Capacitor

DISCLAIMER

Capacitor

Capacitor:

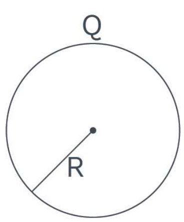
- It is a device which is used to store energy in the form of electric field by storing charge.
- Conductors are used to form capacitors.

Capacitance:

- It is the capacity of capacitor / conductor to store charge.
- It is the amount of charge given to any capacitor/ conductor to rise its potential by 1V.

$$\boxed{C = \frac{Q}{\Delta V} = \frac{Q}{V}}$$

- Unit – Farad [F]
- Capacitance depends on size and shape of the plates and the material between them. It does not depend on Q or V individually.
- Dimension $\rightarrow \frac{[AT]^2}{[ML^2T^{-2}]} = [M^{-1}L^{-2}A^2T^4]$



- Capacitance of spherical conductor

$$V = \frac{KQ}{R} = \frac{Q}{4\pi\varepsilon_0 R}$$

$$\boxed{\therefore C = \frac{Q}{V} = 4\pi\varepsilon_0 R} \quad \left(4\pi\varepsilon_0 = \frac{1}{9} \times 10^{-9}\right)$$

Important points about capacitance of an isolated conductor:-

(i) The capacitor is represented by following symbols

Definitions

- Capacitor:- It is a device which store energy in the form of electric field by storing charge
- Capacitance:- It is the capacity of capacitor to store charge.

Key Points

· Capacitor
· Capacitance
· Farad

(ii) **1 Farad:** 1 Farad is the capacitance of a conductor for which 1 coulomb charge increases potential by 1 volt.

$$1\text{Farad} = \frac{1\text{Coulomb}}{1\text{Volt}}$$

$$1\ \ F = 10^{-6}F, 1nF = 10^{-9}F \text{ or } 1pF = 10^{-12}F$$

(iii) **Capacitance of an isolated conductor depends on following factors:**

- **Shape and size of the conductor:**
  On increasing the size, capacitance increases.
- **On surrounding medium:-**
  With increase in dielectric constant 'K', capacitance increases.
- **Presence of other conductors:-**
  When a neutral conductor is placed nearby a charged conductor then capacitance of conductors increases.

(iv) **Capacitance of a conductor do not depend on**

- Charge on the conductor
- Potential of the conductor
- Potential energy of the conductor.

Concept Reminder

Potential to which a conductor is raised, depends on the amount of charge, geometry of conductor and size of conductor.

Ex. Find the capacitance of sphere of radius 1m.

Sol. $C = 4\pi\varepsilon_0 R$

$$C = 4 \times 3.14 \times 8.85 \times 10^{-12} \times 1 \left(4\pi\varepsilon_0 = \frac{1}{9} \times 10^{-9}\right) = 1.1 \times 10^{-10}F$$

Ex. Find the capacitance of earth.

Sol. $C = 4\pi\varepsilon_0 R$

$$= \frac{1}{9} \times 10^{-9} \times 6400 \times 10^{3}$$

(Radius of Earth $R = 6400\,\text{km}$)

$$\therefore \quad = 7.1 \times 10^{-4}F = 711\ \ F$$

Note:-Theoretically earth has 711 μF capacitance but practically earth has ∞ capacitance.

Practically, $C = \frac{Q}{V} = \frac{Q}{0} = \infty$

Types of Capacitor

(a) Parallel Plate Capacitor
(b) Spherical Capacitor
(c) Cylindrical capacitor

Key Points

- Parallel plate capacitor
- Spherical capacitor
- Cylindrical capacitor

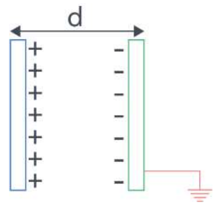
(a) Parallel Plate Capacitor:

A parallel plate capacitor consists of 2 metal plates placed parallel to each other and separated by a distance d that is very small as compared to the dimensions of the plates. The area of each plate is A. The electric field between the plates is given by

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

Where σ is surface charge density on either plate. The potential difference (V) between plates is given by

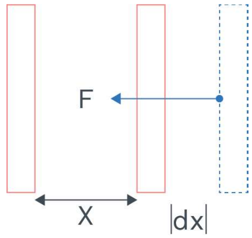
V = Ed.



or, $$V = \frac{\sigma}{\varepsilon_0} d = \frac{Q}{A\varepsilon_0} d$$

Hence, $$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$$

- **Force between plates of capacitor** = Magnitude of electric field by any one plate $E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$



$$F_c = QE$$

$$F_c = \left(\frac{\sigma}{2\varepsilon_0}\right) \times Q$$

$$\boxed{F_c = \frac{Q^2}{2A\varepsilon_0} = \frac{\sigma^2 A}{2\varepsilon_0}}$$

Concept Reminder

If the plates of a parallel plate capacitor are not equal in area, then quantity of charge on the plates will be same but nature of charge will be different.

Rack your Brain

A parallel plate capacitor has capacity C, distance of separation between plates is d and potential difference V is applied between the plates. Find out force of attraction between the plates of parallel plate capacitor.

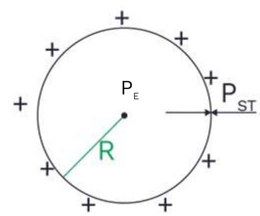
- **Electrostatic pressure**

$$P_E = \frac{F_c}{A} = \frac{\sigma^2 A}{2\varepsilon_0 A}$$

$$\boxed{P_E = \frac{\sigma^2}{2\varepsilon_0}}$$

Note: For soap bubble, if bubble is at equilibrium;

$P_E = P_{ST}$



$$\boxed{\frac{\sigma^2}{2\varepsilon_0} = \frac{4T}{R}}$$

$$\text{Energy density} = \frac{\text{Energy}}{\text{Volume}}$$

$$C = \left(\frac{\varepsilon_0 A}{d}\right) \ \& \ V = Ed$$

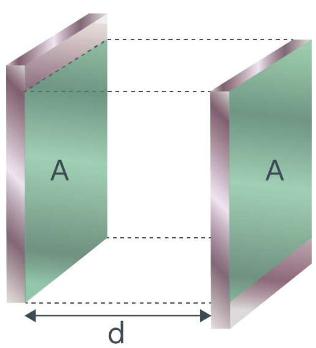
$$\therefore \ \text{Energy} = \frac{A\varepsilon_0}{2d}(Ed)^2 \left(\text{Since Energy} = \frac{1}{2}CV^2\right)$$

$$\boxed{\text{Energy} = \frac{1}{2}\varepsilon_0 E^2 (Ad)}$$

$$\frac{\text{Energy}}{Ad} = \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2$$

$$\left(\text{Energy density} = \frac{1}{2}\varepsilon_0 E^2 = \frac{\sigma^2}{2\varepsilon_0}\right]$$

- If one of the plates of a parallel plate capacitor slides parallel to the other then C decreases (As overlapping area decreases).



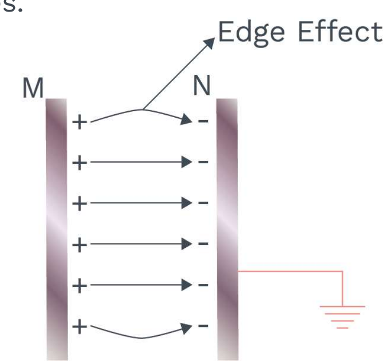
$$C = \frac{\varepsilon_0 A}{d}, \text{ where}$$

Concept Reminder

The plates charged to $+q_1$ and $+q_2$ ($q_2 < q_1$) are brought closer to form a capacitor of capacitance C. The potential difference across plates will be $\left(\frac{q_1 - q_2}{2C}\right)$.

A = overlapping area

- If both the plates of a parallel plate capacitor are touched with each other then, the resultant charge and potential difference becomes 0.
- Electric field between plates of a capacitor is given in diagram. Non-uniformity of electric field at the boundaries or corners of plates of capacitor is negligible if the distance between plates is very small as compared to the length of the plates.



$\vec{E}$ is uniform between the plates.

$\vec{E}$ is non-uniform at the edges.

- Capacitance of parallel plate capacitor does not depend on thickness and nature of metal of plates
- For a parallel plate capacitor:

(i) Intensity of electric field between the plates

$$E = \frac{\sigma}{\varepsilon_0} = \frac{V}{d} \text{(uniform)}$$

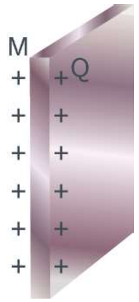
(ii) Force between the plates = $\frac{CV^2}{2d} = \frac{QE}{2} = \frac{Q^2}{2A\varepsilon_0}$

(E $\rightarrow$ Electric field)

(iii) Pressure on the plates = $\frac{\sigma^2}{2\varepsilon_0}$

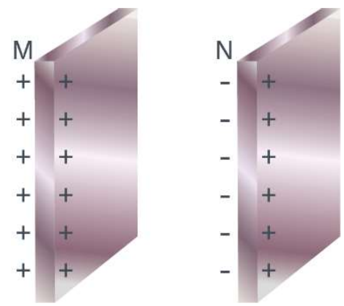
Principle of a Condenser-

It is based on the statement that capacitance can be increased by reducing the potential keeping the electric charge constant. Take a conducting plate M which is given a charge Q such that its potential rises to V then



$$C = \frac{Q}{V}$$

Now place another identical conducting plate N parallel to it such that charge is induced on plate N (as shown in figure). If V_- is the potential at M due to induced negative charge on N and V_+ is the potential at M due to induced positive charge on N, then

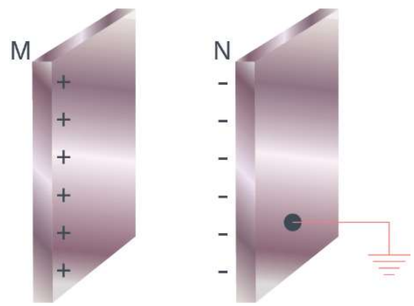


$$C' = \frac{Q}{V'} = \frac{Q}{V + V_+ - V_-}$$

Since V' < V (as the induced negative charge lies closer to the plate 'M' in comparison to induced positive charge).

⇒ C' > C Further, if N is earthed from the outer side (see figure) then $V'' = V_+ - V_-$

(∵ the total positive charge flows to the earth)



$$C'' = \frac{Q}{V''} = \frac{Q}{V_+ - V_-} \Rightarrow C'' >> C$$

If an identical earthed conductor is placed in the vicinity of a charged conductor, then the capacitance of the charged conductor increases appreciably. This is the principle of parallel plate capacitor.

Rack your Brain

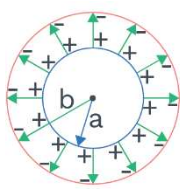
A parallel plate capacitor is charged to a potential difference of V volts. After disconnecting the battery, the distance between the plates is increased using an insulating handle. What will be the effect on potential difference across plates?

(b) Spherical Capacitor:-

A spherical capacitor consists of 2 concentric spheres of radii a and b.

When outer sphere is grounded

The inner sphere is +vely charged to potential V and outer sphere is at zero potential. The inner surface of the outer sphere has an equal negative charge.



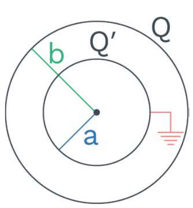
The potential difference between the spheres is

$$V = \frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 b}$$

Hence, capacitance

$$C = \frac{Q}{V} = \frac{4\pi\varepsilon_0 ab}{b - a}$$

When inner sphere is grounded



$$V_{in} = \frac{KQ'}{a} + \frac{KQ}{b} = 0$$

$$\boxed{Q' = \frac{-Qa}{b}}$$

$$V_{out} = \frac{KQ}{b} + \frac{KQ'}{b}$$

$$V_{out} = \frac{KQ}{b}\left(1 - \frac{a}{b}\right)$$

Concept Reminder

Two similar conducting balls having charges +q and −q are placed at a separation d from each other. If radius of each ball is r such that d >> r, then capacitance of two ball system will be

$C = 2\pi\varepsilon_0 r$.

$$V_{out} = \frac{Q(b-a)}{4\pi\varepsilon_0 b^2}$$

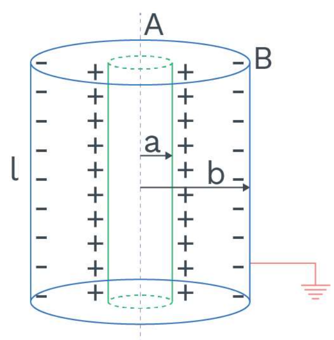
$$C = \frac{Q}{V_{out}} \qquad \boxed{\therefore C = \frac{4\pi\varepsilon_0 b^2}{b-a}}$$

- **Cylindrical Capacitor:-**

  Cylindrical capacitor consists of 2 co-axial cylinders of radii a and b & length l. If a charge q is given to inner cylinder, induced charge -q will reach the inner surface of the outer cylinder. By symmetry, the electric field in the region between the cylinder is radially outward.

