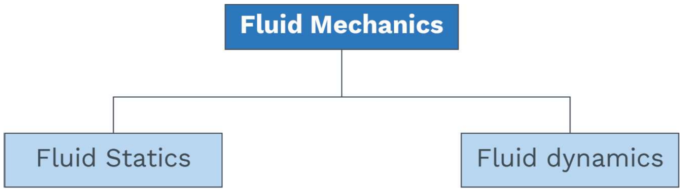
Fluid Mechanics

Fluid Mechanics

HYDROSTATICS

Fluids are the substances that can flow or deforms. Therefore, liquids and gases both are fluids.

Study of fluids at rest is known as fluid statics or hydrostatics and the study of fluid in motion is called fluid dynamics or hydrodynamics. Fluid statics and fluid dynamics collectively known as fluid mechanics.



The intermolecular forces in liquids are comparatively weaker than in solids. Therefore, their shapes can be changed easily. When external forces (shear stress) are present, liquid can flow until it conforms to the boundaries of its container. Most liquids resist compression. Unlike a liquid, a gas does not disperse to fill every space of a container and it forms a free surface.

The intermolecular forces are weakest in gases, so their shapes and sizes can be changed much easily. Gases are highly compressible and occupy the entire space of the container quite rapidly. Unlike liquid, gases can't form free surface.

A. Some important terms:

(i) Density $= \dfrac{\text{Mass}}{\text{Volume}}$

$$\boxed{\rho = \frac{M}{V}}$$

Units:

SI system: $\dfrac{Kg}{m^3}$

Definitions

Study of fluids at rest is called fluid statics or hydrostatics and the study of fluid in motion is called fluid dynamics or hydrodynamics.

Concept Reminder

Fluids have a very small shearing stress i.e., a little application of force along the tangents to their surface, brings them in motion along force.

CGS system: $\frac{gm}{cm^3}$

Note:

(i) ρ_{water} = 1000 kg/m^3 = 1 gm/cc

(ii) ρ_{Hg} = 13600 kg/m^3 = 13.6 gm/cc

Density of mixture: If two immiscible liquids of mass m_1 and m_2 and density ρ_1 and ρ_2 are mixed together then density of mixture is given by:

$$\rho_{mix} = \frac{\text{Total mass of mixture}}{\text{Total volume of mixture}}$$

$$\rho_{mix} = \frac{m_1 + m_2}{V_1 + V_2} = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}}$$

Case-I: If liquid with same masses are mixed i.e. m_1 = m_2 =m then

$\rho_{mix} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$ (Harmonic mean of individual densities)

Case-II: If liquid with same volumes are mixed i.e. V_1 = V_2 = V then

$\rho_{mix} = \frac{\rho_1 + \rho_2}{2}$ (Arithmetic mean of individual densities)

Ex. Two immiscible liquids having density 2 gm/cc and 4 gm/cc are mixed then find density of mixture if-

(a) Same volumes are taken

(b) Same masses are taken

Sol. **(a)** $\rho_{mix} = \frac{\rho_1 + \rho_2}{2} = \frac{2+4}{2} = 3 \text{ gm / cc}$

(b) $\rho_{mix} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} = \frac{2(2)(4)}{2+4} = \frac{8}{3} \text{ gm / cc}$

Concept Reminder

$$\rho_{mix} = \frac{\text{Total mass of mixture}}{\text{Total volume of mixture}}$$

$$\rho_{mix} = \frac{m_1 + m_2}{V_1 + V_2} = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}}$$

Concept Reminder

- If liquid with same masses are mixed i.e. m_1 = m_2 =m then $\rho_{mix} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$.
- If liquid with same volumes are mixed i.e. V_1 = V_2 = V then $\rho_{mix} = \frac{\rho_1 + \rho_2}{2}$.

Ex. Density of mixture is 4 gm/cc when equal volumes are taken and 3 gm/cc when equal masses are taken, then find density of individual liquid.

Sol. Let the density of liquids are ρ_1 and ρ_2

If equal volumes are taken

$$\frac{\rho_1 + \rho_2}{2} = 4 \quad \Rightarrow \rho_1 + \rho_2 = 8 \qquad \text{...(i)}$$

If equal masses are taken

$$\frac{2\rho_1\rho_2}{\rho_1 + \rho_2} = 3 \quad \Rightarrow \rho_1\rho_2 = 12 \qquad \text{...(ii)}$$

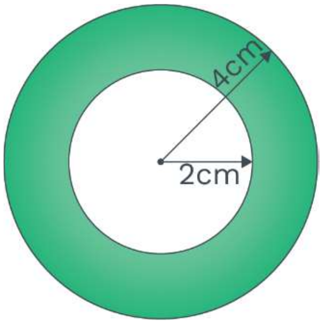
By solving both equations,

ρ_1 = 2 or 6 gm/cc

ρ_2 = 6 or 2 gm/cc

Ex. Inner and outer radius of a hollow sphere are 2cm and 4cm. If mass of hollow sphere is 2.2 kg. Find out the density of material of hollow sphere.

Sol. $V_{material} = V_{out} - V_{in}$



$$= \frac{4}{3}\pi(4^3 - 2^3) = \frac{4}{3}\pi(56)$$

$$\rho_{material} = \frac{\text{mass of material}}{\text{volume of material}}$$

$$= \frac{2.2 \times 1000}{\frac{4}{3} \times \frac{22}{7} \times 56} = \frac{75}{2} \text{ gm / cc}$$

Rack your Brain

If a pressure of 400 atm is exerted on a circular cross-section of radius 0.1 mm. What is force exerted on the area

(ii) Relative density:

$$\text{R.D.} = \frac{\text{Density of substance}}{\text{Density of pure water at } 4^\circ\text{C}}$$

$$\boxed{\text{R.D.} = \frac{\rho}{\rho_w}}$$

R.D. is an unitless and dimensionsless quantity.

(iii) Specific weight:

$$\text{Specific weight} = \frac{\text{Weight of substance}}{\text{Volume of substance}}$$

$$= \frac{mg}{V}$$

$$\boxed{\text{Specific weight} = \rho g}$$

Unit: N/m^3

(iv) Specific Gravity:

$$\text{S.G.} = \frac{\text{Specific weight of substance}}{\text{Specific weight of pure water at } 4^\circ\text{C}}$$

$$\text{S.G.} = \frac{\rho g}{\rho_w g} = \frac{\rho}{\rho_w} = \text{R.D.}$$

Note: The numerical value of specific gravity and relative density are same.

PRESSURE IN A FLUID:-

When a fluid (either gas or liquid) is at rest, it exerts a force perpendicular to any surface in contact with it, such as a container body or wall immersed in the fluid. When the fluid as a whole is at rest condition, the molecules that makes up the fluid are in motion, the force applied by the fluid is due to molecules colliding with their surroundings. If we think of an imaginary surface (plane) within the fluid, the fluid on the two sides of the surface exerts equal and opposite forces on the surface, otherwise the surface would accelerated and the fluid would not remain at rest condition. Assume a small surface of area dA centered on a point on

Concept Reminder

Relative density:

$$\text{R.D.} = \frac{\text{Density of substance}}{\text{Density of pure water at } 4^\circ\text{C}}$$

$$\boxed{\text{R.D.} = \frac{\rho}{\rho_w}}$$

Specific weight:

Specific weight =

$$\frac{\text{Weight of substance}}{\text{Volume of substance}} = \frac{mg}{V}$$

$$\boxed{\text{Specific weight} = \rho g}$$

Specific Gravity:

$$\text{S.G.} = \frac{\text{Specific weight of substance}}{\text{Specific weight of pure water at } 4^\circ\text{C}}$$

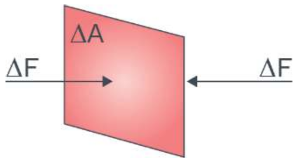
$$\text{S.G.} = \frac{\rho g}{\rho_w g} = \frac{\rho}{\rho_w} = \text{R.D.}$$

Definitions

Pressure is defined as the normal force acting per unit area of a surface i.e., $P = \frac{F_\perp}{A}$.

the fluid, the normal force applied by the fluid on each side is $dF_{\perp}$. The pressure P is explained by that point as the normal force per unit area,

$$P = \frac{dF_{\perp}}{dA}$$



If the pressure is the equal at all points of a finite plane surface with area A, then

$$P = \frac{F_{\perp}}{A}$$

where $F_{\perp}$ is the normal force on single side of the surface. The unit (SI) of pressure is called pascal, where 1 pascal = 1 Pa = 1.0 N/m^2

Single unit used principally in meteorology is the Bar which is equal to 10^5 Pa.

1 Bar = 10^5 Pa

Types of Pressures:

In our day to day activity we commonly encounter the following three types of pressures.

(i) Atmospheric pressure (P_0)

(ii) Gauge pressure (P_{gauge})

(iii) Absolute pressure ($P_{abs.}$)

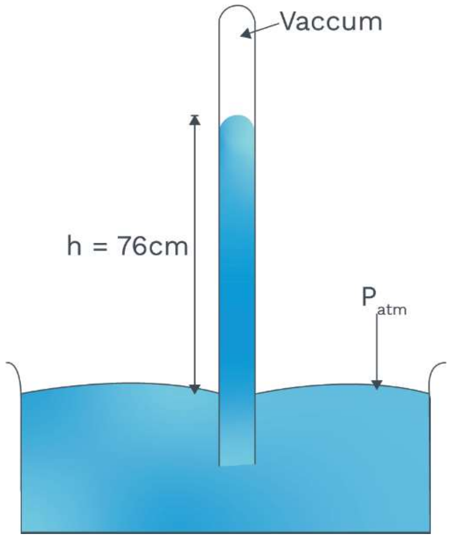
(i) Atmospheric pressure and Torricelli's experiment:

Force exerted by atmospheric column on unit cross-sectional area at mean sea level and at normal temperature is called

KEY POINTS

- Specific weight
- Relative density
- Specific gravity
- Pressure
- Atmospheric pressure
- Gauge pressure
- Absolute pressure

atmospheric pressure (P_0).



P_0 = 101.3 kN/m^2

∴ $P_0 = 1.013 \times 10^5$ N/m^2

A tube of length 1 m and uniform cross section is taken. It is filled with mercury and inverted into a mercury tray. The height of the mercury column in equilibrium inside the tube is 76 cm.

∴ Atmospheric pressure $P_0 = \rho gh$

$= 13.6 \times 10^3 \times 9.81 \times 76 \times 10^{-2}$

$= 1.013 \times 10^5$ N/m^2

Note: The above apparatus is known as a barometer. Barometer is used to measure the atmospheric pressure.

(ii) Gauge Pressure:

Excess Pressure over the atmospheric pressure ($P - P_{atm}$) measured with the help of pressure measuring instruments is called gauge pressure.

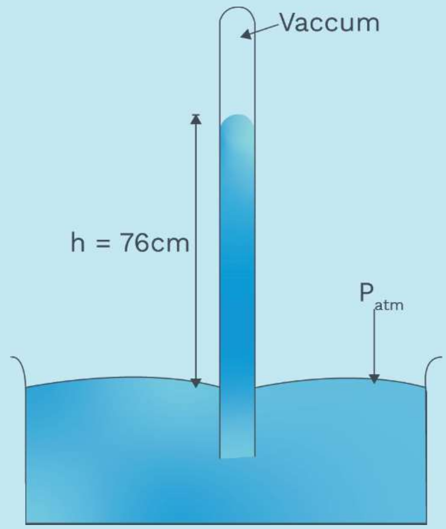
$$P_{gauge} = \frac{F}{A} = \frac{Mg}{A} = \frac{(\text{volume} \times \text{density})g}{A}$$

$$= \frac{(Ah)\rho g}{A}$$

$$P_{gauge} = h\rho g \quad \text{or} \quad P_{guage} \propto h$$

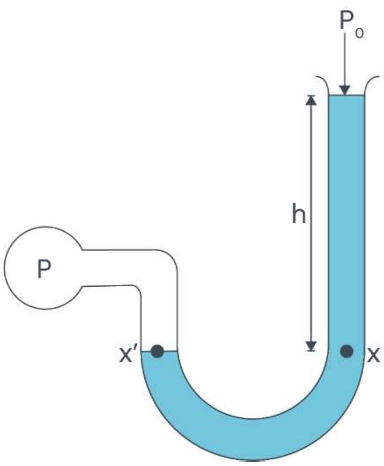
Concept Reminder

Barometer is used to measure the atmospheric pressure.



Definitions

Excess Pressure over the atmospheric pressure ($P - P_{atm}$) measured with the help of pressure measuring instruments is called gauge pressure.



$P_{x'} = P_x$

$P_{x'} = P_0 + h\rho g$

Gauge pressure = $P_{x'} - P_0 = h\rho g$

Note: Gauge pressure is always measured with the help of a "manometer".

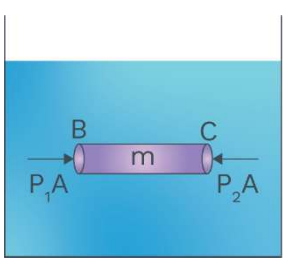
(iii) Absolute Pressure:

Sum of the atmospheric and gauge pressure is called absolute pressure.

$P_{abs} = P_{atm} + P_{guage}$

$P_{abs} = P_0 + h\rho g$

Pressure due to liquid at same horizontal level:



m = mass of liquid element

A = Area of cross section of liquid element

∵ Fluid is at rest

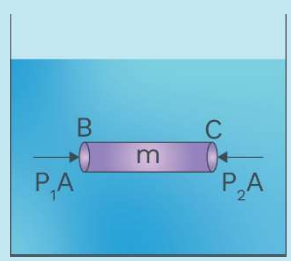
⇒ $P_1A = P_2A$

⇒ $\boxed{P_1 = P_2}$

Concept Reminder

When a force is applied over the head of a pin or a nail, it transmits a large pressure at its tip because of small area, and hence easily penetrate into the wall.

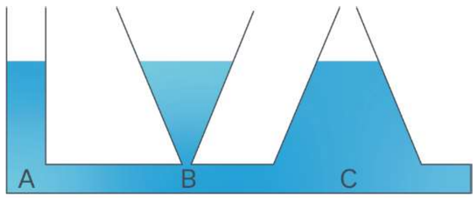
Concept Reminder



Pressure due to liquid at same horizontal level remains same if fluid is at rest.

$P_1 = P_2$

Note: Pressure exerted by a same liquid at any points does not depend on shape and size of the container (it means quantity of liquid). It depends only on the height of liquid column.

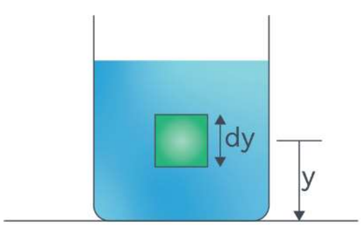


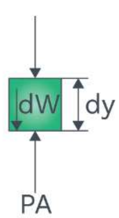
Pressure due to liquid $P_A = P_B = P_C$

Variation in Pressure with depth:-

If the weight (mg) of the fluid can be negligible, the pressure in a fluid is the equal throughout its volume. But often the fluid weight is not negligible and under this condition increasing depth below the surface the pressure increases.

Assume us now derive a general relation between the pressure 'P' at any point in a fluid at rest condition and the elevation y of that point. Let the density 'ρ' and the acceleration due to gravity 'g' are the equal throughout the fluid. If the fluid is in equilibrium condition, every volume element is in equilibrium.





Assume a thin element of fluid with height dy. The top and bottom surfaces each have area (A), and they are at elevations y and y + dy above some reference level (where y = 0). The weight of the fluid element is measured by

$$dW = (\text{volume})\ (g)\ (\text{density}) = (A\ dy)\ (\rho)\ (g)$$

Concept Reminder

To express the absolute pressure, the height of the fluid column is important and not the cross-sectional or base area or shape of the container.

dW = ρg A dy

Find out the other forces in y-direction of this fluid element? Assume the pressure at the bottom surface 'P', the total y component of upward force is PA. The pressure at the top surface is "(P + dP)" and the total y-component of downward force on the top surface is "[(P + dP)A]". The fluid element is in equilibrium condition, so the total y-component of force including the weight and the forces at the tom and bottom surfaces must be zero.

$$\Sigma F_y = 0$$

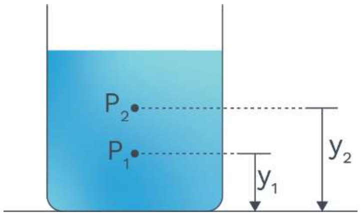
$$\therefore \quad PA - (P + dP)A - \rho g A dy = 0$$

$$\text{or} \quad \frac{dP}{dy} = -\rho g \qquad \text{...(i)}$$

Equation represent that when 'y' increases then P decreases, i.e., as we start moving upward in the fluid, pressure decreases. If 'P_1' and 'P_2' be the pressures at elevations 'y_1' and 'y_2' and if 'ρ' and g are constant, then integrating above equation (i), we get

$$\int_{P_1}^{P_2} dP = -\rho g \int_{y_1}^{y_2} dy$$

$$\text{or} \quad P_2 - P_1 = -\rho g(y_2 - y_1) \qquad \text{...(ii)}$$

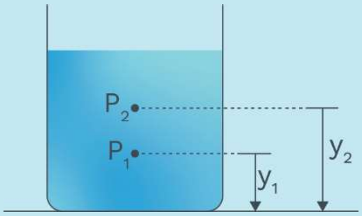


It's often convenient to express equation (ii) in terms of the depth below the surface of a fluid. Assume point 1 at depth h below the surface of fluid and assume P represents pressure at this point. Let point 2 at the surface of the fluid, where the pressure is 'P_0'(subscript zero for zero depth). The depth of point '1' below the surface is

Rack your Brain

Find out the pressure on a swimmer 20 m below the surface of a lake.
(Take g = 10 ms^{-2})

Concept Reminder



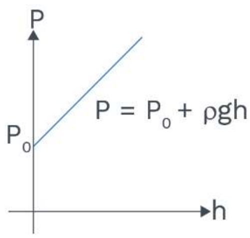
$$P_2 - P_1 = -\rho g(y_2 - y_1)$$

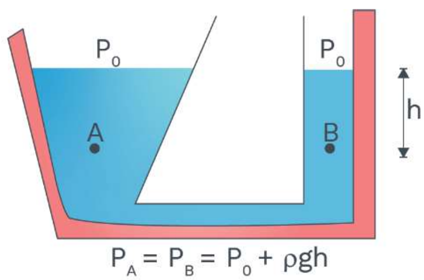
given by, $h = y_2 - y_1$
and Eq. (ii) becomes

$$P_0 - P = -\rho g(y_2 - y_1) = -\rho gh$$

$$\therefore \quad P = P_0 + \rho gh \qquad ...(iii)$$

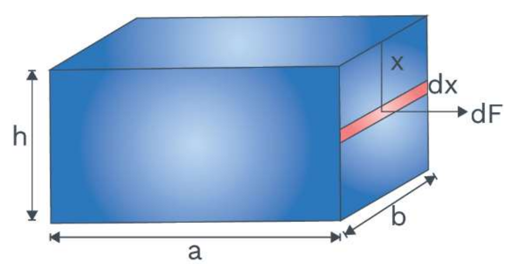
Thus, pressure increases linearly with depth, if 'ρ' and 'g' are uniformly, a graph between P and h is shown given.





