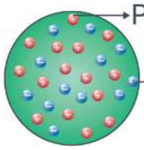
Atomic Physics

DISCLAIMER

Atomic Physics

Thomson's Atomic model :

J.J. Thomson said that atoms are just positively charge lumps of matter with electrons embedded in them like raisins in a fruit cake. Thomson's atomic model called the plum pudding model is illustrated in figure.



Thomson Model of Atom:

- According to Thomson atomic model an atom is a sphere of 10^{-10} m order radius.
- It is a positively charged sphere of matter in which electrons (–ve charge) are embedded like seeds in watermelon.
- Thomson atomic model is also called plum pudding model.
- According to Thomson atomic model atom is electrically neutral.
- Using Thomson atomic model stability of an atom, thermionic emission and ionisation of gases can be explained.

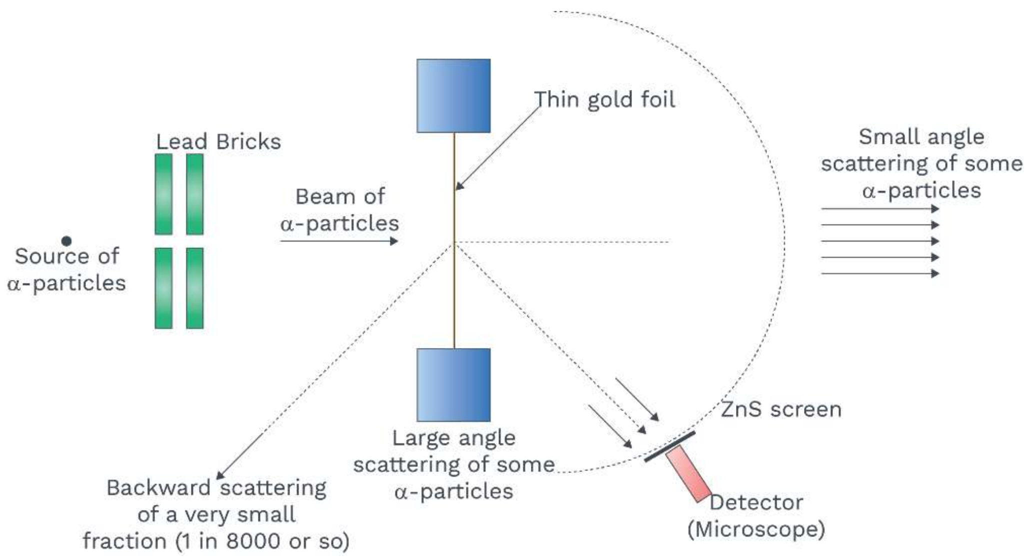
Drawbacks of Thomson atomic model:

- This atomic model is failed to explain linear spectrum of hydrogen atom which is observed in further atomic model.
- It is failed to explain the observations of α -scattering experiment.

Rutherford atomic model:

- **α-Scattering Experiment**

  On the suggestion of Rutherford his two associates Geiger and Marsden in 1911 performed α -scattering experiment from thin gold foil. The systematic practical arrangement is shown in figure.



α -particles coming from the radioactive source are allow to cross a narrow slit, due to which α -particles are converted into a time beam. This beam of α -particles is made to incident on the gold foil and scattered α -particle from the thin gold foil are recorded on the 'ZnS' screen using a microscope along with a counter.

Experimental Observations:

(a) Most of the α-particles were found to pass through the gold- foil without being deviated from their paths.

(b) Some α-particles were found to be deflected through small angles $\theta < 90°$.

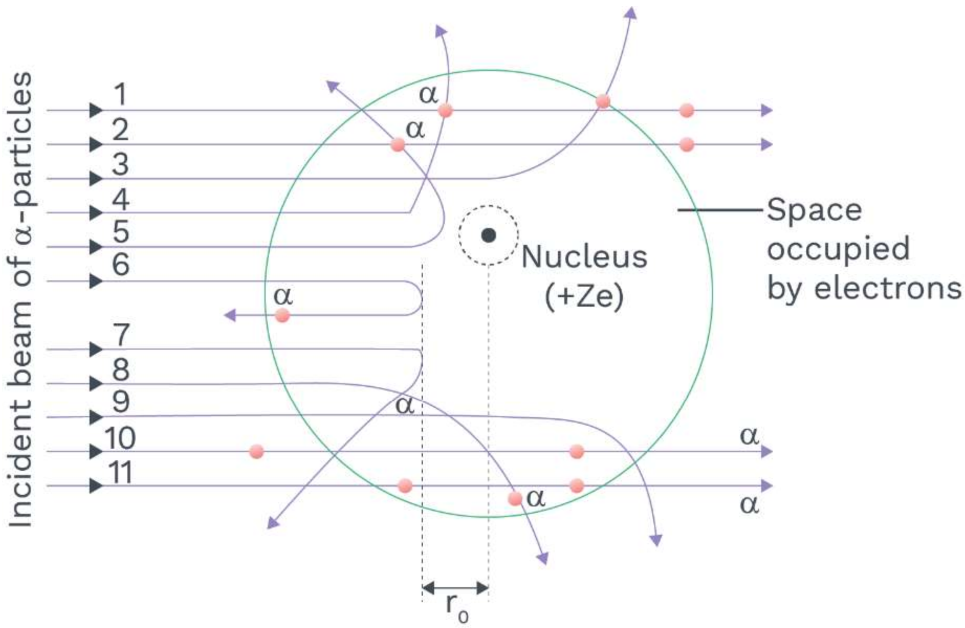
(c) Few α-particles were found to be scattered at fairly large angles from their initial path $\theta > 90°$

Concept Reminder

In Rutherford Model, number of α -particles scattered at an angle θ

as $N(\theta) \propto \dfrac{Z^2}{\sin^4\left(\dfrac{\theta}{2}\right).E^2}$

E = K.E of α -particles.



(d) A very small number of α-particles about 1 in 8000 practically retracted their paths or suffered deflections of nearly 180°.

- The observation (a) indicates that most of the portion of the atom is hollow inside.
- Because α-particle is positively charged, from the observations (b), (c) and (d) atom also have positive charge and the whole positive charge of the atom must be concentrated in small space which is at the centre of the atom is called nucleus. The remaining part of the atom and electrons are revolving around the nucleus in circular objects of all possible radii. The positive charge present in the nuclei of different metals is different. Higher the positive charge in the nucleus, larger will be the angle of scattering of α-particle.

Distance of Closest Approach:

- As the α-particle approaches the nucleus, the electrostatic repulsive force due to the nucleus increases and kinetic energy of the alpha particle goes on converting into the electrostatic potential energy. When whole of the kinetic energy is converted into electrostatic potential energy, the α-particle cannot further move towards the nucleus but returns back on its initial path i.e α-particle is scattered through an angle of 180°. The distance of α-particle from the nucleus in this stage is called as the distance of closest approach and is represented by r_0.

- Let m_α and v_α be the mass and velocity of the α-particle directed towards the centre of the nucleus. Then kinetic energy of the α- particle

$$K = \left(\frac{1}{2}\right) m_\alpha v^2{}_\alpha$$

Because the positive charge on the nucleus is Ze and that on the α-particle 2e, hence the electrostatic potential energy of the α-particle, when it is at a distance r_0 from the centre of the nucleus, is given by

$$U = \frac{1}{4\pi \in_0} \cdot \frac{(2e)(Ze)}{r_0}$$

Because at $r = r_0$ kinetic energy of the α-particle appears as its potential energy, hence, K = U

$$\frac{1}{4\pi \in_0} \cdot \frac{(2e)(Ze)}{r_0} = \frac{1}{2} m_\alpha v_\alpha^2$$

$$r_0 = \frac{1}{4\pi \in_0} \frac{4Ze^2}{m_\alpha v_\alpha^2}$$

- When a mono energetic beam of α particles is projected towards a thin metal foil, some of the particles are found to deviate from their original path. This phenomenon is called α ray scattering.
- It is caused by coulomb repulsive force between α particles and positive charges in atom.
- The number of α -particles scattered at an angle

θ is given by $N = \dfrac{QntZ^2e^4}{(8\pi\varepsilon_0)r^2E^2 \sin^4\left(\dfrac{\theta}{2}\right)}$

Where

Q → Total number of α particles striking the foil
n → number of atoms per unit volume of the foil
r → distance of screen from the foil
t → thickness of the foil
Z → Atomic number of atoms of the metal foil
θ → angle of scattering
E → kinetic energy of α particles

Rack your Brain

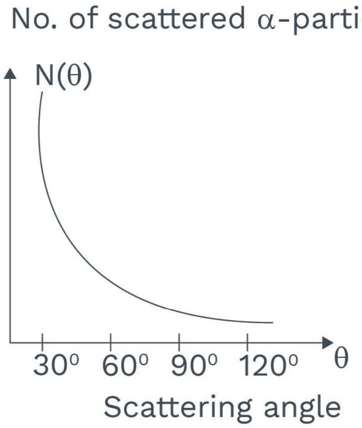
An alpha nucleus of energy $\frac{1}{2}mv^2$ bombards on a heavy nucleus target of charge Ze. Then the distance of closest approach for the alpha nucleus will be proportional to

(1) $\frac{1}{Ze}$ (2) v^2

(3) $\frac{1}{m}$ (4) $\frac{1}{v^2}$

Concept Reminder

If the "plum-pudding" model were correct, the α -particles would be expected to pass nearly straight through the foil.



$$N \alpha t \; ; \quad N \propto Z^2 \; ; \; N \propto \frac{1}{\sin^4 \frac{\theta}{2}}$$

$$N \propto \frac{1}{E^2} \Rightarrow N \propto \frac{1}{v^4}$$

where v is the velocity of α particles falling on the foil.

- **Impact Parameter(b):-**:The perpendicular distance of the initial velocity vector of the α-particle from centre of the nucleus is called "impact parameter".

$$b = \frac{Ze^2 \cot\left(\frac{\theta}{2}\right)}{4\pi \epsilon_0 \times \frac{1}{2} mv^2}$$

As $b \to 0$

$$0 = \cot \frac{\theta}{2}$$

$$\cot 90° = \cot \frac{\theta}{2} \Rightarrow 90° = \frac{\theta}{2} \Rightarrow \theta = 180°$$

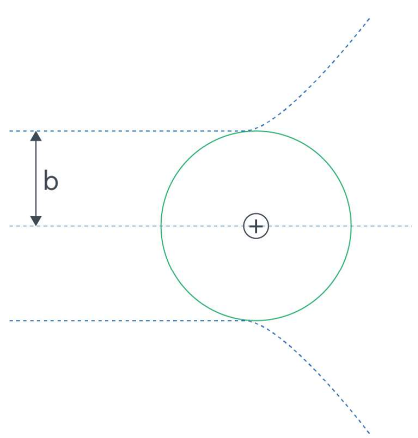
Drawback of Rutherford Atomic Model

- **Regarding stability of atom**

An electron moving in a circular orbit around a nucleus is accelerating and according to electromagnetic theory it should, therefore, emit

Definitions

- The impact parameter b is the perpendicular distance of the initial velocity vector of the α-particle from centre of the nucleus.
- Smaller the b, greater the scattering angle θ.



radiation continuously and thereby lose energy. If this happened the radius of the orbit would decrease, and the electron would spiral into the nucleus in a fraction of second. But we know atoms do not collapse.

- **Regarding explanation of line spectrum:**
  In Rutherford's model, due to continuously changing radii of the circular orbit of electrons, the frequency of revolution of the electrons must be changing. As a result, electrons will radiate electromagnetic waves of all frequencies, i.e., the spectrum of these wave will be 'continuous' in nature. But experimentally, the atomic spectra are not continuous. Instead, they are line spectra.

Bohr's atom model

Bohr modified Rutherford's atomic model on the basis of quantum theory of radiation. His model is based on the following three postulates :

(i) **First postulate:** An electron in an atom could revolve in certain stable orbits without emitting radiant energy. Each possible orbit has definite total energy. These stable orbits are called stationary states of the atom. In these orbits

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)e}{r^2}$$

(ii) **Second postulate :** Electrons cannot revolve in all orbits as suggested by classical theory, but only in those orbits for which angular momentum L is equal to an integral multiple of $h/2\pi$, where h is the Planck's constant. Thus

$$I\omega = mvr = \frac{nh}{2\pi}$$

(a) Angular momentum and Planck's constant have same units and dimensions.

KEY POINTS

- Distance of closest approach
- Impact parameter

(b) The SI unit of Planck's constant is joule-sec and its value is equal to 6.63×10^{-34} joule-sec.

(c) The dimensional formula for angular momentum is

$$= [M^1L^2T^{-1}]$$

(d) As long as the electron revolves in these orbits, it neither loses energy nor gains energy. These orbits are known as stationary orbits.

