# Electro Magnetic Induction

## DISCLAIMER

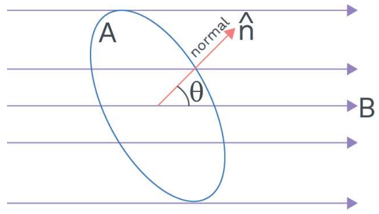
# Electro Magnetic Induction

**Electromagnetic Induction**

- The phenomenon in which electric current is induced by varying magnetic field is called electromagnetic induction.

**Magnetic Flux ($\phi$)**

- The number of magnetic lines passing perpendicular through given area is called magnetic flux.

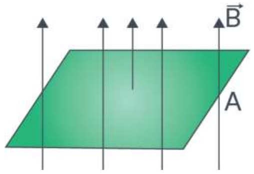



- When a surface of area A is placed in a uniform magnetic field of induction B, such that the unit vector along the normal ($\hat{n}$) makes an angle '$\theta$' with the direction of magnetic field then the flux passing through it is given by $\phi = \vec{B}.\vec{A} = BA\cos\theta$
- If magnetic field is non uniform, then $\phi = \int \vec{B}.\overrightarrow{ds}$

**(Note:** In real sense area is a scalar quantity, but it can be treated as vector whose direction is perpendicular and pointing outward from the surface).

- Magnetic flux is a scalar quantity.
- If a plane surface of area A is imagined in a uniform magnetic field $\vec{B}$, then

**(a)** When a surface is perpendicular to the magnetic field, the lines of force crossing that area, i.e., the magnetic flux is




$\phi = BA$ because $\theta = 0$, $\cos\theta = 1$

**KEY POINTS**

- Electromagnetic Induction
- Magnetic flux
- Area vector

**Definitions**

The phenomenon in which electric current is induced by varying magnetic field is called electromagnetic induction

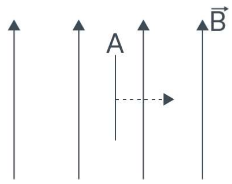
**Definitions**

The number of magnetic lines of force passing normally through given area is called magnetic flux

**Concept Reminder**

Magnetic flux coming out from the entire surface $\phi = \int_S \vec{B}.\overrightarrow{dA}$

**(b)** When the surface is parallel to the magnetic field, then

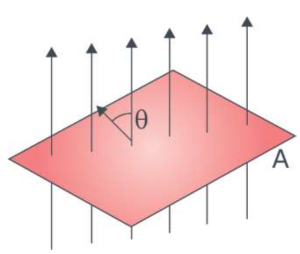



$\theta = 90°, \cos\theta = 0$

$\therefore \phi = BA\cos 90° = 0$

**(c)** when the normal to the surface makes an angle $\theta$ with the magnetic field, the magnetic flux is $\phi = BA\cos\theta$

- If the magnetic field is not uniform and the surface in not plane, then the element $\overline{dA}$ of the surface may be assumed as plane and magnetic field $\vec{B}$ may also be assumed as uniform over this element.
  Thus, the magnetic flux coming out from this element is $d\phi = \vec{B}.\overline{dA}$
  Hence magnetic flux coming out from the entire surface $\phi = \int_S \vec{B}.\overline{dA}$




- For a closed surface the area vector element pointing outward is positive and the area vector element pointing inward is negative.
- Magnetic lines of force are closed curves because free magnetic poles do not exist. Thus, for a closed surface whatever is the number of the lines of force entering it, the same number of lines of force come out from it. As a result, for a

**Concept Reminder**

For a closed surface the are vector element pointing outward is positive and the area vector element pointing inward is negative.

**Concept Reminder**

$$\phi = \oint_S \vec{B}.\overline{dA} = 0$$

The net magnetic flux coming out of a closed surface is equal to zero.

**Concept Reminder**

The dimension formula of magnetic flux is $[\phi] = [M^1L^2T^{-2}A^{-1}]$

closed curve

$$\phi = \oint_S \vec{B}.\overrightarrow{dA} = 0$$

Thus, the net magnetic flux coming out of a closed surface is equal to zero.

- For a normal plane surface in a magnetic field

  $\phi = BA$ hence $B = \dfrac{\phi}{A}$

  Thus, the magnetic flux passing normally from a surface of unit area is equal to magnetic induction B. Therefore $\dfrac{\phi}{A}$ is also called flux density.

- **Unit of magnetic flux -** In M.K.S. system the unit of magnetic flux is Weber (Wb) and in C.G.S. system unit of magnetic flux is Maxwell.

  1 Weber = $10^8$ Maxwell

  The M.K.S unit of flux density or magnetic induction is weber/m$^2$. It is also called tesla.

  1 tesla = 1 weber/m$^2$

  The C.G.S unit of magnetic flux density is gauss.

  1 gauss = 1 Maxwell/cm$^2$

  1 tesla = 1 weber/m$^2$ = $10^4$ gauss

- **Dimensions of magnetic flux:**

  $\phi = BA$

  $$[\phi] = \frac{N}{A-m} \times m^2 = \frac{N-m}{A} = \frac{(kg-m-s^{-2}) \times m}{A}$$

  $$[\phi] = [M^1L^2T^{-2}A^{-1}]$$

**Ex.** The plane of a coil of area 1m$^2$ and having 50 turns is perpendicular to a magnetic field of $3 \times 10^{-5}$ weber/m$^2$. The magnetic flux linked with the coil will be.

**Sol.** $\phi = NBA\cos\theta$

Given; N = 50, B = $3 \times 10^{-5}$ Wb/m$^2$,

A = 1 m$^2$, $\theta = 0^\circ$ (cos 0° = 1)

$\phi = NBA = 50 \times 3 \times 10^{-5} \times 1 = 150 \times 10^{-5}$ weber

$\therefore$ Magnetic flux is $1.5 \times 10^{-3}$ weber.

**Rack your Brain**

A circular disc of radius 0.2 m is placed in a uniform magnetic field of induction $\dfrac{1}{\pi}$ Wb / m$^2$ in such a way that its axis makes an angle of 60° with $\vec{B}$. The magnetic flux linked with the disc is.

(1) 0.08 Wb
(2) 0.01 Wb
(3) 0.02 Wb
(4) 0.06 Wb

**Concept Reminder**

The magnetic flux linked with a coil ($\phi = NBA\cos\theta$) can be changed by

1. Changing N
2. Changing B
3. Changing A
4. Changing angle between B and A

**Ex.** Circular coil of 100 turns & radius 2 cm is placed in uniform magnetic field B = 2T. which makes an angle 60° with the direction of field then find flux passing from this coil?

**Sol.** $\phi = NBA\cos\theta$ Here; $\theta = 30°$

$$\phi = 2\times(\pi\times 4)\times 10^{-4}\times 100\times\frac{\sqrt{3}}{2}$$

$$= 2\times 100\times(\pi\times 4\times 10^{-4})\times\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \phi = 4\sqrt{3}\pi\times 10^{-2}\,\text{Wb}$$

- The magnetic flux linked with a coil ($\phi = NBA\cos\theta$) can be changed by

**(a)** Changing the no. of turns (N)

**(b)** Varying the magnetic field (B)

**(c)** Changing the area bounded by the coil by moving the coil in the magnetic field, into or out of the magnetic field

**(d)** Changing the angle made by the area vector element with the direction of the field

- **The change of flux due to rotation of the coil:** When the coil is rotated from an angle of $\theta_1$ to an angle of $\theta_2$ (both are measured w.r.t normal) in a uniform magnetic field then the initial flux through the coil is

$$\phi_i = NBA\cos\theta_1$$

The final flux through the coil after rotation is

$$\phi_f = NBA\cos\theta_2$$

The change in the flux associated with the coil is

$$\Delta\phi = \phi_f - \phi_i$$

$$\Delta\phi = NBA(\cos\theta_2 - \cos\theta_1)$$

if $\theta_1 = 0°$ and $\theta_2 = 90°$ then $\Delta\phi = -NBA$

if $\theta_1 = 90°$ and $\theta_2 = 180°$ then $\Delta\phi = -NBA$

if $\theta_1 = 0°$ and $\theta_2 = 180°$ then $\Delta\phi = -2NBA$

**Concept Reminder**

When the coil is rotated from an angle of $\theta_1$ to an angle of $\theta_2$ (both are measured w.r.t normal) in a uniform magnetic field then the change in the flux associated with the coil is

$\Delta\phi = \phi_f - \phi_i$

$\Delta\phi = NBA(\cos\theta_2 - \cos\theta_1)$

**Ex.** A rectangular loop of area 0.06 $m^2$ is placed in a uniform magnetic field of 0.3T with its plane

**(i)** normal to the field

**(ii)** inclined 30° to the field

**(iii)** parallel to this field.

Calculate the flux linked with the coil in each case.

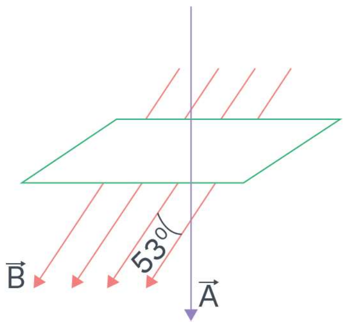
**Sol.** $\phi = NBA\cos\theta$

**(i)** $\phi = 1 \times 0.06 \times 0.3 \times \cos 0^\circ = 0.018$ weber

**(ii)** $\phi = 1 \times 0.06 \times 0.3 \times \cos 60^\circ = 0.009$ weber

**(iii)** $\phi = 1 \times 0.06 \times 0.3 \times \cos 90^\circ = 0$

**Ex.** At a certain location in the northern hemisphere, the earth's magnetic field has magnitude of 42 T and points downwards at 53° to the vertical. Calculate the flux through a horizontal surface of area 2.5 $m^2$. $[\sin 53^\circ = 0.8]$




**Sol.** $\phi_B = BA\cos\theta$

$= 42 \times 10^{-6} \times 2.5 \times \cos 53^\circ = 63$ Wb

**Faraday's Laws Of Electro Magnetic Induction**

**First Law:** When the magnetic flux linked with an electric circuit (coil) changes, then an emf is induced in the circuit (coil).

**Second Law:** The induced emf produced in the coil is equal to the negative rate of change of magnetic flux linked with it.

**Definitions**

**Faraday's Law**

The induced emf produced in the coil is equal to the negative rate of change of magnetic flux linked with it.

**Concept Reminder**

- $e = -\dfrac{d\phi}{dt}$

  e = induced emf

and $\phi$ = flux inked through coil

$$e = -\frac{d\phi}{dt}$$

where $\phi =$ flux through each turn

If the coil contains N turns, an emf appears in every turn all these emf are to be added. Then, the induced emf is given by

$$e = -N.\frac{d\phi}{dt} = -\frac{d}{dt}(N\phi)$$

Where '$N\phi$' is total flux linked with the coil of N turns.

(or)

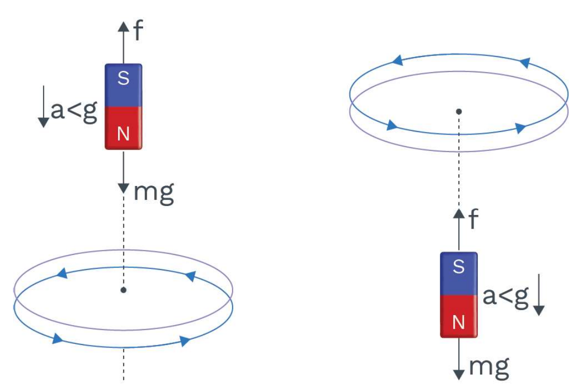
$$e = -\frac{d}{dt}(N\phi) = -\frac{d}{dt}(NBA\cos\theta)$$

The above law is also called Faraday-Neumann's law. –ve sign is in accordance with Lenz's law.

**Lenz's Law And Conservation of Energy**

"The direction of emf induced is always such that it tends to produce a current which opposes the change in magnetic flux."

- Induced emf can exist whether the circuit is open or closed. But induced current can exist only in the closed circuit.
- A metallic ring is held horizontally, and then a bar magnet is dropped through the ring such that its length along the axis of the ring, as shown in figure.



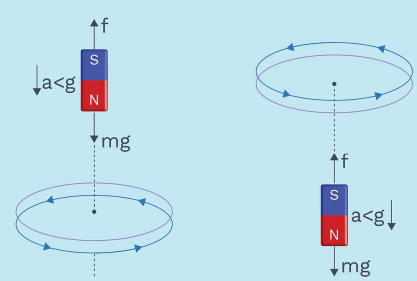

(A) (B)

**Definitions**

**Lenz's Law**

"The direction of the induced emf is always such that it tends to produce a current which opposes the change in magnetic flux."

**Concept Reminder**




(A) (B)

In both the cases $a_{net} < g$

where f = force exerted by the induced magnetic field of ring on the magnet.

In both the cases net force on the magnet is

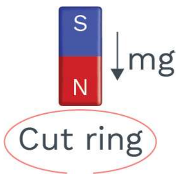
$F_{net} = mg - f$

Hence net acceleration of the fall is

$$a_{net} = g - \frac{f}{m} \Rightarrow a_{net} < g$$

where f = force exerted by the induced magnetic field of ring on the magnet.

- When the magnet is allowed to fall through an open ring (or) cut ring, then




**(a)** an emf is induced

**(b)** No current is induced (since the ring is not closed) and hence no induced magnetic field.

**(c)** No opposition to the motion of the magnet.

**(d)** F = mg

**(e)** $a_{net} = g$ (Magnet falls with an acceleration equal to g)

- If a magnet is permitted to fall through two identical metal coils at different temperatures then magnet falls slowly through the coil at low temperature as its resistance is less. Therefore, more the induced current flows, more is the opposition.
- A magnet allowed to fall through a long cylindrical pipe then the acceleration of magnet is always less than 'g' and the acceleration continuously decreases due to induced currents. But the velocity increases until the magnet moves with acceleration. At a particular instant the acceleration becomes zero and the magnet moves downwards with uniform velocity, called terminal velocity.

**Concept Reminder**

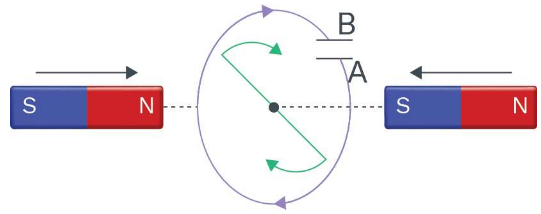
When the magnet is allowed to fall through an open ring (or) cut ring, then

- an emf is induced
- No current is induced

**KEY POINTS**

- Faraday's law
- Lenz's law
- Terminal velocity

- When the two magnets are moved perpendicular to plane of coil as shown, then




**(a)** emf is induced

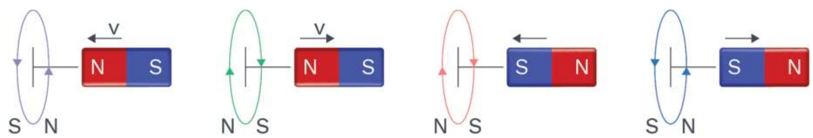
**(b)** Induced current flows from A to B along the coil when A and B are connected through resistor.

**(c)** Electrons flow from B to A along the coil

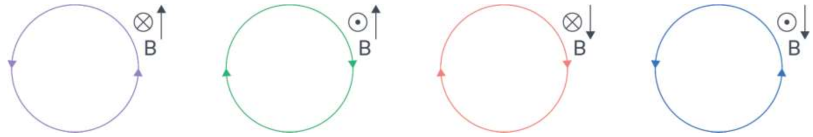
**(d)** Hence plate A will become negatively charged and plate B becomes positively charged.

**Direction of induced current in coils.**

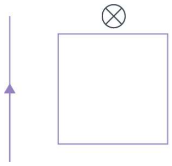
**(1)**




**(2)**

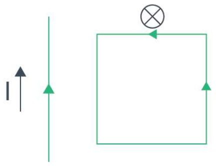



**(3)**



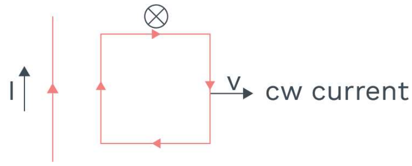
no induced current

**(4)**

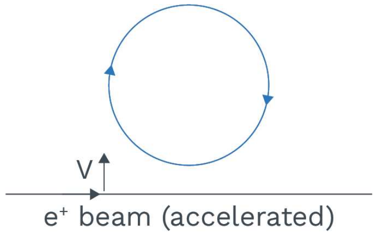



acw current

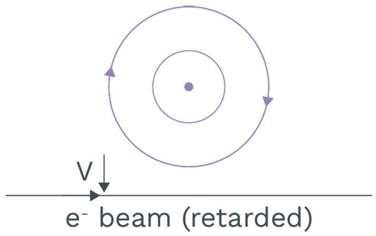
**(5)**


I
v
cw current

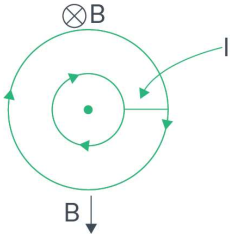
**(6)**


V
e⁺ beam (accelerated)

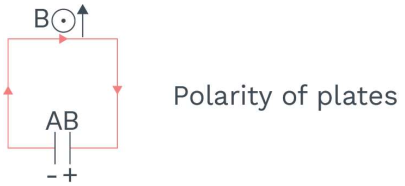
**(7)**


V
e⁻ beam (retarded)

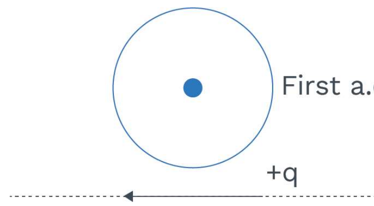
**(8)**


B
I
B

**(9)**

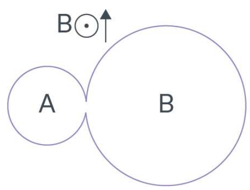

B
AB
- +
Polarity of plates

**(10)**



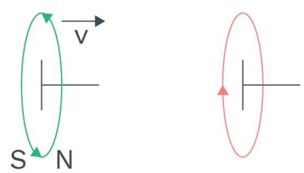

First a.c.w. then c.w.

**(11)**

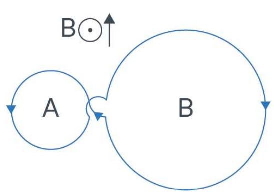



CW in both A and B

**(12)**




**(13)**



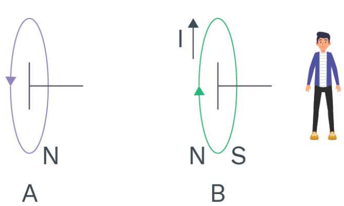

ACW in A and CW in B

$$e = \frac{d\phi}{dt} = A\frac{dB}{dt}$$

$e \propto A$

$e_B > e_A$

**(14)**




ACW in coil A as seen from left side by observer

**Expressions For Induced EMF, Induced Current And Induced Charge**

- According to Faraday's second law and Lenz's law the induced emf is given by $e = -\frac{d\phi}{dt}$ If the coil has N turns then $e = -N\frac{d\phi}{dt}$

  $\Rightarrow$ $\boxed{e = -N\frac{(\phi_2 - \phi_1)}{dt}}$

- As $\phi = BAN\cos\theta$ and $e = -\frac{d\phi}{dt}$

  The emf is induced (or) change in flux is caused by changing B (or) A (or) N (or) $\theta$

- **If 'B' is changed then**

  **(a)** Average induced emf $e = -AN\cos\theta\frac{(B_2 - B_1)}{(t_2 - t_1)}$

  Here $B_1$ is magnetic field induction at an instant $t_1$ and $B_2$ is magnetic field induction at an instant $t_2$

  **(b)** If the plane of the coil is normal to magnetic field, then

  $\theta = 0° \Rightarrow \cos 0° = 1$

  then $e = -AN\frac{(B_2 - B_1)}{(t_2 - t_1)}$

  **(c)** Instantaneous emf $e = -AN\cos\theta\frac{dB}{dt}$

- **If 'A' is changed then**

  **(a)** Average induced emf $e = -BN\cos\theta\frac{(A_2 - A_1)}{(t_2 - t_1)}$

  **(b)** If the plane of the coil is normal to magnetic field, then $\theta = 0° \Rightarrow \cos\theta = 1$

  then $e = -BN\frac{(A_2 - A_1)}{(t_2 - t_1)}$

  **(c)** Instantaneous emf $e = -BN\cos\theta\frac{dA}{dt}$

**Concept Reminder**

If 'B' is changed then
Average induced emf

$e = -AN\cos\theta\frac{(B_2 - B_1)}{(t_2 - t_1)}$

**KEY POINTS**

- Induced EMF
- Induced current
- Induced charge

**Concept Reminder**

If 'A' is charged, then

- Average induced emf

  $e = -BN\cos\theta\frac{(A_2 - A_1)}{(t_2 - t_1)}$

- Instantaneous emf

  $e = -BN\cos\theta\frac{dA}{dt}$

- **If '$\theta$' is changed (i.e., if coil is rotated)**

  **(a)** Average induced emf

  $$e = -BAN\frac{(\cos\theta_2 - \cos\theta_1)}{(t_2 - t_1)}$$

  **(b)** Instantaneous emf $e = -BAN\frac{d}{dt}(\cos\theta)$

  If the coil is rotated with constant angular velocity '$\omega$' then $\theta = \omega t$ and

  $$e = -BAN\frac{d}{dt}(\cos\omega t) = BAN\omega\sin\omega t$$

  $\therefore$ $e = BAN\omega\sin\omega t$

  **(c)** $\omega t = 90°$ if the plane is parallel to the magnetic field, then induced emf is maximum.

  Then Peak emf.