  By using Gauss's theorem, the electric field at a distance r from the axis of the cylinders is given by -

$$E = \frac{1}{2\pi\varepsilon_0 l}\frac{q}{r}$$



Potential difference between cylinders is given by

$$V = \int_b^a \vec{E}.d\vec{r} = -\frac{1}{2\pi\varepsilon_0 l} q \int_b^a \frac{dr}{r}$$

$$= \frac{-q}{2\pi\varepsilon_0 l}\left(\ln\frac{a}{b}\right) = \frac{q}{2\pi\varepsilon_0 l}\ln\left(\frac{b}{a}\right)$$

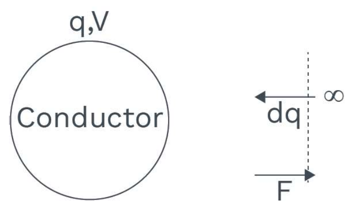
or $C = \frac{q}{V} = \frac{2\pi\varepsilon_0 l}{\left(\ln\frac{b}{a}\right)}$

Potential Energy or Self Energy of An Isolated Conductor:-

Work done in charging the conductor to charge it against its own electric field or total energy stored in electric field of conductor is called self energy or self-potential energy of conductor.

Work done by external agent to charge a conductor

Work has to be done to charge a conductor because charge present on it will repel the next incoming charged element.



Let at an instant charge of conductor is q and potential is V.

Work done by external agent to bring small, charged element dq from infinity to surface of conductor.

$dw = dq.[V_f - V_i]$

but $V_i = V_\infty = 0$ & $V_f = V$

$$\therefore \ dw = dq.[V - 0]$$

$$dw = V.dq$$

By integrating on both side

$$\Rightarrow \int dw = \int V.dq$$

$$\because V = q / C$$

$$\Rightarrow \ W = \int_{q_1}^{q_2} \frac{q}{C}.dq$$

$$\Rightarrow W = \frac{1}{C}\left(\frac{q^2}{2}\right)_{q_1}^{q_2}$$

$$\Rightarrow \ W = \frac{q_2^2 - q_1^2}{2C}$$

$$\because \ q_1 = CV_1, \ q_2 = CV_2$$

Therefore work done by external agent to increases the potential of conductor from V_1 to V_2

$$\boxed{W = \frac{C}{2}(V_2^2 - V_1^2)} \quad \text{for 0 to V}$$

$$W = \frac{C}{2}(V^2 - 0^2)$$

$$W = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} \quad (\text{Since } Q = CV)$$

Rack your Brain

What is energy per unit volume for a capacitor having area A and separation d kept at potential difference V.

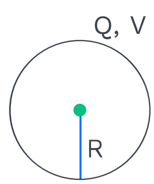
Since electrostatic field is conservative field and in conservative field work done by external agent is stored in form of potential energy.

- Potential energy of conductor will be stored in electric field.
- Potential energy of a conductor which is charged by V potential is given by

$$P.E. = W_{external}$$

$$\boxed{U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}}$$

For example: P.E of a conducting sphere is given by



$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$$

$\because C = 4\pi\varepsilon_0 R$

$$U = \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 R}$$

$$\boxed{U = \frac{KQ^2}{2R}}$$

This P.E will be stored at the surface and outside the surface.

$$\boxed{W_{battery} = QV = CV^2 = \frac{Q^2}{C}}$$

$$W_{battery} = \Delta U + \text{heat}$$

$$\Rightarrow CV^2 = \left(\frac{CV^2}{2} - 0\right) + \text{heat}$$

$$\Rightarrow \boxed{\text{Heat} = \frac{CV^2}{2} = \frac{Q^2}{2C} = \frac{QV}{2}}$$

Concept Reminder

If a capacitor is charged by a battery of potential V, then work done by battery is double of energy stored in capacitor. Half of energy is lost as heat and electro-magnetic radiation due to transient current during charging process.

where Q = Charge on the conductor

V = Potential of the conductor

C = Capacitance of the conductor

- Self energy is stored in term of electric field of the conductor with energy density (Energy per unit volume)

$$\frac{dU}{dV} = \frac{1}{2}\varepsilon_0 E^2$$

[The energy density in a medium is $\frac{1}{2}\varepsilon_0\varepsilon_r E^2$]

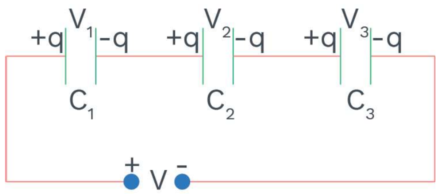
where E is the electric field at the point.

- In charged conductor, energy stored is only Outside the conductor but in case of charged insulating material it is outside as well as inside the insulator.

Combination of Capacitor

- **Capacitor in Series**

Let us consider three capacitors of capacities C_1, C_2 and C_3 connected in series across a source of potential difference 'V' as shown in figure.



In this case, charge acquired by all the three capacitors will be same. As the capacitors are different, the potentials developed across them will be different.

$$q = C_1V_1 = C_2V_2 = C_3V_3$$

$$V_1 = \frac{q}{C_1}, V_2 = \frac{q}{C_2}, V_3 = \frac{q}{C_3}$$

But $V = V_1 + V_2 + V_3$

$$V = q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \quad ....(1)$$

Concept Reminder

Series combinations are used when a high voltage (which a single capacitor cannot bear) is to be divided on several capacitors.

If a single capacitor when connected across the same source draws the same charge, that capacitance is said to be the equivalent capacitance of the three capacitors. If C_S is the equivalent capacitance.

$$C_S = \frac{q}{V}$$

$$V = \frac{q}{C_S} \qquad \text{.... (2)}$$

Substituting (2) in (1)

$$\frac{q}{C_S} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\boxed{\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

In general, $\frac{1}{C_S} = \sum \frac{1}{C_n}$

Key Points

- Combination of capacitors
- Equivalent capacitance

Note:

- In series combination of capacitor equivalent is always less the smallest capacitor of combination.
- In series combination of capacitor the smallest capacitor gets maximum potential.
- In series, ratio of charges on three capacitors is 1 : 1 :1.
- The ratio of potential differences across three capacitors is
- $V_1 : V_2 : V_3 = \frac{Q}{C_1} : \frac{Q}{C_2} : \frac{Q}{C_3} = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$
- P.D across first capacitor is

$$V_1 = \frac{\frac{1}{C_1}}{\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)} V$$

similarly, we can find V_2 and V_3.

Energy stored in the combination: -

$$U_{combination} = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3}$$

$$U_{combination} = \frac{Q^2}{2C_{eq}}$$

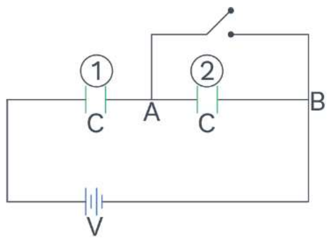
Energy supplied by battery in charging the combination

$$U_{battery} = Q \times V = Q.\frac{Q}{C_{eq}} = \frac{Q^2}{C_{eq}}$$

$$\boxed{\frac{U_{combination}}{U_{battery}} = \frac{1}{2}}$$

Note: Half of the total energy supplied by the battery is stored in kind of electrostatics energy and half of the energy is converted into heat through resistance.

Ex. Consider the circuit as shown below and find the followings: -



(i) Final charge on each capacitor before closing the switch.
(ii) Work done by the battery
(iii) When switch is closed, then find charge on each capacitor.
(iv) Charge flown by the battery after closing the switch.
(v) Heat loss in the circuit after closing the switch.

Sol. (i) $C_{eff} = \frac{C}{2} \Rightarrow Q = \frac{CV}{2}$

(ii) $W = Q_{flow} \times Emf = \frac{CV}{2} \times V = \frac{CV^2}{2}$

(iii) Now switch is closed, then charge on each capacitor on closing switch potential difference between A & B = 0.

$\therefore \quad Q_1 = CV$

$Q_2 = 0$

(iv) Charge flow = $CV - \frac{CV}{2} = \frac{CV}{2}$

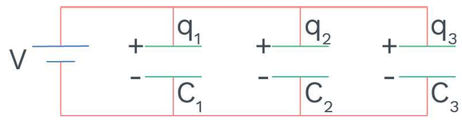
(v) Work done $= \Delta U + \text{heat}$

$$\frac{CV^2}{2} = \left[\frac{CV^2}{2} - \frac{1}{2}\left(\frac{CV^2}{2}\right)\right] + \text{heat}$$

$$\therefore \quad \text{Heat} = \frac{CV^2}{4}$$

Capacitors in Parallel

Let us consider three capacitors of capacities C_1, C_2 and C_3 connected in parallel across a source 'V' as shown.



As all capacitors are connected in parallel, the potential difference across any of the capacitors is same. Here charge gets shared depending upon their capacitances for maintaining same potential.

Concept Reminder

In series connection, the equivalent capacitance is always less than any individual capacitance. And for parallel connection, the equivalent capacitance is always greater than any individual capacitance.

$$V = \frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{q_3}{C_3} \quad \therefore \quad q_1 + q_2 + q_3 = C_1V + C_2V + C_3V$$

$$q = V(C_1 + C_2 + C_3)$$

$$\frac{q}{V} = C_1 + C_2 + C_3 \qquad \text{... (1)}$$

If a single capacitor when connected to the same source draws a charge q then that capacitor is said to be the effective or equivalent capacitor for the three parallel capacitors. If the effective capacitance is C_p,

$$C_P = \frac{q}{V} \qquad \text{... (2)}$$

from (1) & (2)

$$\boxed{C_P = C_1 + C_2 + C_3}$$

In general, $C_P = \sum C_n$

Note:

- The resultant capacity of parallel combination is greater than the largest capacity of the capacitors of the combination.
- In parallel, ratio of P.D. on three capacitors is 1 : 1 : 1.
- The ratio of charges on three capacitors is
  $Q_1 : Q_2 : Q_3 = C_1V : C_2V : C_3V = C_1 : C_2 : C_3$
- The charge on first capacitor is

$$\boxed{Q_1 = \frac{C_1}{C_1 + C_2 + C_3} Q}$$

Similarly, we can find Q_2 and Q_3.

- When ‘n’ identical capacitors each of capacity ‘C’ are first connected in series and next connected in parallel then the ratio of their effective capacities

$$C_s = \frac{C}{n}; C_p = nC \qquad \frac{C_s}{C_p} = 1 : n^2$$

Energy stored in the combination:

$$V_{combination} = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + ...$$

$$= \frac{1}{2}(C_1 + C_2 + C_3....)V^2$$

$$= \frac{1}{2}C_{eq}V^2$$

$$U_{battery} = QV = CV^2$$

$$\frac{U_{combination}}{U_{battery}} = \frac{1}{2}$$

Note: Half of the total energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance.

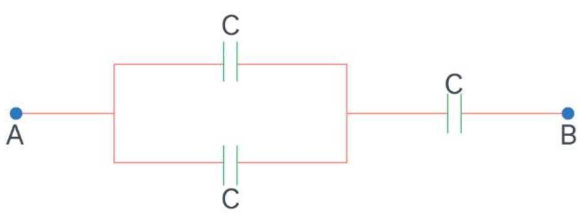
Rack your Brain

A capacitor is charged by a battery. The battery is removed, and another identical uncharged capacitor is connected in parallel. The total electrostatic energy of resulting system.
(1) decreases by factor of 2.
(2) remains same
(3) increases by a factor of 2.
(4) increases by a factor of 4.

Ex. The equivalent capacity between A and B in the given circuit is

Sol. Here 12 μF and 12 μF are short circuited. Hence they are not charged.
∴ Take only 8 μF and 8 μF parallel combination.
C = 8 + 8 = 16 μF.

Ex. Three identical capacitors are connected as shown. Each of them can withstand to a maximum 100 V potential difference. What is the maximum voltage that can be applied across A and B so that no capacitor gets spoiled.



Sol. Let q_{max} be the maximum charge supplied by the battery between A and B so that no capacitor gets spoiled.

For each capacitor

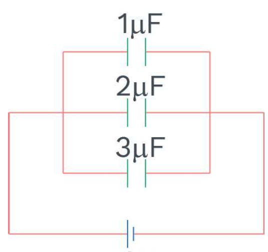
$q_{max} = CV_0 = C(100) = 100\ C$

For the combination $q_{max} = C_{equivalent}\left(V_{max}\right)$

$$100C = \frac{2}{3}C\left(V_{max}\right) \Rightarrow V_{max} = 150V$$

Among 150V, potential difference across parallel combination is 50V and the potential difference across the other capacitor is 100V.

Ex. 3 initially uncharged capacitors are connected to a battery of 10 V in parallel combination. Find out following



(i) charge flowing through battery

(ii) total energy stored in capacitors

(iii) heat produced in the electric circuit

(iv) potential energy in the capacitor of capacitance 3μF.

