Alternating Current

DISCLAIMER

Alternating Current

Introduction

When a resistor is connect across the terminals of a battery, a current is established in the circuit. The current has a unique direction, it goes from the positive terminal to the negative terminal via the external resistor. The magnitude of the current also remains almost constant. This is called direct current (dc).

If the current direction in a resistor or in any other element changes alternately, the current is called an alternating current (ac). In this chapter, we shall study the alternating current that varies sinusoidally with time.

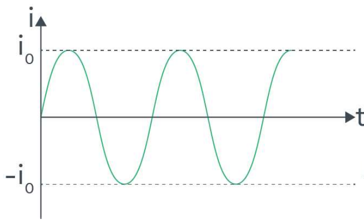
KEY POINTS

- Alternating current
- Alternating voltage
- Direct current
- Ohm's law
- Joule's heating law
- Sinusoidal AC

Alternating Current (A.C.) OR Alternating Voltage (A.V)

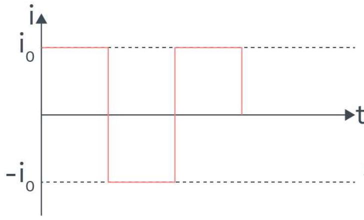
- Electric current or voltage, which keeps on changing in magnitude and direction periodically is defined as alternating current or voltage.
- It obeys Ohm's law and Joule's heating law. It is produced using the principle of electromagnetic induction.
- The alternating voltage or current in general use is sinusoidal in nature. It is produced by rotating a coil in a uniform magnetic field with uniform angular velocity.
- Example of AC:

(1)



Sinusoidal AC

(2)



Square AC

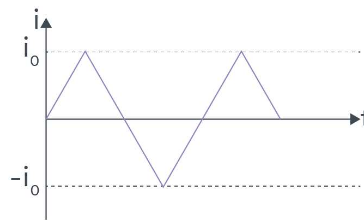
Definitions

Electric current or voltage, which keeps on changing in magnitude and direction periodically is defined as alternating current or voltage.

Concept Reminder

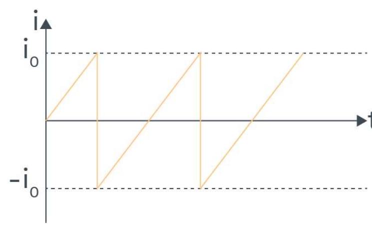
Current whose direction does not change with time through load are known as direct Current.

(3)



Triangular AC

(4)



Saw-tooth AC

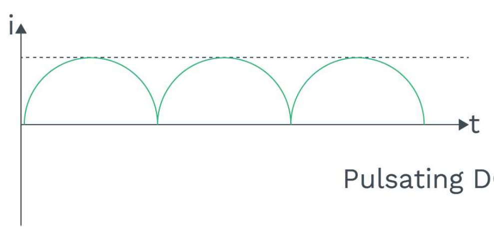
> **Concept Reminder**
>
> Examples of AC
> (i) Sinusoidal AC
> (ii) Square AC
> (iii) Triangular AC
> (iv) Saw-tooth AC

- Example of non-AC

(1)

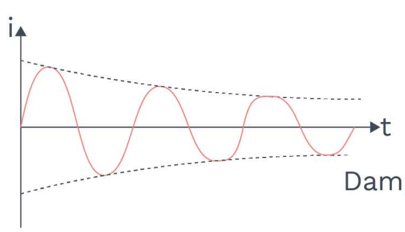
DC

(2)



Pulsating DC

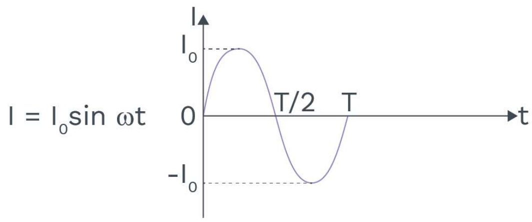
(3)



Damped AC

> **KEY POINTS**
>
> - Pulsating DC
> - Damped AC

Sinusoidal AC



Where:

I = Instantaneous current

I_0= Peak value / Maximum value / Amplitude of current

ω = Angular frequency

$$\omega = 2\pi f = \frac{2\pi}{T}$$

In India: Supply voltage = 220 V, f = 50 Hz

In USA: Supply voltage = 110 V, f = 60 Hz

Advantages Of Alternating Current Over Direct Current

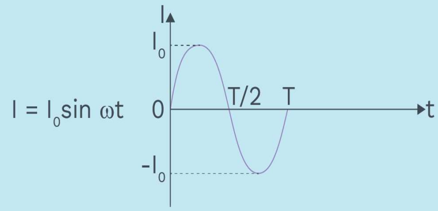
- The cost (amount) of generation of AC is less than that of DC.
- AC can be conveniently converted into dc with the help of rectifiers.
- By supplying AC at high voltages, we can minimize transmission losses or line losses.
- AC is available in a wide range of voltages. These voltages can be easily stepped up or stepped down with the help of transformers.

Disadvantages Of Alternating Current Over Direct Current

- AC is more dangerous than DC.
- AC is transmitted more by the surface of the conductor. This is known as skin effect. Due to this reason that many strands of thin insulated wire, instead of a single thick wire, need to be

Concept Reminder

Sinusoidal AC



Concept Reminder

AC is available in a wide range of voltages. These voltages can be easily stepped up or stepped down with the help of transformers.

Definitions

- The value of current or voltage in an ac circuit at any instant of time is called its instantaneous value.
- Instantaneous current,
  $I = I_0 \sin \omega t$
  (or) $I = I_0 \sin(\omega t + \phi)$
- Instantaneous voltage,
  $E = E_0 \sin \omega t$
  (or) $E = E_0 \sin(\omega t + \phi)$

used.

- For electro refining, electro typing, electroplating, only DC can be used but not AC.

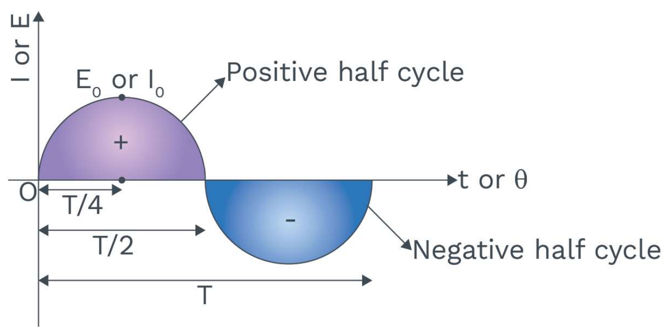
Instantaneous Value Of Current Or Voltage (I or E)

- The value of current or voltage in an ac circuit at any instant of time is called its instantaneous value.
- Instantaneous current,

  $I = I_0 \sin \omega t$ (or) $I = I_0 \sin(\omega t + \phi)$
- Instantaneous voltage,

  $E = E_0 \sin \omega t$ (or) $E = E_0 \sin(\omega t + \phi)$

  Where $(\omega t + \phi)$ is called phase



Ex. Find out instantaneous current value for $I = I_0 \sin \omega t$ at $t = \frac{T}{8}$.

Sol. As $I = I_0 \sin \omega t$

At $t = \frac{T}{8}$

$$I = I_0 \sin\left(\frac{2\pi}{T}\right)\left(\frac{T}{8}\right) = I_0 \sin\frac{\pi}{4}$$

$$I = \frac{I_0}{\sqrt{2}}$$

Ex. Find out instantaneous voltage for V = 200 $\sin(400\pi t)$ at $t = \frac{1}{800}$ sec.

Concept Reminder

AC is transmitted more by the surface of the conductor. This is called skin effect. Due to this reason that several strands of thin insulated wire, instead of a single thick wire is used.

Rack your Brain

A sinusoidal alternating current of peak value I_0 passes through a heater of resistance R. What is the mean power of the heater?

Definitions

The mean value of AC over any half cycle (either positive or negative) is that value of DC which would send same amount of charge through a circuit as is sent by the AC through same circuit at the same time.

Sol. As $V = 200 \sin 400\pi t$

At $t = \frac{1}{800}$ sec

$$V = 200 \sin\left(400\pi \times \frac{1}{800}\right)$$

$\Rightarrow$ $V = 200$ volt

Ex. Find out the time taken by current to reach $\frac{I_0}{\sqrt{2}}$, if the frequency of current is 50Hz.

Sol. At $I = 0$, $\phi = 0°$

At $I = \frac{I_0}{\sqrt{2}}$, $\phi = \frac{\pi}{4}$

As $\Delta t = \frac{T}{2\pi} \times \frac{\pi}{4} = \frac{T}{8}$

Average Value Of Alternating Current

Definition: The mean value of AC over any half cycle (either positive or negative) is that value of DC which would send same amount of charge through a circuit as is sent by the AC through same circuit at the same time.

$$I_{avg} = \bar{I} = \langle I \rangle = \frac{\int_{t_1}^{t_2} I dt}{\int_{t_1}^{t_2} dt} = \frac{\text{Area under } I - t \text{ graph}}{\text{time interval}}$$

Important Formulae:

1. $\langle \sin\theta \rangle = \langle \cos\theta \rangle = 0$ (for full cycle)

2. $\langle \sin\theta \rangle = \langle \cos\theta \rangle = \frac{2}{\pi}$ (for half cycle)

3. $\langle \sin^2\theta \rangle = \langle \cos^2\theta \rangle = \frac{1}{2}$ (for full / half cycle)

Average Value of Sinusoidal AC

Let $I = I_0 \sin\omega t$

Concept Reminder

$$I_{avg} = \bar{I} = \langle I \rangle = \frac{\int_{t_1}^{t_2} I dt}{\int_{t_1}^{t_2} dt}$$

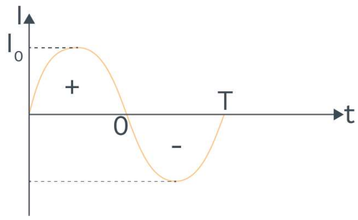
$$= \frac{\text{Area under } I - t \text{ graph}}{\text{time interval}}$$

Concept Reminder

1. $\langle \sin\theta \rangle = \langle \cos\theta \rangle = 0$ (for full cycle)

2. $\langle \sin\theta \rangle = \langle \cos\theta \rangle = \frac{2}{\pi}$ (for half cycle)

3. $\langle \sin^2\theta \rangle = \langle \cos^2\theta \rangle = \frac{1}{2}$ (for full / half cycle)



- **For one complete cycle**

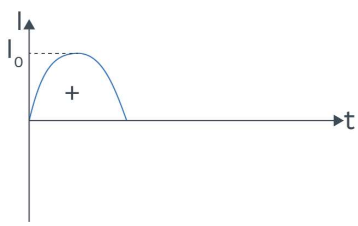
  $< I > = < I_0 \sin\omega t > = I_0 < \sin\omega t >$

  $\Rightarrow \; < I > = 0$

  Note: (a) Therefore batteries cannot be charged by using ac.

  (b) Electrolysis and electroplating cannot be done by using ac.

- **For positive half cycle:**

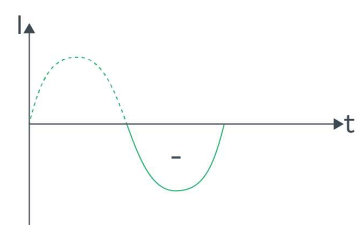


$< I > = < I_0 \sin\omega t > = I_0 < \sin\omega t >$

$$\Rightarrow \boxed{< I > = \frac{2I_0}{\pi}}$$

- **For negative half cycle:**

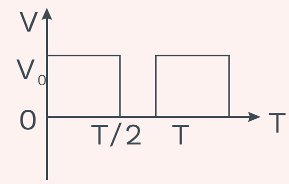
  $< I > = -\frac{2I_0}{\pi}$



- **For Quarter cycle:**

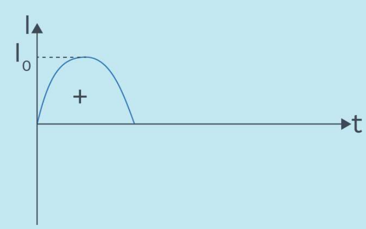
Rack your Brain

Find the average value of potential difference V for given figure.