Further, the pressure is the equal at any two points at the same level in the fluid. The container shape does not matter.

Force on Side Wall of Vessel:-
the force on the side wall of the container cannot be directly calculated as at different depths pressures are different-different. To find this we assume a strip of width dx at a depth x from the surface of the liquid as shown in figure, and on this strip the force due to the liquid is given as:

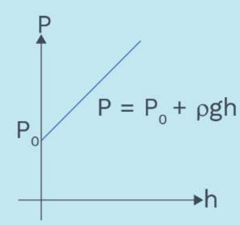


$$dF = x\rho g \times bdx$$

The force direction is acting in the normal to the side wall. Net force can be calculated by integrating equation

Concept Reminder

Pressure increases linearly with depth, if ρ and g are uniform,



$$F = \int dF = \int_0^h x\rho gbdx$$

$$F = \frac{\rho gbh^2}{2} \qquad \text{...(i)}$$

Average Pressure on Side Wall:-

The absolute pressure on the side wall can't be calculated because at different depths on this wall pressure is different. The average pressure on the wall is-

$$<p>_{av} = \frac{F}{bh} = \frac{1}{2}\frac{\rho gbh^2}{bh} = \frac{1}{2}\rho gh \qquad \text{...(ii)}$$

Equation (ii) shows that the average pressure on side vertical wall is half of the net pressure at the bottom surface of the container.

Calculated torque on the Side Wall due to Fluid Pressure:-

The force 'dF', the side wall experiences a torque about the bottom side of the side which is given as

$$d\tau = dF \times (h - x) = x\rho gb\, dx\, (h - x)$$

This net torque is,

$$\tau = \int d\tau = \int_0^h \rho gb(hx - x^2)dx$$

$$= \rho gb\left[\frac{h^3}{2} - \frac{h^3}{3}\right] = \frac{1}{6}\rho gbh^3$$

Ex. Pressure at half depth of a lake of water is equal to 2/3 times of pressure at bottom of lake. Then find the depth of the lake. (Density of water = 1000 kg/m^3)

Sol. $P_A = \frac{2}{3}P_B$

Concept Reminder

Force on Side Wall of Vessel:

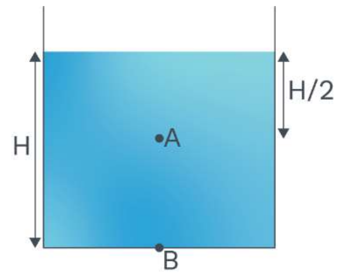
$$F = \frac{\rho gbh^2}{2}$$

Average Pressure on Side Wall:

$$<p>_{av} = \frac{F}{bh} = \frac{1}{2}\frac{\rho gbh^2}{bh} = \frac{1}{2}\rho gh$$

Torque on the Side Wall due to Fluid Pressure:

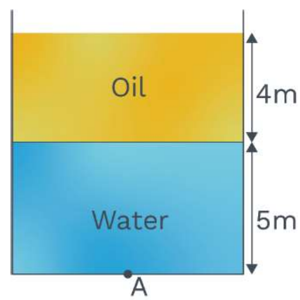
$$\tau = \frac{1}{6}\rho gbh^3$$



$$\Rightarrow \quad \left(P_{atm} + \rho g \frac{H}{2}\right) = \frac{2}{3}(P_{atm} + \rho g H)$$

$$\Rightarrow \quad H = \frac{2P_{atm}}{\rho g} = 20\,m$$

Ex. Two liquids water and oil are filled in a container as shown in figure. Then find the pressure due to liquid at Point A. (R.D.$_{oil}$ = 0.8)



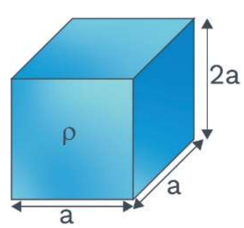
Sol. Pressure due to liquid at Point A

$= \rho_{oil}\, gh_{oil} + \rho_w\, gh_w$

$= 800 \times 10 \times 4 + 1000 \times 10 \times 5$

$= 0.82 \times 10^5$ Pa

Ex. A cuboid (a × a × 2a) is filled with a liquid of density ρ as shown in figure. Neglecting atmospheric pressure, find-



(a) Force on base wall of the cuboid

(b) Force on side wall of the cuboid

Sol. (a) Force on base wall

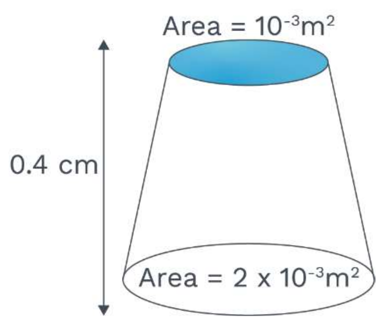
$= P_{\text{base wall}} \times A_{\text{base wall}}$

$= \rho g(2a) \times a^2 = 2\rho g a^3$

(b) Force on side wall = $P_{\text{side wall}} \times A_{\text{side wall}}$

$$= \left[\frac{0 + \rho g(2a)}{2}\right] \times 2a^2 = 2\rho g a^3$$

Ex. A uniformly tapering vessel is filled with a liquid of density 900 kg/m^3. Find out the force that acts on the base of the vessel due to the liquid (g=10 ms^{-2})



Sol. Force = Pressure × Area = ρgh × Area

$= 900 \times 10 \times 0.4 \times 2 \times 10^{-3}$

$= 7.2$ N

PASCAL'S LAW:

Pascal's law is stated in following ways-

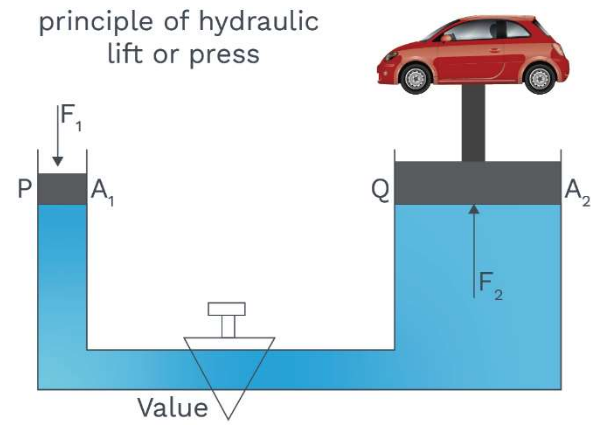
- Liquid exerts same pressures in all direction.
- If the pressure in an enclosed fluids is changed at a particular point, the changes is transmitted to every point of the fluid and to the walls of the container without being diminished in magnitude.

Applications of pascal's law: Hydraulic jacks, hydraulic lifts, hydraulic press, hydraulic brakes,

Concept Reminder

If the pressure in an enclosed fluid is changed at a particular point, the change is transmitted to every point of the fluid and to the walls of the container without being diminished in magnitude.

etc.



Pressure applied = $\frac{F_1}{A_1}$

$\therefore$ Pressure transmitted = $\frac{F_2}{A_2}$

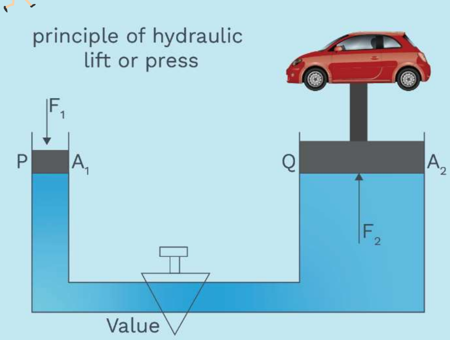
$\because$ Pressure is equally transmitted

$\therefore$ $\frac{F_1}{A_1} = \frac{F_2}{A_2}$

Upward force on A_2 is F_2

$= \frac{F_1}{A_1} \times A_2 = \frac{A_2}{A_1} \times F_1$

Concept Reminder

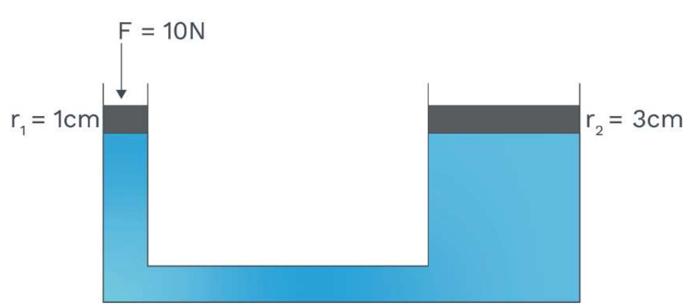


Working principle of hydraulic lift is Pascal's law

$\Rightarrow$ $\frac{F_1}{A_1} = \frac{F_2}{A_2}$

Ex. **(a)** Find the force transmitted on larger piston.

(b) If smaller piston is pushed down through 6cm, then how much the larger piston will move up.



Sol. **(a)** $\frac{F_1}{A_1} = \frac{F_2}{A_2}$

$\frac{10}{\pi(1)^2} = \frac{F_2}{\pi(3)^2}$

$F_2 = 90$ N

(b) $V_1 = V_2$

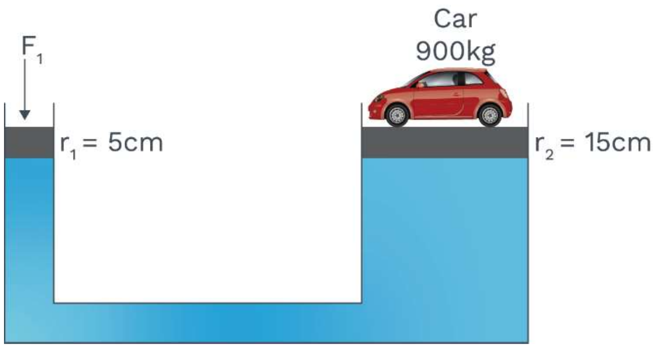
$A_1h_1 = A_2h_2$

$h_2 = \frac{A_1}{A_2}h_1 = \frac{\pi(1)^2}{\pi(3)^2}(6\,\text{cm})$

$= \frac{6}{9} = 0.67$ cm

Ex. (a) Find the force required to lift the car.

(b) Find the pressure required for it.



Sol. **(a)** $\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_1 = F_2\left(\frac{A_1}{A_2}\right)$

$\Rightarrow \quad F_1 = (900 \times 10)\left(\frac{\pi(5)^2}{\pi(15)^2}\right) = 1000$ N

(b) Required pressure $= \frac{F_1}{A_1} = \frac{1000}{\pi(5 \times 10^{-2})^2}$

$= \frac{1000}{3.14 \times 25 \times 10^{-4}} = 1.27 \times 10^5$ Pa

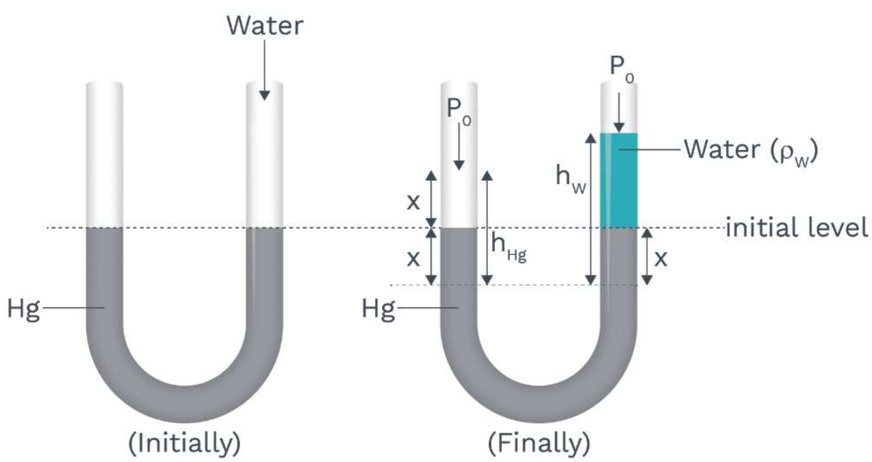
Ex. The neck and bottom of a bottle are 4 cm and 12 cm in radius respectively. If the cork is pressed with a force 10 N in the neck of the bottle, then find out the force exerted on the bottom of the bottle:

Sol. $\frac{F_1}{A_1} = \frac{F_2}{A_2}$

$$\Rightarrow \quad F_2 = \left(\frac{A_2}{A_1}\right)F_1 = \left(\frac{R_2}{R_1}\right)^2 F_1$$

$$\Rightarrow \quad F_2 = \left(\frac{12}{4}\right)^2 \times 10 = 90 \text{ N}$$

U- tube concept:



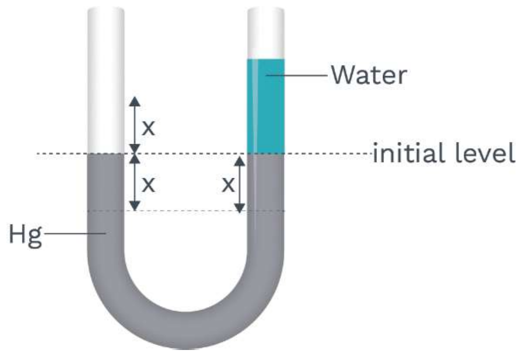
Key Concept For the same stationary liquid, pressure at same horizontal level remains same.

$P_A = P_B$

$P_0 + h_{Hg}\,\rho_{Hg}\,g = P_0 + h_w\,\rho_w\,g$

$$\boxed{h_{Hg}\rho_{Hg} = h_w\rho_w}$$

Ex. An open U-tube of same cross sectional area contains Hg. If 54.4 cm of water column is poured into one limb of the tube. How high does the Hg surface rise in the other limb from initial level. [ρ_w = 1 gm/cc, ρ_{Hg} = 13.6 gm/cc]



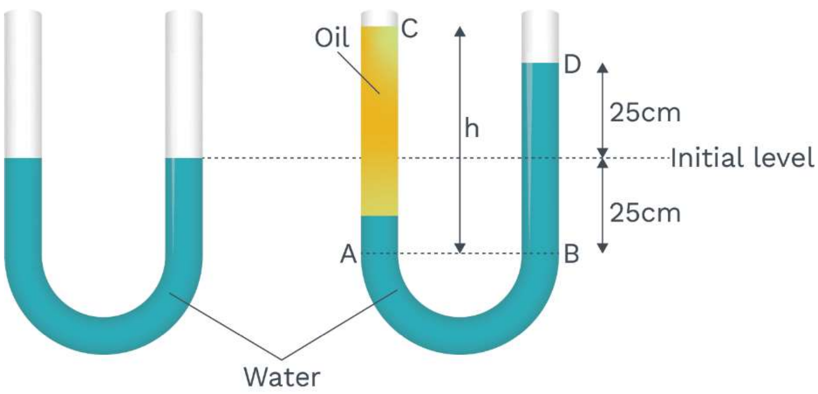
Sol. $h_w \rho_w = h_{Hg}\, \rho_{Hg}$

$\Rightarrow$ (54.4) × 1 = (2x) × (13.6)

x = 2 cm

Ex. A U-tube vessel is partially filled with water. Oil which does not mix with water is next poured into one side, until water rises by 25cm on the other side. If the density of oil is 0.8. Find the height of oil level which stand higher than the water level.

