Circular Motion

Circular Motion

- If a particle moves in a plane such that its distance from a fix-point remains fixed then its motion is known as circular motion with respect to that point
- This point is known as centre and the distance of particle from the point is known as radius.

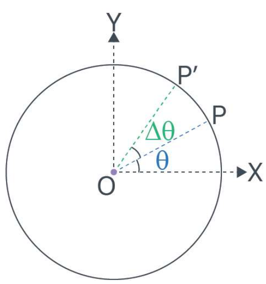
Key Points

- Circular Motion
- Angular Position
- Angular Displacement

Parameter of Circular Motion

(a) Angular Position

The angular position of a point object P at any instant can be marked by an angle θ between OP and OX. This angle θ is known as the angular position of particle.

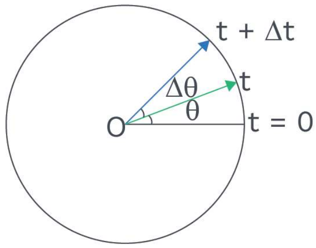


(b) Radius Vector

The line joining the centre of rotation to position of particle in circular path is called radius vector.

(c) Angular Displacement

In a time interval it is an angle traced by radius vector of particle about the centre of circle.

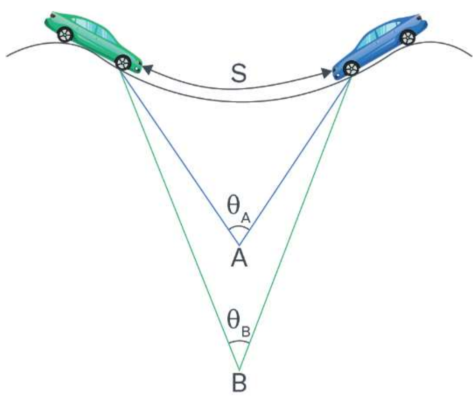


In time interval Δt,

angular displacement is $\Delta\theta$.

Concept Reminder

Angular displacement is an axial vector only when its value is small. If its value becomes large, then it is not a vector.



Concept Reminder

In Rotation, angular displacement of all points of a rigid body is same. In case of a non-rigid body, greater the distance of the point from axis of rotation greater will be its angular displacement.

Important Notes:-

- Angular displacement has no dimension but has SI unit radian. Some other units are degree and revolution.

  $2\pi \text{rad} = 360° = 1\text{rev}$

- Finite angular displacement is a scalar quantity but infinitesimally small angular displacement is a vector quantity. This is because the addition of the infinitesimally small angular displacements is commutative but addition of finite angular displacement is not commutative.

  $d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1$ but $\theta_1 + \theta_2 \neq \theta_2 + \theta_1$

- Direction of small angular displacement is given by right hand thumb rule. According to which when the fingers are directed along the motion of the point, then thumb gives the direction of angular displacement.

(d) Angular Velocity ω

(i) Average Angular Velocity

$$\omega_{av} = \frac{\text{Angular Displacement}}{\text{Total time taken}}$$

$$\Rightarrow \omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

where θ_1 and θ_2 are angular position of the particle at time t_1 and t_2.

Since angular displacement is a scaler, average angular velocity is also a scaler.

(ii) Instantaneous Angular Velocity

It is defined as the limit of average angular velocity as Δt approaches zero. i.e.

$$\vec{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\theta}}{\Delta t} = \frac{d\vec{\theta}}{dt}$$

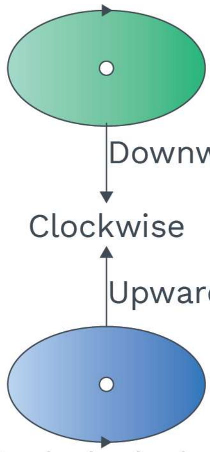
Since infinitesimally small angular displacement $\overrightarrow{d\theta}$ is known as a vector quantity therefore, an instantaneous angular velocity $\vec{\omega}$ is also a vector quantity and its direction is given by right hand thumb rule.

Important Points:

- Angular velocity has dimension of $[T^{-1}]$ and SI unit rad/s.
- For a rigid body, as all points will rotate through same angle in same time, angular velocity is a characteristic of the body as a whole, e.g., angular velocity of all points of earth about earth's axis is $(2\pi / 24)$ rad/hr.
- If T is time period of rotation of an object, then

$$\omega = \frac{2\pi}{T}$$

Right hand Rule: Imagine the axis of rotation to be held in the right hand with fingers curled round the axis and the thumb stretched along the axis. If the curled fingers denote the sense of rotation, then the thumb denoted the direction of the angular velocity (or angular acceleration of infinitesimal angular displacement.



Angular Velocity of hands of a clock

- Angular velocity of seconds hand

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = \frac{\pi}{30} \text{rad s}^{-1}$$

Concept Reminder

- If a body makes n rotations in t second then its angular velocity is $\omega = \left(\frac{2\pi n}{t}\right) \text{rad / s}$

Rack your Brain

Two particles A and B are moving in uniform circular motion in concentric circles of radii r_A and r_B with speed v_A and v_B respectively. Their time period of rotation is the same. Find the ratio of angular speed of A to that of B.

- Angular velocity of minutes hand

$$\omega = \frac{2\pi}{60 \times 60} = \frac{\pi}{1800} \text{rad s}^{-1}$$

- Angular velocity of hours hand

$$\omega = \frac{2\pi}{12 \times 3600} = \frac{\pi}{21600} \text{rad s}^{-1}$$

- In case of self rotation of earth about its own axis

$$\omega = \frac{2\pi}{24 \times 60 \times 60} \text{rad / sec}$$

Ex. A particle is moving in a circle of radius r centred at O with constant speed v. What is the change in velocity in moving from A to B? Given $\angle AOB = 40°$.

Sol. $|\Delta \vec{v}| = 2v \sin(40° / 2) = 2v \sin 20°$

Angular acceleration (α)

The rate of change of angular velocity with time is called angular acceleration.

(i) Average Angular Acceleration:

Let ω_1 and ω_2 be the instantaneous angular speeds at times t_1 and t_2 respectively, then the average angular acceleration α_{av} is defined as

$$\vec{\alpha} = \frac{\vec{\omega}_2 - \vec{\omega}_1}{t_2 - t_1} = \frac{\Delta\vec{\omega}}{\Delta t}$$

(ii) Instantaneous Angular Acceleration:

It is defined as the average angular acceleration limit as Δt approaches zero, i.e.,

$$\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt}$$

since $\vec{\omega} = \frac{\Delta\vec{\theta}}{dt}$

$$\therefore \quad \vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{d^2\theta}{dt^2}$$

Also $\vec{\alpha} = \omega \frac{d\vec{\omega}}{d\theta}$

- **Note :** $\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\theta}}{dt^2} = \omega \frac{d\vec{\omega}}{d\theta}$
- SI units is rad.sec^{-2}.

Key Points

- Angular velocity
- Angular acceleration

- Dimensional formula is $\left[T^{-2}\right]$
- Its direction is in the direction of change in angular velocity and it is given by right hand screw rule. (When angular velocity increases the direction of angular acceleration is in the direction of angular velocity and when angular velocity decreases the angular acceleration is in the opposite direction of angular velocity)
- If a particle rotates with uniform constant angular velocity, then $\alpha = 0$

Ex. When a motor cyclist takes a U-turn in 4s what is the average angular velocity of the motor cyclist.

Sol. When the motor cyclist takes a U-turn, angular displacement, $\theta = \pi$ rad and t = 4 s.

The average angular velocity,

$$\omega = \frac{\theta}{t} = \frac{\pi}{4} = 0.7855\,\text{rad}\,s^{-1}$$

Note : In general we can also use the following equations to solve the problems

$$\omega_{av} = \frac{\Delta\theta}{\Delta t}, \alpha_{av} = \frac{\Delta\omega}{\Delta t}$$

$$\omega_{av} = \frac{d\theta}{dt}, \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \omega\frac{d\omega}{d\theta}$$

$$\boxed{\int d\theta = \int \omega dt, \int d\omega = \int \alpha dt, \int \alpha d\theta = \int \omega d\omega}$$

Concept Reminder

- In case of circular motion, $\omega \perp r$ and $\alpha \perp r$.

$\therefore$ $v = \omega r$ and $a_t = \alpha r$

Relation between linear and angular variables

- Relation between linear and angular displacement is $ds = r d\theta$
- Relation between linear and angular velocities is $v = r\omega$, $\vec{v} = \vec{\omega} \times \vec{r}$ Relation between tangential and angular acceleration is $\vec{a_t} = \vec{\alpha} \times \vec{r}$
- Linear acceleration of a point object moving in a circle. We know $\vec{v} = \vec{\omega} \times \vec{r}$ different w.r.t. time, we get

$$\frac{d\vec{v}}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$$

$$\vec{a} = (\vec{\alpha} \times \vec{r}) + (\vec{\omega} \times \vec{v})$$

but $\vec{\alpha} \times \vec{r} = \vec{a_t}$, it is tangential acceleration.

$\vec{\omega} \times \vec{v} = \vec{a_c}$, it is centripetal acceleration .

- Due to change in direction of velocity there is an acceleration and is always directed towards the centre. This is called centripetal or radial acceleration and the corresponding force acting towards the centre is called centripetal force

Ex. Find out the linear velocity of a person at equator of the earth due to its spinning motion?
(Radius of the earth = 6400km).

Sol. Time period of rotation of earth is 24 hour. Its angular velocity is,

$$\omega = \frac{2\pi N}{t} = \frac{2\pi \times 1}{24 \times 60 \times 60} = \frac{\pi}{43,200} \text{rad s}^{-1}$$

The linear velocity,

$$v = R\omega = 6.4 \times 10^6 \times \frac{\pi}{43,200} = 465.2 \text{ m/s}$$

Motion with constant angular acceleration

A circular motion having constant angular acceleration is analogous to 1-D translational motion having constant acceleration. Therefore, here equation of motion have same form.

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\theta = \left(\frac{\omega + \omega_0}{2}\right)t$$

$$\theta_{n^{th}} = \omega_0 + \frac{\alpha}{2}(\theta_n - \theta_{n-1})$$

Where,

$\omega_0 \Rightarrow$ Initial angular velocity

$\omega \Rightarrow$ Final angular velocity

$\alpha \Rightarrow$ Constant angular acceleration

$\theta \Rightarrow$ Angular displacement

Rack your Brain

An electric fan has blades of length 30 cm measured form axis of rotation. If the fan is rotating at 120 rpm, then find the acceleration of a point on the tip of the blade.

Ex. A fan is rotating with angular velocity 100 rev/sec. When it is switched off then it takes 5 minutes to stop.

(a) How many total number of revolution was made by it before it stops. (Assume uniform angular retardation)

(b) Find the value of angular retardation of the fan.

(c) What is the average angular velocity during this interval.

Sol. (a) $\theta = \left(\frac{\omega + \omega_0}{2}\right)t = \left(\frac{100+0}{2}\right) \times 5 \times 60$

$= 15000 \text{ revolution}$

(b) $\omega = \omega_0 + \alpha t \Rightarrow 0 = 100 - \alpha(5 \times 60)$

$\Rightarrow \alpha = \frac{1}{3} \text{rev / sec}^2$

(c) $\omega_{av} = \frac{\text{Total Angle of Rotation}}{\text{Total time taken}} = \frac{1500}{50 \times 60} = 50 \text{rev. / sec}$

Ex. A fan rotating with $\omega = 100 \text{ rad / s}$, is switched off. After 2n rotation its angular velocity reduces to 50 rad/s. Find out the angular velocity of the fan after n rotations.

Sol. $\omega^2 = \omega_0^2 + 2\alpha\theta \Rightarrow 50^2 = (100)^2 + 2\alpha(2\pi \times 2n)$ ...(1)

If angular velocity after n rotation is ω_n

$\omega_n^2 = (100)^2 + 2\alpha(2\pi.n)$ ... (2)

from equation (1) and (2)

$$\frac{50^2 - 100^2}{\omega_n^2 - 100^2} = \frac{2\alpha(2\pi \times 2n)}{2\alpha \times 2\pi n} = 2$$

$$\Rightarrow \omega_n^2 = \frac{50^2 - 100^2}{2} + 100^2$$

$\omega = 25\sqrt{10} \text{ rad / s}$

Ex. A solid body rotates about a stationary axis with an angular retardation $\alpha = k\sqrt{\omega}$ where ω is the angular velocity of body. Find time after which body will come to rest if at t = 0 angular velocity of body was ω_0.

