2 + v_y^2 + v_z^2$$



$$\Delta P = -2\,mv_0$$

$$\therefore\quad F_{avg} = \frac{\Delta p}{\Delta t} = 2\,mv_x \times (\text{collision frequency})$$

$$\left(t = \frac{2l}{v_x}\right)$$

$$= \frac{2mv_x}{2l} \times v_x \Rightarrow \qquad F = \frac{mv_x^2}{l}$$

$$F_{total\,avg} = F_1 + F_2 + \ldots.. = \frac{m_0 v_{x_1}^2}{l} + \frac{m_0 v_{x_2}^2}{l} + \ldots$$

$$(F_{avg})_x = \frac{m_0}{l}(\Sigma\, v_x^2)$$

$$P = \frac{(F)_\perp}{l^2} = \frac{m_0}{l}\cdot\frac{\Sigma\, v_{x_1}^2}{l^2} = \frac{m_0}{l^3}\Sigma\, v_{x_1}^2$$

Let, $v_x = v_y = v_z$

$$v^2 = 3v_x^2 \qquad \Rightarrow v_x^2 = \frac{v^2}{3}$$

$$P = \frac{m_0 \Sigma\, v_{x_i}^2}{l^3} \Rightarrow P = \frac{m_0\, \Sigma v_i^2}{3\,l^3}$$

$$\Rightarrow \quad P = \frac{m_0 N}{3\,V}\cdot\left(\frac{\Sigma\, v_i^2}{N}\right)$$

$$\text{mean square velocity} = \frac{v_1^2 + v_2^2 + \ldots.}{N}$$

$$\boxed{P = \frac{m}{3\,V} < v^2 >}$$

$$P = \frac{1}{3}\int < v^2 >$$

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + \ldots.}{N}}$$

Formula's Reminder

Pressure inside the container

$$\boxed{\begin{aligned} P &= \frac{1}{3}\frac{mN}{V}V_{rms}^2 \\ &= \frac{1}{2}\rho V_{rms}^2 \end{aligned}}$$

$$\begin{aligned} V_{rms} &= \sqrt{\frac{3\,nRT}{m}} \\ &= \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3PV}{M}} \\ &= \sqrt{\frac{3\,kT}{m}} \end{aligned}$$

Average Speed for Gas:

$$\boxed{\begin{aligned} v_{avg} &= \sqrt{\frac{8}{\pi}\frac{RT}{M_w}} = \sqrt{\frac{8}{\pi}\frac{P}{\rho}} \\ &= \sqrt{\frac{8}{\pi}\frac{PV}{M}} = \sqrt{\frac{8}{\pi}\frac{KT}{m}} \end{aligned}}$$

Most Probable Speed:

$$\boxed{\begin{aligned} v_{mp} &= \sqrt{\frac{2RT}{M_w}} = \sqrt{\frac{2P}{\rho}} \\ &= \sqrt{\frac{2PV}{M}} = \sqrt{\frac{2KT}{m}} \end{aligned}}$$

$$v_{rms} = \sqrt{< v^2 >} \quad \Rightarrow \quad \boxed{P = \frac{m}{3V} \cdot v_{rms}^2}$$

$$PV = nRT \quad \Rightarrow \quad PV = \frac{m}{3} v_{rms}^2$$

$$\Rightarrow \quad nRT = \frac{m}{3} v_{rms}^2$$

$$\boxed{v_{rms} = \sqrt{\frac{3nRT}{m}}}$$

SPEED OF GAS:

1. Root mean square speed [v_{rms}]:

$$\boxed{v_{rms} = \sqrt{\frac{3RT}{M_w}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3KT}{m}}}$$

Note: If $v_{rms} > v_{escaped}$ then gases are escaped from planet i.e. atmosphere is not present at that planet (eg. moon)

If $v_{rms} < v_{escaped}$ then gases are not escaped it means atmosphere is present. (eg. earth)

At T = 0 K then $v_{rms} = 0$, at which molecular motion stop.

If pressure increases then $v_{rms} \rightarrow$ no comment because it also depends on density (ρ).

If temperature increases then v_{rms} increases.

At constant temperature, if pressure increases then v_{rms} constant.

2. Average velocity: [$< \vec{v} >$ and $\vec{v}_{avg}$]

$$\boxed{\vec{v}_{avg} = \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + ... + \vec{v}_N}{N}}$$

3. Average speed: [$<|\vec{v}|>$ or $|\vec{v}_{avg}|$]

$$\boxed{v_{avg} = \frac{v_1 + v_2 + v_3 + ... + v_N}{N}}$$

For gas:

$$\boxed{v_{avg} = \sqrt{\frac{8\ RT}{\pi\ M_w}} = \sqrt{\frac{8\ P}{\pi\ \rho}} = \sqrt{\frac{8\ PV}{\pi\ M}} = \sqrt{\frac{8\ KT}{\pi\ m}}}$$

4. Most probable speed [v_{mp}]:

Most of the gas molecules will be move with speed is called most probable speed.

Key Points

- rms speed
- Average speed
- Most probable speed

Rack your Brain

Gas at a pressure P_0 in contained is a vessel. If the masses of all the molecules are halved and their speeds are doubled, then calculate the resulting pressure P.

$$\boxed{v_{mp} = \sqrt{\frac{2RT}{M_w}} = \sqrt{\frac{2P}{\rho}} = \sqrt{\frac{2PV}{M}} = \sqrt{\frac{2KT}{m}}}$$

Some Important points:

1. $v_{avg} = \sqrt{\frac{8}{\pi}\frac{RT}{M_w}}, \quad v_{rms} = \sqrt{\frac{3RT}{M_w}}, \quad v_{mp} = \sqrt{\frac{2RT}{M_w}}$

   $v_{avg} : v_{rms} : v_{mp} = \sqrt{\frac{8}{\pi}} : \sqrt{3} : \sqrt{2}$

2. $v_{rms} > v_{avg} > v_{mp}$.

3. For all types of speed $v \propto \sqrt{T}$.

4. For one molecule

   $\boxed{v_{rms} = v_{avg} = v_{mp}}$ for 1 molecule

Ex. Find out the temperature of oxygen at which v_{rms} of oxygen is equal to v_{rms} of hydrogen at 27°C.

Sol. $v_{rms} = \sqrt{\frac{3RT}{M_w}} \Rightarrow (v_{rms})_{O_2} = (v_{rms})_{H_2}$

$$\sqrt{\frac{3RT_{O_2}}{32}} = \sqrt{\frac{3R(300)}{2}}$$

$$\Rightarrow \quad T_{O_2} = 4800\,K$$

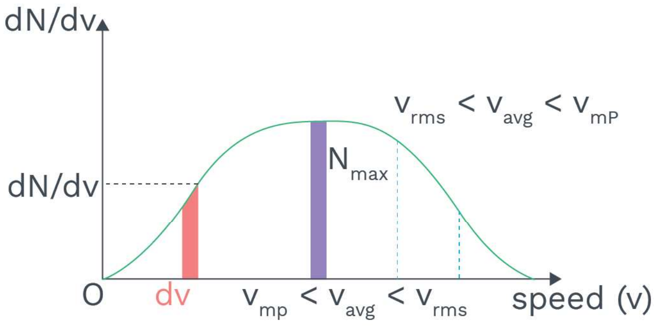
Ex. If mass of gas is double and v_{rms} is half then pressure of gas will be.

