Electromagnetic Waves

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INTRODUCTION:

Maxwell formulated a set of equations including electric field, magnetic field and their sources, the electric charge and electric current densities. These equations are known as equation of Maxwell. Together with the Lorentz force equation, they mathematically express all basic rules of electro-magnetism.

The important prediction to emerge from Maxwell's equations is the existence of electro-magnetic waves, which are (coupled) time varying magnetic and electric field that propagate in free region. The speed of the waves, according to these equations, turned out to be very near to the speed of light (3×10^8 m/s), occured from optical measurements. This led to the remarkable result that light is an electro-magnetic radiation. Maxwell's work thus unified the domain of magnetism, electricity and light.

In this unit, we 1st discuss the need for displacement current and its results. Then we so a descriptive account of electro-magnetic waves. The broad spectrum electromagnetic waves, is stretching from γ rays (wavelength ~ 10^{-12} m) to long radio waves (wavelength ~ 10^6 m).

Light may be described as wave nature. The equation of wave for light propagating in x-direction in vacuum may be written as

$$E = E_0 \sin \omega \left(t - \frac{x}{c} \right)$$

where E is the sinusoidal electric field at position x at t time.
The constant 'c' is the speed of light in vacuum.

Concept Reminder

Electromagnetic waves in the form of visible light enable us to view the world around us.

There is also a sinusoidal magnetic field associated with the electric field when light propagate. The magnetic field is normal to direction of propagation as well as to the electric field E. It is given by

$$B = B_0 \sin\omega\left(t - \frac{x}{c}\right)$$

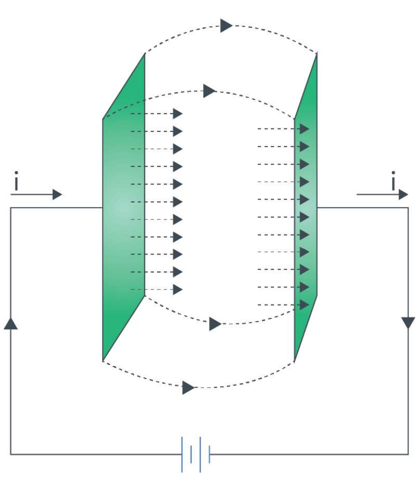
Combination of mutually normal electric and magnetic fields is referred to as an electromagnetic wave in vacuum.

DISPLACEMENT CURRENT:

1. Concept of Displacement Current

When a capacitor is allowed to charge in an electric circuit, the current flows through connecting wires. As capacitor charges, charge accumulates on the two plates of capacitor and as a result, a changing electric field is produced across between the two plate of the capacitor.

According to Maxwell, changing electric field intensity is equivalent to a current through capacitor and that current is known as displacement current (I_d). If + q and –q be the charge on the left and right plates of the capacitor respectively at any instant and if σ be the surface charge density of plate of capacitor then electric field between the plates is given by



Concept Reminder

During charging of capacitor for consistency of ampere circuital law, there must be a current between the plates of capacitor. It is called displacement current.

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

$\Rightarrow$ $Q = \epsilon_0 (EA)$

By definition

ϕ_E = E.A= electric flux,

Therefore, $Q = \epsilon_0 \phi_E$

Also, we know that

$$I = \frac{dQ}{dt}$$

$$\Rightarrow I_D = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 A \frac{dE}{dt}$$

This current is called displacement current.

- **Ampere-Maxwell's law:**
  This law states that " line integral of magnetic field along a closed loop is equal to μ_0 times the total current threading the area bounded by the closed loop".

  $$\oint \vec{B} \cdot d\vec{l} = \mu_0 (I_C + I_D)$$

  I_C = Conduction current (flows in conducting wire)

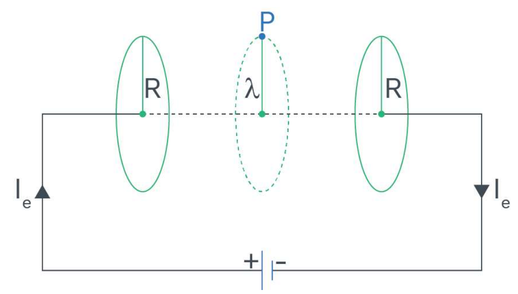
  I_D = Displacement current (between plates of capacitor)
- According to Maxwell during the charging of capacitor a virtual current flow between the plates of capacitor due to variable electric field.
- Generally, it is denoted by 'I_D'.
- Magnitude of conduction current (I_c) is always equal to the magnitude of displacement current (I_D).

  $$\boxed{I_C = I_D}$$

KEY POINTS

- Displacement Current
- Conduction Current

Magnitude of magnetic field between the plates of Parallel plate capacitor (PPC) during charging:



According to Ampere- Maxwell equation

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I_C + \epsilon_0 \frac{d\phi_E}{dt} \right)$$

But in between the plates of PPC $I_C = 0$

$$\therefore \quad \oint \vec{B} \cdot d\vec{l} = \mu_0 \times \epsilon_0 \frac{d\phi_E}{dt}$$

$$\Rightarrow \quad \oint B dl \cos 0 = \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$$

$$\Rightarrow \quad B \oint dl = \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$$

$$\Rightarrow \quad B \times 2\pi r = \mu_0 \epsilon_0 \frac{d\phi_E}{dt} \quad \{\because \oint dl = 2\pi r\}$$

$$\Rightarrow \quad B = \frac{\mu_0 \epsilon_0}{2\pi r} \frac{d\phi_E}{dt}$$

$$\Rightarrow \quad B = \frac{\mu_0 \epsilon_0}{2\pi r} \frac{d(E \times A)}{dt}$$

$$\Rightarrow \quad B = \frac{\mu_0 \epsilon_0 \times A}{2\pi r} \frac{dE}{dt}$$

$$\Rightarrow \quad B = \frac{\mu_0 \epsilon_0 \times \pi R^2}{2\pi r} \frac{dE}{dt}$$

$$\Rightarrow \quad B = \frac{\mu_0 \epsilon_0 R^2}{2r} \frac{dE}{dt}$$

Definitions

Displacement current is a current which is produced due to the rate of change of electric flux with respect to time.