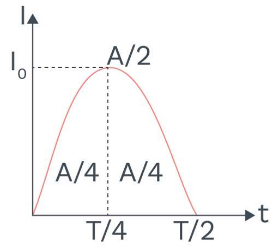


Concept Reminder

For positive half cycle



$$\Rightarrow \boxed{< I > = \frac{2I_0}{\pi}}$$

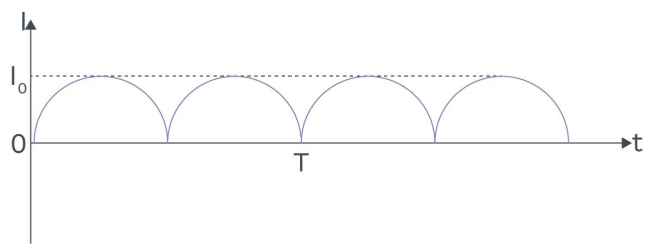


$$< I >= \frac{A_{1/4}}{t_{1/4}} = \frac{\left(\frac{A_{1/2}}{2}\right)}{\left(\frac{t_{1/2}}{2}\right)} = \frac{A_{1/2}}{t_{1/2}}$$

Hence

$$< I >= \frac{2I_0}{\pi}$$

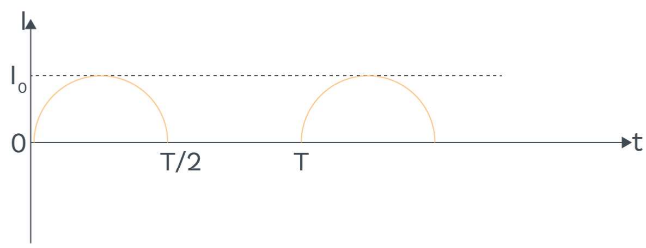
- **Average value for full wave rectifier**



$$< I >= \frac{A}{T} = \frac{2A_{1/2}}{2t_{1/2}} = \frac{A_{1/2}}{t_{1/2}}$$

Hence $< I >_{\text{full cycle}} = < I >_{\text{Half cycle}} = \frac{2I_0}{\pi}$

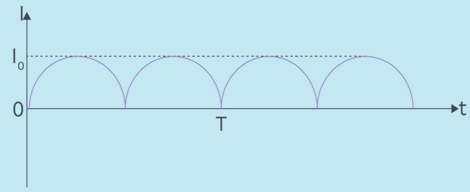
- **Average value for half wave rectifier:**



$$< I >= \frac{A}{t} = \frac{A_{1/2} + 0}{2t_{1/2}} = \frac{1}{2}\frac{A_{1/2}}{t_{1/2}} = \frac{1}{2} < I >_{1/2}$$

Concept Reminder

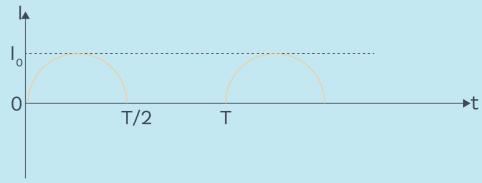
Average value for full wave rectifier



$$< I >_{\text{full cycle}} = < I >_{\text{Half cycle}} = \frac{2I_0}{\pi}$$

Concept Reminder

Average value for half wave rectifier



$$< I >= \frac{1}{2}\left(\frac{2I_0}{\pi}\right) = \frac{I_0}{\pi}$$

Hence $\langle I \rangle = \frac{1}{2}\left(\frac{2I_0}{\pi}\right) = \frac{I_0}{\pi}$

Root Mean Square Value Of Current

It is value of DC current which would produce equal heat in given resistance in given time as is done by the alternating current when passed through the same resistance for the same time.

$$\boxed{I_{rms} = \sqrt{\frac{\int_{t_1}^{t_2} I^2 dt}{\int_{t_1}^{t_2} dt}}} \quad \text{or} \quad \boxed{I_{rms}^2 = \langle I^2 \rangle}$$

The significance of rms voltage and rms current may be shown by considering a resistance R carrying a current $i = i_0 \sin(\omega t + \phi)$

The voltage across the resistor will be

$$V_R = iR = (i_0 R)\sin(\omega t + \phi)$$

The thermal energy developed in the resistance during the time interval t to t + dt is

$$i^2 R dt = i_0^2 R \sin^2(\omega t + \phi) dt$$

The thermal energy produced in one time period is

$$U = \int_0^T i^2 R dt = R\int_0^T i_0^2 \sin^2(\omega t + \phi) dt$$

$$= RT\left[\frac{1}{T}\int_0^T i_0^2 \sin^2(\omega t + \phi) dt\right] = i_{rms}^2 RT$$

- It means the rms value of ac current is that value of steady current, which would generate the equal amount of heat in a given resistance in a given time.
- So, in ac circuits, ac voltage and current are measured in terms of their rms values. Like when we say that the household supply is 220 V ac it means the rms value is 220 V and peak value is $220\sqrt{2} = 311$ V.

Definitions

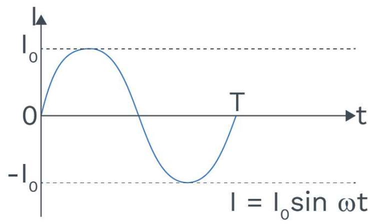
It is value of DC which would produce same heat in given resistance in given time as is done by the alternating current when passed through the same resistance for the same time.

Concept Reminder

$$\boxed{I_{rms} = \sqrt{\frac{\int_{t_1}^{t_2} I^2 dt}{\int_{t_1}^{t_2} dt}}}$$

or $\boxed{I_{rms}^2 = \langle I^2 \rangle}$

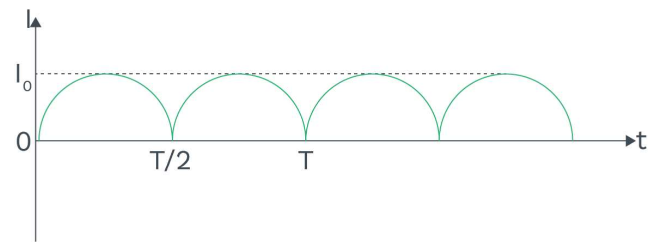
1. RMS value of sinusoidal ac



$$< I^2 >= I_0^2 \sin^2 \omega t >= I_0^2 < \sin^2 \omega t >= I_0^2 \times \frac{1}{2}$$

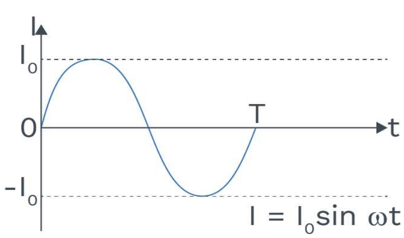
Hence $\boxed{I_{rms} = \frac{I_0}{\sqrt{2}}}$ or $\boxed{I_{rms} = 0.707 I_0}$

2. RMS value for full wave rectifier:



$$I_{rms}^2 = \frac{\left\{(I_{rms})^2_{0-\frac{T}{2}}\right\}\frac{T}{2} + \left\{(I_{rms})^2_{\frac{T}{2}-T}\right\}\frac{T}{2}}{\frac{T}{2}+\frac{T}{2}}$$

$$= \frac{\left(\frac{I_0}{\sqrt{2}}\right)^2 \frac{T}{2} + \left(\frac{I_0}{\sqrt{2}}\right)^2 \frac{T}{2}}{T} = \frac{\frac{I_0^2}{2} \times \frac{T}{2} \times 2}{T} = \frac{I_0^2}{2}$$



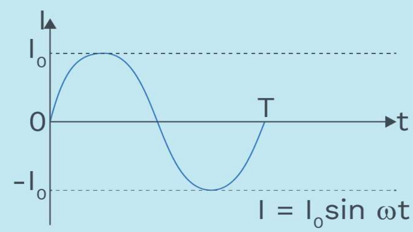
Hence $\boxed{I_{rms} = \frac{I_0}{\sqrt{2}}}$

Rack your Brain

In an electric iron heat produced is same, whether it is connected across an A.C. source or across 50 V constant voltage. Find R.M.S value of the A.C. voltage applied

Concept Reminder

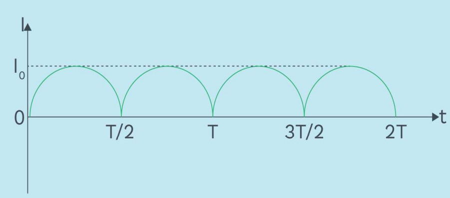
RMS value of sinusoidal ac



$\boxed{I_{rms} = \frac{I_0}{\sqrt{2}}}$ or $\boxed{I_{rms} = 0.707 I_0}$

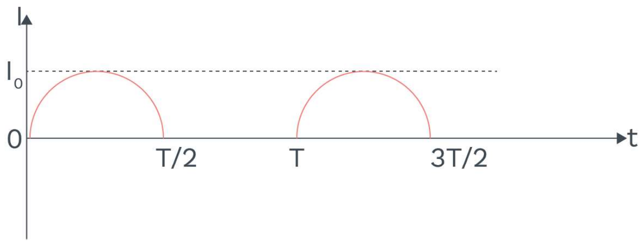
Concept Reminder

RMS value for full wave rectifier



$\boxed{I_{rms} = \frac{I_0}{\sqrt{2}}}$

3. RMS value for half wave rectifier



$$I_{rms}^2 = \frac{\left\{\left(I_{rms}\right)^2_{0-\frac{T}{2}}\right\}\frac{T}{2} + \left\{\left(I_{rms}\right)^2_{\frac{T}{2}-T}\right\}\frac{T}{2}}{\frac{T}{2}+\frac{T}{2}}$$

$$= \frac{\left(\frac{I_0}{\sqrt{2}}\right)^2\frac{T}{2} + (0)^2\frac{T}{2}}{T} = \frac{I_0^2}{4}$$

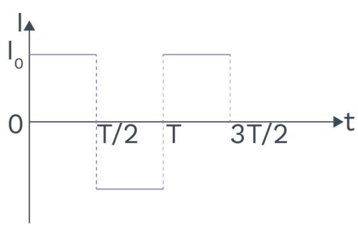
Hence $\boxed{I_{rms} = \frac{I_0}{2}}$

Concept Reminder

RMS value for half wave rectifier

$\boxed{I_{rms} = \frac{I_0}{2}}$

4. RMS value for a square wave AC

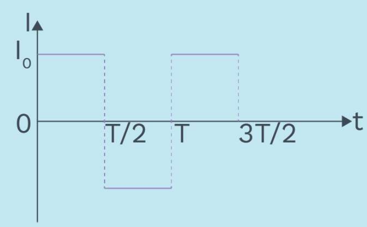


$$I_{rms} = \frac{\left\{\left(I_{rms}\right)^2_{0-\frac{T}{2}}\right\}\frac{T}{2} + \left\{\left(I_{rms}\right)^2_{\frac{T}{2}-T}\right\}\frac{T}{2}}{\frac{T}{2}+\frac{T}{2}} = \frac{I_0^2\frac{T}{2} + I_0^2\frac{T}{2}}{T} = I_0^2$$

Hence $\boxed{I_{rms} = I_0}$

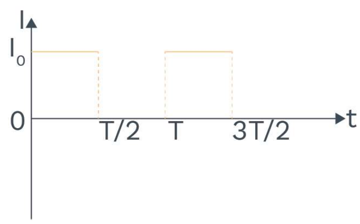
Concept Reminder

RMS value for a square wave AC



$\boxed{I_{rms} = I_0}$

5. RMS value for half square wave

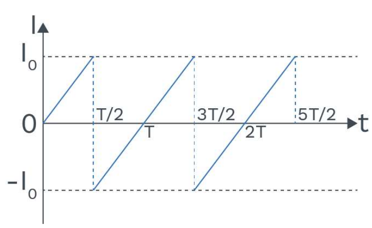


$$I_{rms}^2 = \frac{\left\{\left(I_{rms}\right)^2_{0-\frac{T}{2}}\right\}\frac{T}{2} + \left\{\left(I_{rms}\right)^2_{\frac{T}{2}-T}\right\}\frac{T}{2}}{\frac{T}{2}+\frac{T}{2}}$$

$$= \frac{I_0^2\frac{T}{2} + (0)^2\frac{T}{2}}{T} = \frac{I_0^2}{2}$$

Hence $\boxed{I_{rms} = \frac{I_0}{\sqrt{2}}}$

6. RMS value for sawtooth wave



As

$y = mx$

$$I = \left(\frac{2I_0}{T}\right)t$$

$$I_{rms}^2 = \frac{\int_0^{T/2}\left(\frac{2I_0}{T}t\right)^2 dt}{T/2} = \frac{2}{T}\times\frac{4I_0^2}{T^2}\int_0^{T/2} t^2 dt$$

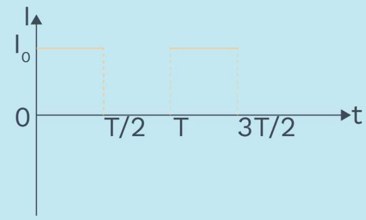
$$= \frac{8I_0^2}{T^3}\left[\frac{1}{3}\left(\frac{T}{2}\right)^3\right] = \frac{8I_0^2}{3T^3}\times\frac{T^3}{8} = \frac{I_0^2}{3}$$

Hence $I_{rms} = \frac{I_0}{\sqrt{3}}$

Ex. If a d.c. current of value a ampere is superimposed on an alternating current

Concept Reminder

RMS Value for half square wave

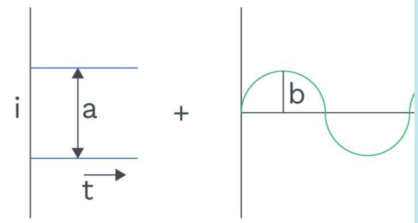


$\boxed{I_{rms} = \frac{I_0}{\sqrt{2}}}$

Rack your Brain

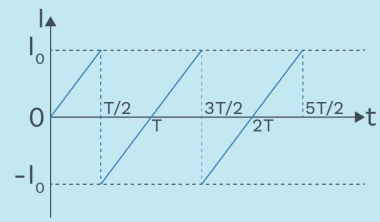
If a direct current of 5 units is superimposed with on alternating current, $I = 3\sin\omega t$ then find effective value of resulting current.