Sol. $P = \frac{1}{3}\rho v_{rms}^2$

$$P = \frac{1}{3}\frac{M}{V}v_{rms}^2 \Rightarrow P \propto Mv_{rms}^2$$

$$P' \propto 2\left(\frac{1}{2}\right)^2 \Rightarrow P' \propto \frac{1}{2} \text{ times.}$$

MAXWELL CURVE OF VELOCITY:



Concept Reminder

Effect of temperature of velocity distribution.

$\frac{dN}{dv} \Rightarrow$ Number of molecules in unit speed interval.

Area = $\frac{dN}{dv} \times dv = dN$, dN = number of molecules present in dv speed interval.

Area covered by $\frac{dN}{dv}$ versus v curve on speed axis is represent the total number of molecule in gas sample.

dN is a number of particles whose speed is between the range v to (v + dv).

$$v_{mp} = \sqrt{\frac{2RT}{m_0}}$$

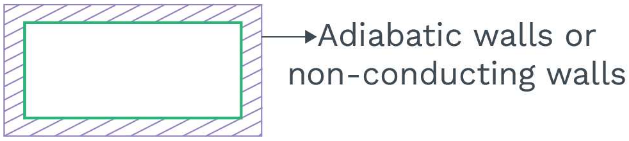
$$\boxed{\frac{dN}{dv} = 4\pi N \left(\frac{M}{2\pi KT}\right)^{3/2} v^2 \cdot e^{-mv^2/2KT}}$$

Rack your Brain

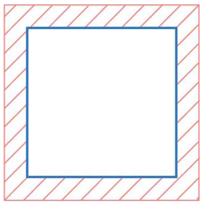
At what temperature will the rms speed of oxygen molecules become just sufficient for escaping from the Earth's atmosphere (Given: mass of oxygen molecule (m) = 2.76 × 10^{-26} kg, Boltzmann's constant $k_B = 1.38 \times 10^{-23}$ JK^{-1})

Some commonly used terms:

- No heat exchange between gas and out side surroundings.



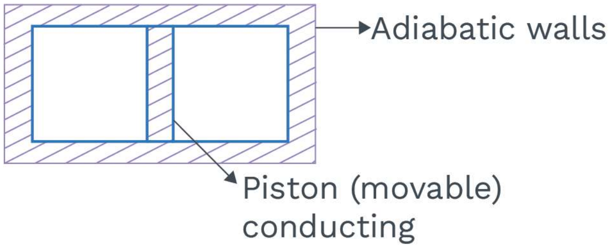
- Heat exchanges between gas and surrounding and finally temperature become same.



Key Points

- Adiabatic wall
- Heat

- Piston (conducting):
  - Movable → temperature and pressure both same.
  Fix → temperature same.
- Piston (non-conducting)
  - Movable → pressure same.
  - Fixed → then it is different chamber (no comment)



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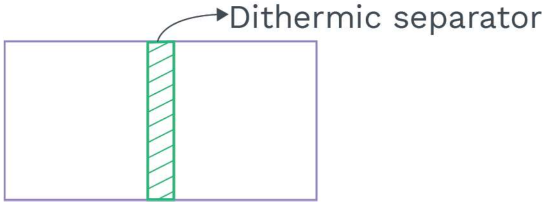
	T	P	V
Non-conducting and fixed	No comment	No comment	No comment
Non-conducting and movable	No comment	Same	No comment
Conducting and fixed	Same	No comment	No comment
Conducting and movable	Same	Same	No comment

- When heat exchange is slow as compared to the conducting walls.



Degree of freedom (Dof):

- It is the number of modes by which a gas particle can keep energy this mode of keeping of energy by a gas molecule is called degree of freedom.

For all type of gas = $f_T = 3$ (Dof)

Monoatomic; Ex. – He, Ar → Dof = 3
(v_x, v_y, v_z)

Diatomic; Ex. – H_2, O_2, N_2 = Dof = 5
$(v_x, v_y, v_z, w_x, w_y)$

Triatomic; Ex. – CO_2, O_3, H_2O →

– Linear → Dof = 5 $(v_x, v_y, v_z, w_y, w_z)$

– Non-linear → Dof = 6 $(v_x, v_y, v_z, w_x, w_y, w_z)$

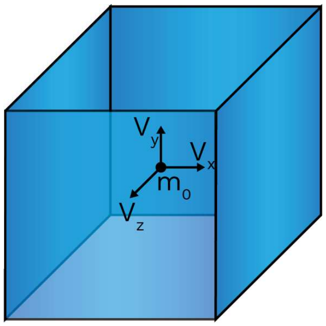
Concept Reminder

A diatomic molecule that free to vibrate will have 7 degrees of freedom.

	T > 5000 K DOF	T > 70 K DOF	NORMAL TEMPERATURE DOF
Monoatomic	3 (3T)	3 (3T)	3 (3T)
Diatomic	7 (3T + 2R + 2V)	3 (3T)	5 (3T + 2R)
Triatomic (Linear)	7 (3T + 2R + 2V)	3 (3T)	5 (3T + 2R)
Triatomic (Non-linear)	8 (3T + 3R + 2V)	3 (3T)	6 (3T + 3R)

T = Translational energy, R = Rotational energy,
V = Vibration energy

- If higher temperature particles starts vibrating about their mean position than add to 2 vibrational energy.



$$P = \frac{1}{3}\rho v_{rms}^2 \quad \Rightarrow \quad PV = \frac{1}{3}M_T v_{rms}^2$$

$$PV = \frac{2}{3}K.E. \quad \Rightarrow \quad KE = \frac{3}{2}PV = \frac{3}{2}nRT$$

Average KE of one mole = $\frac{3}{2}RT$

Average KE of one molecules = $\frac{3}{2}\cdot\frac{RT}{N_A} = \frac{3}{2}KT$

$$\begin{tikzcd} & v_x = \frac{1}{2}KT \\ \frac{3}{2}KT \arrow[ur] \arrow[r] \arrow[dr] & v_y = \frac{1}{2}KT \\ & v_z = \frac{1}{2}KT \end{tikzcd}$$

=

Maxwell's equipartition law:

- Maxwell's equipartition law states that every degree of freedom of a molecules carries same energy and that is equal to $\frac{1}{2}KT$.