  $e_0 = BAN\omega$

  $\therefore$ $e = E_0\sin\omega t$

  This is the principle of AC generator.

**Induced Current**

- If the magnetic flux in a coil of resistance R changes from $\phi_1$ to $\phi_2$ in a time 'dt', then a current 'i' is induced in the coil as $i = \frac{e}{R}$

$$\boxed{i = \frac{N(\phi_2 - \phi_1)}{Rdt}} \qquad \left(\because e = -N.\frac{d\phi}{dt}\right)$$

$\because$ Magnitude of Induced current is given by

$$i = \frac{\text{Induced emf}}{\text{Resistance in the circuit}} = \frac{N}{R}\left(\frac{d\phi}{dt}\right)$$

**Induced Charge**

- The amount of charge induced in a conductor is given as follows

We know, $I = \frac{e}{R}$ (or) $I = \frac{1}{R}\left(-\frac{d\phi}{dt}\right)$

$$\Rightarrow \frac{dq}{dt} = -\frac{1}{R}\frac{d\phi}{dt} \text{ (or) } \frac{dq}{dt} = -\frac{1}{R}d\phi$$

**Concept Reminder**

If '$\theta$' is changed (i.e., if coil is rotated)

- Average induced emf

  $$e = -BAN\frac{(\cos\theta_2 - \cos\theta_1)}{(t_2 - t_1)}$$

- Instantaneous emf

  $$e = -BAN\frac{d}{dt}(\cos\theta)$$

**Concept Reminder**

If the plane is parallel to the magnetic field, then induced emf is maximum.

Therefore, Peak emf.

$e_0 = BAN\omega$

**Concept Reminder**

Magnitude of Induced current is given by

$$i = \frac{\text{Induced emf}}{\text{Resistance in the circuit}}$$

$$= \frac{N}{R}\left(\frac{d\phi}{dt}\right)$$

$\therefore$ Induced charge, $q = -\frac{1}{R}\int_{\phi_i}^{\phi_f} d\phi$

$q = -\frac{1}{R}\left[\phi_f - \phi_i\right]$

or $\boxed{q = \frac{\phi_i - \phi_f}{R}}$ (magnitude of charge)

$\therefore$ In general, induced charge is given by

$$\boxed{q = \frac{\text{change of magnetic flux}}{\text{resistance}}}$$

For N turns, the induced charge is $q = \frac{N}{R}(d\phi)$

- Induced emf is independent of total resistance of the circuit but depends on time of change of flux.
- Induced current depends on both time of change of flux and resistance of circuit.
- Induced charge is independent of time but depends on the resistance of circuit.
- If a magnet is moved towards a stationary coil (i) slowly and (ii) quickly, then
  (a) induced charge is same in both cases.
  (b) induced emf is more in second case.
  (c) induced current is more in second case.

**Ex.** The magnetic flux through a coil normal to its plane is varying according to the relation $\phi_B = \left(5t^3 + 4t^2 + 2t - 5\right)$ Wb. Find the induced current through the coil at t = 2 second. The resistance of the coil is $5\,\Omega$.

**Sol.** $\phi = 5t^3 + 4t^2 + 2t - 5$

$|e| = \frac{d\phi}{dt} = 15t^2 + 8t + 2$

at t = 2 sec

$i \times 5 = 15 \times 4 + 8 \times 2 + 2 \Rightarrow i = 15.6\text{ A}$

**Ex.** A circular coil of 500 turns of wire has an

**Concept Reminder**

Induced charge is given by

$$\boxed{q = \frac{\text{change of magnetic flux}}{\text{resistance}}}$$

**Rack your Brain**

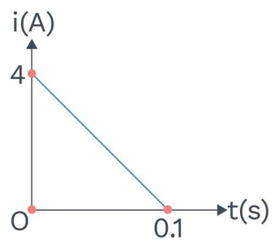
A conducting circular loop is placed in a uniform magnetic field 0.04 T with its plane perpendicular to the magnetic field, the radius of loop starts shrinking at 2 mm/sec. Find the induced emf in the loop when radius is 2 cm.

enclosed area of 0.1 $m^2$ per turn. It is kept normal to a magnetic field of induction 0.2T and rotated by 180° about a diameter normal to the field in 0.1 s. What amount of charge will pass when the coil is connected to a galvanometer with a combined resistance of $50\,\Omega$.

**Sol.** $q = \dfrac{\phi_i - \phi_f}{R} = \dfrac{NBA - (-NBA)}{R} = \dfrac{2NBA}{R}$

$q = \dfrac{2 \times 500 \times 0.2 \times 0.1}{50} = 0.4\,C$

**Ex.** Some magnetic flux is changed from a coil of resistance $10\,\Omega$. Because of this, an induced current is developed in it, which varies with time as shown in figure. What is the magnitude of change in flux through the coil?




**Sol.** The induced charge is $q = \dfrac{\Delta\phi}{R}$

But Area of i-t curve gives charge

$\therefore\ \Delta\phi = R \times \text{Area of } i - t \text{ curve } = \dfrac{1}{2} \times 4 \times 0.1 \times 10$

$\therefore \Delta\phi = 2\text{Wb}$

**Ex.** A long solenoid consisting of 1.5 turns per cm has a small loop of area 2.0$cm^2$ placed inside the solenoid perpendicular to its axis. If amount of current in the solenoid changes steadily from 2.0 A to 4.0 A in 1.0s. The emf induced in the loop is

**Sol.** The magnetic field along the axis of solenoid is $B = {}_0ni$ where n is no. of turns per unit length. flux through the smaller loop placed in solenoid is $\phi = B \times A$. Since current in solenoid is changing, emf induced in loop is

$e = \dfrac{d\phi}{dt} = \dfrac{d}{dt}[{}_0niA]$ **;** $e = {}_0nA\left(\dfrac{di}{dt}\right)$

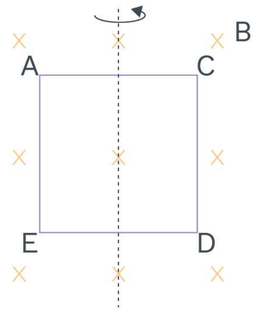
$= 4\pi \times 10^{-7} \times 1.5 \times 10^{2} \times 2 \times 10^{-4} \times \left(\dfrac{4-2}{1-0}\right)$

$= 0.75 \times 10^{-7}\,V$

**Ex.** A square loop ACDE of area $20 cm^2$ and resistance $5\Omega$ is rotated in a magnetic field $B = 2T$ through 180°

(a) in 0.01 s and (b) in 0.02 s.

Find the magnitude of induced emf and $\Delta q$ in both the cases.




**Sol.** Let us take the area vector S perpendicular to plane of loop inwards. So initially dS parallel to B and when it is rotated by 180°, S is anti-parallel to B. Hence, initial flux passing through the loop, $\phi_i = BS\cos 0° = (2)\left(20 \times 10^{-4}\right)(1) = 4 \times 10^{-3}\,Wb$

Flux passing through the loop when it is rotated by 180°, $\phi_f = BS\cos 180$

$$= (2)\left(20 \times 10^{-4}\right)(-1) = -4.0 \times 10^{-3}\,Wb$$

Therefore, change in flux,

$$\Delta\phi_B = \phi_f - \phi_i = -8 \times 10^{-3}\,Wb$$

**(a)** Given $\Delta t = 0.01 s, R = 5\,\Omega$ ; $\therefore |e| = \left|\dfrac{\Delta\phi_B}{\Delta t}\right|$

$$= \frac{8 \times 10^{-3}}{0.01} = 0.8 V;\ \text{or}\ i = \frac{|e|}{R} = \frac{0.8}{5} = 0.16\,A$$

and $\Delta q = i\Delta t = 0.16 \times 0.01; = 1.6 \times 10^{-3}\,C$

**(b)** $\Delta t = 0.02\,s$ ; $\therefore |e| = \left|-\dfrac{\Delta\phi_B}{\Delta t}\right|$

$$= \frac{8 \times 10^{-3}}{0.02} = 0.4\,V;\ i = \frac{|e|}{R} = \frac{0.4}{5} = 0.08\,A$$

and $\Delta q = i\Delta t = (0.08)(0.02) = 16 \times 10^{-3}\,C$

**Ex.** $\phi = t^2e^{-t}$ find time when induced emf is 0V.

**Sol.** $e = -\frac{d\phi}{dt} = -\frac{d(t^2e^{-t})}{dt} = -2te^{-1} - t^2(-1)e^{-t} = 0$

Gives; t = 0, t = 2

**Ex.** Radius of a circular loop is shrinking at rate of 1 mm $s^{-1}$ when loop is placed in uniform transverse magnetic field B = 2T then find induced emf when radius is 2cm.

**Sol.** B = 2T

$\frac{dr}{dt} = -1mm/s = -10^{-3}ms^{-1}$

$r = 2 \times 10^{-2}m$

$e = \frac{-d\phi}{dt} = -\frac{d(BA)}{dt} = -B\frac{dA}{dt}$

$A = \pi r^2$

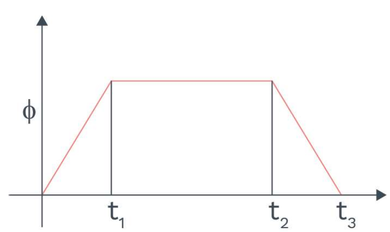
$\frac{dA}{dr} = 2\pi r$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 2\pi(2 \times 10^{-2})(-10^{-3})$

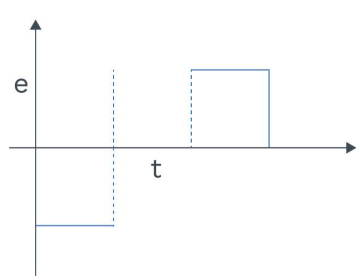
$e = -B\frac{dA}{dt} = +2 \times 10^{-2} \times 4\pi \times 10^{-3}$

$e = 8\pi \times 10^{-5}$ V

**Ex.** Draw e-t curve of given $\phi - t$ curve?




**Sol.** $e = -\frac{d\phi}{dt}$ = -(slope of $e - t$ graph)



**Ex.** If $\phi = t^2$ and $R = 2\Omega$ then find induced heat in 1 sec.

**Sol.** $I = \frac{1}{R}\frac{d(t^2)}{dt} = \frac{1}{R}2t = t$

Induced heat

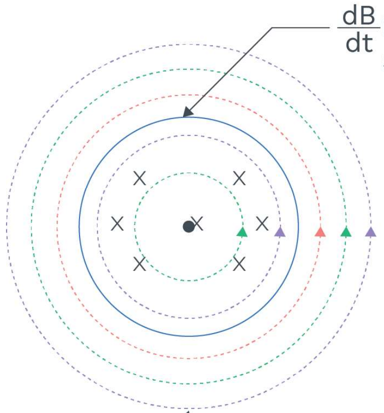
$$H = \int_{t_1}^{t_2} I^2 R.dt$$

$$H = \int_{t_1}^{t_2} t^2 2.dt = \frac{2}{3}[t^3]_0^1 = \frac{2}{3}J$$

**Induced Electric Field**

**Induced electric field and its properties:**

If a magnetic field changes with time in region then an electric field induces within and outside the region.




- This field is not same as the conservative electrostatic field produced by stationary charges.
- These field lines are always in closed curves.
- Relation $\vec{F} = q\vec{E}$ is also valid for induced electric field.
- It is non-conservative in nature, Hence, concept

**Concept Reminder**

When magnetic field changes with time in region then an electric field induces within and outside the region.

**Concept Reminder**

- Induced electric field is non-conservative in nature. Its field lines form closed loop.
- For induced electric field $\oint \vec{E}_{ind}.d\vec{l} \neq 0$

of potential has no meaning in this field.

- For induced electric field $\oint \vec{E}_{ind}.d\vec{l} \neq 0$ (But for electrostatic field $\oint \vec{E}.d\vec{l} = 0$; always)
- If a unit charge goes once around the loop then total work on it by the electric field is same as emf in loop.
- From faraday law of emf

$$e = -\frac{d\phi}{dt} \text{ or } \boxed{\oint \vec{E}_{ind}.d\vec{l} = -\frac{d\phi}{dt}}$$

- Direction of an induced electric field is the same as direction of induced current.

**$E_{ind}$ due to dB/dt:-**

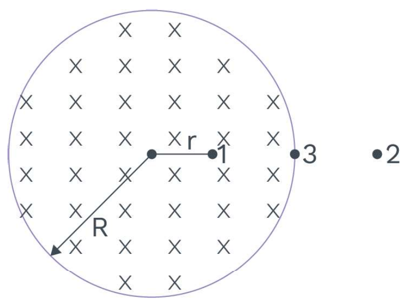
Use $\oint \vec{E}_{ind}.d\vec{l} = -\frac{d\phi}{dt}$

Symmetrical situation $El = \left|\frac{d\phi}{dt}\right| = \frac{AdB}{dt}$

l is length of closed loop

A is area in which magnetic field is changing

**Case I (r < R) (inside):**




$$E(2\pi r) = \frac{AdB}{dt}$$

$$E(2\pi r) = \pi r^2\left(\frac{dB}{dt}\right)$$

$$\boxed{E = \frac{r}{2}\frac{dB}{dt}} \quad (E_{inside} \propto r)$$

**Concept Reminder**

$$\oint \vec{E}_{ind}.d\vec{l} = -\frac{d\phi}{dt}$$

**Rack your Brain**

The magnetic flux through a circuit of resistance R changes by an amount $\Delta\phi$ in time $\Delta t$. Then the total quantity of electric charge Q that passes any point in the circuit during the time $\Delta t$ is represented by

(1) $Q = \frac{1}{R}\frac{\Delta\phi}{dt}$

(2) $Q = \frac{\Delta\phi}{R}$

(3) $Q = \frac{\Delta\phi}{\Delta t}$

(4) $Q = R\frac{\Delta\phi}{\Delta t}$

**Case II (r > R) outside:**

$$E(2\pi r) = A\left(\frac{dB}{dt}\right)$$

$$E(2\pi r) = \pi R^2\left(\frac{dB}{dt}\right)$$

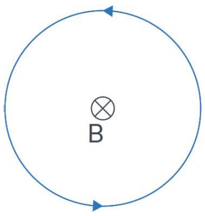
$$\Rightarrow \boxed{E = \frac{R^2}{2r}\frac{dB}{dt}} \left(E_{out} \propto \frac{1}{r}\right)$$

**Case III (r = R) surface:**

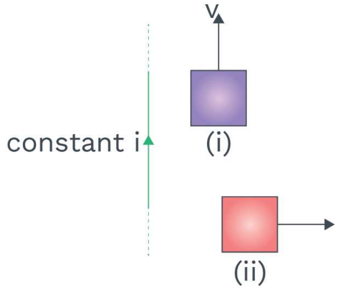
$$E(2\pi R) = \pi R^2\left(\frac{dB}{dt}\right) \Rightarrow \boxed{E = \frac{R}{2}\frac{dB}{dt}}$$

**Ex.** Determine the direction of an induced current in the coil when magnetic field is normal to the plane of coil and is increasing with time.

**Sol.** The inward flux is increasing with time. Therefore, to oppose it outward magnetic field should be induced. Hence current will flow in anticlockwise.



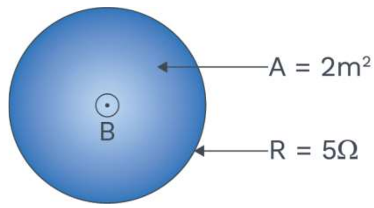

**Ex.** Given figure shows a long current carrying wire and two loops rectangular in shape moving with velocity v. Determine the direction of current in each of these loop.




**Sol.** In 1st loop no emf will be induced since there is no flux change.

In 2nd loop emf will be induced since the coil is moving in a region of decreasing magnetic field intensity inward in direction. Hence, to oppose the flux decrease in inward direction, current will be induced such that direction magnetic field will be inwards. For this current should be in clockwise direction.

**Ex.** Figure represents a coil placed in decreasing magnetic field applied normal to the plane of coil. This magnetic field is decreasing in magnitude at a rate of 10 T/s. Calculate the current in magnitude and direction.




**Sol.** $\phi = B.A$

$$\text{emf} = A.\frac{dB}{dt} = 2 \times 10 = 20\text{ v}$$

$\therefore$ i = 20/5 = 4 A

From the Lenz's law direction of current will be anticlockwise.

**Ex.** The magnetic flux ($\phi$) in a closed circuit having resistance $20\,\Omega$ varies with time (t) according to the equation $\phi = 7t^2 - 4t$ where $\phi$ is in Wb and t is in seconds. Find the magnitude of the induced current at t = 0.25 s.

**Sol.** $\phi = 7t^2 - 4t$

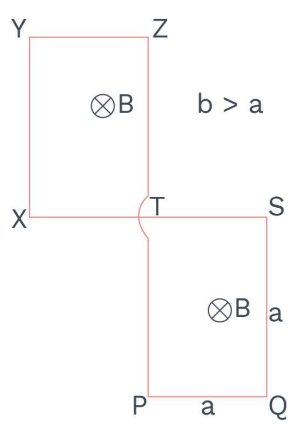
$\Rightarrow$ Induced emf: $|e| = \frac{d\phi}{dt} = 14t - 4$

$\Rightarrow$ Induced current:

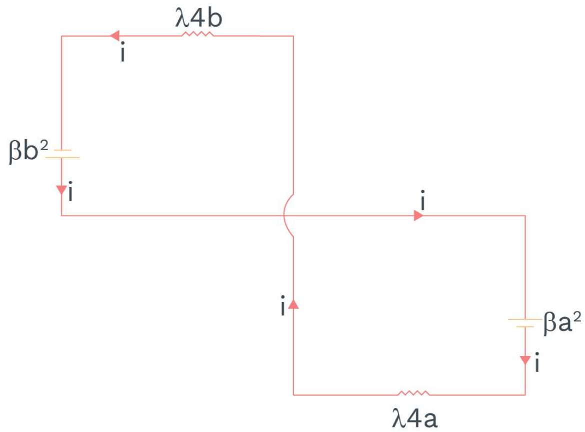
$$i = \frac{|e|}{R} = \frac{|14t-4|}{20} = \frac{|14 \times 0.25 - 4|}{20} \quad (\text{at} \quad t = 0.25\,s)$$

$$= \frac{0.5}{20} = 2.5 \times 10^{-2}\,A = 25\text{ mA}$$

**Ex.** Figure represents a wire frame PQSTXYZ placed in a time varying magnetic field given as $B = \beta t$, where $\beta$ is a positive constant. Given that resistance per unit length of the wire is $\lambda$. Calculate the current induced in the wire and draw its electrical equivalent diagram.



**Sol.** Induced emf in part PQST is $\beta a^2$ (in anticlockwise direction, from Lenz's Law) Similarly induced emf in part TXYZ = $\beta b^2$ (in anticlockwise direction, from Lenz's Law)




Total resistance of the part PQST = $\lambda 4a$ .
Total resistance of the part XYZT = $\lambda$ 4b.
The equivalent circuit is shown in given diagram.
writing KVL along the current flow

$$\beta b^2 - \beta a^2 - \lambda 4ai - \lambda 4bi = 0$$

$$i = \frac{\beta}{4\lambda}(b - a)$$

**Ex.** A space is divided by a line AD into two regions. Region I is field free and the region-II has a uniform magnetic field B directed into the paper. ACD is a semi-circular conducting loop having radius r with its centre at O, the plane of the loop lies in the plane of the paper. The loop is now rotated with a constant angular velocity $\omega$ about an axis passing through O, and normal to the plane of paper in the clockwise direction. Also given that effective resistance of the loop is R. (a) Write an expression for the magnitude of an induced current in the loop. (b) Show the direction of current when the loop is coming into the region II. (c) Also plot a graph between the induced emf and time of rotation for two complete rotation

**Sol.** (a) In time t, the arc swept by this loop in the field, i.e., region II.

$$A = \frac{1}{2}r(r\theta) = \frac{1}{2}r^2\omega t$$

Hence, the flux linked with the rotating loop at time t,

$$\phi = BA = \frac{1}{2}B\omega r^2 t \qquad [\theta = \omega t]$$

Therefore, the induced emf in the loop,

And since, the resistance of the loop is R, the induced current in it,

When this loop is entering the region II, i.e., the field figure (b), the inward flux linked with it will increase, so according to Lenz's law an anticlockwise current will be induced in it.