Sol. $\alpha = -k\sqrt{\omega}$

$$\Rightarrow \frac{d\omega}{dt} = -k\sqrt{\omega}$$

$$\Rightarrow -\int_{\omega_0}^{0} \frac{d\omega}{\sqrt{\omega}} = \int_0^1 k\,dt$$

$$\Rightarrow t = \frac{2\sqrt{\omega_0}}{k}$$

Relative Angular Velocity

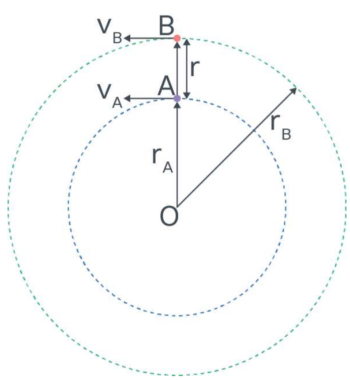
The angular velocity of a particle 'A' with respect to the other moving particle 'B' is defined as the rate at which position vector of 'A' with respect to 'B' rotates that instant. (or we can say, angular velocity of A with origin fixed at particle B). The angular velocity of A with respect to B i.e., ω_{AB} is mathematically define as

$$\omega_{AB} = \frac{\text{Component of relative velocity of A w.r.t. B, perpendicular to line AB}}{\text{seperation between A and B}}$$

$$= \frac{(v_{AB})_{\perp}}{r_{AB}}$$

Important points:

- If any two particles are moving on two different but concentric circles having different velocities then we can say angular velocity of B as watched by A will depend on their positions as well as their velocities. Take a case when A and B are closest to each other moving in same direction as shown in figure. In this situation

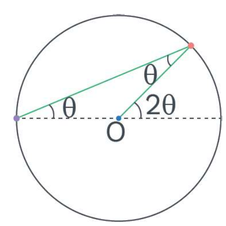


$v_{BA} = v_B - v_A$

Separation between A and B is $r_{BA} = r_B - r_A$

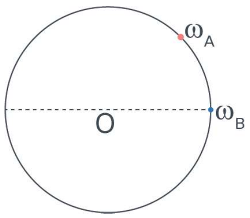
So, $\omega_{BA} = \frac{(v_{BA})}{r_{BA}} = \frac{v_B - v_A}{r_B - r_A}$

- The angular velocity of particle (P) about the point 'O' is ω_0, Let the angular velocity of particle about A is ω_A then $\omega_0 = 2\omega_A$.



Concept Reminder

If $\omega_A = \omega_B$ then $\omega_{rel} = 0$ and so $T = \infty$ i.e., particle will maintain same position relative to each other. This is what actually happens in case of geostationary satellite.

- 

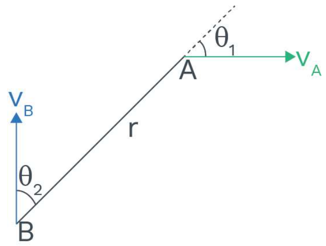
In the above figure. If two particles A and B are moving in same circular path in the same direction, for a person at the centre of the circle.

$\omega_{BA} = \omega_B - \omega_A$

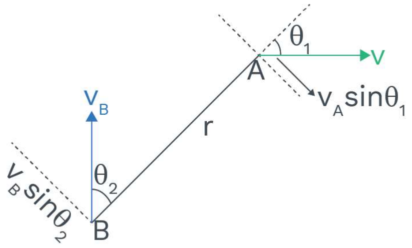
Time taken by one particle to complete one rotation with respect to another particle is

$$T = \frac{2\pi}{\omega_{rel}} = \frac{2\pi}{\omega_B - \omega_A} = \frac{T_A T_B}{T_A - T_B}$$

Ex. Find the angular velocity of A w.r.t. B in the figure given below:

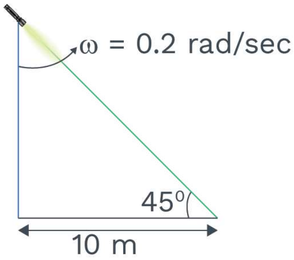


Sol.

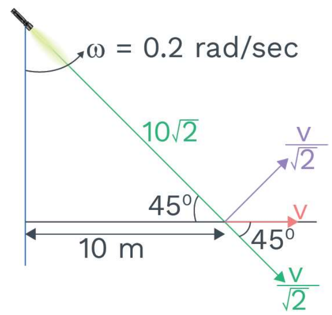


$$\omega = \frac{v_A \sin\theta_1 + v_B \sin\theta_2}{r}$$

Ex. Find speed of point at 10 m away from origin in given figure.



Sol.



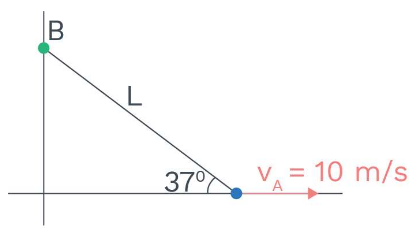
$$\omega = \frac{v_{\perp}}{d}$$

$$\Rightarrow \quad 0.2 = \frac{v}{\sqrt{2}} \times \frac{1}{10\sqrt{2}}$$

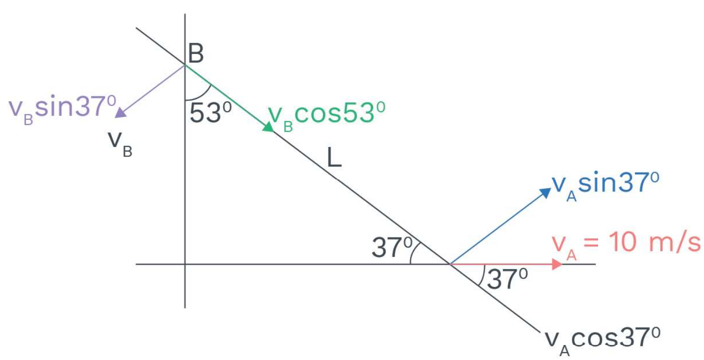
$$\Rightarrow \quad v = 0.2 \times 10 \times 2$$

$$\Rightarrow \quad v = 4\,m/s$$

Ex. Find ω_{AB} is given figure.



Sol.



$$\omega_{AB} = \frac{v_{AB\perp}}{\text{length of line joining}}$$

$$v_A \cos 37° = v_B \cos 53°$$

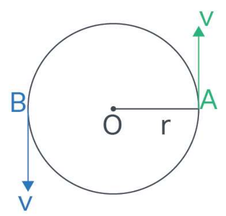
$$v_A \times \frac{4}{5} = v_B \times \frac{3}{5} \Rightarrow 4v_A = 3v_B$$

$$v_B = \frac{4 \times 10}{3} = \frac{40}{3} m/s$$

$$\omega_{AB} = \frac{\vec{v}_{A\perp} - \vec{v}_{B\perp}}{L} = \frac{10 \sin 37° + \frac{40}{3} \sin 53°}{L}$$

$$\omega_{AB} = \frac{10 \sin 37° + \frac{40}{3} \sin 53°}{L} = \frac{50}{3L}$$

Ex. Particles A and B are moving with uniform and same speeds in a circle as shown in given figure, find out the angular velocity of the particle A w.r.t. B, if angular velocity of particle A with respect to O is ω.



Sol. Angular velocity of A with respect to O is

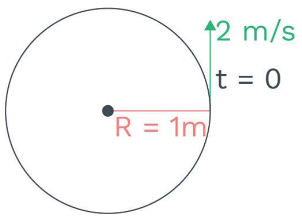
$$\omega_{AC} = \frac{(v_{AO})_\perp}{r_{AO}} = \frac{v}{r} = \omega \Rightarrow \text{Now, } \omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}}$$

$v_{AB} = 2v$, since v_{AB} is perpendicular to r_{AB}.

$$\therefore \quad (v_{AB})_\perp = v_{AB} = 2v \;;\; r_{AB} = 2r$$

$$\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} = \frac{2v}{2r} = \omega$$

Ex. If $\alpha = +2$ rad/sec^2 then find



Rack your Brain

Two stones of masses m and 2m are whirled in horizontal circles, the heavier one in a radius r/2 and the lighter one in radius r. The tangential speed of lighter stone is n times that of value of heavier stone when they experience same centripetal force. Find value of n.

(i) ω at t = 2 sec 2

(ii) θ at t = 2 sec.

Sol. $v = R\omega \Rightarrow \omega = \frac{v}{R} = \frac{2}{1} = 2 \text{ rad / sec}$

$$\omega = \omega_0 + \alpha t$$

$$\Rightarrow \omega = 2 + (2 \times 2)$$

$$\Rightarrow \omega = 6 \text{ rad / sec}$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\theta = 8 \text{ rad}$$

If α is variable

$$\boxed{\alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{\omega . d\omega}{d\theta}}$$

Acceleration in Circular Motion

Two types of acceleration in circular motion are : Tangential acceleration and centripetal acceleration.

- **Tangential acceleration**

  The component of acceleration whose direction along tangent of circle is called tangential acceleration. It is responsible for changing the speed of the particle. It is defined as,

  $$a_t = \frac{dv}{dt} = \frac{d|\vec{v}|}{dt} = \text{Rate of change of speed}$$

  Also $a_t = \alpha r$

- **Centripetal acceleration**

  This acceleration is responsible for change in direction of velocity. Presence of centripetal acceleration in circular motion is necessary.
  Centripetal acceleration is always type of variable acceleration because its direction is variable.
  It is also known as radial acceleration or normal acceleration.

Key Points

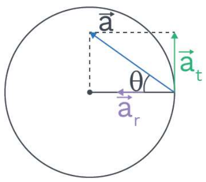
- Tangential acceleration
- Centripetal acceleration

$$\vec{a}_c = \vec{\omega} \times \vec{v}$$

$$\boxed{a_c = \omega v = \omega^2 r = \frac{v^2}{r}}$$

- **Total acceleration**

  Vector sum of tangential acceleration and centripetal acceleration is known as total acceleration.



$$\vec{a} = \frac{d\vec{v}}{dt} = \vec{a}_r + \vec{a}_t$$

$$a = \sqrt{a_t^2 + a_r^2}$$

$$\tan\theta = \frac{a_t}{a_r}$$

Rack your Brain

One end of string of length l is connected to a particle of mass m and the other end is connected to a small peg on a smooth horizontal table. If the particle moves in circle with speed v, then what will be net force acting on that particle.

Uniform circular motion

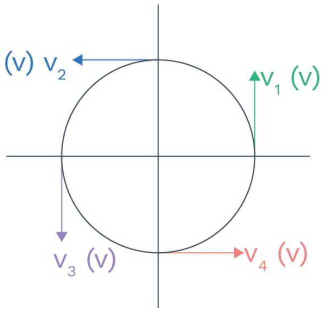
In the above case if $a_c \neq 0$, $a_t = 0$, then the particle under go uniform circular motion (or) When a particle moves in a circular path with constant speed then it is said be in uniform circular motion. in this case the acceleration of the particle

Speed = constant

$\vec{v} \rightarrow$ changes due to change in direction

$$a_c = \frac{v^2}{R} = \omega^2 R$$

Always towards the centre.