One molecule energy = $\frac{f}{2}KT$

One mole energy = $\frac{f}{2}RT$

n mole energy = $\frac{f}{2}nRT$

Internal energy (U)

$= KE + PE = \frac{f}{2}nRT$ (Potential energy = 0)

Rack your Brain

Find the ratio of the specific heats $\frac{C_p}{C_v} = \gamma$ in terms of degrees of freedom (n).

Key Points

- Equipartition law
- Average kinetic energy

Concept Reminder

Different energies of a system of degree of freedom f are as follows:

1. Total energy associated with each molecule = $\frac{f}{2}kT$.
2. Total energy associated with N molecules = $\frac{f}{2}NkT$.
3. Total energy associated with 1 mole = $\frac{f}{2}RT$.
4. Total energy associated with μ moles = $\frac{f}{2}\mu RT$.
5. Total energy associated with each gram = $\frac{f}{2}rT$.
6. Total energy associated with m grams = $\frac{f}{2}mrT$.

Ex. Find out the translatory kinetic energy of He gas molecules 3×10^{22} at 27°C.

Sol. $U = nC_vT$; $\qquad n = \dfrac{3 \times 10^{22}}{6 \times 10^{23}} = \dfrac{1}{20}$

$$U = \frac{1}{20} \times \frac{fR \times 300}{2} = \frac{3 \times 300 \times 2}{40}$$

$$= \frac{90 \times 2}{4} = 45\,\text{cal}$$

Ex. Find out the total kinetic energy of 1 molecule of a gas at NTP.

Sol. Total kinetic energy of 1 molecule of gas = $\dfrac{f}{2}\left(\dfrac{RT}{N_A}\right)$

$$= \frac{3 \times 2 \times 273}{2 \times 6 \times 10^{23}} = \frac{273}{2} \times 10^{-23} = 136.5 \times 10^{-23}\,\text{cal}$$

Ex. For 1 kg diatomic gas pressure is 8×10^4 Pa and density is 4 kg/m^3 then find out total internal energy of gas.

Sol. $\qquad U = nC_vT \quad \Rightarrow \quad U = \dfrac{M}{M_w}\dfrac{fRT}{2}$

$$\Rightarrow \quad U = \frac{Mf}{2}\left(\frac{RT}{M_w}\right) \Rightarrow U = \frac{Mf}{2}\left(\frac{P}{\rho}\right)$$

$$\left(\frac{P}{\rho} = \frac{RT}{M_w}\right)$$

$$U = \frac{1 \times 5 \times 8 \times 10^4}{2 \times 4} = 5 \times 10^4\,\text{J}$$

Ex. If temperature of gas is change from 27°C to 927°C then find out percentage change in internal energy of gas.

Sol. $U \propto T$

$$\Rightarrow \quad \frac{U_i}{U_f} = \frac{T_i}{T_f} \Rightarrow \frac{100}{U_f} = \frac{300}{1200} \Rightarrow \frac{100}{U_f} = \frac{1}{4}$$

$U_f = 400$

$$\%\text{ change in } U = \frac{400 - 100}{100} \times 100 = 300\%$$

Rack your Brain

The molar specific heats of an ideal gas at constant pressure and volume are denoted by C_p and C_v, respectively. If $\gamma = \dfrac{C_p}{C_v}$ and R is the universal gas constant, then find C_v.

Ex. For a gas at 300 K temperature energy is 300 J and speed is 300 m/sec. If temperature is made 2 times of initial then find out the final energy and speed.

Sol. $U \propto T$

$$\frac{U_i}{U_f} = \frac{T_i}{T_f} \Rightarrow \frac{300}{U_f} = \frac{T}{2T} \Rightarrow U_f = 600\,J$$

$$v \propto \sqrt{T}$$

$$\frac{v_i}{v_f} = \sqrt{\frac{T_i}{T_f}} \Rightarrow \frac{300}{v_f} = \sqrt{\frac{1}{2}} \Rightarrow v_f = 300\sqrt{2}\,m/sec$$

Ex. 64 gm oxygen gas filled in a closed vessel at 27°C. temperature then find out that how much heat is require to increase its most probable speed 2 times.

Sol. Closed vessel, $V \Rightarrow$ constant,

$$\Delta Q = nC_v\Delta T, \qquad n = \frac{M}{M_w}, \quad C_v = \frac{5R}{2}$$

$$v_{mp} \propto \sqrt{T} \qquad \Rightarrow \quad T_i = 27°C = 300\,K$$

$$\frac{T_i}{T_f} = \left(\frac{v}{2v}\right)^2 \Rightarrow T_f = 4T_i$$

$$\Rightarrow \quad T_f = 4 \times 300 = 1200\,K$$

$$T_f = 1200\,K \quad \Rightarrow \quad \Delta T = 900\,K$$

$$\Delta Q = \frac{64}{32} \times \frac{5R}{2}(900) \Rightarrow \Delta Q = 9000 \text{ cal}$$

Ex. In an ideal gas at constant volume, pressure is made 2 time and after that at constant pressure volume is made 2 times then find out the specific heat in complete process.

Sol. **Case-I:** V = constant

$$P \propto T$$

$$\frac{P}{2P} = \frac{T}{T'}$$

$$T' = 2T$$

Case-II: P = constant

$$V \propto T$$

$$\frac{V}{2V} = \frac{T'}{T''}$$

$$T'' = 2T' = 4T$$

Now specific heat is,

$$Q = Q_1 + Q_2 \quad \Rightarrow \quad nC\Delta T = nC_v\Delta T_1 + nC_P\Delta T$$

$$C(4T - T) = \frac{3R}{2}[2T - T] + \frac{5R}{2}[4T - 2T]$$

$$C[3T] = \frac{3RT}{2} + \frac{10RT}{2} \Rightarrow C = \frac{13R}{6}$$

Rack your Brain

Calculate the amount of heat energy required to raise the temperature of 1 g of Helium at NTP, from T_1K to T_2K.

Ex. Same amount of same diatomic gas filled in two vessel A and B. A vessel is unclosed by a movable piston and B vessel is unclosed by fixed piston then same amount of heat is given in both vessel then vessel A temperature is increased by 20°C then find out the temperature change of vessel B.

Sol. A, P = constant (C_p), B, V = constant (C_v)

$\Delta Q_1 = nC_p\Delta T_1$ $\Delta Q_2 = nC_v\Delta T_2$

$$\Delta Q_1 = \Delta Q_2 \Rightarrow \frac{n\,7R}{2}(20) = n\frac{5\,R\Delta T_2}{2}$$

$$\Delta T_2 = 28°C$$

Rack your Brain

One mole of an ideal monoatomic gas undergoes a process described by the equation PV^3 = constant. Then find the heat capacity of gas during this process.

Ex. For hydrogen gas gram specific heat is $C_p - C_v = \frac{x}{2}$ and for oxygen gas gram specific heat is $C_p - C_v = \frac{y}{8}$ then find out $\frac{x}{y}$.