Sol. On pouring oil on left side water rises 25 cm from its previous level in the right limb of U-tube, creating a difference of levels of water by 50 cm. Let h cm be the height of oil above level A in the left limb of U-tube. Equating pressures at A and B, we get;



$P_A = P_B$

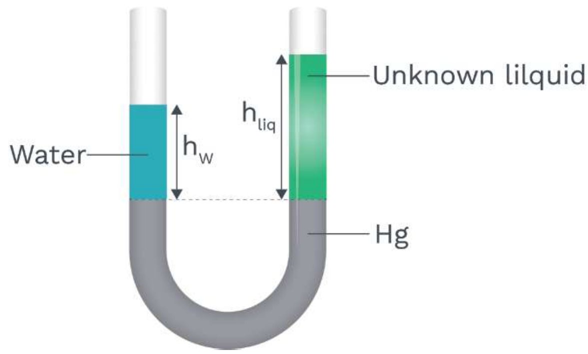
$h \times \rho_{oil} \times g = 50 \times \rho_{water} \times g$

$\therefore \quad h = \dfrac{50 \times \rho_{water}}{\rho_{oil}} = \dfrac{50 \times 1}{0.8} = 62.5$ cm

Difference of levels of oil (C) and water (D) in the two limbs = 62.5 – 50 = 12.5 cm

Ex. An open U-tube contains water and unknown liquid separated by mercury. The mercury columns in two arms are in level with 8 cm of water in one arm and 10 cm of unknown liquid in the other. Find the specific gravity of unknown liquid.

Sol. $h_w \rho_w = h_{liq} \cdot \rho_{liq}$

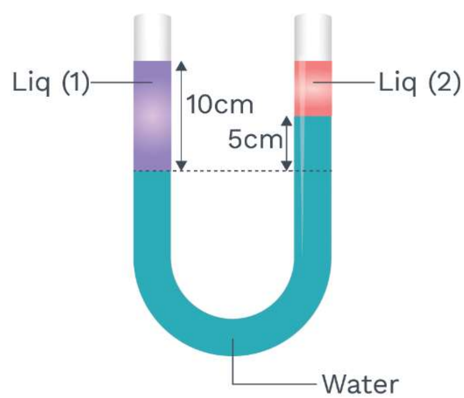


$$(S.G.)_{liq.} = \frac{\rho_{liq}}{\rho_w} = \frac{h_w}{h_{liq}}$$

$$(S.G.)_{liq.} = \frac{8}{10} = 0.8$$

Ex. An open U-tube contains two unknown liquids separated by water as shown in figure. If density of a water is 1 gm/cc, density of liq (1) is 0.8 gm/cc then find density of liq. (2).

Sol. $h_1\rho_1 = h_2\rho_2 + h_w\rho_w$

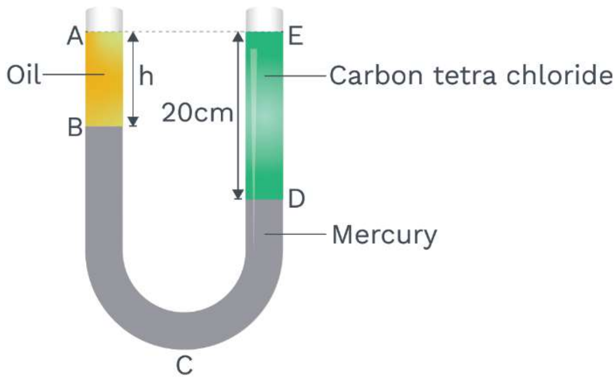


(10) (0.8) = (5) (ρ_2) + (5) (1)

$$\rho_2 = \frac{3}{5} = 0.6 \text{ gm / cc}$$

Ex. Calculate the value of h in U-tube shown in the following figure.

Density of oil = 0.9 g/cm^3
Density of carbon tetrachloride = 1.6 g/cm^3
Density of mercury = 13.6 g/cm^3



Sol. In equilibrium, the pressure of liquid at the same level must be equal. Considering pressure at level D in both arms of U-tube. Pressure of h cm of oil + pressure of (20 – h) cm of mercury = pressure of 20 cm of carbon tetrachloride,

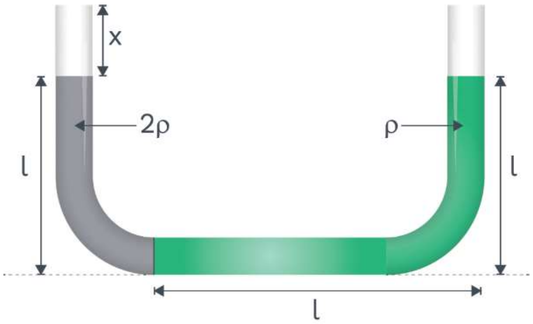
$h \times 0.9 \times g + (20 - h) \times 13.6 \times g = 20 \times 1.6 \times g$

or $\quad 0.9\ h + 272 - 13.6\ h = 32$

or $\quad 12.7\ h = 240$

or $\quad h = \dfrac{240}{12.7} = 18.9$ cm

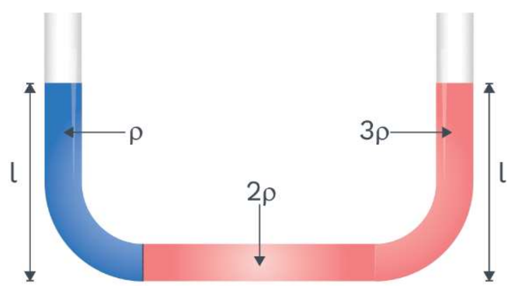
Ex. Two liquid which do not mix chemically are placed in a bent tube as shown in diagram. Calculate the displacement of the liquid in equilibrium.



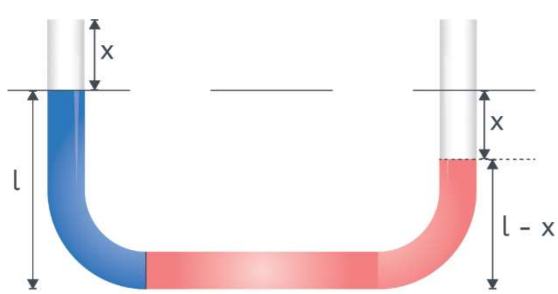
Sol. The pressure at the interface must be equal, calculated via either tube. Since both tube are all open to the atmosphere, we must have.

$$2\rho g(l - x) = \rho g(l + x) \Rightarrow x = \frac{l}{3}$$

Ex. Three liquid which do not mix chemically are placed in a bent tube as shown in figure (initially) then find out the displacement position of the liquid in equilibrium position.



Sol. Assume that level of liquid having 3ρ density displaced below by x as shown in figure.



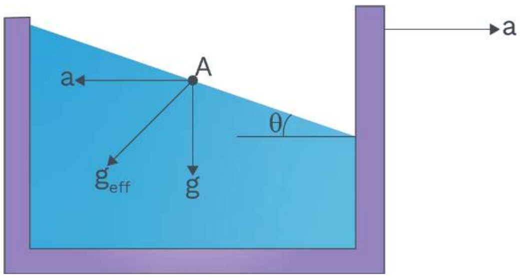
$$\Rightarrow \quad \rho l g + 2\rho g x = 3\rho(l - x)g$$

$$x = \frac{2l}{5}$$

Pressure Distribution in an Accelerated Frame:-

We've discussed that when a liquid is filled in a container, generally its free surface remains horizontal as shown in diagram (a) as for its equilibrium its free surface must be normal to gravity i.e. horizontal. Due to the same reason we call that pressure at each point of a liquid layer surface parallel to its free surface remains constant. Same as situation exist when liquid is in an accelerated frame as shown in diagram (b). Due to acceleration of vessel, liquid filled in it experiences a pseudo force relative to vessel and due to this the free surface of liquid which normal to the gravity now is filled as

$$\theta = \tan^{-1}\left(\frac{a}{g}\right) \quad \text{...(i)}$$



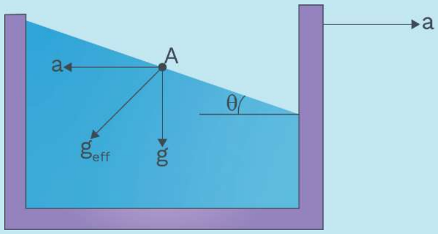
Now from equilibrium of liquid we can say that pressure at each point in a liquid layer parallel to the free surface (which is not horizontal), remains same for example if we find pressure at a point A in the accelerated vessel as shown in figure (a) is given as

Concept Reminder

Air around us also exerts pressure, it is known as atmospheric pressure. Its value is 1.013×10^5 Pa at sea level.

Concept Reminder

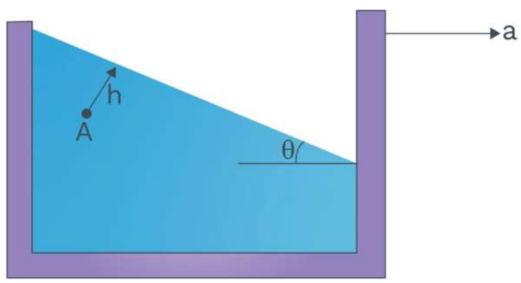
For accelerated system.

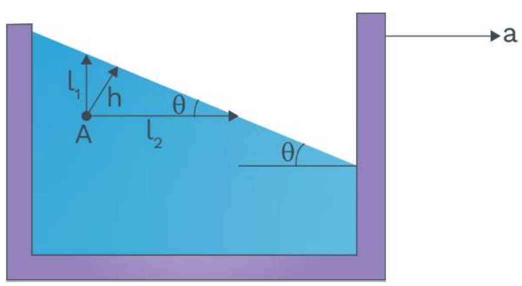


- $\tan\theta = \frac{a}{g}$
- $P_A = P_0 + h\rho\sqrt{a^2 + g^2}$

$$P_A = P_0 + h\rho\sqrt{a^2 + g^2} \quad \text{...(ii)}$$

Where 'h' is the depth of the point A below the free surface of liquid along effective gravity and 'P_0' is the atmospheric pressure applied on free the liquid surface.





The pressure at point A can also calculate in an another way as shown in diagram (b). If l_2 and l_1 are the horizontal and vertical distances of point A from the surface of liquid then pressure at point A can also be given by equation

$$P_A = P_0 + l_1\rho g = P_0 + l_2\rho a \quad \text{...(iii)}$$

Here $l_1\rho g$ is the pressure at A due to the vertical height of liquid above A and according to Pascal's Law pressure at A is given as

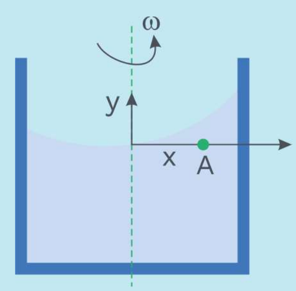
$$P_A = P_0 + l_1\rho g \quad \text{...(iv)}$$

Here we can write l_1 as

$$l_1 = h\sec\theta = \frac{h\sqrt{a^2 + g^2}}{g}$$

Concept Reminder

Pressure in a rotating frame.



- $y = \dfrac{\omega^2 x^2}{2g}$
- $P_A = P_0 + \dfrac{\rho\omega^2 x^2}{2}$

Rack your Brain

Find out angle which the free surface of a liquid filled in a container will make with horizontal if the container is accelerated horizontally with acceleration $\dfrac{g}{\sqrt{3}}$.

or from equation (iv),

$$P_A = P_0 + h\rho\sqrt{a^2 + g^2}$$

Same as if we consider the horizontal distance of point A from free surface of liquid, which is l_2 then due to pseudo acceleration of container the pressure at point A is given as

$$P_A = P_0 + l_2 \rho a \qquad ...(v)$$

Here l_2 is given as

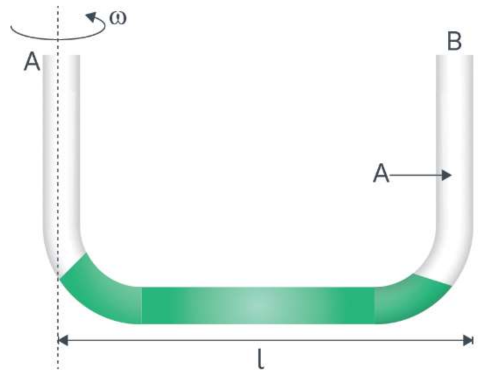
$$l_2 = h \operatorname{cosec}\theta = \frac{h\sqrt{g^2 + a^2}}{a}$$

From equation (iii), we have calculate

$$P_A = P_0 + h\rho\sqrt{g^2 + a^2}$$

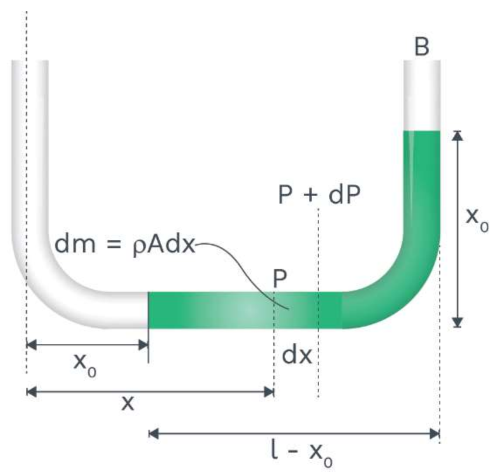
Here we should note that while evaluating pressure at point A from vertical direction we have not mentioned any thing about pseudo acceleration as along vertical length l_1, due to pseudo acceleration at every point pressure must be constant similarly in horizontal direction at every point due to gravity pressure remains constant.

Ex. Given figure shows a tube in which liquid is filled at the level. It is now rotated at an angular frequency ω about an axis passing through arm A find out pressure difference at the liquid interfaces.



Sol. To solve the problem we take a small mass dm from the axis at a distance x in displaced in the condition.

Net inward force = (P + dP) A – PA = AdP



This force is equilibrium by centripetal force

$\Rightarrow \quad AdP = dm\,\omega^2 x = \rho A dx\,\omega^2 x$

$\Rightarrow \quad \int dP = \int_{x_0}^{\ell} \rho\omega^2\, x dx$

$\Delta P = \rho\omega^2 \int_{x_0}^{\ell} x dx = x_0 \rho g$

Barometer in lift:

Case-I: Lift accelerating upwards with acceleration 'a'

$$\boxed{P_{atm} = \rho(g+a)h'}$$

Case-II: Lift accelerating downwards with acceleration 'a'

$$\boxed{P_{atm} = \rho(g-a)h''}$$

Ex. Reading of a mercury barometer inside a stationary lift is 76 cm, then find its reading when it is placed inside a lift accelerating upward with acceleration 9 m/s^2.

Sol. For stationary lift $P_{atm} = \rho gh$

For the lift accelerating upwards

$P_{atm} = \rho(g + a)h$

$\rho gh = \rho(g+a)h'$

$10 \times 76 = (10+9)h' \Rightarrow h' = 40 \text{ cm}$

Buoyancy:

- If a block is partially or wholly immersed in a fluid, it experiences an upward force due to the fluid surrounding it.
- This phenomenon of force exerted by fluid on the block called buoyancy and the force is called buoyant force.
- A block experiences buoyant force whether it floats or sinks, under its own weight or due to other forces applied on it.

Definitions

If a body is partially or wholly immersed in a fluid, it experiences an upward force due to the fluid surrounding it. The phenomenon of force exerted by fluid on the body called buoyancy and the force is called buoyant force.

Archimedes' Principle:-

A block immersed in a fluid experiences an upward direction buoyant force same as to the weight of the fluid displaced by it.

The proof of Archimedes principle is very simple. Imagine a block of arbitrary shape completely immersed in a liquid of density ρ as shown in the figure. A block is being acted upon by the forces from all directions. Let us assume a vertical element of height h and cross-sectional area dA as shown in the diagram (b).

The force acting on the upper surface of the element is 'F_1' (downward) and that on the lower surface is 'F_2' (upward). since $F_2 > F_1$,the net upward force acting on the element is,

$$dF = F_2 - F_1$$

It can be easily represent from the diagram (b), that

$$F_1 = (\rho\, gh_1)dA \text{ and } F_2 = (\rho\, gh_2)dA$$

so $dF = \rho g(h)dA$

Also, $h_2 - h_1 = h$ and $h(dA) = dV$

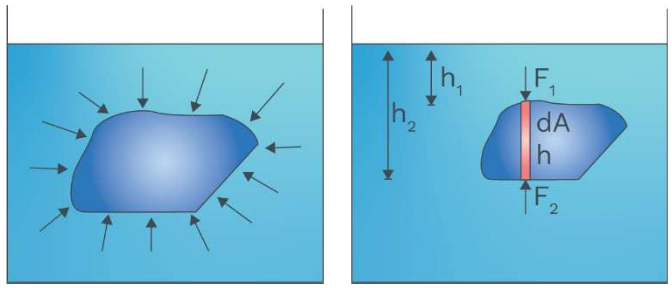
$\therefore$ The net upward force is

Definitions

According to Archimede's principle "The loss of weight of a body submerged (partially or fully) in a fluid is equal to the weight of the fluid displaced".

$$F = \int \rho g dV = \rho V g$$

Therefore, for the entire block, the buoyant force is the weight of the volume of the fluid displaced. The buoyant force acting on the centre of gravity of the displaced fluid.



Note: It buoys because the pressure in the fluid is not uniform: it increases with depth. A block floats on water if it can displace a volume of water whose weight is greater than that of the block. If the density of the matter is less than that of the liquid, it will float even if the matter is a uniform solid, such as a block of wood floats on water surface. If the density of the matter is greater than that of water, such as iron, the block can be made to float provided it is not a uniform solid. An iron built ship is an example to this case.

Principle of Floatation:

When a block of density (ρ) and volume (V) is completely immersed in a liquid of density (s), the forces acting on the block are:

(i) Weight of the block $W = Mg = V\rho g$ directed vertically downwards through the Centre of gravity of the block.

(ii) Buoyant force or Upthrust $Th = V\sigma g$ directed vertically upwards through Centre of buoyancy. The following three cases are possible:

Case-I: Density of the block is greater than that of liquid ($\rho > \sigma$)

Concept Reminder

Upthrust force,

$F = \rho V g$

Where ρ = density of liquid,

V = volume of submerged part of block.