Sol. $C_{eq} = 1 + 2 + 3 = 6\ \ F$

(i) $Q = C_{eq}V = 6 \times 10\ \ C = 60\ \ C$

(ii) $U_{total} = \frac{1}{2} \times 6 \times 10 \times 10 = 300\ \ J$

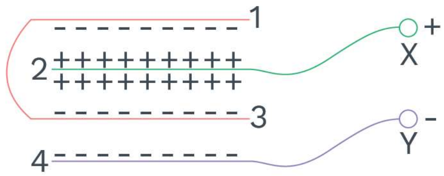
(iii) heat produced

$W - U = CV - U_{total} = 60 \times 10 - 300 = 300\ \ J$

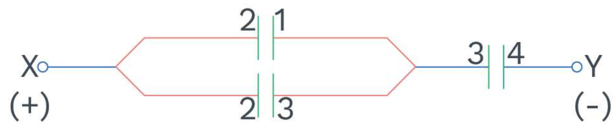
(iv) $U_{3\ \ F} = \frac{1}{2} \times 3 \times 10^{-6} \times 10 \times 10 = 150\ \ J$

Ex. 4 identical metal plates are located in air at equal distance d from one another. The area of each plate is A. Find the equivalent capacitance of the system between X and Y. (X is high potential, Y is low potential)

Sol. Let us give numbers to 4 plates. Here X and Y are connected to the positive and negative terminals of the battery, then the charge distribution will be as shown



Here the given arrangement can be represented as the grouping of 3 identical capacitors each of capacity $\frac{\epsilon_0 A}{d}$. The arrangement will be as shown



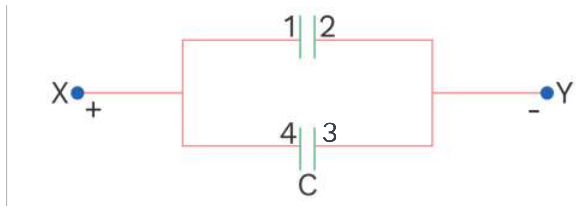
Now the equivalent capacitance between X and Y is

$$C_{XY} = \frac{(C+C)C}{C+C+C} = \frac{2C}{3} = \frac{2\epsilon_0 A}{3d}$$

Ex. Find equivalent capacity between X and Y

Sol. Let give numbers to the four plates.

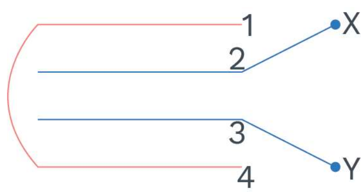
Here the arrangement can be represented as grouping of two identical capacitors each of capacity $\frac{\epsilon_0 A}{d}$. The arrangement will be as shown,



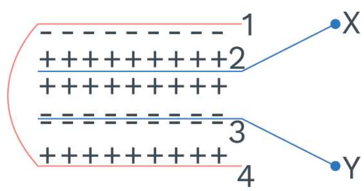
Now the equivalent capacitance between X and Y is

$$C_{XY} = (C + C) = 2C = 2\frac{\epsilon_0 A}{d}$$

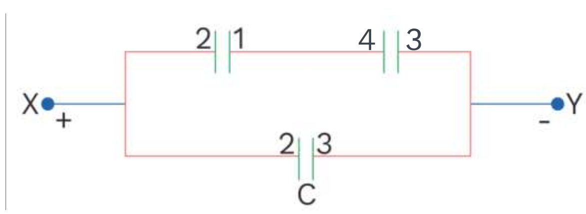
Ex. Find equivalent capacity X and Y. (X is high potential, Y is low potential)



Sol. Let us give numbers to the 4 plates. Here X and Y are connected to the positive and negative terminals of the battery.



Now the arrangement can be represented as the grouping of 3 identical capacitors each of capacity $\frac{\epsilon_0 A}{d}$. The arrangement will be as shown

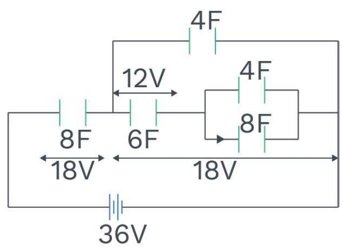


Now the equivalent capacitance between X and Y is

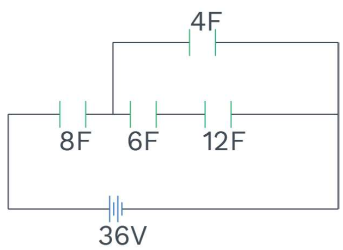
$$C_{XY} = \left(\frac{C.C}{C + C}\right) + C = \frac{C}{2} + C = \frac{3}{2}C$$

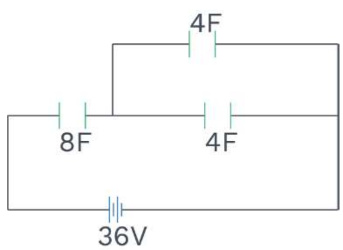
$$C_{XY} = \frac{3}{2}C = \frac{3}{2}\frac{\epsilon_0 A}{d}$$

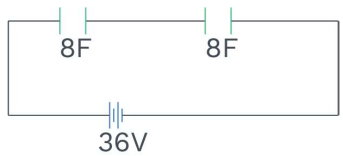
Ex. Find charges and potential difference of each capacitor.



Sol.







$$C_{eq} = \frac{8 \times 8}{8 + 8} = \frac{64}{16} = 4F$$

Net C = 4F

Q = CV = 4 × 36 = 144 C

Q across 8 F = $CV = 8 \times 18 = 144C$

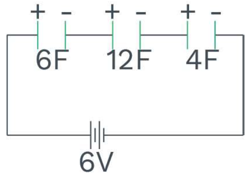
Q across upward 4F = 18 × 4 = 72 C

Q across 6F = 12 × 6 = 72 C

Q across inward 4F = 24C

Q across 8F = 48C

Ex. Find charges and potential difference of each capacitor.



Sol. $\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{12} + \frac{1}{4}$

$\Rightarrow \quad \frac{1}{C_{eq}} = \frac{2+1+3}{12} = \frac{6}{12} = \frac{1}{2}$

$\Rightarrow \quad C_{eq} = 2F$

Net C = 2F

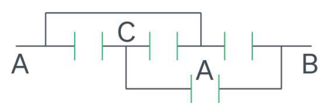
Q = CV = 2 × 6 = 12C

$V_{6F} = \frac{Q}{C} = \frac{12}{6} = 2V$

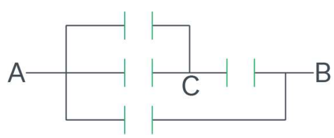
$V_{12F} = \frac{12}{12} = 1V$

$V_{4F} = \frac{12}{4} = 3V$

Ex. Find C_{AB}, capacity of each capacitance is C.

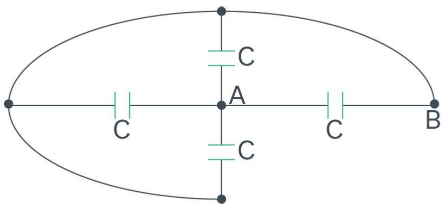


Sol.

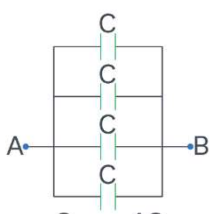


$C_{AB} = \frac{2C}{3} + C = \frac{5C}{3}$

Ex.

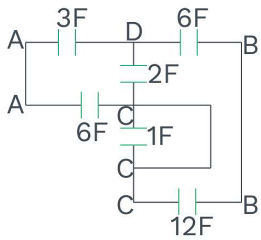


Sol.

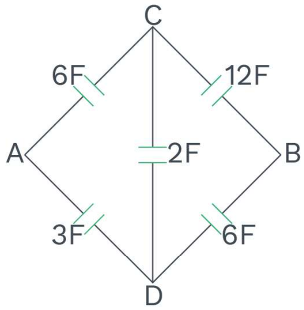


$C_{AB} = 4C$

Ex. Find C_{AB}



Sol.

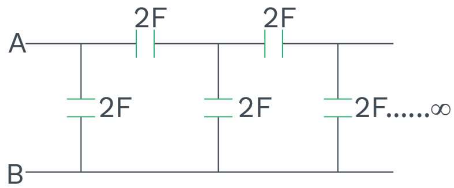


$C_{AB} = 6F$

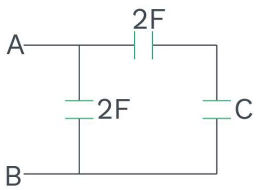
The above circuit forms Wheatstone Bridge circuit so there will be no current in 2F branch and to calculate the equivalent capacitance we can remove this.

$$C_{AB} = \frac{6 \times 12}{6 + 12} + \frac{3 \times 6}{3 + 6} = \frac{6 \times 12}{18} + \frac{3 \times 6}{9} = 4 + 2 = 6F$$

Ex. Find C_{AB}



Sol.



$$2 + \frac{2C}{2 + C} = C$$

$\Rightarrow \quad 2C = (2 + C)(C - 2)$

$\Rightarrow \quad 2C = 2C - 4 + C^2 - 2C$

$\Rightarrow \quad C^2 - 2C - 4 = 0$

$\Rightarrow \quad C = \dfrac{2 \pm \sqrt{4 + 4 \times 4}}{2} = \dfrac{2 + \sqrt{20}}{2}$

$\therefore \quad C = 1 + \sqrt{5} = 3.3\text{ F}$

Ex. 20 capacitors of capacitance 2F & breakdown voltage 5V are connected in series such 10 arrangement are arranged in parallel. Find (i) max. voltage for which no capacitor will fuse (ii) C_{eff}

Sol. $m \times n = 200$ = total capacitors

Given m = 10 = No. of columns

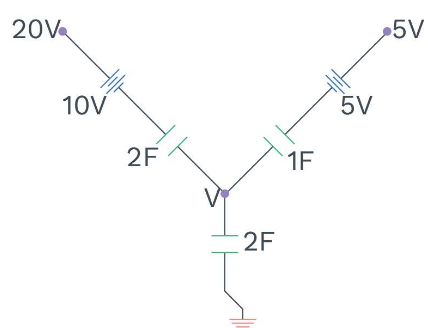
$\therefore \quad n = 20$ = No. of capacitors in a row

(i) $\therefore$ max voltage = nV

$\Rightarrow$ Max voltage = $20 \times 5 = 100\text{ V}$

(ii) $\therefore \; C_{eff} = \dfrac{Cm}{n} = \dfrac{2 \times 10}{20} = 1\text{F}$

Ex. Find potential at A.



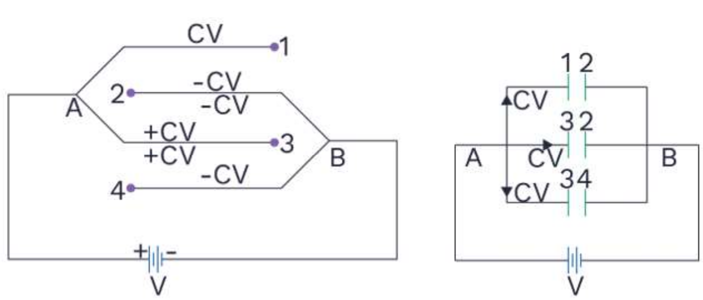
Sol. $(V - 0)2 + (V - 10)2 + (V - 10) \times 1 = 0$

$\Rightarrow \quad 2V + 2V - 20 + V - 10 = 0$

$\Rightarrow \quad 5V - 30 = 0$

$\Rightarrow \quad V = 6V$

Ex. If C is capacitance of each, then find



Sol. $C_{eq} = 3C$

(i) Charge flown $Q = C_{eq}V = 3CV$

(ii) Charge on each plate

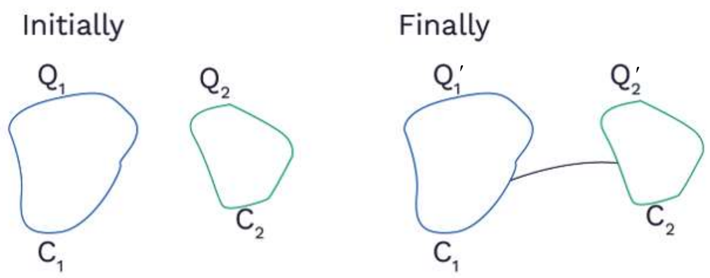
On 1st plate = CV

On 2nd plate = $2(-CV) = -2CV$

On 3rd plate = $2(CV) = 2CV$

On 4th plate = $-CV$

Sharing of Charges on Joining Two Conductors (By a Conducting Wire):



(i) Whenever there is potential difference, then there will be movement of charge.

(ii) If released, positive charge moves from high potential to low potential [if only electric force act on charge].

(iii) If released, negative charge moves from low potential to high potential [if only electric force act on charge].

(iv) The movement of charge will continue till there is potential difference between the conductors or movement of charge stops when potential difference becomes zero.

(v) Formulae related with redistribution of charges:

BEFORE CONNNECTING THE CONDUCTORS		
Parameter	Ist Conductor	IInd Conductor
Capacitance	C_1	C_2
Charge	Q_1	Q_2
Potential	V_1	V_2

Concept Reminder

There is always a loss of potential energy stored in capacitors due to redistribution of charges. This loss is due to heat produced in connecting wire and EM radiation during charge flow.