Sol. $H_2 \Rightarrow C_p - C_v = \frac{x}{2} \Rightarrow \frac{R}{M_w} = \frac{R}{2} = \frac{x}{2} \Rightarrow x = R$

$C_p - C_v = \frac{R}{M_w}$ (for gram specific heat)

$$O_2 \Rightarrow C_p - C_v = \frac{y}{8} \Rightarrow \frac{R}{M_w} = \frac{R}{32} = \frac{y}{8} \Rightarrow y = \frac{R}{4}$$

$$\Rightarrow \frac{x}{y} = \frac{\frac{R}{1}}{\frac{R}{4}} \Rightarrow \frac{x}{y} = 4$$

Gases Mixture:

$$M_{mix} = M_1 + M_2 + M_3 + \dots\dots$$

$$n_{mix} = n_1 + n_2 + n_3 + \dots\dots$$

$$U_{mix} = U_1 + U_2 + U_3 + \dots\dots$$

$$(M_w)_{mix} = \frac{n_1M_{w_1} + n_2M_{w_2} + \dots}{n_1 + n_2 + \dots}$$

$$(C_v)_{mix} = \frac{n_1C_{v_1} + n_2C_{v_2} + \dots}{n_1 + n_2 + \dots}$$

$$(C_P)_{mix} = \frac{n_1C_{P_1} + n_2C_{P_2} + \dots}{n_1 + n_2 + \dots}$$

$$\gamma_{mix} = \left(\frac{C_P}{C_V}\right)_{mix} = \frac{n_1C_{P_1} + n_2C_{P_2} + \ldots.}{nC_{V_1} + nC_{V_2} + \ldots.}$$

$$T_{mix} = \frac{n_1C_{V_1}T_1 + n_2C_{V_2}T_2 + \ldots.}{nC_{V_1} + nC_{V_2} + \ldots.}$$

For same atomic gas:

$$C_{V_1} = C_{V_2}$$

$$T_{mix} = \frac{n_1T_1 + n_2T_2}{n_1 + n_2}$$

Ex. 1 mole He gas and 1 mole H_2 gas are mixed together then find out the γ_{mix}.

Sol. $$\gamma_{mix} = \frac{n_1C_{P_1} + n_2C_{P_2}}{n_1C_{V_1} + n_2C_{V_2}} = \frac{1\times\frac{5R}{2} + 1\times\frac{7R}{2}}{1\times\frac{3R}{2} + 1\times\frac{5R}{2}}$$

$$= \frac{\frac{5R}{2} + \frac{7R}{2}}{\frac{3R}{2} + \frac{5R}{2}} = \frac{\frac{12R}{2}}{\frac{8R}{2}} = \frac{12}{8} = 1.5$$

Ex. 2 mole He gas and 4 mole O_2 gas are filled at temperature the find out U_{mix}.

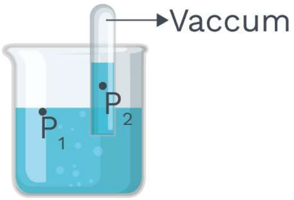
Sol. $U_{mix} = U_1 + U_2$ $\Rightarrow$ $U = nC_vT$

$$U_{mix} = 2\left(\frac{3R}{2}\right)T + 4\left(\frac{5R}{2}\right)T$$

$= 3RT + 10RT$ $\Rightarrow$ $U_{mix} = 3RT + 10RT$

$= 13\ RT$

Pallet Problem:

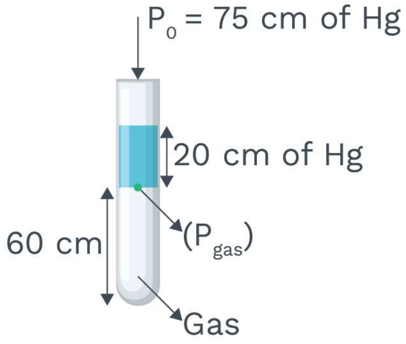


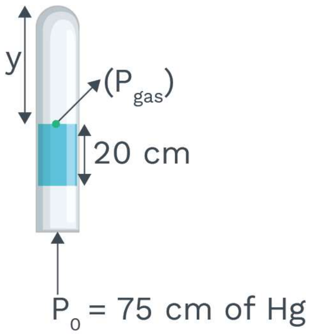
$P_{atm} = P_1 = P_2 = h\rho_{Hg}g$

$1 \times 10^5 = h \times 13600 \times 10$

h = 76 cm → (Barometer)

Ex. Find out pressure of gas (P_{gas}) and new length of gas column in tube if tube is inverted.





Sol. Initially, P_{gas} = 20 cm of Hg + 75 cm of Hg

P_{gas} = 95 cm of Hg

Finally, $P_0 = P_{gas}$ + 20 cm of Hg

$75 = P_{gas}$ + 20 cm of Hg

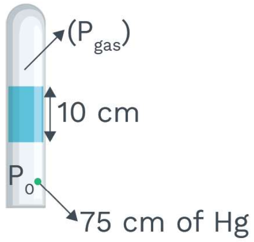
P_{gas} = 55 cm of Hg

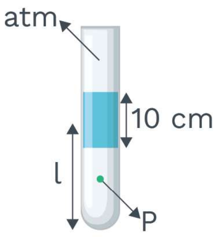
$P_1V_1 = P_2V_2$

$\Rightarrow \quad 95 \times 60 \times A = 55 \times y \times A$

y = 103.63 cm

Ex. Find the new length of gas column in tube if tube is inverted (assume temperature is constant)





Sol. Initially:

$$P_{gas} + 10 = 75 \quad \Rightarrow P_{gas} = 65$$

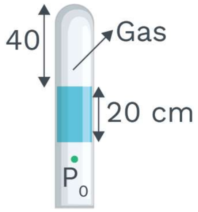
Finally:

$$75 + 10 = P_{gas} \quad \Rightarrow P_{gas} = 85\,\text{cm}$$

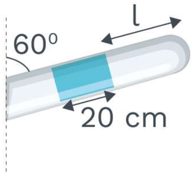
$$P_1V_1 = P_2V_2$$

$$85 \times A \times l = 65 \times 30 \times A \ \Rightarrow l = \frac{1950}{85}\ \text{cm}$$

Ex. Find the new length of gas column in tube if tube is rotated at an angle 60° as shown. (Assume constant temperature)



$\Rightarrow$



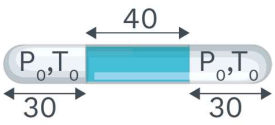
Sol. $P_{gas} = 75 - 20 = 55 \quad [P + 20 \cos 60° = 75]$

$$P + 20 \times \frac{1}{2} = 75 \qquad P = 75 - 10 = 65$$

$$P_1V_1 = P_2V_2 \qquad \Rightarrow 55 \times 40 \times A = 65 \times l \times A$$

$$\Rightarrow \ l = \frac{55 \times 40}{65}\text{cm}$$

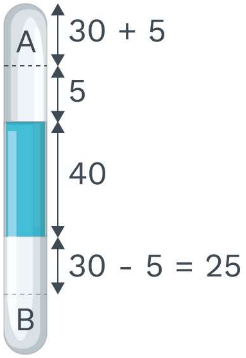
Ex. Assume constant temperature if the tube is changed to vertical position and the pallet comes down by 5 cm then find out P_0.