KEY POINTS

- Buoyancy
- Up thrust force
- Archimedes principle
- Principle of floatation

In this case

W > Th

So, the block will sink to the bottom of the liquid.

$W_{App} = W - Th = V\rho g - V\sigma g$

$= V\rho g\,(1 - \sigma/\rho) = W\,(1 - \sigma/\rho)$

Case-II: Density of the block is equal to the density of liquid ($\rho = \sigma$)

In this case

W = Th

So, the block will float fully submerged in the liquid. It will be in neutral equilibrium.

$W_{App} = W - Th = 0$

Case-III: Density of the block is lesser than that of liquid ($\rho < \sigma$)

In this case, W < Th

So, the block will float partially submerged in the liquid. In this case the volume of liquid displaced by the block (V_{in}) will be less than the volume of body (V). This ensures that Th equally to W.

$\therefore\ W_{App} = W - Th = 0$

The above three cases constitute the laws of floatation which states that a body will float in a liquid if weight of the liquid displaced by the immersed part of the block is at least equal to the weight of the block.

Ex. If a wooden body floats in water with 3/5 of its volume inside the water then what is the density of wooden body?

Sol. In floating condition

W = Th

$\Rightarrow\ \rho_b Vg = \rho_W V_{in} g$

$\Rightarrow\ \rho_b V = 1000 \times \frac{3}{5} V$

$\Rightarrow\ \rho_b = 0.6 \times 10^3\ kg/m^3$

Ex. A wooden body of mass 2 kg and density 5 × 10^3 kg/m^3 is suspended from a string. what will be the

Concept Reminder

Apparent weight of a block when it is fully immersed in fluid is

$$W_{app} = W\left(1 - \frac{\rho}{\sigma}\right)$$

Where,

ρ = density of object

σ = density of liquid.

Concept Reminder

If $\rho = \sigma$

$\Rightarrow\ W_{app} = 0$

Block will floats completely submerged just below the surface of liquid.

tension in the string if the body is completely immersed in water ($g = 10 m/s^2$)

Sol. Tension in the string

$T = W - Th$

$$= mg - \rho_W \cdot V_{in} g = mg - \rho_W \left(\frac{m}{\rho_b}\right) g$$

$$= mg\left(1 - \frac{\rho_W}{\rho_b}\right) = 16\ N$$

Ex. A block of aluminium of mass 1kg and volume $3.6 \times 10^{-4}\ m^3$ is suspended from a string and then completely immersed in a container of water. Find out the decrease in tension in the string after immersion.

Sol. Here, mass of the block, m = 1 kg

Volume of the block, $V = 3.6 \times 10^{-4}\ m^3$

Tension in the string,

$T = mg$

When the block immersed completely in water, its weight becomes

$mg' = mg - \text{Upthrust}$

$= mg - V\rho_{water} g$

∴ Tension in the string,

$T' = mg' = mg - V\rho_{water} g$

∴ Decrease in the tension of the string = T − T'

$= mg - [mg - V\rho_{water} g] = V\rho_{water} g$

$= 3.6 \times 10^{-4} \times 10^3 \times 10 = 3.6\ N$

Ex. A wooden body floats in water with one-third of its volume submerged. The same body floats in oil with $\frac{5}{9}$ of its volume submerged. Find the density of:

(i) Wooden body and **(ii)** Oil (Density of water is $10^3\ kg/m^3$).

Rack your Brain

A wooden cube just floats inside water when a 0.2 kg mass is placed on it. When the mass is removed, the cube is 2 cm above the water level. What is size of cube.

Sol. **(i)** When body floats in water, then

$W = Th$

$\Rightarrow \quad \rho_b Vg = \rho_w \cdot \frac{V}{3} g$

$\Rightarrow \quad \rho_b = \frac{1000}{3} \text{ kg / m}^3$

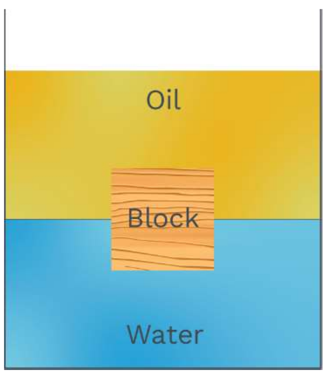
When body floats in oil, then $W = Th$

$\Rightarrow \quad \rho_b Vg = \rho_{oil} \frac{5V}{9} g$

$\Rightarrow \quad \rho_{oil} = \frac{9}{5} \rho_b$

$\Rightarrow \quad \rho_{oil} = 600 \text{ kg / m}^3$

Ex. A cube block of wood having volume V floats at the interface of two immiscible liquids as shown in figure.



Concept Reminder

If $\rho > \sigma$ upthrust is not sufficient enough to balance the weight of the block. Hence, the block sinks.

If $\rho_{oil} = 0.8$ g/cc

$\rho_{block} = 0.85$ g/cc

$\rho_{water} = 1$ g/cc

Then what fraction of the volume of the block will be in the upper liquid.

Sol. Let V_1 volume of block is in upper liquid then

$W = Th_{oil} + Th_{water}$

$\Rightarrow \quad \rho_b Vg = \rho_{oil} V_1 g + \rho W (V - V_1) g$

$\Rightarrow \quad \frac{V_1}{V} = \frac{\rho_w - \rho_b}{\rho_w - \rho_{oil}}$

$\Rightarrow \quad \frac{V_1}{V} = \frac{1 - 0.85}{1 - 0.8} = \frac{3}{4}$

Ex. A cube with an edge of 10cm is immersed in a vessel containing water. A layer of liquid immiscible with water and having a density of 0.8×10^3 kg/m^3 is poured above water. The interface between the liquid is at the middle of the cube height. Find the mass of the cube.

Sol. $W = Th_{liq} + Th_{water}$

$\Rightarrow \quad mg = \rho_\ell V_{in_{liq}} g + \rho_w V_{in_{water}} g$

$\Rightarrow \quad m = (\rho_\ell + \rho_w) V_{in}$

$= (0.8 \times 10^3 + 10^3) \times (10 \times 10 \times 5 \times 10^{-6})$

$= 0.90$ kg

Ex. A boat having a length of 4m and width of 3m is floating in water. The boat sinks by 2cm when a man gets on it. Find the mass of the man.

Sol. In floating condition

$W = Th$

$\Rightarrow \quad mg = \rho_w V_{in} g$

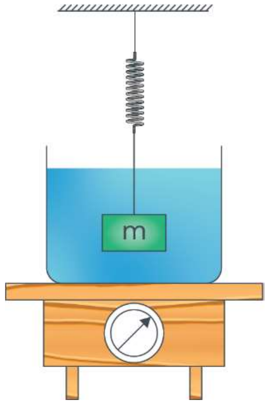
$\Rightarrow \quad m = 1000 \times \left[4 \times 3 \times \frac{2}{100}\right]$

$\Rightarrow \quad m = 240$ kg

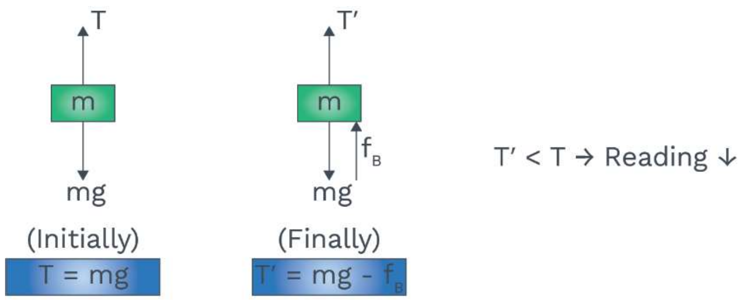
Ex. The spring balance A read 4kg with a block of mass 'm' suspended from it. Another balance B reads 7kg when a beaker with liquid is put on the pan of the balance. The two balances are now arranged that the hanging mass is inside the liquid in the beaker as shown in figure. In this situation-

(a) Reading of A will increases or decreases

(b) Reading of B will increases or decreases



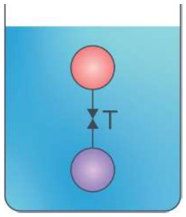
Sol. For spring balance



Due to Reaction of Buoyant force reading of the weighing machine will increase.

Ex. Two spheres of volume 250 cc each but of relative densities 0.8 and 1.2 are connected by a string and the combination is immersed in a liquid. Find the tension in the string.

Sol. The tension on heavier sphere is upwards and on lighter sphere is downwards.



For lighter sphere,

W + T = Th

$$250 \times 10^{-6} \times 800g + T = 250 \times 10^{-6} \times \rho_L \times g$$

For heavier sphere,

W = T + Th

$$250 \times 10^{-6} \times 1200g = T + 250 \times 10^{-6} \rho_L g$$

On solving, T = 0.5 N

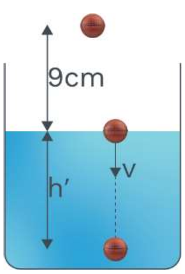
Ex. A ball whose density is 0.4×10^3 kg/m^3 falls into water from a height of 9 cm. To what depth does the ball sink?

Sol. Velocity of ball when ball enter in water

$$v = \sqrt{2gh}$$

In water ball moves with constant retardation

$$a = g\left(\frac{\rho_W}{\rho_B} - 1\right)$$



From 3rd equation of motion

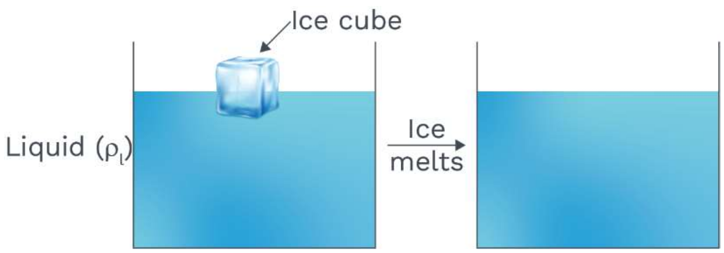
$$v^2 = u^2 + 2as$$

$$0 = 2gh = 2g\left(\frac{\rho_w}{\rho_B} - 1\right)h'$$

$$h' = \frac{h}{\left(\frac{\rho_w}{\rho_B} - 1\right)} = \frac{9}{\left(\frac{10^3}{0.4 \times 10^3} - 1\right)} = \frac{9}{(2.5 - 1)} = 6 \text{ cm}$$

Ex. A ice piece floats in a liquid. What will happen to the level of liquid after the ice melts completing?

Sol.



$$m_{ice}g = \rho_\ell V_{d_1} g$$

$$V_{d_1} = \frac{m_{ice}}{\rho_\ell}, \quad V_{d_2} = \frac{m_{ice}}{\rho_w}$$

If $\rho_\ell > \rho_w \rightarrow$ level $\uparrow$

If $\rho_\ell < \rho_w \rightarrow$ level $\downarrow$

If $\rho_\ell = \rho_w \rightarrow$ level unchanged

Some special points:

- Using Archimedes' principle we can determine the relative density (R D) of a body as

$$\text{R.D.} = \frac{\text{Density of body}}{\text{Density of pure water at 4°C}}$$

$$= \frac{\text{weight of body}}{\text{weight of equal volume of water}}$$

$$= \frac{\text{weight of body}}{\text{force of upthrust due to water}}$$

$$= \frac{\text{wt. of body}}{\text{loss of wt. in water}}$$

$$= \frac{\text{wt. of body in air}}{\text{wt. in air} - \text{wt. in water}} = \frac{W_A}{W_A - W_W}$$

- If a body is weighed in air (W_A), in water (W_W) and in a
Specific gravity of liquid

$$= \frac{\text{loss of weight in liquid}}{\text{loss of weight in water}} = \frac{W_A - W_L}{W_A - W_W}$$

- The weight of a plastic bag full of air is same as that of empty bag because the force of upthrust is equal to the weight of the air enclosed.

Concept Reminder

The relative density (R.D.) of a body is given as

$$\text{R.D.} = \frac{W_A}{W_A - W_W}$$

Ex. A sample of metal weight 300 gm in air, 270 gm
liquid. Then find:

(1) Relative density of metal

(2) Relative density of unknown liquid.

Sol. (1) $(\text{R.D.})_{\text{metal}} = \frac{W_A}{W_A - W_W} = \frac{300}{300 - 270} = 10$

(2) $(\text{R.D.})_{\text{liquid}} = \frac{W_A - W_L}{W_A - W_W} = \frac{300 - 210}{300 - 270} = 3$

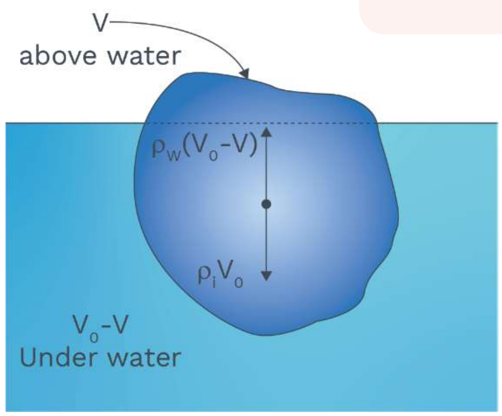
Rack your Brain

A body floats with one-third of its volume outside the water. In another liquid, if floats with one-forth of its volume inside the liquid. If density of water is 1 g/cc, then the density of liquid is:

(1) $\frac{5}{4}$ g / cc (2) $\frac{3}{5}$ g / cc

(3) $\frac{8}{3}$ g / cc (4) $\frac{7}{3}$ g / cc

Ex. An iceberg with a density of 920 kgm^{-3} floats on
fraction of the iceberg is visible.



Sol. Assume V be the volume of the iceberg above the water surface, then the volume under water will be $V_0 - V$. Under floating conditions, the weight ($\rho_i V_0 g$) of the iceberg is balanced by the buoyant force $\rho_w(V_0 - V)g$. Thus,

$$\rho_i V_0 g = \rho_w (V_0 - V) g$$

or $\rho_w V = (\rho_w - \rho_i) V_0$

or $\rho_w V = (\rho_w - \rho_i) V_0$

or $$\frac{V}{V_0} = \left(\frac{\rho_w - \rho_i}{\rho_w}\right)$$

Therefore $\rho_w = 1025 \text{ kg m}^{-3}$ and $\rho_i = 920 \text{ kg m}^{-3}$, therefore,

$$\frac{V}{V_0} = \frac{1025 - 920}{1025} = 0.10$$

That's 10% of the total volume is visible.

Ex. When a 2.5 kilogram crown is immersed in water, it has an apparent weight of 22 N. Find out the density of the crown?

Sol. Let W = actual weight of the crown,

W' = apparent weight of the crown

ρ = density of crown

ρ_0 = density of water

The buoyant force is given by

$$F_B = W - W'$$

or $\rho_0 V g = W - W'$

Since $W = \rho V g$, therefore,

$$V = \frac{W}{\rho g}$$

Remove V from the above two equations, we get

$$\rho = \frac{\rho_0 W}{W - W'}$$

Here $W = 25 \text{ N};\ W' = 22 \text{ N};\ \rho_0 = 10^3 \text{ kg m}^{-3}$

$$\therefore \quad \rho = \frac{(10^3)(25)}{25 - 22} = 8.3 \times 10^3 \text{ kg m}^{-3}$$

Rack your Brain

A metallic sphere has a cavity inside it. The sphere weighs 40 g wt in air and 20 g wt in water. If density of the material of the sphere is 8 g/cc then find volume of cavity.

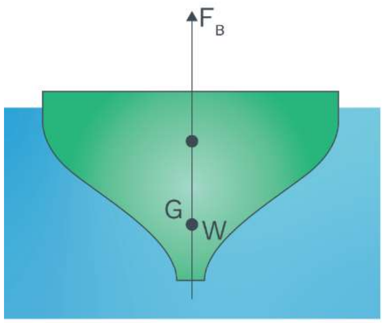
Stability of a Floating Body:-

The stability of a floating block depends on the effective point of application of the buoyant force. The weight of the block acts at its centre of gravity. The buoyancy force acts at the centre of mass of gravity of the displaced liquid. This is known as the centre of buoyancy. Under

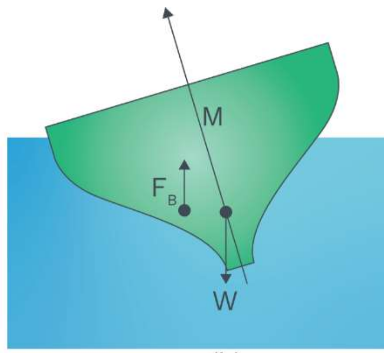
Definitions

The buoyant force acts at the centre of gravity of the displaced liquid. This is called the centre of buoyancy.

equilibrium the centre of gravity G and the centre of buoyancy B lies along the vertical axis of the block as shown in the figure(a).



(a)

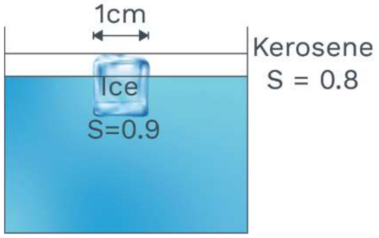


(b)

(a) The buoyant force acts at the centre of gravity of the displaced fluid.
(b) When the boat tilts, the line of action of the buoyant force intersects the axis of the boat at the metacentre M. In a stable boat, M is above the centre of gravity of the boat.