In uniform circular motion

(a) magnitude of velocity does not change

(b) direction of velocity changes

(c) velocity changes

(d) angular velocity is constant

(e) centripetal acceleration changes (only in direction)
(f) linear momentum changes
(g) angular momentum w.r.t to centre does not change

Non Uniform Circular motion

In a circular motion if $a_c \neq 0$, $a_t \neq 0$ then the particle undergo non uniform circular motion, in this case the acceleration of particle is given by

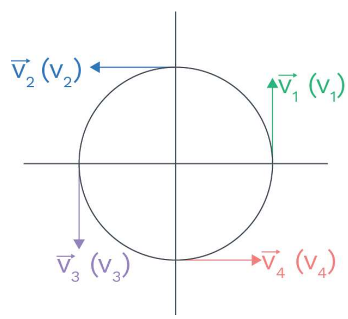
$$a = \sqrt{a_c^2 + a_t^2}$$

$$\boxed{\vec{a} = \frac{v^2}{R}\hat{r} + \left(\frac{dv}{dt}\right)\hat{t}}$$

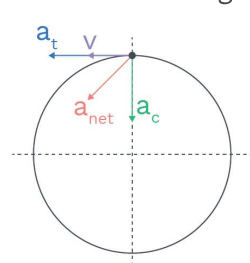
Key Points

- Uniform circular motion
- Non-uniform circular motion

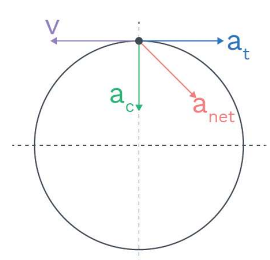
where $\hat{r}$ represents radial direction and $\hat{t}$ represents tangential direction.

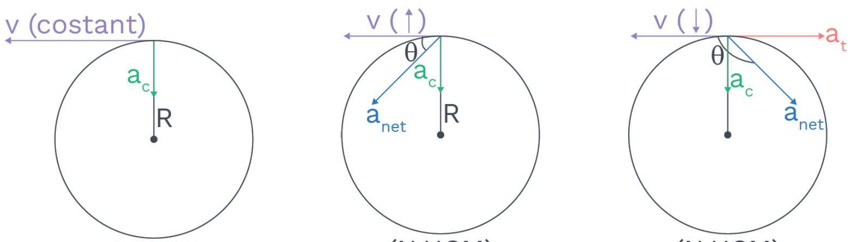


- Non-uniform circular motion with increasing speed $\theta < 90°$



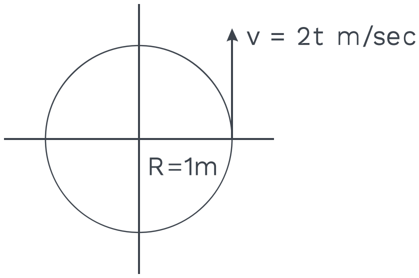
- Non-uniform circular motion with decreasing speed $\theta > 90°$



- 

(U.C.M)	(N.UCM)	(N.UCM)
$a_t = 0$	$\theta < 90^0$	$\theta > 90^0$
$a_c = a_{net}$	with (v↑)	with (v↓)
$\theta = 90^0$		

Ex. Find out



(i) at t = 2 sec.
(ii) a_c at t = 1 sec
(iii) α at t = 1 sec
(iv) a_t at t = 2 sec
(v) a_{net} at t = 1 sec

Sol. (i) $\omega = \frac{v}{R} \Rightarrow \omega = \frac{2t}{1} = 2t$ at t = 2 sec

$= 2 \times 2 = 4$ rad/sec^2

(ii) $a_c = \omega^2 R = (2t)^2.1 = 4t^2$ at t = 1 sec

$= 4 \times (1)^2 = 4$ rad/sec^2

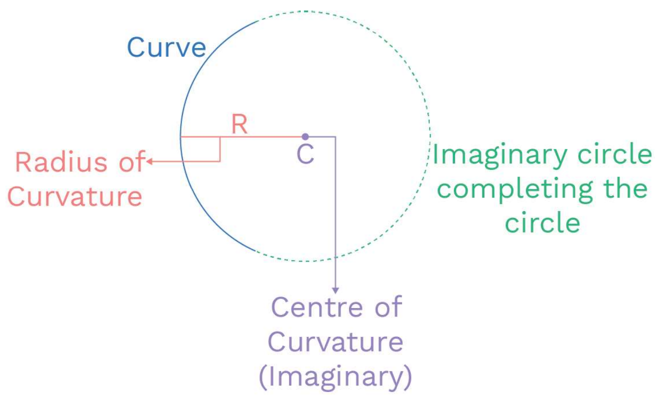
(iii) $\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(2t) = 2\,rad/s^2$

(iv) $a_t = R.\alpha = 1 \times 2 = 2m/s^2$

(v) $a_{net} = \sqrt{a_c^2 + a_t^2} = \sqrt{(4)^2 + (2)^2}$

$= \sqrt{16+4} = 2\sqrt{5}\ rad/s^2$

Radius of curvature:



Concept Reminder

In projectile motion, radius of curvature.

(i) At highest point $R_H = \dfrac{u^2 \cos^2\theta}{g}$

(ii) A point of projection

$$R_P = \frac{u^2}{g\cos\theta}$$

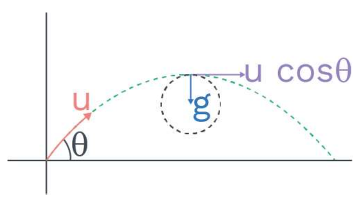
where u = initial speed

θ = Angle of projection

For a curve, it equals the radius of the circular arc which best approximates the curve at that point, for surfaces, the radius of curvature is the radius of a circle that best fits a normal section.

Ex. Find R_C at the point when it is at the highest point of its trajectory.

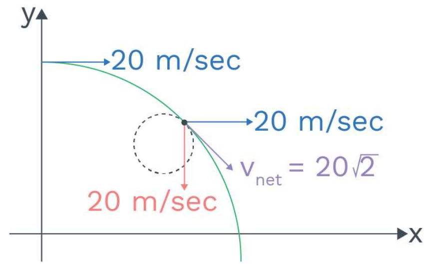
Sol.



$$R_C = \frac{(\text{speed})^2}{a_c}; R_C = \frac{u^2\cos^2\theta}{g}$$

Ex. A particle is projected horizontal with speed of 20 m/s. Find R_C at t = 2 sec.

Sol.



x	y
$u_x = 20$	$u_y = 0$
$a_x = 0$	$a_y = 10$
$v_x = 20$	$v_y = u_y + a_y t$
	$v_y = 0 + 10 \times 2 = 20\,m/s$

$$R_c = \frac{(20\sqrt{2})^2}{g/\sqrt{2}} = \frac{400 \times 2 \times \sqrt{2}}{10}$$

$$R_c = 80\sqrt{2}$$

Concept Reminder

If kinetic energy of particle along a circle of radius r depends on distance covered s as k = As, where A is constant, then force acting on it will be

$$F = 2As\sqrt{1 + (s/r)^2}$$

Dynamics of Circular Motion

When net resultant force acting on a body is zero then it will move in a straight line (with constant speed). Hence if a body is moving in a circular path or any curved path, there must be some force acting on the body. If speed of body is constant, the net force acting on the body is along the inside normal to the path of the body and it is called centripetal force.

Centripetal force (F_c).

- It is the force required to keep the body in uniform circular motion. This force changes the direction of linear velocity but not its magnitude,

$$F_C = \frac{mv^2}{r} = mr\omega^2 = mv\omega \quad (\because v = r\omega)$$

- Direction of centripetal force is always perpendicular to the direction of linear velocity. Work done by centripetal force is always zero, as it is perpendicular to velocity and instantaneous displacement .
- When speed of the body varies then, in addition to centripetal force which acts along inside perpendicular to velocity, there is also a force acting along the tangent of the path of the body which is called tangential force.

Tangential force (F_t) = $Ma_t = M\frac{dv}{dt} = M\alpha r$

where α is the angular acceleration.

Centrifugal force

- The Pseudo force which acts radially outward on the body moving along a circle is called centrifugal force.

Even through centripetal and centrifugal forces are equal in magnitude and opposite in direction, they do not form action – reaction pair because they act on the same body in two different frames.

Important Point

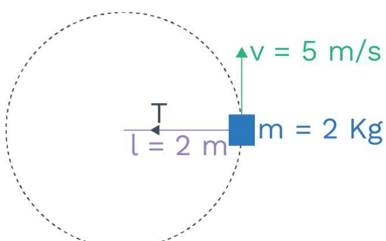
- $\frac{mv^2}{r}$ is not a fundamental force . It is the value of the net force acting along inside perpendicular to velocity which is a reason for circular motion. This force can be any force like friction, tension, normal, gravitational force, spring force or a combination of these forces.
- To solve problem in uniform circular motion we identify all the forces acting along the normal (towards centre), calculate their resultant and equate it to $\frac{mv^2}{r}$.
- If circular motion is non-uniform then after above step we identify all type of forces acting along the tangent to the circular path, calculate their resultant and equate it to $\frac{mdv}{dt}$ or $\frac{md|\vec{v}|}{dt}$.

Key Points

- Centripetal force
- Centrifugal force

Ex. An object having mass 2kg is tied to a rope of length 2m, the other end of which is fixed. The object is moved on a smooth horizontal table with constant speed 5 m/s. Find the tension in the string.

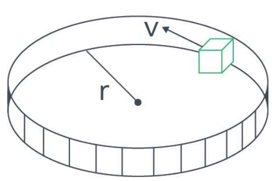
Sol.



here centripetal force is provided by tension.

$$T = \frac{mv^2}{r} = \frac{2 \times 5^2}{2} = 25\,N$$

Ex. An object of mass m moves with speed v against a smooth, fixed vertical circular groove which have radius r kept on fixed smooth horizontal surface.



Find out:

(i) the normal reaction of the surface of floor on the object.

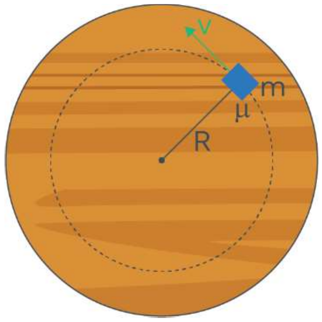
(ii) the normal reaction of the vertical wall on the object.

Sol. For given case, centripetal force is provided by normal reaction of vertical wall.

(i) Normal reaction of floor $N_F = mg$

(ii) Normal reaction of vertical wall = $\frac{mv^2}{r}$

Ex. The coefficient of friction between block surface and table is μ. Find the tension in the string if the block moves on the horizontal table with speed v in circle of radius R.



Sol. The magnitude of centripetal force is $\frac{mv^2}{R}$

(i) If limiting friction is more than or equal to $\frac{mv^2}{R}$, then static friction provides centripetal force, so tension is equal to zero.

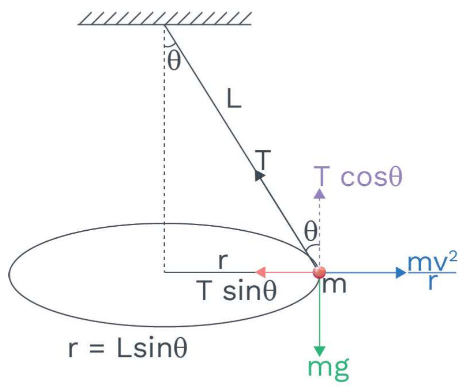
(ii) If limiting friction is less than $\frac{mv^2}{R}$, then friction as well as tension both combine to provide the necessary centripetal force. $T + f_2 = \frac{mv^2}{R}$

For this case friction is equal to limiting friction, $f_2 = \mu mg$

$\therefore$ Tension = $T = \frac{mv^2}{R} - \mu mg$

Conical Pendulum

Consider a small particle of mass m tied to a string is whirled along a horizontal circle, as shown in figure then the arrangement is called a 'conical pendulum'. In case of conical pendulum the vertical component of tension balances the weight while its horizontal components provides the necessary centripetal force. Thus,



$$T\sin\theta = \frac{mv^2}{r} \text{ and } T\cos\theta = mg \Rightarrow v = \sqrt{rg\tan\theta}$$

$\therefore$ Angular speed $\omega = \frac{v}{r} = \sqrt{\frac{g\tan\theta}{r}}$

So, the time period of pendulum is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r}{g\tan\theta}} = 2\pi\sqrt{\frac{L\cos\theta}{g}}$$

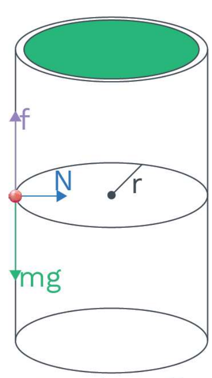
Concept Reminder

In case of conical pendulum, if $\theta = 90°$ then $v = \infty$ and $T = \infty$. However, as velocity and tension cannot be infinite so this situation is not physically possible.