$i = b \sin \omega t$ flowing through a cable, what is the e
in the given circuit?



Concept Reminder

RMS value for saw tooth wave



$$I_{rms} = \frac{I_0}{\sqrt{3}}$$

Sol. As current at any instant time in the circuit will

$i = i_{dc} + i_{ac} = a + b \sin \omega t$

So, $i_{eff} = \left[\dfrac{\int_0^T i^2 dt}{\int_0^T dt}\right]^{1/2} = \left[\dfrac{1}{T}\int_0^T (a + b\sin\omega t)^2 dt\right]^{1/2}$

i.e.,

$$i_{eff} = \left[\frac{1}{T}\int_0^T (a^2 + 2ab\sin\omega t + b^2\sin^2\omega t)dt\right]^{1/2}$$

but as $\frac{1}{T}\int_0^T \sin\omega t dt = 0$ and $\frac{1}{T}\int_0^T \sin^2 \omega t dt = \frac{1}{2}$

So, $i_{eff} = \left[a^2 + \frac{1}{2}b^2\right]^{1/2}$

Ex. If the voltage equation in an ac circuit is represented by,

$V = 220\sqrt{2}\sin(314t - \phi)$ then calculate

(a) peak and rms value of the voltage,

(b) average voltage,

(c) frequency of ac.

Sol. (a) For ac voltage,

$$V = V_0 \sin(\omega t - \phi)$$

The peak value of voltage is

$$V_0 = 220\sqrt{2} = 311 V$$

The rms value of voltage is

$$V_{rms} = \frac{V_0}{\sqrt{2}}; V_{rms} = 220\,V$$

(b) Average voltage in a full cycle is zero.
Average voltage in half cycle is

$$V_{avg} = \frac{2}{\pi}V_0 = \frac{2}{\pi} \times 311 = 198.1\,V$$

(c) As $\omega = 2\pi f, 2\pi f = 314$

$$\text{i.e., } f = \frac{314}{2 \times \pi} = 50\,Hz$$

Ex. The current in a circuit is presented by
$i = i_0\,(t/T)$ for some time. Find out the rms current for the period $t = 0$ to $t = T$.

Sol. The mean square current is

$$(i^2)_{avg} = \frac{1}{T}\int_0^T i_0^2 (t/T)^2 dt = \frac{i_0^2}{T^3}\int_0^T t^2 dt = \frac{i_0^2}{3}$$

Thus, the rms current is

$$i_{rms} = \sqrt{i_{avg}^2} = \frac{i_0}{\sqrt{3}}$$

Ex. Find out average and RMS value of following currents

(i) $I = a\sin\omega t + b\cos\omega t$

(ii) $I = a\sin\omega t\cos\omega t$

Sol. (i) $I = a\sin\omega t + b\cos\omega t$

$$I_{max} = \sqrt{a^2 + b^2}$$

$$< I > = < a\sin\omega t + b\cos\omega t >$$

$$= a < \sin\omega t > + b < \cos\omega t >$$

$$\Rightarrow \quad < I > = 0$$

$$I_{rms}^2 = < I^2 >$$

$$= a^2 < \sin^2\omega t > + b^2 < \cos^2\omega t > + ab < \sin 2\omega t >$$

$$= \frac{a^2}{2} + \frac{b^2}{2} + 0$$

$$\Rightarrow \quad I_{rms} = \sqrt{\frac{a^2}{2} + \frac{b^2}{2}}$$

(ii) $I = a\sin\omega t\cos\omega t$

or $I = \frac{a}{2}\sin 2\omega t \Rightarrow I_{max} = \frac{a}{2}$

$$< I >= \frac{a}{2} < \sin 2\omega t >= 0$$

$$I_{rms}^2 =< I^2 >= \frac{a^2}{4} < \sin^2 2\omega t >$$

$$= \frac{a^2}{4} \times \frac{1}{2} = \frac{a^2}{8}$$

$$I_{rms} = \frac{a}{2\sqrt{2}}$$

Ex. $I = \sqrt{t}$ m

Find rms between 0 to 2 sec.

Sol. $I_{rms} = \sqrt{\frac{\int_0^2 I^2 dt}{\int_0^2 dt}}$

$$= \sqrt{\frac{\int_0^2 t dt}{[t]_0^2}}$$

$$= \sqrt{\frac{\left(\frac{t^2}{2}\right)_0^2}{[2-0]}} = \sqrt{\frac{1}{4}\left(2^2 - 0\right)}$$

$$= \sqrt{4.\frac{1}{4}} = 1$$

KEY POINTS

- Root mean square value
- Average value
- Hot wire instruments
- Phase
- Phase difference

Measurement of A.C.

Alternating current and voltage are measured by AC ammeter and AC voltmeter respectively. Working of these instruments is principle on heating effect of current, hence they are also known as hot wire instruments.

Concept Reminder

Instruments based on magnetic effects of current (moving coil galvanometer) cannot measure alternating current. They measure mean value of current which is zero for ac.

Phase and phase difference

(a) Phase

$I = I_0 \sin(\omega t \pm \phi)$

Initial phase = ϕ (it not depend on time)

Instantaneous phase = $\omega t \pm \phi$ (it changes

with time)
Phase decides both value and sign.
Unit: radian

(b) Phase difference

Voltage

Current

- Phase difference of I w.r.t. V
- Phase difference of V w.r.t. I

Concept Reminder

The alternating currents do not have a fixed value of current and they don't have even a fixed direction. Normally EMF is chosen to be reference = . If a current lead by then or if current lags by then

Lagging and leading Concept

(a) V leads I or I lags It means, V reach maximum before I

Let if then

and if then

In both cases V leads I.

(b) V lags I or I leads It means, V reach maximum after I

Let if then

and if then

In both cases V lags I.

Phasor and Phasor diagram:

Phasors are sinusoidal vectors that rotate about origin with an angular speed equal to angular frequency of related physical quantities.

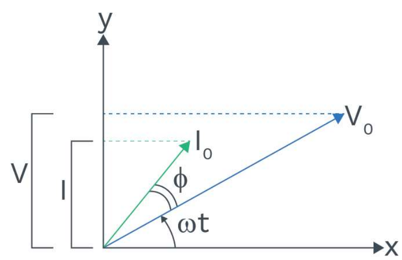
Properties:

1. They rotate anticlockwise.

2. The length of phasors represents the maximum value (amplitude) of quantity.

3. Vertical component represents its instantaneous value.

Let $V = V_0 \sin\omega t$ and $I = I_0 \sin(\omega t + \phi)$



Definitions

Phasors are sinusoidal vectors that rotate about origin with an angular speed equal to angular frequency of related physical quantities.

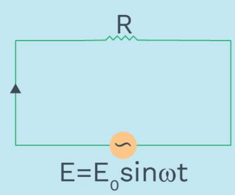
Different Type of ac-circuit:

In order to study the behaviour of A.C. circuits we classify them into two categories:

(a) Simple circuits containing only one basic element i.e., resistor (R) or inductor (L) or capacitor (C).

(b) Complicated circuit containing any two of the three circuit elements R, L and C or all the three elements.

Concept Reminder



$$I = \frac{E}{R} = \frac{E_0}{R}(\sin\omega t) = I_0 \sin\omega t$$

A.C. Through A Resistor

- A pure resistor of resistance R is connected across an alternating source of emf

Rack your Brain

A capacitor is connected to a 200 V, 50 Hz ac supply. The r.m.s value of current in the circuit is nearly.

(1) 1.7 A (2) 2.05 A

(3) 2.5 A (4) 25.1 A

- The instantaneous value of alternating emf is
- The instantaneous value of alternating current is
- Peak value of current,
- **Phasor diagrams:**

Concept Reminder

Phasor diagram of purely resistive AC circuit.

- Emf and current will be in phase .
- Emf and current have same frequency.

- Peak emf is more than peak current.
- The value of impedance (Z) is equal to R and reactance (X) is zero.
- Apart from instantaneous value, current in the circuit is independent of frequency and decreases with increase in R (similar to dc circuits).

In the a.c. circuit having R only, as current and voltage are in the same phase, hence in fig. both phasors E_0 and I_0 are in the same direction, making an angle ωt with OX. Their projections on Y-axis represent theinstantaneous values of alternating current and voltage.

i.e. $I = I_0 \sin\omega t$ and $E = E_0 \sin\omega t$

Since $I_0 = \frac{E_0}{R}$, hence $\frac{I_0}{\sqrt{2}} = \frac{E_0}{R\sqrt{2}} \Rightarrow I_{rms} = \frac{E_{rms}}{R}$

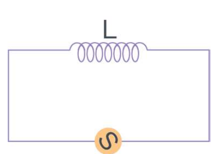
Power

- Power factor $\cos\phi = \cos 0° = 1$
- Instantaneous power $P_i = E_0 I_0 \sin^2\omega t$
- Average power over time 'T' sec

$$= P_{avg} = E_{rms}I_{rms}\cos\phi = E_{rms}.I_{rms} = \frac{E_{rms}^2}{R}$$

AC Circuit Containing Pure Inductance

A circuit containing a pure inductance L (having zero ohmic resistance) connected with a source of alternating emf. Let the alternating e.m.f. $E = E_0 \sin\omega t$



$E = E_0 \sin\omega t$

When AC flows through the circuit, emf induced across inductance $= -L\frac{dI}{dt}$

Concept Reminder

Average power over time 'T' sec in purely resistive circuit

$= P_{avg} = E_{rms}I_{rms}\cos\phi$

$= E_{rms}.I_{rms} = \frac{E_{rms}^2}{R}$

Rack your Brain

A coil of self-inductance L is connected in series with bulb B and an AC source. Brightness of bulb decreases when

(1) a capacitance of $X_C = X_L$ is included in circuit.

(2) an iron rod is inserted in coil.

(3) frequency of AC source is decreased.

(4) number of turns in coil is reduced.

Note: Negative sign indicates that induced emf acts in opposite direction to that of applied emf.

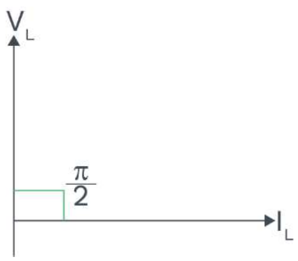
Because there is no other circuit element present in the circuit other than inductance so with the help of Kirchhoff's circuital law

$$E + \left(-L\frac{dI}{dt}\right) = 0 \Rightarrow E = L\frac{dI}{dt}$$

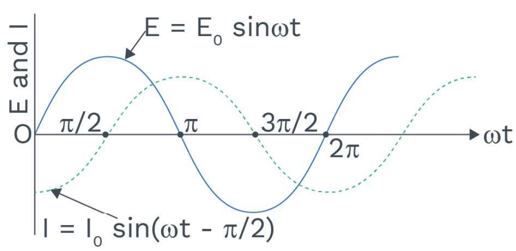
so, we get $I = \frac{E_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$

Maximum current $I_0 = \frac{E_0}{\omega L} \times 1 = \frac{E_0}{\omega L}$,

Hence, $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

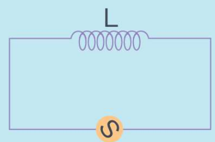


In a pure inductive circuit current always lags behind the emf by $\frac{\pi}{2}$ or alternating emf leads the a. c. by a phase angle of $\frac{\pi}{2}$.



Expression $I_0 = \frac{E_0}{\omega L}$ resembles the expression $I_0 = \frac{E_0}{R}$. This non-resistive resistance to to the

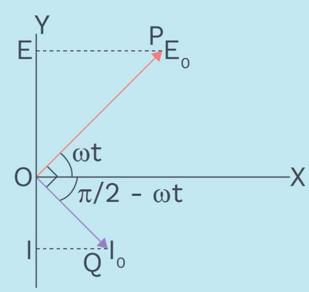
Concept Reminder



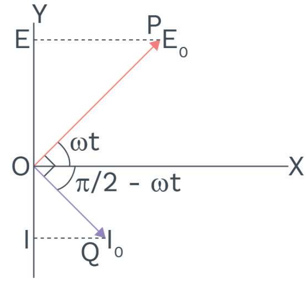
$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

Concept Reminder

In a pure inductive circuit current always lags behind the emf by $\frac{\pi}{2}$ or alternating emf leads the a. c. by a phase angle of $\frac{\pi}{2}$.



the flow of A.C. in a circuit is called the inductive reactance (X_L) of the circuit.