**(c)** Taking an induced emf to be negative when flux linked with the loop is increasing and +ve when decreasing, the emf versus time graph will be, as shown in figure (c)

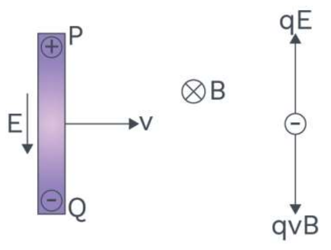
**Ex.** Determine the direction of an induced current in the wire AB, when it is rotated anticlockwise through angle , initially it is placed as shown in given figure

**Sol.**

| Range of Angle Rotated | Direction of induced current |
|---|---|
| 0 – 90 | A to B |
| 90 – 180 | A to B |
| 180 – 270 | B to A |
| 270 – 360 | B to A |

## Motional emf

When a rod is moving with velocity v in a magnetic field B, as shown in figure, all the free electrons inside rod will experience a magnetic force in downward direction and therefore, all free electrons will collected at the lower end and there will be a deficiency of free electrons and therefore, a surplus of +ve charge at the upper end. These charges at ends will produce an electric field in downward direction which will provide an upward force on electron. When the rod has been moving for quite some time enough charges will get collected at the ends so that the both forces qE and qvB will balance each other. Thus E = v B.




$V_P - V_Q = vBl$

A moving rod in magnetic field equivalent to the following diagram, electrically.

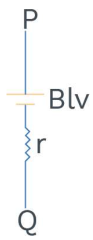
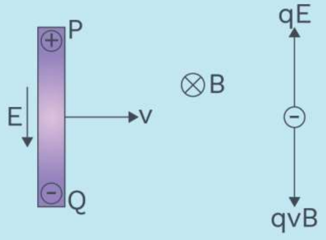



Figure represents a closed coil ABCA moving in a uniform magnetic field B with a velocity v. The magnetic flux passing through the coil is a

### Concept Reminder




$V_P - V_Q = vBl$

### KEY POINTS

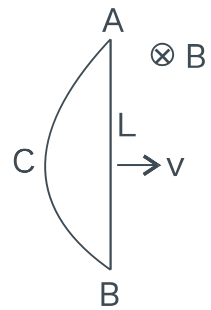
- Motional EMF
- Induced electric field

### Rack your Brain

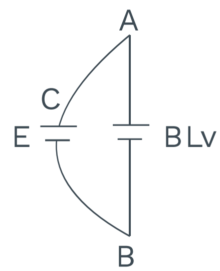
A metal ring is held horizontally and bar magnet is dropped through the ring with its length along the axis of ring. The acceleration of falling magnet is

(1) more than g
(2) less than g
(3) equal to g
(4) either (a) or (b)

constant and hence, the induced emf is zero.

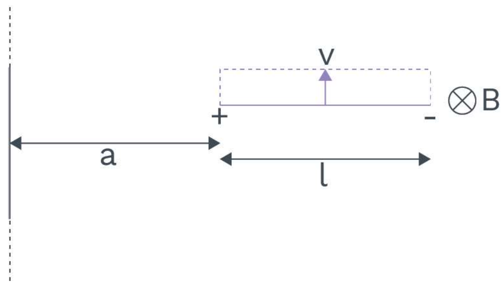



Now take rod AB, which is a part of the coil. So, emf induced in the rod is B L v
Suppose an emf induced in part ACB is E, as shown below.




Since the emf in the coil is 0, Emf (in ACB) + Emf (in BA) = 0
or we can write $-E + vBL = 0$
or $E = vBL$
Thus, an emf induced in any path joining A and B is same, it the magnetic field is uniform. Also, an equivalent emf between A and B is BLv (here the two emf's are in parallel)

**Ex.** A rod of length l is placed normal to a long wire carrying current i. This rod is moved parallel to the wire with a velocity v. Calculate the emf induced in the rod, if its closest end is at a distance 'a' from the wire.

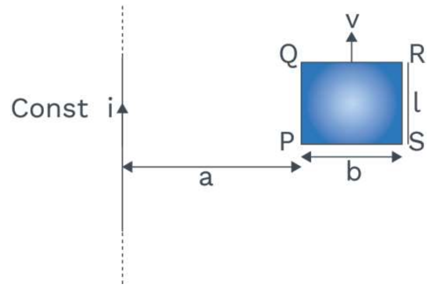


**Sol.** Take a segment of rod of length dx, at a distance x from the wire. Emf induced in the segment

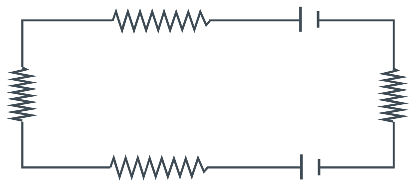
$$de = \frac{\mu_0 i}{2\pi x} dx.v$$

$$e = \int_a^{a+l} \frac{\mu_0 i v dx}{2\pi x} = \frac{\mu_0 i v}{2\pi} \ln\left(\frac{l+a}{a}\right)$$

**Ex.** A loop rectangle in shape is moving parallel to a long wire carrying current i with a velocity v. Determine the emf induced in the loop, if its nearest end is at a distance 'a' from the wire. Find induced current in circuit.




**Sol.** Using result of above question.

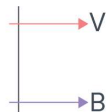


$$e = \frac{\mu_0 i v}{2\pi} \ln\left(\frac{a+b}{a}\right)$$

$$V_Q - V_R = e, \; V_P - V_S = e$$

$$\Rightarrow \quad i = \frac{e-e}{4r} = 0$$

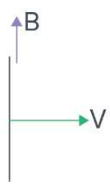
**Ex.** Determine the value of emf induced in the rod for the following cases. The figures are self-explanatory. B= magnetic field and V is velocity of conductor.

(a) 


**Ans.** emf = 0

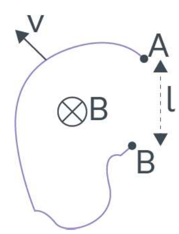
(b) 


**Ans.** emf = 0

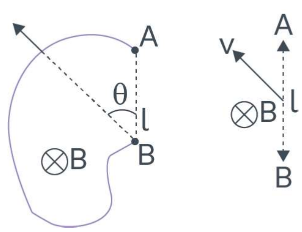
(c) 


**Ans.** emf = 0

**Ex.** Figure represents an irregular shape AB moving with velocity v in the direction as shown. Calculate the emf induced in the wire.

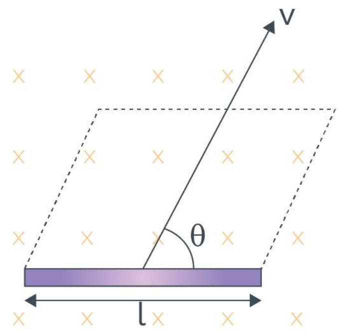



**Sol.** The same amount of emf will be induced in the straight imaginary wire joining A and B, which is $|\varepsilon| = Bvl\sin\theta$.




**Ex.** A 0.4-meter-long straight conductor moves in a magnetic field of value 0.9 Wb/m$^2$ with a velocity of 7 m/sec. Calculate the emf induced in the conductor under the condition when it is maximum.

**Sol.** If a rod of length $l$ is moved with velocity $\vec{v}$ and angle $\theta$ to the length of the rod in a field $\vec{B}$ which is perpendicular to the plane of the motion, the flux linked with the area generated by the motion of rod in time t,



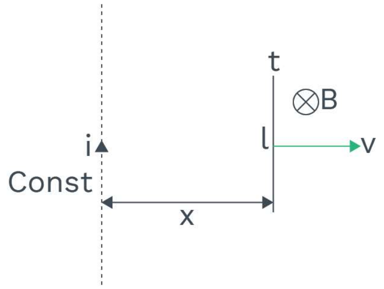

$\phi = Bl(v \sin\theta)t$ so, $|\varepsilon| = \frac{d\phi}{dt} = Bvl \sin\theta$

This will be maximum when $\theta = max = 1$, i.e., the rod is moving perpendicular to its length and then $(\varepsilon)_{max} = Bvl$

$\varepsilon_{max}$ = 0.9 × 7 × 0.4 = 2.52 V

**Ex.** A rod of length l is placed parallel to a long wire carrying constant current i. Rod is moving away from the wire with a velocity v. Calculate the emf induced in wire if its distance from the long wire is x.

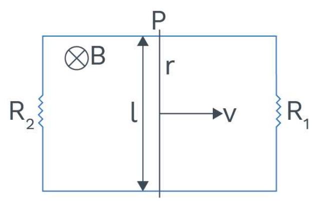
**Sol.** Induced emf is equal to the rate with which magnetic field lines are cut. In time dt the area swept by the rod is lvdt. The magnetic field lines cut in this time is



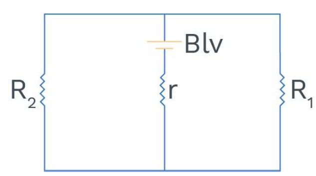

$\phi = Bl\, vdt = \frac{\mu_0 il\, vdt}{2\pi x}$

∴ The rate at which magnetic field lines are cut $= \frac{\mu_0 i l v}{2\pi x}$

**Ex.** A conducting rod PQ of mass m and resistance r is moving on two fixed, resistance less, smooth conducting rails which is closed on both sides by resistances $R_1$ and $R_2$). Determine the current in the rod at the instant its velocity is v.




**Sol.** The equivalent circuit of above figure

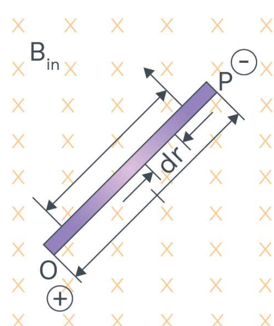



$$i = \frac{Blv}{r + \frac{R_1 R_2}{R_1 + R_2}}$$

**Induced emf due to rotation**

**Rotation of a rod**

Suppose a conducting rod of length l is rotating in a uniform magnetic field.

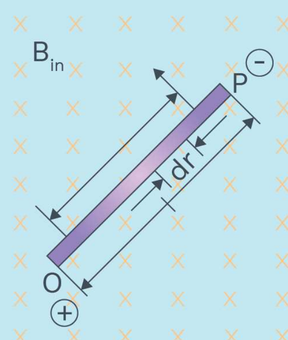




small segment just like a rod

emf induced in a small segment

**Concept Reminder**




- Induced emf $= \frac{1}{2} B\omega l^2$

An emf induced in a small segment of length dr, of the rod $= vB\,dr = r\omega B\,dr$

$\therefore$ emf induced in the rod

$$= \omega B\int_0^l r\,dr = \frac{1}{2}B\omega l^2$$

equivalent of this rod is as following

or

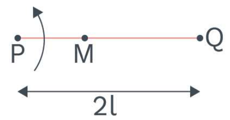
$$\varepsilon = \frac{d\phi}{dt} = \frac{\text{flux through the area swept by the rod in time dt}}{dt}$$

$$\boxed{e = \frac{1}{2}B\omega l^2}$$

**Concept Reminder**

The motional emf can be discussed independently from Faraday's law by using Lorentz force on moving charges.

**Ex.** A rod PQ having length 2l is rotating about one end P in a uniform magnetic field B which is normal to the plane of rotation of the rod. Point M is the midpoint of the rod. Calculate the induced emf between M & Q if that between P & Q = 100 V.



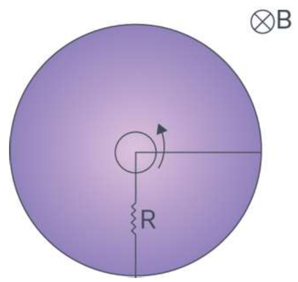

**Sol.** $E_{MQ} + E_{PM} = E_{PQ}$

corner $\rightarrow \dfrac{Bwl^2}{2} = 100$

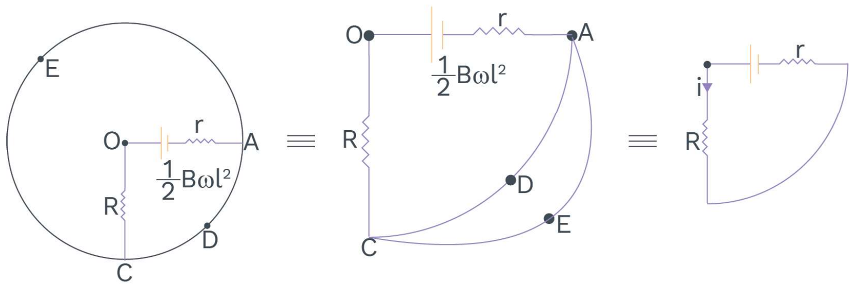
$$E_{MQ} + \frac{B\omega\left(\frac{l}{2}\right)^2}{2} = \frac{B\omega l^2}{2}$$

$$E_{MQ} = \frac{3}{8}B\omega l^2 = \frac{3}{4}\times 100\text{ V} = 75\text{ V}$$

**Ex.** A rod having length L and resistance r is rotating about one end as shown in given figure. Its other end touches a conducting ring of negligible resistance. A resistance R is connected in between centre and periphery of ring. Draw the electrical equivalence and find the current in the resistance R. There is a uniform magnetic field B directed as shown.



**Sol.**

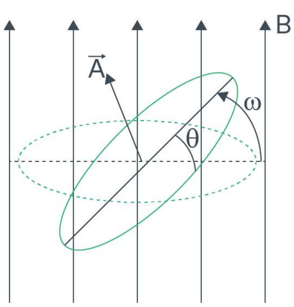



$$\text{Current } i = \frac{\frac{1}{2}B\omega l^2}{R+r}$$

**By Changing The Angle**

Let us take the case when the magnitude of the magnetic field strength and area of the coil remains same. If the coil is rotated relative to the direction of field, an induced current is developed which lasts as long as this coil is rotating.

We have, $\phi = BA\cos\theta$ [where B is the magnetic field strength, A is the magnitude of the are a vector & $\theta$ is the angle between them] If the angular velocity with which the coil is rotating is $\omega$, then $\theta = \omega t$

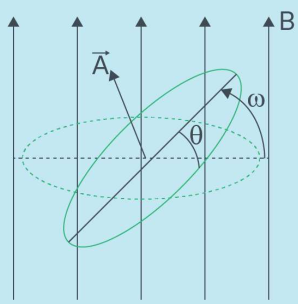



Induced emf in the coil

$$\varepsilon = -\frac{d\phi}{dt} = BA\omega\sin\omega t$$

Induced current in the coil $\boxed{= i = \frac{|\varepsilon|}{R} = \frac{B\omega A}{R}\sin\omega t}$

**Concept Reminder**



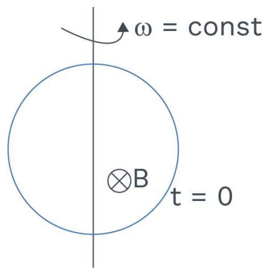

Induced emf in the coil

$$\varepsilon = -\frac{d\phi}{dt} = BA\omega\sin\omega t$$

Induced current in the coil

$$i = \frac{|\varepsilon|}{R} = \frac{B\omega A}{R}\sin\omega t$$

**Ex.** A ring is rotating with angular velocity $\omega$ about an axis in the plane of the ring and which passes through the centre of this ring. A constant magnetic field B exists normal to the plane of the ring. Calculate the emf induced in the ring as a function of time.




**Sol.** At any time t, $\phi = BA\cos\theta = BA\cos\omega t$

Now induced emf in the loop

$$e = \frac{-d\phi}{dt} = BA\,\omega\sin\omega t$$

If there are N turns

$\text{emf} = BA\omega N\sin\omega t$

$BA\,\omega N$ is the amplitude of the emf

$e = e_m\sin\omega t$

$$i = \frac{e}{R} = \frac{e_m}{R}\sin\omega t = i_m\sin\omega t$$

$$i_m = \frac{e_m}{R}$$

A rotating coil produces a sinusoidally varying current or alternating current. This is the basic principle which is always used in generator.

**Induced Electric Field Due to A Time Varying Magnetic Field**

Take a consider a conducting loop placed at rest in a magnetic field $\vec{B}$. Let this field is constant till t = 0 and then changes with time. An induced current produces in the loop at t = 0.

**Rack your Brain**

A straight-line conductor of length 0.4 m is moved with speed of 7 m/s perpendicular to a magnetic field of intensity 0.9 $Wb/m^2$. The induced emf across the conductor is

(1) 5.04 V (2) 25.2 V
(3) 1.26 V (4) 2.52 V

**Concept Reminder**

If the velocity of rod is taken as the reference direction, then induced emf $e = B_\perp l_\perp v$, where $B_\perp$ is the component of magnetic field perpendicular to velocity and $l_\perp$ is component of length perpendicular to velocity.

All free electrons were at rest till t = 0. Remember, we are not interested in any random motion of the electrons. The magnetic field can never exert force on electrons at rest. Hence, the magnetic force cannot Begin the induced current. The free electrons may be forced to move only by an electric field. Hence, we conclude that an electric field appears at initial time.

This electric field is produced by changing magnetic field and not by any charged particles. The electric field produced by changing magnetic field is non electrostatic and non-conservative in nature. We can never define a potential related to this field. We known it as induced electric field. The induced electric field lines are closed curves. There are no beginning and ending points of these field lines.

If $\vec{E}$ is the induced electric field, the force on the charge q placed in the field of $q\vec{E}$. The work done per unit charge when the charge moves by $d\vec{l}$ is $Ed\vec{l}$. An emf developed in the loop is, therefore,

$$\varepsilon = \oint \vec{E}.d\vec{l}$$

Using Faraday's law of induction,

$$\varepsilon = -\frac{d\phi}{dt}$$

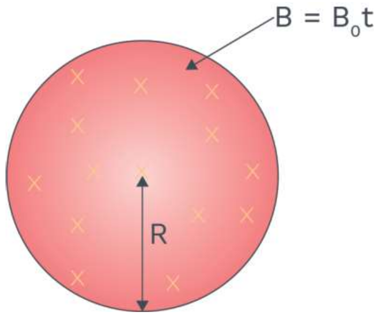
or $$\oint \vec{E}.d\vec{l} = -\frac{d\phi}{dt} \quad \text{... (6)}$$

A presence of a conducting loop is not always necessary to have an induced electric field. As long as $\vec{B}$ keeps changing, an induced electric field is producing. If a loop is present then free electrons start drifting and due to this an induced current produces.

**Concept Reminder**

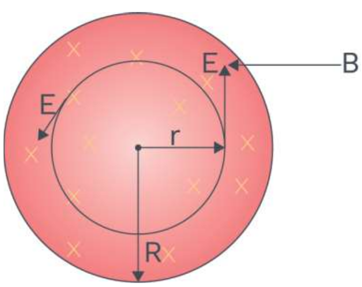
An electric field is induced in any region of space in which a magnetic field is changing with time. Also, a magnetic field is induced in any region of space in which electric field is changing with time.

**Ex.** Determine an electric field at a distance r from axis of changing cylindrical magnetic field B, which is parallel to axis of cylinder?




**Sol.** (i) If r < R

let at a distance r electric field is E

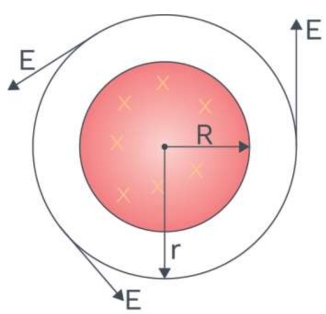



$$\varepsilon = \left|\oint \vec{E}.d\vec{l}\right| = \left|-\frac{d\phi}{dt}\right|$$

$$E.2\pi r = \left|-\frac{d[B.(\pi r^2)]}{dt}\right|$$

$$E = \frac{r}{2}\frac{dB}{dt} = \frac{B_0 r}{2}$$

**(ii)** When r > R




**Concept Reminder**

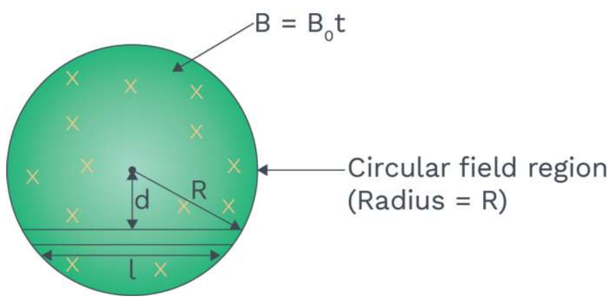
The electric field produced by stationary charges is called electrostatic field and for such field $\oint \vec{E}.d\vec{l} = 0$ i.e., electrostatic field is conservative.

$$\left|\oint \vec{E}.d\vec{l}\right| = \left|-\frac{d\phi}{dt}\right|$$

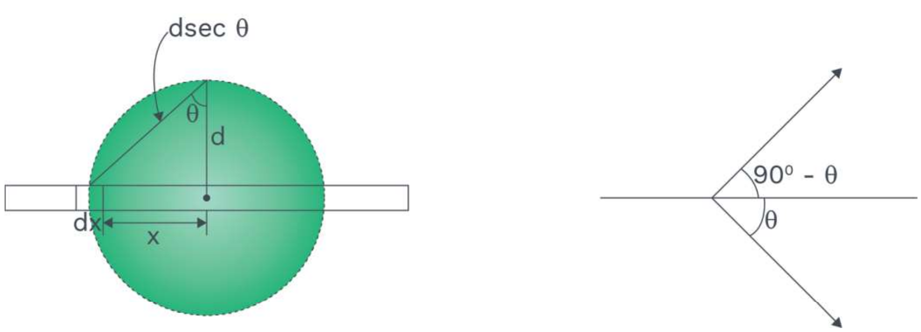
$$E.2\pi r = \frac{d(B\pi R^2)}{dt}$$

$$\Rightarrow \quad E = \frac{R^2}{2r}.\frac{dB}{dt} = \frac{B_0 R^2}{2r}$$

**Ex.** Calculate the e.m.f induced in the rod as shown in the figure.




**Sol.**




$$E = \frac{r}{2}\frac{dB}{dt} = \frac{d\sec\theta}{2}B_0$$

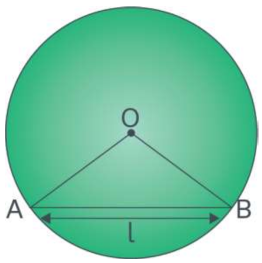
$$dE = E.dx\cos\theta = \frac{d\sec\theta B_0 dx\cos\theta}{2}$$

$$\Rightarrow \quad \int_0^E dE = \frac{B_0 d}{2}\int_{-l/2}^{l/2} dx \Rightarrow E = \frac{B_0 l d}{2}$$

**Another Method**

Induced emf in OA & OB is 0 (since $\vec{B}$ & $d\vec{l}$ are perpendicular to each other)

Total induced emf in OAB is same as in AB




Area $= \frac{1}{2} ld$

$\phi = \frac{B_0 ldt}{2}$

$\frac{d\phi}{dt} = \frac{B_0 ld}{2}$

**Rack your Brain**

The total charge, induced in a conducting loop when it is moved in magnetic field depends on

(1) The rate of charge of magnetic flux

(2) Initial magnetic flux only

(3) The total charge in magnetic flux

(4) Final magnetic flux only.