(iii) Third postulate : When an electron jumps from higher energy level to lower energy level, the energy difference is released in the form of light of frequency ν, i.e.

$$E_2 - E_1 = h\nu$$

where E_2 is the energy of the outer orbit and E_1 is the energy of the inner orbit.

Orbital radius of revolving electron

Let us consider an electron is revolving around the nucleus in n^{th} orbit inside an atom. In which its revolving speed is v_n and radius of the orbit is 'r_n'. Then from the first postulate of Bohr model.

$$\frac{mv_n^2}{r_n} = \frac{KZe^2}{r_n^2} \quad \text{.... (i)}$$

Now, acceleration to second postulate of Bohr model

$$mv_nr_n = \frac{nh}{2\pi} \Rightarrow v_n = \frac{nh}{2\pi mr_n} \quad \text{.... (ii)}$$

Using equation (ii) in to equation (i)

$$m \times \frac{n^2h^2}{4\pi^2m^2r_n^2} = \frac{Kze^2}{r_n} \Rightarrow \frac{n^2h^2}{4\pi^2mr_n} = Kze^2$$

$$\Rightarrow r_n = \frac{n^2h^2}{4\pi^2m \times Kze^2}$$

$$r_n = \frac{n^2h^2}{4\pi^2mKe^2 \times z}$$

Concept Reminder

Each atom has definite stable orbits. Electrons can exist in these orbits only. These stable orbits are called stationary states of the atom.

KEY POINTS

- Bohr atomic model
- Bohr's postulate
- Bohr radius

$$\Rightarrow r_n = \frac{h^2}{4\pi^2 mKe^2} \times \frac{n^2}{z}$$

$$\Rightarrow r_n = \frac{h^2}{4\pi^2 m \times \frac{1}{4\pi\varepsilon_0} \times e^2} \times \frac{n^2}{z} \qquad \left\{\because K = \frac{1}{4\pi\varepsilon_0}\right\}$$

$$\Rightarrow r_n = \frac{\varepsilon_0 h^2}{\pi me^2} \times \frac{n^2}{z}$$

Here, $\frac{\varepsilon_0 h^2}{\pi me^2}$ is a constant whose value is 0.529 Å.

$$\boxed{\therefore r_n = 0.529 \frac{n^2}{z} Å}$$

Special Point

- For 'H' atom Z = 1

$$\therefore \quad r_n = 0.529 \times \frac{n^2}{1} Å$$

Radius of the first orbit of the 'H' atom is called Bohr radius.

Hence, $\boxed{\text{Bohr radius} = 0.529Å \cong 53\,\text{pm}}$

Ex. Find the radius of second orbit for the Li^{++} atom?

Sol. For Li^{++} :- Z = 3

n = 2

we know that

$$r_n = 0.529 \frac{n^2}{z} Å \qquad \Rightarrow \qquad r_2 = 0.529 \times \frac{4}{3} Å$$

Orbital speed of revolving electron

- **From first postulate of Bohr model**

$$\frac{mv_n^2}{r_n} = \frac{KZe^2}{r_n^2} \qquad \text{.... (i)}$$

And from the second postulate of Bohr model

$$mv_n r_n = \frac{nh}{2\pi}$$

or dividing equation (i) & (ii)

Concept Reminder

If the difference between $(n+1)^{th}$ Bohr radius and nth Bohr radius is equal to $(n-1)^{th}$ Bohr radius then value of n is 4.

Rack your Brain

For which one of the following, Bohr model is not Valid?

(1) Hydrogen atom

(2) Singly ionized helium atom He^+

(3) Deuteron atom

(4) Singly ionized neon atom (Ne^+)

$$\frac{mv_n^2}{r_n \times mv_n \times r_n} = \frac{Kze^2}{r_n^2} \times \frac{2\pi}{nh}$$

$$\Rightarrow \quad v_n = \frac{2Kze^2}{nh}$$

$$\Rightarrow \quad v_n = 2\pi \times \frac{1}{4\pi\varepsilon_0} \times z \times e^2$$

$$\Rightarrow \quad \boxed{v_n = \frac{e^2}{2\varepsilon_0 h} \times \frac{z}{n}}$$

Here, $\frac{e^2}{2\varepsilon_0 h}$ is constant which has value 2.189×10^6 m/sec.

$$\therefore \quad \boxed{v_n = 2.189 \times 10^6 \times \frac{z}{n} m / sec}$$

1. For 'H' atom z = 1

So, $v_n = 2.189 \times 10^6 \times \frac{1}{n} m / sec$

2. For 'H' atom

$$v_1 : v_2 : v_3 = \frac{1}{1} : \frac{1}{2} : \frac{1}{3} \quad \left\{ v \propto \frac{1}{n} \right\}$$

$$\Rightarrow \quad \boxed{v_1 : v_2 : v_3 = 6 : 3 : 2}$$

3. Speed of the electron in the first orbit of 'H' atom

$$v_1 = 2.189 \times 10^6 \; m / sec$$

and $\frac{v_1}{c} = \frac{1}{137} = \alpha$

Fine structure constant (α)**:**

The ratio of speed of the electron in first orbit of 'H' atom to the speed of light $\left(\frac{v_1}{c}\right)$ is known as time structure constant.

So, $\boxed{\alpha = \frac{v_1}{c} = \frac{1}{137}}$

Concept Reminder

- Ratio of magnetic dipole moment to the angular momentum for hydrogen like atoms is $\frac{e}{2m}$
- Bohr magneton is magnetic moment of electron in I[st] orbit of hydrogen atom and is equal to

$$\frac{eh}{4\pi m} = 9.27 \times 10^{-24} JT^{-1}$$

Orbital frequency of revolving electron:

We know that

$$T_n = \frac{2\pi r_n}{v_n}$$

So, the frequency of revolution of the electron in n^{th} orbital

$$f_n = \frac{v_n}{2\pi r_n} \quad \text{.... (i)}$$

But we know, $$v_n = \frac{e^2}{2\varepsilon_0 h} \times \frac{z}{n}$$

and $$r_n = \frac{\varepsilon_0 h^2}{\pi m e^2} \times \frac{n^2}{z}$$

using above values in equation (i)

$$f_n = \frac{e^2}{2\varepsilon_0 h} \times \frac{z}{n \times 2\pi} \times \frac{\pi m e^2 z}{\varepsilon_0 h^2 n^2}$$

$$\Rightarrow f_n = \frac{m e^4 z^2}{4\varepsilon_0^2 h^3 n^3}$$

$$\Rightarrow \boxed{f_n \propto \frac{z^2}{n^3}}$$

Special Point

- $$\because \quad f_n = \frac{v_n}{2\pi r_n}$$

  $$\Rightarrow f_n \propto \frac{v_n}{r_n}$$

  $$\Rightarrow f_n \propto \frac{z}{n} \times \frac{z}{n^2} \left\{ \because v_A \propto \frac{z}{n} \text{ and } r_n \propto \frac{n^2}{z} \right\}$$

  $$\Rightarrow f_n \propto \frac{z^2}{n^3}$$

- Time period $T_n \propto \dfrac{n^3}{z^2}$

Total energy of the revolving electron

- Total energy of the revolving electron in n^{th} orbit will equal to sum of K.E. and P.E.

KEY POINTS

- Orbital speed
- Fine structure constant
- Total energy of revolving electron

Hence, T.E. = K.E. + P.E

$$E_n = K_n + U_n \quad \text{.... (i)}$$

From the first postulate of Bohr model

$$\frac{mv_n^2}{r_n} = \frac{Kze^2}{r_n^2} \Rightarrow mv_n^2 = \frac{Kze^2}{r_n}$$

$$\Rightarrow \frac{1}{2}mv_n^2 = \frac{1}{2}\frac{Kze^2}{r_n}$$

$$\Rightarrow K_n = \frac{1}{2}\frac{Kze^2}{r_n} \quad \text{..... (ii)}$$

and P.E. of the electron revolving in n^{th} orbit-

$$U_n = \frac{kq_1q_2}{r_n}$$

Here $q_1 = +ze$ and $q_2 = -e$

$$\therefore U_n = \frac{K \times ze \times (-e)}{r_n}$$

$$U_n = \frac{-Kze^2}{r_n} \quad \text{..... (iii)}$$

Using equation (ii) and (iii) into equation (i)

$$E_n = \frac{1}{2}\frac{Kze^2}{r_n} + \left(\frac{-Kze^2}{r_n}\right)$$

$$\Rightarrow \boxed{E_n = -\frac{1}{2}\frac{Kze^2}{r_n}}$$

$$\Rightarrow E_n = \frac{-1}{2} \times \frac{1}{4\pi\varepsilon_0} \times \frac{ze^2}{r_n}$$

$$\Rightarrow E_n = \frac{-1}{8\pi\varepsilon_0} \times \frac{ze^2}{r_n} \quad \text{..... (iv)}$$

But we know that

$$r_n = \frac{\varepsilon_0 h^2}{\pi me^2} \times \frac{n^2}{Z}$$

Using this value in equation (iv) –

$$E_n = \frac{-1}{8\pi\varepsilon_0} \times \frac{ze^2}{\varepsilon_0 h^2 n^2} \times \pi me^2 z$$

Rack your Brain

The total energy of an electron in an atom in an orbit is –3.4 eV. Find out its kinetic energy and potential energy.

$$\Rightarrow \quad E_n = \frac{-mz^2e^4}{8\varepsilon_0^2h^2n^2}$$

$$\Rightarrow \quad \boxed{E_n = \frac{-me^4}{8\varepsilon_0^2h^2} \times \frac{z^2}{n^2}} \qquad .....(v)$$

Here, $m = 9.1 \times 10^{-31}$ kg

$e = 1.6 \times 10^{-19}$ C

$\varepsilon_0 = 8.85 \times 10^{-12} \dfrac{C^2}{N \times m^2}$

$h = 6.62 \times 10^{-34}$ J sec.

Put these values in equation (v) and we get,

$$\therefore \quad E_n = -13.6 \times 1.6 \times 10^{-19} \times \frac{Z^2}{n^2} \text{ Joule}$$

$\{\because 1\text{eV} = 1.6 \times 10^{-19} \text{ Joule}\}$

$$\Rightarrow \quad \boxed{E_n = -13.6 \times \frac{z^2}{n^2} \text{eV}}$$

Special point

- For 'H' atom (z = 1)
- $E_n = -13.6 \times \dfrac{1}{n^2}$ e.V , $\boxed{-2K_n = 2E_n = U_n}$
- Total energy of revolving electron is negative which represent that electron is bound with nucleus and to remove electron from its state external energy is required.

Concept Reminder

The magnetic field induction produced at the centre of orbit due to an electron revolving in n^{th} orbit of hydrogen atom is proportional to n^{-5}.

Energy level diagram for hydrogen atom

For hydrogen atom total energy of revolving electron

$$E_n = -13.6 \times \frac{1}{n^2} \text{e.V}$$

For $n = 1, E_1 = -13.6$ e.V.

For $n = 2$

$$E_2 = \frac{-13.6}{4} = -3.4 \text{ e.V.}$$

For $n = 3$

$$E_3 = \frac{-13.6}{9} = -1.51 \text{ e.V.}$$

Concept Reminder

The ground state of H-atom, n = 1 has lowest energy as we move to state n > 1 the energy becomes less negative i.e., energy increases.

For n = 4

$$E_4 = \frac{-13.6}{16} = -0.85 \text{ e.V.}$$

For n = 5

$$E_5 = \frac{-13.6}{25} = -0.54 \text{ e.V.}$$

$$E_\infty = \frac{-13.6}{\infty} = 0$$

Energy level diagram for 'H' atom

Definitions valid for single electron system :-

(i) **Ground state :** The lowest energy state of any atom or an ion is called ground state of the atom.

Ground level energy of H atom = –13.6 eV

Ground level energy of He^+ ion = –54.4 eV

Ground level energy of Le^{++} ion = –122.4 eV

(ii) **Excited state :** The state of atom other than the ground state are called its excited states.

n = 2	first excited state.
n = 3	second excited state
n = 4	third excited state
$n = n_0 + 1$	n_0^{th} excited state

(iii) **Ionization energy (I.E) :** Minimum energy required to move an electron from ground state to n = ∞ is called the ionisation energy

KEY POINTS

- Energy level diagram
- Ground state
- Excited state
- Ionization energy
- Ionization potential

of the atom or ion

Ionization energy of H atom = 13.6 eV

Ionization energy of He^{+} atom = 54.4 eV

Ionization energy of Le^{++} atom = 122.4 eV

(iv) Ionization potential (I.P):- Potential difference through which a free electron must be accelerated from rest such that its kinetic energy becomes equal to ionisation energy of the atom is called 'ionisation potential (I.P)' of the atom.