Rack your Brain

A parallel plate capacitor of capacitance 20 μF is being charged by a voltage source whose potential is charging at the rate of 3 V/s. Find conduction current through the connecting wires and the displacement current through the plates of the capacitor.

Maxwell's Equations:

The 4 basic laws of magnetism and electricity i.e. Gauss's law in electrostatics, Gauss's law in magnetism, Faraday's law of electro-magnetic induction and Maxwell-Ampere's circuital law are called Maxwell's equations.

1. $\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$ (Gauss's law for electricity)
2. $\oint \vec{B} \cdot d\vec{A} = 0$ (Gauss's law for magnetism)
3. $\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$ (Faraday's law)
4. $\oint \vec{B} \cdot d\vec{l} = \mu_0(I_C + I_D)$ (Ampere-Maxwell law)

Nature of Electromagnetic Waves:

Concept of displacement current led to the result that an electric field varying with time at a point produces magnetic field at that given point. This symmetry in the law of electricity & magnetism leads to the result that a electric field varying with time gives rise to a time varying magnetic field and vice versa. Electric and magnetic fields in electro-magnetic wave are of transverse nature.

Magnetic and electric fields in electro-magnetic wave satisfy following wave equation, which can be obtained from Maxwell's (3) & (4) equations.

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

and

$$\frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}$$

In free space

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m/s}$$

Electric field ($\vec{E}$) and magnetic fields ($\vec{B}$) in a EM wave are related by the following expression in free space.

$$E = cB$$

or

$$c = \frac{E}{B}$$

Ex. Green light of mercury has a wavelength 5.5×10^{-5} centimetre.

(a) What is the frequency in MHz and period in ms in vacuum?

(b) What is its wavelength in glass, if refractive index of glass is 1.5?

Given: $c = 3 \times 10^8$ m/s.

Sol. Here,

Wavelength $\lambda = 5.5 \times 10^{-5}$ cm $= 5.5 \times 10^{-7}$ m

Velocity of light, $c = 3 \times 10^8$ metre/second

(a) If υ is the frequency, then

$$\upsilon = \frac{c}{\lambda} = \frac{3 \times 10^8}{5.5 \times 10^{-7}} \text{Hz}$$

$$= \frac{3 \times 10^8}{5.5 \times 10^{-7} \times 10^6} = 5.45 \times 10^8 \text{ MHz}$$

Time period,

$$T = \frac{1}{\upsilon} = \frac{\lambda}{c} = \frac{5.5 \times 10^{-7}}{3 \times 10^8}$$

$= 1.8 \times 10^3 \times 10^6$ ms $= 1.8 \times 10^{-12}$ ms

(b) Now, refractive index μ

$$= \frac{\text{Velocity of light in vacuum}}{\text{Velocity of light in glass}} = \frac{c}{v}$$

Therefore, velocity of light in glass

$$v = \frac{c}{\mu} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ m/s}$$

The wavelength of light in glass,

$$\lambda = \frac{v}{\upsilon} = 2 \times 10^8 \times 1.8 \times 10^{-15} = 3.6 \times 10^{-7} \text{m}$$

Ex. Derive the magnetic field between the circular plates of a capacitor when r < R (R=radius of plates).

Sol. Consider a loop of radius r (<< R) between the two circular plates placed coaxially. The area of loop = pr^2.

By symmetry magnetic field $\vec{B}$ is equal in magnitude at all points on the loop. If i_D is is the displacement current crossing the loop and I_D is the total displacement current between plates

$$i_D = \frac{I_D}{\pi R^2} \times \pi r^2$$

Using Ampere Maxwell's law we have,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_D$$

or $$2\pi r B = \mu_0 I_D \frac{\pi r^2}{\pi R^2}$$

or $$B = \frac{\mu_0 I_D r}{2\pi R^2}$$

Ex. In a plane electro-magnetic wave the electric field oscillates sinusoidally at a frequency 3×10^{10} Hz and amplitude 50 Volt/metre.
(a) What is the wavelength of electro-magnetic wave?
(b) What is the amplitude of oscillating magnetic field?

Sol. Here, $f = 3 \times 10^{10}$ Hz, $E_0 = 50$ Volt/metre

(a) Wavelength,

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^{10}} = 10^{-2} \text{m} = 1 \text{ cm}$$

(b) Amplitude of oscillating magnetic field,

$$B_0 = \frac{E_0}{c} = \frac{50}{3 \times 10^8} = 1.67 \times 10^{-7} \text{ T}$$

SOURCES OF ELECTROMAGNETIC WAVE

1. A variable electric field produces variable magnetic field (Ampere–Maxwell law) and variable magnetic field produces variable electric field (Faraday's law). In LC oscillator charge oscillates between plates of capacitor which produces time variable sinusoidal electric and magnetic field and their frequency is given by

$$f = \frac{1}{2\pi\sqrt{LC}} \text{ (for L-C oscillator)}$$

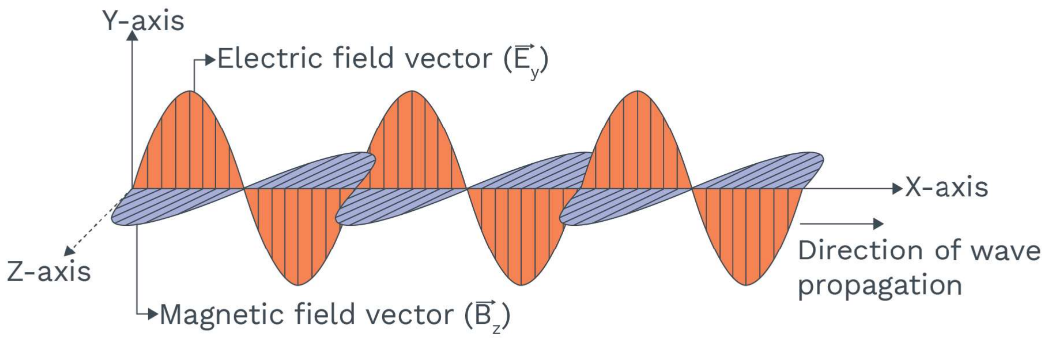
2. Electromagnetic wave (x-rays) is produced when high speed electron enters target of high atomic weight.
3. Electromagnetic wave (gamma rays) is produced during de-excitation of nucleus in radioactivity.