**Ex.** A thin, non-conducting ring having mass m and carrying charge q, can rotate freely about its axis. At an instant t = 0 this ring was at rest and there was no magnetic field present. Then suddenly a magnetic field B was applied normal to the plane. Find the angular velocity acquired by the ring.

**Sol.** Because the sudden change of flux, an electric field is set up and the ring experiences an impulsive torque and suddenly acquires an angular velocity.

$$\varepsilon(\text{induced emf}) = -\frac{d\phi}{dt} = -\frac{d}{dt}\int \vec{B}.dA$$

Also $\varepsilon = \oint \vec{E}.d\vec{l}$ where E is an induced electric field.

$$\therefore \quad \oint \vec{E}.d\vec{l} = -\frac{d}{dt}\int \vec{B}.d\vec{A} \Rightarrow E.2\pi r = -\frac{d}{dt}(B\pi r^2)$$

Force experienced by the ring $= q|\vec{E}|$

Torque experienced by the ring $\tau = (qE)r = \frac{qr^2}{2}\frac{dB}{dt}$

∴ Angular impulse experienced by the ring

$$= \int \tau\, dt = \frac{qr^2}{2}\int \frac{dB}{dt}dt = qr^2\frac{B}{2}$$

Also, angular impulse acquired $= l\omega$ where l is moment of inertia of the ring about its axis $= mr^2$

$$\therefore \quad mr^2\omega = qr^2B/2$$

⇒ Angular velocity acquired by the ring $\omega = qB/2m$

**Moving Conducting Rod In Earth's Magnetic Field**
**(Assume angle of declination is zero)**

**Case-I** Placed Horizontally and moves in horizontal plane.

If its end in
- → E – W direction ⇒ $B_V$ cuts
- → N – S direction ⇒ $B_V$ cuts

} Dynamic emf := $e_d = B_V vl$

**Case-II** Hold vertically and moves in horizontal plane:-

If it moves on
- → E – W line ⇒ $B_H$ cuts } Dynamic emf := $e_d = B_H vl$
- → N – S line ⇒ No flux cutting ⇒ No Dy. EMI

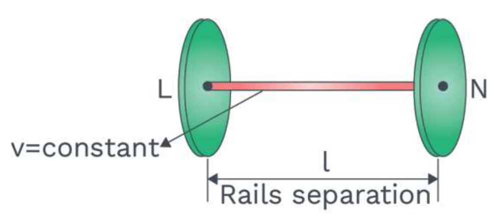
**Case-III** Placed horizontally and allow to fall under gravity in vertical plane :-

If its end in
- → E – W direction ⇒ $B_V$ cuts } Dynamic emf := $e_d = B_H vl$
- → N – S direction ⇒ No flux cutting ⇒ No Dyn. EMI

- **Applications ($H_z \to$ Horizontal, $V_t \to$ Vertical)**

**(i) Moving Train ($H_z$ – $H_z$): –**

Induced emf across axle of moving train is: -

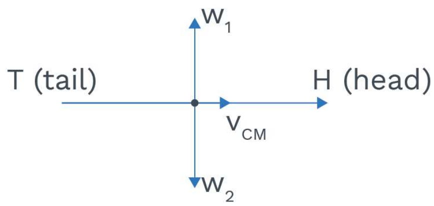



where $B_V = B\sin\theta$, $\theta$ angle of dip at that place

$v \to m/sec$

**(ii) Moving Aeroplane:**

Motion of aeroplane can be dealt as motion of two metal rods (H-T) and ($w_1$–$w_2$) which are perpendicular to each other. For (H-T) conductor $\vec{l} \parallel \vec{v}_{cm}$, so (H-T) conductor never do flux cutting hence no induced emf across (H-T) of aeroplane for its any sort of motion, only ($w_1$–$w_2$) conductor can do flux cutting.




$w_1$ & $w_2$ = wings

**(a) When aeroplane flying at a certain height i.e., parallel to earth surface ($H_z$ – $H_z$):**

If wings ($w_1 - w_2$) in
- E – W direction ⇒ $B_V$ cuts
- N – S direction ⇒ $B_V$ cuts

Induced emf across wings of aeroplane given as (both cases): -

$\boxed{e_{w_1w_2} = B_V l w_1 w_2 v}$, where $B_V = B\sin\theta$ [$\theta$ angle of dip.]

**(b) When Aeroplane dives vertically ($H_z$ – $V_t$): -**

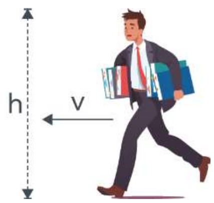
If wings ($w_1 - w_2$) in
- E – W direction ⇒ $B_H$ cuts
- N – S direction ⇒ No flux cutting ⇒ No Dy. EM

Induced emf across wings of aeroplane given as (only in one case)

$\boxed{e_{w_1w_2} = B_H l w_1 w_2 v}$, where $B_H = B\cos\theta$ [$\theta$ angle of dip.]

**(iii) Human body ($v_t$ – $H_z$):**

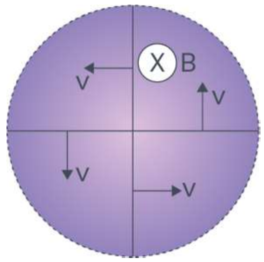
A human body of height 'h' moves with constant velocity v then induced emf between his head and feet, if it moves along:

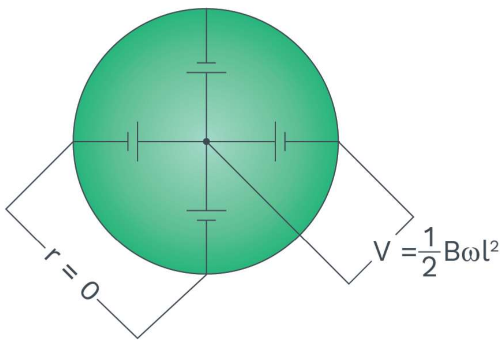



E – W line ⇒ $B_H$ cuts } Dynamic emf ⇒ $\boxed{e_d = B_H vh}$

*N – S line ⇒ No flux cutting ⇒ No Dyn. EMI

**Motion of wheel of cycle**



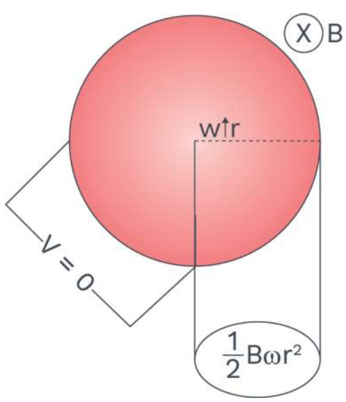




- P.D b/w two points on circumference is zero.
- P.D b/w centre & a point on circumference is 1/2 $B\omega L^2$.

  Length of spoke is l.
- Induced emf does not depend on no, of spoke.

  Similarly, if a metallic disc is Rotating.

  About its axis in uniform transverse magnetic field.



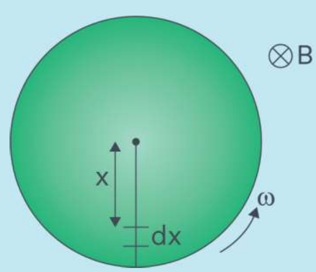

**Concept Reminder**

If case of electromagnetic induction, line integral of induced electric field $\vec{E}$ around a closed path is not zero i.e., induced electric field is non-conservative. In such a field, work done in moving a charge round a close path is not zero.

**Concept Reminder**

Emf between the centre and the edge of disc.

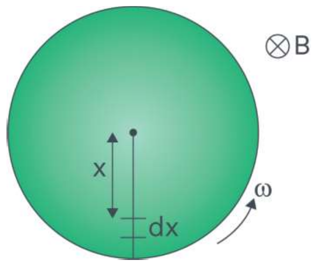
$$= \int_0^r B\omega x dx = \frac{B\omega r^2}{2}$$



**EMF Induced In A Rotating Disc:**

Consider a disc of radius r rotating in a magnetic field B.

Consider an element dx at a distance x form the centre. This element is moving with speed $v = \omega x$.




∴ Induced emf across dx

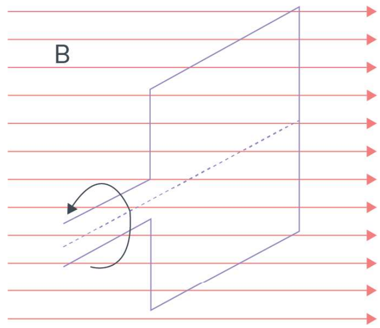
$= B(dx) = vBdx\omega x = B\omega xdx$

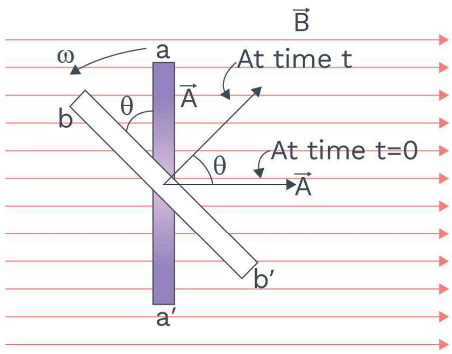
∴ An emf developed between the centre and edge of disc.

$$= \int_0^r B\omega xdx = \frac{B\omega r^2}{2}$$

**Rotation Of A Rectangular Coil In Uniform Magnetic Field**

- If the figure a conducting rectangular coil of area A and turns N is shown. It is rotating in a uniform magnetic field B about a horizontal axis perpendicular to the field with an angular velocity $\omega$. The flux of magnetic field linked with this coil is continuously changing due to rotation.




**Concept Reminder**

The work done by the induced electric field in one revolution is $W = q_0E$, where E is induced emf.

**Rack your Brain**

A wire loop is rotated in a magnetic field. The frequency of change of direction of induced emf is

(1) four times per revolution
(2) Six times per revolution
(3) Once per revolution
(4) Twice per revolution

$\theta$ is the angle between the perpendicular to the plane of the coil and the direction of magnetic field.

- The magnetic flux passing through the rectangular coil depends upon the orientation of the plane of the coil about its axis.
- Magnetic flux passing through the coil $\phi = \vec{B}.\vec{A} = BA\cos\theta = BA\cos\omega t$

  If N are number of turns in the coil, then the flux linked with the coil $\phi = BAN\cos\omega t$

**Concept Reminder**

Moving charges in static field and static charges in a time varying field seems to be a symmetric situation for Faraday's law.

- Since $\phi$ depends upon the time t, the rate of change of magnetic flux

$$\frac{d\phi}{dt} = -BAN\omega\sin\omega t$$

- According to Faraday's law, the emf induced in the coil

$$e = -\frac{d\phi}{dt}$$

or $\quad e = BAN\omega\sin\omega t$

BAN$\omega$ is the maximum value of emf induced,

Thus writing

$$BAN\omega = e_0$$

$\therefore \quad e = e_0\sin\omega t$

This equation represents the instantaneous value of emf induced at time t.

- If the total resistance of circuit along with the coil is R, then the induced current due to alternating voltage

$$I = \frac{e}{R} = \frac{e_0}{R}\sin\omega t$$

or $\quad I = I_0\sin\omega t$

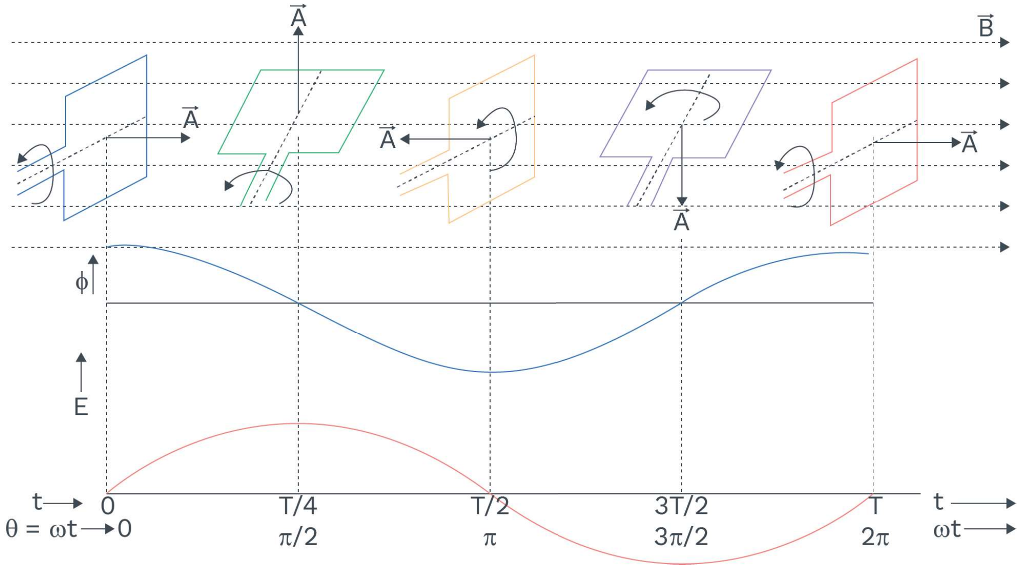
Where $I_0 = \frac{e_0}{R}$ is the maximum value of current.

- The magnetic flux linked with coil and the emf induced at different positions of the coil in one rotational cycle are shown in the following table:

| Time | Position of Coil |
|---|---|
| t = 0 | Plane of the coil normal to $\vec{B}(\theta = 0)$ |
| t = T/4 | Plane of the coil parallel to $\vec{B}(\theta = 90°)$ |
| t = T/2 | Plane of the coil normal to $\vec{B}$ again $(\theta = 180°)$ |
| t = 3T/4 | Plane of the coil parallel to $\vec{B}(\theta = 360°)$ |

| Magnetic flux | Induced emf |
|---|---|
| $\phi = NBA = \text{maximum flux}$ | $e = 0$ |
| $\phi = 0$ | $e = NBA\ \omega = \text{maximum}$ |
| $\phi = -NBA$ | $e = 0$ |
| $\phi = 0$ | $e = -NBA\ \omega$ |
| $\phi = NBA$ | $e = 0$ |

The variations of magnetic flux linked with the coil and induced e.m.f at different times given in the above table are shown in the following figure.



- The phase difference between the instantaneous magnetic flux and induced emf is $\pi / 2$.
- The ratio of $e_{max}$ and $\phi_{max}$ is equal to the angular velocity of the coil, Thus

$$\frac{e_{max}}{\phi_{max}} = \frac{NBA\omega}{NBA} = \omega$$

- If $\theta = \frac{\pi}{4} = 45°$, then $\Rightarrow \phi = \frac{NBA}{\sqrt{2}}$ and $e = \frac{NBA\,\omega}{\sqrt{2}}$

In this case the ratio of the induced emf and the magnetic flux is equal to the angular velocity of the coil.

Thus $\frac{e}{\phi} = \frac{NBA\,\omega}{\sqrt{2}} / \frac{NBA}{\sqrt{2}} = \omega$

- The direction of an induced emf in the coil changes during one cycle so it is called alternating emf and current induced due to it is called alternating current. This is the principle of AC generator.

**Ex.** The phase difference between the emf induced in the coil rotating in a uniform magnetic field and the magnetic flux associated with it, is

**Sol.** $\phi = NAB\cos\omega t$ and $e = NAB\,\omega\sin\omega t$

Hence the phase difference between $\phi$ and $e$ will be $\frac{\pi}{2}$.

**Ex.** A coil has 20 turns and area of each turn is 0.2 $m^2$. If the plane of the coil makes an angle of 60° with the direction of magnetic field of 0.1 tesla, then the magnetic flux associated with the coil will be

**Sol.** $\phi = n(BA\cos\theta)$

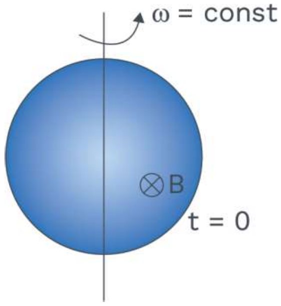
$= 20 \times 0.1 \times 0.2\cos(90° - 60°)$

$= 20 \times 0.1 \times 0.2 \times \frac{\sqrt{3}}{2} = 0.346$ weber

**Concept Reminder**

The phase difference between the instantaneous magnetic flux and induced emf is $\pi / 2$.

**Ex.** A ring is rotating with angular velocity $\omega$ about an axis in the plane of the ring which passes through the centre of this ring. A constant magnetic field B exists normal to the plane of the ring. Calculate the emf induced in the ring as a function of time.




**Sol.** At any time t, $\phi = BA\cos\theta = BA\cos\omega t$

Now induced emf in the loop

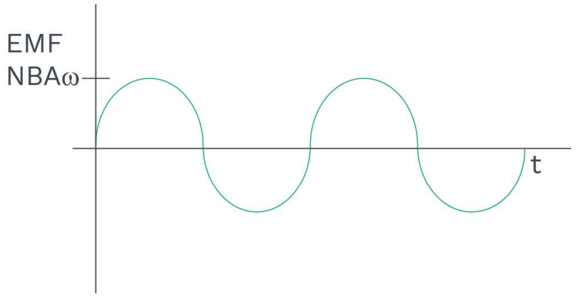
$$e = \frac{-d\phi}{dt} = BA\,\omega\sin\omega t$$

If there are N turns emf = $BA\omega N\sin\omega t$

$BA\,\omega N$ is the amplitude of the emf

$e = e_m \sin\omega t$

$$i = \frac{e}{R} = \frac{e_m}{R}\sin\omega t = i_m \sin\omega t$$




$$i_m = \frac{e_m}{R}$$

A rotating coil thus produces a sinusoidally varying current or alternating current. This is also the basic principle used in generator.

**Rack your Brain**

In a region of magnetic induction $B = 10^{-2}$ tesla, a circular coil of radius 30 cm and resistance $\pi^2\,\Omega$ is rotated about an axis which is perpendicular to the direction of B and which forms a diameter of the coil. If the coil rotates at 200 rpm, the amplitude of alternating current induced in coil is

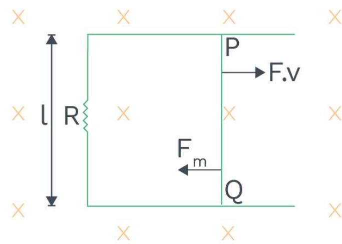
(1) $4\pi^2$ mA

(2) 30 mA

(3) 6 mA

(4) 200 mA

## Energy Consideration




- A conductor PQ is moved with a constant velocity v on parallel sides of a U-shaped conductor in the magnetic field as shown in figure.
  Let R be the resistance of the closed loop. The value of emf induced in this rod is e = Blv

  The current in the circuit is $i = \frac{e}{R} = \frac{Blv}{R}$

  As current flows in the conductor PQ from Q to P of the conductor. So, an equal and opposite force F has to be applied on the conductor to move the conductor with a constant velocity v.

  Thus, $F = F_m = \frac{B^2l^2v}{R}$ The rate at which work done is done by the applied force to move the rod is,

  $$P_{applied} = Fv = \frac{B^2l^2v^2}{R}$$

  The rate at which energy is dissipated in the circuit is,

  $$\boxed{P_{dissipated} = i^2R = \left(\frac{Bvl}{R}\right)^2 R = \frac{B^2l^2v^2}{R}}$$

  This is just equal to the rate at which work is done by the applied force.

**Ex.** A conductor of length 0.1 m carrying a current of 50 A is placed perpendicular to magnetic field of 1.25 mT. Find the mechanical power to move the conductor with a speed of 1 $ms^{-1}$ is

**Sol.** Power P = Fv; P = Bilv ; l = 0.1m ; i = 50

**Concept Reminder**

Usually in AC generators, magnet is held stationery and coil is rotated but, in some generator, the coils are held stationary and it is the electromagnets which are rotated.