I.P. of H atom = 13.6 V

I.P. of He^{+} ion = 54.4 V

(v) Excitation energy : Necessary amount of energy required to move an electron from ground energy level of atom to any other exited state of atom is called 'excitation energy' of that state.

Energy in ground state of H atom = –13.6 eV

Energy in first excited state of H-atom = –3.4 eV

I^{st} excitation potential = 10.2 V.

(vi) Excitation potential : Potential difference through which an electron must be accelerated from rest so that its kinetic energy of that electron becomes equal to excitation energy of any state is called excitation potential of that state.

I^{st} excitation energy = 10.2 eV

I^{st} excitation potential = 10.2 V

Binding energy or separation energy :

The amount of energy required to move e^{-} from any state of n = ∞ to some particular n is called binding energy of that state.

Binding energy of ground state of H-atom = 13.6 eV

Rack your Brain

The radius of the I^{st} permitted Bohr orbit for the electron, in a hydrogen atom equals 0.51 Å and its ground state energy equals –13.6 eV. If the electron in hydrogen atom is replaced by muon (μ^{-}) [charge same as e^{-} and mass $207m_e$], then find Bohr radius and ground state energy.

Explanation of linear spectrum of hydrogen atom using Bohr model.

- According to Bohr model revolving electron in stationary orbits do not releases radiations or energy. But by absorbing energy electron jumps from energy level to higher energy level. At higher energy level it stays for a short time duration (10^{-3} sec) and it returns in the lower energy levels by releasing energy in the form of radiations.
- Wavelength of these emitted radiations can be find out using third postulate of Bohr model.
- According to third postulate of Bohr model –

$$E_{n_2} - E_{n_1} = \frac{hc}{\lambda} \qquad \text{.... (i)}$$

But we know that

$$\left\{E_n = \frac{-me^4}{8\varepsilon_0^2 h^2} \times \frac{z^2}{n^2}\right\}$$

Hence, total energy of revolving electron in 'n_2' orbit

$$E_{n_2} = \frac{-me^4}{8\varepsilon_0^2 h^2} \times \frac{z^2}{n_2^2} \qquad \text{.... (ii)}$$

And total energy of revolving electron in 'n_1' orbit-

$$\Rightarrow E_{n_1} = \frac{-me^4}{8\varepsilon_0^2 h^2} \times \frac{z^2}{n_1^2} \qquad \text{... (iii)}$$

Using equation (ii) and equation (iii) into equation (i)

$$\boxed{\frac{-me^4}{8\varepsilon_0^2 h^2} \times \frac{z^2}{n_2^2} - \left(\frac{-me^4}{8\varepsilon_0^2 h^2} \times \frac{z^2}{n_1^2}\right) = \frac{hc}{\lambda}}$$

$$\Rightarrow \frac{hc}{\lambda} = \frac{me^4}{8\varepsilon_0^2 h^2} \times \frac{z^2}{n_1^2} - \frac{me^4}{8\varepsilon_0^2 h^2} \times \frac{z^2}{n_2^2}$$

$$\Rightarrow \frac{hc}{\lambda} = \frac{me^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] z^2$$

$$\Rightarrow \frac{1}{\lambda} = \frac{me^4 z^2}{8\varepsilon_0^2 h^3 c}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

KEY POINTS

- Excitation energy
- Excitation potential
- Binding energy
- Line spectrum

Concept Reminder

When a hydrogen atom is raised from the ground state to an excited state, then its P.E increases and K.E decreases.

Here $\frac{me^4}{8\varepsilon_0^2 h^3 c} = R$ is a constant which is called Rydberg constant

$$\boxed{R = 1.09737 \times 10^7 m^{-1}}$$

$$\Rightarrow \frac{1}{\lambda} = Rz^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For 'H' atom z = 1

$$\therefore \boxed{\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]}$$

It is called Rydberg formula. Using this formula if wavelength of emitted radiations for the 'H' atom is calculated, then it acquires in describe form. So, the emitted spectrum for 'H' atom will be linear. It will not be continuous.

Special Point

- $\frac{1}{\lambda} = \bar{\nu}$ (wave number)

  $$\therefore \bar{\nu} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

- $C = \nu\lambda$

  $$\Rightarrow \nu = \frac{c}{\lambda}$$

  'ν' is the frequency of emitted radiation.

  $$\Rightarrow \boxed{\nu = RC\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]}$$

  The hydrogen spectrum (some selected lines)

Rack your Brain

The energy of hydrogen atom in n^{th} orbit is E_n then what will be the energy of n^{th} orbit of singly ionized helium atom.

KEY POINTS

- Rydberg's constant
- Emission spectral

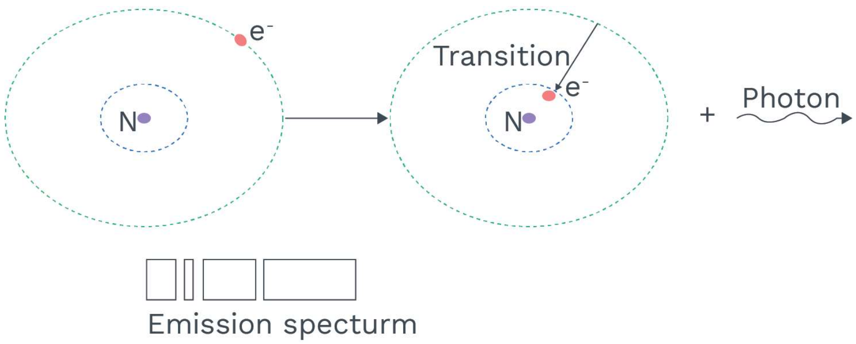
NAME OF SERIES	NUMBER OF LINE	QUANTUM NUMBER			
		n_i (Lower state)	n_f (Upper state)	Wavelength (nm)	Energy
Lyman	I	1	2	121.6	10.2 eV
	II	1	3	102.6	12.09 eV
	III	1	4	97	12.78 eV
	Series Limit	1	∞(series limit)	91.2	13.6 eV
Balmer	I	2	3	656.3	1.89 eV
	II	2	4	486.1	2.55 eV
	III	2	5	434.1	2.86 eV
	Series Limit	2	∞(series limit)	364.6	3.41 eV
Paschen	I	3	4	1875.1	0.66 eV
	II	3	5	1281.8	0.97 eV
	III	3	6	1093.8	1.13 eV
	Series Limit	3	∞(series limit)	822	1.51 eV

Emission Spectrum of Hydrogen atom :

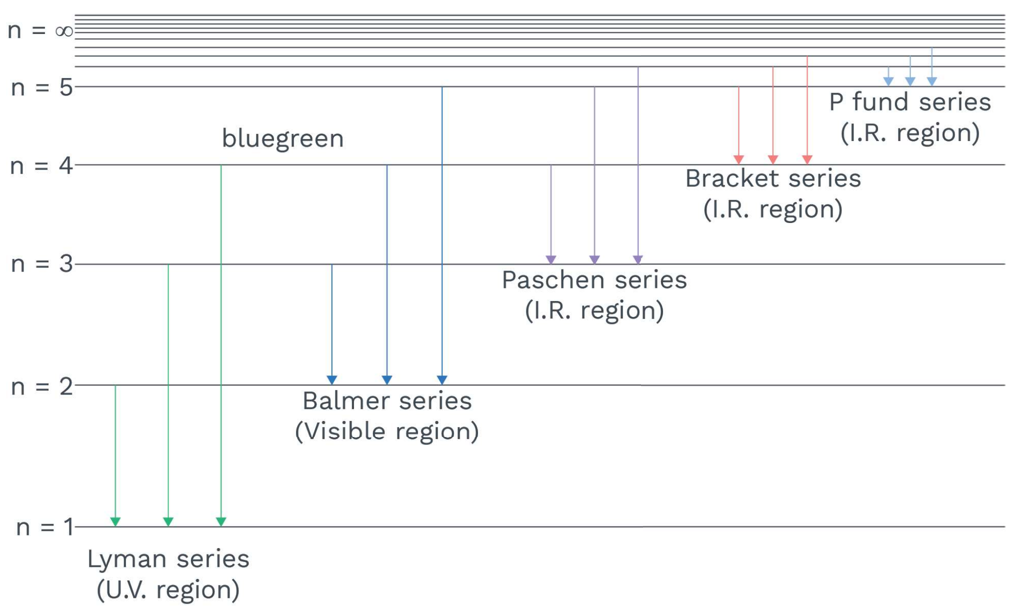
Electron in hydrogen atom, can be in excited state for very small time of the order of 10^{-8} second. This is because in the presence of conservative force, particles of a system always try to occupy stable equilibrium position and hence minimum potential energy, which is least in ground state. Because of instability, when an electron in excited state makes a transition to lower energy state a photon is emitted. Collection of such emitted photon frequencies is called an emission spectrum. This is as shown in figure.

Concept Reminder

When an electron is excited to n^{th} energy state in hydrogen, the possible number of spectral lines emitted are $\frac{n(n-1)}{2}$



The Spectral Series of Hydrogen Atom as shown in figure, are explained below.



(a) Lyman Series: Lines corresponding to transition from outer energy levels n_2 = 2, 3, 4, ... ∞ to first orbit (n_1 = 1) constitute Lyman series. The wave numbers of different lines are given by,

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n_2^2}\right]$$

Line corresponding to transition from n_2 = 2 to n_1 = 1 is first line; its wavelength is maximum.

Concept Reminder

The ratio of maximum to minimum possible radiation energy in Bohr's hydrogen model is equal to $\frac{4}{3}$.

$$\frac{1}{\lambda_{max}} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = 1.1 \times 10^7\left[\frac{1}{1} - \frac{1}{4}\right]$$

$$\therefore \quad \lambda_{max} = 1212Å$$

Similarly transition from $n_2 = \infty$ to $n = 1$ gives line of minimum wavelength.

$$\frac{1}{\lambda_{min}} = R\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = 1.1 \times 10^7$$

$$\therefore \lambda_{min} = 912\ Å$$

- Lyman series lies in ultraviolet region of electromagnetic spectrum.
- Lyman series is obtained in emission as well as in absorption spectrum.

(b) Balmer Series: Lines corresponding to $n_2 = 3, 4, 5, \ldots\ \infty$ to $n_1 = 2$ constitute Balmer series. The wave numbers of different lines are given by $\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right]$

- Line corresponding to transition $n_2 = 3$ to $n_1 = 2$ is first line, wavelength corresponding to this transition is maximum. Line corresponding to transition $n_2 = \infty$ to $n_1 = 2$ is last line; wavelength of last line is minimum.

$$\frac{1}{\lambda_{max}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$\therefore \quad \lambda_{max} = 6568\ Å$$

$$\frac{1}{\lambda_{min}} = R\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right]$$

$$\therefore \quad \lambda_{min} = 3636\ Å$$

- Balmer series lies in the visible region of electromagnetic spectrum. The wavelength of $L\alpha$ line is 656.8 nm (red). The wavelength of $L\beta$ line is 486 nm (blue green). The wavelength of $L\gamma$ line is 434 nm (violet). The remaining lines of Balmer series closest to violet light wavelength. The

Rack your Brain

Find the ratio of wavelengths of last line of Balmer series and the last line of Lyman series.

speciality of these lines is that in going from one end to other, the brightness and the separation between them decreases regularly.

- This series is obtained only in emission spectrum. Absorption lines corresponding to Balmer series do not exist, except extremely weakly, because very few electrons are normally in the state n = 2 and only a very few atoms are capable of having an electron knocked from the state n = 2 to higher states. Hence photons that correspond to these energies will not be strongly absorbed. In highly excited hydrogen gas there is possibility for detecting absorption at Balmer-line wavelengths

(c) Paschen Series: Lines corresponding to n_2 = 4,5,6,.......∞ to n_1 = 3 constitute Paschen series. The wave number of different lines are given by $\bar{\nu} = R\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right]$

- Line corresponding to transition n_2 = 4 to n_1 = 3 is first line, having maximum wavelength. Line corresponding to transition $n_2 = \infty$ to n_1 = 3 is last line, having minimum wavelength

$$\frac{1}{\lambda_{max}} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right] \quad \therefore \lambda_{max} = 19747\,\text{Å}$$

$$\frac{1}{\lambda_{min}} = R\left[\frac{1}{3^2} - \frac{1}{\infty}\right] = 1.1 \times 10^7 \times \left[\frac{1}{9} - 0\right]$$

$$\therefore \lambda_{min} = 8202\,\text{Å}$$

- 'Paschen series' lies in the infrared region of electromagnetic spectrum.
- This series is obtained only in the emission spectrum

(d) Bracket Series: The series corresponds to transitions from n_2 = 5,6,7......., ∞ to n_1 = 4. The wave number are given by, $\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{n_2^2}\right]$

KEY POINTS

- Lyman series
- Balmer series
- Paschen series
- Bracket series
- pfund series

- Line corresponding to transition from $n_2 = 5$ to $n_1 = 4$ has maximum wavelength and $n_2 = \infty$ to $n_1 = 4$ has minimum wavelength.

$$\frac{1}{\lambda_{max}} = R\left[\frac{1}{4^2} - \frac{1}{5^2}\right]$$

$$\therefore \lambda_{max} = 40477\,\text{Å}$$

$$\frac{1}{\lambda_{min}} = R\left[\frac{1}{4^2} - \frac{1}{\infty^2}\right]$$

$$\therefore \lambda_{min} = 14572\,\text{Å}$$

- This series lies in the 'infrared region' of electromagnetic spectrum.