Definitions

The waves that are produced by accelerated charged particles and composed of electric and magnetic field vibrating transversely and sinusoidally perpendicular to each other and to direction of propagation are called electromagnetic waves.

CHARACTERISTICS OF ELECTROMAGNETIC WAVE

1. The EMW's consists of sinusoidally time varying electric field and magnetic field at right angle to each other as well as at normal angle to the direction of propagation.



$E = E_0 \sin(\omega t - kx)$, along y-axis

$B = B_0 \sin(\omega t - kx)$, along z-axis

Propagation of wave is along x-axis

$$\omega = \frac{2\pi}{T} = 2\pi f \quad \text{and} \quad K = \frac{2\pi}{\lambda}$$

2. Direction of propagation of EMW is given by direction of $\vec{E} \times \vec{B}$.

3. $\vec{E}$ and $\vec{B}$ become maximum at same place and at the same time , but perpendicular to each other as well as to direction of propagation. Therefore, the phase difference between the two fields is zero.

4. For EMW's relation between magnitude of $\vec{E}$ and $\vec{B}$ is

$$c = \frac{E}{B}$$

5. Velocity of EMW's in vacuum

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m / sec}$$

Concept Reminder

A charge moving with uniform velocity (i.e., steady current) produces both electric and magnetic field but here magnetic field does not change with time hence it does not produce time varying electric field.

Velocity in medium

$$v = \frac{1}{\sqrt{\mu_r\mu_0\varepsilon_r\varepsilon_0}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}} = \frac{c}{n}$$

where $n = \sqrt{\mu_r\varepsilon_r}$ = Refractive index of medium.

6. EMW's are transverse wave and non-mechanical waves.

7. EMW's possess energy and they can carry energy from one place to other.

- Energy Density or energy per unit volume in Electro-magnetic Waves

(a) Average energy density of electric field is-

$$u_E = \frac{1}{2}\varepsilon_0 E_{rms}^2$$

$$= \frac{1}{2}\varepsilon_0\left(\frac{E_0}{\sqrt{2}}\right)^2 = \frac{1}{4}\varepsilon_0 E_0^2$$

(b) Average energy density of magnetic field is given by-

$$u_B = \frac{B_{rms}^2}{2\mu_0} = \frac{(B_0/\sqrt{2})^2}{2\mu_0} = \frac{B_0^2}{4\mu_0}$$

(c) $u_E = u_B$

(d) Total average energy density

$$u_E + u_B = 2u_E = 2u_B = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}\frac{B_0^2}{\mu_0}$$

The units of u_E and u_B are J/m^3.

8. EMW's exert pressure on a surface when they incident on it called radiation pressure. If ΔU is the energy of EMW incident on a surface, then momentum delivered to this surface

$$\Delta P = \frac{\Delta U}{C} \text{ (for complete absorption)}$$

$$\Delta P = 2\frac{\Delta U}{C} \text{ (for complete reflection)}$$

9. The energy of electro-magnetic wave crossing per unit time per unit area normal to the direction of wave propagation is called the

Rack your Brain

The ratio of contributions made by the electric field and magnetic field components to the intensity of an electromagnetic wave is (C = speed of electromagnetic wave)

(1) C : 1 (2) 1 : 1
(3) 1 : C (4) 1 : C^2

intensity of electro-magnetic wave. The intensity of electro-magnetic wave is-

$$I = \frac{P_{av}}{4\pi r^2} = u_{av} \times c = \frac{1}{2}\varepsilon_0 E_0^2 c$$

$$= \frac{1}{2\mu_0} B_0^2 c = \frac{1}{2\mu_0} \cdot \frac{E_0^2}{c}$$

10. Momentum of Electromagnetic Wave:

- The e.m. wave during its propagation has linear momentum with it.
- If the e.m. wave incident on a material surface is completely absorbed, then wave will deliver energy U and momentum $p = \frac{U}{c}$ to the completely absorbed surface.
- If incident wave is totally reflected from the surface, then the momentum delivered to totally reflected surface is $p = \frac{U}{c} - \left(-\frac{U}{c}\right) = \frac{2U}{c}$. Due to which the e.m. waves incident on a surface exerts a force on the surface.

11. Poynting Vector-

- When an e.m. wave advances, the electro-magnetic energy flows in the direction of $\vec{E} \times \vec{B}$. The total energy flowing normally per second per unit area into the surface in free space is called Poynting vector $\vec{S}$, where

$$\vec{S} = c^2 \varepsilon_0 (\vec{E} \times \vec{B}) = \frac{(\vec{E} \times \vec{B})}{\mu_0}$$

The S.I. unit of Poynting vector S is watt/(metre)^2.

12. Radiant Flux Density-

- The average value of poynting vector $(\vec{S})$ over a convenient time interval in the propagation of electromagnetic wave is known as radiant flux density.
- When energy of electro-magnetic wave is incident on a surface, the flux density is called intensity of wave
(denoted by I). So
$I = |\vec{S}|$

KEY POINTS

- Poynting vector
- Radiant flux density

Concept Reminder

An electron orbiting around nucleus in a stationary orbit does not emit electromagnetic wave. It will emit radiation only during transition from higher energy orbit to lower energy orbit.