**Concept Reminder**

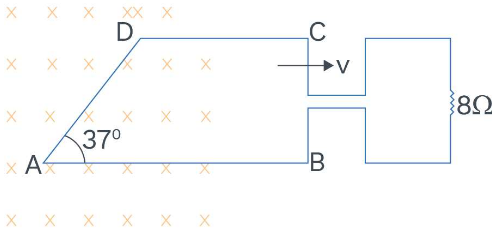
$$\boxed{P_{dissipated} = i^2R = \left(\frac{Bvl}{R}\right)^2 R = \frac{B^2l^2v^2}{R}}$$

$B = 1.25 \times 10^{-3}\,;\, v = 1 m / sec$

$\therefore p = Bilv$

$= 1.25 \times 10^{-3} \times 50 \times 0.1 \times 1 = 6.25 \times 10^{-3} = 6.25\,mW$

**Ex.** The loop ABCD is moving with velocity 'v' towards right. The magnetic field is 4T. The loop is connected to a resistance of $8\,\Omega$ .If steady current of 2A flows in the loop then value of 'v' if loop has a resistance of $4\,\Omega$, is:
(Given AB = 30cm, AD = 30 cm)




**Sol.** An induced emf in the loop is Blv

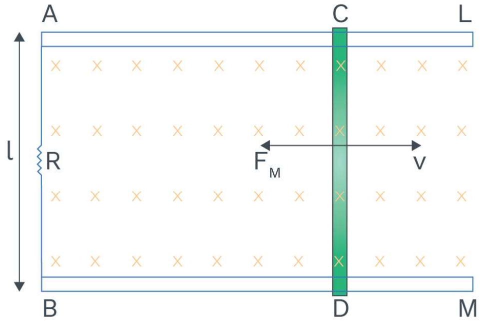
$$e = B(AD)\sin 37^\circ v = 4 \times 0.3 \sin 37^\circ v$$

$$R = (4+8) = 12\,\Omega\,;\ \text{Hence}\ i = \frac{e}{R} = \frac{Blv}{R}$$

$$\Rightarrow \quad 2 = \frac{4 \times 0.3 \times \sin 37^\circ v}{(4+8)}\,;\ v = \frac{100}{3} m / s$$

**Ex.** Two parallel wires AL and BM are placed at a separation of $l$ are connected by a resistor R. In this region a magnetic field B present which is normal to the plane containing the wires. Another wire CD now connects both the wires perpendicularly and made to move with velocity v. Find the work done per second required to move the wire CD. Neglect value of resistance of all the wires.

**Sol.** If a rod of length $l$ moves in a magnetic field with induced in it. Due to this induced emf, a current $i = \frac{\varepsilon}{R} = \frac{Bvl}{R}$ will flow in circuit as shown in given figure. Because of this induced current, the wire will experience a magnetic force



$$F_M = Bil = \frac{B^2 l^2 v}{R}$$

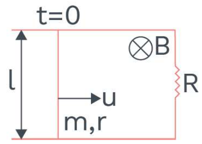
which opposes its motion, so to maintain the motion of wire CD a force F = $F_M$ should be applied in the direction of motion.

The work done per second, i.e., power needed to slide the wire is given by

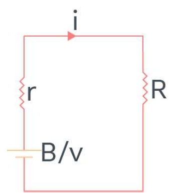
$$P = \frac{dW}{dt} = Fv = F_M v = \frac{B^2 v^2 l^2}{R}$$

**Note:** The power delivered by external agent is converted into joule heating. This means magnetic field helps in converting mechanical energy into joule heating.

**Ex.** A rod having mass m and resistance r is placed on fixed, resistance less, smooth conducting rails which is closed by a resistance R and it is given an initial velocity u Determine velocity as a function of time.




**Sol.** Suppose at some instant the velocity of the rod be v. The emf induced in the rod will be vBl. The electrically equivalent circuit is shown in the following diagram.




∴ Current in the circuit $i = \frac{Blv}{R+r}$

At time t

Magnetic force acting on the rod is $F = ilB$, opposite to the motion of the rod.

$$ilB = -m\frac{dv}{dt} \quad \text{.... (1)}$$

$$i = \frac{Blv}{R+r} \quad \text{.... (2)}$$

Now solving these two equations

$$\frac{B^2 l^2 v}{R+r} = -m.\frac{dv}{dt}$$

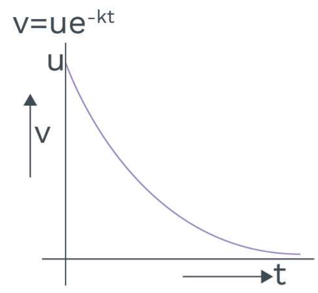
$$\Rightarrow \quad -\frac{B^2l^2}{(R+r)m}.dt = \frac{dv}{v}$$

$$\text{let} \Rightarrow \quad -K.dt = \frac{dv}{v}$$

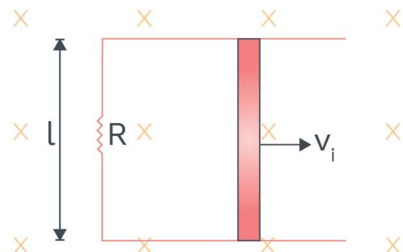
$$\int_u^v \frac{dv}{v} = \int_0^t -K.dt$$

$$\ln\left(\frac{v}{u}\right) = -Kt$$

$$\Rightarrow \quad \mathcal{A} = \quad ^{-Kt}$$




**Ex.** A bar having mass m and length l moves on two frictionless parallel rails in the presence of a uniform magnetic field directed into the plane of the paper. The bar is given an initial velocity $v_i$ to the right and released. Determine the velocity of bar, induced emf across the bar and the current in the circuit as a function of time.




**Sol.** The induced current is in the anticlockwise direction and magnetic force acting on the bar is given by $F_B = -ilB$. The –ve sign tells us that the force is towards the left and retards motion.

$$F = ma$$

$$-ilB = m.\frac{dv}{dt}$$

Since, the force depends on current and the current depends on the speed therefore, the force is not constant and the acceleration of the bar is not constant. The induced current is given by

$$i = \frac{Blv}{R}; -ilB = m.\frac{dv}{dt}$$

$$-\left(\frac{Blv}{R}\right)lB = m.\frac{dv}{dt} \Rightarrow \frac{dv}{v} = -\frac{B^2l^2}{mR}dt$$

$$\int_{v_1}^{v} \frac{dv}{v} = -\frac{B^2l^2}{mR}\int_0^t dt \ ; \ \ln\left(\frac{v}{v_1}\right) = -\frac{B^2l^2}{mR}t = \frac{-t}{T}$$

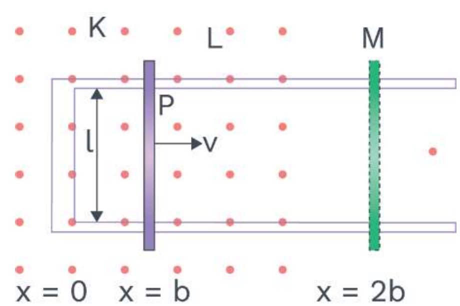
where $T = \frac{mR}{B^2l^2} \Rightarrow v = v_i e^{\frac{t}{T}}$

The speed of the bar decreases exponentially with time under the action of magnetic retarding force.

$$\text{emf} = iR = Blv_i e^{\frac{t}{T}};$$

current: $i = \frac{Blv}{R} = \frac{Bl}{R}v_1 e^{\frac{t}{T}}$

**Ex.** The arm PQ of rectangular conductor is moved from x = 0, outwards in uniform magnetic field which extends from x = 0 to x = band is zero for x > b as shown. Only the arm PQ has resistance r. Take the situation when the arm PQ is pulled outwards from x = 0 to x = 2b and is then pushed back to x = 0 with constant speed v. Write down an expressions for the flux, an induced emf, the force required to pull the arm and power dissipated as Joule heat. Draw the variation of these quantities with distance.



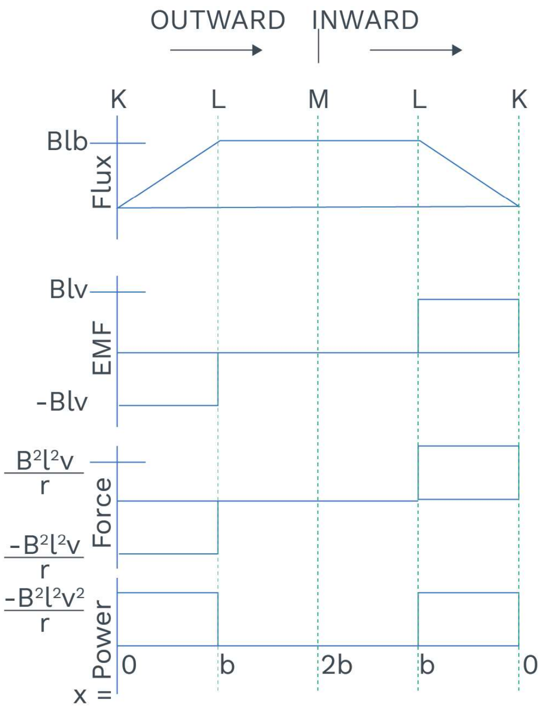

**Sol.** Let us first consider the forward motion of arm from x= 0 to x = 2b. The flux $\phi_B$ linked with circuit SPQR is

And induced emf is,

$$\varepsilon = -\frac{d\phi_B}{dt} = -Blv \qquad 0 \le x < b$$

$$\varepsilon = 0 \qquad b \le x < 2b$$

If the induced emf is non-zero, the current I is $I = \frac{Blv}{r}$(in magnitude)




The force needed to keep the arm PQ in constant motion is IBl. Its direction is to the left.

$$F = \frac{B^2l^2v}{r} \; 0 \le x < b : F = 0 \; b \le x < 2b$$

The Joule heating loss is

| | |
|---|---|
| IBl | $0 \le x < b$ |
| $P_J = 0$ | $b \le x < 2b$ |

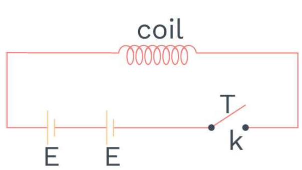
One obtains same expressions for inward motion from x = 2b to x = 0. One can appreciate this whole process by examining the sketch of various quantities displayed in Fig.

**Eddy Currents**

- If bulk pieces of conductors are placed in changing magnetic flux, induced currents are produced in them.
- The flow patterns of induced currents resemble the whirling eddies in water. This effect was discovered by Foucault and these currents are called eddy currents (or) Foucault currents.
- A Cu plate is allowed to swing like a simple pendulum between the pole pieces of a strong magnet, its motion is damped and the plate comes to rest in the magnetic field due to eddy currents in the plate.

- If rectangular slots are made in the Cu plate area available to the flow of eddy currents is less. So, electromagnetic damping is reduced and the plate swings more freely.
- The eddy currents heat up the metallic cores and dissipate electrical energy in the form of heat in the devices like transformers, electric motors and other such devices.
- The eddy currents are minimized by laminating metal to make a metal core. The laminations are separated by an insulating material like lacquer.
- The plane of the laminations must be arranged parallel to the magnetic field, so that they cut across the eddy current paths reduces the strength of the eddy current.
- **Advantages**:
  Eddy currents are used in
  (a) Magnetic brakes in trains.
  (b) Electromagnetic damping.
  (c) Induction furnace.
  (d) Electric power meters.

**Self-Induction:**




- When a current flowing in a coil changes, the magnetic flux linked with the coil changes. Then emf induced in the coil is called self-induced emf and the phenomenon is called self-induction.
- If 'i' is the current flowing through the coil and '$\phi$' is magnetic flux linked with the coil, then

$$\phi \propto i \qquad \Rightarrow \phi = Li \qquad \therefore \ L = \frac{\phi}{i}$$

- Self-induced emf is given by

$$\boxed{e = \frac{-d\phi}{dt} = -L\frac{di}{dt}}$$

**Definitions**

- When bulk pieces of conductors are subjected to changing magnetic flux, induced currents are produced in them
- The flow patterns of induced currents resemble the whirling eddies in water. This effect was discovered by Foucault and these currents are called eddy currents (or) Foucault currents.

**Concept Reminder**

The eddy currents heat up the metallic cores and dissipate electrical energy in the form of heat in the devices like transformers, electric motors and other such devices.

**Concept Reminder**

The eddy currents are minimized by using. laminations of metal to make a metal core. The laminations are separated by an insulating material like lacquer.

- Self-inductance of a coil is magnetic flux linked with the coil when unit current flows through it(or) emf induced in the coil when current changes in it at the rate of 1 A/sec.
- S.I. Unit of self-inductance: Henry.
  Other Units: weber/ampere, volt-second/ampere, $J / amp^2, Wb^2 / J, volt\ sec^2\ C^{-1}$.
  Dimensional formula of L is $[ML^2T^{-2}I^{-2}]$
- A coil having high self-inductance is called inductor.
- Self-induction is also known as inertia of electricity as it opposes the growth or decay of the current in the circuit.
- Inductance may be viewed as electrical inertia. It is analogous to inertia in mechanics. It does not oppose the current but is opposes the change in current.

**Self-Inductance Of A Flat Circular Coil:**

Let us take a circular coil having radius r and containing N-turns. Suppose it carries a current 'i'. The magnitude of magnetic field at its centre due to this current $B = \frac{\mu_0 Ni}{2r}$

And total flux = $NBA = N\left(\frac{\mu_0 Ni}{2r}\right)\pi r^2 = \frac{\mu_0 \pi N^2 ri}{2}$

Now comparing with $N\phi_B = Li$ we get

$$L = \frac{\mu_0 \pi N r}{2}$$

**Self-Inductance Of A Solenoid:**

Consider a long solenoid of length l, area of cross-section A and number of turns per unit length n and length is very large when compared with radius of cross section.

Let I be the current flowing through the solenoid. The magnetic field inside the long solenoid is uniform and is given by $B = \mu_0 nI$

**Definitions**

If current flowing in a coil changes, the magnetic flux linked with the coil changes. Then emf induced in the coil is called self-induced emf and the phenomenon is called self induction.

**Concept Reminder**

Self-induced emf is given by

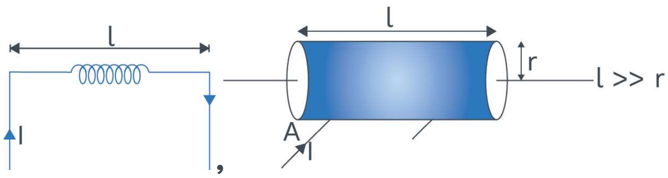
$$e = \frac{-d\phi}{dt} = -L\frac{di}{dt}$$

**Concept Reminder**

Self-induction is also known as inertia of electricity as it opposes the growth or decay of the current in the circuit.

Total number of turns in the solenoid of length l is N = nl.

The magnetic flux linked with every turn of this solenoid $B \times A = \mu_0 nIA$




∴ Total magnetic flux linked with whole solenoid, $\phi$ = magnetic flux with each turn × number of turns in the solenoid.

$\phi = \mu_0 nIA \times nl = \mu_0 n^2 IAl$ ... (1)

But $\phi = LI \Rightarrow LI = \mu_0 n^2 IAl$ from (1) & (2)

∴ $\boxed{L = \mu_0 n^2 Al}$ Since $n = \frac{N}{l}, L = \mu_0 \frac{N^2}{l} A$

- **S.I. unit:** Henry

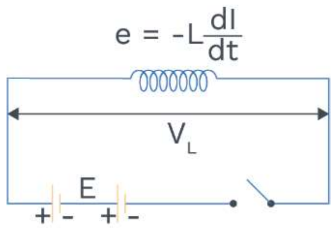
**Concept Reminder**

Self-Inductance Of A Solenoid

∴ $\boxed{L = \mu_0 n^2 Al}$

**Energy Stored In An Inductor**

- Consider an ideal inductor of inductance 'L' connected with a battery. suppose I be the current in the circuit at any instant 't'




This induced emf is given by $e = -L\frac{dI}{dt}$

-ve sign shows that 'e' opposes the change of current I in the inductor. To drive the expression of current through the inductor against the induced emf 'e', the external voltage is applied. Here, an external voltage is emf of the battery = E

According to Kirchhoff's voltage law, E + e = 0

$E = -e; E = L\frac{dI}{dt}$

**Concept Reminder**

Dimensional formula of self-inductance or mutual inductance is $[ML^2T^{-2}A^{-2}]$

Let an infinitesimal charge dq be driven through the inductor in time dt. So, the rate of work done by the external voltage is given by

$$\frac{dW}{dt} = EI = L\frac{dI}{dt} \times I = LI\frac{dI}{dt}$$

The total work done in establishing a current through the inductor from 0 to I is given by

$$W = \int dW = \int_0^1 LI\,dI;\ W = L\left(\frac{I^2}{2}\right) = \frac{1}{2}LI^2$$

$$\boxed{W = \frac{1}{2}LI^2}$$

The work done in maintaining the current through an inductor is stored as the potential energy (U) in the magnetic field. Hence energy stored in the inductor is given by

- The equation $U = \frac{1}{2}LI^2$ is similar to the expression for kinetic energy $E = \frac{1}{2}mv^2$. It shows that L is analogues to mass 'm' and self-inductance is called electrical inertia.
- The self-inductance of a coil is numerically same as twice the energy stored in it when unit current flows through it. i.e., When i = 1A, L = 2U
- Induced power $P = e \times i = Li\left(\frac{di}{dt}\right)$
- In case of solenoid $L = \mu_0 n^2 Al$
- Magnetic energy stored per unit volume $u_B = \frac{\frac{1}{2}Li^2}{Al} \Rightarrow \mu_B = \frac{1}{2}\mu_0 n^2 i^2$. Hence $u_B = \frac{B^2}{2\mu_0}$
- The magnetic energy stored per unit volume similar to electrostatic energy stored per unit volume in a parallel plate capacitor $u_B = \frac{1}{2}\varepsilon_0 E^2$.

  In both cases the energy is proportional to the square of field strength.

**Concept Reminder**

Energy stored in inductor is $U = \frac{1}{2}Li^2$

**Concept Reminder**

The equation $U = \frac{1}{2}LI^2$ is similar to the expression for kinetic energy $E = \frac{1}{2}mv^2$. It shows that L is analogues to mass 'm' and self-inductance is called electrical inertia.

**Ex.** The self-inductance of a coil having 200 turns is 10 milli henry. Find the magnetic flux through the cross-section of the coil corresponding to current of 4 milli ampere. Also calculate the total flux linked with each turn.

**Sol.** Total magnetic flux linked with the coil, $N\phi = LI = 10^{-\mathcal{A}} \times 4 \times 10^{-} = 4 \times 10^{-}$ Wb

$$\text{Flux per turn, } \phi = \frac{4 \times 10^{-5}}{200} = 2 \times 10^{-7}\,\text{Wb}$$

**Ex.** A coil of inductance 0.2 Henry is connected to 600-volt battery. At what rate, will the current in the coil grow when circuit is completed?

**Sol.** As the battery and inductor are in parallel, at any instant, emf of the battery and self emf in the inductor are equal

$$|e| = L\frac{dI}{dt} \text{ or } \frac{dI}{dt} = \frac{|e|}{L} = \frac{600\text{V}}{0.2\,\text{H}} = 3000\,\text{A s}^{-1}$$

**Ex.** An inductor of 5H inductance carries a steady current of 2A. How can a 50V self-induced emf be made to appear in the inductor

**Sol.** $L = 5\text{H}; |e| = 50\,\text{V}$; Let us produce the required emf by reducing current to zero

$$\text{Now, } |e| = L\frac{dI}{dt} \text{ or } dt = \frac{LdI}{|e|} = \frac{5 \times 2}{50}\,\text{s}$$

$$= \frac{10}{50}\,\text{s} = \frac{1}{8}\,\text{s} = 0.2\,\text{s}$$

So, the desired emf can be produced by reducing the given current to zero in 0.2 second

**Ex.** Two different coils have self-inductances $L_1$ =16 mH and $L_2$ = 12 mH. At some instant, the current in both the coils is increasing at same rate of power supplied to the two coils is the same. Determine the ratio of (i) induced voltage (ii) current (iii) energy stored in the two coils at that instant.

**Sol.** **(i)** $$V_1 = L_1\frac{dI}{dt}; V_2 = L_2\frac{dI}{dt}; \frac{V_1}{V_2} = \frac{L_1}{L_2} = \frac{16}{12} = \frac{4}{3}$$

**(ii)** $$P = V_1I_1 = V_2I_2 \Rightarrow \frac{I_1}{I_2} = \frac{V_1}{V_2} = \frac{3}{4}$$

**(iii)** $$\frac{U_1}{U_2} = \frac{\frac{1}{2}L_1I_1^2}{\frac{1}{2}L_2I_2^2} = \left(\frac{L_1}{L_2}\right)\left(\frac{I_1}{I_2}\right)^2 = \frac{4}{3}\left(\frac{3}{4}\right)^2 = \frac{3}{4}$$

**Ex.** The network shown is a part of the closed circuit in which the current is changing. At an instant, current in it is 5A. Potential difference between points A and B if the current is


(1) Increasing at 1A/sec

(2) Decreasing at 1A/sec

**Sol.** (1) The coil can be imagined as a cell of emf $e = L\left(\frac{di}{dt}\right) = 5 \times 1 = 5V$ ; $\therefore$ Equivalent circuit is


$V_A - 5(1) - 15 - 5 = V_B$

Hence $V_A - V_B = 5 + 15 + 5 = 25\ V$

(2) The coil can be imagined as a cell of emf $e = L\left(\frac{di}{dt}\right) = 5 \times 1 = 5V$ ;

$\therefore$ Equivalent circuit is


$V_A = 5(1) - 15 + 5 = V_B$

Hence $V_A - V_B = 5 + 15 - 5 = 15\ V$

**Ex.** Current in a coil changes from 10A to 20A in 0.5 sec. If coefficient. of self-induction is $10^{-2}$ H, then emf induced.