$X_L = \omega L = 2\pi f L$ where f = frequency of A.C.

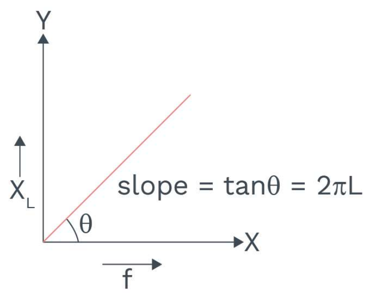
- **Unit of X_L:**

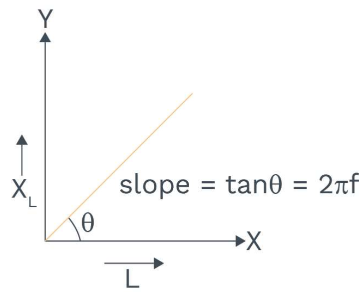
  (ωL) = Unit of L × Unit of $(\omega = 2\pi f)$ = henry × sec^{-1}

  $$= \frac{volt}{ampere / sec} \times sec^{-1} = \frac{volt}{ampere} = ohm$$

- **Inductive reactance $X_L \propto f$**

  Higher the frequency of A.C. higher is the inductive reactance offered by an inductor in an A.C. circuit.





- **For DC circuit, f = 0**

  $\therefore \quad X_L = \omega L = 2\pi f L = 0$

Concept Reminder

$X_L = \omega L = 2\pi fL$

- For constant f

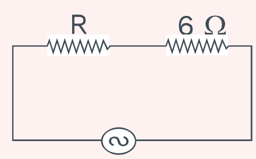
  $X_L \propto L$

- For constant L

  $X_L \propto f$

Rack your Brain

Two resistors are connected across an AC source of 5V. The P.D across 6Ω is 3V. If resistor R is replaced by a pure inductor of such magnitude that the value of current remains same, then what will be voltage across inductor.



Hence, inductor offers no opposition to the flow of DC whereas a resistive path to AC.

- **Power Supplied to Inductor**

The instantaneous power produced by the inductor is $P_L = EI = I_0 \sin\left(\omega t - \frac{\pi}{2}\right) \times E_0 \sin(\omega t)$

$$= -I_0 E_0 \cos(\omega t)\sin(\omega t) = -\frac{I_0 E_0}{2}\sin(2\omega t)$$

So, the average power over a full cycle is

$P_{avg} = E_{rms}.I_{rms}\cos\phi = 0 \quad (\because \Delta\phi = 90°)$

Or $P_{avg} = -\frac{I_0 E_0}{2}\langle \sin(2\omega t)\rangle = 0$

Since the average of $\sin(2\omega t)$ over a full cycle is zero.

Thus, the average power delivered to an inductor over one full cycle is zero.

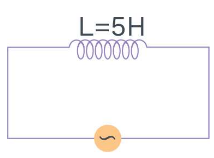
Ex. An inductor of inductance 5 H is connected to an AC source having voltage $V = 10\sin(10t + \frac{\pi}{6})$ Find

(i) Inductive Reactance (X_L)

(ii) Peak & Rms voltage (V_0& V_{rms})

(iii) Peak & Rms current (I_0& I_{rms})

(iv) Instantaneous current (I(t))



Sol. (i) $x_L = \omega L = 10 \times 5 = 50$

(ii) $V_0 = 10$

$$V_{rms} = \frac{10}{\sqrt{2}}$$

KEY POINTS

- Inductance
- Inductive reactance
- Instantaneous voltage
- Instantaneous current
- Capacitance

Concept Reminder

Energy is absorbed by inductor from the source when current through inductor is rising. The energy is returned by inductor to source when current is decreasing. So, average power over complete cycle in purely inductive circuit is zero.

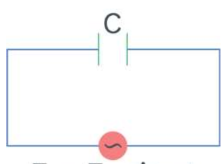
(iii) $I_0 = \frac{V_0}{X_L} = \frac{1}{5}$

$I_{rms} = \frac{1}{5\sqrt{2}}$

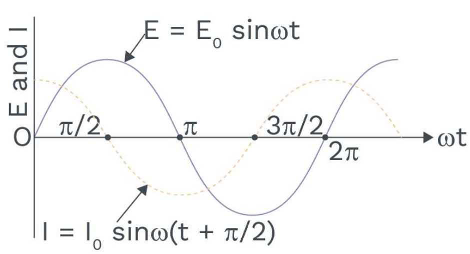
(iv) $I(t) = \frac{1}{5}\sin(10t + \frac{\pi}{6} - \frac{\pi}{2})$

AC Circuit Containing Pure Capacitance

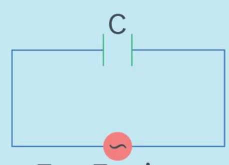
A circuit containing an ideal capacitor of capacitance C connected with a source of alternating emf as shown in fig. The alternating e.m.f. in the circuit $E = E_0 \sin\omega t$. When alternating e.m.f. is applied across the capacitor, a similarly varying alternating current flows in the circuit.



The two plates of the capacitor become alternately positively and negatively charged and the magnitude of the charge on the plates of the capacitor varies sinusoidally with time. Also, the electric field between the plates of the capacitor varies sinusoidally with time.



Concept Reminder

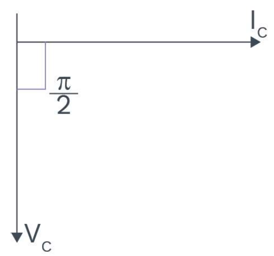


$$\Rightarrow I = \frac{E_0}{(1/\omega C)} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

Rack your Brain

In an AC circuit an alternating voltage $200\sqrt{2}\sin 100t$ volts are connected to a capacitor of capacity $1\mu F$. Find the r.m.s value of current.



Let at any instant t charge on the capacitor = q
Instantaneous potential difference across the capacitor E = q/C

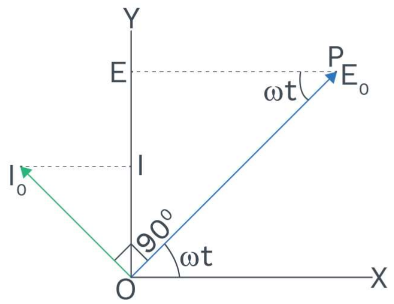
$\Rightarrow \; q = CE \Rightarrow q = CE_0 \sin\omega t$

The instantaneous value of current

$$I = \frac{dq}{dt} = \frac{d}{dt}\left(CE_0 \sin\omega t\right) = CE_0\omega\cos\omega t$$

$$\Rightarrow \; I = \frac{E_0}{\left(1/\omega C\right)}\sin\left(\omega t + \frac{\pi}{2}\right) = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

where $I_0 = \omega CE_0$

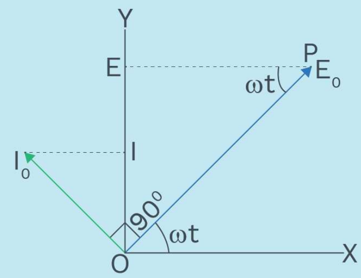


In a pure capacitive circuit, the current always leads the e.m.f. by a phase angle of $\frac{\pi}{2}$. The alternating emf lags behind the alternating current by a phase angle of $\frac{\pi}{2}$.

- **Important Points**
  E/I is the resistance R, when both E and I are in phase, in present case they differ in phase by $\frac{\pi}{2}$,

Concept Reminder

In a pure capacitive circuit, the current always leads the e.m.f. by a phase angle of $\frac{\pi}{2}$.



Concept Reminder

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

- If C = constant

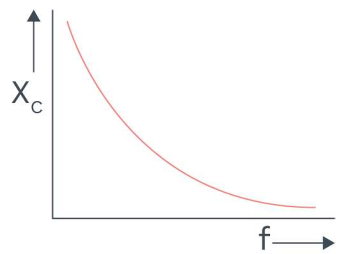
  $X_C \propto \frac{1}{f}$

- If f = constant

  $X_C \propto \frac{1}{C}$

hence $\frac{1}{\omega C}$ is not the resistance of the capacitor, the capacitor offer opposition to the flow of A.C. This non-resistive opposition to the flow of A.C. in a pure capacitive circuit is known as capacitive reactance X_C.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$



Capacitive reactance 'X_C' is inversely proportional to frequency of A.C.
X_C decreases as the frequency increases.

- **Unit of X_C:** ohm
- **For dc circuit f = 0**

$\therefore \quad X_C = \frac{1}{2\pi fC} = \infty$ but has a very small value for ac.

This shows that capacitor blocks the flow of dc but provides an easy path for ac.

- **Power Supplied to Capacitor:**

The instantaneous power produced by the capacitor is

$$P_c = IE = I_0 \cos(\omega t) E_0 \sin(\omega t)$$

$$= I_0 E_0 \cos(\omega t)\sin(\omega t) = \frac{I_0 E_0}{2}\sin(2\omega t)$$

Hence, the average power over a full cycle is zero

since $< \sin(2\omega t) >= 0$ over a complete cycle.

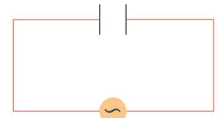
$$P_{avg} = E_{rms}.I_{rms} \cos\phi = E_{rms}.I_{rms} \cos 90^\circ = 0$$

$\therefore$ no power is consumed in a purely capacitive circuit.

Ex. A capacitor of capacitive reactance 5Ω is connected with A.C. source having voltage $V = 3\sin(\omega t + \pi/6)$. Find

(I) Rms and Peak voltage rms

(II) Peak current and instantaneous current



Sol. On comparing with

$V = V_0 \sin(\omega t + \phi) \Rightarrow V_0 = 3$

$$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{3}{\sqrt{2}}$$

$$\Rightarrow \quad I_{rms} = \frac{V_{rms}}{X_c} = \frac{3}{5\sqrt{2}}$$

$$I_{rms} = \frac{V_0}{X_c} = \frac{3}{5}$$

$$\Rightarrow \quad I(t) = I_0 \sin(\omega t + \frac{\pi}{6} + \frac{\pi}{2})$$

Ex. An alternating voltage $E = 200\sqrt{2}\sin(100t)$ volt is connected to a $1\mu F$ capacitor through an ac ammeter. What will be the reading of the ammeter?

Sol. Comparing $E = 200\sqrt{2}\sin(100t)$ with $E = E_0 \sin\omega t$; $E_0 = 200\sqrt{2}V$ and $\omega = 100\,(rad/s)$

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 10^{-6}} = 10^4\,\Omega$$

$$I_{rms} = \frac{E_{rms}}{Z} = \frac{E_0}{\sqrt{2}X_C} = \frac{200\sqrt{2}}{\sqrt{2} \times 10^4} = 20\,mA$$

Ex. Find the maximum value of current when a coil of inductance 2H is connected to 150V, 50 cycles / sec supply.

Sol. Here L = 2H, E_{rms} = 150 V, f = 50 Hz

$$X_L = L\omega = L \times 2\pi f = 2 \times 2 \times 3.14 \times 50 = 628\,\Omega$$

RMS value of current through the inductor,

$$I_{rms} = \frac{E_{rms}}{X_L} = \frac{150}{628} = 0.24\,A$$

Maximum value (or peak value) of current is given by $I_{rms} = \frac{I_0}{\sqrt{2}}$

or $I_0 = \sqrt{2}\, I_{rms} = 1.414 \times 0.24 = 0.339\,A$

Ex. An inductor of 1 Henry is connected across a 220 V, 50 Hz supply. The peak value of the current is approximately.