Sol. For upper part

$$P_1V_1 = P_0V_0 \quad \Rightarrow \quad P_1 35\,A = P_0 30\,A$$

$$\Rightarrow \quad P_1 = \frac{30}{35}P_0 \qquad \text{...(i)}$$



For lower part

$$P_2V_2 = P_0V_0$$

$$\Rightarrow \quad P_0 30\,A = P_2 25\,A$$

$$\Rightarrow \quad P_2 = \frac{30}{25}P_0 \qquad \text{...(ii)}$$

Again

$$P_1 + 40 = P_2 \qquad \text{...(ii)}$$

From (i) and (ii),

$$\frac{30}{35}P_0 + 40 = \frac{30}{25}P_0 \quad \Rightarrow 40 = 30P_0\left(\frac{1}{25} - \frac{1}{35}\right)$$

$$\Rightarrow \quad \frac{4}{3} = P_0\left(\frac{10}{25 \times 35}\right) \quad \Rightarrow P_0 = \frac{4 \times 25 \times 35}{10 \times 3} = \frac{700}{6}$$

Boyle's Law:

"For a gas which having sufficiently low density, its pressure in inversely proportional to its volume, at constant temperature."

$$P \propto \frac{1}{V} \qquad \Rightarrow \quad PV = \text{constant}$$

$$\Rightarrow \quad P_1V_1 = P_2V_2$$

(A) But volume $V = \dfrac{m}{\rho}$

From equation (1),

$$\therefore P\left(\frac{m}{\rho}\right) = \text{constant}$$

If mass m is constant, then $\dfrac{P}{\rho} =$ constant $\Rightarrow \dfrac{P_1}{\rho_1} = \dfrac{P_2}{\rho_2}$

(B) Number of molecules in unit volume $n = \dfrac{N}{V} \Rightarrow V = \dfrac{N}{n}$ from equation (1),

$$P\left(\frac{N}{n}\right) = \text{constant}$$

If N is constant then, $\dfrac{P}{n} =$ constant

$$\Rightarrow \frac{P_1}{n_1} = \frac{P_2}{n_2} = c$$

Charle's Law:-

"For a gas having sufficiently low density, if pressure is constant, then its volume is directly proportional to its absolute temperature."

$$V \propto T \qquad \Rightarrow \quad \frac{V}{T} = \text{constant}$$

$$\Rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

(A) But density $\rho = \dfrac{m}{V} \Rightarrow V = \dfrac{m}{\rho}$; m = constant

So, $\dfrac{m}{\rho T} =$ constant

$$\Rightarrow \boxed{\rho T = \text{constant}}\ \rho_1T_1 = \rho_2T_2$$

(B) For a given gas at constant pressure, volume at 0°C is V_0 then Volume at t °C is

$$V_t = V_0\left(1+\frac{t}{273.15}\right).$$

Gaylussac's Law:-

"For a gas having sufficiently low density, if volume is constant, then its pressure is directly proportional to its absolute temperature."

$$P \propto T \qquad \Rightarrow \frac{P}{T} = \text{constant}$$

$$\Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

(A) For a given mass of gas, if volume is constant and pressure at 0°C is P_0 so pressure at t °C is

$$P_t = P_0\left[1+\frac{t}{273.15}\right]$$

Avogadro's hypothesis:

"For given constant temperature and pressure, number of molecules per unit volume is same for all gases." Number of molecules in 1 mole gas is $N_A = 6.023 \times 10^{23}$

Graham's law for gas expansion:

"If two different gases are mixed at constant temperature and pressure, then rate of mixing is inversely proportional to square root of density of gas."

$$r \propto \frac{1}{\sqrt{\rho}}, \quad \rho = \text{density}$$

$$r \propto \frac{1}{\sqrt{M}}, \quad M = \text{Molecular mass of gas}$$

$$\therefore \frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{M_2}{M_1}}$$

If volume V of given gas mixes in time t then

$$r = \frac{V}{t}$$

$$\frac{r_1}{r_2} = \frac{V_1}{V_2} \times \frac{t_2}{t_1}$$

Ideal Gas Equation:

$PV = \mu RT$ for μ mole gas

$PV = \left(\frac{R}{N_A}\right)T$ for 1 mole gas

$= k_B T$

$k_B = \frac{R}{N_A} = 1.38 \times 10^{-23}\ JK^{-1} =$ **Boltzmann's Constant**

$= Nk_B T$ for N molecules

$\Rightarrow PV = \left(\frac{R}{M}\right)T$ for 1 g gas

$PV = rT$ where, r = specific Gas constant

$PV = mrT$ for m g gas.

where $r = \frac{R}{M} =$ Gas constant per unit mass.

unit of $r = Jg^{-1}k^{-1}$

Van-der-waal's correction:

(A) Correction in Volume: $(V - b)$

Where $b = \frac{1}{8}\frac{RT_c}{P_c}$

(B) Correction in pressure: $\left(P + \frac{a}{V^2}\right)$

Where $a = \frac{27}{64}\frac{R^2T_c^2}{P_c}$

Where P_c = Critical pressure, T_c = Critical temperature & V_c = Critical volume.

For one mole gas $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

for μ mole gas $\left(P + \frac{\mu^2 a}{V^2}\right)(V - \mu b) = \mu RT$

Pressure of Gas:

$PV = Nk_B T$

N = Number of molecules

$$P = \frac{N}{V} k_B T$$

$$P = n k_B T$$

Where $n = \frac{N}{V} =$ Number of molecule in unit volume.

$$PV = \mu RT$$

$$PV = \frac{M}{M_0} RT$$

$$\mu = \frac{M}{M_0} = \frac{\text{Mass of gas}}{\text{Molecular mass of gas}}$$

$$P = \frac{M}{V} \frac{R}{M_0} T$$

$$P = \frac{\rho RT}{M_0}$$

where $\rho = \frac{M}{V} =$ Density of gas

$$P = \frac{1}{3} \rho < v^2 > \quad \text{where } < v^2 > = v_x^2 + v_y^2 + v_z^2$$

$$< v^2 > = < v_x^2 > + < v_x^2 > + < v_x^2 >$$

$$v_{rms} = \sqrt{\frac{3P}{\rho}}$$

where $\rho = \frac{M}{V} =$ Density of gas

$$P = \frac{1}{3} \rho < v^2 >, \text{ where } < v^2 > = v_x^2 + v_y^2 + v_z^2$$

$$< v^2 > = < v_x^2 > + < v_x^2 > + < v_x^2 >$$

$$v_{rms} = \sqrt{\frac{3P}{\rho}}, \quad < v^2 > = 3 < v_x^2 >$$

Where $\sqrt{< v^2 >} = v_{rms}$

$$= \sqrt{\frac{3k_B T}{m}}$$

where m = mass of one molecule.