**Sol.** $e = 10^{-2} \times \frac{10}{0.5}$

$e = 0.2\ V$

**Ex.** Alternating current of peak value 2 Amp and frequency 50Hz is flowing in a coil of self-inductance $10^{-2}$ H then find max. value of induced emf.

**Sol.** $I_0 = 2A, \nu = 50\,Hz, \omega = 100\pi$

$I = I_0 \sin\omega t$

$= 2\sin(100\pi \times t)$

$\frac{dI}{dt} = 200\pi\cos(100\pi \times t)$

$e = L\frac{dI}{dt} = 10^{-2} \times 200\pi\cos(100\pi t)$

We can find $e_{\mathcal{A}}$ If $\cos(100\pi t) = 1$

$e_{max} = 200\pi \times 10^{-2}$

$e_{max} = 2\pi$ V

**Ex.** In previous ques. find induced emf when current is half of its max. value.

**Sol.** $I = I_0 \sin(100\pi t)$

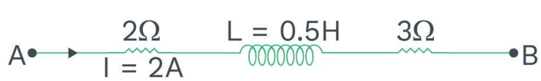
$\frac{I_0}{2} = I_0 \sin(100\pi t)$

$t = \frac{1}{600}\,sec.$

$e = L\frac{dI}{dt} = 10^{-2} \times 1 \times 200\pi\cos(100\pi\frac{1}{600})$

$e = \frac{\sqrt{3}}{2}\pi \times 2 = \sqrt{3}\pi$ V

**Ex.** Find potential diff. b/w A and B ($V_A - V_B$).

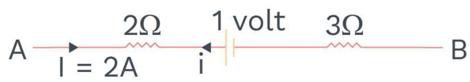



**(i)** I = constant

$V_A - 4 - 6 = V_B$

$V_A - V_B = 10\,V$

**(ii)** $I\uparrow\uparrow; \frac{dI}{dt} = 2A/\sec.$

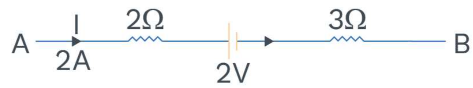



$e = \frac{LdI}{dt} = 0.5 \times 2 = 1V$

$V_A - 4 - 1 - 6 = V_B$

$V_A - V_B = 11V$

**(iii)** $I\downarrow\downarrow; \frac{dI}{dt} = 4A/\sec$

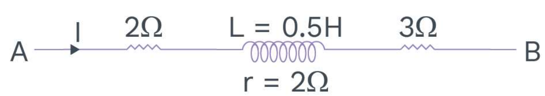



$e = 0.5 \times 4 = 2V$

$V_A - 4 + 2 - 6 = V_B$

$V_A - V_B = 8V$

**Ex.**




If $I = t^2 + 2$; then find potential difference between point A and B. (t = 2 sec.)

**Sol.** $I = 2^2 + 2 = 6A$

$\frac{dI}{dt} = 2t = 4A/\sec.$

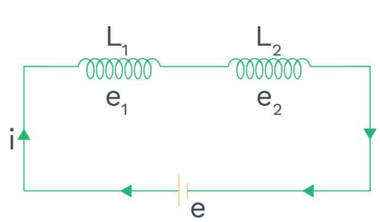

$e = L\frac{dI}{dt} = 0.5 \times 4 = 2\text{ volt}$

$V_A - 12 - 2 - 12 - 18 = V_B$

$V_A - V_B = 44\text{ volt}$

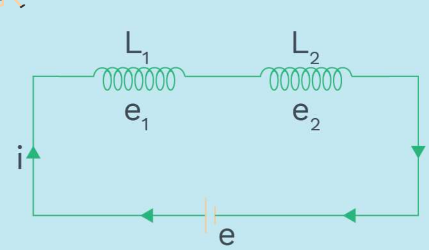
**Inductors In Series:**

If two coils of inductances $L_1$ and $L_2$ are connected in series, then the potential divides.




i.e., $e = e_1 + e_2 \text{(or)} L_S\frac{di}{dt} = L_1\frac{di}{dt} + L_2\frac{di}{dt}$

**Concept Reminder**




$L_S = L_1 + L_2$

Since in series, $\frac{di}{dt}$ is same for all coils

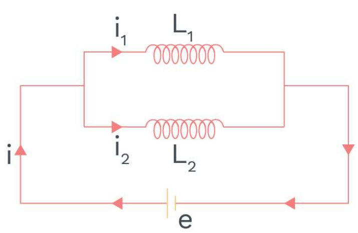
$\therefore \quad L_S = L_1 + L_2$

If n coils of inductances $L_1, L_2, L_3 \,.......L_n$ are connected in series, then effective inductance of the arrangement,

$$\boxed{L = L_1 + L_2 + L_3 + ...... + L_n}$$ (When coils are far away)

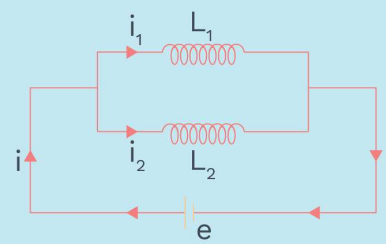
**Inductors In PARALLEL:**

If two coils of inductances $L_1$ and $L_2$ are connected in parallel, then the current divides.




i.e., $i = i_1 + i_2$ (or) $\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \Rightarrow \frac{e}{L_P} = \frac{e_1}{L_1} + \frac{e_2}{L_2}$

**Concept Reminder**



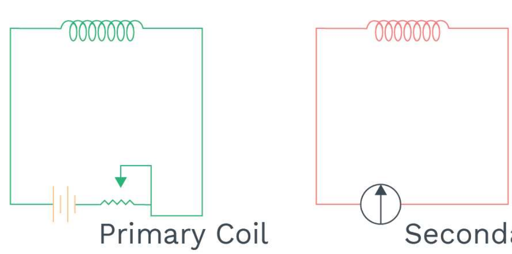

$$\frac{1}{L_P} = \frac{1}{L_1} + \frac{1}{L_2}$$

However, in parallel as potential difference remains same i.e., $e = e_1 = e_2$, so

If n coil of inductances $L_1, L_2, L_3 ........L_n$ are connected in parallel then effective inductance of the arrangement,

$$\boxed{\frac{1}{L_P} = \frac{1}{L_1} + \frac{1}{L_2} + ..... + \frac{1}{L_n}}$$ (When coils are far away)

**Mutual Induction**




If current in a primary coil is changed w.r.t time, then flux related to secondary coil is also changed so an emf is induced in secondary coil and this is called Mutual induction.

- Dimensional formula of self-inductance or mutual inductance is $[ML^2T^{-2}A^{-2}]$.
- The value of mutual inductance depends on
  **1)** Distance between the two coils
  **2)** Number of turns of coils
  **3)** Geometrical shape of the coil
  **4)** Material of the core medium between the coils
  **5)** Orientation of the coils i.e., angle between the axes of the coils.

**Coefficient of Mutual Induction: -**

$$N_2\phi_2 \propto I_1$$

$$N_2\phi_2 = MI_1$$

$$M = \frac{N_2\phi_2}{I_1}$$

$$M = \frac{N_2B_1A_2}{I_1}$$

Unit of M is Henry.
M does not depend on current.

**Induced emf in secondary coil-**

$$N_2\phi_2 = MI_1$$

$$N_2\frac{d\phi_2}{dt} = -M\frac{dI_1}{dt}$$

$$e_2 = -M\frac{dI_1}{dt}$$

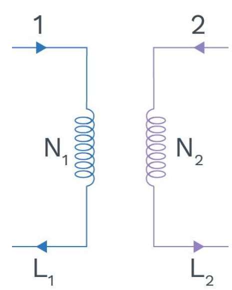
Induced emf in secondary ← $e_2$; $\frac{dI_1}{dt}$ → Rate of change of current in primary coil

**Relation Between $L_1$, $L_2$ And M:**

The flux linked with coil 1 is $N_1\phi_1 = L_1i_1 \Rightarrow L_1 = \frac{N_1\phi_1}{i_i}$

The flux linked with coil 2 is

$$N_2\phi_2 = L_2i_2 \Rightarrow L_2 = \frac{N_2\phi_2}{i_2}$$




M on 1 because of 2; $M_{12} = \frac{N_1\phi_1}{i_2}$

M on 2 because of 1; $M_{21} = \frac{N_2\phi_2}{i_1}$

- If the flux in linkage is maximum, then $M_{12} = M_{21} = M$; $M_{12} \times M_{21} = \frac{N_2\phi_2}{i_1} \times \frac{N_1\phi_1}{i_2}$

$M^2 = L_1L_2$;

$\therefore \quad M = \sqrt{L_1L_2}$

This is the maximum mutual inductance when all the flux linked with one coil is also completely linked with the other.

In general, only a fraction of the total flux will be linked with the coil due to the flux leakage.

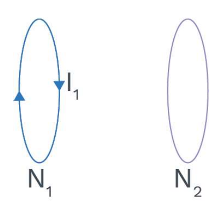
$\therefore \quad \boxed{M = K\sqrt{L_1L_2}}$

Where K-coefficient of coupling $(K \le 1)$

For tight coupling (or) if the coils are closely wound, then K=1.

$\therefore \quad M_{max} = \sqrt{L_1L_2}$

**Ex.** Find coefficient of Mutual induction of two circular rings of same radius and turns $N_1$ and $N_2$.




**Sol.** $M = \frac{N_2B_1A_2}{I_1}$

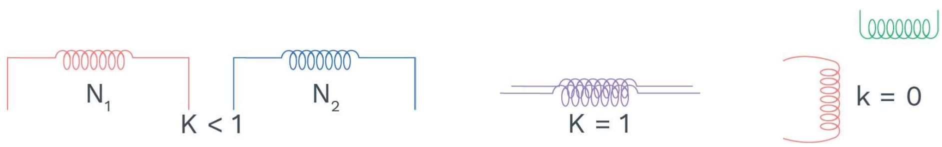
$$B_1 = \frac{N_1 \mu_0 I_1}{2R}$$

$$M = \frac{N_2 A_2}{I_1} \times \frac{N_1 \mu_0 I_1}{2R} \qquad \boxed{M = \mu_0 N_1 N_2 \frac{\pi R^2}{2R}}$$

**Coupling Coefficient or Coupling Factor (K)**

It is the ratio of flux in one turn of secondary to that of flux in one turn of primary coil.

$$K = \frac{\phi_S}{\phi_P} \le 1$$




K is unit less

**Ex.** Self-inductance of two coils is 2H & 8H. If 50% flux of primary coil is linked with secondary coil, then find coefficient of mutual inductance.

**Sol.** $M = \frac{1}{2}\sqrt{2 \times 8} = \frac{4}{2} = 2H$

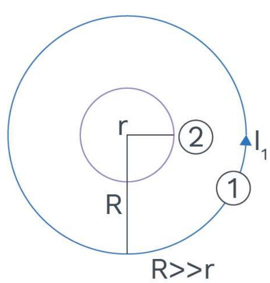
**Ex.** Two coaxial solenoids are assembled by winding thin insulated wire over a pipe having a cross-sectional area A = 10 cm$^2$ and of length = 20 cm. Given that one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is $\left(\mu_0 = 4\pi \times 10^{-7} TmA^{-1}\right)$

**Sol.** $M = \frac{\mu_0 N_1 N_2 A}{L}$

$$M = \frac{4\pi \times 10^{-7} \times 3 \times 10^2 \times 4 \times 10^2 \times 10^{-3}}{2 \times 10^{-1}}$$

$$= 2.4\,\pi \times 10^{-4} H$$

**Ex.** Find mutual induction for given figure.



**Sol.** $M = \frac{N_2 B_1 A_2}{I_1}$

$B_1 = \frac{\mu_0 I_1}{2R}$

$A_2 = \pi r^2$

$M = N_2 \frac{\mu_0 I_1}{2R} \frac{\pi r^2}{I_1} (N_2 = N_1 = 1)$

$M \propto \frac{r^2}{R}$

**Ex.** Alternating current $I_1 = 2\sin(100\,\pi t)$ is flowing in a primary coil if coefficient of mutual induction is $10^{-2}$ H, then find max. value of induced emf in secondary coil.

**Sol.** $e_2 = -M\frac{dI_1}{dt}\left(\frac{dI_1}{dt} = 200\,\pi\cos(100\pi t)\right)$

$|e_2| = 10^{-2} \times 200\,\pi\cos(200\,\pi t)$ $\qquad \left(\frac{dI_1}{dt}\right)_{max} = 200\,\pi$

$e_2 = 2\pi\,(max.)$

**Ex.** Resistance of primary and secondary coil $R_1 = 10\Omega, R_2 = 5\Omega$ coefficient of mutual induction is $M = 10^{-2}$ H. If current $I_2 = 0.5$ Amp is flowing in secondary coil, then find rate of current change in primary coil.

**Sol.** $e_2 = -10^{-2} \times \frac{dI_1}{dt}$

$= \frac{0.5 \times 5}{10^{-2}} = 2.5 \times 10^2 = \frac{dI_1}{dt}$

**Ex.** Determine the mutual inductance between two coils if a current of 2A changes to 6A in 2 seconds and induces an emf of 20 mV in the secondary coil

**Sol.** $|e| = M\frac{dI}{dt}$

$20 \times 10^{-3} = M\frac{(6-2)}{2}$ (or) $M = 10\,mH$

**Ex.** When the coefficient of mutual induction of primary and secondary induction coils is 6H and a current of 5A is cut off in 1/5000 second, calculate the emf induced in the secondary coil.

**Sol.** $|e| = M\frac{dI}{dt}; e = 6 \times \frac{5}{1/5000} V = 15 \times 10^4 V$

**Ex.** A solenoid 50 cm long and has a radius of 2 cm. It has 500 turns. Around its central section a coil of 50 turns is wound. Calculate the mutual inductance of the system.

**Sol.** $N_P = 500, N_S = 50; A = \pi \times 0.02 \times 0.02 m^2$

$\mu_0 = 4\pi \times 10^{-7} Hm^{-1}, l = 50 cm = 0.5 m$

Now, $M = \frac{\mu_0 N_P N_S A}{l}$

$$= \frac{4\pi \times 10^{-7} \times 500 \times 50 \times \pi \times (0.02)^2}{0.5} H$$

$= 789.8 \times 10^{-7} H = 78.98 \mu H$

**Ex.** A solenoidal coil has 50 turns per cm along its length and a cross-sectional area of $4 \times 10^{-4} m^2$. 200 turns of another wire is wound round the first solenoid co-axially. Both the two coils are electrically insulated from each other. Calculate the mutual inductance between the two coils.

**Sol.** $n_1$ = 50 turns per cm = 5000 turns per metre

$n_2 l = 200, A = 4 \times 10^{-4} m^2; M = \mu_0 n_1 (n_2 l) A$

$= 4\pi \times 10^{-7} \times 5000 \times 200 \times 4 \times 10^{-4} H$

$= 5.03 \times 10^{-4} H$

**Ex.** A toroid solenoid with an air core has an average radius of 0.15 m, area of cross-section $12 \times 10^{-4} m^2$ and 1200 turns. Obtain the self-inductance of the toroid. neglect field variation across the cross section of the toroid.

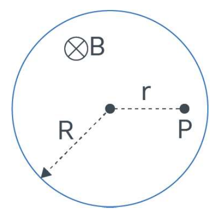
**Sol.** $B = \mu_0 n_1 I = \frac{\mu_0 N_1 I}{l} = \frac{\mu_0 N_1 I}{2\pi r}$

Total magnetic flux, $\phi_B = N_1 BA = \frac{\mu_0 N_1^2 IA}{2\pi r}$

But $\phi_B = LI \quad \therefore L = \frac{\mu_0 N_1^2 A}{2\pi r}$

$$L = \frac{4\pi \times 10^{-7} \times 1200 \times 1200 \times 12 \times 10^{-4}}{2\pi \times 0.15} H = 2.3 \times 10^{-3} H = 2.3 mH$$

**Ex.** A uniform magnetic field B of induction B is confined in a cylindrical region of radius R. When the field is increasing at constant rate of $\frac{dB}{dt} = \alpha t\ T/s$, then the electric field intensity induced at point P, at a distance r from an axis as shown in the figure is proportional to:

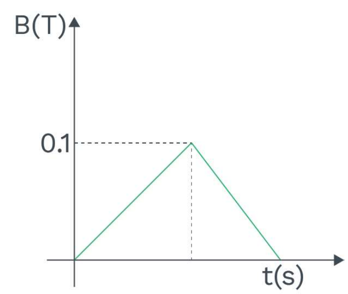



**Sol.** For $r < R; e = \int E.ds = \frac{d\phi_B}{dt}$

$$E.2\pi r = -A\left(\frac{dB}{dt}\right); E.2\pi r = -\pi r^2\left(\frac{dB}{dt}\right)$$

$$E = -\frac{r}{2}\left(\frac{dB}{dt}\right); E = -\frac{r}{2}\alpha t\ ; E \propto r$$

**Ex.** A closed loop of cross-sectional area $10^{-2} m^2$ which has inductance L= 10 mH and negligible resistance is placed in a time-varying magnetic field. Given figure shows the variation of B with time for the interval 4s. The field is normal to the plane of the loop (given at t = 0, B = 0, I = 0). The value of the maximum current induced in the loop is




**Sol.** Induced emf (e) $= L\frac{di}{dt}$

$$\Rightarrow \quad A\left(\frac{dB}{dt}\right) = L\frac{di}{dt} \Rightarrow di = \frac{A}{L}\left(\frac{dB}{dt}\right) \times dt$$

$$\int_0^I di = \int_0^B \left(\frac{A}{L}\right) dB; I = \frac{A}{L}B;$$

$$\Rightarrow \quad I_{max} = \frac{A}{L} B_{max}$$

$$= \frac{10^{-2}}{10 \times 10^{-3}} \times 0.1 = 0.1A = 100\,mA$$

**Ex.** A magnetic field is directed into the page and changes with time according to the expression $B = \left(0.03t^2 + 1.4\right)T$, where t is in seconds. The field has a circular cross - section of radius R -2.5 cm. What is the magnitude and direction of electric field at P, when t = 3.0s and r = 0.02 m.

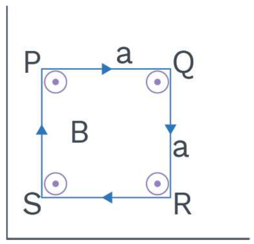
**Sol.** $e = \oint E.dl = \frac{+d\phi}{dt}$

$$E(2\pi r) = A.\frac{dB}{dt} = \pi r^2 \times \frac{d}{dt}(0.03t^2 + 1.4)$$

$$E = \frac{\pi r^2}{2\pi r} \times (0.06t) = \frac{r}{2}(0.06\,t)$$

$$|E| = \frac{0.02}{2} \times 0.06 \times 3 = 18 \times 10^{-4} N/C$$

**Ex.** A wire is bent to form of a square having side 'a' in a varying magnetic field $\vec{B} = \alpha B_0 t\hat{k}$. If the resistance per unit length is $\lambda$, then find the following




**(i)** The direction of induced current B

**(ii)** The current in the loop

**Sol. (i)** Direction of current is close wise.

**(ii)** $|e| = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = a^2(\alpha B_0.t) = a^2 \alpha B$

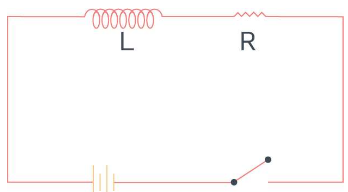
Current: $i = \frac{e}{R} = \frac{a^2 \alpha B_0}{4a\lambda} = \frac{a \alpha B_0}{4\lambda} \quad (\because R = 4a\lambda)$

**R-L Series circuit for DC supply (At the time of switch on)**

$$I = \frac{E}{R}\left(1 - e^{-t/\lambda}\right)$$

$$I = I_0\left(1 - e^{-t/\lambda}\right)$$

$I_0 = \frac{E}{R}$ (Steady state current or saturation current**)**

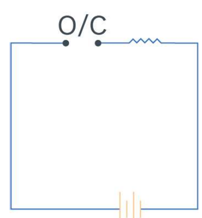



$$\lambda \rightarrow \text{time constant} = \frac{L}{R}(\text{sec.})$$

**(1)** at $t = 0$ (switch)

$I = I_0(1 - e^{0})$

$I = \text{zero}$




**(2)** at $t = \infty$ (after sometime)

$I = I_0(1 - e^{-\infty})$

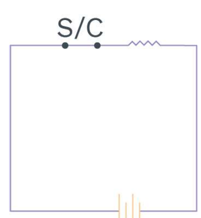
$I = I_0 = \frac{E}{R}$

at $t = \lambda$

$I = I_0\left(1 - e^{-\lambda/\lambda}\right)$

$e^{-1} = 0.37$

$I = 0.63\ I_0 = 0.63\frac{E}{R}$




**(3)** Half time ($T_h$)

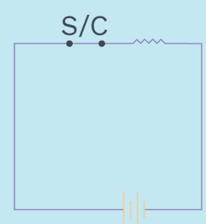
$\frac{I_0}{2} = I_0(1 - e^{-T_h/\lambda})$

$\frac{I_0}{2} = e^{-T_h/\lambda}$

$\log_e \frac{1}{2} = \log_e e^{-T_h/\lambda}\ \log\frac{1}{2} = -\frac{T_h}{\lambda}$

$T_h = (\log_e 2)\lambda$

**Concept Reminder**




at $t = \infty$ (after sometime)

$I = I_0(1 - e^{-\infty})$

$I = I_0 = \frac{E}{R}$

**Definitions**

The time in which current reaches to 63% of its maximum value of (0.63 $I_0$) of $I_0$(0.63 E/R) is called time constant.

$$T_h = 0.693\lambda$$

$$T_h \simeq 0.7\lambda$$

The time in which current reaches to 63% of its maximum value of (0.63 $I_0$) of $I_0$(0.63 E/R) is called time constant.