Sol. Peak value of current

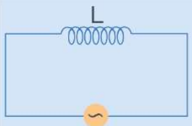
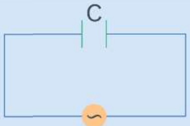
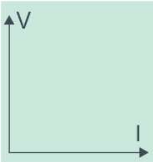
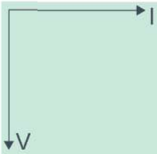
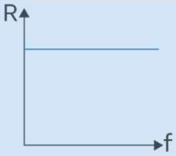
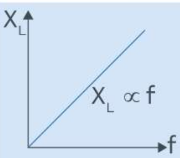
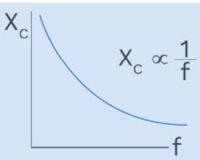
$$i_0 = \frac{E_0}{X_L} = \frac{\sqrt{2}E_{rms}}{\omega L} = \frac{\sqrt{2}E_{rms}}{2\pi fL} = \frac{\sqrt{2}(220)}{2\pi \times 50 \times 1} = 0.99\text{ A}$$

Ex. A capacitor of 2 F is connected in a radio circuit. The source frequency is 1000 Hz. If the current through the capacitor branch is 2 mA then the voltage across the capacitor is

Sol. $V_C = IX_C = I \times \frac{I}{\omega C} = \frac{I}{2\pi fC}$

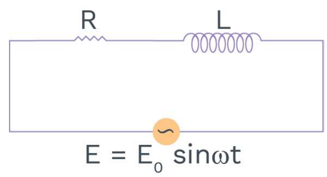
$$= \frac{2 \times 10^{-3}}{2\pi \times 10^3 \times 2 \times 10^{-6}} = 0.16\text{ V}$$

TE
Circuit
Supply Volta
Current
Peak Curren
Impedance (
$Z = \frac{V_0}{I_0} = \frac{V_{rms}}{I_{rms}}$
Phase differ
Phasor diagr
Variation of
G, S_L, S_C (mho, sieme
Behaviour of D.C. and A.C
Ohm's law

INDIVIDUAL COMPONENTS (R OR L OR C)			
RM	R	L	C
			
ge	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$
	$I = I_0 \sin \omega t$	$I = I_0 \sin (\omega t - \frac{\pi}{2})$	$I = I_0 \sin (\omega t + \frac{\pi}{2})$
t	$I_0 = \frac{V_0}{R}$	$I_0 = \frac{V_0}{\omega L}$	$I_0 = \frac{V_0}{1/\omega C} = V_0 \omega C$
(Ω)	$\frac{V_0}{I_0} = R$	$\frac{V_0}{I_0} = \omega L = X_L$	$\frac{V_0}{I_0} = \frac{1}{\omega C} = X_C$
-	R = Resistance	X_L = Inductive reactance	X_C = Capacitive reactance
ence	Zero (in same phase)	$+\frac{\pi}{2}$ (V leads I)	$-\frac{\pi}{2}$ (V lags I)
am			
Z with f			
ns)	G=1/R = conductance	$S_L = 1/X_L$ Inductive susceptance	$S_C = 1/X_C$ Capacitive susceptance
device in	Same in A.C and D.C.	L passes DC easily (because $X_L = 0$) while gives a high impedance for the A.C. of high frequency ($X_L \propto f$)	C - blocks DC (because $X_C = \infty$) while provides an easy path for the A.C. of high frequency $\left[X_C \propto \frac{1}{f}\right]$
	$V_R = IR$	$V_L = IX_L$	$V_C = IX_C$

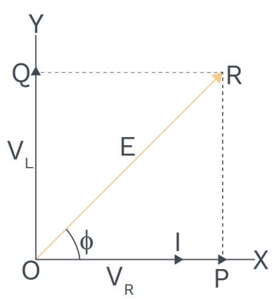
Resistance And Inductance In Series (R-L circuit)

A RL circuit containing a series combination of a resistance R and an inductance L, connected with a source of alternating e.m.f. E as shown in figure.



- **Phasor diagram for L-R circuit**

Let in a L-R series circuit, applied alternating emf is $E = E_0 \sin \omega t$. As R and L are joined in series, hence current flowing through both will be same at each instant. Assume I be the current in the circuit at any instant and V_L and V_R the potential differences across L and R respectively at that instant.



Then $V_L = IX_L$ and $V_R = IR$

Now, V_R is in phase with the current while V_L leads the current by $\frac{\pi}{2}$.

So, V_R and V_L are mutually perpendicular

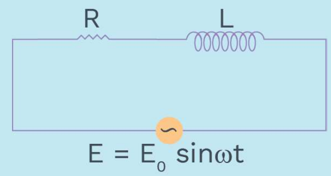
(Note: $E \neq V_R + V_L$)

The vector OP represents V_R (which is in phase with I), while OQ represents V_L (which leads I by 90°).

The resultant of V_R and V_L = the magnitude of vector $E = \sqrt{V_R^2 + V_L^2}$

Thus $E^2 = V_R^2 + V_L^2 = I^2(R^2 + X_L^2) \Rightarrow I = \dfrac{E}{\sqrt{R^2 + X_L^2}}$

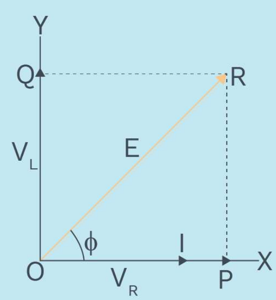
Concept Reminder



$$I = \frac{E}{\sqrt{R^2 + X_L^2}}$$

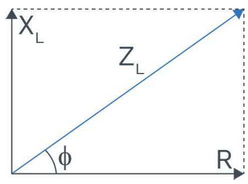
Concept Reminder

In L-R circuit the applied emf E leads the current I or conversely the current I lags behind the e.m.f. E by a phase angle ϕ.



$$\tan \phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{\omega L}{R}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$



The phasor diagram shown in fig. also shows that in L-R circuit the applied emf E leads the current I or conversely the current I lags behind the e.m.f. E by a phase angle ϕ.

$$\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{\omega L}{R} \Rightarrow \phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

- **Inductive Impedance Z_L:**
  In L-R circuit the maximum value of current $I_0 = \frac{E_0}{\sqrt{R^2 + \omega^2 L^2}}$. Here $\sqrt{R^2 + \omega^2 L^2}$ represents the effective opposition offered by L-R circuit to the flow of a.c. through it. It is known as impedance of L-R circuit and is represented by Z_L.

$$Z_L = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + (2\pi f L)^2}$$

The reciprocal of impedance is called admittance

$$Y_L = \frac{1}{Z_L} = \frac{1}{\sqrt{R^2 + \omega^2 L^2}}$$

Ex. When 100-volt dc is applied across a coil, a current of 1 amp flows through it; when 100V ac of '50' Hz is applied to the same coil, only '0.5' amp flows. Find out the resistance of inductance of the coil.

Sol. When dc is applied, $\omega = 0$, so Z = R

and hence $i = \frac{V}{R}$ i.e., $R = \frac{V}{i} = \frac{100}{1} = 100\,\Omega$

For a coil, i.e., L – R circuit.

$$i = \frac{V}{Z} \text{ with } Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2}$$

$$Z = \frac{V}{I} = \frac{100}{0.5} = 200\Omega$$

Rack your Brain

An inductor 20 mH, a capacitor 100 F and a resistor 50Ω are connected in series across a source of emf, $v = 10\sin 314t$. Find power loss is circuit.

Concept Reminder

For L-R ac circuit:
Impedance

$$Z_L = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + (2\pi f L)^2}$$

KEY POINTS

- Impedance
- Admittance

but $Z = \sqrt{R^2 + \omega^2 L^2}$ i.e., $\omega^2 L^2 = Z^2 - R^2$

i.e., $(2\pi fL)^2 = 200^2 - 100^2$

$= 3 \times 10^4$ (as $\omega = 2\pi f$)

So, $L = \dfrac{\sqrt{3} \times 10^2}{2\pi \times 50} = \dfrac{\sqrt{3}}{\pi} H = 0.55 H$

Ex. An inductance of $0.05/\pi$ Henry and a 12 ohm resistance with resistance (negligible) are connected in series combination. Across the end of this electric circuit is connected a 130 V alternating voltage of frequency 50 cycles/second. Find out the alternating current in the circuit and potential difference across the inductance and that across the resistance.

Sol. The impedance of the circuit is

$$Z = \sqrt{(R^2 + \omega^2 L^2)} = \sqrt{[R^2 + (2\pi fL)^2]}$$

$$= \sqrt{[(12)^2 + \{2 \times 3.14 \times 50 \times 0.05 / 3.14\}^2]}$$

$$= \sqrt{(144 + 25)} = 13 \text{ ohm}$$

Current in the circuit =

$$i = E/Z = \frac{130}{13} = 10 \text{ amp}$$

Potential difference across resistance

$$V_R = iR = 10 \times 12 = 120 \text{ volt}$$

Inductive reactance of coil $X_L = \omega L = 2\pi fL$

$$\therefore \quad X_L = 2\pi \times 50 \times \left(\frac{0.05}{\pi}\right) = 5 \text{ ohm}$$

Potential difference across inductance

$$V_L = i \times X_L = 10 \times 5 = 50 \text{ volt}$$

Resistance and Capacitor in Series (R-C circuit)

A circuit containing a series combination of a resistance R and a capacitor C, connected with a source of e.m.f. of peak value E_0 as shown

Rack your Brain

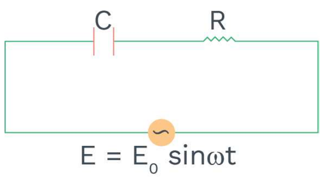
The instantaneous values of alternating current and voltages in a circuit are given as

$$i = \frac{1}{\sqrt{2}} \sin(100\pi t) \text{ amperes}$$

$$E = \frac{1}{\sqrt{2}} \sin\left(100\pi t + \frac{\pi}{3}\right) \text{ volts}$$

Find average power consumed by circuit.

in fig.



- **Phasor diagram for R-C circuit**

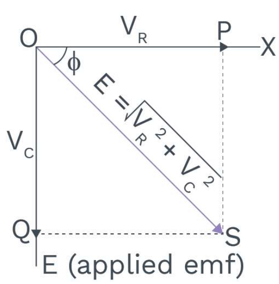
Current through both the resistance and capacitor will be same at every instant and the instantaneous potential differences across C and R are

$$V_C = IX_C \text{ and } V_R = IR$$

where X_C = capacitive reactance and I = instantaneous current.

Now, V_R is in phase with I, while V_C lags behind I by 90°.

The phasor diagram is shown in fig.



The vector OP represents V_R (which is in phase with I) and the vector OQ represents V_C (which lags behind I by $\frac{\pi}{2}$).

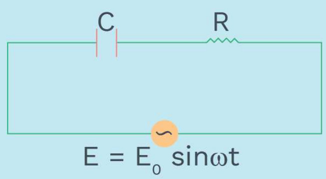
The vector OS represents the resultant of V_R and V_C

= the applied e.m.f. E.

Hence $V_R^2 + V_C^2 = E^2 \Rightarrow E = \sqrt{V_R^2 + V_C^2}$

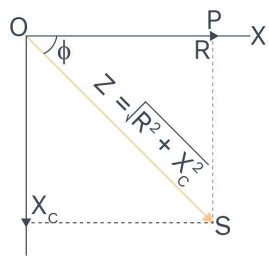
$$E^2 = I^2(R^2 + X_C^2) \Rightarrow I = \frac{E}{\sqrt{R^2 + X_C^2}}$$

Concept Reminder



$$E = \sqrt{V_R^2 + V_C^2}$$

$$I = \frac{E}{\sqrt{R^2 + X_C^2}}$$



The term $(R^2 + X_C^2)$ represents the effective resistance of the R-C circuit and called the capacitive impedance Z_C of the circuit.

Hence, in C-R circuit $Z_C = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

- **Capacitive Impedance Z_C:**

In R-C circuit the term $\sqrt{R^2 + X_C^2}$ effective opposition offered by R-C circuit to the flow of a.c. through it. It is known as impedance of R-C circuit and is represented by Z_C. The phasor diagram also shows that in R-C circuit the applied e.m.f. lags behind the current I (or the current I leads the emf E) by a phase angle ϕ given by

$$\tan\phi = \frac{V_C}{V_R} = \frac{X_C}{R} = \frac{1/\omega C}{R} = \frac{1}{\omega CR}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

Ex. An A.C. source of angular frequency 'ω' is fed across a resistor 'R' and a capacitor 'C' in series. The current registered is 'I'. If now the angular frequency of the source is changed to $\omega/3$ (but voltage is same), the current in the circuit is found to be halved. Find out the ratio of reactance to resistance at the original frequency.

Sol. At angular frequency ω, the current in R-C circuit is given by

$$i_{rms} = \frac{E_{rms}}{\sqrt{\{R^2 + (1/\omega^2 C^2)\}}} \quad \text{... (i)}$$

Concept Reminder

In R-C circuit the applied e.m.f. lags behind the current I (or the current I leads the emf E) by a phase angle ϕ given by

$$\tan\phi = \frac{V_C}{V_R} = \frac{X_C}{R} = \frac{1/\omega C}{R} = \frac{1}{\omega CR}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

When frequency is changed to $\omega/3$, the current is halved. Thus

$$i_{rms} = \frac{E_{rms}}{\sqrt{\{R^2 + 1/(\omega/3)^2C^2\}}} = \frac{E_{rms}}{\sqrt{\{R^2 + (9/\omega^2C^2\}}} \quad \text{... (ii)}$$

From equations (i) and (ii), we have

$$\frac{1}{\sqrt{\{R^2 + (1/\omega^2C^2)\}}} = \frac{2}{\sqrt{\{R^2 + (9/\omega^2C^2\}}}$$

Solving this equation, we get $3R^2 = \frac{5}{\omega^2C^2}$

Hence, the ratio of reactance to resistance is $\frac{(1/\omega C)}{R} = \sqrt{\left(\frac{3}{5}\right)}$

Ex. A 100 V, 50 W lamp is to be connected to an ac main of 200 V, 50 Hz. Calculate the capacitance is essential to be put in series with the lamp?