AFTER CONNNECTING THE CONDUCTORS		
Parameter	I^{st} Conductor	II^{nd} Conductor
Capacitance	C'_1	C'_2
Charge	Q'_1	Q'_2
Potential	V	V

After connecting, Potential will be same

$$\Rightarrow V = \frac{Q_1'}{C_1} = \frac{Q_2'}{C_2} \Rightarrow \frac{Q_1'}{Q_2'} = \frac{C_1}{C_2}$$

But $Q_1' + Q_2' = Q_1 + Q_2$ (charge is conserved)

$$\therefore \ V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$$\therefore \ \boxed{Q_1' = \frac{C_1}{C_1 + C_2}(Q_1 + Q_2)}$$

and $$\boxed{Q_2' = \frac{C_2}{C_1 + C_2}(Q_1 + Q_2)}$$

Heat loss during redistribution:

$$\boxed{\Delta H = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}\left(V_1 - V_2\right)^2}$$

The loss of energy is in form of Joule heating in the wire.

Note: Always put Q_1, Q_2, V_1 and V_2 with proper sign convection.

Ex. When 30μC charge is given to isolated conductor of capacitance 5μF. Find out the following -

(i) Potential of the conductor

(ii) Energy stored in the electric field of conductor

(iii) Now if this conductor is now connected to another isolated conductor by a conducting wire (at very large distance) of total charge 50 μC and capacity 10 μF then

(a) Find out the 'common potential' of both the conductors

(b) Obtain the heat dissipated during the process of charge of distribution.

(c) Find the ratio of final charges on conductors.

(d) Obtain the final charges on each conductor.

Sol. $Q_1 = 30\ \ C,\ C_1 = 5\ \ C$

(i) $V_1 = \frac{Q_1}{C_1} = \frac{30}{5} = 6V$

(ii) $U = \frac{1}{2}\frac{Q_1^2}{C_1} = \frac{1}{2}\frac{(30 \times 10^{-6})^2}{(5 \times 10^{-6})} = 90\ \ J$

(iii) $Q_2 = 50\ \ C, C_2 = 10\ \ F,\ V_2 = \frac{Q_2}{C_2} = \frac{50}{10} = 5V$

(a) Common potential $V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{30 + 50}{5 + 10} = \frac{16}{3}V$

(b) $\Delta H = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2 = \frac{1}{2}\left(\frac{5 \times 10}{5 + 10}\right)(6 - 5)^2$

$= \frac{1}{2} \times \frac{50}{15} \times 1 = \frac{5}{3}\ \ J$

(c) $\frac{Q_1'}{Q_2'} = \frac{C_1}{C_2} = \frac{5}{10} = \frac{1}{2}$

(d) $Q_1' = C_1V = 5 \times \frac{16}{3} = \frac{80}{3}\ \ C$

$Q_2' = C_2V = 10 \times \frac{16}{3} = \frac{160}{3}\ \ C$

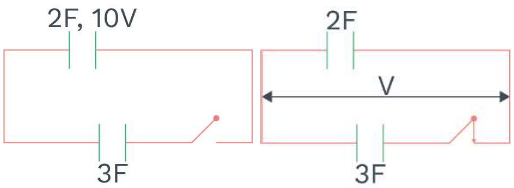
Ex. 2F capacitor is charged by 10 V battery. Now it is connected in parallel with 3F capacitor. Find

(i) Common potential

(ii) Charge transferred

(iii) Heat loss

Sol.



(i) By using conservation of charge,

$2 \times 10 + 0 = (2 + 3)V$

$20 = 5V$

$V = 4V$

(ii) charge transferred = Final charge – Initial charge

$= (2 \times 10) - (2 \times 4) = 20 - 8 = 12\,C$

(iii) Heat loss

Heat loss = $U_i - U_f$

$$= \frac{(20)^2}{2\times 2} - \left(\frac{(8)^2}{2\times 2} + \frac{(12)^2}{2\times 3}\right)$$

$$= 100 - (16 + 24)$$

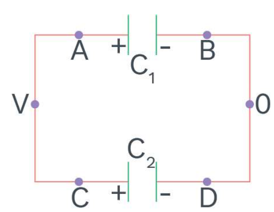
$$= 100 - 40 = 60\ \text{J}$$

Ex. Find the following if 'A' is connected with C and B is connected with D.

(i) How much electric charge flows in circuit.

(ii) How much heat is produced in circuit.

Sol. (i)



Let potential of 'B' and 'D' is zero and common potential on capacitors is V, then at 'A' and 'C' it will be V.

Before connecting total charge is

$$Q_1 + Q_2 = 2\times 20 + 3\times 10 = 70\ \mu C$$

After connecting,

$$Q_1' + Q_2' = 2V + 3V = 5V$$

[Using q = CV for each capacitor]

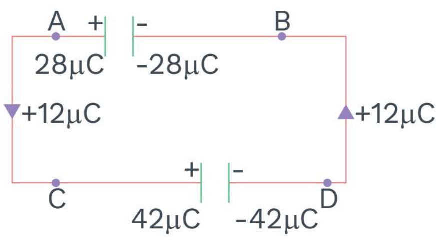
By charge conservation,

3V + 2V = 70

$5V = 70\ V \Rightarrow V = \frac{70}{5} = 14\ \text{Volt}$

Charge flow = 40 – 28 = 12 μC (Using ΔQ for $2\ f$ capacitor)

Now final charge on each plate is shown in the figure.



(ii) Heat produced-

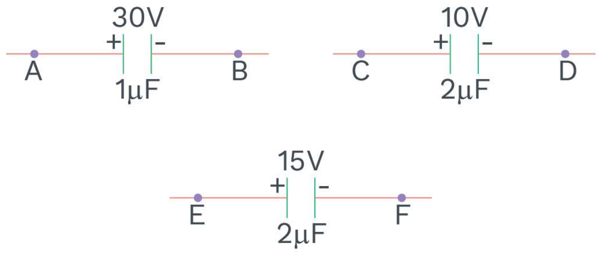
$$= \frac{1}{2}\times 2\times(20)^2 + \frac{1}{2}\times 3\times(10)^2 - \frac{1}{2}\times 5\times(14)^2$$

$$= 400 + 150 - 490$$

$$= 550 - 490 = 60 \text{ J}$$

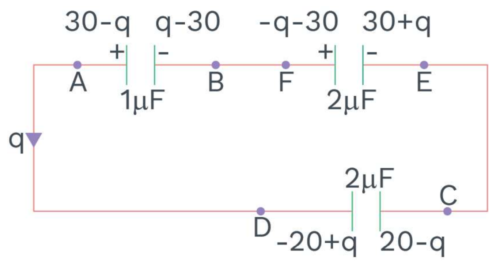
Note: When capacitor plates are joined then the charge remains conserved.

Ex. 3 capacitors as given of capacitance 1μF, 2μF & 2μF are charged upto potential difference 30 V, 10 V and 15 V respectively. If terminal A is connected with D, C is connected with E and F is connected with B. Then find charge flow in the circuit and find the final charges on capacitors.



Sol. Let charge flow is q. Now applying Kirchhoff's voltage low

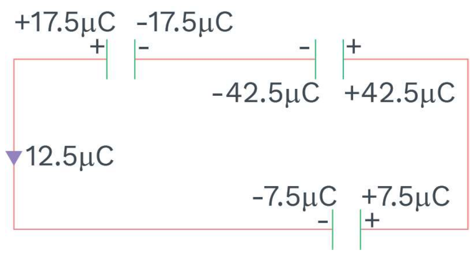
$$-\frac{(q-20)}{2} - \frac{(30+q)}{2} + \frac{30-q}{1} = 0$$



$-2q = -25$ μC

$q = 12.5$ μC

Final charges on plates



Coalescence of Charged Drops:

There are 'n' charged drops of radius 'r' and charge 'q'. The drops merge to form a bigger drop. If capacity of small drop is 'C' then

- capacity of bigger drop is $C' = n^{\frac{1}{3}} \times C$
- Potential of bigger drop is

$$V' = \frac{Q}{C'} = \frac{nq}{n^{1/3}.C} = \frac{n^{2/3}q}{C} = n^{2/3}V.$$

- Energy of bigger drop is

$$U = \frac{Q^2}{2C'} = \frac{n^2q^2}{2n^{1/3}.C} = \frac{n^{5/3}q^2}{2C} = n^{5/3}U.$$

- Surface charge density of bigger drop is

$$\sigma' = \frac{Q}{4\pi R^2} = \frac{nq}{4\pi n^{2/3}.r^2} = \frac{n^{1/3}q}{4\pi r^2} = n^{1/3}.\sigma$$

S.NO.	QUANTITY	FOR EACH CHARGED SMALL DROP	FOR THE BIG DROP
1	Radius	r	$R = n^{1/3}r$
2	Charge	q	$Q = n \times q$
3	Capacity	C	$C' = n^{1/3} \times C$
4	Potential	V	$V' = n^{2/3} \times V$
5	Energy	U	$U' = n^{5/3} \times U$
6	Surface Charge	σ	$\sigma' = n^{1/3} \times \sigma$

Concept Reminder

During coalescence of n charged drops, total charge remains conserved. As charged conservation is a fundamental conservation law.

Dielectrics:

- A dielectric is an insulating material in which electrons are tightly bound to the nuclei of the atoms.
  Ex: glass, mica, paper etc
- There are two types of dielectrics

(1) Non-polar dielectrics
(2) Polar dielectrics

Non-polar Dielectrics:-

- In non-polar dielectrics materials the centre of positive charge and centre of negative charge of each molecule coincide
- Under ordinary conditions Non-polar molecule will have zero dipole moment.
- When a Non-polar dielectric is subjected to electric field, the positive charge of each molecule is shifted in the direction of electric field and negative charge in the opposite direction.
  Ex: oxygen, nitrogen

Polar Dielectrics

- In polar dielectrics the centre of positive charges and centre of negative charges of each molecule do not coincide.
- Each molecule has a permanent dipole moment.
- When polar dielectric is subjected to external electric field, the electric field exerts torque on the dipoles, tending to align them in the direction of the field.
  Ex: H_2O, NH_3, HCl, etc
- If a dielectric is charged by induction, then induced charge q^1 is less than inducing charge q. Induced charge,

$$\boxed{q^1 = -q\left[1-\frac{1}{K}\right]}$$

where K is dielectric constant.

- Electric field due to induced charges on the dielectric is

$$E_{ind} \text{ or } E_p = E_0 - \frac{E_0}{K} = E_0\left(1-\frac{1}{k}\right).$$

Polarization vector $(\vec{P})$ **-**

It is a vector quantity which describes the extent to which molecules of dielectric become polarized by an electric field or gets oriented in the direction of field.
$\vec{P}$ = the dipole moment per unit volume of

Concept Reminder

Polar dielectrics have permanent dipole moments due to separation between centres of their positive and negative charges.

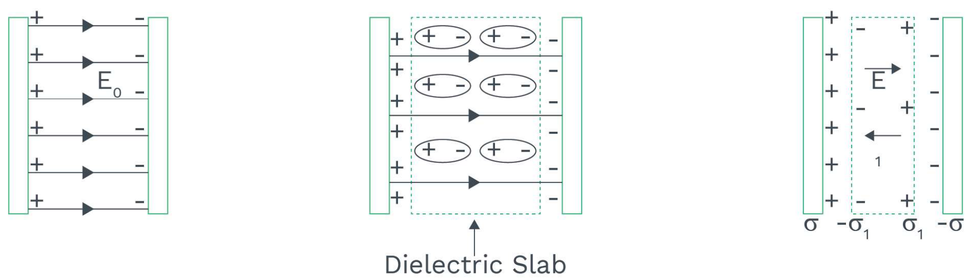
Definitions

Polarization:- The phenomenon of reorientation of polar molecules or the shifting of the positive and negative charge distributions an atom of non-polar molecule in dielectric material in presence of external electric field is called polarization.

dielectric = n $\vec{P}$ when n is number of atoms per unit volume of dielectric and $\vec{P}$ is dipole moment of an atom or molecule.

$$\vec{P} = n\vec{P} = \frac{q_i d}{Ad} = \left(\frac{q_i}{A}\right) = \sigma_1$$ = induced surface charge density.

Unit of $\vec{P}$ is C/m² Dimension : $[L^{-2}T^{1}A^{1}]$



Let E_0, V_0, C_0 be the electric field, potential difference and capacitance in the absence of dielectric.

Let E, V, C be the corresponding quantities in the presence of the dielectric respectively.