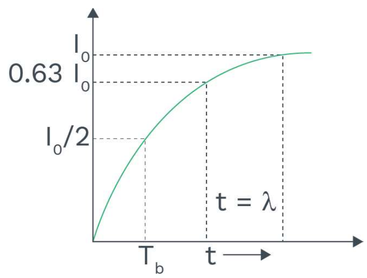
Rate of current change

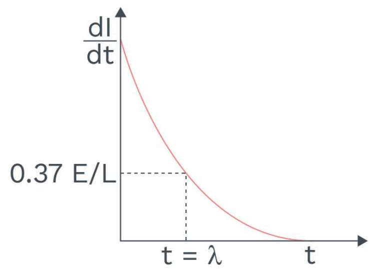
$$I = I_0(1 - e^{-t/\lambda})$$

$$\frac{dI}{dt} = \frac{E}{L}e^{-t/\lambda}$$

Initial rate of current change at t = 0

$$\left(\frac{dI}{dt}\right)_{max} = \frac{E}{L}$$




**Ex.** In R–L series DC circuit E = 8V, R = 2Ω and L = 2mH then find saturation current, max. rate of current change, time constant& half time.

**Sol.** $i_s = \frac{8}{2} = 4\,\text{amp.}$

$$\left(\frac{dI}{dt}\right)_{max} = \frac{E}{L} = \frac{8}{2\times10^{-3}} = 4\times10^3\,A/s \quad \lambda = \frac{L}{R} = \frac{2\times10^{-3}}{2} = 10^{-3}/s$$

$$T_h = 0.7\times\lambda = 0.7\times10^{-3}\,s$$

**Ex.** In R–L series DC circuit E = 20 V, L = 5 mH and R = 1Ω. If battery is connected at t = 0 sec. then find time when I is 12.6 A.

**Sol.** $I_0 = \frac{20}{1} = 20$

I = 12.6 A

I is 63% of maximum current

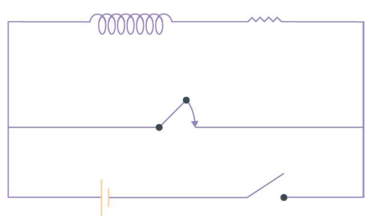
so, time taken $t = \lambda$

$$\frac{L}{R} = 5\,ms$$

**Note**: Don't apply exponential equation in previous question. Find time when current in 10A. I is 50% maximum current.

$$t = T_{1/2} = \frac{0.693\lambda}{5} = 3.5\,ms$$

**R-L Series DC circuit (at the time of switch off)**



$$I = I_0 e^{-t/\lambda}$$

**(1)** At t = 0 (switch off)

$$I = I_0 e^{o}$$

$$I = I_0$$

**(2)** At $t = \infty$ (after sometime)

$$I = I_0 e^{\infty} = 0$$

**(3)** At $t = \lambda$

$$I = I_0 e^{-1} = \frac{I_0}{e} = 0.37\,I_0$$

**Concept Reminder**

**(1)** at t = 0 (switch off)

$$I = I_0 e^{o}$$

$$I = I_0$$

**(2)** at $t = \infty$ (after sometime)

$$I = I_0 e^{\infty} = 0$$

**Growth And Decay Of Current In L-R Circuit**

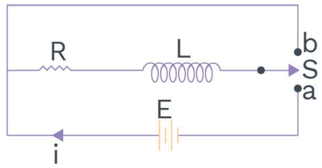
**Growth of Current**

Take a circuit containing a resistance R, an inductance L, a two-way key and a battery of e.m.f E are connected in series as shown in given figure. If the switch S is connected to a, the current in the circuit start to grow from zero value. An inductor opposes the growth of the current. This is because of the fact that when the current grows through an inductor, a back e.m.f. is produce which opposes the growth of current in the circuit. Therefore, the rate of growth of current is reduced. During this growth of current in the circuit, suppose i be the current in the circuit at any instant t. Applying Kirchhoff's

**Rack your Brain**

Two coils of self-inductance 2 mH and 8 mH are placed so close together that the effective flux in one coil is completely linked with other. Find mutual inductance between these coils.

voltage law in the circuit, we obtain




$$E - L\frac{di}{dt} = Ri \text{ or } E - Ri = L\frac{di}{dt}$$

or $\frac{di}{E - Ri} = \frac{dt}{L}$

Multiplying by –R on both the sides, we get

$$\frac{-R\,di}{E - Ri} = \frac{-Rdt}{L}$$

Integrating the above equation, we have

$$\log_e(E - Ri) - -\frac{R}{L}t + A \quad \text{... (1)}$$

here A is an integration constant. The value of this constant can be determine by applying the condition that current i is 0 just at start i.e., at t = 0. Hence

$$\log_e E = 0 + A$$

or $A = \log_e E$ ... (2)

Placing the value of A from equation (2) in equation (1), we get

$$\log_e(E - Ri) = -\frac{R}{L}t + \log_e E$$

or $\log_e\left(\frac{E - Ri}{E}\right) = -\frac{R}{L}t$

or $\left(\frac{E - Ri}{E}\right) = \exp\left(-\frac{R}{L}t\right)$

or $1 - \frac{Ri}{E} = \exp\left(-\frac{R}{L}t\right)$

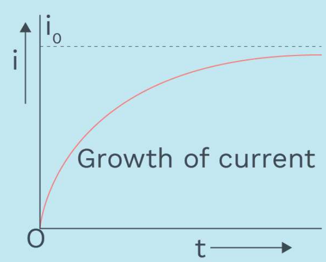
or $\frac{Ri}{E} = \left\{1 - \exp\left(-\frac{R}{L}t\right)\right\}$

**Rack your Brain**

For a coil having L = 2 mH, current flow through it is $I = t^2e^{-t}$ then, the time of which emf becomes zero.

(1) 2 s (2) 1 s
(3) 4 s (4) 3 s

**Concept Reminder**



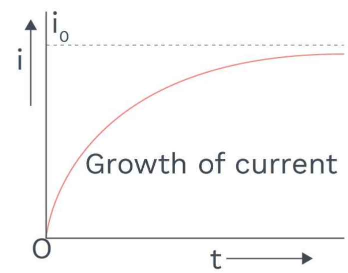
$$\therefore \quad i = \frac{E}{R}\left\{1 - \exp\left(-\frac{R}{L}t\right)\right\}$$

Maximum value of current in the circuit $i_0$ = E/R. So

$$i = i_0\left\{1 - \exp\left(-\frac{R}{L}t\right)\right\} \qquad \text{... (3)}$$

Equation (3) gives the value of current in the circuit at any instant of time t. It can be observed from equation (3) that i is equal to $i_0$, when

$$\exp\left(-\frac{R}{L}t\right) = 0 \text{ i.e., at } t = \infty$$




Therefore, the current never achieves the value $i_0$ but it approaches this value asymptotically. A graph between current and time is shown in given figure.

- We observe the following points

**(i)** When t = (L/R) then

$$i = i_0\left\{1 - \exp\left(-\frac{R}{L}t\right)\right\}$$

$$= i_0\{1 - \exp(-1)\} = i_0\left(1 - \frac{1}{e}\right) = 0.63\, i_0$$

Hence, after an interval of (L/R) second, the current reaches to a value which is 63% of the maximum current. This value of (L/R) is Called time constant of L-R circuit and is represented by $\tau$. Therefore, the time constant of a circuit can be defined as time in which the current gains value from zero to 63% of its final value. In terms of $\tau$,

**Concept Reminder**

Just when the switch is closed, there is no current through the inductor, while after a long time (steady state), inductor offers no opposition and behaves like a zero-resistance path.

$$i = i_0\left(1 - e^{\frac{-t}{\tau}}\right)$$

**(ii)** Rate of growth of the current (di/dt) is given by $\frac{di}{dt} = \frac{d}{dt}\left[i_0\left\{1 - \exp\left(-\frac{R}{L}t\right)\right\}\right]$

$$\Rightarrow \quad \frac{di}{dt} = i_0\left(\frac{R}{L}\right)\exp\left(-\frac{R}{L}t\right) \qquad .... (4)$$

From equation (3), $\exp\left(-\frac{R}{L}t\right) = \frac{i_0 - i}{i_0}$

$$\therefore \quad \frac{di}{dt} = i_0\left(\frac{R}{L}\right)\left(\frac{i_0 - i}{i_0}\right) = \frac{R}{L}(i_0 - i) \qquad .... (5)$$

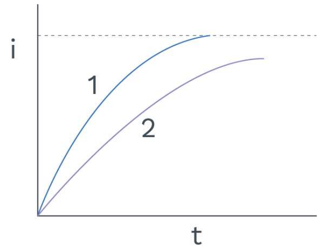
This presents that rate of growth of the current decreases as i reach to $i_0$. For any other value of current, rate of growth of current depends upon the ratio of R/L. Hence, more is the value of time constant, lower will be the rate of growth of current.

**Note:**

- Final current in the circuit is $\frac{\varepsilon}{R}$, it doesn't depend on L.
- After one time constant, the value of current in the circuit is 63% of the final current.
- Larger value of time constant in circuit implies slow rate of change of current.
- When there is any change in the circuit containing inductor, then there is no instantaneous effect on the flux of inductor.

  $L_1 i_1 = L_2 i_2$

**Ex.** Which of the two curves as shown has less time constant?

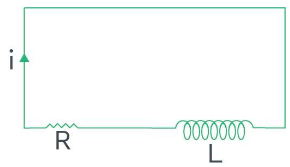



**Rack your Brain**

An inductor may store energy in
(1) Its electric field
(2) Its coil
(3) Its magnetic field
(4) Both in electric and magnetic field.

**Sol.** curve one

**Decay of the Current in L-R Circuit:** Suppose the circuit is disconnected from battery and switch S is thrown to point b in the figure. Then current will begin to fall. If inductance is not present, the current would have fallen from maximum $i_0$ to zero immediately. But because of the presence of inductance, which opposes the decay of the current, the rate of decay of current is decreased. Consider that during the decay of current, i be the value of current at any instant of time t. Applying Kirchhoff's voltage law in the circuit, we get




$$-L\frac{di}{dt} = Ri \qquad \text{or} \qquad \frac{di}{dt} = -\frac{R}{L}i$$

Integrating this expression, we get

$$\log_e i = -\frac{R}{L}t + B$$

Where B is a constant of integration. This value of B can be obtained by using the condition that when t = 0, $i = i_0$

$\therefore \quad \log_e i_0 = B$

Substituting the value of B, we get

$$\log_e i = -\frac{R}{L}t + \log_e i_0$$

or $\log_e \frac{i}{i_0} = -\frac{R}{L}t$

or $(i / i_0) = \exp\left(-\frac{R}{L}t\right)$ .... (6)

or $i = i_0 \exp\left(-\frac{R}{L}t\right) = i_0 \exp\left(-\frac{t}{\tau}\right)$

**Concept Reminder**

The current in an inductor never changes instantaneously, but after the current settles down to a constant value, the inductor plays no role in the circuit.

**Rack your Brain**

If the number of turns per unit length of 2 coil of solenoid is doubled, the self-inductance of the solenoid will

(1) remain unchanged
(2) be halved
(3) be doubled
(4) becomes four time

where $\tau = L/R$ = inductive time constant of circuit. It is certain from equation that the value of current in circuit decays exponentially as shown in figure.

- We observe the following points

**(i)** After time t = L/R, the current in the circuit is given by $i = i_0 \exp\left(-\frac{R}{L} \times \frac{L}{R}\right) = i_0 \exp(-1)$

$= (i_0 / e) = i_0 / 2.718 = 0.37 i_0$

Therefore, after a time (L/R) second, the current reduces to 37% of the maximum current $i_0$. (L/R) is known as time constant $\tau$. This is known as the time during which the current decays to 37% of the maximum value of current during decay.

**Concept Reminder**

If we attempt to stop a current already flowing say, by opening a switch then an emf is induced that attempts to keep the current flowing possibly creating a spark across the switch.

(ii) The rate of decay of a current which given by

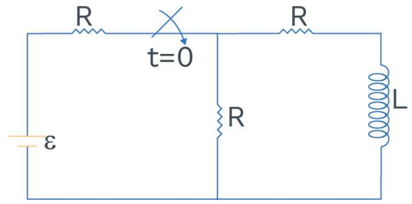
$$\frac{di}{dt} = \frac{d}{dt}\left\{i_0 \exp\left(-\frac{R}{L}t\right)\right\}$$

$$\Rightarrow \quad \frac{di}{dt} = \frac{R}{L} i_0 \exp\left(-\frac{R}{L}t\right) = -\frac{R}{L} i \qquad \text{... (7)}$$

or $\quad -\frac{di}{dt} = \frac{R}{L} i$

This equation tells that when L is small, the rate of decay of current will be more i.e., the current will decay out more rapidly.

**Ex.** In the following circuit switch is closed at t = 0. In beginning there is no current in inductor. Determine current through the inductor coil as a function of time.




**Sol.** At any time t

$-\varepsilon + i_1 R - (i - i_1) R = 0$

$-\varepsilon + 2i_1 R - i R = 0$

$i_1 = \frac{iR + \varepsilon}{2R}$

Now, $-\varepsilon + i_1R + iR + L.\frac{di}{dt} = 0$

$$-\varepsilon + \left(\frac{iR+\varepsilon}{2}\right) + iR + L\frac{di}{dt} = 0 \Rightarrow -\frac{\varepsilon}{2} + \frac{3iR}{2} = -L\frac{di}{dt}$$

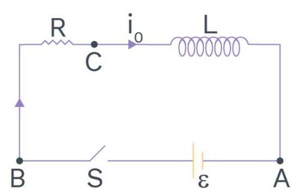
$$\left(\frac{-\varepsilon + 3iR}{2}\right)dt = -L.di \Rightarrow -\frac{dt}{2L} = \frac{di}{-\varepsilon + 3iR}$$

$$-\int_0^t \frac{dt}{2L} = \int_0^i \frac{di}{-\varepsilon + 3iR} \Rightarrow -\frac{t}{2L} = \frac{1}{3R}\ln\left(\frac{-\varepsilon + 3iR}{-\varepsilon}\right)$$

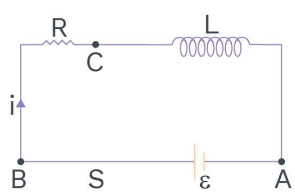
$$-\ln\left(\frac{-\varepsilon + 3iR}{-\varepsilon}\right) = \frac{3Rt}{2L}$$

$$i = +\frac{\varepsilon}{3R}\left(1 - e^{-\frac{3Rt}{2L}}\right)$$

**Ex.** Figure shows a circuit consisting of a ideal cell, an inductor L and a resistor R, connected in series. Let the switch S be closed at t = 0. Suppose at t = 0 current in the inductor is $i_0$ then find out equation of current as a function of time




**Sol.** Let an instant t current in the circuit is i which is increasing at the rate di/dt. Writing KVL along the circuit, we have



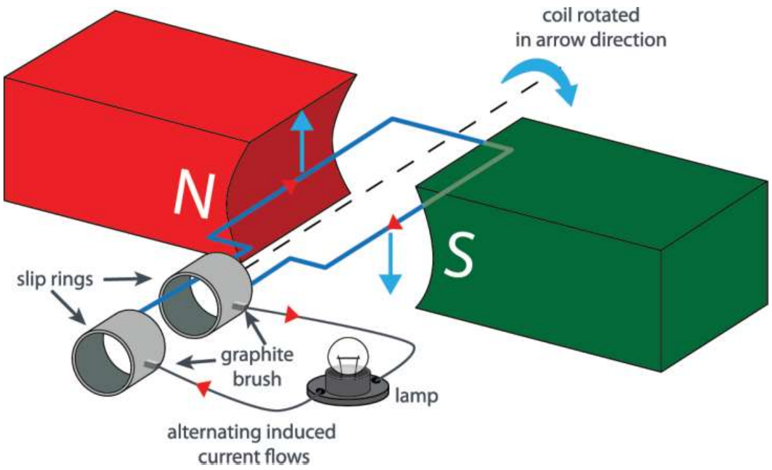

$$\varepsilon - L\frac{di}{dt} - iR = 0 \Rightarrow L\frac{di}{dt} = \varepsilon - iR$$

$$\Rightarrow \quad \int_{i_0}^{i} \frac{di}{\varepsilon - iR} = \int_0^t \frac{dt}{L} \Rightarrow \ln\left(\frac{\varepsilon - iR}{\varepsilon - i_0R}\right) = -\frac{Rt}{L}$$

$$\Rightarrow \quad \varepsilon - iR = (\varepsilon - i_0R)e^{-Rt/L} \Rightarrow i = \frac{\varepsilon - (\varepsilon - i_0R)e^{-Rt/L}}{R}$$

**AC Generator / Dynamo-**

It converts Mechanical energy into Electrical energy.




$\phi = NBA\cos\theta$

$\phi = NBA\cos\omega t\ (\theta = \omega t)$

$\frac{d\phi}{dt} = -NBA\omega\sin\omega t$

$e = \frac{-d\phi}{dt} = NBA\omega\sin\omega t$

$e = e_0\sin\omega t$

$e_0 = e(max) = NBA\omega$

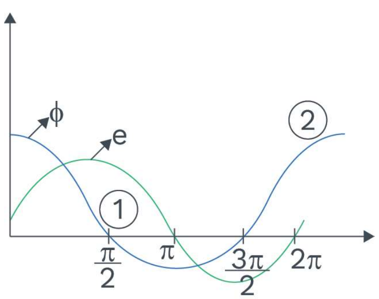
$I = \frac{e}{R} = \frac{NBA\omega}{R}\sin\omega t$

$\phi = 0 \Rightarrow e = max$

$\phi = max \Rightarrow e = 0$

**Concept Reminder**

AC generator converts mechanical energy into electrical energy.



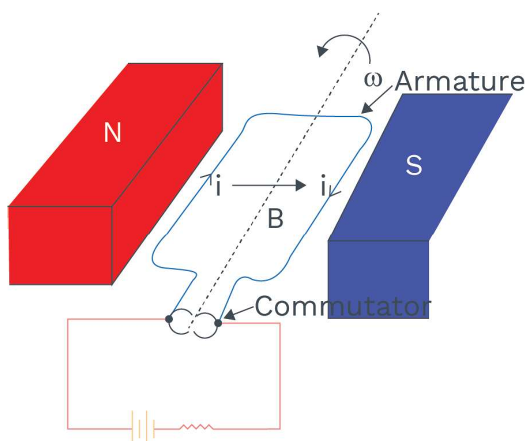

Phase difference b/w flux & emf is $\pi / 2$

* For two pole generators frequency of AC is equal to rotational frequency of Armature.
for multiple generation frequency of AC is equal to –

$$f = \frac{np}{2}$$

n $\rightarrow$ rotational frequency of armature P $\rightarrow$ no. of poles.

**DC MOTOR:**




- It is a device, that converts electrical energy into mechanical energy (rotational energy).
- **Principle:** When current carrying coil is placed in magnetic field it experiences a torque.
- **Working**: When DC motor is connected to a dc source, a current flow in coil, which reverses its direction in regular intervals so that magnetic torque act on coil in same direction. The direction of current is reversed by Commutator.
  Due to rotation of coil in magnetic field, magnetic flux linked with coil changes with time hence E.M.F. is induced, which opposes the current in the coil. This e.m.f. is known as back emf. If coil of N turns, each of area A rotates with constant angular velocity. Then peak value of back e.m.f. is given by:

$$e_0 = NBA\omega \qquad \text{i.e., } e_0 \propto \omega$$

**Concept Reminder**

for multiple generation frequency of AC is equal to –

$$f = \frac{np}{2}$$

n $\rightarrow$ rotational frequency of armature
P $\rightarrow$ no. of poles.

**Definitions**

**DC MOTOR:**
It is a device, that converts electrical energy into mechanical energy (rotational energy).

**Concept Reminder**

Working principle of DC motor is that when a current carrying coil is placed in magnetic field it experiences a torque.