Sol. As resistance of the lamp $R = \frac{V_s^2}{P} = \frac{100^2}{50} = 200\,\Omega$ and the maximum current $i = \frac{V}{R} = \frac{100}{200} = \frac{1}{2}A$; so, when the lamp is put in series with a capacitance and run at 200 V ac, from V =iZ we have,

$$Z = \frac{V}{i} = \frac{200}{(1/2)} = 400\Omega$$

Now as in case of C-R circuit, $Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

i.e., $R^2 + \left(\frac{1}{\omega C}\right)^2 = 160000$

or $\left(\frac{1}{\omega C}\right)^2 = 16 \times 10^4 - (200)^2 = 12 \times 10^4$

Concept Reminder

In purely inductive or capacitive circuit,

$\cos\phi = 0 \Rightarrow \phi = \frac{\pi}{2}$. Average power loss = 0. Although current is flowing in circuit, power will be zero.

$$\text{So, } \frac{1}{\omega C} = \sqrt{12} \times 10^2$$

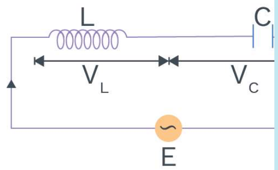
$$\text{or } C = \frac{1}{100\pi \times \sqrt{12} \times 10^2} \text{F}$$

$$\text{i.e., } \quad C = \frac{100}{\pi\sqrt{12}} \text{ F} = 9.2 \text{ F}$$

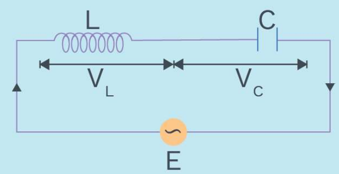
L-C Series Circuit With Alternating Voltage

- Let an alternating source of emf $E = E_0 \sin \omega t$ is conn

pure capacitor of capacitance (C) and an inductor of



Concept Reminder

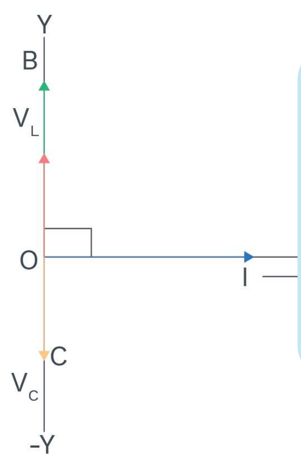


- If $X_L > X_C$

  $V = V_L - V_C$
- If $X_L < X_C$

  $V = V_C - V_L$

- Let I be the rms value of current flowing in the circui
- The P.D across 'L' is $V_L = I.X_L$
- The current I lags V_L by an angle $\pi / 2$.
- The P.D across capacitance is $V_C = I.X_C$
- The current I leads V_C by an angle $\pi / 2$.
- The voltage V_L and V_C are represented by OB and OC respectively.



Concept Reminder

In L - C, circuit, the impendence

$$Z = \left|\omega L - \frac{1}{\omega C}\right|,$$

And Current $I = \frac{E}{Z}$.

The resultant P.D of V_L and V_C is

$$V = V_L - V_C = I(X_L - X_C)$$

$$= I\left[\omega L - \frac{1}{\omega C}\right] = IZ_{LC}$$

- From the above equations, Impedance of L-C circuit is

$$Z_{LC} = \left[(\omega L) - \frac{1}{\omega C}\right]$$

- If $\omega L > \frac{1}{\omega C}$ i.e., $X_L > X_C$ then $V_L > V_C$ resulting potential difference $V = V_L - V_C$.

  Now current lags behind voltage by $\pi/2$.

- If $\omega L < \frac{1}{\omega C}$ then $V_L < V_C$ resultant potential difference $(V) = V_C - V_L$

  Now current leads emf by $\pi/2$.

- If $\omega L = \frac{1}{\omega C}$ then $Z = \omega L - \frac{1}{\omega C} = 0$

  Current $I = \frac{E}{Z} = \infty$

- In L - C, circuit, the phase difference between voltage and current are always $\pi/2$.
- Power factor $\cos\phi = \cos\pi/2 = 0$

  So, power consumed in L-C circuit is

$$P = V_{rms} \times I_{rms} \times \cos\phi = 0$$

$\therefore$ In L - C circuit no power is consumed.

Note:

- In L - C, circuit, the impendence $Z = \left|\omega L - \frac{1}{\omega C}\right|$,

  Current $I = \frac{E}{Z}$.

  So, the impedance and current vary with frequency.

- At a particular angular frequency, $\omega L = \frac{1}{\omega C}$ and current $I = \frac{E}{Z}$ becomes maximum (I_0) and resonance occurs.

  At resonance Z = 0 and $I_0 = \frac{E_0}{Z} = \infty$.

Resonant angular frequency $\omega_0 = \dfrac{1}{\sqrt{LC}}$

Resonant frequency $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

Ex. A 0.21 H inductor and a 12-ohm resistance are connected in series to a 220 V,50 Hz ac source. Find out the current in the circuit and the phase angle between the current and the source voltage.

Sol. Here

$$X_L = \omega L = 2\pi fL = 2\pi \times 50 \times 0.21 = 21\pi\Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{12^2 + (21\pi)^2} = \sqrt{144 + 4348}$$

$$Z = \sqrt{4492} \simeq 67.02\,\Omega\,; \; I = \frac{V}{Z} = \frac{220}{67.02} = 3.28\,A$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{21\pi}{12}\right)$$

The current will lag the applied voltage by an angle $\tan^{-1}\left(\dfrac{21\pi}{12}\right)$.

Ex. A 10 F capacitor is in series with a $50\,\Omega$ resistance and the combination is connected to a 220V, 50 Hz line. Calculate (i) the capacitive reactance, (ii) the impedance of the circuit and (iii) the current in the circuit.

Sol. Here, $C = 10\ \ F = 10 \times 10^{-6} = 10^{-5} F$

$R = 50\,\text{ohm}, E_{rms} = 220\,V, \nu = 50\,Hz$

(i) Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$

$$= \frac{1}{2 \times 3.14 \times 50 \times 10^{-5}} = 318.5\,\Omega$$

(ii) Impedance of CR circuit.

$$Z_{CR} = \sqrt{R^2 + X_C^2}$$

$$= \sqrt{(50)^2 + (318.5)^2} = 322.4\,\Omega$$

(iii) Current, $I_{rms} = \dfrac{E_{rms}}{Z_{CR}} = \dfrac{220}{322.4} = 0.68\,A$

Ex. A resistance of 220Ω and a coil has an inductance of 0.7 H is joined in series with. When an alternating e.m.f of 220 V at 50 cps is applied to it, then the wattless component of the current in the circuit is

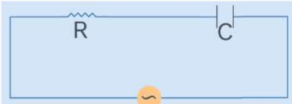
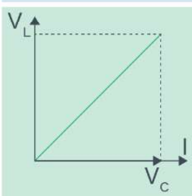
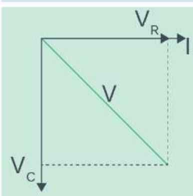
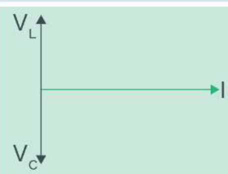
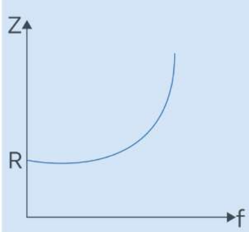
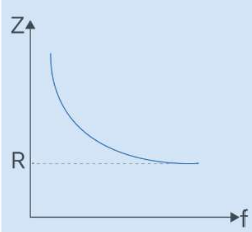
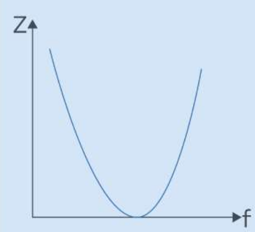
Sol. $\tan\phi = \frac{X_L}{R} = \frac{\omega_L}{R} = \frac{2\pi \times 50 \times 0.7}{220} = 1$

$\therefore \quad \phi = 45°, Z = \sqrt{R^2 + X_L^2}$

$= \sqrt{220^2 + 220^2} = 220\sqrt{2}\Omega$

Wattless component of current = $I_V \sin\phi$

$= \frac{E_V}{Z}\sin 45° = \frac{220}{220\sqrt{2}} \times \frac{1}{\sqrt{2}} = 0.5\text{ A}$

COMBINATION OF COMPONENTS (R-L OR R-C OR L-C)			
TERM	**R-L**	**R-C**	**L-C**
Circuit	 I is same in R & L	 I is same in R & C	 I is same in L & C
Phasor diagram	 $V^2 = V_R^2 + V_L^2$	 $V^2 = V_R^2 + V_C^2$	 $V = V_L - V_C$ ($V_L > V_C$) $V = V_C - V_L$ ($V_C > V_L$)
Phase difference in between V & I	V leads I (ϕ = 0 to $\frac{\pi}{2}$)	V lags I (ϕ = $-\frac{\pi}{2}$ to 0)	V lags I ($\phi = -\frac{\pi}{2}$, if $X_C > X_L$ V lags I ($\phi = +\frac{\pi}{2}$, if $X_L > X_C$
Impedance	$Z = \sqrt{R^2 + X_L^2}$	$Z = \sqrt{R^2 + (X_C)^2}$	$Z = \lvert X_L - X_C \rvert$
Variation of Z with f	As f ↑, Z ↑ 	As f ↑, Z ↓ 	As f ↑, Z first ↓ then ↑ 
At very low f	$Z \simeq R$ ($X_L \to 0$)	$Z \simeq X_C$	$Z \simeq X_C$
At very high f	$Z \simeq X_L$	$Z \simeq R$ ($X_C \to 0$)	$Z \simeq X_L$

Ex. Find out the required inductance to put in series of bulb (10W, 60V) to run it safely across an alternating supply of 100V, 60Hz.

Sol. For LR-circuit

$$V = \sqrt{V_R^2 + V_L^2}$$

$$R = \frac{60 \times 60}{10} = 360\,\Omega$$

$$V_L = \sqrt{(100)^2 - (60)^2} = 80\text{ V}$$

$$\text{As } V_L = I X_L$$

$$80 = \left[\frac{V}{Z}\right] X_L = \left[\frac{100}{\sqrt{(360)^2 + X_L^2}}\right] X_L$$

$$25X_L^2 = 16\left[(360)^2 + X_L^2\right]$$

$$9X_L^2 = 16 \times (360)^2$$

$$X_L = 480\,\Omega$$

$$\text{Finally, } X_L = \omega L = 480\,\Omega$$

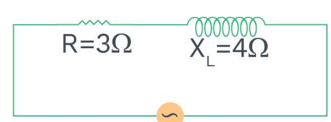
$$L = \frac{480}{2\pi \times 60} = \frac{4}{\pi}\text{Henry}$$

Ex. For the RL-circuit calculate the following:

(i) Impedance of circuit

(ii) Phase difference between V & I

(iii) I_{rms}



Sol. **(i)** $Z = \sqrt{R^2 + X_L^2} = \sqrt{9 + 16} = 5\Omega$

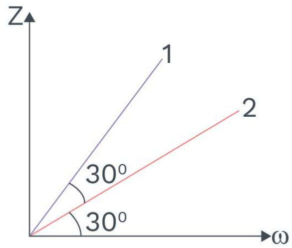
(ii) $\tan\phi = \frac{X_L}{R} = \frac{4}{3} \Rightarrow \phi = 53°$

(iii) $I_{rms} = \frac{V_{rms}}{Z} = \frac{\left(\frac{100}{\sqrt{2}}\right)}{5} = 10\sqrt{2}\text{A}$

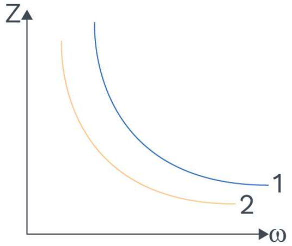
Ex. Across two different inductors same alternating source is connected. For this Z vs ω graph is given. Find out $\frac{L_1}{L_2}$

Sol. As $\tan\phi = \frac{Z}{\omega} = \frac{\omega L}{\omega} = L$

So $\frac{L_1}{L_2} = \frac{\tan 60^\circ}{\tan 30^\circ} = \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} = 3$



Ex. Two capacitors C_1 & C_2 are connected across the same source one by one and for them $Z-\omega$ graphs are shown in figure. Select the correct one



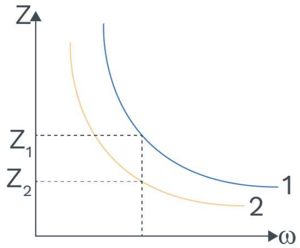
(i) $C_1 = C_2$
(ii) $C_1<C_2$
(iii) $C_1>C_2$
(iv) None of these

Sol. For same value of ω

$Z_1 > Z_2$

or $\frac{1}{\omega C_1} > \frac{1}{\omega C_2}$

or $\boxed{C_1 < C_2}$



Ex. For the given RC-circuit. What will happen regarding brightness of bulb if following changes are done

(i) Frequency of ac-source is increased.
(ii) A mica slab is inserted in between plates of capacitor

Sol. (i) As $X_C = \frac{1}{\omega C}$

$f\uparrow \longrightarrow \omega\uparrow \longrightarrow X_C\downarrow \longrightarrow I\uparrow$

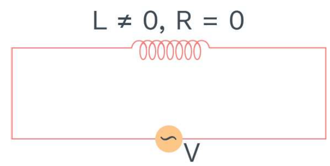
In this way brightness will increase

(ii) If the mica slab is placed than

$C' > C \Rightarrow X_C \downarrow \Rightarrow$ brightness$\uparrow$

Ex. An alternating emf V = 100 sin (100 πt) volt is connected to a choke coil of negligible resistance. The electric current in circuit oscillates with amplitude of 10A. Find the inductance of choke coil.