Electric field in the absence of dielectric

$$E_0 = \frac{V_0}{d} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

Electric field in the presence of dielectric

$$E = E_0 - E_i = \frac{\sigma - \sigma_i}{\varepsilon_0} = \frac{Q - Q_i}{A\varepsilon_0} = \frac{V}{d}$$ Capacitance in the absence of dielectric

$$C_0 = \frac{Q}{V_0}$$

Capacitance in the presence of dielectric

$$C = \frac{Q - Q_i}{V}$$

Dielectric constant or relative permittivity

$$K \text{ or } \varepsilon_r = \frac{E_0}{E} = \frac{V_0}{V} = \frac{C}{C_0} = \frac{Q}{Q - Q_i} = \frac{\sigma}{\sigma - \sigma_i} = \frac{\varepsilon}{\varepsilon_0}$$

From $K = \frac{Q}{Q - Q_i} \Rightarrow Q_i = Q\left(1 - \frac{1}{K}\right)$ and

Key Points

- Dielectric
- Polar dielectric
- Non-polar dielectric
- Polarization
- Dielectric constant

$$K = \frac{\sigma}{\sigma - \sigma_i} \Rightarrow \sigma_i = \sigma\left(1 - \frac{1}{K}\right)$$

Dielectric strength of Air: A conducting sphere cannot hold very large quantity of charge. It can hold a maximum charge Q such that the electric intensity on the surface is equal to dielectric strength of air $\left(3 \times 10^6 \, Vm^{-1}\right)$

i.e. $\frac{1}{4\pi \epsilon_0} \frac{Q}{R^2} = 3 \times 10^6 \, Vm^{-1}$

- **Electrical Susceptibility**
  It defines the ease with which any substance can be polarised.

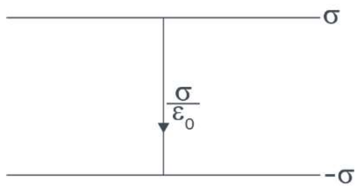
$$\chi = \frac{E_{induced}}{E_0}$$

Concept Reminder

If a very high electric field is applied in a dielectric it may behave like a conductor due to the detachment of the electrons from their parent atoms. This phenomenon is known as dielectric breakdown.

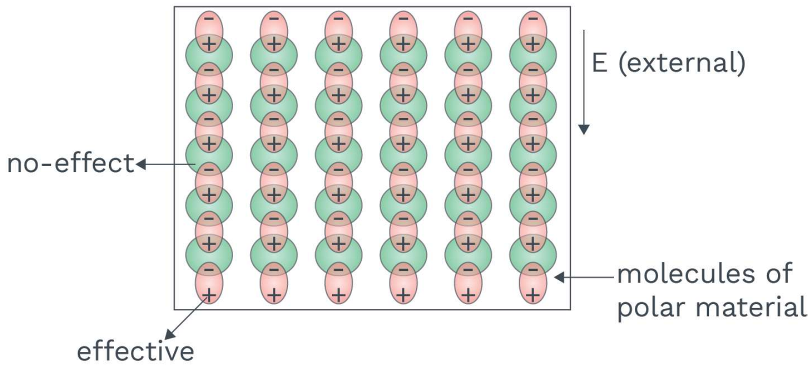
Capacitors With Dielectric

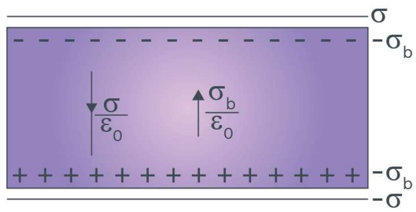
(i) In absence of dielectric



$$E = \frac{\sigma}{\varepsilon_0}$$

(ii) When a dielectric materials fills the area between the plates then molecules having dipole-moment align the direction of electric field.





σ_b = induced charge density (called bound charge as it is not due to free electrons)

(iii) If the medium is filled between the plates completely by dielectric then capacitance will be

$$C = \frac{\sigma A}{V} = \frac{\sigma A}{\frac{\sigma}{K\varepsilon_0}.d} = \frac{AK\varepsilon_0}{d}$$

Here capacitance is increased by factor K.

$$\boxed{C = \frac{AK\varepsilon_0}{d}}$$

Rack your Brain

The capacitance of a parallel plate capacitor with air as medium is 6 μF. With the introduction of dielectric medium, the capacitance becomes 30 μF. Find the permittivity of the medium $\left(\varepsilon_0 = 8.85 \times 10^{-12} C^2N^{-1}m^{-2}\right)$

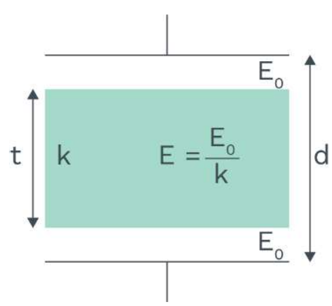
(iv) **If capacitor is partially filled with dielectric**

When the capacitor is filled partially with dielectric between plates, and thickness of dielectric slab is t(t < d) :

For a capacitor, the field E_0 is given by $E_0 = \frac{\sigma}{\varepsilon_0}$, exists in the space d.

On inserting a slab of thickness t, a field $E = \frac{E_0}{\varepsilon_r}$ appears inside the slab and a field E_0 exists in the remaining space (d – t). If 'V' is the potential difference between the plates then

$V = E_0(d - t) + Et$



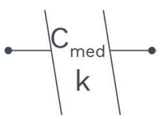
$$\Rightarrow \quad V = E_0\left[d - t + \left(\frac{E}{E_0}\right)t\right]$$

$$\because \quad \frac{E_0}{E} = k = \text{Dielectric constant}$$

$$\Rightarrow \quad V = \frac{\sigma}{\varepsilon_0}\left[d - t + \frac{t}{k}\right] = \frac{q}{A\varepsilon_0}\left[d - t + \frac{t}{k}\right]$$

$$\Rightarrow \quad C = \frac{q}{V} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{k}\right)} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{k}\right)} \qquad \text{... (i)}$$

- If the dielectric medium is present between the entire space. then t = d

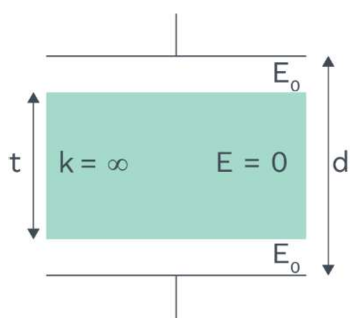


Now from equation (i) $C_{medium} = \dfrac{\varepsilon_0 kA}{d}$

- If capacitor is partially filled with a conducting slab of thickness t (t < d).

$\because \quad k = \infty$ for conductor,

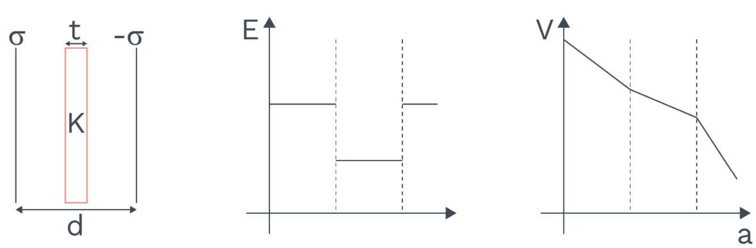
$$\text{so } C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\infty}\right)} = \frac{\varepsilon_0 A}{(d - t)}$$



Concept Reminder

The capacitance of a parallel plate capacitor remains same, when a very thin metallic sheet is placed in the space between the plates, parallel to the plates.

- **Partially filled parallel plate Capacitor**



(v) Comparison of E (electric field), σ (surface charges density), Q (charge), C (capacitance) before and after inserting a dielectric slab between the plates of a parallel plate capacitor.

- **Battery remains connected.**
  Case (1): Without dielectric

Case I

$$C = \frac{\varepsilon_0 A}{d}$$

$$Q = CV$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{CV}{A\varepsilon_0} = \frac{V}{d}$$

Here potential difference between the plates

Ed = V

$$\Rightarrow E = \frac{V}{d}$$

$$\Rightarrow \frac{V}{d} = \frac{\sigma}{\varepsilon_0}$$

Case(2): When dielectric is inserted between the plates.

Case II

$$C' = \frac{A\varepsilon_0 K}{d}$$

$$Q' = C'V \qquad E' = \frac{\sigma'}{K\varepsilon_0} = \frac{CV}{A\varepsilon_0} = \frac{V}{d} \text{also}$$

Here potential difference between the plates

E'd = V

$$\Rightarrow E' = \frac{V}{d}$$

Equating both (by case-1 & case-2)

$$\frac{\sigma}{\varepsilon_0} = \frac{\sigma'}{K\varepsilon_0} \quad ; \quad \sigma' = K\sigma$$

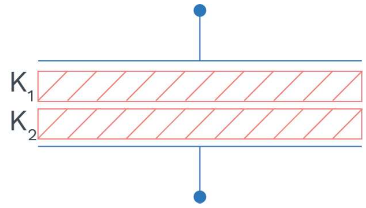
In the presence of dielectric substance, i.e. in case II capacitance of capacitor is more.

(vi) Energy density in a dielectric $= \frac{1}{2}\varepsilon_0 kE^2$

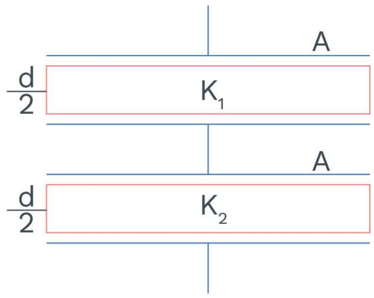
Rack your Brain

A parallel plate capacitor with oil between the plates (dielectric constant of oil, k = 2) has a capacitance C. Find the capacitance of the capacitor when oil is removed.

Ex. When the space between the plates of a parallel plate condenser is completely filled with two slabs of dielectric constants K_1 and K_2 and each slab having area A and thickness equal to $\frac{d}{2}$ as shown in the figure then find out the effective dielectric constant.



Sol.



Capacity of the upper half $C_1 = \frac{2K_1 \epsilon_0 A}{d}$

Capacity of the lower half $C_2 = \frac{2K_2 \epsilon_0 A}{d}$

C_1 and C_2 may be supposed to be connected in series.

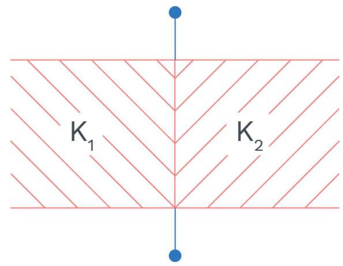
Effective capacity

$$C = \frac{C_1C_2}{C_1 + C_2} = \frac{\varepsilon_0 A}{d}\left(\frac{2K_1K_2}{K_1 + K_2}\right) = C_0\left(\frac{2K_1K_2}{K_1 + K_2}\right)$$

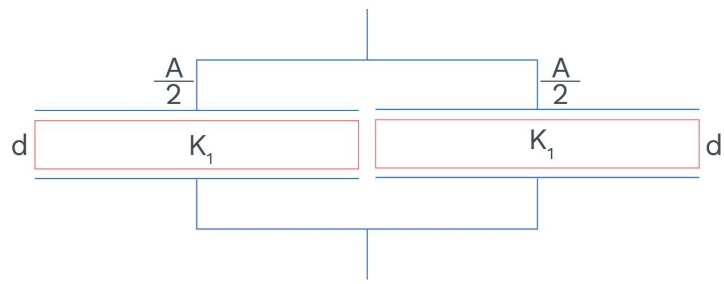
Here C_0 is the capacity of the condenser with air medium.

Effective dielectric constant $K = \left(\frac{2K_1K_2}{K_1 + K_2}\right)$

Ex. When the space between the plates of a parallel plate condenser is completely filled with two slabs of dielectric constants K_1 and K_2 and each slab having area $\frac{A}{2}$ and thickness equal to distance of separation d as shown in the figure. The effective dielectric constant is



Sol.



Capacity of the left half $C_1 = K_1 \frac{\varepsilon_0 A}{2d}$

Capacity of the right half $C_2 = K_2 \frac{\varepsilon_0 A}{2d}$

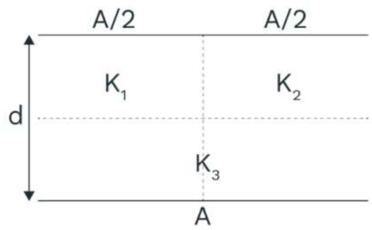
C_1 & C_2 are connected in parallel then effective capacity,

$$C = C_1 + C_2 = \frac{\varepsilon_0 A}{d}\left(\frac{K_1 + K_2}{2}\right)$$

$C = C_0\left(\frac{K_1 + K_2}{2}\right)$ where C_0 is capacity of capacitor without dielectric.