Death Well or Rotor:

In case of 'death well' a person drives a motorcycle on the vertical surface of a large well while in case of a rotor a person hangs resting against the wall without any support from the bottom at a certain angular speed of rotor. In death well walls are at rest and person revolves while in case of rotor person is at rest and the walls rotate.

In both cases friction balances the weight of person while reaction provides the centripetal force for circular motion, i.e.,



Death Well

Rack your Brain

A gramophone record is revolving with an angular velocity ω. A coin is placed at a distance r from the centre of the record. The static coefficient of friction is μ. The coil will revolve with the record if

(1) $r = \mu g \omega^2$ (2) $r < \frac{\omega^2}{\mu g}$

(3) $r \le \frac{\mu^2}{\omega^2}$ (4) $r \le \frac{\mu g}{\omega^2}$

$$f = mg \text{ and } N = \frac{mv^2}{r} = mr\omega^2 \text{ and}$$

$$f = \mu N = mg$$

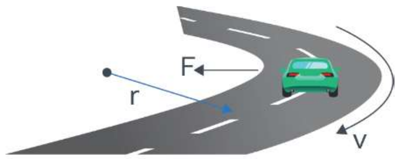
Circular Turning on Roads:

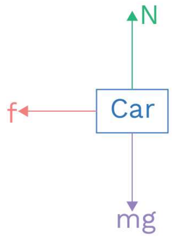
When vehicles move through turnings, they go along a nearly circular arc. There should be some force which will provide the required centripetal acceleration. If these vehicles move in a circular path in horizontal plane, the resultant force is also horizontal. This necessary centripetal force may be provided to the vehicles by following ways.

1. **By friction force only**
2. **By banking of roads only.**
3. **By friction force and banking of roads both.**

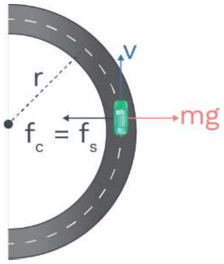
By Friction

When a vehicle say a car takes a turn on a level road, the portion of the turn can be considers as an arc of a circle of radius r (see fig.) If this car makes the turn at a constant speed v, then there must be some centripetal force acting on the car. This force is generated by the friction between the tyre and the road. (car has a tendency to slip radially outward, so frictional force acts inwards) μ is coefficient of static friction and N = mg is the normal reaction of the surface.





Free body diagram of car



Thus, $f = \frac{mv^2}{r}$

Further, limiting value of f is μN or $f_L = \mu N = \mu mg \quad (\because N = mg)$

Therefore, for a safe turn without sliding $\frac{mv^2}{r} \le f_L$

or $\quad \frac{mv^2}{r} \le \mu mg$

or $\quad \mu \ge \frac{v^2}{rg} \qquad$ or $v \le \sqrt{\mu rg}$

In this case, two situations can arise. If μ and r are known to us, then speed of the vehicle should not exceed $\sqrt{\mu rg}$ and if we know value of v and r then coefficient of friction should be greater than $\frac{v^2}{rg}$.

Ex. A car is moving with speed 30 km/h in a circle of radius 60 m. What is the minimum value of μ_s for the car to make the turn without skidding?

Sol. The minimum μ_s should be that

$$\mu_s mg = \frac{mv^2}{r} \quad \text{or} \quad \mu_s = \frac{v^2}{rg}$$

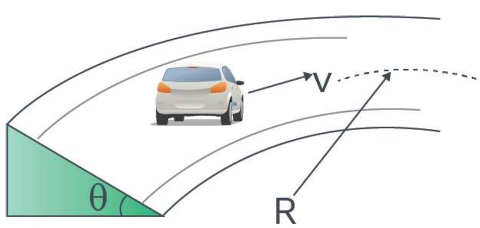
Here $v = 30\frac{km}{h} = \frac{30 \times 1000}{3600} = \frac{25}{3} m/s$

$$\Rightarrow \mu_s = \frac{25}{3} \times \frac{25}{3} \times \frac{1}{60 \times 10} = 0.115$$

For all values of μ_s greater than or equal to the above value, the car can make the turn without skidding. If the speed of the car is large so that minimum μ_s is greater than the standard values (rubber tyre on dry concrete μ_s = 1 and on wet concrete μ_s = 0.7), then the car will skid.

By Banking of road

If a cyclist takes a turn, he can bend from his vertical position. This is not quite possible in the case of car, truck or train.



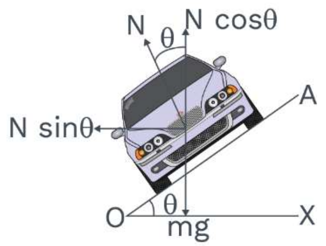
Rack your Brain

A car is negotiating a curve road of radius R. The road is banked at an angle θ. The coefficient of friction between the tyres of the car and road is μ_s. Find out the maximum safe velocity of car on this road.

The tilting of the vehicle is achieved by raising the outer edge of the circular track, slightly above the inner edge. This is known as banking of curved track. The angle of inclination of surface with the horizontal is called the angle of banking. If driver moves with slow velocity that friction does not play any role in negotiating the turn. Number of forces acting on the vehicle are:

(i) Weight of the vehicle (mg) in the downward direction.

(ii) Normal reaction (N) perpendicular to the inclined surface of the road.



Resolve N in two components.

N cosθ, vertically upwards which balances weight of the vehicle.

∴ $N\cos\theta = mg$ ....(i)

N sin θ, in horizontal direction which provides necessary centripetal force.

∴ $N\sin\theta = \frac{mv^2}{r}$ ....(ii)

on dividing eqn. (ii) by eqn. (i)

$$\frac{N\sin\theta}{N\cos\theta} = \frac{\frac{mv^2}{r}}{mg} \qquad \text{or} \quad \tan\theta = \frac{v^2}{rg}$$

∴ $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

Where m denotes mass of given vehicle, r is radius of curvature of the road, v is speed of the vehicle and θ is the banking angle.

Factors that decide the value of angle of banking are as follows:

There is no need of mass of the vehicle to express the value of angle of banking i.e. angle of banking ⇒ does not dependent on the mass of the vehicle.

∵ $v^2 = gr\tan\theta$

∴ $v = \sqrt{gr\tan\theta}$ (maximum safe speed)

- In actual practice, some frictional forces are always present. So, the maximum safe velocity is always much greater than that given by the above equation. While construction the curved track, the value of θ is calculated for fixed values of v_{max} and r. This explains why along the curved roads, the speed limit at which the curve is to be negotiated is clearly incited on sign boards.

The outerside of the road is raised by h = b × θ. When θ is small, then $\tan\theta \approx \sin\theta = \frac{h}{b}$;

Also $\tan\theta = \frac{v^2}{rg}$

$\therefore \quad \frac{v^2}{rg} = \frac{h}{b} \quad$ or $h = \frac{v^2}{rg} \times b$

Ex. A circular track has radius 600 m and is designed for cars to move at an average speed of 180 km/hr. Find the angle of banking of this circular track?

Sol. Let the angle of banking be θ. The forces on the car are

(a) the weight of the car is Mg downward and

(b) the normal force is N.

For proper banking, static frictional force is not needed.

For vertical direction the acceleration is zero.

So, $N\cos\theta = Mg$ .....(i)

For horizontal direction, the acceleration is v^2/r towards the centre, so that

$N\sin\theta = Mv^2 / r$ .....(ii)

From (i) and (ii), $\tan\theta = v^2 / rg$

Putting the values,

$$\tan\theta = \frac{180(km/hr)^2}{(600m)(10m/s^2)} = 0.4167$$

$\theta = 22.6°$

Ex. At what should a highway be banked for cars travelling at a speed of 100 km/h if the radius of the road is 400 m and no frictional forces are involved?

Sol. The banking should be done at an angle θ such that

$$\tan\theta = \frac{v^2}{rg} = \frac{\frac{250}{9} \times \frac{250}{9}}{400 \times 10}$$

or $\tan\theta = \frac{625}{81 \times 40} = 0.19$

or $\theta = \tan^{-1} 0.19 \approx 0.19\,\text{radian} \approx 0.19 \times 57.3° \approx 11°$

Ex. The radius of curvature of a railway line at a place when the train is moving with a speed of 36 kmh^{-1} is 1000 m, the distance between the two rails being 1.5 metre. Calculate the elevation of the outer rail above the inner rail so that there may be no side pressure on the rails.

Sol. Velocity, $v = 36\,kmh^{-1} = \frac{36 \times 1000}{3600} ms^{-1} = 10\,ms^{-1}$

radius, $r = 1000\,m; \tan\theta = \frac{v^2}{rg} = \frac{1}{98}$

Let h be the height through which outer rail is raised. Let l be the distance between the two rails.

Then, $\tan\theta = \frac{h}{l}$ $[\because \theta \text{ is very small}]$

or $h = l\tan\theta$

$h = 1.5 \times \frac{1}{98} m = 0.0153\,m$ $[\because l = 1.5\,m]$

Ex. A train rounds an unbanked circular bend of radius 30 m at a speed of 54 km h⁻¹. The mass of the train is 106 kg. Find the value centripetal force required for this propose? The engine or the rails? The outer of inner rails? Which rail will wear out faster, the outer or the inner rail? What is the angle of banking that is required to prevent wearing out of the rails?

Sol. r = 30 m, v = 54 km h^{-1} = $\frac{54 \times 5}{18} ms^{-1} = 15\,ms^{-1}$

$m = 10^6\,kg$, θ = ?

(i) The centripetal force is provided by the lateral thrust by the outer rail on the flanges of the wheel of the train. The train causes an equal and opposite thrust on the outer rail (Newton's third law of motion). Thus, the outer rails wears out faster. Thus, the outer rails wears out faster.

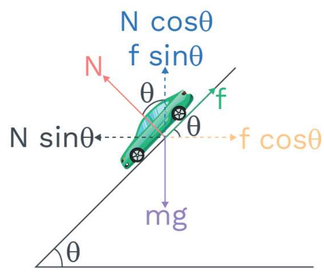
(ii) $\tan\theta = \frac{v^2}{rg} = \frac{15 \times 15}{60 \times 9.8} = 0.7653$

$\therefore \theta = \tan^{-1}(0.7653) = 37.43°$

By friction and banking of roads both

If friction is present between the surface of road and the tyres, the components of friction and normal reaction provide the centripetal force.

Case-I : If $N\sin\theta > \frac{mv^2}{r}$, the vehicle processes the tendency to slip down the plane. The minimum speed for avoiding slipping down the plane can be obtained by taking friction up the plane.



Concept Reminder

In given case., If μ = 0 then $v_{max} = (rg\tan\theta)^{1/2}$. This is optimum speed. A this speed, frictional force is not required at all to provide centripetal force.

To find minimum speed we can use fig (i),

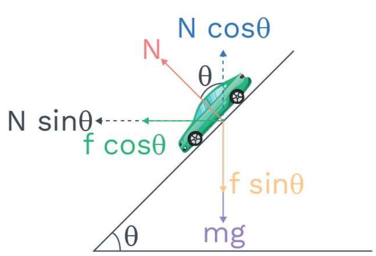
$$N\sin\theta - f\cos\theta = \frac{mv_{min}^2}{r} \quad .... (1)$$

$$N\cos\theta + f\sin\theta = mg \quad .... (2)$$

From (1) & (2) we get

$$v_{min} = \sqrt{\frac{rg(\sin\theta - \mu_s\cos\theta)}{(\cos\theta + \mu_s\sin\theta)}} = \sqrt{\frac{rg(\tan\theta - \mu_s)}{(1 + \mu_s\tan\theta)}}$$

Case-II: If $N\sin\theta < \frac{mv^2}{r}$,the vehicle possesses the tendency to skid up the plane. The safe maximum speed for avoiding skidding can be obtained by taking friction acting down the plane.