$$= \sqrt{\frac{3RT}{M_0}}$$

where M_0 = molecular mass

Internal energy $E_{int} = \frac{f}{2}\mu RT$

Here f = Degrees of freedom
Monoatomic gas (He, Ne, Ar, ...) f = 3
Diatomic gas (H2, O2, N2, ...) f = 5 (Rigid rotator)
CO_2 f = 7

EXAMPLES

Q1 **The rms speed of H_2 molecules of an ideal H_2 gas kept in a gas chamber at 0°C is 3180 m/s. Find the pressure of the hydrogen gas.**
(Density of hydrogen gas is 8.99×10^{-2} kg/m³, 1 atmosphere = 1.01×10^5 N/m²)

Sol. As $P = \frac{1}{3}\rho v_{rms}^2 = \frac{1}{3}\left(8.99 \times 10^{-2}\right) \times \left(3180\right)^2$

$= 3.03 \times 10^5$ N/m² = 3.0 atm.

Q2 **A cylinder contained 10 kg of gas at pressure 10^7N/m². If final pressure is 2.5×10^6 N/m find the quantity of gas taken out of cylinder (assume the temperature of gas is constant).**

Sol. At constant T for the given volume of gas

$$\frac{P_1}{P_2} = \frac{m_1}{m_2}$$

$$\Rightarrow \frac{10^7}{2.5 \times 10^6} = \frac{10}{m_2} \Rightarrow m_2 = \frac{2.5 \times 10^6 \times 10}{10^7} = 2.5 \text{ kg}$$

Amount of gas taken out of the cylinder

= 10 – 2.5 = 7.5 kg.

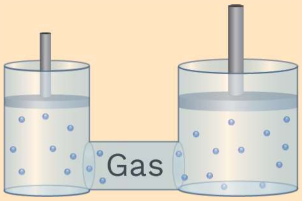
Q3 **A fish released an air bubble of volume V_0 at a depth h in a lake. The bubble comes to the surface. Take constant temperature and standard atmospheric pressure P above the lake. Calculate the volume of the bubble just before touching the surface (density of water is ρ, gravity = g).**

Sol. According to Boyle's law product of pressure and volume will remains constant at the top and bottom.

If P is the atmospheric pressure at the top of lake and the volume of bubble is V then from $P_1V_1 = P_2V_2$

$$\left(P + h\rho g\right)V_0 = PV \Rightarrow V = \left(\frac{P + h\rho g}{P}\right)V_0$$

Q4 **A gas is filled in the cylinder shown in given figure. The 2 pistons are joined by a string. If the gas is heated, the positions will**



Sol. If temperature of gas increases then it expands. As cross-sectional area of right piston is more, so that greater force will work on it (because F = PA). So position will move towards right.

Q5 **At 0°C pressure measured by barometer is 760 mm. What will be pressure at 100°C?**

Sol. From Gay Lussac's low $\frac{P_2}{P_1} = \frac{T_2}{T_1} = \left(\frac{100+273}{0+273}\right) = \frac{373}{273}$

$$\Rightarrow P_2 = \left(\frac{373}{273}\right) \times 760 = 1038 \text{ mm.}$$

Q6 **The capacity of a vessel is 3 litres. It contains 6 gram oxygen, 8 gram nitrogen and 5 gram CO_2 mixture at room temperature (27°C). If R = 8.31 J/mole kelvin, then find the pressure in the vessel in N/m^2.**
Hint : Use gas equation

Sol. Dalton's law

$$P = P_1 + P_2 + P_3 = \frac{\mu_1 RT}{V} + \frac{\mu_2 RT}{V} + \frac{\mu_3 RT}{V}$$

$$= \frac{RT}{V}\left[\mu_1 + \mu_2 + \mu_3\right] = \frac{RT}{V}\left[\frac{m_1}{M_1} + \frac{m_2}{M_2} + \frac{m_3}{M_3}\right]$$

$$= \frac{8.31 \times 300}{3 \times 10^{-3}}\left[\frac{6}{32} + \frac{8}{28} + \frac{5}{44}\right]$$

$$= 498 \times 10^3 \cong 500 \times 10^3 \cong 5 \times 10^5 \text{N / m}^2.$$

Q7 **Two gases X and Y has temperature, pressure and volume T, P and V respectively. When these gases are mixed then the volume and temperature of mixture become V and T respectively. Then the pressure and mass of the mixture will be**

Sol. From Dalton's law,

Pressure of mixture = $P_1 + P_2 = P + P = 2P$

Same as mass also will become double i.e. 2M.

Q8 **When volume of system is increased two times and temperature is decreased half of its initial temperature, then pressure becomes**

Sol. From $PV = \mu RT$ we get

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)\left(\frac{V_1}{V_2}\right) = \left(\frac{T_1/2}{T_1}\right)\left(\frac{V_1}{2V_1}\right) = \frac{1}{4} \Rightarrow P_2 = \frac{P_1}{4}$$

Q9 **A flask is filled with 13 gram of an ideal gas at 27°C and its temperature is raised to 52°C. What mass of the gas that must be released to keep the temperature of gas in the flask at 52°C and pressure remaining the same.**

Sol. $PV \propto$ Mass of gas × Temperature. In this question pressure and volume remains constant so $M_1T_1 = M_2T_2$ = constant

$$\therefore \frac{M_2}{M_1} = \frac{T_1}{T_2} = \left(\frac{27+273}{52+273}\right) = \frac{300}{325} = \frac{12}{13}$$

$$\Rightarrow M_2 = M_1 \times \frac{12}{13} = 13 \times \frac{12}{13}\,\text{gm} = 12\ \text{gm}$$

i.e. the mass of gas released from the flask = 13 gm – 12 gm = 1 gm.