(e) Pfund Series: This series corresponds to transitions from $n_2 = 6, 7, 8, ..., \infty$ to $n_1 = 5$. The wave numbers are given by

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{n_2^2}\right].$$

- Line corresponding to transition from $n_2 = 6$ to $n_1 = 5$ has maximum wavelength and $n_2 = \infty$ to $n_1 = 4$ has minimum wavelength.

$$\frac{1}{\lambda_{max}} = R\left[\frac{1}{5^2} - \frac{1}{6^2}\right]$$

$$\therefore \quad \frac{1}{\lambda_{max}} = 74563\,\text{Å} \qquad \frac{1}{\lambda_{min}} = R\left[\frac{1}{5^2} - \frac{1}{\infty^2}\right]$$

$$\therefore \quad \lambda_{min} = 22768\,\text{Å}$$

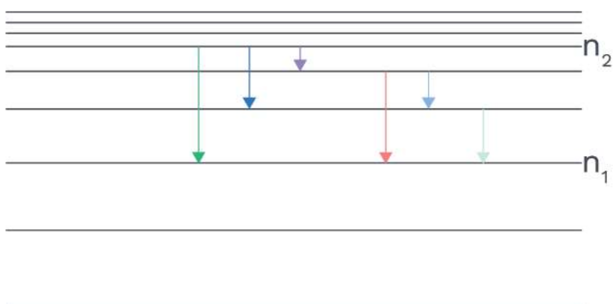
- This series lies in infrared region of electromagnetic spectrum.

Note : In an atom emission transition may start from any higher energy level and end at any energy level below of it. Hence in emission spectrum the total possible number of emission lines from some excited state n_2 to another energy state $n_1 (< n_2)$ is

$$\frac{(n_2 - n_1)(n_2 - n_1 + 1)}{2}$$

Rack your Brain

Hydrogen atom in ground state is excited by a monochromatic radiation of $\lambda = 975\text{Å}$. What will be the number of spectral lines in emission spectrum?



Note 1 : for $n_2 = 4$, and $n_1 = 1$, the number of possible lines are 6.
Note 2 : If ΔE is the energy difference between two given energy states, then due to transition between these two states wavelength of emitted photon is $\lambda(\text{Å}) = \dfrac{12400}{\Delta E(eV)}$

S.No.	NAME OF THE SERIES	FINAL STATE (n_1)	INITIALSTATE (n_2)	FORMULA	SERIES LIMIT	MAXIMUN WAVELENGTH	REGION
1	Lyman	$n_1 = 1$	2,3,4,.....∞	$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n_2^2}\right]$	$\lambda = \frac{1}{R} = 911A^0$	$\lambda = \frac{4}{3R}$	UV
2	Balmer	$n_1 = 2$	3,4,5,.....∞	$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right]$	$\lambda = \frac{4}{R}$	$\lambda = \frac{36}{5R}$	Visible
3	Paschen	$n_1 = 3$	4,5,6,.....∞	$\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right]$	$\lambda = \frac{9}{R}$	$\lambda = \frac{144}{7R}$	Near IR
4	Brackett	$n_1 = 4$	5,6,7,.....∞	$\frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{n_2^2}\right]$	$\lambda = \frac{16}{R}$	$\lambda = \frac{400}{9R}$	Middle IR
5	Pfund	$n_1 = 5$	6,7,8,.....∞	$\frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{n_2^2}\right]$	$\lambda = \frac{25}{R}$	$\lambda = \frac{9000}{11R}$	Far IR

Limitation of Bohr's model :

Despite its considerable achievements, the Bohr's model has certain short coming.

- It could not interpret the details of optical spectra of atoms containing more than one electron.
- It involves the concept of orbit which could not be checked experimentally.
- It could be successfully applied only to single electron atoms (e.g., H, He^+, Li^{2+}, etc.)
- Bohr's atomic model could not explain the binding of atoms into molecules.
- No justification was given for the "principle of quantization of angular momentum".

- Bohr's model could not explain the reason why atoms should combine to form chemical bonds and why do the molecules become more stable on such combinations.
- Bohr had assumed that an electron in the atom is located at definite distance from the nucleus and is revolving with a definite velocity around it. This is against the Heisenberg uncertainty principle. With the advancements in quantum mechanics, it become clear that there are no well defined orbits; rather there are clouds of negative charge.

Ex. Calculate the kinetic energy (K.E.), potential energy (P.E.) and total energy in first and second orbit of hydrogen atom if potential energy in first orbit is taken to be zero.

Sol. $E_1 = -13.60$ eV; $K_1 = -E_1 = 13.60$eV

$U_1 = 2E_1 = -27.20$ eV $E_2 = -3.40$eV

$K_2 = 3.40$eV and $U_2 = -6.80$eV

Now, $U_1 = 0$, i.e., potential energy has been increased by 27.20eV. So, we will increase U and E in all energy states by 27.20eV while kinetic energy will remain unchanged.

Hence K(eV), U(eV), E(eV)

For First orbit are 13.6, 0, 13.6

For Second orbit 3.40, 20.40, 23.80

Ex. Consider a H-like atom whose energy in nth excited state is given by $E_n = \frac{13.6\,Z^2}{n^2}$ when this excited atom makes transition from excited state to ground state most energetic photons have energy $E_{max} = 52.224$ eV and least energetic photons have energy $E_{min} = 1.224$eV. Calculate the atomic number of atom and the state of excitation.

Sol. Maximum energy is liberated for transition $E_n \rightarrow 1$ and minimum energy for En $\rightarrow E_{n-1}$

Hence, $\frac{E_1}{n^2} - E_1 = 52.224\,\text{eV}$ .... (1)

and $\frac{E_1}{n^2} - \frac{E_1}{(n-1)^2} = 1.224$ eV ..... (2)

Solving above equations simultaneously, we get

$E_1 = -54.4\text{eV}$ and n = 5

Now $E_1 = -\frac{13.6Z^2}{1^2} = 54.4\,\text{eV}$

Hence, Z = 2 i.e, gas is helium originally excited to n = 5 energy state.

Ex. A H-atom in a state of binding energy 0.85 eV makes a transition to a state of excitation energy of 10.2 eV.

(i) What is the initial state of H-atom?

(ii) What is the final state of H-atom?

(iii) What is the wavelength of photon emitted?

Sol. (i) Let n_1 be initial state of electron. Then $E_1 = -\frac{13.6}{n_1^2}\,\text{eV}$

Here $E_1 = -0.85\,\text{eV}$, therefore $-0.85 = \frac{13.6}{n_1^2} \Rightarrow n_1 = 4$

(ii) Let n_2 be the final excitation state of the electron. Since excitation energy is always measured with respect to the ground state, therefore.

$\Delta E = 13.6\left[1 - \frac{1}{n_2^2}\right]$, here $\Delta E = 10.2\,\text{eV}$, therefore,

$$10.2 = 13.6\left[1 - \frac{1}{n_2^2}\right] \Rightarrow n_2 = 2$$

Thus, the electron jumps from $n_1 = 4$ to $n_2 = 2$.

(iv) The wavelength of the photon emitted for a transition between $n_1 = 4$ to $n_2 = 2$, is given by

$$\frac{1}{\lambda} = R_\infty\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right]$$

(or) $$\frac{1}{\lambda} = 1.09 \times 10^7\left[\frac{1}{2^2} - \frac{1}{4^2}\right] \Rightarrow \lambda = 4860\ \text{Å}$$

Ex. The radius of the inner-most electron orbit of a H-atom is 5.3×10^{-11} m. What are the radii of n = 2 and n = 3 orbits?

Sol. Given, the radius of the innermost e^- orbit of a hydrogen $r_1 = 5.3 \times 10^{-11}$ m.

As we know that

$r_n = n^2 r_1$ for n = 2,

radius $r_2 = 2^2 r_1 = 4 \times 5.3 \times 10^{-11} = 2.12 \times 10^{-10}$ m.

For n = 3, radius $r_3 = 3^2 r_1 = 9 \times 5.3 \times 10^{-11} = 4.77 \times 10^{-10}$ m

Ex. A 12.5 eV e^- beam is used to bombard gaseous hydrogen (H) at room temperature (27°C). What series of wavelength will be emitted?

Sol. Energy of e^- beam E = 12.5 eV

= $12.5 \times 1.6 \times 10^{-10}$ J

Planck's constant (h) = 6.63×10^{-34} J-s

Speed of light c = 3×10^8 m/s

Using the relation

$$E = \frac{hc}{\lambda} = \frac{6.32 \times 10^{-34} \times 3 \times 10^8}{12.5 \times 1.6 \times 10^{-19}}$$

= 0.993×10^{-7} metre = 993×10^{-10} m = 993 Å.

This wavelength falls in the range of 'Lyman series' (912 Å to 1216 Å)

So, we conclude that Lyman series of wavelength 993 Å is emitted.

Ex. Calculate (a) the wavelength and (b) the frequency of the Hβ line of the Balmer series for hydrogen.

Sol. (a) Hβ line of Balmer series corresponds to the transition from n = 4 to n = 2 level. The corresponding wavelength for Hβ line is,

$$\frac{1}{\lambda} = (1.097 \times 10^7)\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$

$$\frac{1}{\lambda} = 0.306 \times 10^7$$

$\therefore \quad \lambda = 4.9 \times 10^{-7}$ m

(b) $$\nu = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{4.9 \times 10^{-7}}$$

$$\nu = 6.12 \times 10^{14} \text{ Hz}$$

Ex. Find the largest and shortest wavelengths in the Lyman series for hydrogen. In what region of the electromagnetic spectrum does each series lie?

Sol. The transition equation for Lyman series is given by, $\frac{1}{\lambda} = R\left[\frac{1}{(1)^2} - \frac{1}{n^2}\right]$

n = 2, 3, ...

For largest wavelength, n = 2

$$\frac{1}{\lambda_{max}} = 1.097 \times 10^7\left(\frac{1}{1} - \frac{1}{4}\right) = 0.283 \times 10^7$$

$\therefore \quad \lambda_{max} = 1.2154 \times 10^{-7}\text{m} = 1215\text{ Å}$

The shortest wavelength corresponds to n = ∞

$$\therefore \quad \frac{1}{\lambda_{max}} = 1.097 \times 10^7 \left(\frac{1}{1} - \frac{1}{\infty}\right)$$

or $\quad \lambda_{min} = 0.911 \times 10^{-7} m = 911 Å$

Both of these wavelengths lie in ultraviolet (UV) region of electromagnetic spectrum.

Ex. How may different wavelengths be observed in the spectrum from a hydrogen sample if atoms are excited to states with quantum number (n)?

Sol. From the nth state, the atom may go to $(n - 1)^{th}$ state, ..., 2^{nd} state or 1^{st} state. So there are (n – 1) possible transitions starting from the nth state. The atoms reaching $(n - 1)^{th}$ state may make (n – 2) different transitions. Similarly for other lower states. The total number of possible transitions is

(n – 1) + (n – 2) + (n – 3) +...........2 + 1

$$= \frac{n(n-1)}{2}$$

Ex. Find the kinetic energy potential energy and total energy in first and second orbit of hydrogen atom if potential energy in first orbit is taken to be zero.

Sol. $E_1 = -13.60$ eV, $\quad K_1 = -E_1 = 13.60$ eV

$U_1 = 2E_1 = -27.20$ eV

$$E_2 = \frac{E_1}{(2)^2} = -3.40\,eV$$

$K_2 = 3.40$ eV and $U_2 = -6.80$ eV

Now $U_1 = 0$, i.e., potential energy has been increased by

27.20 eV while kinetic energy will remain unchanged. So values of kinetic energy, potential energy and total energy in first orbit are 13.60 eV, 0, 13.60 respectively and for second orbit these values are 3.40 eV, 20.40 eV and 23.80 eV

Ex. An e^- is orbiting in a circular orbit of radius r under the influence of a constant magnetic field of strength B. If Bohr's postulate regarding the quantisation of angular momentum holds good for this electron, find

(a) Allowed values of the radius 'r' of the orbit.

(b) Kinetic energy of the electron in orbit

(c) Potential energy of interaction between magnetic moment of the orbital current due to the e^- moving in its orbit and magnetic field B.