To find maximum safe speed, we have to consider figure (ii).

$$N\sin\theta + f\cos\theta = \frac{mv_{max}^2}{r} \quad ... (1)$$

$$N\cos\theta - f\sin\theta = mg \quad ... (2)$$

From (1) and (2) we get

$$v_{max} = \frac{rg(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)} = \sqrt{\frac{rg(\tan\theta + \mu_s)}{(1 - \mu_s\tan\theta)}}$$

Ex. Two cars that have masses m_1 and m_2 are travelling in circles of radii r_1 and r_2 respectively and their speeds are such that they make complete circle in the same time t. The ratio of their centripetal acceleration is

Sol. As their time period of revolution is same, angular speed is also same. centripetal acceleration is

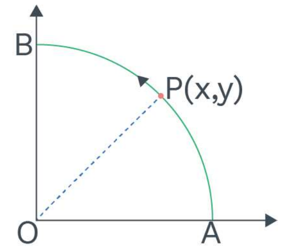
$$a = \omega^2 r\ ;\ \frac{a_1}{a_2} = \frac{\omega^2 r_1}{\omega^2 r_2} = \frac{r_1}{r_2}$$

Ex. A car is moving on a curved path of radius 18m without being in danger of skidding. The friction coefficient between the tyres of the car and the surface of the curved path is 0.2. What is the maximum speed in kmph of the car for safe driving? ($g=10\ ms^{-2}$)

Sol. Maximum speed. $v = \sqrt{\mu_s g r}$

$$v = \sqrt{0.2 \times 10 \times 18} = \sqrt{36} = 6\,ms^{-1}$$

Ex. A point P moves in a counter clock wise direction on a circular path as shown in fig. The movement of 'P' is such that it sweeps out a length $S = t^3 + 5$, where 'S' is in metres and 't' is in seconds. The radius of the path is 20m. The acceleration of 'P' when t = 2s is nearly.



Sol. $S = t^3 + 5 \Rightarrow v = \dfrac{dS}{dt} = 3t^2$

For $t = 2s, v = 3 \times 4 = 12\,ms^{-1}$

Tangential acceleration, $a_t = \dfrac{dv}{dt} = 6t$

For $t = 2s, a_t = 12\,ms^{-2}$

Centripetal acceleration

$$a_c = \frac{v^2}{R} = \frac{144}{20} = 7.2\,ms^{-2}$$

net acceleration = $\sqrt{a_t^2 + a_c^2} \approx 14\,ms^{-2}$

Note:

- The expression $\tan\theta = \frac{v^2}{rg}$ can also gives the angle of banking for an aircraft, i.e., an angle through which aircraft should tilt while negotiating a curve, to avoid deviation from the circular path.
- The expression $\tan\theta = \frac{v^2}{rg}$ can also gives the angle at which a cyclist should lean inward, when rounding a corner. In this case, θ is the angle which the cyclist must make with the vertical.

Rotating Bowl : (Friction absent)

$$N\sin\theta = ma_c = m\omega^2 R\sin\theta$$

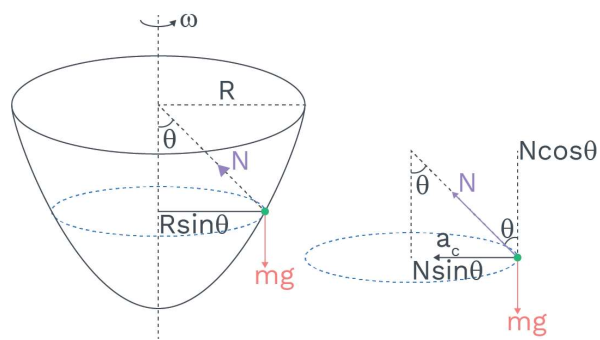
$$N\cos\theta = mg$$

$$\tan\theta = \frac{m\omega^2 R\sin\theta}{mg}$$

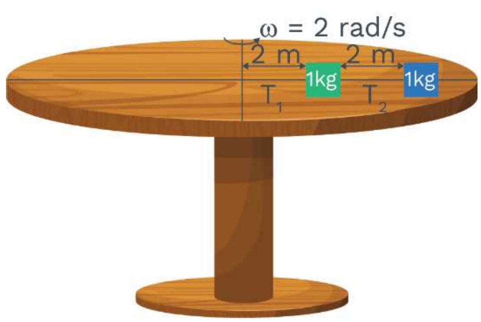
$$\frac{\sin\theta}{\cos\theta} = \frac{\omega^2 R\sin\theta}{g}$$

$$\boxed{\omega = \sqrt{\frac{g}{R\cos\theta}}}$$

$$T = \frac{2\pi}{\omega};\ \boxed{T = 2\pi\sqrt{\frac{R\cos\theta}{g}}}$$



Ex.



Frictionless table find T_1 & T_2.

Sol.

,

,

,

,

Ex. Normal reaction in following cases are

(a) $N_A = mg$

(b)

(c)

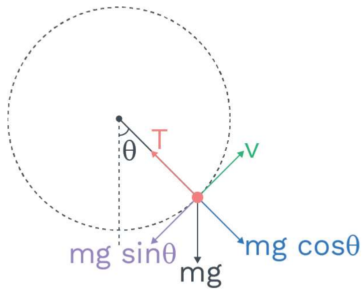
Concept Reminder

In Vertical circle, the torque about centre is . and hence v are not constant. This is an example of non-uniform circular motion.

Motion in a Vertical Circle

Consider the motion of a particle tied to a string of length l and whirled in circle in a vertical plane. If at any time 't' the body is at angular position , as shown in the figure, then the forces that are acting on it are tension T

in the string along the radius towards the centre and the weight of the body mg acting vertically downwards.
Applying Newton's law along radial direction



$$T - mg\cos\theta = m.a_r = \frac{mv^2}{l}$$

or $$T = \frac{mv^2}{l} + mg\cos\theta \quad \text{.... (1)}$$

A particle will complete the whole vertical circle only and only if tension is never zero (except momentarily, if at all) if tension is zero at any point, string will becomes slack. Due to which the only force that acts on the body is gravitational force. Therefore, its motion will become same as that of projectile.

Using equation ...(1),

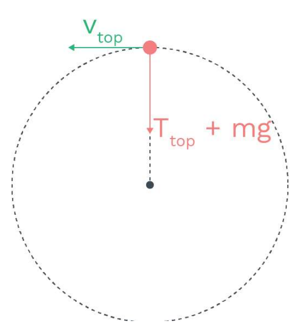
It is known that tension will decrease when θ increases because cos θ is a decreasing function and v decreases with height. Therefore, tension is minimum at the top most point. i.e.,

$T_{min} = T_{topmost}$.

T > 0 at all points. $\Rightarrow$ $T_{min} > 0$.

But if tension becomes zero for a moment at topmost poin the particle would still be able to complete that verticle circle.

So, condition for completing the circle (or looping the loop) is $T_{min} \geq 0$ or $T_{top} \geq 0$.



$$T_{top} + mg = \frac{mv_{top}^2}{l} \quad .....(2)$$

$$\Rightarrow \quad \frac{mv_{top}^2}{l} \geq mg$$

$$\Rightarrow \quad v_{top} \geq \sqrt{gl} \quad .....(3)$$

Condition for looping the loop is $v_{top} \geq \sqrt{gl}$.

If we take speed at the lowest point of motion is u, then by using conservation of mechanical energy between lowest most point and top most point.

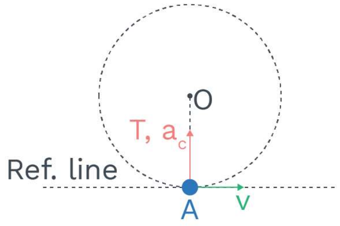
$$\frac{1}{2}mu^2 = \frac{1}{2}mv_{top}^2 + mg.2l$$

using equation ... (3) for v_{top} we get

$$u \geq \sqrt{5gl}$$

i.e., for looping the loop, speed at lowest most point must be $\geq \sqrt{5gl}$.

- If velocity at lowest most point is enough for just looping the loop, then value of various quantities. (True for a particle attached to a string or a mass moving on a smooth vertical circular track.)



P.E. = 0

$$v = \sqrt{5gl}$$

$$T - mg = \frac{mv^2}{l}$$

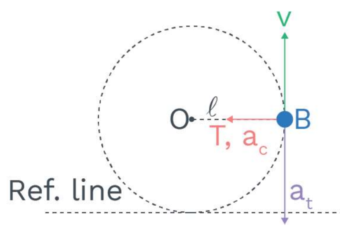
can be obtained by putting $\theta = 0$ in equation .... (1)

$$\Rightarrow T = 6mg$$

$$a_c = 5g$$

$$a_t = 0$$

$$a_{net} = 5g$$



P.E. = mgl

By energy conservation, $v = \sqrt{3gl}$

$T = mv^2 / l$

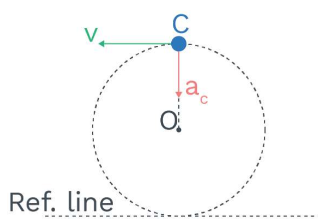
could also be find by putting $\theta = 90°$ in equation .... (1)

T = 3mg

$a_c = 3g$

$a_t = g$

$a_{net} = g\sqrt{10}$



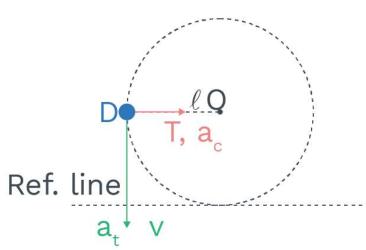
By be energy conservation

$v = \sqrt{gl}$

T = 0

$a_c = g$

$a_t = 0 \Rightarrow a_{net} = g$



P.E. = mgl

Rack your Brain

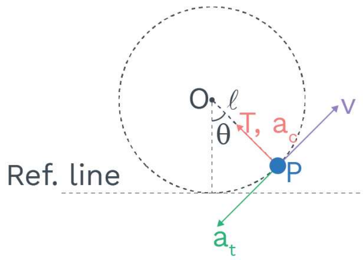
A child is sitting on a swing. Its minimum and maximum heights from ground 0.75 m and 2m respectively. Find out its maximum speed.

by energy conservation $v = \sqrt{3gl}$

$T = 3\ mg$

$a_c = 3g$

$a_t = g$



$\text{P.E.} = mgl(1 - \cos\theta)$

by energy conservation

$v = \sqrt{gl(3 + 2\cos\theta)}$

$T = 3mg\,(1 + \cos\theta)$

$a_c = g(3 + 2\cos\theta)$

$a_t = g\sin\theta$

		A	B,D	C	P(general point)
1	Velocity	$\sqrt{5gl}$	$\sqrt{3gl}$	$\sqrt{gl}$	$\sqrt{gl(3 + 2\cos\theta)}$
2	Tension	6 mg	3 mg	0	$3mg\,(1 + \cos\theta)$
3	Potential Energy	0	mgl	2mgl	$mgl(1 - \cos\theta)$
4	Radial acceleration	5g	3g	g	$g(3 + 2\cos\theta)$
5	Tangential acceleration	0	g	0	$g\sin\theta$

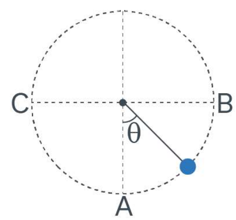
Note:

From above table we can see,

$T_{bottom} - T_{top} = T_C - T_A = 6mg$, this value of difference in tension remain same even if $v > \sqrt{5gl}$

Condition For Oscillation Or Leaving The Circle:

In case of non-uniform circular motion in a vertical plane if velocity of body at lowest point is lesser than $\sqrt{5gl}$, the particle will not complete verticle circle. In this case, the motion of the particle which depend on 'whether tension becomes zero before speed becomes zero or vice versa.



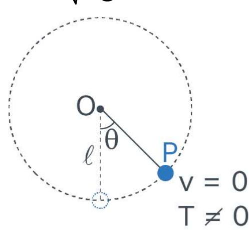
Rack your Brain

A stone is tied to a string of length l and is whirled in vertical circle with other end of string as the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. Find the magnitude of the change in velocity as it reaches a position where string is horizontal.