- Equation of motor (D.C.):

  $E - e = iR$

  At t = 0, when motor is switched ON, $\omega = 0$

  $\Rightarrow$ e = 0 and $i = \frac{E}{R} = \text{Maximum}$

  as $t \uparrow, \omega \uparrow \Rightarrow e \uparrow$ but $i \downarrow$

  and when motor rotate with maximum angular speed, current is minimum.

  $$i_{min} = \frac{E - e_{maximum}}{R}$$

- Efficiency of motor:

  $$\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{ei}{Ei} \times 100$$

  $$\boxed{\eta = \frac{e}{E} \times 100}$$

**Note:** When $e = \frac{E}{2}$, then efficiency of motor is maximum

$$\eta_{maximum} = 50\%$$

**Transformer**

**Principle: Mutual Induction**

**1.** Rate of change of flux in one turn of primary coil is equal to rate of flux change in one turn of secondary coil.

$$\left(\frac{d\phi}{dt}\right)_p = \left(\frac{d\phi}{dt}\right)_s$$

Coupling factor, K = 1

**2.** For an ideal transformer $P_{in} = P_{out}$

$\eta\% = 100\%$

**3.** Frequency of AC in primary coil is equal to freq. of AC in secondary coil.

**Definitions**

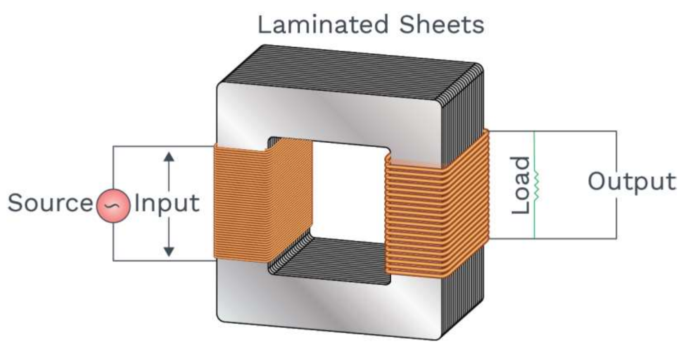
Due to rotation of coil in magnetic field, magnetic flux linked with coil changes with time hence E.M.F. is induced, which opposes the current in the coil. This e.m.f. is known as back emf.

**Concept Reminder**

- Efficiency of motor:

  $$\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{ei}{Ei} \times 100$$

  $$\boxed{\eta = \frac{e}{E} \times 100}$$




$$E_1 = N_1 \frac{d\phi}{dt}, \qquad E_2 = N_2 \frac{d\phi}{dt}$$

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}, \qquad E \propto N$$

$$N_2 > N_1 \text{ step up}, \qquad N_1 > N_2 \text{ step down}$$

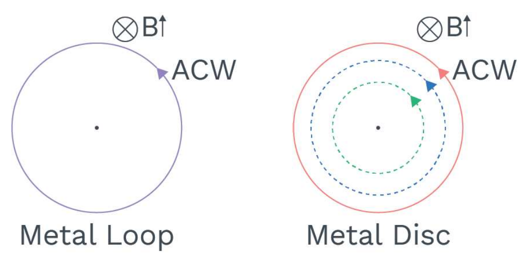
**Concept Reminder**

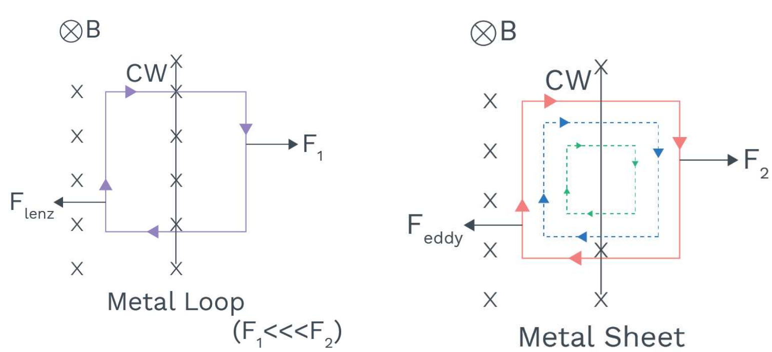
Transformers are based upon mutual induction which transform an alternating voltage from one to another of greater or smaller value.

**Losses in Transformer: -**

1. **Joule Heating or copper losses: -** ($I^2Rt$)
2. **Flux Leakage: -** Flux leakage can be reduced by soft iron core because magnetic permeability of soft iron is very high.
3. **Hysteresis Loss: -** It can be reduced by soft iron core because soft iron is a soft ferromagnetic material. So, hysteresis losses are very low.
4. **Eddy current loss: -**

   **Eddy currents (or Foucault's currents) [Experimental verification by Foucault]**



## Examples

Q1

1. **The north pole of a magnet is moved down along the axis of a horizontal circular coil (figure). Because of this the flux through the coil changes from 0.4 Wb to 0.9 Wb in an interval of half of a second. What is the average emf induced during this period? Also tell if induced current is clockwise or anti-clockwise as you look into the coil from the side of the magnet?**

**Sol:**

$$\varepsilon = \frac{\phi_1 - \phi_2}{t} = \frac{0.4 - 0.9}{1/2} = -\frac{0.50}{1/2}$$

$$\varepsilon = -1 \text{ volt} \Rightarrow |\varepsilon| = 1 \text{ volt.} \quad \text{(Anticlockwise)}$$

Q2

**The magnetic flux through a closed conducting loop of resistance 0.4 Ωchanges with time according to the equation $\Phi = 0.20\ t^2 + 0.40\ t + 0.60$ where t is time (in seconds). what is (i) the induced emf at t = 2s. (ii) the average amount of induced emf in t = 0 to t = 5 s. (iii) charge passed through this loop in t = 0 to t = 5s (iv) the average current in interval of time t = 0 to t = 5 s (v) the heat produced in t = 0 to t = 5s.**

**Sol:** $\varepsilon = -\frac{d\phi}{dt} = -(0.4t + 0.4)$

(i) $\varepsilon_{t=2} = -1.2 \text{ volt}$

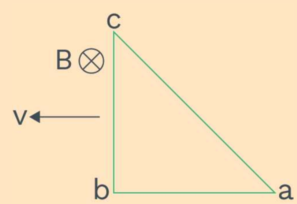
(ii)

(iii) $\Delta q = \frac{\Delta \phi}{R} = 17.5 \text{ C.}$

(iv) $< i > = \frac{\Delta q}{\Delta t} = 3.5$ Anticlockwise

(v) $H = \int \frac{\varepsilon^2}{R}.dt = \int_0^5 \frac{[0.4t + 0.4]^2}{R}.dt = \frac{86}{3} J$

**Q3** **A right-angled triangle abc which is made up of a metallic wire is moving with a uniform speed of v in its plane as shown in the given figure.**




**A uniform magnetic field B exists in the perpendicular direction of plane of this triangle. Find an induced emf**

**(a) in loop abc** **(b) in segment bc**

**(c) in segment ac and** **(d) in segment ab.**

**Sol:** (a) $\varepsilon_{abc} = 0$ [$\because$ effective length is zero]

(b) $\varepsilon_{bc} = BvL_{bc}$

(c) $\varepsilon_{ac} = BvL_{bc}$

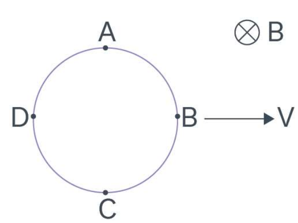
(d) $\varepsilon_{ab} = 0.$

**Q4** **A circular conducting-ring having radius r moves in its plane with a constant velocity v. In this region a uniform magnetic field B exists in direction normal to the plane of the ring. Take different pairs of diametrically opposite points on this ring.**

**(a) Between which pair of points is emf maximum?**

**(b) Between which pair of points is emf minimum? Find the value of this minimum emf?**

**Sol:**




$\varepsilon_{max} = \varepsilon_{AC} = Bv\,(2r)$

$\varepsilon_{min} = \varepsilon_{BD} = 0$

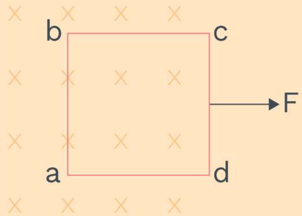
**Q5** **A square frame made up of wire abcd of side 1 m has a total resistance of 4 ohm. It is pulled out from a magnetic field B = 1 T by using a force of 1 N (as shown in figure). It is found that this frame moves with constant speed. What is**

**(a) this constant speed?**

**(b) an emf induced in the loop?**

**(c) the potential difference between points a and b and**

**(d) the potential difference between points c and d?**




**Sol:** (a) $F = ILB = \frac{\varepsilon}{R}(LB) = \frac{B^2L^2v}{R} = 1 = v/4$

$v = 4$ m/s

(b) $\varepsilon = BvL = 1 \times 4 \times 1 = 4$ Volt

(c) $V_{ab} = \varepsilon - IR_{ab} = 4 - (4/4)(1) = 3$Volt

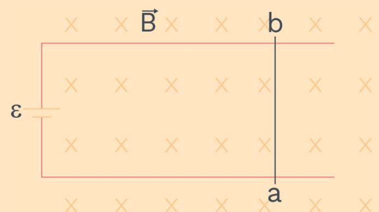
(d) $V_{cd} = IR_{cd} = (4/4)(1) = 1$ Volt.

**Q6** **Given figure shows a smooth pair of thick metallic rails connected across a battery of emf $\varepsilon$ of a negligible internal resistance. A wire ab having length $\ell$ and resistance r can slide smoothly on these rails. The entire system is present in a horizontal plane and in uniform vertical magnetic field B. At some instant of time t, the wire is given a small velocity v towards right.**

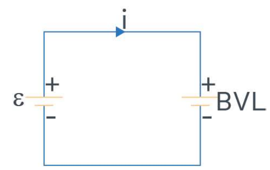
**(a) What is the current in wire at this instant.**

**(b) What is the force acting on the wire at this instant.**

**(c) Prove that after some time the wire ab will slide with a constant velocity. Find this velocity.**



**Sol:**




(a) $i = \frac{\varepsilon - BvL}{r}$ Clockwise

(b) $F = iLB = \left(\frac{\varepsilon - BvL}{r}\right)LB$

(c) F is directed right so v will keep on increasing after sometime current will not flow in circuit then it will move with constant velocity. v will be maximum (or constant) when F = 0.

So $\varepsilon = BvL$, $v = \frac{\varepsilon}{BL}$

**Q7** **Consider the situation of the previous problem.**
**(a) Determine the force required to move the sliding wire with a constant velocity v.**
**(b) If the force required just after t = 0 is $F_0$, find the time at which force needed will be $F_0/2$.**

**Sol:** (a) $F = iLB = \frac{BvL}{2r(L+vt)} \times LB = \frac{B^2L^2v}{2r(L+vt)}$

(b) at t = 0, $F_0 = \frac{B^2L^2v}{2rL}$

$$\frac{1}{2}\left(\frac{B^2L^2v}{2rL}\right) = \frac{B^2L^2v}{2r(L+vt)}$$

$$t = \frac{L}{v}.$$

**Q8** **A thin wire of negligible mass and a small spherical bob constitute a simple pendulum having effective length $\ell$. When this pendulum is made to swing through a semi-vertical angle $\theta$ with vertical, under gravity in a plane perpendicular to uniform magnetic field of induction B, find the maximum potential difference between the ends of the wire.**

**Sol:** Here $mg\ell\left(1-\cos\theta\right)=\frac{1}{2}mv^2$

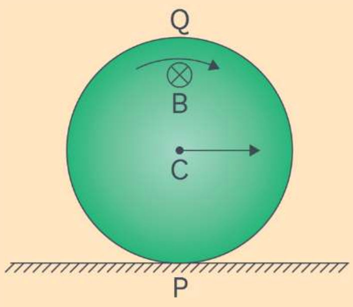
$$v^2=2g\ell\left(1-\cos\theta\right)$$

$$\omega_{max}=\frac{v}{\ell}=\sqrt{\frac{2g\left(1-\cos\theta\right)}{\ell}}=\sqrt{\frac{2g\times2\sin^2(\theta/2)}{\ell}}=2\sin(\theta/2)\sqrt{\frac{g}{\ell}}$$

$$\varepsilon_{max}=\frac{1}{2}B\ell^2\times2\sin(\theta/2)\sqrt{\frac{g}{\ell}}$$

$$\varepsilon_{max}=B\ell\sqrt{g\ell}\sin\frac{\theta}{2}.$$

**Q9** **A conducting disc of radius R is rolling without slipping on a horizontal surface with a constant velocity v. The uniform magnetic field of magnitude B is applied perpendicular to the plane of the disc. Calculate the EMF induced between (at this moment)**




**(a) P and Q** **(b) P and C** **(c) Q and C**

**(C is centre, P and Q are opposite points on vertical diameter of the disc)**

**Sol:** (a) $\varepsilon_{PQ} = \frac{1}{2}B\omega(2R)^2$

$= \frac{1}{2}B\left(\frac{v}{R}\right)(2R)^2 = 2BvR.$

(b) $\omega_{PC} = \frac{1}{2}B\omega R^2 = \frac{1}{2}B\left(\frac{v}{R}\right)R^2 = \frac{BvR}{2}.$

(c) $\varepsilon_{QC} = 2BvR - \frac{BvR}{2} = \frac{3}{2}vBR.$

**Q10** **A closed coil of 50 turns is rotating in a uniform magnetic field B = 2 × $10^{-4}$ T about a diameter which is normal to the field. The angular velocity of its rotation is 300 revolutions per minute. The area of the coil is 100 $cm^2$ and its resistance is 4 ohm. Calculate**

**(a) The average emf produced in half a turn from a position when the coil is perpendicular to the magnetic field.**

**(b) The average emf in a full turn.**

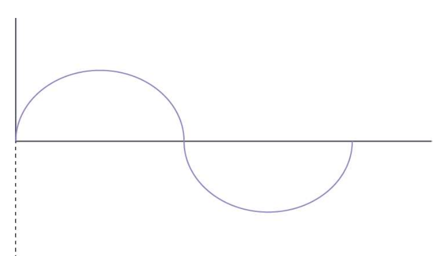
**(c) The net charge flown in part (a).**

**(d) An emf induced as a function of time 't' given that it is zero at t = 0 and is increasing in +ve direction.**

**(e) The maximum amount of induced emf.**

**(f) The average of squares of emf induced over a long period.**

**Sol:** For half cycle,

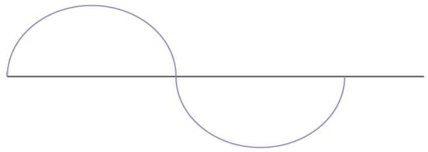


(a) $< \varepsilon > = \frac{2\varepsilon_0}{\pi}$

$< \varepsilon > = \frac{2NBA\omega}{\pi} = \frac{2 \times 50 \times 2 \times 10^{-4} \times 100 \times 10^{-4} \times 2\pi \times 300}{60 \times \pi}$

$< \varepsilon > = 2 \times 10^{-3}$ volt.

(b) For complete cycle



$<\varepsilon> = 0$

(c) $\Delta q = \frac{\Delta\phi}{R} = \frac{2NBA}{R} = \frac{2 \times 50 \times 2 \times 10^{-4} \times 100 \times 10^{-4}}{4} = 5 \times 10^{-5} C$

(d) $\varepsilon = NBA\omega \sin\omega t$.

(e) $\varepsilon_{max} = NBA\omega = \pi \times 10^{-3} V$

(f) $< \varepsilon^2 >= \frac{N^2B^2A^2\omega^2}{2} = \frac{\pi^2}{2} \times 10^{-6} V.$

**Q11** **The current flowing in an ideal, long solenoid is changed at a uniform rate of 0.01 A/s. This solenoid has 2000 turns/m and radius is 6.0 cm.**

**(a) Consider a circle having radius 1.0 cm inside this solenoid with its axis coinciding with axis of solenoid. Calculate the change in the magnetic flux through this circle in 2.0 seconds.**

**(b) What the electric field induced at a point which lies on the circumference of the circle?**

**(c) Determine the electric field induced at a point outside the solenoid at a distance 8.0 cm from its axis.**

**Sol:** (a) $B = \mu_0 nI$

$$\varepsilon = \frac{d\phi}{dt} = A\frac{dB}{dt} = \pi\left(1 \times 10^{-2}\right)^2 \times \mu_0 \times \frac{2000}{1} \times \frac{dI}{dt}$$

$$\varepsilon = \pi\mu_0 \times 10^{-4} \times 2000 \times 0.01$$

$$\Delta\phi = 2 \times \frac{d\phi}{dt} = 4\pi \times 10^{-3} \times \mu_0$$

$= 16\pi^2 \times 10^{-10}$ Weber.

(b) $E = \frac{\varepsilon}{2\pi r} = \frac{2\pi \times 10^{-3} \times \mu_0}{2\pi \times 1 \times 10^{-2}}$

$= 0.1\mu_0 = 4\pi \times 10^{-8}$ V/m.

(c) $E' \times 2\pi r' = A' \frac{dB}{dt}$

$$E' \times 2\pi \times 8 \times 10^{-2} = \pi \times \left(6 \times 10^{-2}\right)^2 \frac{dB}{dt}$$

$$E' = \frac{36}{8}.E$$

$$\Rightarrow \frac{18}{4}E = \frac{18}{4} \times 4\pi \times 10^{-8}$$

$\Rightarrow$ E = 18π × $10^{-8}$ V/m.

**Q12** **Calculate the energy stored in the magnetic field inside a volume of 1.00 mm³ at a distance of 10.0 cm from a long wire which is carrying a current of 4 A.**

**Sol:** $B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 4}{2\pi \times 10 \times 10^{-2}}$

$$\text{Energy} = \frac{1}{2} \times \frac{B^2}{\mu_0} \times \text{volume}$$

$$= \frac{B^2}{2\mu_0} \times \left[1 \times 10^{-3}\right]^3.$$

**Q13** **The given network shown in figure is a part of a circuit. Find the potential difference ($V_B - V_A$), if the current I is 5 ampere and also it is decreasing at a rate of $10^3$ (A/s)?**


**Sol:** $\frac{di}{dt} = 10^3$ A/s

$\therefore$ Induced emf across inductance $|e| = L\frac{di}{dt}$

$$|e| = (5\times10^{-3})(10)^3 \text{ V} = 5\text{V}$$

Since, the value of current is decreasing, the polarity of this emf would be so as to increase the existing current. The circuit can be redrawn as


Now $V_A - 5 + 15 + 5 = V_B$

$\therefore$ $V_A - V_B = -15$ V

or $V_B - V_A = 15$ V

**Q14 A solenoid has an inductance of 10 Henry and a resistance of 2 Ω. It is connected to a 10-volt battery. Find the time taken for the magnetic energy to reach 1/4th of its maximum value?**

**Sol:** $U = \frac{1}{2}Li^2$ i.e. $U \propto i^2$

U will reach $\Rightarrow$ of its maximum value when current is reached half of its maximum value. In L-R circuit, equation of current growth is written as

$$i = i_0\left(1 - e^{-t/\tau}\right)$$

Here $i_0$ = Maximum value of current

$\tau$ = Time constant = L/R

$$\tau = \frac{10\,\text{henry}}{2\,\text{ohm}} = 5\,s^{-1}$$

Therefore $i = i_0/2 = i_0\left(1 - e^{-t/5}\right)$

or $\frac{1}{2} = 1 - e^{-t/5}$ or $e^{-t/5} = \frac{1}{2}$

or $-t/5 = \ln\left(\frac{1}{2}\right)$

or t/5 = ln (2) = 0.693

t = (5) (0.693) s

or t = 3.465 s.

**Q15** **A coil of resistance 4 ohm is connected with a 0.4 V battery. The current flowing through the coil is 63 mA. After 1 sec. the battery is connected. Find the inductance of the coil.** $\left[e^{-1} \simeq 0.37\right]$

**Sol:**

$$I = I_0\left(1 - e^{-t/\tau}\right)$$

$$e^{-t/\tau} = 1 - \frac{I}{I_0} = 0.37 = e^{-1}$$

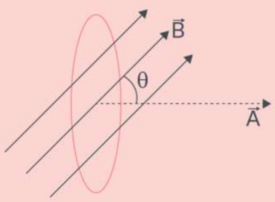
$$\frac{tR}{L} = 1 \qquad \Rightarrow L = tR$$

$$L = 4H.$$

## Mind Map

### Magnetic Flux

$\phi = \vec{B}.\vec{A}$

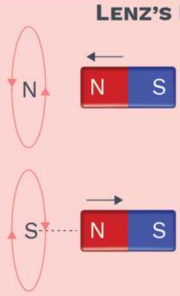



### Faraday's Law of EMI

(i) Induced emf $e = -N\frac{d\phi}{dt}$

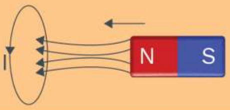
(ii) Induced current $i = -\frac{Nd\phi}{Rdt}$

### Lenz's Law

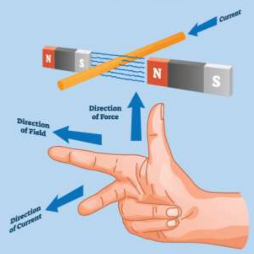



It is based on conservation of energy

### Electromagnetic Induction




### Fleming's Right Hand Rule




### Energy Stored in Inductor

$U = \frac{1}{2}Li^2$

### (i) Self Induction

$\phi = LI$

$e = -L\frac{dI}{dt}$

### (ii) Mutual Induction

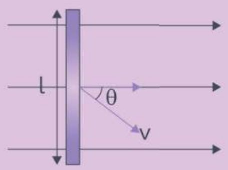
$\phi_2 = LI_1$

$e_2 = -M\frac{dI_1}{dt}$

### Motional EMF

**Translatory motion**

$e = Blv\sin\theta$




**Rotational motion**

$e = \frac{Bl^2\omega}{2}$