Effective dielectric constant $K = \frac{K_1 + K_2}{2}$

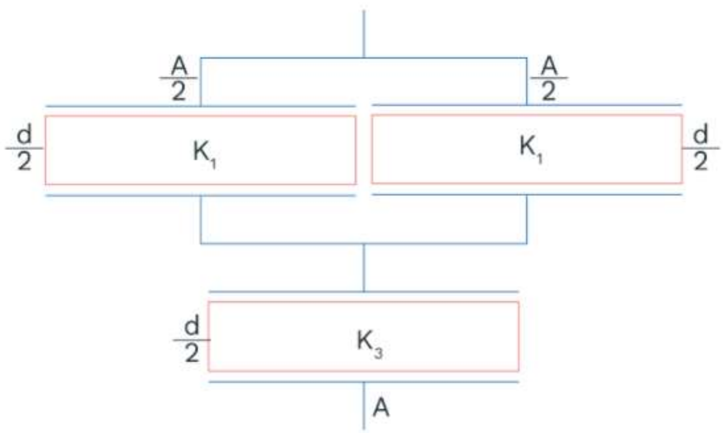
Ex. A parallel plate capacitor of area A, plate separation d and capacitance C is filled with 3 different dielectric materials having dielectric constants K_1, K_2 and K_3 as given in fig. If a single dielectric substance is to be used to have the same effective capacitance as the given combination then its dielectric constant K is given by:



Sol. Let $C = \frac{\epsilon_0 A}{d}$; $C_1 = K_1 \frac{\epsilon_0 \frac{A}{2}}{\frac{d}{2}} = K_1 C$

$$C_2 = K_2 \frac{\epsilon_0 \frac{A}{2}}{\frac{d}{2}} = K_2 C \; ; \; C_3 = \frac{K_3 \epsilon_0 A}{\frac{d}{2}} = 2K_3 C$$

The equivalent circuit as shown



$$\frac{1}{C_{eq.}} = \frac{1}{C_3} + \frac{1}{C_1 + C_2} \; , \; \frac{1}{K_{eq.} C_{eq.}} = \frac{1}{2K_3 C} + \frac{1}{(K_1 + K_2) C}$$

$$\frac{1}{K_{eq}} = \frac{1}{K_1 + K_2} + \frac{1}{2K_3}$$

Ex. The parallel plates of a capacitor have an area 0.2 (metre)^2 and area 10^{-2} metre apart. The initial potential difference between them is 3000 Volt, and it decreases to 1000 Volt when a sheet of dielectric material is inserted between the plates filling the full space. Compute: (ϵ_0 = 9 x 10^{-12} S.I. units)

(i) Original capacitance C_0.

(ii) The charge Q on each plate.

(iii) Capacitance C after insertion of the dielectric. (iv) Dielectric constant K.

(v) Permittivity $\in$ of the dielectric.

(vi) The original field E_0 between the plates.

(vii) The electric field E after insertion of the dielectric.

Sol. (i) $C_0 = \frac{\epsilon_0 A}{d} = \frac{0.2 \epsilon_0}{10^{-2}} = 20 \epsilon_0 = 20 \times 9 \times 10^{-12} = 180\,pF$

(ii) $Q = C_0 V = 180 \times 10^{-12} \times 3000 = 5.4 \times 10^{-7}\,C$

(iii) $C_1 = \frac{Q}{V_1} = \frac{5.4 \times 10^{-7}}{1000} = 540\,pF$

(iv) $K = \frac{C_1}{C_0} = \frac{540}{180} = 3$

(v) $\in = \in_r \in_0 = K \in_0 = 3 \times 9 \times 10^{-12} = 27 \times 10^{-12}$

(vi) $E_0 = \frac{V}{d} = \frac{3000}{10^{-2}} = 3 \times 10^5\ V/m$

(vii) $E = \frac{V_1}{d} = \frac{1000}{10^{-2}} = 1 \times 10^5\ V/m$

Rack your Brain

Three capacitors each of capacitance C and of breakdown voltage V are joined in series. Find the capacitance and breakdown voltage of the combination.

Important Points

1. Initially capacitor is charged to some potential. Now the separation between the plates of capacitor is increased.

- If battery is disconnected, then.

(i)	Capacitance	decreases	because $C = \frac{\varepsilon_0 A}{d}$
(ii)	Charge	(same)	
(iii)	Potential	increases	because Q = CV
(iv)	$\vec{E}$	(same)	because V/d increases in same ratio
(v)	Energy	increases	because $\frac{Q^2}{2C}$ (Q = same, C decreases)
(vi)	Energy density	(same)	because $= \frac{\sigma^2}{2\varepsilon_0}$

- If battery remains connected, then.

(i)	Capacitance	decreases	because $C = \frac{\varepsilon_0 A}{d}$
(ii)	Charge	decreases	Q = CV
(iii)	Potential	same	
(iv)	Electric field	decreases	$\vec{E} = \frac{V}{d}$
(v)	Energy	decreases	$U = \frac{CV^2}{d}$
(vi)	Energy density	decreases	$Ed = \frac{\sigma^2}{2\varepsilon_0}$

2. A capacitor is charged, and a slab of dielectric constant K is inserted between the plates of the capacitor.

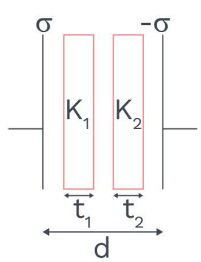
- If battery is disconnected, then.

(i)	Capacitance	increases by K times	$C' = kC_0$
(ii)	Charge	Same	Q = CV
(iii)	Potential	decreases by K times	
(iv)	Electric Field	decreases by K times	$\left(E = \frac{\sigma}{\varepsilon_0} \text{ or } \frac{V}{d}\right)$
(v)	Energy	decreases by K times	$\frac{Q^2}{2C}$
(vi)	Energy Density	decreases by K times	$\left(\frac{U}{V}\right)$

- If battery remains connected, then

(i)	Capacitance	increases by K times	$C' = kC_0$
(ii)	Charge	increases by K times	$Q = CV$
(iii)	Potential	Same	
(iv)	Electric Field	Same	$\left(\frac{V}{d} = \text{same}\right)$
(v)	Energy	increases by K times	$\left(U = \frac{1}{2}CV^2\right)$
(vi)	Energy Density	increases K times	$\left(\frac{U}{V}\right)$

Ex.

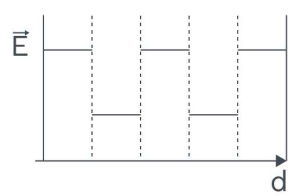


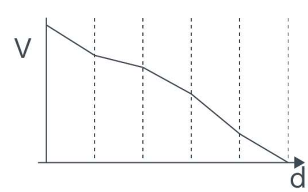
(i) Draw $\vec{E}$ V/S distance graph

(ii) Draw V V/S distance graph

(iii) Find the C. [If $K_1 > K_2$]

Sol.

(i) 

(ii) 

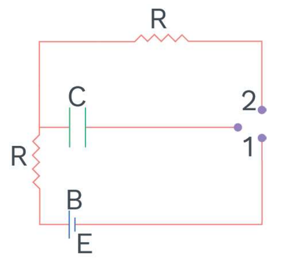
(iii) $$C = \frac{\varepsilon_0 A}{d - t_1 - t_2 + \frac{t_1}{K_1} + \frac{t_2}{K_2}}$$

R-C Circuits

Charging of a condenser

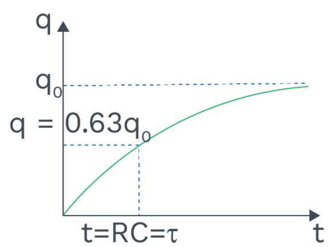
(i) In the following circuit. If key 1 is closed, then the capacitor gets charged. Finite time is taken in the charging process. The quantity of electric charge at any instant of time t is given by

$$\boxed{q = q_0[1 - e^{-(t/RC)}]}$$



Where q_0 = maximum final value of charge at t = ∞. According to this equation, the quantity of charge on the capacitor increases exponentially with increase of time.

(ii) If $t = RC = \tau$ then



$$q = q_0[1 - e^{-(RC/RC)}] = q_0\left[1 - \frac{1}{e}\right]$$

or $q = q_0(1 - 0.37) = 0.63\,q_0$

$= 63\%$ of q_0

Rack your Brain

A capacitor of capacitance C is charged in series with a resistor R with help of battery of emf V volts. Find charge on capacitor at

(i) t = 0 (ii) t = RC

(iii) t = ∞

(iii) Time t = RC is known as 'time constant'.
i.e. the 'time constant' is that time during which the charge rises on the capacitor plates to 63% of its maximum value.

(iv) The potential difference across the capacitor plates at any instant of time is given by

$$\boxed{V = V_0[1 - e^{-(t/RC)}]}\ \text{volt}$$

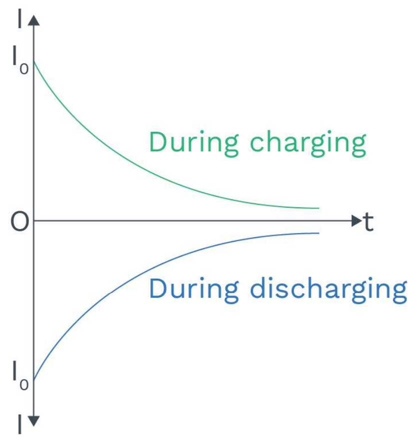
(v) The potential-time curve is also similar to that of charge-time. During charging process an electric current flows in circuit for a short interval of time which is known as the 'transient current'. The value of 'transient current' at any instant of time is given by

$$\boxed{I = I_0[e^{-(t/RC)}] \text{ ampere}}$$

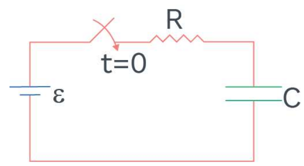
According to above equation the current falls in the circuit exponentially.

(vi) If $t = RC = \tau$ $I = I_0 e^{(-RC/RC)} = \frac{I_0}{e} = 0.37 I_0 = 37\% \text{ of } I_0$

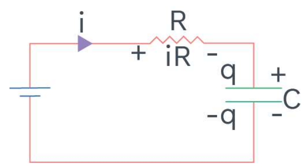
i.e. 'time constant' is the time during which current in the circuit falls to 37% of its maximum value.



Derivation



it is given that initially capacitor is not charged.



Let at any time t.
Applying Kirchhoff voltage law

$$\varepsilon - iR - \frac{q}{C} = 0$$

$$\Rightarrow \quad iR = \frac{\varepsilon C - q}{C}$$

$$\Rightarrow \quad i = \frac{\varepsilon C - q}{CR}$$

$$\Rightarrow \quad \frac{dq}{dt} = \frac{\varepsilon C - q}{CR} \quad \Rightarrow \quad \frac{CR}{\varepsilon C - q}.dq = dt$$

$$\Rightarrow \quad \int_0^q \frac{dq}{\varepsilon C - q} = \int_0^t \frac{dt}{CR}$$

$$\Rightarrow \quad \left[-\ln\left(\varepsilon C - q\right)\right]_0^q = \frac{1}{RC}.t$$

$$\Rightarrow \quad -\ln(\varepsilon C - q) + \ln \varepsilon C = \frac{t}{RC}$$

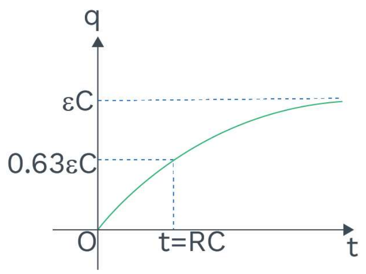
$$\ln \frac{\varepsilon C}{\varepsilon C - q} = \frac{t}{RC}$$

$$\varepsilon C - q = \varepsilon C.e^{-t/RC}$$

$$\boxed{q = \varepsilon C(1 - e^{-t/RC})}$$

$$\boxed{q = q_0(1 - e^{-t/RC})}$$

RC = ‘time constant’ of the RC series circuit.



Key Points

- Charging of capacitor
- Discharging of capacitor
- Time constant of circuit

After one time constant

$$q = \varepsilon C\left(1 - \frac{1}{e}\right) = \varepsilon C(1 - 0.37) = 0.63\,\varepsilon C$$

Current at any time t

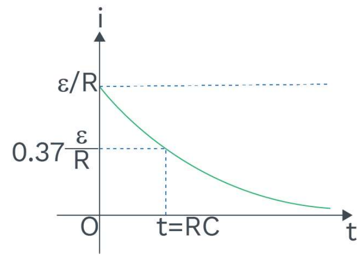
$$i = \frac{dq}{dt} = \frac{d}{dt}\left[\varepsilon C\left(1 - e^{-t/RC}\right)\right]$$

$$\Rightarrow \quad i = \varepsilon C\left[0 - e^{-t/RC}\left(-\frac{1}{RC}\right)\right]$$

$$\Rightarrow \quad i = \frac{\varepsilon C}{RC}\left(e^{-t/RC}\right)$$

$$\boxed{i = \frac{\varepsilon}{R}\left(e^{-t/RC}\right)}$$

$$i = i_{max}e^{-t/RC} \qquad \begin{bmatrix} \because t = 0, i = \varepsilon / R \\ i_{max} = \frac{\varepsilon}{R} \end{bmatrix}$$

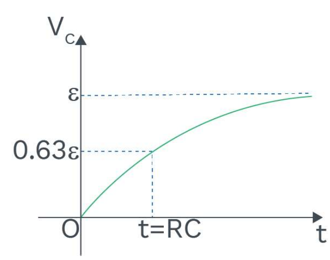


Concept Reminder

During charging of a capacitor in RC circuit, At t = 0, capacitors behaves like a closed switch (short circuit).
At $t = \infty$, capacitor blocks current or behaves like an open switch.