Case I (Speed equals zero before tension)

In this particular case the ball never rises above the level of the centre O i.e. the boy is confined to move within 'C' and 'B', $(|\theta| < 90°)$ for this the speed at A,

$v < \sqrt{2gl}$



For Oscillation $0 < v_L < \sqrt{2gl}$

$0 < \theta < 90°$

For this case tension cannot be zero, since a component of gravity will acts in a direction radially outwards.

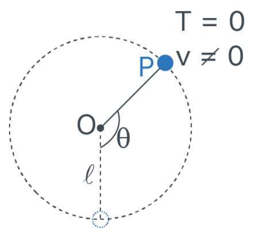
Hence the string will not go slack, and the ball will reverse back as soon as its speed becomes zero.

Its motion will be oscillatory motion.

Case II (Tension becomes zero before speed does)

For this case the ball rises above the level of centre O i.e. it goes beyond point B ($\theta > 90°$) for this $v > \sqrt{2gl}$ (as proved in example) given above.

For this case a component of gravity will always act towards centre, hence centripetal acceleration or speed will remain nonzero. Hence tension becomes zero first.



For leaving the circular path after which motion converts into projectile motion.

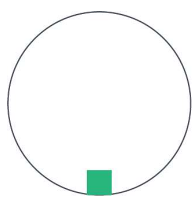
$\sqrt{2gl} < v_L < \sqrt{5gl}$

$90° < \theta < 180°$

When tension becomes zero at any point on its path, string will go slack and eventually, the only force that acts on the body is gravity. Hence its afterward motion will be similar to that of a projectile. For this case motion is a combination of circular and projectile motion.

Condition For Looping The Loop In Some Other Cases

Case 1 : An object of mass m moving on a smooth vertical circular track.



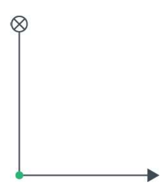
Mass moving along a smooth vertical circular loop.

condition for just looping the loop, normal at highest point = 0

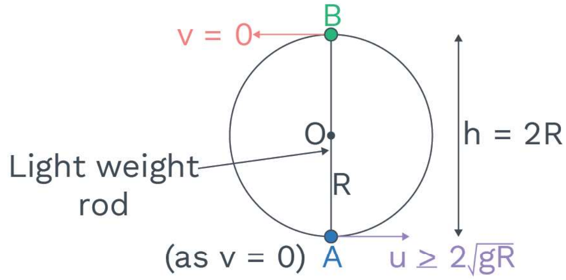
By calculation similar to article (motion in vertical circle)

Minimum horizontal velocity at lowest point = $\sqrt{5gl}$

Case 2 : A point mass m fixed to a light rod is rotating in vertical circle.



Condition for just looping the loop, velocity v = 0 at highest point (even if tension is zero, rod won't slack, and a compressive force can appear in the rod).



By energy conservation,

velocity at lowest point = $\sqrt{4gl}$

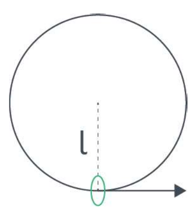
$v_{min} = \sqrt{4gl}$ (for completing the circle)

Case 3 : A bead is attached to a ring and rotated in vertical plane.

Condition to just complete the loop, velocity v is zero at topmost point (even if normal becomes zero, the bead will not lose contact with the track, normal can act radially outward).

By energy conservation,

velocity at lowest point = $\sqrt{4gl}$

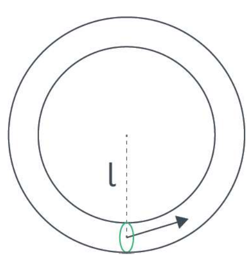


$v_{min} = \sqrt{4gl}$ (for completing the circle)

Case 4 : An object is rotating between smooth surfaces of pipe.

The condition to just complete the loop, velocity v is 0 at topmost point (even if normal becomes zero, the bead will not lose contact with the track, normal can act radially outward).

By energy conservation,



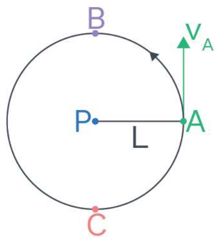
velocity at lowest point = $\sqrt{4gl}$

$v_{min} = \sqrt{4gl}$ (to complete the circle)

Ex. If a point of mass M is tied to a light inextensible string fixed at point 'P' and particle is projected at 'A' with velocity $v_A = \sqrt{4gL}$ as shown.

Find :

(i) velocity at points B and C



(ii) tension in the string at 'B' and 'C'

Consider particle is projected in the vertical plane.

Sol. $v_B = \sqrt{2gL}$ (from energy conservation)

$v_C = \sqrt{6gL}$

$T_B = Mg$

$T_C = 7Mg$ (where M is mass of the particle)

Ex. A car is travelling with uniform speed on a circular bridge of radius R which subtends an angle of 90° at centre of circular arc. What is the minimum possible speed so that the car can cross the bridge without losing the contact any where.

Sol. Let the car losses the contact at angle θ with the vertical

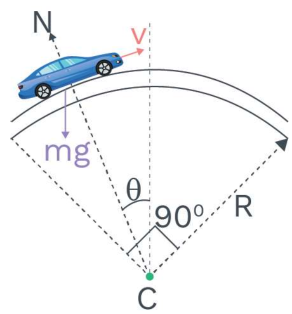
$$mg\cos\theta - N = \frac{mv^2}{R}$$

$$N = mg\cos\theta - \frac{mv^2}{R} \quad \text{... (1)}$$

for losing the contact N = 0,

$\Rightarrow \quad v = \sqrt{Rg\cos\theta}$ (from equation (1))

for lowest speed, $\cos\theta$ should have minimum value so that θ should be maximum.



$$\theta_{max} = 45° \Rightarrow \cos 45° = \frac{1}{\sqrt{2}}$$

$$v_{min} = \left(\frac{Rg}{\sqrt{2}}\right)^{1/2}$$

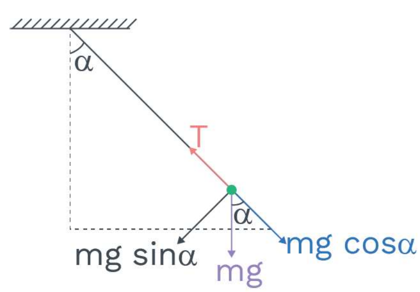
Remember, if car cannot lose the contact at initial or final point, car cannot be lose the contact anywhere.

Ex. A simple pendulum is made by attaching a bob of mass m to a string of length L fixed at its upper end. The bob of this pendulum oscillates in a vertical circle. It is noticed that the speed of this bob is v when the string subtends an angle α with the vertical. What is the tension in the string and the magnitude of net force on the bob at the instant.

Sol. (i) Following are the forces acting on the bob are :

(a) the tension force T

(b) the gravitational force mg



since the bob is moving in a circle of radius L with centre at O, centripetal force of magnitude $\frac{mv^2}{L}$ is required towards O. This force will be provided by the resultant of T and mg cos α. Thus,

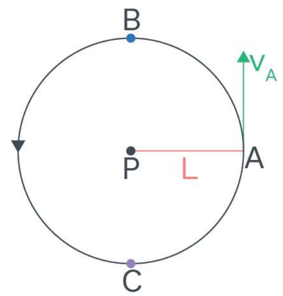
or $$T - mg\cos\alpha = \frac{mv^2}{L}$$

$$T = m\left(g\cos\alpha + \frac{v^2}{L}\right)$$

(ii) $a_{net} = \sqrt{a_t^2 + a_r^2} = \sqrt{(g\sin\alpha)^2 + \left(\frac{v^2}{L}\right)^2}$

$$|\vec{F}_{net}| = ma_{net} = m\sqrt{g^2\sin^2\alpha + \frac{v^4}{L^2}}$$

Ex. If a particle of mass M is tied to a light inextensible string fixed at point P and particle is projected at A with velocity $v_A = \sqrt{4gL}$ as shown. Find:



(i) velocity at points B and C

(ii) tension in the string at B and C

Assume particle is projected in the vertical plane.

Sol. $v_B = \sqrt{2gL}$ (from energy conservation)

$v_C = \sqrt{6gL}$

$T_B = Mg$

$T_C = 7Mg$ (where M $\Rightarrow$ Mass of the particle)

Ex. A body having weight 0.4 kg is whirled in a vertical circle with a string making 2 revolutions/second. Given that the radius of this circle is 1.2 m. Find out the tension

(a) At the top of the circle

(b) At the lowermost point of the circle.

Given : g = 10 m s^{-2} and π = 3.14.

Sol. Mass, m = 0.4 kg;

time period = $\frac{1}{2}$ second and radius, r = 1.2 m

Angular velocity, $\omega = \frac{2\pi}{1/2} = 4\pi \text{ rad s}^{-1} = 12.56 \text{ rad s}^{-1}$.

(a) At the top of the circle,

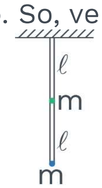
$$T = \frac{mv^2}{r} - mg = mr\omega^2 - mg = m(r\omega^2 - g)$$

$$= 0.4(1.2 \times 112.56 \times 12.56 - 9.8)\text{N} = 71.2\text{ N}$$

(b) At the lowest point, $T = m\left(r\omega^2 + g\right) = 80\text{ N}$

Ex. Two-point mass m are connected the light rod of length l and rod is free to rotate in vertical plane. Find out the minimum horizontal velocity is given to mass so that it completes the circular motion in vertical lane.

Sol. In this case, tension in the rod at the top most point of circle can be zero or –ve to complete the loop. So, velocity at the highest point is zero.

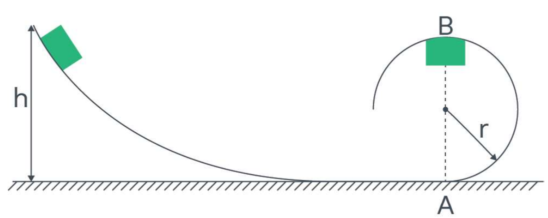


From energy conservation

$$2\left(\frac{1}{2}mv^2\right) = mg(2l) + mg(4l) + 0$$

$$\Rightarrow \quad v = \sqrt{6gl}$$

Ex. A block of mass m is released from rest from the highest point of a smooth vertical track, which further ends in a circle having radius r as shown.



(i) Find out the minimum value of h so that the block complete the circle.

(ii) Find normal reaction when the block is at the points A and B for h = 3r.

(iii) Find the velocity of the block when it loses the contact with the track for h = 2r.

Sol. (i) For completing the circle, velocity at lowest point of circle (say A) is $\sqrt{5gr}$

from energy conservation $mgh = \frac{1}{2}m\left(\sqrt{5gr}\right)^2 \Rightarrow h = \frac{5r}{2}$

(ii) $h = 3r$

By using energy conservation at point A and B of circle.

$mg.3r = \frac{1}{2}mv_A^2 \Rightarrow v_B = \sqrt{2gr}$

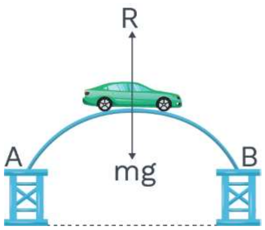
Hence, normal reaction at A and B is

$$N_A - mg = \frac{mv_A^2}{r} \Rightarrow N_A = 7mg$$

$$N_B - mg = \frac{mv_B^2}{r} \Rightarrow N_B = mg$$

Ex. Prove that a motor car travelling over a convex bridge feels lighter than the exact car resting on the same convex bridge.

Sol. The motion of a motor car over a convex bridge AB is the same as motion along the segment of a circle AB as shown in figure. In this case, centripetal force is provided by the difference of weight mg of the car and the normal reaction force R of the bridge.



Clearly R < mg, i.e., the weight of the moving car is less than the weight of the stationary car.

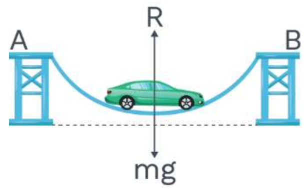
$$\therefore \quad mg - R = \frac{mv^2}{r}$$

or $$R = mg + \frac{mv^2}{r}$$

Clearly R < mg, i.e., the weight of the moving car is less than the weight of the stationary car.

Ex. Prove that a motor car moving over a concave bridge is heavier than the same car resting on the same bridge.