When the block tilts to one side, the centre of buoyancy shifts relative to the centre of gravity as shown in the figure (b). The two forces acting along different vertical lines. As a result the buoyant force applied a torque about the centre of mass of gravity.
The action line of the buoyant force crosses the axis of the block at the point M, called the metacentre. If 'G' is below 'M', the torque will tend to restore the body to its equilibrium. If G is above 'M', the torque will tend to rotate the block away from its equilibrium position and the body will be unstable.

Ex. An ice cube of side 1 centimetre is floating at the interface of water and kerosene in a beaker of base area 10 cm^2. The kerosene level is just covering the top surface of the ice cube.



(a) The depth of submergence in the and water that in the kerosene.
(b) Calculate the change in the total level of the liquid when the whole ice convert into water.

Sol. (a) Condition of floating

or $0.8\,\rho_w g h_k + \rho_w g h_w = 0.9\,\rho_w g h$

$0.8\,h_k + h_w = (0.9)h$ ...(i)

Where h_w and h_k be the submerged depth of the ice in the water and kerosene, respectively.

Also, $h_k + h_w = h$ ...(ii)

Solving equation (i) and (ii),

$h_w = 0.5$ cm, $h_k = 0.5$ cm

(b) $\underset{\text{(ice)}}{1\text{ cm}^3} \xrightarrow{\text{melts}} \underset{\text{(water)}}{0.9\text{ cm}^3}$]

Fall in the level of kerosene

$$\Delta h_k = \frac{0.5}{A}$$

Rise in the level of water

$$\Delta h_w = \frac{0.9 - 0.5}{A} = \frac{0.4}{A}$$

Net fall in the overall level

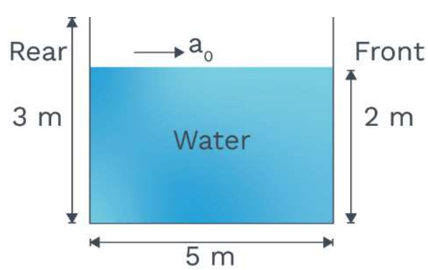
$$\Delta h = \frac{0.1}{A} = \frac{0.1}{10} = 0.01\text{ cm} = 0.1\text{ mm}$$

Ex. An open rectangular tank size 5 m × 4 m × 3 m high containing water upto a height of 2 metre is accelerated horizontally along the longer side.

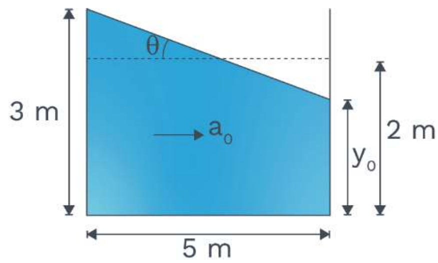
(a) Calculate the maximum acceleration of time that can be given without spilling the water.

(b) Find out the percentage of water spilt over, if this acceleration is increased by '20' percentage.

(c) Initially, the tank is closed at the top surface and is accelerated horizontally by 9 m/s^2, find out the gauge pressure at the bottom of the front and rear walls of the tank. (Take g = 10 m/s^2)



Sol. **(a)** Water volume inside the tank remains constant



$$\left(\frac{3+y_0}{2}\right)5\times 4 = 5\times 2\times 4$$

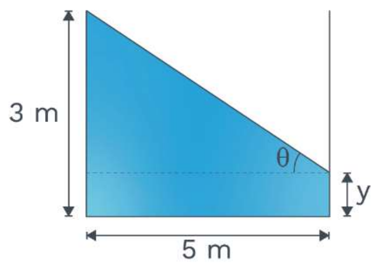
or $\quad y_0 = 1$ m

$\therefore \quad \tan\theta_0 = \dfrac{3-1}{5} = 0.4$

Since, $\tan\theta_0 = \dfrac{a_0}{g}$, therefore $a_0 = 0.4\ g = 4\ m/s^2$

(b) When acceleration is increased by 20%

$a = 1.2\ a_0 = 0.48\ g$



$\therefore \quad \tan\theta = \dfrac{a}{g} = 0.48$

Now, $y = 3 - 5\tan\theta = 3 - 5(0.48) = 0.6$ m

Fraction of water spilt over

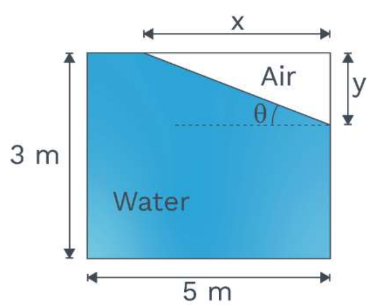
$$= \frac{4\times 2\times 5 - \dfrac{(3+0.6)}{2}\times 5\times 4}{2\times 5\times 4} = 0.1$$

Percentage of water spilt over = 10%

(c) $a' = 0.9\,g$

$\tan\theta' = \dfrac{a'}{g} = 0.9$

Volume of air remains constant



Fluid Mechanics

$$4 \times \frac{1}{2} yx = (5)(1) \times 4$$

Since $y = x \tan\theta'$

$\therefore \quad \frac{1}{2}x^2 \tan\theta' = 5$

or $x = 3.33$ metre; $y = 3.0$ metre

Gauge pressure at the bottom of the

(i) Front wall p_f = zero.

(ii) Rear wall $p_r = (5\tan\theta')\rho wg$

$= 5(0.9)(10^3)(10) = 4.5 \times 10^4$ Pa

FLUID DYNAMICS

When a fluid moves in such a way that there are relative motions among the fluid particles, the fluid is said to be flowing.

Types of Fluid flow:

Fluid flow can be classified as:

- **Steady flow:-**
  Steady flow is explained as that type of flow in which the fluid velocity at a point do not change with time. The fluid particle may have a different velocity at some other point.
  In steady flow condition all the particles passing through a given point follow the same path and therefore a unique line of flow. This line or path is known as a streamline. Streamlines do not intersect each other, if they do so any particle at the point of intersection can move in either directions and consequently the flow cannot be steady.
- **Laminar and Turbulent Flow:**
  Laminar flow is the flow in which fluid particles move along well-defined streamlines which are parallel and straight. In laminar flow type the velocities at different points in the fluid may be different magnitudes, but their directions are parallel. Thus, the particles move in laminae or layers sliding smoothly over the adjacent surface.

KEY POINTS

- Fluid flow
- Steady flow
- Streamline
- Laminar flow
- Turbulent flow
- Compressible flow
- Incompressible flow
- Rotational flow
- Irrotational flow

Definitions

Steady flow is defined as that type of flow in which the fluid velocity at a point do not change with time.

Concept Reminder

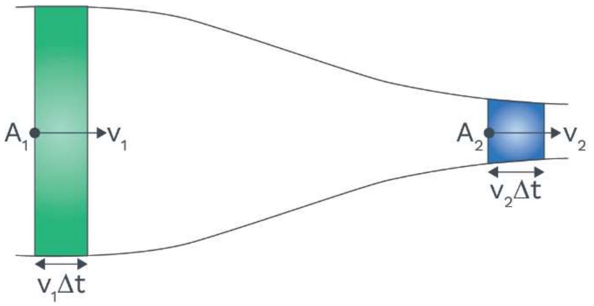
In laminar flow the velocities at different points in the fluid may have different magnitudes, but their directions are parallel.

Turbulent flow is an irregular type flow in which the particles move in zig-zag way due to which eddy formation take place which are responsible for high energy losses.

- **Compressible and Incompressible Flow:**
  In compressible flow the density of fluid varies from point to point i.e., the fluid density is not constant, whereas in an incompressible flow the density of the fluid remains uniform throughout. Liquids are practically incompressible while gases are highly compressible.
- **Rotational and Ir-rotational Flow:**
  Rotational flow is the flow in which the fluid particles while flowing along different path-lines also rotate about their own axes. In ir-rotational flow the particles do not rotate about their axes.

Equation of continuity:

- The continuity equation is the mathematical expression of the law of conservation of mass in fluid dynamics.
- In the steady flow mass of the fluid entering into a tube of flow in a particular time interval is equal to the mass of fluid leaving the tube.



Concept Reminder

According to equation of continuity "For the streamiline flow of an incompressible fluid through a pipe of varying cross-section, Av remains constant throughout the flow."

$$\frac{m_1}{\Delta t} = \frac{m_2}{\Delta t} \quad \text{or} \quad \rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

For incompressible fluid, $\rho_1 = \rho_2$

or $A_1 v_1 = A_2 v_2$

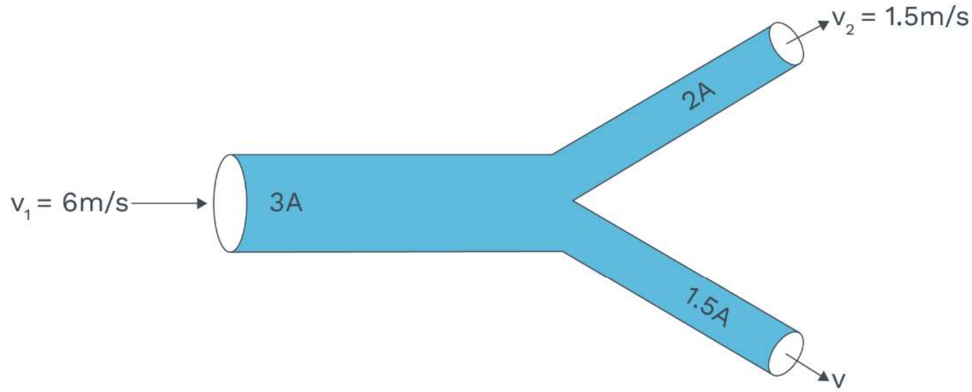
or $Av = \text{constant}$

Volume flux = Rate of flow = Volume of liquid flowing per second

$$Q = \frac{dV}{dt} = Av$$

Unit of (Q) m^2/s or $\frac{\text{Litre}}{\text{sec}}$ or $\frac{m\ell}{s}$

Ex. An liquid (incompressible) flows through a horizontal tube as shown in the following figure. Then find the velocity v of the fluid.

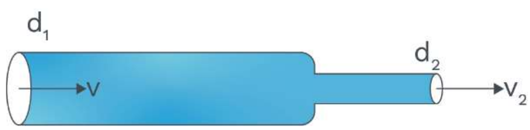


Sol. $(3A)\ (v_1) = (2A)\ (v_2) + (1.5A)\ v$

$(3A)\ (6) = (2A)\ (1.5) + (1.5A)\ v$

$v = 10\ m/s$

Ex. An incompressible liquid flows in a tube from left to right as shown in figure. Diameters d_1 and d_2 are the diameters of the portions of the tube as shown. Find the ratio of speeds v_1 and v_2.



Sol. $A_1v_1 = A_2v_2$

$(\pi r_1^2)v_1 = (\pi r_2^2)v_2$

$$\frac{v_1}{v_2} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{d_2}{d_1}\right)^2$$

Bernoulli's Theorem:

- Bernoulli's theorem is the mathematical expression of the law of mechanical energy conservation in fluid dynamics.
- Bernoulli's theorem is applicable to ideal fluids. Characteristics of an ideal fluid are:

(i) The fluid is incompressible.

Rack your Brain

The cylindrical tube of a spray pump has radius R, one end of which has n fine holes, each of radius r. If speed of liquid in tube is v, then find the speed of the ejection of the liquid through the holes.

(ii) The fluid is non-viscous.
(iii) The fluid flow is steady.
(iv) The fluid flow is irrotational.

- Every volume at a point in an ideal fluid flow is associated with three kinds of energies:

(i) Kinetic Energy per unit volume:
If a liquid of mass (m) and volume (V) is flowing with velocity (v) then

$$K.\ E. = \frac{1}{2}mv^2$$

and kinetic energy per unit volume

$$= \frac{K.E.}{Volume} = \frac{1}{2}\frac{m}{V}v^2 = \frac{1}{2}\rho v^2$$

(ii) Potential Energy per unit volume:
If a liquid of mass (m) and volume (V) is at a height (h) above the surface of the earth then its
P.E. = mgh
and potential energy per unit volume

$$= \frac{P.E.}{Volume} = \frac{m}{V}gh = \rho gh$$

(iii) Pressure Energy per unit volume:
If liquid moves through a distance (l) due to pressure P on area A then
Pressure energy = Work done = force × displacement = pressure × area × displacement

$$= PAl = PV\ [\because Al = volume\ V]$$

Pressure energy per unit volume

$$= \frac{Pressure\ energy}{Volume} = P$$

Freely Falling Liquid:
When liquid falls freely under gravity, the area of cross section of the stream continuously decreases, as the velocity increases.

Concept Reminder

Kinetic energy per unit volume

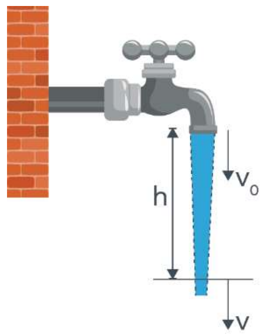
$$= \frac{K.E.}{Volume} = \frac{1}{2}\frac{m}{V}v^2 = \frac{1}{2}\rho v^2.$$

Potential energy per unit volume

$$= \frac{P.E.}{Volume} = \frac{m}{V}gh = \rho gh$$

Pressure energy per unit volume

$$= \frac{Pressure\ energy}{Volume} = P$$



For example, we consider water coming out from a tap, as shown in figure. Let its speed near the mouth of tap is v_0 and at a depth h it is v, then we have

$$v^2 = v_0^2 + 2gh$$

If cross section of tap is A then according to the equation of continuity, the cross section (say a) at point on surface (h height below) can be given as

$$v_0 A = a\sqrt{v_0^2 + 2gh}$$

or $$a = \frac{v_0 A}{\sqrt{v_0^2 + 2gh}}$$

BERNOULLI'S EQUATION

It relates the variables describing the steady laminar of liquid. It is based on energy conservation.

Assumptions:

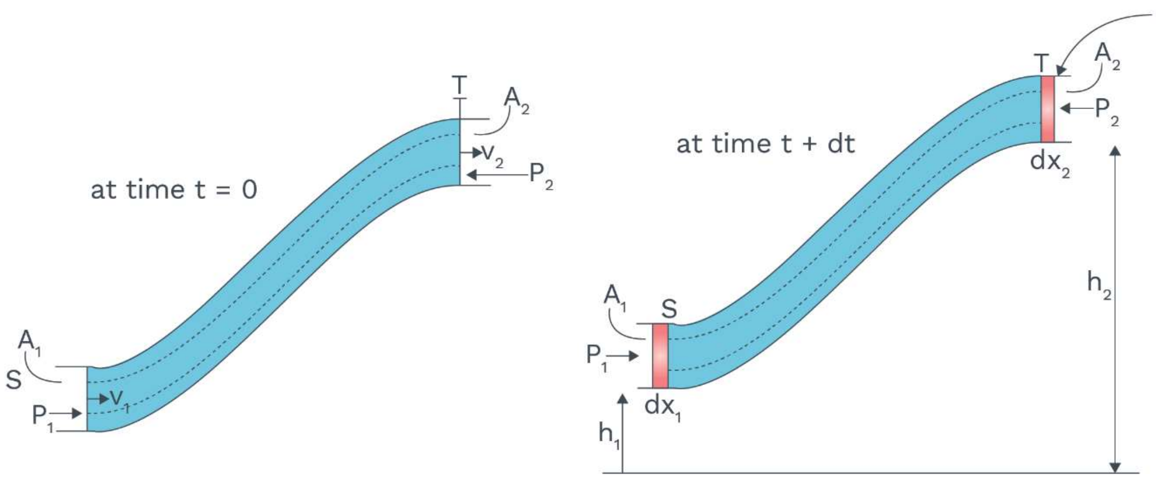
The fluid is incompressible, non-viscous, non-rotational and streamline flow fluid.

KEY POINTS

- Bernoulli's theorem
- Kinetic energy per unit volume
- Potential energy per unit volume
- Pressure energy per unit volume
- Bernoulli's equation

Rack your Brain

A wind with speed 40 m/s blows parallel to the roof of a house. The area of the roof is 250 m^2. Assuming that the pressure inside the house is atmospheric pressure. Find force exerted by wind on roof and direction of force. (ρ_{air} = 1.2 kg/m^3)



Therefore mass of the fluid entering from side S

$dm_1 = \rho A_1 dx_1 = \rho dv_1$

The work-done in this displacement dx_1 at point S is

$$W_{P_1} = F_1 dx_1 = P_1 A_1 dx_1$$

$$W_{P_1} = P_1 dV_1 \qquad (\because \ A_1 dx_1 = dV_1)$$

At the same time the amount of fluid moves out of the tube at point T is

$$dm_2 = \rho dV_1$$

According to equation of continuity

$$\frac{dm_1}{dt} = \frac{dm_2}{dt}$$

$\Rightarrow$ $dV_1 = dV_2 = dV$

The work done in the displacement of dm_2 mass at point T

$$W_{P_2} = P_2 dV_2$$

Now applying work energy theorem.

$$W_{P_1} + W_{P_2} = (K_f + U_f) - (K_i + U_i)$$

$$\Rightarrow P_1 dV - P_2 dV$$

$$= \left(\frac{1}{2}\rho dV\, v_2^2 + \rho dV\, gh_2\right) - \left(\frac{1}{2}\rho dV\, v_1^2 + \rho dV gh_1\right)$$

Concept Reminder

According to Bernoulli's theorem,

$P + \frac{1}{2}\rho v^2 + \rho gh$ = constant (Energy per unit volume)

or $\frac{P}{\rho} + \frac{v^2}{2} + gh$ = constant (energy per unit mass)

or $\frac{P}{\rho g} + \frac{v^2}{2g} + h$ = constant (energy per unit weight)

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 + \rho g h_2 - \frac{1}{2}\rho v_1^2 - \rho g h_1$$

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v^2$$

$$\Rightarrow \boxed{P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}}$$

$$\frac{P}{\rho g} + h + \frac{1}{2}\frac{v^2}{g} = \text{constant}$$

Where $\frac{P}{\rho g}$ = pressure head

h = Gravitational head

$\frac{1}{2}\frac{v^2}{g}$ = volume head.