Sol.

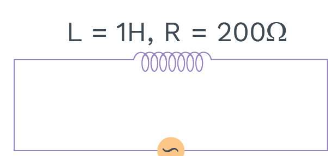


As $I_0 = 10\text{ A}$ $\quad\Rightarrow\quad Z = \frac{V_0}{I_0} = \frac{100}{10} = 10\Omega$

$\because \quad Z = X_L = \omega L$ $\quad\Rightarrow\quad L = \frac{Z}{\omega} = \frac{10}{100\pi} = \frac{1}{10\pi}\text{Hz}$

Ex. An inductor of resistance $200\,\Omega$ and self-inductance 1 H is connected to an ac-source of frequency $\frac{100}{\pi}$ Hz. Find out the phase difference between voltage and current in circuit.

Sol.



As $\omega = 2\pi f = 2\pi \times \frac{100}{\pi} = 200\text{ rad/s}$

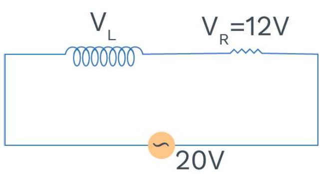
Since, $f = \frac{100}{\pi}\text{Hz}$

For LR-circuit

$\tan\phi = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{200 \times 1}{200} = 1 \Rightarrow \boxed{\phi = 45^\circ}$

Ex. An ac source of 20V is applied across a circuit consisting of a resistance and a coil with negligible resistance. If the voltage across the resistance is 12V. Find out voltage across coil.

Sol.



As $V = \sqrt{V_L^2 + V_R^2}$

$\Rightarrow$

$V_L = \sqrt{V^2 - V_R^2} = \sqrt{(20)^2 - (12)^2} = \sqrt{256} = 16\text{ V}$

Ex. When 100V DC is applied across a choke coil then current of 1A flows in it. If 100V ac is applied across the same source, then electric current drops to 0.5A. Find the inductance of coil.

(Given = frequency of ac source is 50 Hz)

Sol. In dc $R = \frac{V}{I} = \frac{100}{1} = 100\,\Omega$

In ac $Z = \frac{V}{I} = \frac{100}{0.5} = 200\,\Omega$

$\because \quad Z = \sqrt{X_L^2 + R^2}$

$\Rightarrow$

$X_L = \sqrt{Z^2 - R^2} = \sqrt{(200)^2 - (100)^2} = 100\sqrt{3}\,\Omega$

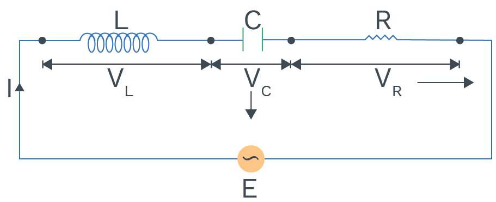
Again $L = \frac{X_L}{\omega} = \frac{100\sqrt{3}}{2\pi \times 50} = \frac{\sqrt{3}}{\pi}\text{H}$

Concept Reminder

The only element which dissipates energy in ac circuit is resistor.

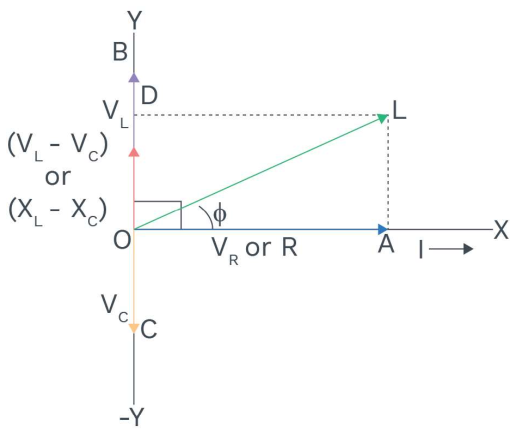
LCR Series Circuit

- A circuit containing pure inductor of inductance (L), pure capacitor of capacitance (C) and resistor of resistance (R), all joined in series, is shown in figure.
- Let E be the r.m.s value of the applied alternating emf to the LCR circuit.



- The potential difference across L,
  $V_L = IX_L$ ....(i)
- The potential difference across C,
  $V_C = IX_C$ ..... (ii)
- The potential difference across R,
  $V_R = IR$ .....(iii)

Phasor Diagram



- Since V_L and V_C are in opposite phase, so their resultant $(V_L - V_C)$ is represented by OD (Here $V_L > V_C$)
- The resultant of V_R and $(V_L - V_C)$ is given by OL. The magnitude of OL is given by

$$OL = \sqrt{(OA)^2 + (OD)^2}; = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$E = I\sqrt{R^2 + (X_L - X_C)^2}$$

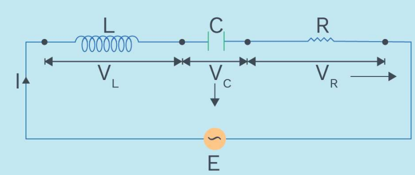
$$Z = \frac{E}{I} = \sqrt{R^2 + (X_L - X_C)^2}$$

Rack your Brain

A series LCR circuit is connected to a source of AC current. At resonance, the phase difference between the applied voltage and current in circuit is

(1) π (2) zero
(3) $\pi / 4$ (4) $\pi / 2$

Concept Reminder



- $V = \sqrt{V_R^2 + \left(V_L - V_C\right)^2}$
- $I = \dfrac{E}{\sqrt{R^2 + \left(X_L - X_C\right)^2}}$
- $Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$

∴ Impedance (Z) of LCR circuit is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\therefore \quad I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{E}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

- Let ϕ be the phase angle between E and I, then from Phasor diagram

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R}$$

$$\tan\phi = \frac{\left(L\omega - \frac{1}{C\omega}\right)}{R}$$

∴ Current in L-C-R series circuit is given by

$$I = \frac{E}{Z} = \frac{E_0}{Z}\sin(\omega t \pm \phi)$$

$$\text{(or) } I = I_0 . \sin(\omega t \pm \phi)$$

- If X_L and X_C are equal then Z = R i.e., expression for pure resistance circuit.
- If $X_L = 0$ then $Z = \sqrt{R^2 + X_C^2}$ i.e., expression for series RC circuit.
- Similarly, if $X_C = 0$ then $Z = \sqrt{R^2 + X_L^2}$ i.e., expression for series RL circuit.

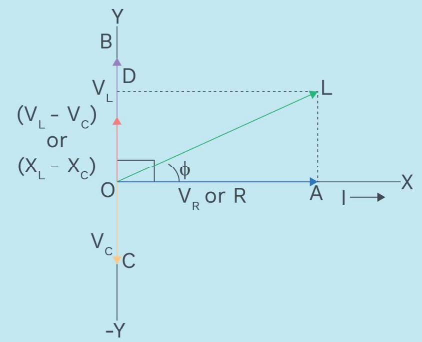
Also, $\cos\phi = \frac{R}{Z}$

Case (i): If $X_L > X_C$ then ϕ is +ve. Then the voltage leads the current by a phase angle

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Case (ii): If $X_L < X_C$ then ϕ is –ve. In this case the current leads the emf by a phase angle

Concept Reminder



$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R}$$

Concept Reminder

In Series LCR

- If $X_L > X_C$

  Current lags behind the emf.
- If $X_L < X_C$

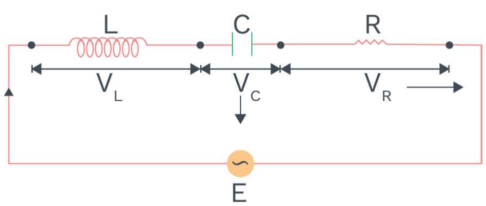
  Current leads the emf.

$$\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

Case (iii): If $X_L = X_C$ then ϕ is 0. In this case the current and emf are in phase.

- If $X_L > X_C$, then the circuit will be inductive
- If $X_L < X_C$, then the circuit will be capacitive
- If $X_L = X_C$, then the circuit will be purely resistive.
- The LCR circuit can be inductive or capacitive or purely resistive depending on the value of frequency of alternating source of emf.
- At some frequency of alternating source, $X_L > X_C$ and for some other frequency, $X_L < X_C$. There exists a particular value of frequency where $X_L = X_C$ (This situation is explained under resonance of LCR series circuit)

Note: Relation between applied pd & pd across the components in L - C - R circuit



$V = IZ$

(Only before steady state)

$$V = I\sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$V^2 = V_R^2 + \left(V_L - V_C\right)^2$$

where $V_L = IX_L = I\omega L$ and $V_C = IX_C = \frac{1}{\omega C}$ and

$V_R = IR$

Note: Rules to be followed for various combinations of ac circuits

- Compute effective resistance of the circuit as R
- Calculate the net reactance of the circuit as

$X = X_L - X_C$ where $X_L = \omega L, X_C = \frac{1}{\omega C}$

Rack your Brain

In LCR circuit, when L is removed from circuit, the phase difference between the voltage and current is $\pi/3$. If instead, C is removed, the phase difference again $\pi/3$. Find power factor of circuit.

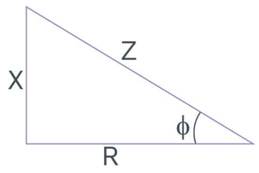
Concept Reminder

The power factor in a LCR circuit is a measure of how close the circuit is to expanding the maximum power.

- Resistance offered by all the circuited elements to the flow of ac is impedance (Z)

  $\therefore \quad Z = \sqrt{R^2 + X^2} = \sqrt{R^2 + (X_L - X_C)^2}$

- Calculate the peak value of current as $I_0 = \frac{E_0}{Z}$
- The phase difference between emf & current can be known by constructing an ac triangle as



Ex. In a circuit R, L and C are connected in series with an alternating voltage source of frequency f. The current leads the voltage by 45°. The value of C is:

Sol. As current leads the voltage by 45°,

$\therefore \quad \tan\theta = \frac{X_C - X_L}{R} = \tan 45° = 1$

$\therefore \quad X_C - X_L = R \text{ or } X_C = X_L + R$

or $\quad \frac{1}{\omega C} = \omega L + R \Rightarrow C = \frac{1}{\omega(\omega L + R)}$

$C = \frac{1}{2\pi f(2\pi f L + R)}$

Ex. In a series LCR circuit, the voltage across the resistance, capacitance and inductance is 10V each. If the capacitance is short circuited, then the voltage across the inductance will be

Sol. As $\quad V_R = V_L = V_C; R = X_L = X_C$

$Z = R; V = IR = 10 \text{ volt}$

when capacitor is short circuited,

$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + R^2} = R\sqrt{2}$

New current, $I' = V / Z = \frac{V}{R\sqrt{2}} = \frac{10}{R\sqrt{2}}$

Potential drop across inductance

$= I'X_L = I'R = \frac{10 \times R}{R\sqrt{2}} = \frac{10}{\sqrt{2}} \text{ volt}$

Ex. A resistance of $10\,\Omega$, an inductance of $\frac{200}{\pi}$mH and a capacitance of $\frac{10^{-3}}{\pi}$F are connected in series with an AC source of 220 V, 50 Hz. The phase angle of the circuit is?

Sol. Here, $L = \frac{200}{\pi} mH = \frac{200 \times 10^{-3}}{\pi} H = \frac{0.2}{\pi} H$

$$C = \frac{10^{-3}}{\pi} F, R = 10\Omega; E_v = 220V, n = 50\,Hz$$

$$X_L = \omega L = 2\pi n L = 2\pi \times 50 \times \frac{0.2}{\pi} = 20\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi n C} = \frac{\pi}{2\pi \times 50 \times 10^{-3}} = 10\,\Omega$$

$$\tan\phi = \frac{(X_L - X_C)}{R} = \frac{20 - 10}{10} = 1 \Rightarrow \phi = \frac{\pi}{4}$$

Ex. In a series LCR circuit, $R = 200\,\Omega$, the voltage and the frequency of the main supply is 220 V and 50 Hz. respectively. On remove the capacitance from the circuit, the current lags behind the voltage by 30°. Removing the inductor from the circuit, the current leads the voltage by 30°. The power dissipated in the LCR circuit will be?