A harmonic electromagnetic wave travelling along X-axis in free region can be described by periodic variation of electric & magnetic field along y-axis and z-axis with the equations-

$$\vec{E} = \vec{E}_0 \cos(kx - \omega t)$$

and $$\vec{B} = \vec{B}_0 \cos(kx - \omega t)$$

- Then radiant flux density $\vec{S}$ is given as

$$\vec{S} = c^2\varepsilon_0(\vec{E} \times \vec{B}) = c^2\varepsilon_0\vec{E}_0 \times \vec{B}_0 \cos^2(kx - \omega t)$$

Hence,

$$|\vec{S}| = c^2\varepsilon_0 \, |\vec{E}_0 \times \vec{B}_0| \cos^2(kx - \omega t)$$

- The average value of Poynting vector $\vec{S}$ over a single period 'T' is given by

$$S = I = c^2\varepsilon_0 \, |\vec{E}_0 \times \vec{B}_0| \frac{1}{T}\int_0^T \cos^2(kx - \omega t)dt$$

$$= c^2\varepsilon_0 E_0 B_0 \sin 90° \left[\frac{1}{2}\right]$$

$$\left[\text{since } \frac{1}{T}\int_0^T \cos^2(kx - \omega t)dt = \frac{1}{2}\right]$$

$$= c^2\varepsilon_0 E_0\left(\frac{E_0}{c}\right)\left(\frac{1}{2}\right) = \frac{1}{2}c\varepsilon_0 E_0^2 = \frac{1}{2c\mu_0}E_0^2$$

Ex. A point source of electro-magnetic radiation has an average power output of 800 W. Calculate the maximum value of electric field (E) at a distance 3.5 m from the source.

Sol. Intensity of electromagnetic wave given is by

$$I = \frac{P_{av}}{4\pi r^2} = \frac{E_m^2}{2\mu_0 c}$$

$$E_m = \sqrt{\frac{\mu_0 c P_{av}}{2\pi r^2}} = \sqrt{\frac{(4\pi \times 10^{-7}) \times (3 \times 10^8) \times 800}{2\pi \times (3.5)^2}}$$

$$= 62.6 \text{ V/m}$$

Rack your Brain

Light with an average flux of 20 W/cm² falls on a non-reflecting surface at normal incidence having surface area 20 cm². Find out energy received by the surface during time spam of 1 minutes.

Ex. In an electromagnetic wave, the amplitude of electric field is I V/m. The frequency of wave is 5×10^{14} Hz. The wave is propagating along z-axis. The average energy density of electric field, in Joule/m^3, will be.

Sol. Average energy density is given by

$$u_E = \frac{1}{2}\varepsilon_0 E_{rms}^2 = \frac{1}{2}\varepsilon_0\left(\frac{E_0}{\sqrt{2}}\right)^2$$

$$= \frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{4}(8.85 \times 10^{-12} \times (1)^2)$$

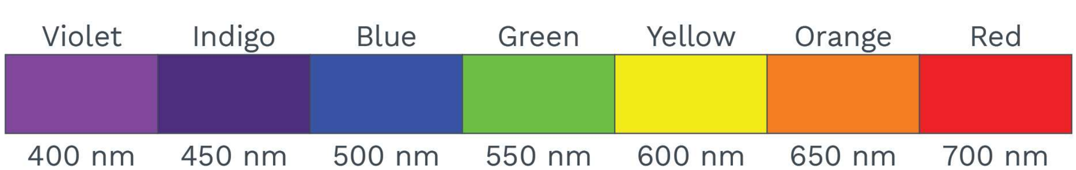
$$= 2.2 \times 10^{-12} \text{ J/m}^3.$$

ELECTROMAGNETIC SPECTRUM:

At Maxwell's predicted the existence of e.m. waves, the only familiar e.m. waves were the visible light. The existence of ultraviolet waves and infrared waves was hardly established. By the end of the 19^{th} century, X -rays and gamma rays had also been discovered. We now know that, e.m. waves include visible light waves, X-waves, gamma rays, radio waves, microwaves, ultraviolet and infrared waves. The classification of electro-magnetic waves according to frequency is the electromagnetic spectrum is shown in table along with applications. There is no particular sharp division between one kind of wave and the next. The classification is roughly based on how the waves are produced or detected.

KEY POINTS

- Electromagnetic Spectrum



Spectrum of Visible Light

TYPE	WAVELENGTH RANGE	PRODUCTION	APPLICATION
Radio	> 0.1 m	Rapid acceleration and deceleration of electrons in aerials	In radio and T.V. communaction system, Cellur phones
Microwave	0.1 m to 1 mm	Klystron, magnetron value, Gun diode	In aircraft and satellite communication and to cook food (microwave oven)
Infra-red	1 mm to 700 mm	Vibration of atoms and molecules	In Physiotherapy, green house, military purpose and in agriculture
Visible Light	700 nm to 400 nm	Electrons in atoms emit light when the move from one energy level to a lower energy level	
Ultraviolet	400 nm to 1 nm	Inner shell electrons in atoms moving from one energy level to a lower level	In welding, eye surgery and to kill germs
X-rays	1 nm to 1 pm	X-ray tubes or inner shell electrons of atom	To destroy living tissue and organisms
Gamma rays	< 1 pm	Radioactive decay of the nucleus	To destroy cancer cells

- **Radio Waves:**
  Radio waves are produced by the accelerated movement of electric charges in conducting wires. They are used in radio and TV communication systems. Radio waves generally in the frequency range from 500 kHz to about 1000 MHz. The amplitude modulated band is from "530 kHz to 1710 kHz". Higher frequencies up to 54 MHz are used for short wave bands. Telivision waves range from 54 MHz to 890 MHz. The FM radio band extends from 88 MHz to 108 MHz. Mobile phones use radio waves to transmit audio communication in the UHF band.
- **Microwaves:**
  Microwaves (short-wavelength radio waves), with frequencies in the gigahertz (GHz) range, are generated by special vacuum tubes (called klystrons, magnetrons and Gunn diodes). Due to their short wavelengths, they are suitable for the radar systems used in aircraft navigation. Radar also provides the basis for the speed guns used to time fast balls, tennis serves, and automobiles. Microwave ovens are an interesting domestic appliance of micro waves. In such ovens, the frequency of the microwaves is selected to match resonant frequency of molecules of water so that energy from the waves is transferred sufficiently to the kinetic energy of the molecules. This raises the temperature of any food containing water molecules.