This equation is the Bernoulli's equation and expresses principle of conservation of mechanical energy in case of moving fluids.

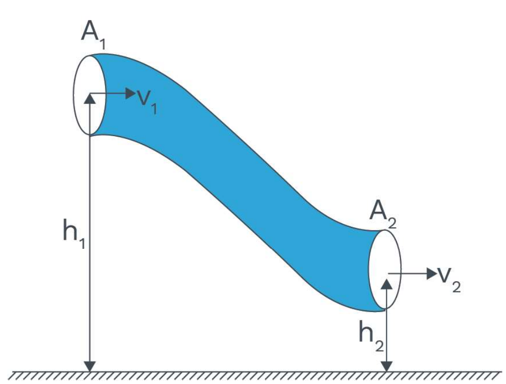
The sum addition of pressure energy, kinetic energy and potential energy per unit volume remains constant along a streamline in an ideal fluid flow i.e.,

$P + \frac{1}{2}\rho v^2 + \rho g h$ = constant (Energy per unit volume)

or $\frac{P}{\rho} + \frac{v^2}{2} + gh$ = constant (energy per unit mass)

or $\frac{P}{\rho g} + \frac{v^2}{2g} + h$ = constant (energy per unit weight)

Ex. Water is moving with a velocity of '5' m/sec through a pipe with cross sectional area of 4cm². The water gradually descends 10m as the pipe increase in area to 5cm². If the pressure at the upper level is 2 atm then find the pressure at lower level. (g = 10 m/s²)



Sol. Given,

$A_1 = 4\ cm^2$, $A_2 = 5\ cm^2$

$h_1 - h_2 = 10$ m

$v_1 = 5$ m/sec

From equation of continuity,

$A_1v_1 = A_2v_2$

(4) (5) = (5) v_2

$v_2 = 4$ m/sec

Applying Bernoulli's theorem –

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$P_2 = P_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) + \rho g(h_1 - h_2)$$

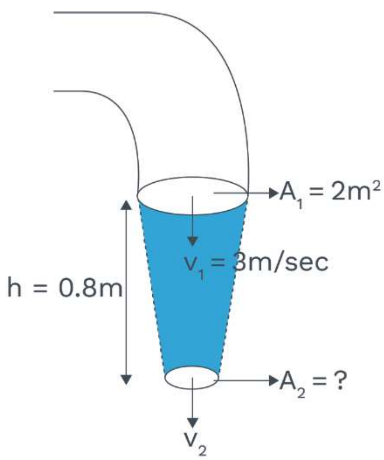
$$P_2 = (2\times 10^5) + \frac{1}{2}(10^3)(5^2 - 4^2) + (10^3)(10)(10)$$

$$P_2 = 2\times 10^5 + 4.5\times 10^3 + 1\times 10^5$$

$$P_2 = 3.045\times 10^5\ N/m^5$$

$P_2 = 3.045$ atm

Ex. Water from a tap come out vertically downwards with an initial speed of 3 m/sec. The cross sectional area of tap is 2 m^2. Let us assume that the pressure is constant throughout the stream of water and that the flow is steady. Find the cross sectional area of stream 80 cm below the tap. (g = 10 m/s^2)



Sol. By 3rd equation of motion

$v^2 = u^2 + 2as$

$v_2^2 = (3)^2 + 2(10)(0.8)$

v_2 = 5 m/sec

By equation of continuity-

$A_1v_1 = A_2v_2$

(2) (3) = (A_2) (5)

$$A_2 = \frac{6}{5} = 1.2 \text{ m}^2$$

Ex. The pressure of water in a water pipe when tap is opened and closed is 2 × 10^5 N/m^2 and 2.5 × 10^5 N/m^2 respectively. Find the velocity of water flowing from open tap.

Sol. Applying Bernoulli's theorem –

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$2.5 \times 10^5 + 0 + 0 = 2 \times 10^5 + \frac{1}{2} \times 10^3 \times v^2 + 0$$

v = 10 m/sec

Ex. In a horizontal pipe, the flowing oil pressure falls by 8 N/m^2 between two points separated by 1m. Find the change in kinetic energy per unit mass of oil at these points (ρ_{oil} = 800 kg/m^3)

Sol. From Bernoulli's theorem –

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\frac{1}{2}\rho(v_2^2 - v_1^2) = P_1 - P_2 = 8 \text{ N/m}^2$$

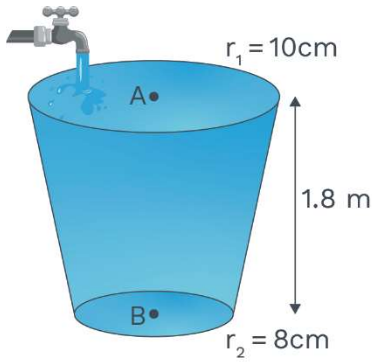
∴ Change n kinetic energy per unit mass

$$= \frac{1}{2}(v_2^2 - v_1^2) = \frac{P_1 + P_2}{\rho}$$

$$= \frac{8}{800} = 0.01 \text{ J/kg}$$

Ex. In a bucket shown in figure, water is kept. Find the speed at which water will be coming out if the bottom surface is removed and its top level is maintain by pouring the liquid through the tap. (density of water = 1000 kg/m^3, g = 10 m/s^2)

Sol. Applying Bernoulli's theorem at points A and B-



$$P_0 + \frac{1}{2}\rho v_A^2 + \rho g h = P_0 + \frac{1}{2}\rho v_B^2$$

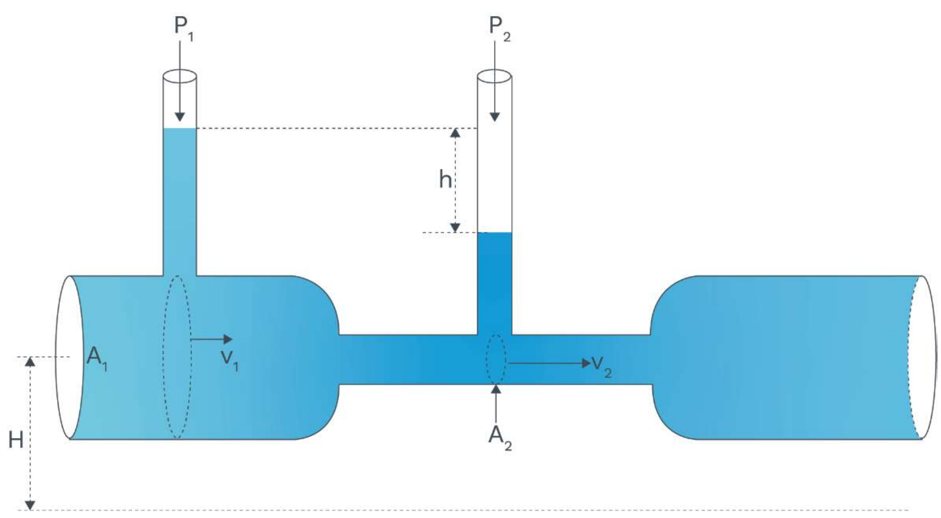
$$\therefore \quad v_B = \sqrt{\frac{2gh}{1-\left(\frac{r_2}{r_1}\right)^2}}$$

$$= \sqrt{\frac{2 \times 10 \times 1.8}{1-(0.8)^2}} = 10 \text{ m / s}$$

Applications of Bernoulli's theorem:

(i) Venturimeter or Venturi Tube or Flowmeter:

Venturimeter is used to measure the flow velocities in an incompressible fluid. As shown in figure if P_1 and P_2 are the pressures and v_1 and v_2 are the velocities of the fluid of density ρ at points 1 and 2 on the same horizontal level and A_1 and A_2 be the respective areas, then from equation of continuity.



$$A_1v_1 = A_2v_2$$

or $$v_2 = \left[\frac{A_1}{A_2}\right]v_1 \quad ...(i)$$

From Bernoulli's equation for horizontal flow,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

or $$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho\left[\frac{A_1^2}{A_2^2}\right]v_1^2 \text{ [from eq. (i)]}$$

or $$P_1 - P_2 = \frac{1}{2}\rho v_1^2\left[\frac{A_1^2}{A_2^2} - 1\right]$$

But $P_1 - P_2 = \rho gh$ ($\because$ difference in heights between the liquid surfaces in the two arms is h)

$\therefore$ $$\rho gh = \frac{1}{2}\rho v_1^2\left[\frac{A_1^2}{A_2^2} - 1\right]$$

$\therefore$ $$v_1 = \sqrt{2gh}\left[\frac{A_1^2}{A_2^2} - 1\right]^{-1/2} = A_2\sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

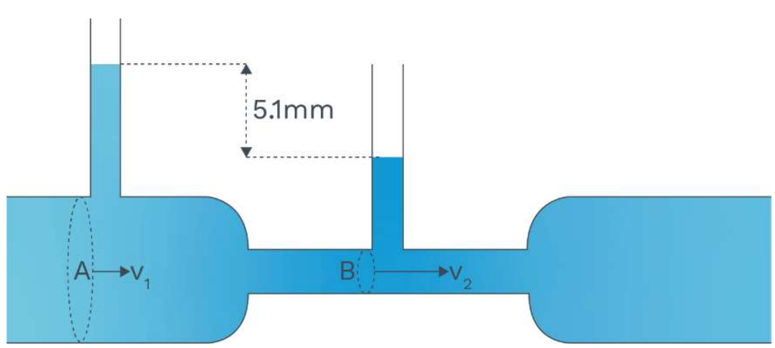
If Q be the volume of liquid flowing per unit time then

$$Q = A_1v_1 = A_2v_2 = A_1A_2\sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

Thus, at a point where the cross-sectional area is smaller velocity is greater and pressure is lower and vice versa.

> **Concept Reminder**
>
> In venturimeter,
>
> - $P_1 - P_2 = \frac{1}{2}\rho v_1^2\left[\frac{A_1^2}{A_2^2} - 1\right]$
> - $v_1 = \sqrt{2gh}\left[\frac{A_1^2}{A_2^2} - 1\right]^{-1/2}$

Ex. Water is flowing in a venturimeter as shown in the diagram. The cross-sectional area at points 'A' and 'B' are 10 m^2 and 6 m^2 respectively and pressure difference at points A and B is 2 atm then find out the flow speed of water and rate of flow of water at point A. ($g = 10 m/s^2$)



Fluid Mechanics

Sol. $P_A - P_B = \rho g h$

$2 \times 10^5 = (10)^3 (10)h$

$h = 20$ m

$$v_1 = A_2\sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

$$v_1 = 6\sqrt{\frac{2 \times 10 \times 20}{(10)^2 - (6)^2}} = 15 \text{ m / sec}$$

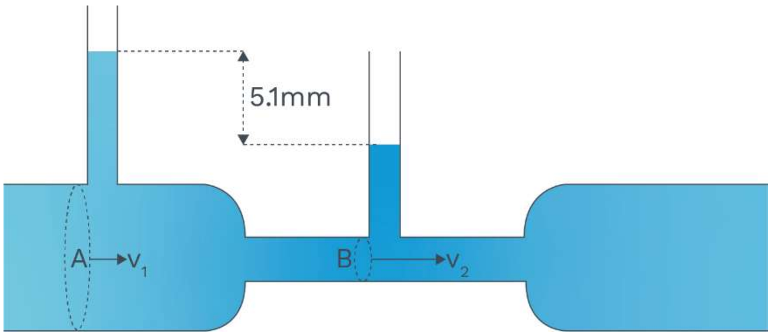
Rate of flow $= A_1 v_1$

$= (10)(15) = 150 \text{ m}^3/\text{sec}$

KEY POINTS

- Venturimeter
- Speed of efflux
- Torricelli's law

Ex. Difference in the reading of tubes of venturimeter is 5.1 mm; then find speed of the fluid in narrow section when speed at wider section is 2 cm/sec. ($g = 10 \text{ m/s}^2$)



Sol. Applying Bernoulli's theorem at points A and B

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\frac{1}{2}\rho(v_2^2 - v_1^2) = P_1 - P_2$$

$$\frac{1}{2}\rho(v_2^2 - v_1^2) = \rho g h$$

$$\therefore \quad v_2^2 = v_1^2 + \rho g h$$

$$v_2^2 = (2 \times 10^{-2})^2 + 2(10)(5.1 \times 10^{-3})$$

$$v_2 = 32 \text{ cm/sec}$$

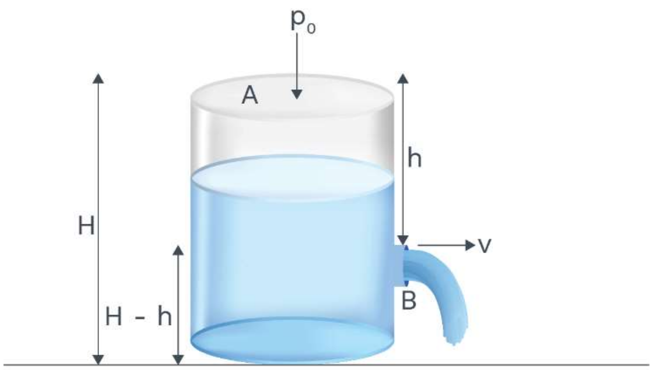
(ii) Speed of efflux (Torricelli's Law)

As shown in the figure the area of cross-

Concept Reminder

The word efflux means the out flow of the fluid.

section of the vessel A is very large as compared to that orifice B, therefore speed of liquid flow at A is zero i.e. $v_A \approx 0$. The fluid at sections A and B are at the same pressure P_0 (atmospheric pressure). Applying Bernoulli's theorem at A and B.



$$P_0 + \rho g H + \frac{1}{2}\rho v_A^2 = P_0 + \rho g(H-h) + \frac{1}{2}\rho v_B^2$$

or $\frac{1}{2}\rho v_B^2 = \rho g h$ or $\boxed{v_B = \sqrt{2gh}}$

This equation is same as that of the velocity acquired by a freely falling body after falling through h height and is known as **Torricelli's law**.

Writing the equation of uniformly accelerated motion in the vertical direction

$$H - h = 0 + \frac{1}{2}gt^2 \quad \left(\text{from } s_y = u_y t + \frac{1}{2}a_y t^2\right)$$

$\Rightarrow$ $\boxed{t = \sqrt{\frac{2(H-h)}{g}}}$

t = time of flight as in case of horizontal projection from the top of a tower.

Horizontal range,

$$R = v_x t = \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}}$$

or $\boxed{R = 2\sqrt{h(H-h)}}$

Concept Reminder

If container is closed and P_2 is pressure above liquid in container then,

$$v = \sqrt{\frac{2}{\rho}(P_2 - P_0) + 2gh}$$

Range will be maximum when

$h = H - h$ or $h = \frac{H}{2}$

$\therefore \quad R_{max} = 2\sqrt{\frac{H}{2}\left[H - \frac{H}{2}\right]} = H$

Reaction force on vessel:

By Newton's third law –

$$F = v\left(\frac{dm}{dt}\right)$$

$$F = v\frac{d(\rho V)}{dt}$$

$$F = v\rho\frac{dV}{dt}$$

$$F = v\rho(Av)$$

$$\boxed{F = \rho A v^2}$$

Concept Reminder

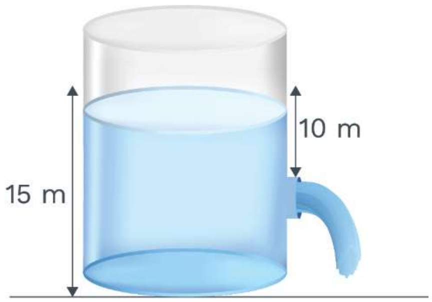
When container is open then,

$P_2 = P_0$

$\therefore \quad v = \sqrt{2gh}$

This is known as Torricelle's equation.

Ex. A cylindrical container of large area has a small hole of cross section area 1 cm^2 at depth 10 metre from the top water surface. ($g = 10 m/s^2$). Find out the following-

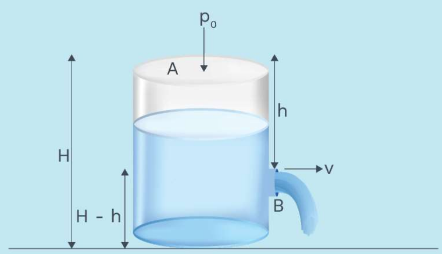


(1) Initial speed of efflux
(2) Time of flight
(3) Range
(4) Reaction Force on container
(5) Rate of flow

Sol. (1) $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 10} = 10\sqrt{2}$ m / sec

(2) $T = \sqrt{\frac{2(H-h)}{g}} = \sqrt{\frac{2(15-10)}{g}} = 1$ sec

Concept Reminder



Time of flight of efflux

$$t = \sqrt{\frac{2(H-h)}{g}}$$

Range of efflux

$$R = \sqrt{2h(H-h)}$$

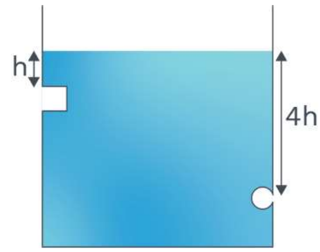
(3) $R = vT = (10\sqrt{2})(1) = 10\sqrt{2}\ m$

(4) $F = \rho A v^2 = (10^3)(1 \times 10^{-4})(10\sqrt{2})^2 = 20\,N$

(5) Rate of flow = Av

$= (10^{-4})(10\sqrt{2}) = \sqrt{2} \times 10^{-3}\ m^3 / sec$

Ex. A container of large area has a square shaped hole of side L and a circular hole of radius R at depth h and 4h respectively from the top of water surface. If rate of flow from both holes are same then find the relation between h and L.