Voltage across capacitor after one time constant (t = RC) V = 0.63 ε

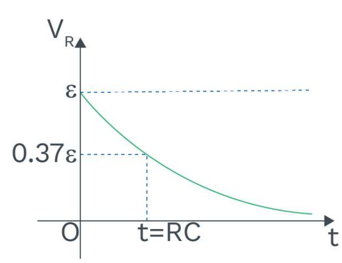
Q = CV, VC = $\varepsilon(1 - e^{-t/RC})$



Voltage across the resistor

$$\boxed{V_R = iR = \varepsilon e^{-t/RC}}$$

By energy conservation,
Heat dissipated = work done by battery – ΔU capacitor



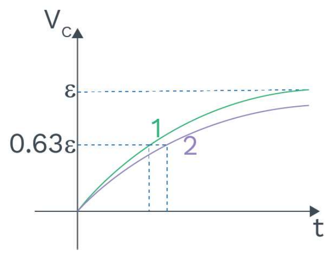
$$= C\varepsilon(\varepsilon) - \left(\frac{1}{2}C\varepsilon^2 - 0\right) = \frac{1}{2}C\varepsilon^2$$

Heat generated in RC circuit during charging

$$\text{Heat} = H = \int_0^\infty i^2 R\,dt = \int_0^\infty \frac{\varepsilon^2}{R^2} e^{\frac{-2t}{RC}} R\,dt$$

$$= \frac{\varepsilon^2}{R} \int_0^\infty e^{-2t/RC}\,dt = \frac{\varepsilon^2}{R}\left[\frac{e^{\frac{-2t}{RC}}}{-2/RC}\right]_0^\infty$$

$$= -\frac{\varepsilon^2 RC}{2R}\left[e^{\frac{-2t}{RC}}\right]_0^\infty = \frac{\varepsilon^2 C}{2} = \frac{1}{2}CV^2$$



In the figure time constant of (2) is more than (1)

Note:-

In any circuit when there is only one capacitor then $Q = Q_{st}\left(1 - e^{-t/\tau}\right)$

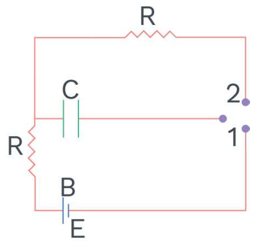
Q_{st} = steady state charge on capacitor

$\tau = R_{eff}.\ C$

$R_{efffective}$ is the resistance between the capacitor when battery is replaced by its internal resistance.

Discharging of a Capacitor

(i) In the circuit if key 1 is opened and key 2 is closed then the capacitor gets discharged.



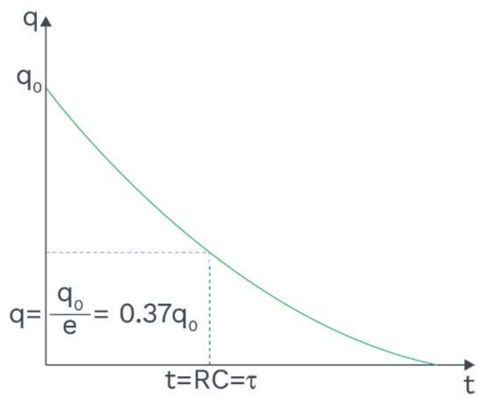
Concept Reminder

The dimension of RC is of time i.e., $[M^0L^0T^1]$

(ii) The quantity of charge on the capacitor at any instant of time t is given by

$q = q_0\ e^{-(t/RC)}$

i.e. the charge falls exponentially.



(iii) If $t = RC = \tau$ = time constant, then

$q = \frac{q_0}{e} = 0.37q_0 = 37\%$ of q_0

i.e. the 'time constant' is that time during which the charge on capacitor plates discharge process falls to 37%

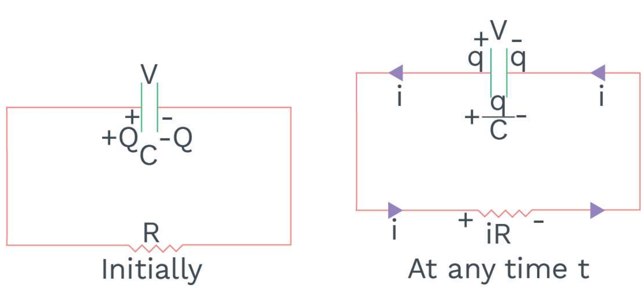
(iv) The dimensions of RC are those of time i.e. $[M^0L^0T^1]$ and the dimensions of $\frac{1}{RC}$ are those of frequency i.e. $[M^0L^0T^{-1}]$.

(v) The potential difference across the plates of capqacitor at any time t is given by

$V = V_0e^{-(t/RC)}$ Volt.

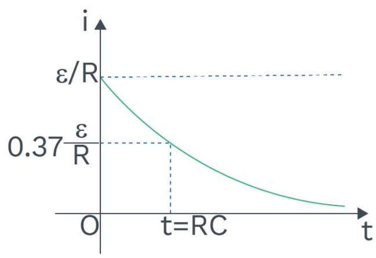
(vi) The transient current at any instant of time is given as $I = -I_0\, e^{-(t/RC)}$ ampere. i.e. the current in the circuit decreases exponentially but its direction is opposite to that of charging current.

Derivation of expression of discharging circuit:-



Applying K.V.L.

$$+\frac{q}{C} - iR = 0\,, \quad i = \frac{q}{CR}$$



$$-\frac{dq}{dt} = \frac{q}{RC}$$

Negative sign indicates that value of charge decreases with respect to time.

$$\int_Q^q -\frac{dq}{q} = \int_0^t \frac{dt}{RC}$$

$$\left[\ln q\right]_Q^q = -\frac{t}{RC}$$

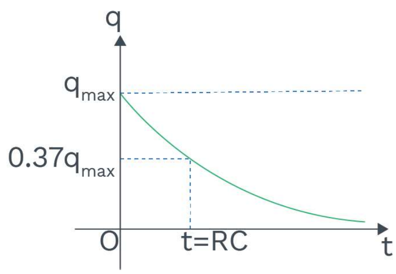
$$\ln q - \ln Q = -\frac{t}{RC}$$

$$\ln\left(\frac{q}{Q}\right) = -\frac{t}{RC}$$

$$\frac{q}{Q} = e^{-t/RC}$$

$$\boxed{q = Q.e^{-t/RC}}$$

$$\boxed{q = q_{max}e^{-t/RC}}$$

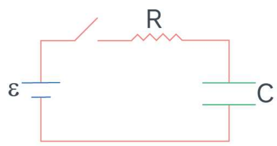


Circuit Solution for R-C circuit at T = 0 (Initial State) and at T = ∞(Final State)

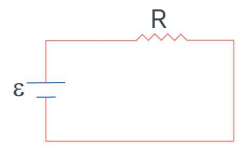
Note:

(i) Charge on the capacitor doesn't change instantaneously or suddenly if there is a resistance in the path (series) of the codensor.

(ii) When an uncharged codensor is connected with battery shows then its initial charge is zero hence potential difference across it is zero initially. At this time the capacitor can be considered as a conducting wire.

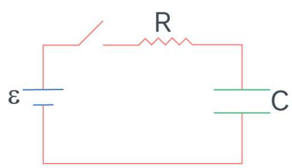


Before connection

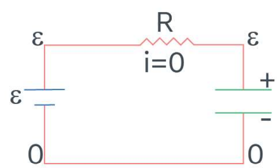


Just after connection at t = 0

(iii) The current will become zero finally (that means in steady state) in the branch which contains capacitor. So, finally the capacitor can be treated as an open circuit.



Before connection



After connection at t = ∞

Ex. Find current in the circuit and charge on capacitor which is initially uncharged in the following situations.

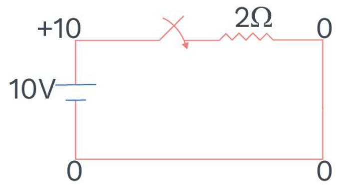
(1) Just after the switch is closed.

(2) After a long time when switch was closed.

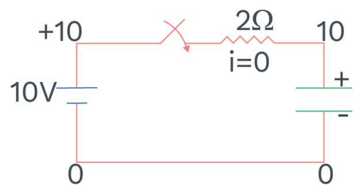
Sol. (1) For just after closing the switch : potential difference across capacitor = 0

$\therefore \quad Q_C = 0$

$\therefore \quad i = \dfrac{10}{2} = 5A$



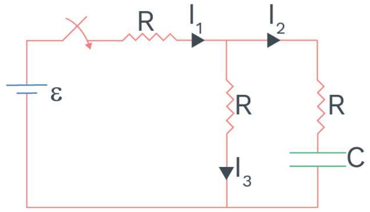
(2) After a long time at steady state current i = 0



and potential difference across capacitor = 10 V

$\therefore \quad Q_C = 3 \times 10 = 30C$

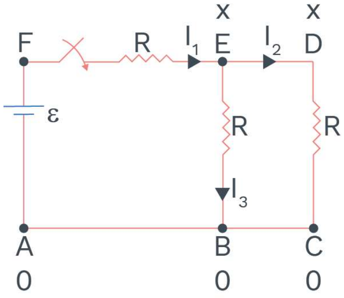
Ex. Find out current I_1, I_2, I_3, charge on capacitor and $\frac{dQ}{dt}$ of capacitor in the circuit which is initially uncharged in the following situations.



(1) Just after the switch is closed

(2) After a long time when switch is closed.

Sol. (1) Initially the capacitor is uncharged, so its behaviour is like a conductor. Assume potential at A be zero so at B and C also it becomes zero and at F it is ε. Let potential at E is x so at D also x. Apply Kirchhoff's I st law at point E:



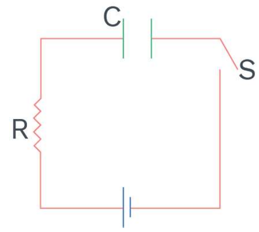
Alternatively

(2) at t = (finally)

Capacitor completely charged so there will be no current through it.

,

Ex. A codensor of capacitance C, a resistor of resistance R and a battery of emf ε are connected in series at t = 0. What is the maximum value of



(a) the potential difference across the resistor
(b) the current in the circuit
(c) the potential difference across the capacitor
(d) the energy stored in the capacitors.
(e) the work done by the battery
(f) the energy converted into heat.

Sol. (a) $V_{max} = \varepsilon$ [at t = 0]

(b) $i_{max} = \frac{\varepsilon}{R}$ (at t = 0)

(c) $V_C = \varepsilon$ (at $t = \infty$)

(d) $U_C = \frac{1}{2}C\varepsilon^2$ (at $t = \infty$)

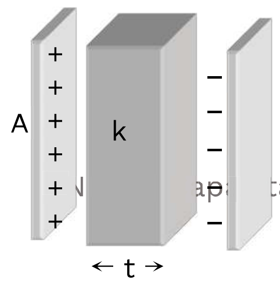
(e) $W_{battery} = \varepsilon_a = \varepsilon^2 C$

(f) $\Delta H = W_{battery} = U_c = \frac{1}{2}\varepsilon^2 C$

EXAMPLES

Q1 **Separation between the plates of a parallel plate capacitor is d and the area of each plate is A. When a slab of material of dielectric constant k and thickness $t(t < d)$ is introduced between the plates, its capacitance becomes.**

Sol.