- **Infrared waves:**
  These are produced by hot bodies and molecules. Infrared wave band lies adjacent to the low-frequency or long wavelength end of the visible light spectrum. These are sometimes referred to as heat radiations. This is because aqua molecules present in most materials readily absorb infrared waves (many other molecules, for example: CO_2, NH_3, also absorb infrared radiations). After absorption, their intermolecular motion increases, that is, they heat up and heat their surroundings. Infrared lamps are used in medical therapy. These radiation plays an important role in maintaining the earth's warmth or average temperature through the greenhouse effect. The incoming visible light is absorbed by the earth's surface and re-radiated as smaller frequency radiations. This radiation is trapped by greenhouse gases like carbon dioxide and H_2O vapour. Infrared detectors are used in Earth satellites, both for military purposes and to observe growth of crops. Electronic devices (for example semiconductor light emitting diodes) also emit infrared and are widely used in the remote switches of household electronic systems such as TV sets, video recorders and hi-fi systems.
- **Visible Rays:**
  'visible light rays' is the important familiar form of e.m. waves. It is the part of the spectrum that is detected by the human eye. It runs from about 4×10^{14} Hz to about 7×10^{14} Hz or a wavelength range of about 700–400 nm. Visible light emitted or reflected from objects around us provides us information about the world. Our eyes are sensitive to this range of wavelengths. Different animals are sensitive to different range of wavelengths. Example- snakes can detect infrared waves, and the 'visible light' range of many insects extends well into the ultraviolet.

Rack your Brain

If we consider the radiation emitted by human body, which one of the following statements is true?

(1) The radiation emitted is in the infrared region.
(2) The radiation is emitted only during the day.
(3) The radiation is emitted during the summers and absorbed during the winters.
(4) The radiation emitted lies in the ultraviolet region and hence is not visible.

- **Ultraviolet Rays:**

  UV rays covers wavelengths ranging from about 4×10^{-7} m (400 nm) down to 6×10^{-10} m(0.6 nm). Ultraviolet (UV) radiation is produced by special lamps and very hot bodies. The sun is an important origin of UV light. But fortunately, most of it is absorbed in the ozone layer in the atmosphere at an altitude of about 40 – 50 km. UV light in large quantities has harmful effects on humans. Exposure to UV radiation induces the production of more melanin, causing tanning of the skin. UV radiation is absorbed by ordinary glass. Hence, one can't get tanned or sunburn through glass windows. Welders wear special glass goggles or face masks with glass windows to protect their eyes from large amount of UV produced by welding arcs. Due to its shorter wavelengths, UV radiations can be focussed into very narrow beams for high precision applications such as LASIK (Laser assisted in situ keratomileusis) eye surgery. UV lamps are used to kill germs in aqua purifiers. Ozone layer in the atmosphere plays a protective role, and hence its depletion by chloro-fluoro-carbons (CFCs) gas (such as freon) is a matter of international concern.

- **X-rays:**

  Beyond the UV region of the e.m. spectrum lies the X-ray region. We are familiar with X-rays because of its medical uses. It covers wavelengths from 10^{-8} m (10 nm) down to 10^{-13} m (10^{-4} nm). One common way to generate X-rays is to bombard a metal target by high energy electrons. X-rays are used as a diagnostic tool in medicine and as a treatment for certain forms of cancer. Because X-rays damage or destroy living tissues and organisms, care must be taken to avoid unnecessary or over exposure.

Concept Reminder

The study of X-rays has revealed the atomic structure and crystal structure. Whereas the study of γ-rays provides us valuable information about structure of atomic nuclei.

- **Gamma Rays**
  Gamma rays lie in the upper frequency range of the electro-magnetic spectrum and have wavelengths of from 10^{-10} m to less than 10^{-14} m. This high frequency radiation is produced in nuclear reactions and also emitted by radioactive nucleus. They are used in medicine to destroy cancer cells. As mentioned earlier, the demarcation between different region is not sharp and there are overlaps.
- The complete spectrum of electromagnetic spectrum and the modern terminology used for various sections of the spectrum are given in table. Various regions don't have sharply defined boundaries.