Q10 **If the intermolecular forces vanish away, the volume occupied by molecules contained in 4.5 kilogram water at standard temperature and pressure will be given by**

Sol. Without intermolecular force water behaves like an ideal gas.
No of moles

$$\mu = \frac{\text{Mass of water}}{\text{Molecular wt. of water}} = \frac{4.5\ \text{kg}}{18 \times 10^{-3}\ \text{kg}} = 250,$$

T = 273 K and P = 10^5 N/m^2 (STP)
From

$$PV = \mu RT \Rightarrow V = \frac{\mu RT}{P} = \frac{250 \times 8.3 \times 273}{10^5} = 5.66\ m^3.$$

Q11 **Two ideal gases at temperature T_1 and T_2 are mixed. There is no loss of energy. If masses of molecules of the two gases are m_1 and m_2 and number of their molecules are n_1 and n_2 respectively. Find the temperature of the mixture.**

Sol. Total energy of molecules of first gas $= \frac{3}{2} n_1 k T_1$,

Total energy of molecules of second gas $= \frac{3}{2} n_2 k T_2$

Let temperature of mixture be T then total energy of molecules of mixture

$$= \frac{3}{2} k \left(n_1 + n_2\right) T$$

$$\therefore \frac{3}{2}\left(n_1 + n_2\right) kT = \frac{3}{2} k \left(n_1 T_1 + n_2 T_2\right) \Rightarrow T = \frac{n_1 T_1 + n_2 T_2}{\left(n_1 + n_2\right)}$$

Q12 **The velocities of ten particles in ms^{-1} are 0, 2, 3, 4, 4, 4, 5, 5, 6, 9. Calculate**
(1) rms speed
(2) most probable speed

Sol. (1) rms speed,

$$v_{rms} = \left[\frac{(0)^2 + (2)^2 + (3)^2 + (4)^2 + (4)^2 + (4)^2 + (5)^2 + (6)^2 + (9)^2}{10}\right]^{1/2}$$

$$= \left[\frac{228}{10}\right]^{1/2} = 4.77\ ms^{-1}$$

(2) most probable speed v_{mp} = 4 m/s

Q13 **Let A and B are the two gases and given :**

$$\frac{T_A}{M_A} = 4.\frac{T_B}{M_B};$$

where T and M stands for the temperature and the molecular mass.
If C_A and C_B are the rms speed for the given gases A and B respectively, then the ratio $\frac{C_A}{C_B}$ will be.

Sol. As $v_{rms} = \sqrt{\frac{3RT}{M}}$ $\therefore \frac{C_A}{C_B} = \sqrt{\frac{T_A / T_B}{M_A / M_B}}$

$$= \sqrt{4} = 2 \left[\text{As } \frac{T_A}{T_B} = 4\frac{M_A}{T_B} \text{ given} \right]$$

Q14 **Find the ratio of rms speeds of the gases in the mixture of nitrogen oxygen.**

Sol. We use $v_{rms} = \sqrt{\frac{3RT}{M}}$

$$\therefore \frac{v_{N_2}}{v_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{N_2}}} = \sqrt{\frac{32}{28}} = \sqrt{\frac{8}{7}}$$

Q15 **Find the ratio of specific heat at constant pressure to the specific heat constant volume for NH_3.**

Sol. For polyatomic gas ratio of specific heat $\gamma < 1.33$

Because we know that as the atomicity of gas increases its value of γ decreases.

Q16 **At constant volume the specific heat of a gas is $\frac{3R}{2}$, then find the value of γ.**

Sol. Specific heat at constant volume $C_v = \frac{R}{\gamma - 1} = \frac{3R}{2}$ (given)

$$\therefore \quad \gamma - 1 = R / C_v = 2/3 \Rightarrow \gamma = \frac{5}{3}.$$

Q17 **A gas, is heated at constant pressure. Find the fraction of heat supplied used for external work.**

Sol. We know fraction of given energy that goes to increase the internal energy is

$$= \frac{\text{Total heat supplied to increase internal energy}}{\text{Total heat supplied}} = \frac{C_v}{C_p} = \frac{1}{\gamma}$$

So we can say the fraction of given energy that supplied for external work

$$= 1 - \frac{1}{\gamma}.$$

Q18 **The average degrees of freedom per molecule for a gas is 6. The gas performs 25 J of work when it expands at constant pressure. Find the heat absorbed by gas.**

Sol. As f = 6 (given) $\therefore \gamma = 1 + \frac{2}{f} = 1 + \frac{2}{6} = \frac{4}{3}$

Fraction of energy for external work $\frac{\Delta W}{\Delta Q} = \left(1 - \frac{1}{\gamma}\right)$

$$\Rightarrow \frac{25}{\Delta Q} = \left(1 - \frac{1}{4/3}\right) = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\Rightarrow \Delta Q = 25 \times 4 = 100 \text{ Joule.}$$

Q19 **If two moles of diatomic gas and one mole of monoatomic gas are mixed with then find the ratio of specific heats.**

Sol. $\mu_1 = 1,\ \gamma_1 = \frac{5}{3}$ (for monoatomic gas) and $\mu_2 = 2,\ \gamma_2 = \frac{7}{5}$ (for diatomic gas)

From formula

$$\gamma_{\text{mixture}} = \frac{\frac{\mu_1\gamma_1}{\gamma_1 - 1} + \frac{\mu_2\gamma_2}{\gamma_2 - 1}}{\frac{\mu_1}{\gamma_1 - 1} + \frac{\mu_2}{\gamma_2 - 1}} = \frac{\frac{1\times\frac{5}{3}}{\frac{5}{3}-1} + \frac{2\times\frac{7}{5}}{\frac{7}{5}-1}}{\frac{1}{\frac{5}{3}-1} + \frac{2}{\frac{7}{5}-1}}$$