(d) Total energy of the allowed energy levels.

Sol. (a) radius of circular path

$$r = \frac{mv}{Be} \qquad \text{... (i)}$$

$$mvr = \frac{nh}{2\pi} \qquad \text{.... (ii)}$$

Solving the two equations, we get

$$r = \sqrt{\frac{nh}{2\pi Be}} \quad \text{and} \quad v = \sqrt{\frac{nhBe}{2\pi m^2}}$$

(b) $K = \frac{1}{2}mv^2 = \frac{nhBe}{4\pi m}$

(c) $M = iA = \left(\frac{e}{T}\right)(\pi r^2) = \frac{evr}{2} = \frac{e}{2}\sqrt{\frac{nh}{2\pi Be}}\sqrt{\frac{nhBe}{2\pi m^2}} = \frac{nhe}{4\pi m}$

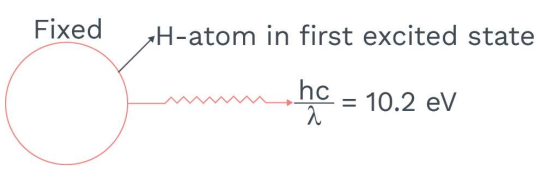
Now potential energy $U = -M.B = \frac{nheB}{4\pi m}$

(d) $E = U + K = \frac{nheB}{2\pi m}$

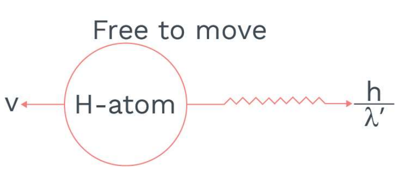
Calculation of recoil speed of atom on emission of a photon:-

momentum of photon = $mc = \frac{hc}{\lambda} = 10.2\,\text{eV}$

(a)



(b)



Concept Reminder

Conservation of linear momentum is a fundamental conservation law. It is applicable everywhere including in an atom.

m = mass of atom

According to momentum conservation law

$$mv = \frac{h}{\lambda'} \qquad \text{.... (i)}$$

According to energy conservation

$$\frac{1}{2}mv^2 + \frac{hc}{\lambda'} = 10.2\,\text{eV}$$

Since mass of atom is very large than photon hence $\frac{1}{2}mv^2$ can be neglected

$$\frac{hc}{\lambda'} = 10.2\,eV \qquad \Rightarrow \frac{h}{\lambda'} = \frac{10.2}{c}eV$$

$$mv = \frac{10.2}{c}eV \qquad \Rightarrow v = \frac{10.2}{cm}$$

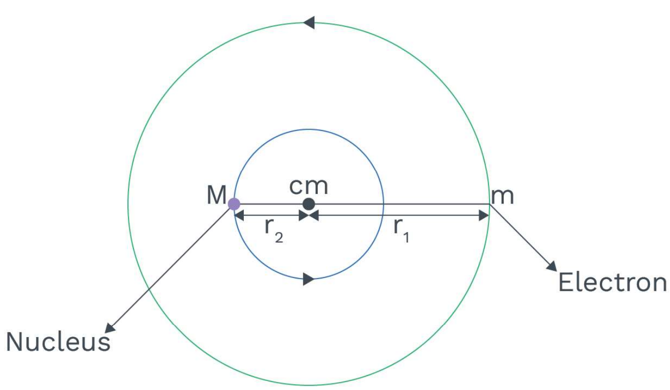
Recoil speed of atom = $\frac{10.2}{cm}$

Effect of finite mass of nucleus on Bohr's model of an atom

(i) In the atomic spectra of hydrogen and hydrogen like atoms a very small deviation with Bohr's model occurs.

(ii) This is in the assumption that the nucleus is infinitely massive when compared to mass of electron so that it remains stationary during the rotation of electron around it.

(iii) Infact the nucleus is not infinitely massive and hence both the nucleus and electron revolve around their centre of mass with same angular velocity ω



(iv) let m be the mass of electron, M be the mass of the nucleus, Z be its atomic number and r be the separation between them. If r_1, r_2 are distances of centre of mass from electron and nucleus respectively then $r_1 + r_2 = r$ and

KEY POINTS

- Recoil speed
- Reduced mass

Rack your Brain

Why nearly all H atoms are in the ground state at room temperature.

$$r_1 = \frac{Mr}{M+m}, r_2 = \frac{mr}{M+m}$$

(v) For both the electron and the nucleus the necessary centripetal force to revolve in circular orbits is provided by the electrostatic force between them.

i.e., $mr_1\omega^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$

$$m\frac{Mr}{M+m}\omega^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$$

i.e., $r^3\omega^2 = \frac{Ze^2}{4\pi\varepsilon_0}$ ..... (1)

where $= \frac{Mm}{M+m}$ called reduced mass

(vi) From Bohr's theory of quantization of angular momentum, total angular momentum of the system

$$L = I_1\omega + I_2\omega = \frac{nh}{2\pi} \qquad mr_1^2\omega + Mr_2^2\omega = \frac{nh}{2\pi}$$

$$m\frac{M^2r^2}{(M+m)^2}\omega + \frac{Mm^2r^2}{(M+m)^2}\omega = \frac{nh}{2\pi}$$

$$\frac{mMr^2\omega}{(M+m)^2}(M+m) = \frac{nh}{2\pi}$$

i.e. $r^2\omega = \frac{nh}{2\pi}$ ..... (2)

(vii) A system of this type is equivalent to a single particle of mass μ revolving around the position of the heavier particle(nucleus) in an orbit of radius r

From (1) and (2) $r = \frac{\varepsilon_0 n^2h^2}{\pi z\ \ e^2}$

(viii) Radius of orbit of such a particle in a quantum state n is $r_n = \frac{\varepsilon_0 n^2h^2}{\pi z\mu e^2} \Rightarrow r_n \propto \frac{n^2}{\mu z}$

Concept Reminder

In case, when we take into account mass of nucleus we have to replace mass of electron by reduced mass in all formulae.

(ix) Potential energy of the system $PE = \frac{-Ze^2}{4\pi\varepsilon_0 r}$

and kinetic energy of the system

$$KE = \frac{1}{2}I_1\omega^2 + \frac{1}{2}I_2\omega^2 = \frac{1}{2}(I_1 + I_2)\omega^2$$

$$= \frac{1}{2}\left(mr_1^2 + Mr_2^2\right)\omega^2$$

$$= \frac{1}{2}\left[m\frac{M^2r^2}{(M+m)^2} + M\frac{m^2r^2}{(M+m)^2}\right]\omega^2$$

$$= \frac{1}{2}\frac{mMr^2\omega^2}{(M+m)^2}(M+m) = \frac{1}{2}\left(\frac{mM}{M+m}\right)r^2\omega^2$$

i.e. $KE = \frac{1}{2}\mu r^2\omega^2 = \frac{1}{2}\frac{Ze^2}{4\pi\varepsilon_0 r}\left(\because \mu r^3\omega^2 = \frac{Ze^2}{4\pi\varepsilon_0}\right)$

∴ Total energy of the system (or equivalent particle of mass μ)

E = PE + KE

$$E = -\frac{Ze^2}{8\pi\varepsilon_0 r} = -\frac{Ze^2}{8\pi\varepsilon_0} \times \frac{\pi Z\mu e^2}{n^2h^2\varepsilon_0} = -\frac{Z^2\mu e^4}{8\varepsilon_0^2n^2h^2}$$

i.e Energy of n^{th} quantum state

$$E_n = \frac{-Z^2\mu e^4}{8\varepsilon_0^2n^2h^2} = \frac{-mZ^2\mu e^4}{8\varepsilon_0^2n^2h^2 \times m} = \frac{-me^4}{8\varepsilon_0^2h^2} \times \left(\frac{Z^2\mu}{n^2m}\right)$$

$$E_n = -13.6\left(\frac{z^2}{n^2}\right) \times \frac{\mu}{m}eV = -13.6\left(\frac{Z^2}{n^2}\right) \times \frac{M}{M+m}eV$$

$$= \frac{-13.6}{1+\frac{m}{M}}\left(\frac{Z^2}{n^2}\right)eV$$

The formulae for r_n and E_n can be obtained simply by replacing m by μ in the formulae for stationary nucleus, If λ is wave length of photon emitted due to transition from a quantum state n_2 to a quantum state n_1,

then $\frac{hc}{\lambda} = \frac{me^4z^2}{8\varepsilon_0^2h^2} \times \frac{1}{\left(1+\frac{m}{M}\right)}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Concept Reminder

$$R_M = \frac{R_\infty}{1+\frac{m}{M}}$$

where R_∞ = Rydberg's constant for infinitely heavy nucleus and R_M = Rydberg's constant for M mass of nucleus.

$$\Rightarrow \quad \frac{hc}{\lambda} = \frac{me^4 z^2}{8\varepsilon_0^2 h^2} \times \frac{1}{\left(1+\frac{m}{M}\right)}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \quad \frac{1}{\lambda} = \frac{R_0 Z^2}{\left(1+\frac{m}{M}\right)}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$ where R_0 is Rydberg's constant when the nucleus is stationary.

Excitation by collision

(i) When an atom is bombarded by particles like electron, proton, neutron, α – particle etc, the loss in KE of the system during collision may be used in excitation of the atom.

(ii) If loss in KE of the system during collision (during deformation phase) is less than the energy required to excite the e^- to next higher energy state, electron can't be excited and the loss of KE of the system during deformation phase again converts into KE of the system and totally there will be no loss of KE of the system and hence the collision is elastic.

(iii) If loss in KE of the system during deformation phase is more than or equal to the energy required to excite the electron to next higher state, excitation of the electron may take place and hence kinetic energy of the system may not be conserved hence the collision may be inelastic or even perfectly inelastic.

(iv) If loss in KE is sufficient even ionization may take place. Even though the possible loss in KE is greater than or equal to excitation energy of electron, excitation may not take place necessarily and hence collision may be elastic.

(v) Consider a particle of mass m moving with velocity u which strikes a stationary

Concept Reminder

A positronium atom is a system that consists of a positron (instead of proton) and an electron that orbit each other. The ground state energy of this atom is –6.8 eV.

hydrogen like atom of mass M which is in ground state.

(vi) Loss in KE will be maximum in perfectly inelastic collision. In this case if V is common velocity after collision, from conservation of linear momentum

$$mu + 0 = (M + m)V \Rightarrow V = \frac{mu}{M + m}$$

$\therefore$ Maximum possible loss in KE is

$$\Delta K = \frac{1}{2}mu^2 - \frac{1}{2}(M + m)V^2$$

$$\text{i.e } \Delta K = \frac{1}{2}\left(\frac{Mm}{M + m}\right)u^2$$

(vii) If ΔE is minimum excitation energy (ex: n = 1 to n = 2 in ground state) and if Δ E< Δ K E then electrons can't be excited hence there will be no loss of KE of the system hence the collision is elastic.

(viii) If $\Delta E = \Delta$ K E the electron may get excited, and the collision may be perfectly in elastic.

(ix) If $\Delta E > \Delta$ K E the electron may get excited to higher energy states or even removed from the atom and may have some kinetic energy. In this case the collision may be inelastic or may be perfectly inelastic as there is loss in KE of the system, or even elastic if excitation does not take place.

Explanation of second postulate of Bohr by De-Broglie Hypothesis

OR

Explanation of Bohr's quantization by De-Broglie

- According to de-Broglie hypothesis wave nature is associated with every moving particle. Revolving electron around the nucleus inside the atom is also a moving particle. So, a wave must be associated with it.
- According to De-Broglie the wave associated with the revolving electron is a stationary wave for

Concept Reminder

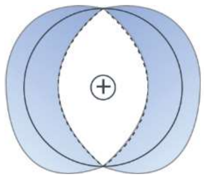
There are two ways to excite an electron in an atom:

(1) By supplying energy to an electron through electromagnetic photons.

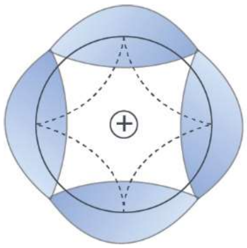
(2) The energy can be supplied to an electron by atomic collision.

which circumference of the circular orbit must integral multiple of its wavelength.

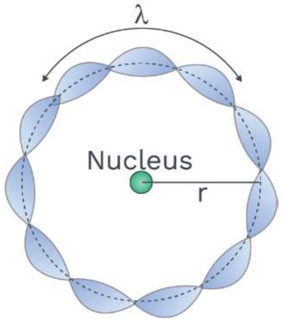
For n = 1



For n = 2



For n = n



In generalize form

$$2\pi r = n\lambda \qquad \text{..... (i)}$$

But according to De-Broglie hypothesis the wavelength of the wave associated with the moving particle.

$$\lambda = \frac{h}{mv} \qquad \text{.... (ii)}$$

Using equation (ii) into equation (i)

$$2\pi r = n\frac{h}{mv} \Rightarrow \boxed{mvr = \frac{nh}{2\pi}}$$

It's the second postulate of Bohr model.