S.No.	RADIATION	DISCOVER	HOW PRODUCED	WAVELENGTH RANGE	FREQUENCY RANGE	ENERGY RANGE	PROPERTIES	APPLICATION
1	γ-rays	Henry Becquerel and Madame Curie	Due to decay of radioactive nuclei.	10^{-14} m to 10^{-10} m	3×10^{22} Hz to 3×10^{18}	10^{7} eV - 10^{4} eV	(a) High penetrating power (b) Uncharged (c) Low ionising power	(a) Gives information on nuclear structure (b) 1Medical treatment etc.
2	X-rays	Roentgen	Due to collisions of high energy electrons with heavy targets	6×10^{-12} m to 10^{-9} m	5×10^{19} Hz to 3×10^{17} Hz	2.4×10^{5} eV to 1.2×10^{3} eV	(a) Low Penetrating power (b) Other properties similar to γ-rays except wavelength	(a) Medical diagnosis and treatment (b) Study of crystal structure (c) Industrial radiography
3	Ultraviolet rays	Ritter	By ionised gases, sun lamp, spark, etc.	6×10^{-10} m to 3.8×10^{-7} m	5×10^{17} Hz to 7×10^{14} Hz	2×10^{3} eV to 3 eV	(a) All properties of light (b) Photoelectric effect	(a) To detect adulteration, writing and signature (b) Sterilization of water due to its distructive action on bacteria
4	Visible light Subparts of visible specturm (a) Violet (b) Blue (c) Green (d) Yellow (e) Orange (f) Red	Newton	Outer orbit electron transitions in atoms, gas discharge tube, incandescent solids and liquids.	3.8×10^{-7} m to 7.8×10^{-7} m 3.9×10^{-7} m to 4.55×10^{-7} m 4.55×10^{-7} m to 4.92×10^{-7} m 4.92×10^{-7} m to 5.77×10^{-7} m 5.77×10^{-7} m to 5.97×10^{-7} m 5.97×10^{-7} m to 6.22×10^{-7} m 6.22×10^{-7} m to 6.22×10^{-7} m	8×10^{14} Hz to 4×10^{14} Hz 7.69×10^{14} Hz to 6.59×10^{14} Hz 6.59×10^{14} Hz to 6.10×10^{14} Hz 6.10×10^{14} Hz to 5.20×10^{14} Hz 5.20×10^{14} Hz to 5.03×10^{14} Hz 5.03×10^{14} Hz to 4.82×10^{14} Hz 4.82×10^{14} Hz to 3.84×10^{14} Hz	3.2 eV to 1.6 eV	(a) Sensitive to human eye	(a) To see objects (b) To study molecular structure

S.No.	RADIATION	DISCOVER	HOW PRODUCED	WAVELENGTH RANGE	FREQUENCY RANGE	ENERGY RANGE	PROPERTIES	APPLICATION
5	Infra-Red waves	William Herschell	(a) Rearrangement of outer orbital electrons in atoms and molecules. (b) Change of molecular vibrational and rotational energies (c) By bodies at high temperature	7.8×10^{-7} m to 10^{-3} m	4×10^{14} Hz to 3×10^{11} Hz	1.6 eV to 10^{-3} eV	(a) Thermal effect (b) All properties similar to those of light except λ	(a) Used in industry, medicine and astronomy (b) Used for fog or haze photography (c) Elucidating molecular structure.
6	Microwaves	Hertz	Special electronic devices such as klystron tube	10^{-3} to 0.3 m	3×10^{11} Hz to 10^{9} Hz	10^{-3} eV to 10^{-5} eV	(a) Phenomena of reflection, refraction and diffraction	(a) Radar and telecommunication. (b) Analysis of fine details of molecular structure
7	Radio waves Subparts of Radio-spectrum	Marconi	Oscillating circuits	0.3 to few kms	10^{9} Hz to few Hz	10^{-3} eV to »0	(a) Exhibit waves like properties more than particle like properties.	(a) Radio communication
(A)	(a) Super High Frequency (SHF) (b) Ultra High Frequency (UHF) (c) Very high frequency (VHF)			0.01 m to 0.1 m 0.1 m to 1 m 1 m to 10 m	3×10^{10} Hz to 3×10^{9} Hz 3×10^{9} Hz to 3×10^{8} Hz 3×10^{8} Hz to 3×10^{7} Hz		Radar, Radio and satelite communication (Microwaves), Radar and Television broadcast short distance communication, Television communication.	
(B)	(a) High Frequency (HF) (b) Medium Frequency (MF) (c) Low Frequency (LF) (d) Very Low Frequency (VLF)			10 m to 100 m 100 m to 1000 m 1000 m to 10000 m 10000 m to 30000 m	3×10^{7} Hz to 3×10^{6} Hz 3×10^{6} Hz to 3×10^{5} Hz 3×10^{5} Hz to 3×10^{4} Hz 3×10^{4} Hz to 10^{4} Hz		Medium distance communication Telephone communication, Marine and navigation use, long range communication. Long distance communication.	

Examples

Q1 **A plane e.m. wave propagating in the x-direction has a wavelength of 6 mm. The electric field is along the y-direction and maximum magnitude of field is 33 V/m. Obtain suitable equations for the electric and magnetic field as a function of x and t.**

Sol: Here, $\lambda = 6.00 \text{ mm} = 6 \times 10^{-3}\text{m}$,

$E_0 = 33 \text{ V/m}$

$$\omega = 2\pi\nu = \frac{2\pi c}{\lambda} = \frac{2\pi \times 3 \times 10^8}{6 \times 10^{-3}}$$

$$= \pi \times 10^{11} \text{ rad/s.}$$

The maximum magnetic field,

$$B_0 = \frac{E_0}{c} = \frac{33}{3 \times 10^8} = 11 \times 10^{-8} \text{ T}$$

For the equation for the electric field, along y-axis in the electromagnetic wave is

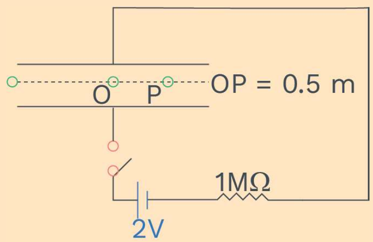
$$E_y = E_0 \sin\omega\left(t - \frac{x}{c}\right) = 33 \sin\left\{\pi \times 10^{11}\left(\frac{t - x}{c}\right)\right\}$$

For the equation for the magnetic field along z-axis in the electromagnetic wave is

$$B = B_0 \sin\omega\left(t - \frac{x}{c}\right)$$

$$= 11 \times 10^{-8} \sin\left\{\pi \times 10^{11}\left(\frac{t - x}{c}\right)\right\}$$