Sol. The motion of the motor car over a concave bridge AB is the motion along the segment of a circle AB as shown in figure. The centripetal force is provided by the difference of normal reaction R of the bridge and weight mg of the car.

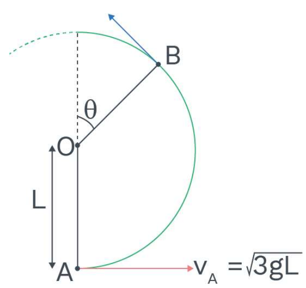


$$\therefore \quad R - mg = \frac{mv^2}{r}$$

or $$R = mg + \frac{mv^2}{r}$$

clearly R > mg, i.e., the weight of the moving car is greater than the weight of the stationary car.

Ex. A particle in projected with velocity $\sqrt{3gL}$ at point A (lowest point of the circle) in the vertical plane. Find the maximum height about horizontal level of point A if the string slacks at the point B as shown.



Sol. As tension at B ; T = 0

$$\therefore \quad mg\cos\theta = \frac{mv_B^2}{L}$$

$$\therefore \quad v_B = \sqrt{gL\cos\theta} \qquad \text{.... (1)}$$

Now by equation of energy between A and B.

$$0 + \frac{3}{2}mgL = \frac{1}{2}mv_B^2 + mgL(1 + \cos\theta)$$

put $v_B = \sqrt{gL\cos\theta}$

$$\therefore \quad \cos\theta = \frac{1}{3}$$

$\therefore$ height attended by particle after the point B where the string slacks is;

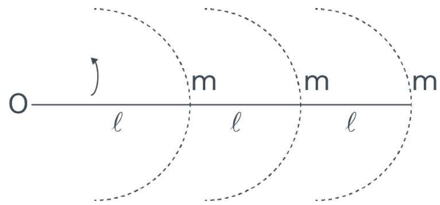
$$h' = \frac{v_B^2 \sin^2\theta}{2g} = \frac{gL\cos\theta(1-\cos^2\theta)}{2g} = \frac{4L}{27}$$

$\therefore$ Maximum height about point A is given by ;

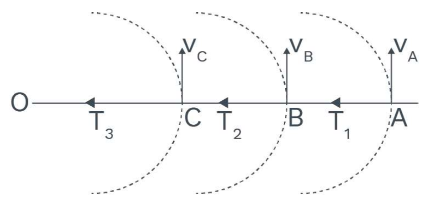
$$H_{max} = L + L\cos\theta + h'$$

$$= L + \frac{L}{3} + \frac{4L}{27} = \frac{40L}{27}$$

Ex. Three identical particles are connected by three strings as shown in fig. These particles are revolving in a horizontal plane. The velocity of outer most particle is v. Then $T_1 : T_2 : T_3$ will be –
(Where T_1 is tension in the outer most string etc.)



Sol.



Required centripetal force = $\frac{mv_A^2}{3l}$

(net force towards centre = T_1)
This will provide required centripetal force particle at A,

$$\therefore \quad T_1 = \frac{mv_A^2}{3l}$$

Similarly, $T_2 = \frac{mv_B^2}{2l}$ and $T_3 = \frac{mv_C^2}{l}$

Remember ω i.e. angular velocity, of all the particles is same

$$\omega = \frac{v_A}{3l} = \frac{v_B}{2l} = \frac{v_C}{l}$$

$$\Rightarrow \quad v_B = \frac{2}{3}v_A \text{ and } v_C = \frac{v_A}{3}$$

$$\therefore \quad T_1 : T_2 : T_3 = \frac{1}{3} : \frac{2}{9} : \frac{1}{9} = 3 : 2 : 1$$

EXAMPLES

Q1 **A particle moves clockwise in a circle of radius 1 m with centre at (x, y) = (1m, 0). It starts at rest at the origin at time t = 0. Its speed is increasing at the constant rate of $\left(\frac{\pi}{2}\right)$ m/s². **

(a) Find out how long does it take to travel halfway around the circle?

(b) What is the speed at that time?

(c) What is the net acceleration at that time?

Sol. R = 1m,

$a_t = \frac{dv}{dt} = \frac{\pi}{2}$ m/s²

at t = 0 , u = 0, $\omega_0 = 0$

$\alpha = \frac{a_t}{R} = \frac{\pi}{2}$ rad/s²

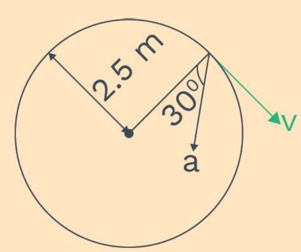
(a) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \Rightarrow \pi = 0 + \frac{1}{2}\frac{\pi}{2}t^2 \Rightarrow t = 2\,\text{sec}$

(b) $v = u + a_t t = 0 + \frac{\pi}{2} \times 2 = \pi\, m/s$

(c) $a_t = \frac{\pi}{2} m/s^2, a_c = \frac{v^2}{r} = \pi^2\, m/s^2$

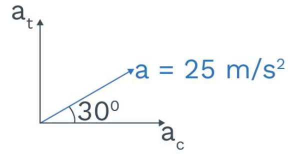
$a = \sqrt{a_1^2 + a_c^2} = \sqrt{\frac{\pi^2}{4} + \pi^4} = \frac{\pi}{2}\sqrt{1 + 4\pi^2}\; m/s^2$

Q2 **Figure shows the direction of total acceleration and velocity of a particle moving clockwise in a circle of radius 2.5 m at a given instant of time. At this time if magnitude of net acceleration is 25 m/sec², find:**



(a) The radial acceleration, **(b) The speed of the particle and**

(c) Its tangential acceleration



(a) $a_c = a\cos 30° = 25\frac{\sqrt{3}}{2} m/s^2$

(b) $a_c = \frac{v^2}{R} \Rightarrow v^2 = a_c R = 25\frac{\sqrt{3}}{2} \times 2.5$

$v = \left(125\frac{\sqrt{3}}{4}\right)^{1/2} m/s$

(c) $a_t = a\sin 30° = \frac{25}{2} m/s^2$

Q3 **A block having mass 'm' moves on a circle in horizontal plane with the wall of a cylindrical room having radius R. The floor of the room on which the block moves is smooth but the coefficient of friction between the block surface and the wall is μ. The block is given an initial speed v_0 then as a function of the instantaneous speed 'v' write**

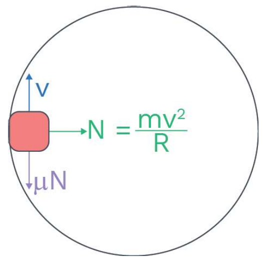
(i) normal force by the wall on the block,

(ii) the frictional force by the wall and

(iii) the tangential acceleration of the block.

(iv) find the speed of the block after one revolution.

Sol.



(i) Normal reaction by wall on the block is $N = \frac{mv^2}{R}$

(ii) Friction force on the block by the wall is $f = \mu N = \frac{\mu m v^2}{R}$

(iii) The tangential acceleration of the block = $\frac{f}{m} = \frac{\mu v^2}{R}$

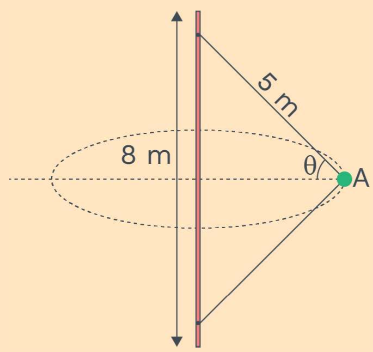
(iv) $\dfrac{dv}{dt} = -\dfrac{\mu v^2}{R}$

or $\quad v\dfrac{dv}{ds} = -\dfrac{\mu v^2}{R} \Rightarrow \displaystyle\int_{v_0}^{v} \dfrac{dv}{v} = -\int_0^{2\pi R} \dfrac{\mu}{R} ds$

integrating we get

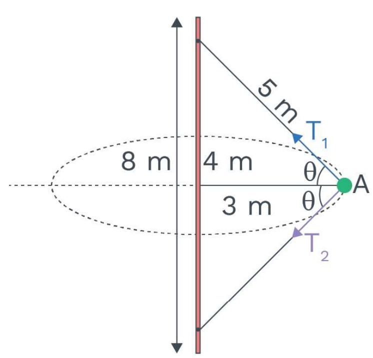
$\ln\dfrac{v}{v_0} = -\mu 2\pi \quad$ or $\quad v = v_0 e^{-2\mu\pi}$

Q4 **A 4 kg small sphere is fixed to a vertical rod by means of two strings of same length. When this system rotates about the axis of the rod, the strings are extended as shown in figure.**



(a) Find out number of revolutions per minute should the system make in order for the tension in the upper chord to be 20 kgf?
(b) Find out the tension in the lower chord?

Sol. Centripetal acceleration



$M\omega^2 r = T_1 \cos\theta + T_2 \cos\theta \qquad \text{.... (1)}$

Apply Newton law in vertical direction

$T_1 \sin\theta = mg + T_2 \sin\theta$ ..... (2)

given m = 4 kg, T_1 = 20 kgf = 200 N, r = 3m

$\cos\theta = \frac{3}{5}, \sin\theta = \frac{4}{5}$

Put in equation (2)

T_2 = 150 N

Put in equation (1) we get,

$\omega^2 = \frac{210}{4 \times 3} = \frac{35}{2}$

$\omega = \sqrt{\frac{35}{2}}\ \text{rad/s}$

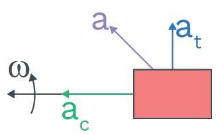
Q5 **A block having mass m is kept on a horizontal ruler. The coefficient of friction between the ruler and the block is μ. The ruler is fixed at one end and the block is at a distance L from the fixed end. The ruler is rotated about the fixed end in the horizontal plane through the fixed end. (A) What can the maximum angular speed before which the block does not slip? (B) find at what angular speed will the block slip, if the angular speed of the ruler is uniformly increased from zero at a constant angular acceleration α.**

Sol. (a) Ruler rotate with constant angular velocity for just slipping

$m\omega^2 L = \mu mg$

$\omega = \sqrt{\frac{\mu g}{L}}$

(b) Angular velocity increase with constant angular acceleration α



$a_c = \omega^2 L, a_t = \alpha L$

$a = \sqrt{a_c^2 + a_t^2} = \sqrt{\left(\omega^2 L\right)^2 + (\alpha L)^2}$ .... (1)

for just slipping $\Rightarrow$ ma = μmg

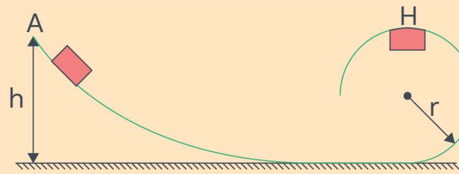
$\Rightarrow$ a = μg ... (2)

form (1) & (2)

$$\mu g = \sqrt{(\omega^2 L)^2 + (\alpha L)^2}$$

$$\omega = \left[\left(\frac{\mu g}{L}\right)^2 - \alpha^2\right]^{1/4}$$

Q6 **A small block having mass m is allowed to slide on an inclined frictionless track from rest position as shown in the figure.**



(i) **Find the minimum height h, so that body may successfully complete the loop of radius 'r'.**

(ii) **If h is double of that minimum height, find the resultant force on the block at position H**

Sol. (i) for complete the loop minimum velocity at lowest point is v = $\sqrt{5gr}$

from energy conservation

$$\frac{1}{2}mv^2 = mgh$$

$$\frac{1}{2}m(\sqrt{5gr})^2 = mgh \Rightarrow h = \frac{5}{2}r$$

(ii) h is double then velocity at h position is

mg2h – mg 2r = $\frac{1}{2}mv^2$ (from energy conservation)

$$v = \sqrt{6gr}$$

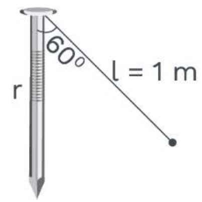
Normal reaction at highest point.

$$F_R = N + mg = \frac{m(\sqrt{6gr})^2}{r}$$

$$F_R = 6mg$$

Q7 **A nail is situated at a some fixed distance vertically below the point of suspension of a simple pendulum. The pendulum bob is released from the point where the string makes an angle of 60° from the vertical. Calculate the distance of the nail from the point of suspension such that the bob will just perform revolution with the nail as centre. Assume the length of pendulum to be 1m.**

Sol.



velocity at lowest point

$mgl(1 - \cos 60°) = \frac{1}{2}mv^2$ (from energy conservation)

$v = \sqrt{gl}$

for completing the loop.