$$= \frac{5/2+7}{3/2+5} = \frac{19}{13}$$

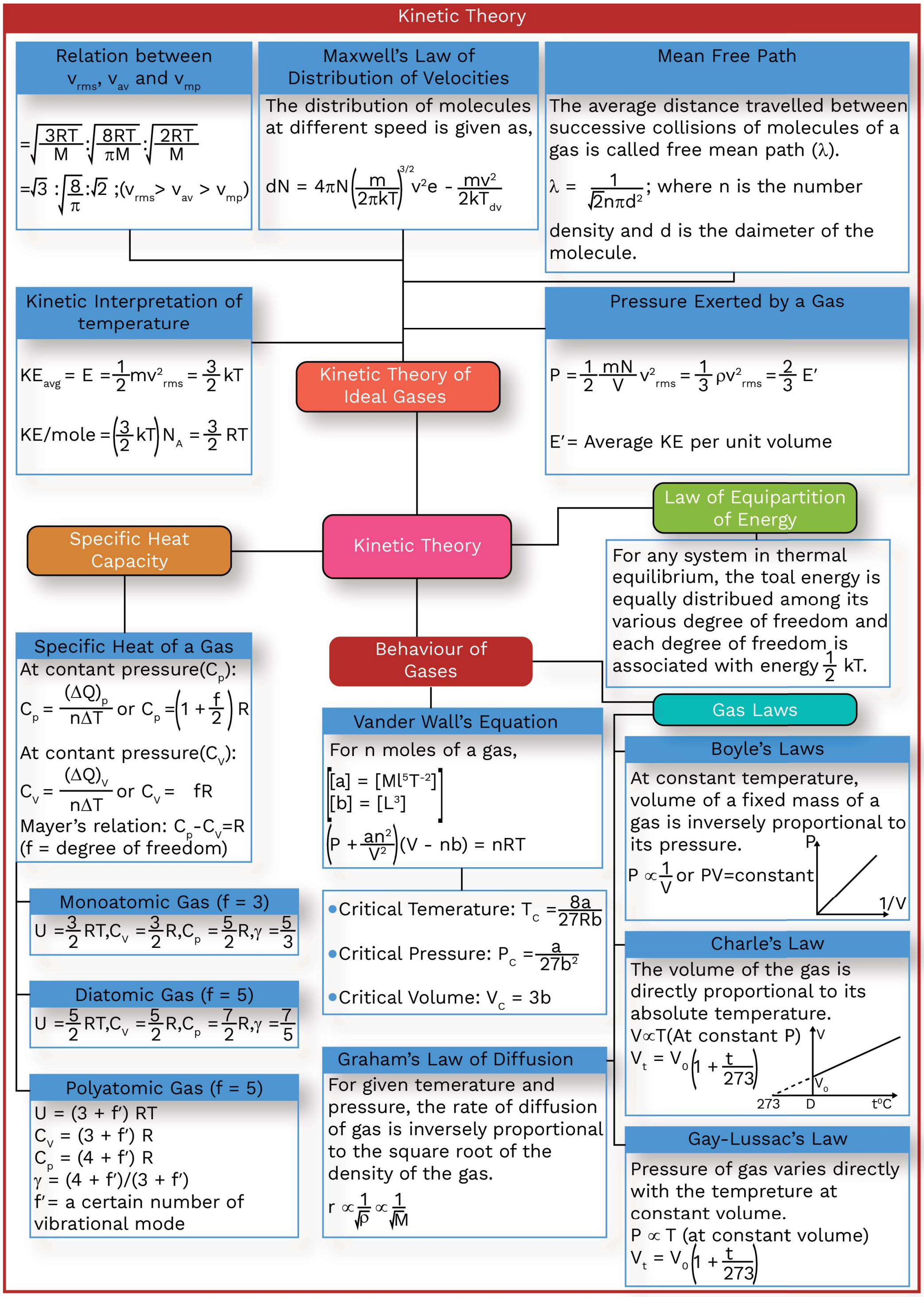
Q20 **The mean free path of N_2 molecules is 0.8×10^{-7}m at a pressure of 1.0 atm and temperature 0°C. If the number of density of molecules is 2.7×10^{25} per m^3, then find the molecular diameter.**

Sol. Mean free path $\lambda = 0.8 \times 10^{-7}$ m number of molecules per unit volume n = 2.7×10^{25} per m^3

Substituting these value in $\lambda = \frac{1}{\sqrt{2}\pi n d^2}$

we get $d = \sqrt{1.04 \times 10^{-19}} = 3.2 \times 10^{-10}\text{m} = 3.2\text{Å}$

Mind Map



Kinetic Theory

Relation between v_{rms}, v_{av} and v_{mp}

$$=\sqrt{\frac{3RT}{M}}:\sqrt{\frac{8RT}{\pi M}}:\sqrt{\frac{2RT}{M}}$$

$$=\sqrt{3}:\sqrt{\frac{8}{\pi}}:\sqrt{2}\ ;(v_{rms} > v_{av} > v_{mp})$$

Maxwell's Law of Distribution of Velocities

The distribution of molecules at different speed is given as,

$$dN = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e - \frac{mv^2}{2kT_{dv}}$$

Mean Free Path

The average distance travelled between successive collisions of molecules of a gas is called free mean path (λ).

$\lambda = \frac{1}{\sqrt{2}n\pi d^2}$; where n is the number density and d is the daimeter of the molecule.

Kinetic Interpretation of temperature

$$KE_{avg} = E = \frac{1}{2}mv^2_{rms} = \frac{3}{2}kT$$

$$KE/mole = \left(\frac{3}{2}kT\right)N_A = \frac{3}{2}RT$$

Kinetic Theory of Ideal Gases

Pressure Exerted by a Gas

$$P = \frac{1}{2}\frac{mN}{V}v^2_{rms} = \frac{1}{3}\rho v^2_{rms} = \frac{2}{3}E'$$

E′= Average KE per unit volume

Kinetic Theory

Law of Equipartition of Energy

For any system in thermal equilibrium, the toal energy is equally distribued among its various degree of freedom and each degree of freedom is associated with energy $\frac{1}{2}$ kT.

Specific Heat Capacity

Specific Heat of a Gas

At contant pressure(C_p):

$$C_p = \frac{(\Delta Q)_p}{n\Delta T} \text{ or } C_p = \left(1 + \frac{f}{2}\right)R$$

At contant pressure(C_V):

$$C_V = \frac{(\Delta Q)_V}{n\Delta T} \text{ or } C_V = \quad fR$$

Mayer's relation: $C_p - C_V = R$
(f = degree of freedom)

Monoatomic Gas (f = 3)

$$U = \frac{3}{2}RT, C_V = \frac{3}{2}R, C_p = \frac{5}{2}R, \gamma = \frac{5}{3}$$

Diatomic Gas (f = 5)

$$U = \frac{5}{2}RT, C_V = \frac{5}{2}R, C_p = \frac{7}{2}R, \gamma = \frac{7}{5}$$

Polyatomic Gas (f = 5)

$U = (3 + f')\ RT$
$C_V = (3 + f')\ R$
$C_p = (4 + f')\ R$
$\gamma = (4 + f')/(3 + f')$
f′= a certain number of vibrational mode

Behaviour of Gases

Vander Wall's Equation

For n moles of a gas,

$$\begin{bmatrix}[a] = [ML^5T^{-2}] \\ [b] = [L^3]\end{bmatrix}$$

$$\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

- Critical Temerature: $T_c = \frac{8a}{27Rb}$
- Critical Pressure: $P_c = \frac{a}{27b^2}$
- Critical Volume: $V_c = 3b$

Graham's Law of Diffusion

For given temerature and pressure, the rate of diffusion of gas is inversely proportional to the square root of the density of the gas.

$$r \propto \frac{1}{\sqrt{\rho}} \propto \frac{1}{\sqrt{M}}$$

Gas Laws

Boyle's Laws

At constant temperature, volume of a fixed mass of a gas is inversely proportional to its pressure.

$P \propto \frac{1}{V}$ or PV=constant

Charle's Law

The volume of the gas is directly proportional to its absolute temperature.

$V \propto T$(At constant P)

$$V_t = V_0\left(1 + \frac{t}{273}\right)$$

Gay-Lussac's Law

Pressure of gas varies directly with the tempreture at constant volume.

$P \propto T$ (at constant volume)

$$V_t = V_0\left(1 + \frac{t}{273}\right)$$