Q2 **A parallel plate capacitor of radius 1 m has capacitance of 1 nF. At time t = 0, it is connected for charging in series with a resistor R = 1 M Ω across a 2 V battery (as shown). Obtain the magnetic field at a point P. halfway between the centre and the periphery of the plates, after t = 10^3 s. (The charge on the plate capacitor at time t is q(t) = CV $\left[1-\exp\left(\frac{-t}{\tau}\right)\right]$, where the time constant τ is equal to CR)**



Sol: Time constant of the RC circuit is given by

$$\tau = CR = 10^{-3}\ s$$

Then we have

$$\text{Charge } q(t) = CV\left[1-\exp\left(\frac{-t}{\tau}\right)\right]$$

$$= 2\times10^{-9}\left[1-\exp\left(\frac{-t}{10^{-3}}\right)\right]$$

Electric field between the plates at time t is

$$E = \frac{q(t)}{\varepsilon_0 A} = \frac{q}{\pi\varepsilon_0}$$

$$A = \pi(1)^2 m^2 = \text{area of the plates} \qquad \text{...(i)}$$

Consider a circular loop of radius 0.5 m parallel to the plates passing through point P. The magnetic field at all point on loop is along the loop and of the same value. The flux Φ_E through this loop is

$$\Phi_E = E \times \text{area of the loop}$$

$$= E\times\pi\times\left(\frac{1}{2}\right)^2 = \frac{\pi E}{4} = \frac{q}{4\varepsilon_0} \qquad \text{...(ii) follows from (i)}$$

The displacement current

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \frac{1}{4}\frac{dq}{dt} = 0.5 \times 10^{-6} \exp(-1)$$

At time $t = 10^{-3}$ s.

After applying Ampere-Maxwell's law for the loop, then we get,

$$B \times 2\pi \times \left(\frac{1}{2}\right) = \mu_0 (i_c + i_d)$$

$$= 0.5 \times 10^{-6} \mu_0 \exp(-1)$$

or, $B = 0.74 \times 10^{-3}$ T

Q3 **Light with an energy flux of 18 Watt/cm² falls on a non-reflecting surface at perpendicular incidence. The surface has area of 20 cm², then calculate the average force exerted on surface during a 30 minute time span.**

Sol: Total energy incident on the surface is

$U = (18\ W/cm^2) \times (20\ cm^2) \times (30 \times 60)$

$= 6.48 \times 10^5$ J

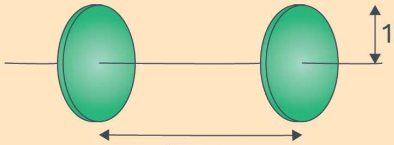
So, total momentum delivered (for complete absorption) is

$$p = \frac{U}{c} = \frac{6.48 \times 10^5 J}{3 \times 10^8 m/s} = 2.16 \times 10^{-3}\ kg\,m/s$$

The average force exerted on the surface is

$$F = \frac{p}{t} = \frac{2.16 \times 10^{-3}}{0.18 \times 10^4} = 1.2 \times 10^{-6}\ N$$

Q4 **As shown in figure a capacitor made of 2 circular plates each of radius 12 cm and separated by 5.0 mm. The capacitor is charged by an external battery source. The charging current is constant and it is equal to 0.15 A.**



(a) Calculate the capacitance, the rate of change of potential difference between the plates of capacitor.
(b) Calculate the displacement current across the plates of capacitor.
(c) Is Kirchhoff's 1st rule valid at each plate of the capacitor? Explain.
Given: $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$.

Sol: Here,

$d = 5.0\ mm = 5.0 \times 10^{-3}\ m;$

$R = 12\ cm = 12 \times 10^{-2}\ m,\ I = 0.15\ A$

$\varepsilon_0 = 8.85 \times 10^{-12} C^2N^{-1}m^{-2}$

Now, $A = \pi R^2 = \pi(1210^{-2})^2$

$= 1.44\,\pi \times 10^{-2}\ m^2$

(a) The capacitance of parallel plate capacitor is,

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 1.44\,\pi \times 10^{-2}}{5 \times 10^{-3}}$$

$= 80.1 \times 10^{-12}\ V$

Now, $Q = CV$

Therefore,

$$\frac{dQ}{dt} = C\frac{dV}{dt}$$

or $$I = C\frac{dV}{dt}$$

or $$\frac{dV}{dt} = \frac{I}{C} = \frac{0.15}{80.1 \times 10^{-12}} = 1.873 \times 10^9\ Vs^{-1}$$

(b) Conduction current = displacement current i.e. 0.15 A.

(c) Yes. Kirchhoff's law will be valid at each plate of the capacitor, provided electric current means the sum of the conduction and displacement current.

Q5 **A plane e.m. wave of frequency 25 MHz travels in free region toward the x-direction. At a particular point in free space and time, E = 6.3 $\hat{j}$ V/m. What is B at this point?**

Sol: The magnitude of B is

$$B = \frac{E}{c}$$

$$= \frac{6.3\,V/m}{3 \times 10^8 m/s} = 2.1 \times 10^{-8} T$$

To find the direction of wave, we note that E is toward y-direction and the wave propagates along x-axis. Therefore, B should be in a direction perpendicular to both y-axis and x-axis. E × B should be along x-axis. Since, $(+\hat{j}) \times (+\hat{k}) = i$, B is along the z-direction.