Concept Reminder

The angular momentum of an electron in an orbit is quantized because it is a necessary condition for the compatibility with wave nature of electron.

X-RAYS

X-rays are electromagnetic radiations of very short wavelength (0.1 Å to 100 Å) and high energy which are radiated when fast moving electrons or cathode rays strike a target of high atomic mass.

Discovery of X-Rays:

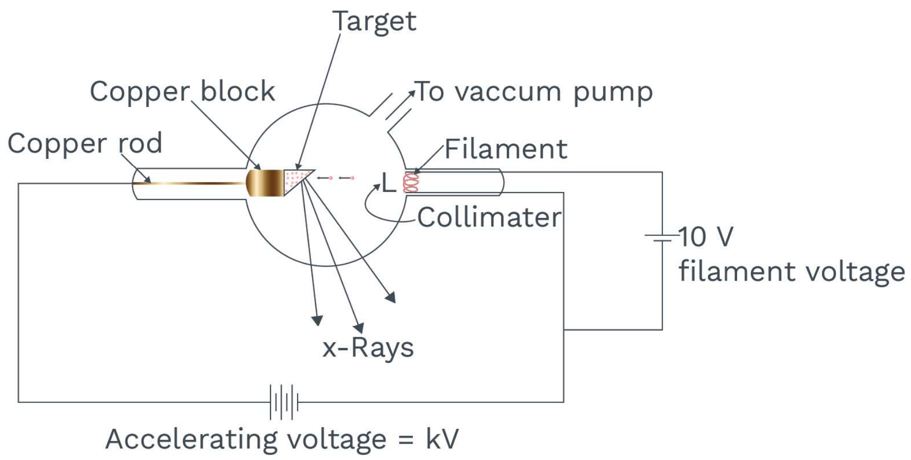
X-rays were discovered by Roentgen (1895) who found that a discharge tube, operating at low pressure and high voltage, emitted a radiation that caused a florescent screen in the neighbourhood to glow brightly. Crystals of barium platinocyanide also showed fluorescence. Results were same, if the discharge tube is wrapped in black paper, to prevent visible light. This indicated that some unknown radiation (X-rays) were responsible for fluorescence. Roentgen then confirmed that X-rays are emitted, when cathode rays (electrons) strike the wall of discharge tube.

Production of X-Rays (Coolidge's Tube):

X-rays are produced when energetic (fast moving) electrons strike a target such as a metal piece. When electrons collide with the atoms of solid, they lose their kinetic energy which is converted into radiant energy in the form of X-rays. The figure shows essential features of a modern X-ray tube developed by Coolidge.

Concept Reminder

Prof. Wilhelm Konrad Roentgen, a German Scientist, in 1895, observed that when fast moving cathode rays strike a metal piece of hight atomic weight and high melting point, a new kind of rays are produced. And he called them X-rays.



Coolidge's X-ray tube consists of a glass bulb exhausted to nearly perfect vacuum. The cathode C is the source of electrons by using a heated Filament getting supply from battery. The anode is made of solid copper bar A. A high melting metal like platinum or tungsten is embedded at the end of copper rod and serves as target T. A high d.c. voltage V(50 kV) is maintained between cathode and anode.

The energetic electrons strike the target and the X-rays are produced. Only about 1-10% of the energy of the electrons is converted to X-rays and the rest is converted into heat. The target T as a result becomes very hot and therefore should have high melting point. The heat generated is dissipated through the copper rod and the anode is cooled by water flowing through the anode.

The nature of emitted X-rays depends on:

(i) the current in the filament F

(ii) the voltage between the filament and the anode.

- An increase in the filament current increases the number of electrons it emits. Larger number of electrons means an intense beam of X-rays is produced. This way we can control the quantity of X-rays i.e. Intensity of X-rays.
- An increase in the voltage of the tube increase the kinetic energy of electrons ($eV = 1/2\ mv^2$). When such highly energetic beam of electrons are suddenly stopped by the target, an energetic beam of X-rays is produced. This way we can control the quality of X-rays i.e. penetration power of X-rays.
- Based on penetrating power, X-rays are classified into two types. HARD-rays and SOFT-X-rays. The first one having high energy and hence high

penetration power are HARD-X-rays and the other with low energy and hence low penetration power are SOFT-X-rays.

Properties of X-Rays:

- These are highly penetrating rays and can pass through several materials which are opaque to ordinary light.
- They ionize the gas through which they pass. While passing through a gas, they knock out electrons from several of the neutral atoms, leaving these atoms with +ve charge.
- They cause fluorescence in several materials. A plate coated with barium platinocyanide, (zinc sulphide) etc becomes luminous when exposed to X-rays.
- They affect photographic plates especially designed for the purpose.
- They are not deflected by electric and magnetic fields, showing that they are not charged particles.
- They show all the properties of the waves except refraction. They show diffraction patterns when passed through a crystal which behaves like a grating.

Application of X-rays:

X-rays have important and useful applications in surgery, medicine, engineering and studies of crystal structures.

1. **Scientific Applications:**
   The diffraction of X-rays at crystals opened new dimension to X-rays crystallography. Various diffraction patterns are used to determining internal structure of crystals. The spacing and dispositions of atoms of a crystal can be precisely determined used Bragg's Law: .
2. **Industrial Applications:**
   Since X-rays can penetrate through various materials, they are used in industry to detect defects in metallic structures in Big machines, railway tracks and bridges. X-rays are used to analyse the composition of alloys and pearls.
3. **In Radio Therapy:**
   X-rays can cause damage to the tissues of body (cells are ionized and molecules are broken). So X-rays damages the malignant growths like cancer and tumours which are dangerous to life, when it used in proper and controlled intensities.

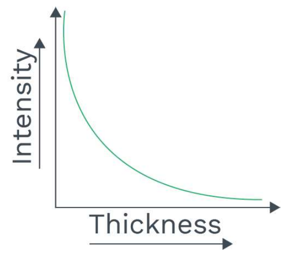
4. **In Medicine and Surgery:**

X-rays are absorbed more in heavy elements than the lighter ones. Since bones (containing calcium and phosphorus) absorb more X-rays than the surrounding tissues (containing light elements like), their shadow is casted on the photographic plate. So the cracks or Fracture in bones can be easily located. Similarly intestine and digestive system abnormalities are also detected by X-rays.

X-ray Absorption:

The intensity of X-rays at any point may be defined as the energy falling per second per unit area held perpendicular to the direction of energy flow. The intensity of a X-rays beam decreases during its passage through the sheet of any material. The decrease in the intensity of X-rays is due to the absorption of X-rays by the material.

- The intensity of X-ray beam is defined as amount of energy carried per unit area per sec perpendicular to direction of flow of energy.



- When a beam of X-rays with incident intensity I_0 passes through material then intensity of emergent X-rays (I) is

$$I = I_0 e^{-\mu x}$$

where μ is absorption coefficient and x is thickness of medium.

- The absorption coefficient of the material is defined as reciprocal of thickness after which intensity of X-rays falls to $\frac{1}{e}$ times the original intensity.

KEY POINTS

- Soft X-rays
- Hard X-rays
- Absorption coefficient

Rack your Brain

X-rays incident on a material.

(1) Will exert a force on it.

(2) Will transfer energy to it.

(3) May cause emission of electrons.

(4) All of these

at $\mu = \frac{1}{x} \Rightarrow I = \frac{I_0}{e}$

- The absorption coefficient depends on wavelength of X-rays (λ), the atomic number (Z) of material and density (ρ) of material.
  Absorption coefficient $\mu = CZ^4\lambda^3\rho$

  (i) $\mu \propto \lambda^3$ (ii) $\mu \propto \frac{1}{\nu^3}$

  (iii) $\mu \propto Z^4$ (iv) $\mu \propto \rho$
- Best absorber of X-rays is lead while lowest absorption takes place in air.
- Half Thickness ($X_{1/2}$) : The thickness of given sheet which reduces the intensity of incident X-rays to half of its initial value is called half thickness.

  at $x = x_{1/2}$

  So, $x_{1/2} = \frac{0.693}{\mu}\left(\frac{I}{I_0}\right) = \left(\frac{1}{2}\right)^{x/x_{1/2}}$
- For photographing human body parts $BaSO_4$ is used.
- The patients are asked to drink $BaSO_4$. solution before X-rays examination because it is a good absorber of X-rays.

Ex. The absorption coefficients of Al for soft X-rays is 1.73 per cm. Find the percentage of transmitted X-rays from a sheet of thickness 0.578 cm.

Sol. $I = I_0 e^{-\mu x}$

So, $\frac{I}{I_0} = e^{-\mu x} = e^{-1.73 \times 0.578}$

or $\frac{I}{I_0} = e^{-1} = \frac{1}{e} = \frac{1}{2.718} = 37\%$

Ex. When X-rays of wavelength 0.5Å pass through 10 mm thick Al sheet then their intensity is reduced to one sixth. Find the absorption coefficient for Aluminium.

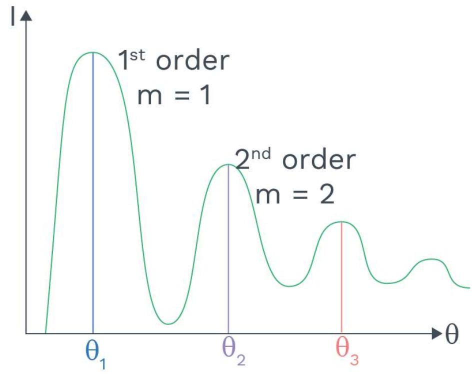
Sol. $\mu = \frac{2.303}{x}\log_{10}\left(\frac{I_0}{I}\right) = \frac{2.303}{10}\log_{10} 6$

$= \frac{2.303 \times 0.7781}{10} = 0.198\ /\ \text{mm}$

Rack your Brain

The interplanar distance in a crystal is 2.8×10^{-8} m. What is value of maximum wavelength that can be diffracted.

X-Ray Spectra and Origin of X-Rays:
Experimental observation and studies of spectra of X-rays reveal that X-rays are of two types and so are their respective spectra's.

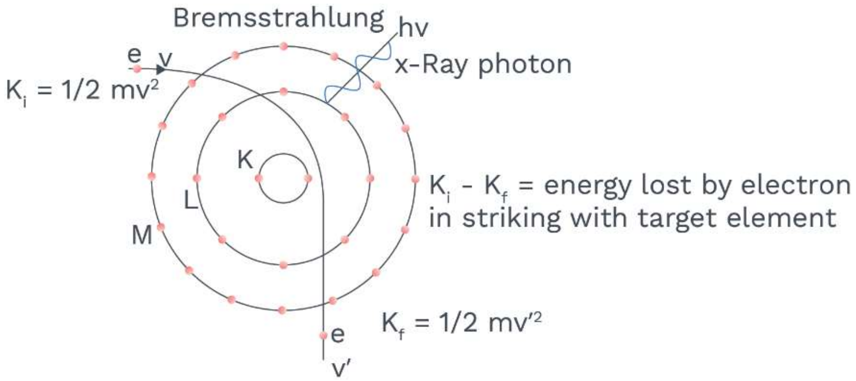


Concept Reminder

X-ray spectrum has mainly two members, viz., continuous and characteristic spectrum. Continuous spectrum was called Bremsstrahlung (braking radiation). characteristic spectrum was explained by Moseley.

(i) Characteristic X-rays
(ii) Continuous X-rays.

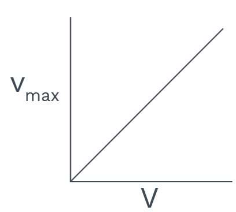
Continuous x-ray:
When high energy electrons (accelerated by Coolidge tube potential) strike the target element they are defected by coulomb attraction of nucleus & due to numerous glancing collisions with the atoms of the target, they lose energy which appears in the form of electromagnetic waves (bremsstrahlung or braking radiation) & the remaining part increases the kinetic energy of the colliding particles of the target. The energy received by the colliding particles goes into heating the target. The electron makes another collision with its remaining energy.



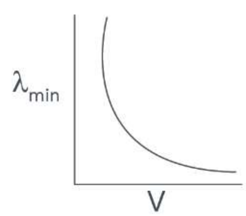
1. When high energetic electrons enter into target material, they are decelerated. In this process emission of energy take place. Spectrum of this energy is continuous. This is also called bremsstrahlung.
2. Continuous spectrum (ν or λ) depends upon potential difference between filament and target.
3. It does not depends upon nature of target material.
4. If V is the potential difference & ν is the frequency of emitted x-ray photon then.