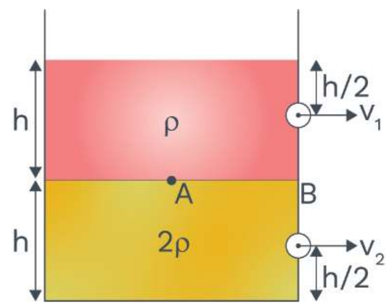


Sol. $A_S v_S = A_C v_C$

$(L^2)\sqrt{2gy} = (\pi R^2)\sqrt{2h(4y)}$

$R = \dfrac{L}{\sqrt{2\pi}}$

Ex. Two immiscible liquid of densities ρ and 2 ρ are filled in a container as shown in the diagram then find out the ratio of v_1 and v_2.



Sol. $v_1 = \sqrt{2g\left(\dfrac{h}{2}\right)} = \sqrt{gh}$

Applying Bernoulli's theorem between A and B

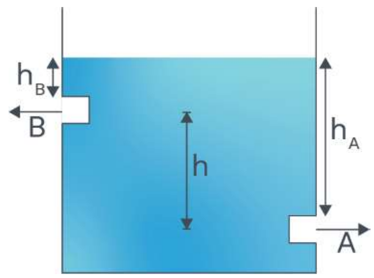
$$P_0 + \rho g h + 2\rho g \frac{h}{2} = P_0 + \frac{1}{2} 2\rho v_2^2$$

$$v_0 = \sqrt{2gh}$$

$$\therefore \quad \frac{v_1}{v_2} = \frac{1}{\sqrt{2}}$$

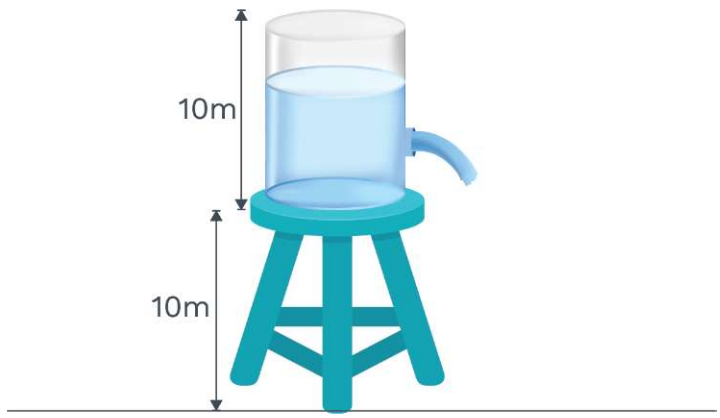
Ex. There are two identical small holes on the opposite sides of a tank containing a liquid. The tank is open at the top. The difference in heights between the two holes is h. Find the resultant force of reaction of liquid flowing out of vessel.

Sol. Net reaction force, $F = F_B - F_A$



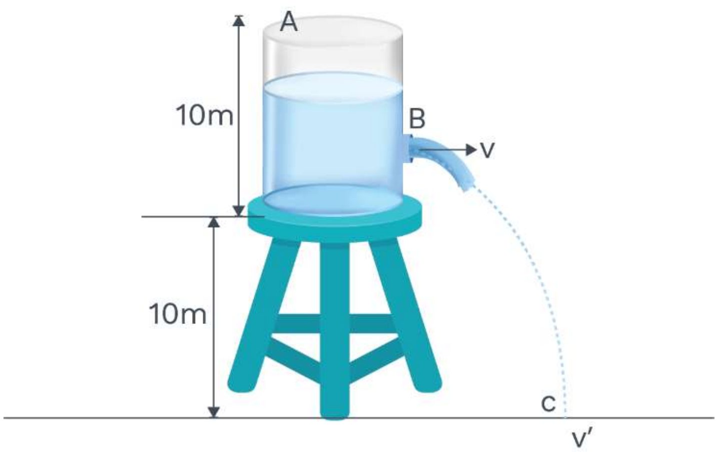
$= \rho a(v_B^2 - v_A^2) = \rho a(2gh_B - 2gh_A)$
$= 2\ \rho agh$

Ex. A cylindrical tank 1m in radius rests on a platform 10 metre high. Initially case the tank is filled with water to a height of 10 m. A small plug whose area is 10^{-4} m^2 is removed from an orifice located on the side of the tank at the bottom. Calculate the:



(i) Initial speed with which water flows out from the orifice.
(ii) Initial speed with which the water strikes the ground.

Sol. **(i)** Applying Bernoulli's theorem between the water surface and the orifice,



$$P_0 + \frac{1}{2}\rho(0)^2 + \rho gh = P_0 + \frac{1}{2}\rho v^2 + \rho g(0)$$

$$\rho gh = \frac{1}{2}\rho v^2$$

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 10} = 10\sqrt{2}\ \text{m/s}$$

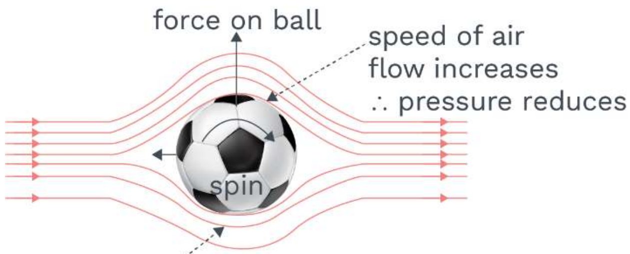
(ii) Let v' be the initial velocity with which the water strikes the ground.
Then, applying Bernoulli's theorem between the top of the tank and the ground level, we get

$$v' = \sqrt{2g(H+h)} = \sqrt{2 \times 10 \times 20} = 20\ \text{m/s}$$

(iii) Magnus Effect (Observed in a Spinning Ball)
Tennis and cricket players usually experience that when a ball is thrown spinning it moves along a curved path. This is called swing of the ball. This is due to the air which is being dragged round by the spinning ball. When the ball spins, the layer of the air around it also moves with the ball. So, as shown in figure the resultant velocity of air increases on the upper side and reduces on the lower side. Hence, according to Bernoulli's theorem the pressure on the upper side becomes lower than that on the lower side. This difference of pressure exerts a force on the ball due to which it moves along a curved path. This effect is known as Magnus-effect.

Definitions

- The upward force experienced by a body when it moves through a fluid is called dynamic lift.

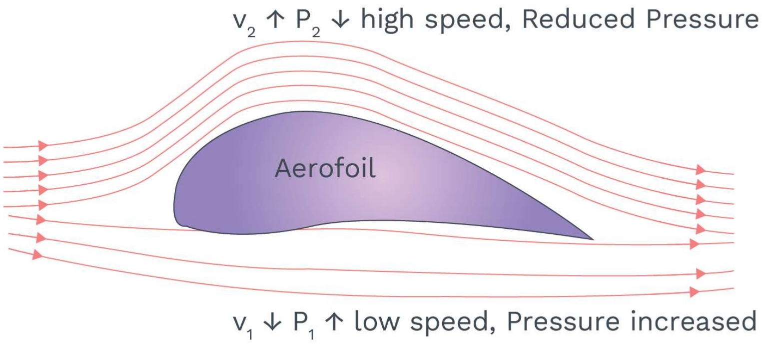


motion of a spin ball

(iv) Aerofoil

This is a structure which is shaped in such a way so that its motion relative to a fluid produces a force perpendicular to the flow. As shown in the figure the shape of the aerofoil section causes the fluid to flow faster over the top surface than below the bottom i.e. the streamlines are closer above than below the aerofoil. By Bernoulli's theorem the pressure at above reduced whereas that underneath it gets increased. Thus a resultant upward thrust is generated normal to the flow and it is this force which provides most of the upward lift for an aeroplane.

Examples of aerofoils are aircraft wings, turbine blades and propellers.



Applying Bernoulli's theorem between upper and bottom surface of aerofoil.

Concept Reminder

when spinning ball experiences a dynamic lift and deviated from its path, then that effect is known as "magnus effect".

Definitions

- An aerofoil is a streamlined shaped, solid body that is capable of generating a dynamic lift as it moves through a fluid.

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow \quad P_1 - P_2 = \frac{1}{2}\rho_{air}(v_2^2 - v_1^2)$$

Net upward lifting force,

$$\boxed{F = (P_1 - P_2)A = \frac{1}{2}\rho_{air}\, A(v_2^2 - v_1^2)}$$

Ex. An aeroplane of weight 5×10^5 N and total wing area of 250 m^2 is in a level flight at some height. Find the difference in pressure between the upper and lower surfaces of its wings. ($g = 10\ m/s^2$)

Sol. $\Delta P = \frac{Mg}{A} = \frac{5 \times 10^5}{250} = 2 \times 10^3 Pa = 2\,kPa$

Ex. The flow speeds of air on the lower and upper surfaces of the wing of an aeroplane are v and 2v respectively. The density of air is ρ and surface area of wing is A. Find the dynamic lift on the wing.

Sol. Dynamic lift = Pressure difference × Area

$= (P_{lower} - P_{upper})A$

According to Bernoulli's theorem

$$= \left(\frac{1}{2}\rho v_{upper}^2 - \frac{1}{2}\rho v_{lower}^2\right)A$$

$$= \frac{1}{2}\rho A((2v)^2 - (v)^2) = \frac{3}{2}\rho A v^2$$

(v) Sprayer or Atomizer

This is an instrument used to spray a liquid in the form of small droplets (fine spray). It consists of a vertical tube whose lower end is dipped in the liquid to be sprayed, filled in a vessel. The upper end opens in a horizontal tube. At one end of the horizontal tube there is a rubber bulb and the other end has is a fine bore (hole).

Concept Reminder

Streamline shape is a special shape given to bodies moving in fluids, so as to produce minimum drag.

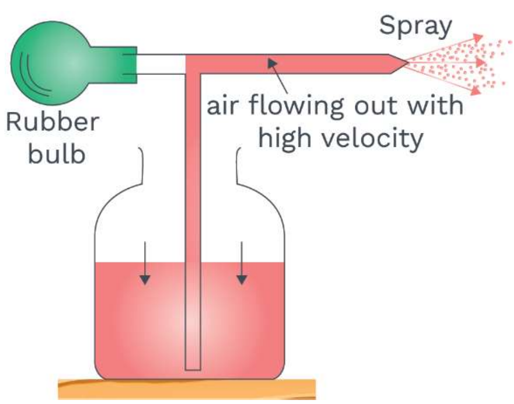
Rack your Brain

In magnus effect, lift occurs towards the direction where:

(1) Relative velocity of fluid is greater
(2) Relative velocity of fluid is smaller
(3) Pressure is greater
(4) Kinetic energy of fluid is smaller

KEY POINTS

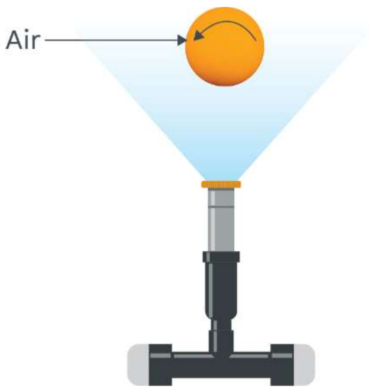
- Dynamic lift
- Magnus effect
- Aerofoil
- Atomizer



When the rubber bulb is squeezed, air rushes out through the horizontal tube with very high velocity and thus the pressure reduces (according to Bernoulli's theorem). Consequently, the liquid in the vessel rises up and mixes with air in the form of small droplets which gets ejected in the form of a fine spray.
Example: paint guns, perfume or deodorant sprayer, etc.

(vi) Motion of the Ping-Pong Ball

When a ping-pong ball is placed on a vertical stream of water-fountain, it rises upto a certain height above the nozzle of the fountain and spins about its axis. The reason for this is that the air streams of water rise up from the fountain with large velocity so that the air-pressure decreases. Therefore, whenever the ball tends to fall out from the stream, the outer air which is at atmospheric pressure pushes it back into the stream (in the region of low pressure). Thus, the ball remains more or less stable in the fountain.

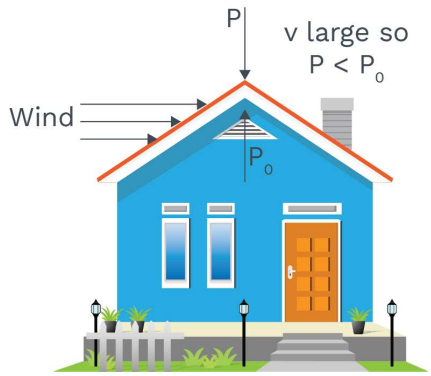


Concept Reminder

Weakly built roots of houses are blown away during wind storms. This is a consequence of Bernoulli's principle.

(vii) Blowing-off of Tin Roof Tops in Wind Storm

When wind blows with a high velocity above a tin roof, it causes lowering of pressure above the roof, while the pressure below the roof is still atmospheric. Due to this pressure-difference the roof is lifted up and is blown away during storms.



(viii) Pull-in or Attraction Force by Fast Moving Trains

If we are standing on a platform and a train passes through the platform with very high speed we are pulled towards the train. This is because as the train moves at high speed, the pressure close to the train decreases. Thus the air away from the train which is still at atmospheric pressure pushes us towards the train. The reason behind flying-off of small papers, straws and other light objects towards the train is also the same.

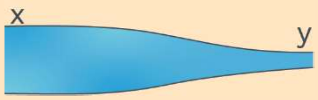
Examples

Q1 **Liquid (water) flows through a horizontal tube of variable cross-section (figure). The area of cross-section at x and y are 40 mm² and 20 mm² respectively. If 10 cc of water enters per second through x, find:**

(i) The speed of water at x

(ii) The speed of water at y

(iii) The pressure difference $P_x - P_y$.



Sol: $A_1v_1 = A_2v_2$

(i) $10\ cm^3/s = 40\ mm^2 \times v_x$ or $v_x = 25\ cm/s$

(ii) $10\ cm^3/s = 20\ mm^2 \times v_y$ or $v_y = 50\ cm/s$

$$P_x + \frac{1}{2}\rho v_x^2 = P_y + \frac{1}{2}\rho v_y^2$$

$$P_x - P_y = \frac{1}{2}\rho v_y^2 - \frac{1}{2}\rho v_x^2 = \frac{1}{2} \times 1000\left[\left(\frac{1}{2}\right)^2 - \left(\frac{1}{4}\right)^2\right]$$

or $P_x - P_y = 93.75\ N/m^2$.

Q2 **Suppose the tube in the previous problem is kept vertical with y upward. Water enters through y at the rate of 10 cm³/s. Repeat part (iii). Note that the speed decreases as the water falls down.**

Sol: $A_1v_1 = A_2v_2$

$$P_y + \frac{1}{2}\rho v_y^2 + \rho gh = P_x + \frac{1}{2}\rho v_x^2 + 0$$

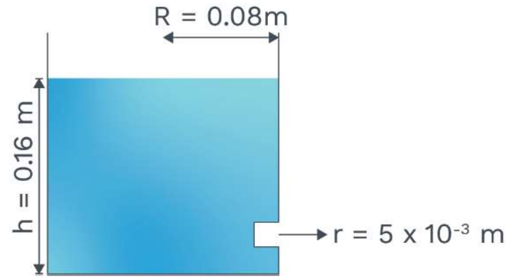
$$P_x - P_y = \frac{1}{2}\rho\left(v_y^2 - v_x^2\right) + \rho gh$$

$$P_x - P_y = \frac{1}{2}1000\left[\left(\frac{1}{2}\right)^2 - \left(\frac{1}{4}\right)^2\right] + 1000 \times 10 \times \frac{15}{1600}$$

$P_x - P_y = 187.5\ N/m^2$.

Q3 **A cylindrical tank of height 0.4 m is open at the top and has a diameter 0.16 m. Water is fill up to a height of 0.16 m. How long it will take to empty the tank through a hole of radius 5×10^{-3} m at its bottom?**

Sol:



$A_1v_1 = A_2v_2$

$\pi R^2\, dh/dt = \pi r^2 v$ ...(i)

$v = \sqrt{2gh}$ ...(ii)

From equation (ii) put the value of v in equation (i)

$\pi R^2 dh / dt = \pi r^2\sqrt{2gh}$

$$\Rightarrow \int \frac{R^2 dh}{r^2\sqrt{2gh}} = \int dt$$

$$\frac{R^2}{r^2\sqrt{2gh}}\int_h^0 \frac{dh}{\sqrt{h}} = \int_0^T dt$$

$$T = \frac{R^2}{r^2}\sqrt{\frac{2h}{g}}$$

On solving

T = 46.26 second.