Thus, $B = 2.1 \times 10^{-8}\,\hat{k}\,T$

Q6 **The magnetic field in a plane e.m. wave is given by $B_y = 2 \times 10^{-7}$ sin (0.5 × 10^3 x + 1.5 × 10^{11}t) Tesla. Calculate the frequency and wavelength of the wave?**

Sol: Comparing the given equation with

$$B_y = B_0 \sin\left[2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right)\right]$$

We get, $\lambda = \frac{2\pi}{0.5 \times 10^3} m = 1.26\,cm,$

and $\frac{1}{T} = \nu = \left(1.5 \times 10^{11}\right) / 2\pi = 23.9\,GHz$

Q7 **The magnetic field in a plane e.m. wave is given by $B_y = 2 \times 10^{-7} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)$ Tesla. Write expression for the electric field.**

Sol: $E_0 = B_0 c = 2 \times 10^{-7}$ Tesla $\times 3 \times 10^8$ m/s $= 6 \times 10^1$ Volt/metre

Electric field component is normal to the direction of wave propagation and the direction of magnetic field. Therefore, equation of the electric field component along the z-axis is obtained as

$$E_z = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t) \text{ Volt/metre}$$

Q8 **Light with an energy flux of 18 W/cm² falls on a non-reflecting surface at normal incidence. If surface has an area of 20 cm², then find average force exerted on the surface during a 30 min time span.**

Sol: The total energy falling on the surface is

$$U = (18 \text{ W/cm}^2) \times (20 \text{ cm}^2) \times (30 \times 60) = 6.48 \times 10^5 \text{ J}$$

Therefore, the total momentum delivered (for complete absorption) is

$$p = \frac{U}{c} = \frac{6.48 \times 10^5 \text{ J}}{3 \times 10^8 \text{ m/s}} = 2.16 \times 10^{-3} \text{ kg m/s}$$

The average force exerted on the surface is

$$F = \frac{p}{t} = \frac{2.16 \times 10^{-3}}{0.18 \times 10^4} = 1.2 \times 10^6 \text{N}$$

Q9 **Calculate electric and magnetic fields produced by the radiation coming from a 100 W bulb at a distance of 3 m. Assume that efficiency of the bulb is 2.5% and it is a point source.**

Sol: The bulb radiates light in all possible directions uniformly. At a distance of 3 m, surface area of the surrounding sphere is

$$A = 4\pi r^2 = 4\pi(3)^2 = 113\,m^2$$

Intensity at this distance is

$$I = \frac{\text{Power}}{\text{Area}} = \frac{100\,W \times 2.5\%}{113\,m^2}$$

$$= 0.022\ W/m^2$$

Half of this intensity is provided by electric field and half by magnetic field.

$$\frac{1}{2}I = \frac{1}{2}\left(\varepsilon_0 E_{rms}^2 c\right)$$

$$= \frac{1}{2}\left(0.022\,W/m^2\right)$$

$$E_{rms} = \sqrt{\frac{0.022}{\left(8.85 \times 10^{-12}\right)\left(3 \times 10^8\right)}}\,V/m = 2.9\ V/m$$

The value of electric field E found above is the r.m.s. value of the electric field. Since, electric field in a light beam is sinusoidal, the peak electric field, E_0 is,

$$E_0 = \sqrt{2}E_{rms} = \sqrt{2} \times 2.9\,V/m$$

$$= 4.07\ V/m$$

So, we see that the electric field strength of the light that we use for reading is fairly large. Compare it with electric field strength E of TV or FM waves, which is of the order of a few micro-volts per metre.

Now, let calculate the strength of the magnetic field. It is

$$B_{rms} = \frac{E_{rms}}{c} = \frac{2.9\,Vm^{-1}}{3 \times 10^8\ ms^{-1}} = 9.6 \times 10^{-9}\,T$$

The field in the light beam is sinusoidal, peak (maximum) magnetic field is $B_0 = \sqrt{2}\ B_{rms} = 1.4 \times 10^{-8}$ Tesla. The energy in the magnetic field is equal to the energy in the electric field, the magnetic field strength is evidently very weak.

Mind Map

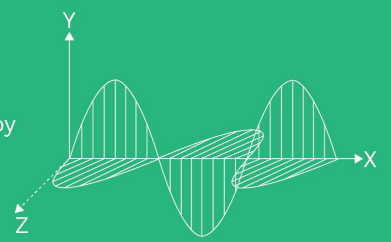
ELECTROMAGNETIC WAVE

An electromagnetic wave is a wave radiated by an accelerated charge as coupled electric and magnetic field oscillating perpendicular to each other and to the direction of propagation of wave.

Magnitude of $\vec{E}$ and $\vec{B}$ are related as $\frac{\varepsilon_0}{B_0} = c$

Speed of an electromagnetic wave in free space is given by

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$



Ampere's Circuital Law

The line integral of magnetic field around any close path in vaccum is equal to μ_0 times the total current passing through that close path.

$$\oint \vec{B}.\vec{dl} = \mu_0 I$$

Modified Ampere's Law

This law implies the fact that not only a conduction current but a displacement current, associated with a changing electric field, also produces a magnetic field.

$$\oint \vec{B}.\vec{dl} = \mu_0(i_C + i_D) = \mu_0\left(I_C + \varepsilon_0 \frac{d\phi_E}{dt}\right)$$

Displacement Current

It is the current which is produced when electric field and hence electric flux changes with time.

$$I_D = \varepsilon_0 \frac{d\phi_E}{dt}$$

Maxwell's Four Equations

1. $\oint_s \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$

2. $\oint_s \vec{B}.\vec{ds} = 0$

3. $\oint_c \vec{E}.\vec{dl} = \frac{-d\phi_B}{dt} = -\frac{d}{dt}\oint_s \vec{B}.\vec{ds}$

4. $\oint \vec{B}.\vec{dl} = \mu_0(I_C + I_D)$

$$= \mu_0\left(I_C + \varepsilon_0 \frac{d\phi_E}{dt}\right)$$

$$= \mu_0\left(I_C + \varepsilon_0 \frac{d}{dt}\oint_s \vec{E}.\vec{ds}\right)$$

Properties of Electromagnetic Waves

- Do not carry any charge.
- Do not get deflected by electric and magnetic field.
- Travel with speed of light in vaccum.
- Frequency does not change when it goes from one medium to another, but its wavelength changes.
- Transverse in nature.
- Do not require any material medium for propagation.