$v = \sqrt{5g(l - r)} = \sqrt{gl}$

$r = \frac{4}{5}m$

Q8 **A person stands on a spring balance at the equator. By what percentage is the balance reading less than his true weight?**

Sol. At equator $T + m\omega^2 R = mg$

$$\% \frac{\Delta T}{T} = \frac{\omega^2 R}{g}$$

$$= \left(\frac{4\pi^2 \times 6400 \times 1000}{(24 \times 60 \times 60)^2 \times 9.8}\right) \times 100 = 0.35\%$$

Q9 **A car goes on a horizontal circular road of radius R, the speed increasing at a constant rate $\frac{dv}{dt}$ = a. Given that coefficient of friction between the road and the tyre is μ. Find out the speed at which the car will skid.**

Sol. Net force on car = frictional force f

$\therefore \quad f = m\sqrt{a^2 + \frac{v^4}{R^2}}$ (where m is mass of the car) .... (1)

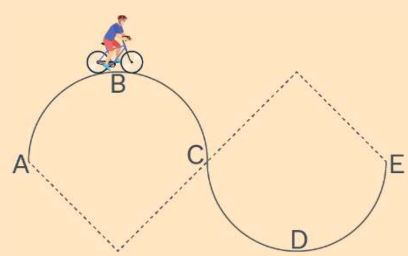
For skidding to just occur

$f = \mu N = \mu mg$ .... (2)

$\therefore$ From (1) and (2)

$v = \{R^2[\mu^2 g^2 - a^2]\}^{1/4}$

Q10 **A track have two circular parts ABC and CDE of equal radius 100 m and joined smoothly as shown in fig. Each part makes a 90° angle at its centre. A cycle of weight 100 kg including the rider travels with a constant speed of 18 km/h on the track. (a) What will be the normal contact force by the road on the cycle when it is at B and D. (b) Find out the force exerted by friction by the track on the tyres when the cycle is at B, C and D. (c) Find out the normal reaction force between the road and the cycle just before and just after the cycle crosses C. (d) Find out the minimum coefficient of friction between the tyre of car and road which can ensure that the cyclist will move with constant speed ? Take g = 10m/s².**



Sol. Constant speed = 18 km/hr = 5m/sec.

m = 100 kg, r = 100 m

(a) at B $\quad mg - N_B = \frac{mv^2}{r} = \frac{100 \times 5^2}{100} = 25$

$N_B = 975$ N

at D, $N_D - mg = \frac{mv^2}{r} \Rightarrow N_D = 1025$ N

(b) at B & D friction force act is zero.

at C, $f = mg \sin 45 = 100 \times 10 \frac{1}{\sqrt{2}}$ $\quad (\because v = \text{constant})$

= 707 N

(c) for BC part

$$mg\cos 45\ N_{BC} = \frac{mv^2}{R} \Rightarrow N_{BC} = 682\,N$$

for CD part

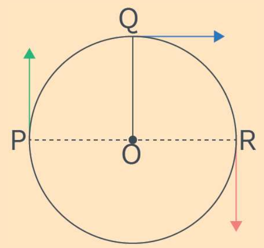
$$N_{CD} - mg\cos 45 = \frac{mv^2}{R} \Rightarrow N_{CD} = 732\,N$$

(d) $f \le \mu N \Rightarrow \mu \ge \frac{f}{N}$

position where its maximum and N is minimum which is in part BC at C position.

$$\mu \ge \frac{mg\sin 45^\circ}{mg\cos 45^\circ - \frac{mv^2}{r}} \Rightarrow \mu \ge \frac{707}{682} = 1.037$$

Q11 **Three-point masses P, Q, R moving in a circle of radius 'r' with different but uniform speeds. They all start to move at t = 0 from their initial positions as shown in the given figure. Given that angular velocities (in rad/sec) of P, Q and R are** $5\pi, 2\pi \,\&\, 3\pi$ **respectively, in the same sense. The time at which they all meet is:**



Sol. $\omega_{QP} = 2\pi - 5\pi = -3\pi\,rad/s$

$\omega_{RP} = 3\pi - 5\pi = -2\pi\,rad/s$

Time when Q particle reaches at P = $t_1 = \frac{\pi/2}{3\pi} = \frac{1}{6}$ sec.

$$t_2 = \frac{5\pi/2}{3\pi} = \frac{5}{6}\,sec.$$

$$t_3 = \frac{9\pi/2}{3\pi} = \frac{3}{2}\,sec.$$

Time where R particle reaches at P.

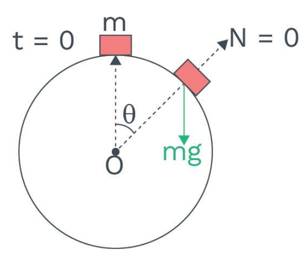
$$t_1 = \frac{\pi}{2\pi} = \frac{1}{2}\,\text{sec}.$$

$$t_2 = \frac{3\pi}{2\pi} = \frac{3}{2}\,\text{sec}.$$

Common time to reaches at P is $\frac{3}{2}$ sec.

Q12 **A particle of mass m begins to slide down a fixed smooth sphere from the top with negligible initial velocity. What is its tangential acceleration when it breaks off the sphere?**

Sol.



at loose contact N = 0

$$mg\cos\theta = \frac{mv^2}{R} \quad \text{... (1)}$$

from energy conservation

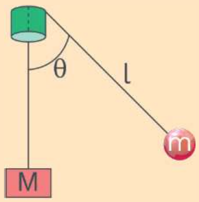
$$mgR(1-\cos\theta) = \frac{1}{2}mv^2 \quad \text{... (2)}$$

from (1) & (2)

$$\cos\theta = \frac{2}{3} \Rightarrow \sin\theta = \frac{\sqrt{5}}{3}$$

tangential acceleration $k = g\sin\theta = \frac{\sqrt{5}g}{3}$

Q13 **A large mass M hangs stationary at the end of a light string that passes through a smooth fixed ring to a small mass m that moves around in a horizontal circular path. If l is defined is the length of the string from m to the top end of the tube and θ is angle between this part and vertical part of the string as shown in the figure, then time taken by m to complete one circle is equal to**



Sol. For M to be stationary

$T = Mg$ .... (1)

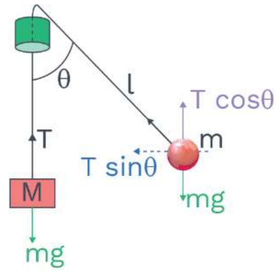
Also, for mass m,

$T \cos \theta = mg$ .... (2)

$T \sin \theta = \dfrac{mv^2}{l \sin\theta}$ .... (3)

Dividing (3) by (2)

$$\tan\theta = \frac{v^2}{gl\sin\theta} \Rightarrow v = \sqrt{\frac{gl}{\cos\theta}}.\sin\theta$$



$$\text{Time period} = \frac{2\pi R}{v} = \frac{2\pi l \sin\theta}{\sqrt{\frac{gl}{\cos\theta}}.\sin\theta}$$

From (1) and (2)

$$\cos\theta = \frac{m}{M}$$

$$\text{then time period} = 2\pi\sqrt{\frac{lm}{gM}}$$

Q14

A stone having mass 1 kg tied to a light inextensible string of length $L = \frac{10}{3}$ m, whirling in a circular path. If the ratio of maximum tension in the string to the minimum tension in the string is 4 and g is 10 m/s², find out the speed of the stone at the highest point of the circle.

Sol. Maximum tension in string at lowest

$$T_{max} = \frac{mv_{LP}^2}{L} + mg \quad \text{... (1)}$$

minimum tension in string at highest point.

$$T_{min} = \frac{mv_{HP}^2}{L} - mg \quad \text{... (2)}$$

from energy conservation

$$\frac{1}{2}mv_{LP}^2 = 2mgL + \frac{1}{2}mv_{HP}^2 \quad \text{.... (3)}$$

from (1) & (3)

$$T_{max} = \frac{1}{L}mv_{HP}^2 + 5\,mg \quad \text{... (4)}$$

from (2) & (4)

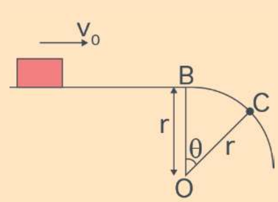
$$4 = \frac{T_{max}}{T_{min}} = \frac{\frac{mv_{HP}^2}{L} + 5mg}{\frac{mv_{HP}^2}{L} - mg}$$

$$\Rightarrow \quad 3mv_{HP}^2 = 9\,mgL$$

$$\Rightarrow \quad v_{HP} = \sqrt{3gL} = 10 \text{ m/s}$$

Q15

A small frictionless block slides with velocity $0.5\sqrt{gr}$ on the horizontal surface as shown in the Figure. The block leaves the surface at point C. The angle θ in the Figure is:



Sol. Given $v_B = 0.5\sqrt{gr}$

Assume block leave the contact at C, N = 0

$$\frac{mv_C^2}{r} = mg\cos\theta \qquad \text{... (1)}$$

from energy conservation $\frac{1}{2}mv_B^2 + mgr(1-\cos\theta) = \frac{1}{2}mv_C^2$ ... (2)

from equation (1) and (2).

$$\frac{1}{2}m\left(\frac{1}{4}gr\right) + mgr(1-\cos\theta) = \frac{1}{2}mgr\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{3}{4}$$

$$\Rightarrow \quad \theta = \cos^{-1}\frac{3}{4}$$

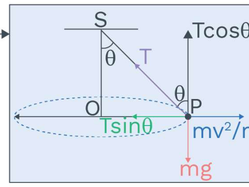
MIND MAP

CIRCULAR MOTION

Dynamics of Circular Motion

Conical Pendulum : Tension in the string

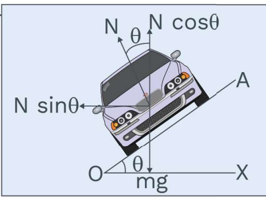
$T = mg\sqrt{1 + \left(\frac{v^2}{rg}\right)^2}$



Time period $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

Linear velocity $v = \sqrt{gr\tan\theta}$

Angular velocity $\omega = \sqrt{\frac{g}{l\cos\theta}}$

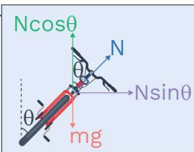


Banking of a road : Optimum speed of a vehicle on a banked road.

$v = \sqrt{rg\tan\theta}$

Maximum safe speed on a banked frictional road

$v = \sqrt{\frac{rg(\mu + \tan\theta)}{1 - \mu\tan\theta}}$



Bending of a Cyclist : The cyclist should bend through an angle

$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

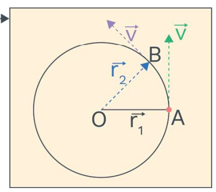
Kinematics of Uniform Circular Motion

Uniform Circular Motion : A particle moves in a circle at a constant speed

Angular Displacement :
It is denoted by θ.
SI unit : rad or degree

Angular Velocity (ω) :

$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$



Relation between :
s and θ ⟹ $s = r\theta$
v and ω ⟹ $\vec{v} = \vec{\omega} \times \vec{r}$

Kinematics of Non-uniform Circular Motion

Non-uniform Circular Motion : Speed of the particle in a horizontal circular motion changes with respect to time.

Tangential acceleration :
$\vec{a}_t = \vec{\alpha} \times \vec{r}$

Centripetal acceleration :
$\vec{a}_c = \vec{\omega} \times \vec{v}$

Forces :
Centripetal force

$F_c = ma_c = \frac{mv^2}{r} = mr\omega^2$

Tangential force, $F_t = ma_t$

Net force, $F_{net} = m\sqrt{a_c^2 + a_t^2}$

As $F = \frac{mv^2}{r} \neq 0$, Particle is not in equilibrium

$\tau_{net} = 0$ as central force is involved, i.e., $p \neq$ constant but L = constant