Q4

An open tank 10 m long and 2m deep is filled upto height 1.5 m of oil of specific gravity 0.80. The tank is accelerated uniformly from rest to a speed of 10 m/sec. What is the shortest time in which this speed may be attained without spilling any oil. [g = 10 m/s^2]

Sol:

$$\tan\theta = \frac{0.5}{5} = \frac{1}{10}$$

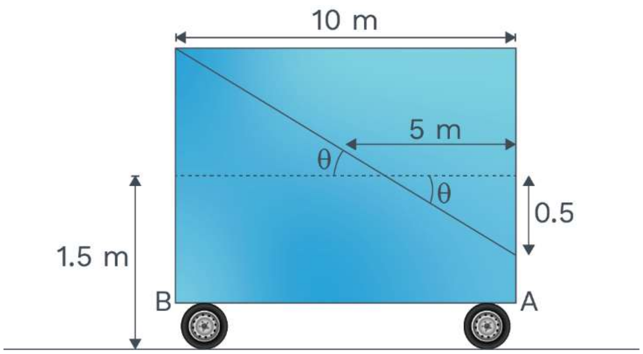
$$\frac{a}{g} = \frac{1}{10}$$

$a = 1\ m/s^2$

$v = u + at$

$10 = 0 + 1 \times t$

$t = 10$ second.



Q5

A particle of density 'd' is dropped onto a horizontal solid surface plane. It bounces elastically from the surface plane and returns to its original position in a time 't_1'. Next, the particle is released and it falls through the same height before striking the surface plane of a liquid of density d_L.

(a) If $d < d_L$, pickup an expression (in terms of d, t_1 and dL) for the time t_2 the particle takes to come back to the position from which it was released.

(b) Is the motion of the particle simple harmonic?

(c) If $d = d_L$, how does the speed of the particle depend on its depth inside the liquid?

Neglect all frictional and other dissipative forces. Let us assume the depth of the liquid to be large.

Sol: In elastic collision with the surface plane, direction of velocity is reversed but its magnitude remains equal. Therefore, time of fall = time of rise.

or time of fall = $t_{1/2}$

Hence, velocity of the ball just before it collides with liquid is

$$v = g\frac{t_1}{2} \qquad ...(1)$$

Retardation inside the liquid

$$a = \frac{\text{upthrust} - \text{weight}}{\text{mass}}$$

$$= \frac{Vd_L g - Vdg}{Vd} = \left(\frac{d_L - d}{d}\right) g \left(V = \text{volume of ball}\right) \qquad ...(2)$$

Time taken to come to rest condition under this retardation will be

$$t = \frac{v}{a} = \frac{gt_1}{2a} = \frac{gt_1}{2\left(\frac{d_L - d}{d}\right)g} = \frac{dt_1}{2\left(d_L - d\right)}$$

Equal time to come back on the liquid surface.

Hence,

(a) t_2 = time the particle takes to came back to the position from where it was released

$$= t_1 + 2t = t_1 + \frac{dt_1}{d_L - d} = t_1\left[1 + \frac{d}{d_L - d}\right]$$

$$\text{or} \quad t_2 = \frac{t_1 d_L}{d_L - d}$$

(b) The motion of the particle is periodic but not simple harmonic because the acceleration of the particle is g in air and $\left(\frac{d_L - d}{d}\right)g$ inside the liquid which is not proportional to the displacement, which is necessary and sufficient condition for SHM.

(c) When $d_L = d$, retardation or acceleration inside the liquid becomes zero (upthrust = weight). Therefore, the particle will continue to move with constant velocity $v = gt_1/2$ inside the liquid.

Q6 **Two identical cylindrical vessels with their bases at the same level each contain a liquid of density ρ as shown in figure. The height of the liquid in one vessel is h_2 and other vessels h_1, the area of either base is A. Find the work done by gravity in equalizing the levels when the two vessels are connected.**

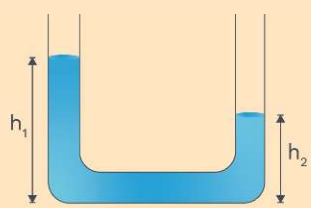


Figure (1)

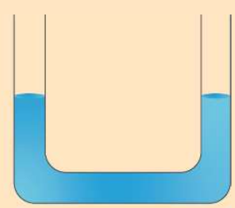
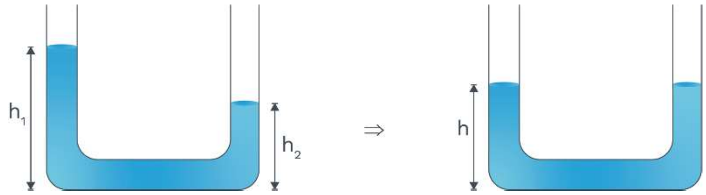


Figure (2)

Sol: Let h be level in equilibrium. Equating the volumes, we have



$$Ah_1 + Ah_2 = 2Ah$$

$$\therefore \quad h = \left(\frac{h_1 + h_2}{2}\right)$$

Work done by gravity = $U_i - U_f$

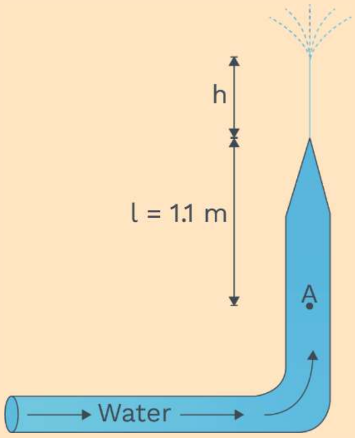
$$W = \left(m_1 g \frac{h_1}{2} + m_2 g \frac{h_2}{2}\right) - (m_1 + m_2) g \frac{h}{2}$$

$$= \frac{Ah_1 \rho g h_1}{2} + \frac{Ah_2 \rho g h_2}{2} - \left[Ah_1\rho + Ah_2\rho\right] g \left(\frac{h_1 + h_2}{4}\right)$$

Simplifying this, we get

$$W = \frac{\rho A g}{4}(h_1 - h_2)^2.$$

Q7 **Water shoots out of a pipe and nozzle as shown in the figure. The cross-sectional area for the tube at point A is four times that of the nozzle. The pressure of water at point A is 41 × 10^3 Nm^{-2} (gauge). Find the height 'h' above the nozzle to which water jet will shoot. Ignore all the losses occurred in the above process. [g = 10 m/s^2].**



Sol:

$A_1v_1 = A_2v_2$

$Av_A = \frac{A}{4}v_n$

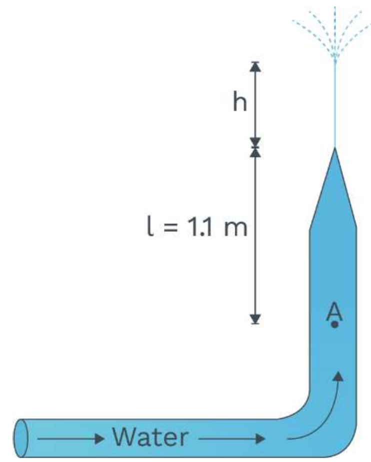
$v_n = 4v_A$ ...(i)

$P_A + \frac{1}{2}\rho v_A^2 = \frac{1}{2}\rho v_n^2 + \rho gh$ ...(ii)

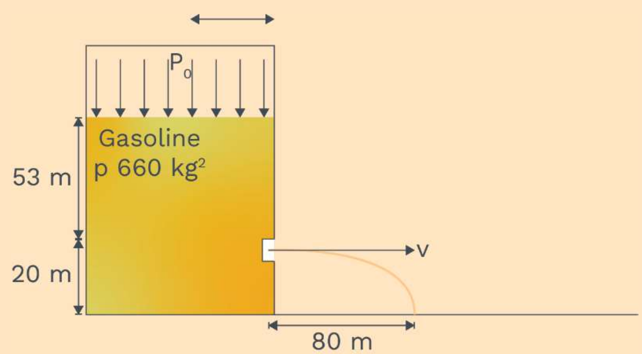
$v_n^2 / 2g = h$ ...(iii)

On solving (i), (ii) & (iii) equation we get

$h = 3.2\,m.$



Q8 **A tank containing gasoline is sealed and the gasoline is under pressure P_0 as shown in the figure. The stored gasoline has a density of 660 kg m³. A sniper fires a rifle bullet into the gasoline tank, making a small hole 53 m below the surface of gasoline. The total height of gasoline is 73 m from the base. The jet of gasoline shooting out of the hole strikes the ground at a distance of 80 m from the tank initially. Find the pressure P_0 above the gasoline surface. The local atmospheric pressure is 10^5 Nm⁻².**



Sol:

vt = 80 m ...(i)

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 20}{10}}$ = 2 second ...(ii)

From (i) and (ii) equation

v = 40 m/s. ...(iii)

$P_0 + \rho gh = P_{atm} + \frac{1}{2}\rho v^2$ ...(iv)

On solving (iv) equation

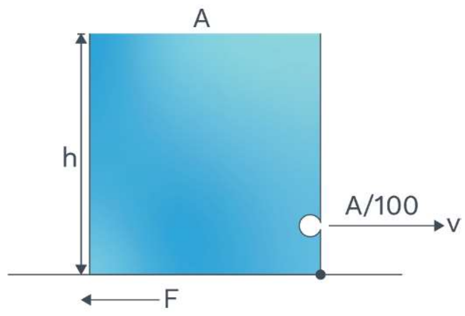
$P_0 = 2.78 \times 10^8 Nm^{-2}$.

Q9 **A large open top container of negligible mass and uniform cross-sectional area A has a small hole of cross-sectional area $\frac{A}{100}$ in its side wall near the bottom. Container is kept on a smooth horizontal surface and contains a liquid of density 'ρ' and mass 'm_0'. Assuming that the liquid (density 'ρ') starts flowing out horizontally through the hole at t = 0, Find out**

(a) The acceleration of the container and

(b) Its velocity when 75 % of the liquid has drained out.

Sol: **(i)** Mass of water = (Volume) (density)



$\therefore \quad m_0 = (AH)\rho$

$\therefore \quad H = \frac{m_0}{A\rho}$

Velocity of efflux, $v = \sqrt{2gh} = \sqrt{2g\frac{m_0}{A\rho}}$

$= \sqrt{\frac{2m_0 g}{A\rho}}$

Thrust force on the container due to draining out of liquid from the bottom is given by,

F = (density of liquid) (area of hole) (velocity of effux)^2

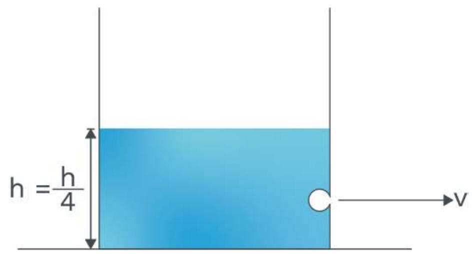
($F = \rho a v^2$)

$$F = \rho(A/100)v^2 = \rho(A/100)\left(\frac{2m_0 g}{A\rho}\right)$$

$$F = \frac{m_0 g}{50}$$

$\therefore$ Acceleration of the container $a = F/m_0 = g/50$

(ii) Velocity of efflux when 75% liquid has been drained out i.e. height of liquid, $h = \frac{H}{4} = \frac{m_0}{4A\rho}$



$$v = \sqrt{2gh}$$

$$= \sqrt{2g\left(\frac{m_0}{4A\rho}\right)}$$

$$v = \sqrt{\frac{m_0}{4A\rho}} \text{ (relative to container)}$$

Q10 **A cylindrical vessel filled with water upto a height of 2 m stands on horizontal plane. The side wall of the vessel has a plugged circular hole touching the bottom. Find the minimum diameter of the hole so that the vessel begins to move on the floor if the plug is removed. The coefficient of friction between the bottom of the vessel and the plane is 0.4 and total mass of water plus vessel is 100 kg.**

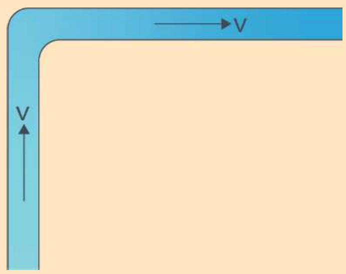
Sol:

$$\rho a v^2 = \mu N$$

$$1000 \times a\left(2g \times 2\right) = 0.4 \times 100 \times 10$$

$$1000 \times \pi \frac{d^2}{4} \times 4g = 400 \text{ or } d = \frac{0.2}{\sqrt{\pi}} = 0.133 \text{ m.}$$

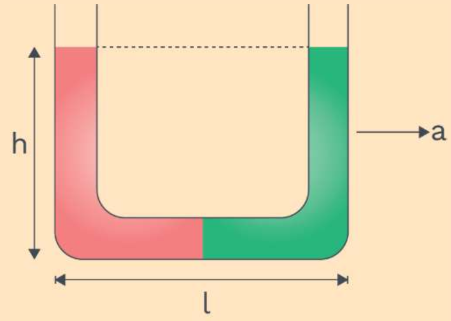
Q11 **A fire hydrant (as shown in the figure) delivers water of density ρ at a volume rate L. The water travels vertically upward through the hydrant and then does 90° turn to emerge horizontally at speed v. The pipe and nozzle have uniform cross-section throughout. The force applied by the water on the corner of the hydrant is**



Sol:

$$F_{th} = \frac{\sqrt{2}dp}{dt} = \sqrt{2}\,v\frac{dm}{dt} = \sqrt{2}\,v\left[\rho L\right] = \sqrt{2}\rho\, vL$$

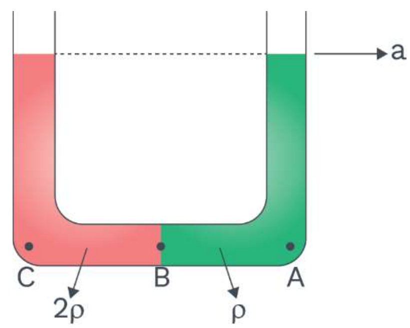
Q12 **A U-tube of base length "l" filled with same volume of two liquids of densities ρ and 2ρ is moving with an acceleration "a" on the horizontal plane as shown in the figure. If the height difference between the two surfaces (open to atmosphere) becomes zero, then the height h is given by**



Sol: For the given situation, liquid of density 2ρ should be behind that of ρ.

From right limb

$$P_A = P_{atm} + \rho gh$$



$$P_B = P_A + \rho a\frac{\ell}{2} = P_{atm} + \rho gh + \rho a\frac{\ell}{2}$$

$$P_C = P_B + (2\rho)a\frac{\ell}{2} = P_{atm} + \rho gh + \frac{3}{2}\rho a\ell \quad ...(1)$$

But from left limb

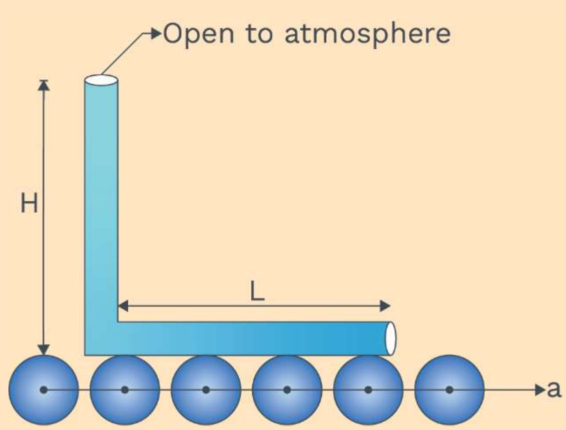
$$P_C = P_{atm} + (2\rho)gh \quad ...(2)$$

From (1) and (2)

$$P_{atm} + \rho gh + \frac{3}{2}\rho a\ell = P_{atm} + 2\rho gh$$

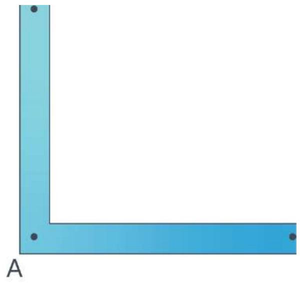
$$\Rightarrow \quad h = \frac{3a}{2g}\ell$$

Q13 **A narrow tube completely filled with a liquid is lying on a series of cylinders as shown in figure. Assuming no sliding between any surfaces, the value of acceleration of the cylinders for which liquid will not come out of the tube from anywhere is given by**



Sol: No sliding $\Rightarrow$ pure rolling

Therefore, acceleration of the tube = 2a (since COM of cylinders are moving at 'a')

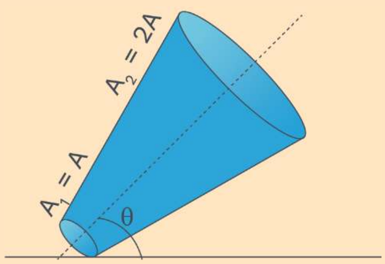


$P_A = P_{atm} + \rho(2a)L$ (From horizontal limb)

Also; $P_A = P_{atm} + \rho gh$ (From vertical limb

$\Rightarrow a = \frac{gH}{2L}$

Q14 A portion of a tube is shown in the figure. Fluid is flowing from cross-section area A_1 to A_2. The two cross-sections are at distance 'ℓ' from each other. The velocity of the fluid at section A_2 is $\sqrt{\frac{g\ell}{2}}$. If the pressures at A_1 and A_2 are same, then the angle made by the tube with the horizontal will be.



Sol:

$A_1v_1 = A_2v_2$ or $A \times v_1 = 2A\sqrt{\frac{g\ell}{2}}$ or $v_1 = \sqrt{2g\ell}$

$\frac{1}{2}\rho\left[v_1^2 - v_2^2\right] = \rho g\ell \sin\theta$

$\Rightarrow \frac{1}{2}\left[2g\ell - \frac{g\ell}{2}\right] = g\ell \sin\theta$

On solving, $\sin\theta = \frac{3}{4}$.

Mind Map

APPLICATIONS OF PASCAL'S LAW