Variation of frequency (ν) and wavelength (λ) of x-rays with potential difference is plotted as shown in figure:

$$eV = \frac{1}{2}mv^2 = h\nu_{max} = \frac{hc}{\lambda_{min}}$$

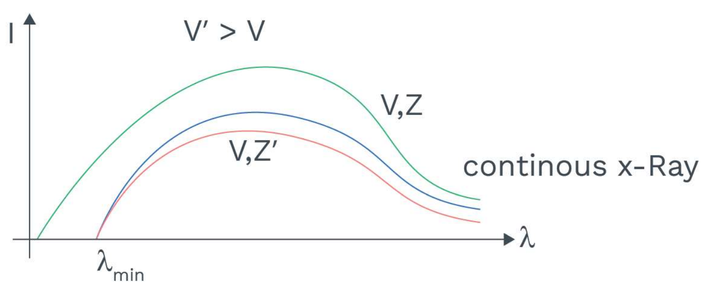


$$\nu_{max} = \frac{eV}{h}$$



$$\lambda_{min} = \frac{hc}{eV}$$

Variation of Intensity of x-rays with λ is plotted as shown in figure:



The minimum wavelength corresponds to the maximum energy of the x-rays which in turn is equal to the maximum kinetic energy eV of the striking electrons thus

$$eV = \frac{1}{2}mv^2 = h\nu_{max} = \frac{hc}{\lambda_{min}}$$

$$\Rightarrow \lambda_{min} = \frac{hc}{eV} = \frac{12400}{V(\text{involts})}\text{Å}$$

We see that cut off wavelength λ_{min} depends only on accelerating voltage applied between target and filament. It does not depend upon material of target, it is same for two different metals (Z and Z')

Ex. An X-ray tube operates at 20 kV. A particular electron loses 5% of its kinetic energy to emit an X-ray photon at the first collision. Find the wavelength corresponding to this photon.

Sol. Kinetic energy acquired by the electron is K = 20×10^3 eV.

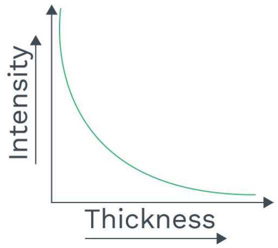
The energy of the photon = $0.05 \times 20 = 10^3$ eV .

Thus, $\frac{h\nu}{\lambda} = 10^3 \text{eV}$

$$\lambda = \frac{1242\,\text{eV} - \text{nm}}{10^3\ \text{eV}} = 1.24\,\text{nm}$$

Characteristic X-rays

The sharp peaks obtained in graph are known as characteristic x-rays because they are characteristic of target material. $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, ... = characteristic wavelength of material having atomic number Z are called characteristic x-rays and the spectrum obtained is called characteristic spectrum. If target of atomic number Z' is used then peaks are shifted.



Characteristic x-ray emission occurs when an energetic electron collides with target and remove an inner shell electron from atom, the

Rack your Brain

The minimum wavelength of the x-rays produced by electrons accelerated through potential difference of V volts is directly proportional to

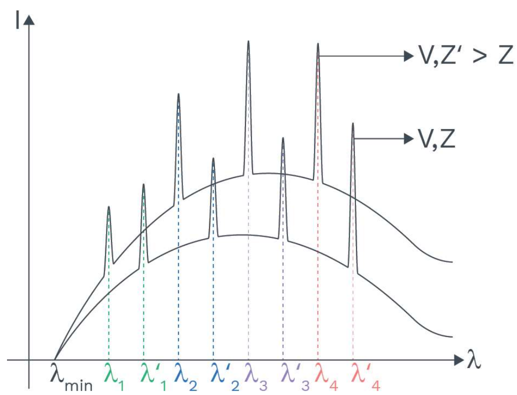
(1) $\frac{1}{\sqrt{V}}$ (2) $\frac{1}{V}$

(3) $\sqrt{V}$ (4) V^2

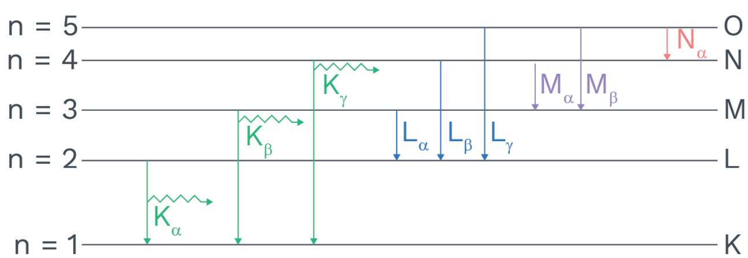
KEY POINTS

- Continuous X-rays
- Characteristic X-rays
- Moseley's law

vacancy created in the shell is filled when an electron from higher level drops into it. Suppose vacancy created in innermost K-shell is filled by an electron dropping from next higher-level L-shell then Kα characteristic x-ray is obtained. If vacancy in K-shell is filled by an electron from M-shell, Kβ line is produced and so on



Similarly Lα, Lβ,.....Mα, Mβ lines are produced.



Continuous and characteristic x-rays

(i) When an accelerated electron hits the target, the electron loses its energy in two processes. One process gives rise to continuous X-rays and the other process gives rise to characteristic X-rays.

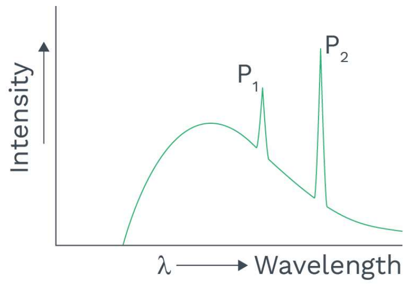
(ii) When the electron loses its kinetic energy in several collisions with the atoms, a fraction can range from zero to one.

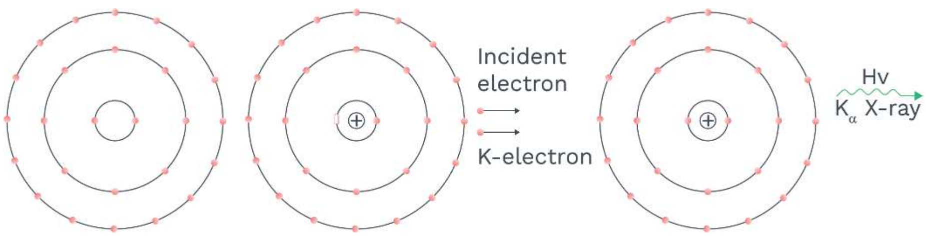
(iii) Corresponding to the maximum kinetic energy lost by an electron, we have the largest frequency of the X-ray or the shortest wavelength.

$$\frac{hc}{\lambda_{min}} = K_{max} = eV \Rightarrow \lambda_{min} = \frac{hc}{eV}$$

The emitted wavelengths range from a minimum value all the way to infinity. Thus the name continuous spectrum.

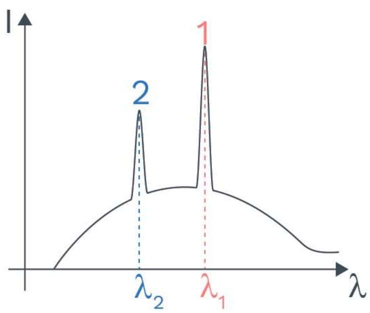
(iv) The other possibility is that the accelerated electron might knock out the inner electron of the target atom, whereby a vacancy is created in the inter orbit. Electrons from the higher orbit jump in to fill this vacancy, while releasing the energy difference as electromagnetic radiations. These wavelengths are characteristic of the material from which they are emitted. Hence the name Characteristic Spectra.





(v) To discuss energy transitions in X-rays, we take the ground state atom with all electrons intact as the zero-level. The atom with a vacancy in the K-shell is the one with the highest energy, as a lot of energy is required to dismiss a K-shell electron.

Ex. Find which is K_α and K_β



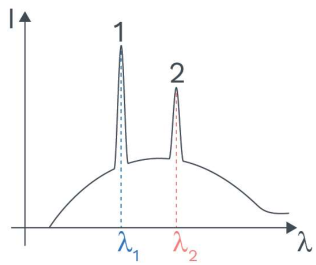
Sol. $\Delta E = \dfrac{hc}{\lambda}, \qquad \lambda = \dfrac{hc}{\Delta E}$

since energy difference of K_α is less than K_β

$\Delta E_k\alpha < \Delta E_k\beta$

1 is K_β and 2 is K_α

Ex. Find which is K_α and L_α

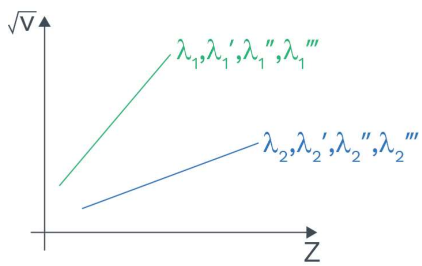


Sol. $\therefore$ $\Delta E_K\alpha > \Delta E_L\alpha$

1 is K_α and 2 is L_α

MOSELEY'S LAW:

Moseley measured the frequencies of characteristic x-rays for a large number of elements and plotted the square root of frequency against position number in periodic table. He discovered that plot is very closed to a straight line not passing through origin.



Z_1	I_1	I_2
Z_2	I_1'	I_2'
Z_3	I_1''	I_2''
Z_4	I_3'''	I_2'''

Wavelength of characteristic wavelengths

Moseley's observations can be mathematically expressed as

$$\sqrt{\nu} = a(Z - b)$$

a and b are positive constants for one type of x-rays & for all elements (independent of Z). Moseley's Law can be derived on the basis of Bohr's theory of atom, frequency of x-rays is given by

$$\sqrt{\nu} = \sqrt{CR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}.(Z - b)$$

by using the formula $\frac{1}{\lambda} = Rz^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ with modification for multi electron system.

b → known as screening constant or shielding effect, and (Z – b) is effective nuclear charge.

for Kα line

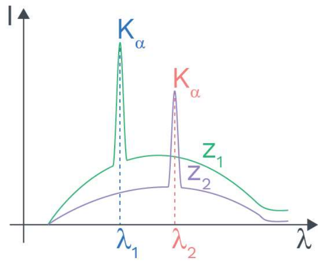
$$n_1 = 1,\ n_2 = 2$$

$$\therefore \quad \sqrt{\nu} = \sqrt{\frac{3RC}{4}}(Z - b)$$

$$\Rightarrow \quad \sqrt{\nu} = a(Z - b)$$

Here, $a = \sqrt{\frac{3RC}{4}}$, [b = 1 for K_α lines]

Ex. Find in Z_1 and Z_2 which one is greater.



Sol. $\because \quad \sqrt{\nu} = \sqrt{cR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}.(Z - b)$

If Z is greater then ν will be greater, λ will be less

$\therefore \quad \lambda_1 < \lambda_2$

$\therefore \quad Z_1 > Z_2$.

Ex. A cobalt target is bombarded with electrons and the wavelength of its characteristic spectrum are measured. A second, fainter, characteristic spectrum is also found because of an impurity in the target. The wavelength of the K_α lines are 178.9 pm (cobalt) and 143.5 pm (impurity). What is the impurity?

Sol. Using Moseley's law and putting c/λ for ν (and assuming b = 1), we obtain

$$\sqrt{\frac{c}{\lambda_{c_0}}} = aZ_{c_0} = a \quad \text{and} \quad \sqrt{\frac{c}{\lambda_x}} = aZ_x = a$$

Dividing yields $\sqrt{\frac{\lambda_{c_0}}{\lambda_x}} = \frac{Z_x - 1}{Z_{c_0} - 1}$.

Substituting gives us $\sqrt{\frac{178.9\,\text{pm}}{143.5\,\text{pm}}} = \frac{Z_x - 1}{27 - 1}$

Solving for the unknown, we find Z_x = 30.0; the impurity is zinc.

EXAMPLES:

Q1 **Find the radius and energy of a He^+ ion in the states (a) n = 2, (b) n = 3.**

Sol: r = 0.529;

(a) $r\,(n = 2) = 0.529 \times \frac{2^2}{2} = 1.058$ Å

$E(n = 2) = -13.6 \times \frac{2^2}{2^2} = -13.6$ eV

(b) $r\,(n = 3) = 0.529 \times \frac{3^2}{2} = 2.38$ Å

$E\,(n = 3) = -13.6 \times \frac{2^2}{3^2} = -6.04$ eV.