Potential difference between the plates $V = V_{air} + V_{medium}$

$$= \frac{\sigma}{\varepsilon_0} \times (d - t) + \frac{\sigma}{K\varepsilon_0} \times t$$

$$\Rightarrow \quad V = \frac{\sigma}{\varepsilon_0}(d - t + \frac{t}{K})$$

$$= \frac{Q}{A\varepsilon_0}(d - t + \frac{t}{K})$$

Hence capacitance $C = \frac{Q}{V} = \frac{Q}{\frac{Q}{A\varepsilon_0}(d - t + \frac{t}{K})}$

$$= \frac{\varepsilon_0 A}{(d - t + \frac{t}{k})} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{k}\right)}$$

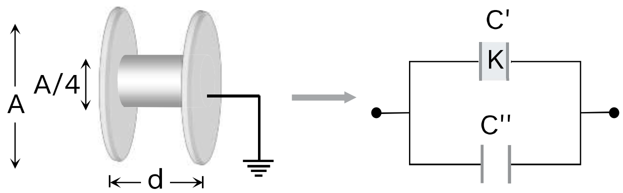
Q2 **The two metallic plates of radius r are placed at a distance d apart and its capacity is C. If a plate of radius $r/2$ and thickness d of dielectric constant 6 is placed between the plates of the condenser, then its capacity will be**

Sol. Area of the given metallic plate $A = \pi r^2$

Area of the dielectric plate $A' = \pi\left(\frac{r}{2}\right)^2 = \frac{A}{4}$

Uncovered area of the metallic plates $A'' = A - A'$

$$= A - \frac{A}{4} = \frac{3A}{4}$$



The given situation is equivalent to a parallel combination of two capacitor. One capacitor (C') is filled with a dielectric medium (K = 6) having area $\frac{A}{4}$ while the other capacitor (C'') is air filled having area $\frac{3A}{4}$

Hence $C_{eq} = C' + C'' = \frac{K\varepsilon_0 (A/4)}{d} + \frac{\varepsilon_0 (3A/4)}{d}$

$$= \frac{\varepsilon_0 A}{d}\left(\frac{K}{4} + \frac{3}{4}\right) = \frac{\varepsilon_0 A}{d}\left(\frac{6}{4} + \frac{3}{4}\right) = \frac{9}{4}C \quad \left(\because C = \frac{\varepsilon_0 A}{d}\right)$$

Q3 **A parallel plate capacitor of plate area A and plate separation d is charged to potential V and then the battery is disconnected. A slab of dielectric constant k is then inserted between the plates of the capacitors so as to fill the space between the plates. If Q, E and W denote respectively, the magnitude of charge on each plate, the electric field between the plates (after the slab is inserted) and work done on the system in question in the process of inserting the slab, then state incorrect relation from the following**

Sol. After inserting the dielectric slab

New capacitance $C' = K.C = \frac{K\varepsilon_0 A}{d}$

New potential difference $V' = \frac{V}{K}$

New charge $Q' = C'V' = \frac{\varepsilon_0 AV}{d}$

New electric field $E' = \frac{V'}{d} = \frac{V}{Kd}$

Work done (W) = Final energy – Initial energy

$$W = \frac{1}{2}C'V'^2 - \frac{1}{2}CV^2 = \frac{1}{2}(KC)\left(\frac{V}{K}\right)^2 - \frac{1}{2}CV^2 = \frac{1}{2}CV^2\left(\frac{1}{K}-1\right) = -\frac{1}{2}CV^2\left(1-\frac{1}{K}\right)$$

$$= -\frac{\varepsilon_0 AV^2}{2d}\left(1-\frac{1}{K}\right) \text{ so } |W| = \frac{\varepsilon_0 AV^2}{2d}\left(1-\frac{1}{K}\right).$$

Q4 **A conducting sphere of radius 10cm is charged 10 μC. Another uncharged sphere of radius 20 cm is allowed to touch it for some time. After that if the sphere are separated, then surface density of charges, on the spheres will be in the ratio of.**

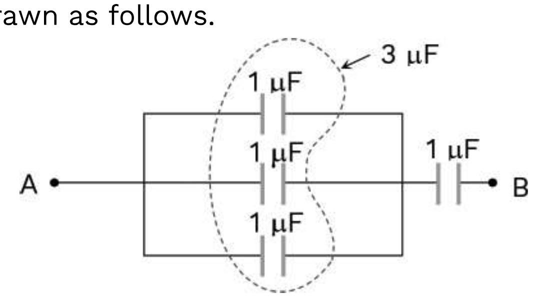
Sol. After redistribution new charges on spheres are $Q'_1 = \left(\frac{10}{10+20}\right) \times 10 = \frac{10}{3}\mu C$

and $Q'_2 = \left(\frac{20}{10+20}\right) \times 10 = \frac{20}{3}\mu C$

Ratio of charge densities $\frac{\sigma_1}{\sigma_2} = \frac{Q'_1}{Q'_2} \times \frac{r_2^2}{r_1^2} = \frac{10/3}{20/3} \times \left(\frac{20}{10}\right)^2 = \frac{2}{1}$ $\left\{\sigma = \frac{Q}{4\pi r^2}\right\}$

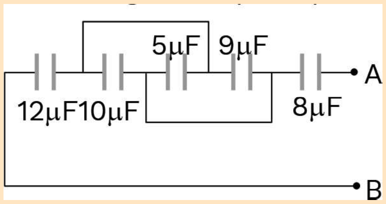
Q5 **Three capacitors each of capacitance $1\mu F$ are connected in parallel. To this combination, a fourth capacitor of capacitance $1\mu F$ is connected in series. The resultant capacitance of the system is.**

Sol. The circuit can be drawn as follows.

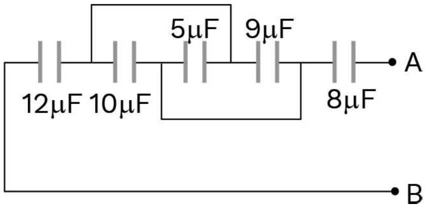


$$\Rightarrow C_{AB} = \frac{3\times 1}{3+1} = \frac{3}{4}\mu F$$

Q6 **The capacities and connection of five capacitors are shown in the adjoining figure. The potential difference between the points A and B is 60 volts. Then the equivalent capacity between A and B and the charge on 5 μF capacitance will be respectively**



Sol. The given circuit can be redrawn ass follows

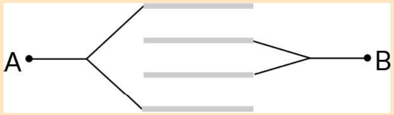


Equivalent capacitance of the circuit $C_{AB} = 4\mu F$ Charge given by the battery

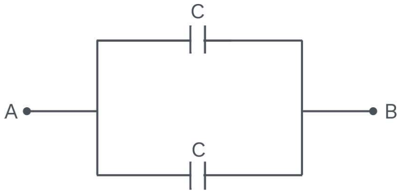
$Q = C_{eq}V = 4 \times 60 = 240\mu C$

Charge in 5μF capacitor $Q' = \frac{5}{(10+5+9)} \times 240 = 50\mu C$

Q7 **Four plates of equal area A are separated by equal distances d and are arranged as shown in the figure. The equivalent capacity is**



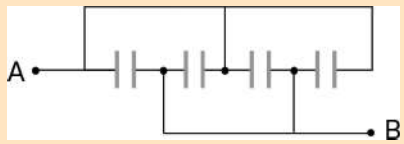
Sol. The given circuit is equivalent to a parallel combination two identical capacitors



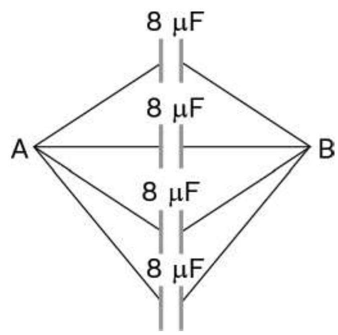
Hence equivalent capacitance between A and B is

$$C = \frac{\varepsilon_0 A}{d} + \frac{\varepsilon_0 A}{d} = \frac{2\varepsilon_0 A}{d}$$

Q8 **Four condensers are joined as shown in the adjoining figure. The capacity of each is 8 μF. The equivalent capacity between the points A and B will be**

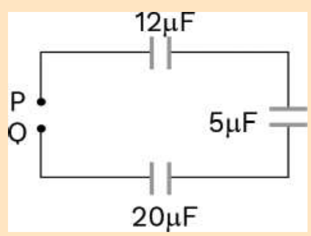


Sol. **Sol.** Given circuit can be drawn as



Equivalent capacitance = $4 \times 8 = 32\mu F$

Q9 **In the circuit diagram shown in the adjoining figure, the resultant capacitance between P and Q is**

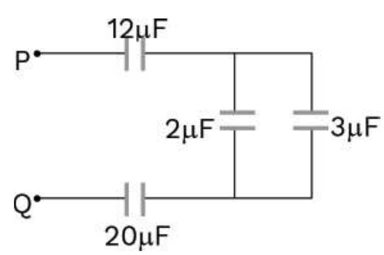


Sol. The given circuit can be drawn as

where, $C = (3 + 2)\mu F = 5\mu F$

$$\frac{1}{C_{PQ}} = \frac{1}{5} + \frac{1}{20} + \frac{1}{12} = \frac{20}{60} = \frac{1}{3}$$

$\Rightarrow C_{PQ} = 3\mu F$



Q10 **Two metallic spheres of radii $1\,cm$ and $2\,cm$ are given charges 10^{-2} C and $5 \times 10^{-}$ C respectively. If they are connected by a conducting wire, the final charge on the smaller sphere is**

Sol. $Q_1 = 10^{-2}C$, $Q_2 = 5 \times 10^{-2}C$

Total charge of the system Q = 6×10^{-6} C

Charge on small sphere

$$Q'_1 = \frac{Q r_1}{r_1 + r_2} = \frac{6 \times 10^{-2} \times 1}{1 + 2} = 2 \times 10^{-2}C$$

Q11 **A parallel plate capacitor is charged and the charging battery is then disconnected. If the plates of the capacitor are moved further apart by means of insulating handles, then**

Sol. When the battery is disconnected, the charge will remain same in any case.

Capacitance of a parallel plate capacitor is given by $C = \frac{\varepsilon_0 A}{d}$

When d is increased, capacitance will decreases and because the charge remains the same, so according to q = CV, the voltage will increase, Hence the electrostatics energy stored in the capacitor will increase.

Q12 **The radii of two metallic spheres P and Q are r_1 and r_2 respectively. They are given the same charge. If $r_1 > r_2$. then on connecting them with a thin wire, the charge will flow**

Sol. Since charge flows from high potential to lower potential.

If positive charge is given, then $V_1 < V_2$ as $r_1 > r_2$

So positive charge flows from $Q \rightarrow P$

If negative charge is given, then $V_1 > V_2$

So negative charge flows form $P \rightarrow Q$.

Since it is not given that whether the charge given is positive or negative, hence the information is incomplete.

Q13 **The respective radii of the two spheres of a spherical condenser are 12 cm and 9 cm. The dielectric constant of the medium between them is 6. The capacity of the condenser will be**

Sol. $C = 4\pi\varepsilon_0 K\left[\frac{ab}{b-a}\right] = \frac{1}{9\times10^9}.6\left[\frac{12\times9\times10^{-4}}{3\times10^{-2}}\right]$

$= 24\times10^{-11} = 240\,\text{pF}$

Q14 **The area of each plate of a parallel plate capacitor is 100 cm²and the distance between the plates is 1 mm. It is filled with mica of dielectric 6. The radius of the equivalent capacity of the sphere will be**

Sol. $C = \frac{\varepsilon_0 AK}{d} = 4\pi\varepsilon_0 r$

r = Radius of sphere of equivalent capacity

$\Rightarrow r = \frac{AK}{4\pi d} = \frac{100\times10^{-4}\times6}{1\times10^{-3}\times4\times3.14} = \frac{15}{3.14} = 4.77\,\text{m}$

Q15 **A capacitor is charged by using a battery which is then disconnected. A dielectric slab is then slipped between the plates, which results in**

Sol. Battery in disconnected so Q will be constant as $C \propto K$. So with introduction of dielectric slab capacitance will increase using Q = CV, V will decrease and using $U = \frac{Q^2}{2C}$, energy will decrease.

MIND MAP

CAPACITOR

Capacity of a Capacitor, $C = \frac{Q}{V}$

where C is a +ve constant for a conductor and is
called capacity (capacitance) of a capacitor.
SI unit of Capacity is Farad

Conductor	**Capacitor**
Q → Charge on conductor	Q → Charge on positive plate
V → Potential of conductor	V → Potential difference between plates

1 Farad is a very big unit. $\mu F(10^{-6}F)$, $nF(10^{-9}F)$, $pF(10^{-12}F)$,

Heat Generated in Circuit $H = \Sigma W_b - \Sigma \Delta E$

$\Delta E = (E_f - E_i)$

ΣW_b = Work done by all batteries

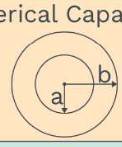
$\Sigma \Delta E$ = Energy change in all capacitors

Capacity of a Capacitor depends on

(1) Shape and size of the plates
(2) Distance between plates
(3) Medium between the plates

Parallel Plate Capacitor $C = \frac{A\varepsilon_0}{d}$

Spherical Capacitor



$C = \frac{4\pi\varepsilon_0 ab}{b - a}$

Cylindrical Capacitor

$C = \frac{2\pi\varepsilon_0 l}{\ln\left(\frac{b}{a}\right)}$

Work Done By Battery

$W = QV$

Series Combination (Charge is same), $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ...$

Parallel Combination (V is same), $C_{eq} = C_1 + C_2 + ...$

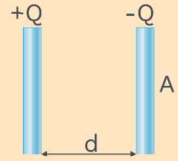
Mesh Analysis
- Assume potential using Junction Law.
- Apply KVL

Nodal Analysis
- Assume potential in a circuit.
- Apply Junction Law

Energy Stored in a Capacitor $= \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$

Electrostatic Force b/w the Plates of a Parallel Plate Capacitor

$F = \frac{Q^2}{2A\varepsilon_0}$



Bound Charges (Induced Charges) on Dielectric $q = Q\left(1 - \frac{1}{K}\right)$

Heat Loss $= \frac{Q_0^2}{2C}$