Q2 **A positive hydrogen like ion having electron at its ground state emits it, if a photon of wavelength 228 Å or less is absorbed by it. Identify the ion.**

Sol: BE of last electron,

$E = 13.6 \times Z^2 = \frac{hc}{\lambda}$

$\Rightarrow \quad Z^2 = \frac{1.24 \times 10^4}{13.6 \times 228} \approx 4$

$\Rightarrow \quad Z = 2$

Q3 **Find the temperature at which the average kinetic energy of the molecules of hydrogen equals the binding energy of its electron in ground state, assuming average kinetic energy of hydrogen gas molecule = $\frac{3}{2}kT$.**

Atomic Physics

Sol: 1.5 KT = 13.6 eV

$$T = \frac{13.6 \times 1.6 \times 10^{-19}}{1.5 \times 1.38 \times 10^{-23}} K = 1.05 \times 10^5 K$$

Q4 **A monochromatic light source of frequency ν illuminates a metallic surface and emits photoelectrons. The photoelectrons having maximum energy are just able to ionize the hydrogen atom in ground state. When the whole experiment is repeated with an incident radiation of frequency $\left(\frac{5}{6}\right)\nu$ the photoelectrons so emitted are able to excite the H-atom beam which then emits a radiation of wavelength of 1215 Å.**

Find the frequency $\left(\frac{5}{6}\right)\nu$.

Sol: $K_{1(max)}$ = maximum K.E. of ejected electron in first case

= 13.6 eV

$\therefore \quad E_\lambda = \phi + K_{1(max)}$

$\Rightarrow \quad h\upsilon = \phi + 13.6$ eV ....(i)

$$K_{2(max)} = E_\lambda - \phi = h.\frac{5}{6}.\upsilon - \phi$$

$$= \frac{5}{6}h\upsilon - (h\upsilon - 13.6\,eV) \quad \text{......... (ii)}$$

The energy of radiation of λ = 1215 Å is

$$= \frac{12400}{1215} eV = 10.2 \text{ eV}$$

This is the energy of the e^-

$$10.2 \text{ eV} = -\frac{h\upsilon}{6} + 13.6eV$$

$$\Rightarrow \quad \upsilon = \frac{(13.6 - 10.2) \times 6eV}{h} = 5 \times 10^{15} \text{ Hz.}$$

Q5 **Find the smallest wavelength in emission spectra of (a) hydrogen, (b) He^+**

Sol: The smallest wavelength corresponds to transition from n = 1 to n = ∞.

(a) $\Delta E = 13.6$ eV

$$\Rightarrow \lambda = \frac{1.24 \times 10^4}{13.6} \text{ Å} = 911 \text{ Å} = 91 \text{ nm.}$$

(b) $\Delta E = 13.6 \times 2^2$ ev

$$\lambda = \frac{124 \times 10^4}{13.6 \times 4} \text{ Å} = 228 \text{ Å} \cong 23 \text{ nm.}$$

Q6 **Calculate the angular frequency of revolution of an electron occupying the second Bohr orbit of He^+ ion.**

Sol: $v = 2.19 \times 10^6 \times \frac{z}{n}$ m/sec

$r = 0.529 \times 10^{-10} \times \frac{n^2}{z}$ m On putting values

$$\frac{v}{r} = \frac{2.19 \times 10^6}{0.529 \times 10^{-10}} \times \frac{Z^2}{n^3} = 2.07 \times 10^{16} \text{ s}^{-1}$$

Q7 **Find out the quantum number n corresponding to the excited state of He^+ ion, if on transition to ground state that ion emits 2 photons in succession with wave lengths 108.5nm and 30.4 nm.**

Sol: Total energy of radiation

$$E = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$

$$13.6 \times 4 \times \left(1 - \frac{1}{n^2}\right) = \left(\frac{1240}{108.5} + \frac{1240}{30.4}\right) eV$$

$$1 - \frac{1}{n^2} = \frac{1}{13.6 \times 4} \times 52.22$$

$\Rightarrow$ $n = 5.$

Q8 **Consider a gas of hydrogen like ions in an excited state 'A'. It emits photons of wavelength equal to wavelength of the 1st line of the Lyman series together with photons of 5 other wavelengths. Identify the gas and find out the principal quantum number of the state A.**

Sol: Since there are six transition

$${}^nC_2 = 6 \Rightarrow n = 4$$

since one of the photons matches with first line of Lyman series

$$13.6\left(1 - \frac{1}{4}\right) = 13.6\ z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{Z^2} - \frac{1}{(2z)^2} = \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Q9 **A stationary hydrogen atom emits a photon corresponding to first line of the Lyman series. What velocity does the atom acquire?**

Sol: Energy of the first time in Lyman series

$$E = 13.6 \times \left(1 - \frac{1}{4}\right) eV = 10.2 \text{ eV}$$

velocity of atoms $= \frac{p}{m} = \frac{E}{c} \cdot \frac{1}{m} = \frac{10.2 \times 1.6 \times 10^{-19}}{3 \times 10^{8} \times 1.67 \times 10^{-24}}$ m/s = 3.25 m/s

Q10 **From the condition of the above problem, find out how much (in %) the energy of the emitted photon differs from energy of the corresponding transition in a H-atom.**

Sol: From momentum conservation

$$mv = \frac{\text{Å}}{C}$$

From energy consideration

$$E_H = \frac{1}{2} mv^2$$

$$= \frac{1}{2} \times \frac{(1.67 \times 10^{-27}) \times (3.25)^2}{1.6 \times 10^{-19}} eV = 5.5 \times 10^{-8} \text{ eV}$$

% change in energy $= \frac{E_H}{E} \times 100$

$$= \frac{5.5 \times 10^{-8}}{10.2} \times 100 = 0.55 \times 10^{-6} \%$$

Q11 **Consider a gas consisting Li^{+2} (which is hydrogen like ion).**
(a) Find out the wavelength of radiation required to excite the electron in Li^{++} from n = 1 and n = 3. (Ionisation energy of the hydrogen atom = 13.6 eV).
(b) Find out spectral lines are observed in the emission spectrum of the above excited system.

Sol: (a)

$$\Delta E = 13.6 \times 3^2 \times \left(\frac{1}{1} - \frac{1}{3^2}\right) \text{eV}$$

$$= 13.6 \times 8 \text{ eV}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{12400}{13.6 \times 8} \text{ Å} = 113.7 \text{ Å}$$

(b) no. of spectral lines

$= {}^3C_2 = 3.$

Q12 **A free atom of iron emits a photon of energy 6.4 keV. Then find the recoil kinetic energy of the atom. (Take mass of iron atom = 9.3×10^{-26} kg).**

Sol: Momentum of recoiling atom

= momentum of the photon

$$= \frac{E}{c}$$

K.E. of recoiling atom $= \frac{P^2}{2m} = \frac{E^2}{2mc^2}$

$$= \left(\frac{6.4 \times 10^3 \times 1.6}{3 \times 10^8}\right)^2 \times \frac{1}{2 \times (9.3 \times 10^{-26})} \text{J}$$

$= 0.63 \times 10^{-22}$ J $= 3.9 \times 10^{-4}$ eV.

Q13 **At what minimum kinetic energy must a hydrogen atom move for it's inelastic head on collision with another stationary hydrogen atom so that one of them emits a photon? Both atoms are assumed to be in the ground state prior to the collision.**

Sol: The loss in kinetic energy should be 10.2 eV so that the hydrogen atom may be excited to first excited state. For minimum kinetic energy case, the two atoms will move together.

$$mv_0 = (m + m)\,v \qquad \Rightarrow v = v_0/2$$

$$\text{loss in kinetic energy} = \frac{1}{2}\,mv_0^2 - \frac{1}{2}(2m)\left(\frac{v_0}{2}\right)^2$$

$$10.2\text{eV} = \frac{1}{4}.mv_0^2 = \frac{1}{2}.T$$

$$\therefore \quad T = 2 \times 10.2\text{ eV} = 20.4\text{ eV}$$

Q14 **On increasing the operating voltage in a x-ray tube by 1.5 times, the shortest wavelength decreases by 26 pm. Find the original value of operating voltage.**

Sol:

$$KE = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{KE}$$

$$\lambda_1 - \lambda_2 = 1.24 \times 10^4 \left(\frac{1}{V} - \frac{1}{1\cdot 5V}\right) \text{Å}$$

$$26 \times 10^{-2} = 1.24 \times 10^4 \times \frac{1}{3V}$$

$$\therefore\ V = \frac{1.24 \times 10^4}{3 \times 26 \times 10^{-2}} = 15.9 \times 10^3 \text{ Volt}$$

Q15 **Characteristic X-rays of frequency 4.2×10^{18} Hz are emitted from a metal due to transition from L- to K-shell. Find out the atomic number of the metal using Moseley's law. ($R = 1.1 \times 10^7\ m^{-1}$).**

Sol:

$$\Delta E = h\nu = Rhc\,(Z - b)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For K-series b = 1

∴ $\nu = Rc\,(z - 1)^2$

Substituting the values

$$4.2 \times 10^{18} = (1.1 \times 10^7)(3 \times 10^8)(z - 1)^2 \left(\frac{1}{1} - \frac{1}{4}\right)$$

∴ $(z - 1)^2 = 1697$

or $z - 1 \approx 41$ or $z = 42$

Mind Map

ATOMS

Rutherford's Model

Postulates - Atoms have a central, massive, positively charged core around which electrons revlove.

Size of nucleus = 1 fermi = 10^{-15}m

Drawbacks - Dosen't explain the stability of atom.

Dosen't explain the atomic spectra.

$r = \frac{e^2}{4\pi\varepsilon_0 mv^2}$ $\quad K = \frac{e^2}{8\pi\varepsilon_0 r}$ $\quad U = \frac{-e^2}{4\pi\varepsilon_0 r}$

Bhor's Model

Postulates -

Electrons revolve around the nucleus in stationary orbits.

Angular momentum :- $mv_n r_n = \frac{nh}{2\pi}$

An electron can make transitions to a lower energy state.

Energy of the photon released - $hf = E_i - E_f$

For H-like atoms - $v_n = \left[\left(2.18 \times 10^6 \times \frac{Z}{n}\right)\right]$ m/s $\quad r_n = \left[\left(0.53 \times \frac{n^2}{Z}\right)\right]$ A $\quad K.E. = \frac{KZ^2}{2r}$

$P.E. = \frac{-KZ^2}{r}$ $\quad E = -13.6\frac{Z^2}{n^2}$ eV

Atomic Spectra $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

For Lyman series (U.V. region) - $n_1 = 1$ and $n_2 = 2, 3, 4, ...$

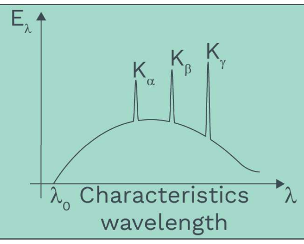
For Balmer series (Visible region) - $n_1 = 2$ and $n_2 = 3, 4, 5, ...$

For Paschan series (Infrared region) - $n_1 = 3$ and $n_2 = 4, 5, 6, ...$

X-Rays

$\lambda_{min} = \frac{hc}{eV}$ $\quad \sqrt{f} = \sqrt{\frac{3Rc}{4}}(Z - 1)$

$\lambda_{min} = \frac{12400}{V}$ A $\quad \sqrt{f} = a(Z - 1)$



ATOMIC STRUCTURE

Rutherford's Gold Foil Experiment

1. Most of the α-particles (nearly 99.9%) went straight without suffering any deflection.
2. A few of them got deflected.

Value of charge on electron = 1.602×10^{-19} C

The distance of closest approach for α-particle is $r = \frac{2KZe^2}{K.E._\alpha}$

Energy During Transitions

$\Delta E = E_{final\ state} - E_{initial\ state}$

$\Delta E = 13.6 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] Z^2$ (in eV)

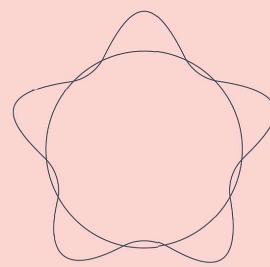
$E = -13.6 \times \frac{Z^2}{n^2}$ eV

-0.85 ev — n = 4,3rd exited state

-1.51 ev — n = 3,2nd exited state

-3.4 ev — n = 2,1st exited state

-13.6 ev — n = 1, ground state



For nth shell
$2\pi r = n\lambda$

Bohr's Postulate

Electrons can revolve only in those orbits whose angular momentum (mrv) is integral multiple of $\frac{h}{2\pi}$ $\Rightarrow$ $mvr = \frac{nh}{2\pi}$

$$\frac{k(Ze)e}{r^2} = \frac{mv^2}{r}$$

Radius of Bohr orbit :

$r_n = \frac{n^2h^2}{4\pi^2 mKZe^2}$, $r = 0.529 \frac{n^2}{Z}$ A

Bohr Radius 'a_0' = 0.529 A

Energy of Electrons : $E = -KE = \frac{PE}{2}$

$E_n = Rhc \frac{Z^2}{n^2}$

R is 'Rydberg Constant' = 1.1×10^7 m^{-1}

$E = -21.8 \times 10^{-19} \times \frac{Z^2}{n^2}$ J

Velocity of Electrons : $v_n = \frac{2\pi KZe^2}{nh}$

$v_n = 2.2 \times 10 \frac{Z}{n}$ m/s