Sol. Here, $R = 200\,\Omega, E_v = 220V$

In L-R circuit, $\tan 30° = \frac{X_L}{R}$

In C-R circuit, $\tan 30° = \frac{X_C}{R}$

$$\therefore \quad \frac{X_L}{R} = \frac{X_C}{R} \text{ or } X_L = X_C$$

In L-C-R circuit, if θ is the phase difference between voltage and current, then

$$\tan\theta = \frac{X_L - X_C}{R} = \frac{0}{200} = 0 \Rightarrow \theta = 0°$$

i.e., current and voltage are in the same phase.

$\therefore$ Average power $= E_V I_V \cos\theta = \dfrac{E_V^2}{R}$

$(\because \theta = 0) \; = \dfrac{(220)^2}{200} = 242\text{W}$

Ex. A 750 Hertz - 20 volt source is connected to a resistance of 100 ohm, an inductance of 0.1803 Henry and a capacitance of 10 F all in series. What is the time in which the resistance (Thermal capacity = 2 joule/ °C) will get heated by 10°C?

Sol. Here, $\nu = 750\,\text{Hz}, E_V = 20\,\text{V}, R = 100\,\Omega$

$L = 0.1803\,\text{H}, C = 10 \;\; \text{F} = 10^{-5}\text{F}, t = ?$

$\Delta\theta = 10°\text{C}$, thermal capacity $= 2\text{J} / °\text{C}$

$X_L = \omega L = 2\pi\nu L = 2 \times 3.14 \times 750 \times 0.1803$

$= 849.2$ ohm

$X_L = \dfrac{1}{\omega C} = \dfrac{1}{2\pi \times 750 \times 10^{-5}} = 21.2\,\Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{100^2 + (849.2 - 21.2)^2} = 834\,\Omega$

Power dissipated $= E_V I_V \cos\phi$

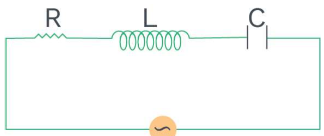
$= E_V \left(\dfrac{E_V}{Z}\right)\left(\dfrac{R}{Z}\right) = \dfrac{20^2 \times 100}{(834)^2} = 0.0575\text{W}$

$t = \dfrac{\text{thermal capacity} \times \Delta\theta}{\text{power}} = \dfrac{2 \times 10}{0.0575}$

$= 347.8$ sec

Series L-C-R Circuit

1. Circuit diagram



I same for R, L & C

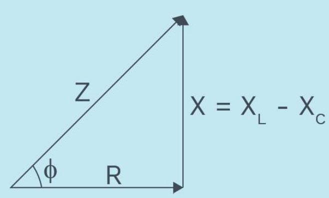
Concept Reminder

If you remember the formula for series LCR circuit, then to deal with series LR circuit, omit the term X_C from formula of series LCR circuit.

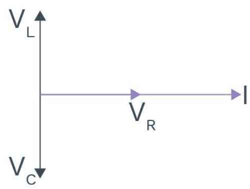
Concept Reminder

For Series LCR circuit

Impedance triangle



2. Phasor diagram

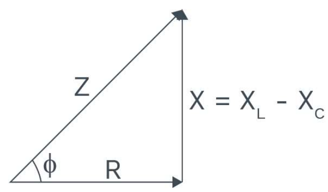


(i) If $V_L > V_C$ then $V_L - V_C$

(ii) If $V_C > V_L$ then $V_C - V_L$

(iii) $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

(IV) Impedance triangle

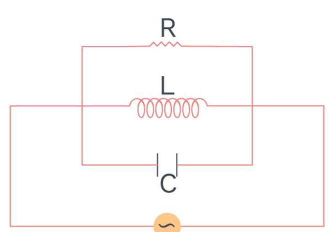


Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{V_L - V_C}{V_R}$$

Parallel L-C-R Circuit

1. Circuit diagram

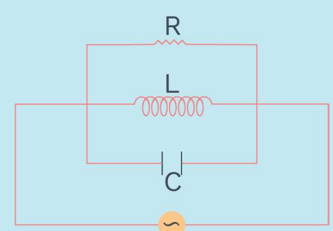


V same for R, L and C

2. Phasor diagram

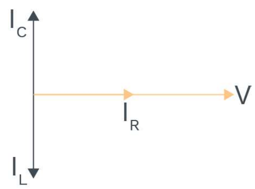
Concept Reminder

Parallel LCR circuit

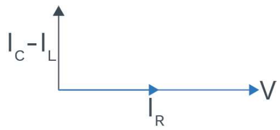


V same for R, L and C

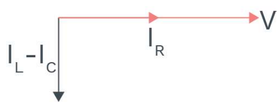
$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}$$



(i) $I_C > I_L$ then $I_C - I_L$



(ii) $I_L > I_C$ then $I_L - I_C$



Resonance

A electric circuit is said to be resonant when the natural frequency of circuit is equal to frequency of the applied voltage. For resonance both L and C must be present in circuit.

There are two types of resonance:

(i) Series Resonance (ii) Parallel Resonance

Definitions

A circuit is said to be resonant when the natural frequency of circuit is equal to frequency of the applied voltage.

4. Series Resonance

(a) At Resonance

(i) $X_L = X_C$

(ii) $V_L = V_C$

(iii) $\phi = 0$ (V and I in same phase)

(iv) $Z_{min} = R$ (impedance minimum)

(v) $I_{max} = \frac{V}{R}$ (current maximum)

(b) Resonance frequency

$$\because X_L = X_C \Rightarrow \omega_r L = \frac{1}{\omega_r C} \Rightarrow \omega_r^2 = \frac{1}{LC}$$

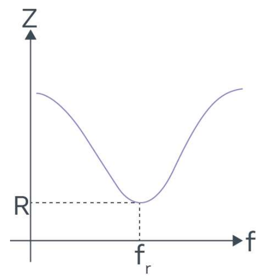
$$\omega_r = \frac{1}{\sqrt{LC}} \Rightarrow f_r = \frac{1}{2\pi\sqrt{LC}}$$

Concept Reminder

Resonance frequency

$$\omega_r = \frac{1}{\sqrt{LC}} \Rightarrow f_r = \frac{1}{2\pi\sqrt{LC}}$$

(c) Variation of Z with f



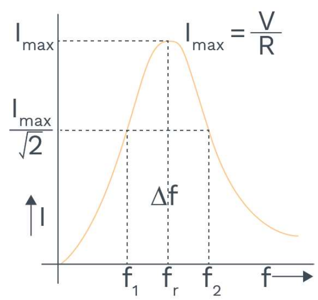
(i) If $f < f_r$ then $X_L < X_C$
circuit nature capacitive, ϕ (negative)

(ii) At $f = f_r$ then $X_L = X_C$
circuit nature, Resistive, ϕ = zero

(iii) If $f > f_r$ then $X_L > X_C$
circuit nature is inductive, ϕ (positive)

(d) Variation of I with f

as f increase, Z first decreases then increase

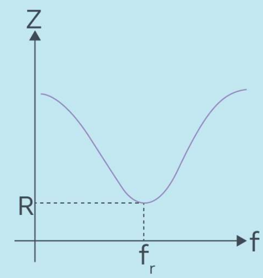


as f increase, I first increase then decreases.

- At resonance, impedance of the series resonant circuit is minimum, so it is called 'acceptor circuit' as it most readily accepts that current out of many currents whose frequency is equal to its natural frequency. In radio or TV tuning we receive the desired station by making the frequency of the circuit equal to that of the desired station.

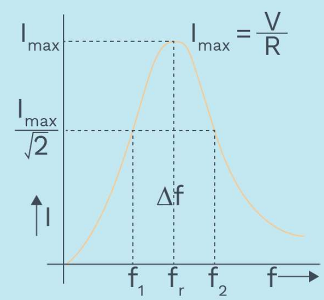
Concept Reminder

In series LCR circuit, variation of Z with f is shown below.



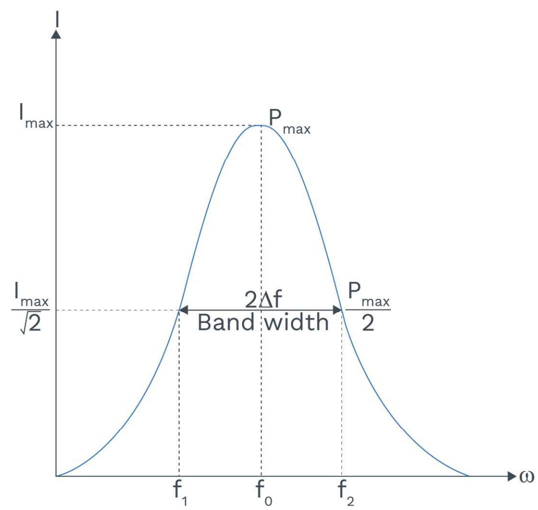
Concept Reminder

In series LCR circuit, the variation of current with f is shown below.



Half Power Frequency

There are two such frequencies of applied ac source in LCR-series circuit where power consumption is exactly half of the maximum at resonance such frequencies are called half power frequencies. Hence f_1& f_2 are called half power frequencies.



$$f_1 = f_0 - \Delta f$$

$$f_2 = f_0 + \Delta f$$

$$\& \boxed{f_0 = \sqrt{f_1 f_2}} \text{ or } \boxed{\omega_0 = \sqrt{\omega_1 \omega_2}}$$

At ω_1 & ω_2

$$\Rightarrow P = \frac{P_{max}}{2}$$

$$I_{half}^2 R = \frac{I_{max}^2 R}{2} \Rightarrow \boxed{I_{half} = \frac{I_{max}}{\sqrt{2}}}$$

Again $\frac{V}{Z} = \frac{1}{\sqrt{2}} \frac{V}{R} \Rightarrow Z = \sqrt{2}R$ or $Z^2 = 2R^2$

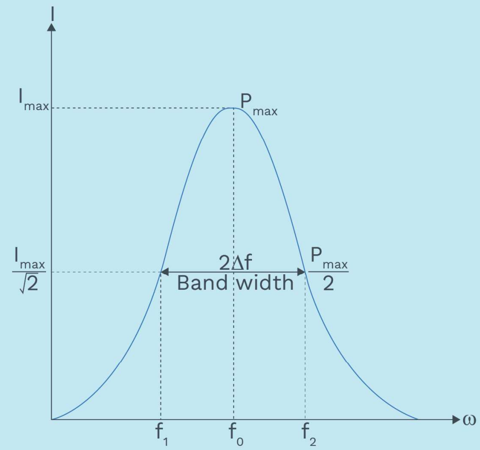
$$R^2 + (X_L - X_C)^2 = 2R^2 \Rightarrow X_L - X_C = R$$

or $\omega L - \frac{1}{\omega C} = R$ or $\omega - \frac{1}{LC\omega} = \frac{R}{L}$

Definitions

There are two such frequencies of applied ac source in LCR-series circuit where power consumption is exactly half of the maximum at resonance such frequencies are called half power frequencies.

Concept Reminder



$$\boxed{f_0 = \sqrt{f_1 f_2}} \text{ or } \boxed{\omega_0 = \sqrt{\omega_1 \omega_2}}$$

Concept Reminder

The difference $\omega_2 - \omega_1 = 2\Delta\omega$ is often called the bandwidth of the circuit. The quantity $\left(\frac{\omega_0}{2\Delta\omega}\right)$ is regarded as measure of sharpness of resonance.

or $\omega - \dfrac{\omega_0^2}{\omega} = \dfrac{R}{L}$

then $(\omega_0 + \Delta\omega) - \omega_0^2(\omega_0 + \Delta\omega)^{-1} = \dfrac{R}{L}$

or $(\omega_0 + \Delta\omega) - \dfrac{\omega_0^2}{\omega_0}\left[1 + \dfrac{\Delta\omega}{\omega_0}\right]^{-1} = \dfrac{R}{L}$

$$(\omega_0 + \Delta\omega) - \frac{\omega_0^2}{\omega_0}\left[1 - \frac{\Delta\omega}{\omega_0}\right] = \frac{R}{L}$$

$$(\omega_0 + \Delta\omega) - \omega_0\left[1 - \frac{\Delta\omega}{\omega_0}\right] = \frac{R}{L}$$

$$\Rightarrow \omega_0 + \Delta\omega - \omega_0 + \Delta\omega = \frac{R}{L}$$

$$\Rightarrow \boxed{\Delta\omega = \frac{R}{2L}}$$

Quality factor or Q-factor

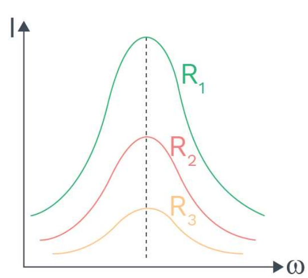
This factor gives relative information about stored energy and lost energy per cycle. Hence

$$Q = 2\pi\left[\frac{\text{Maximum energy stored per cycle}}{\text{Maximum energy loss per cycle}}\right]$$

$$= \frac{\omega_0}{\text{Band width}} = \frac{\omega_0}{2\Delta\omega}$$

Since $\Delta\omega = \dfrac{R}{2L}$

So $\boxed{Q = \dfrac{\omega_0 L}{R}}$ or $\boxed{Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}}$

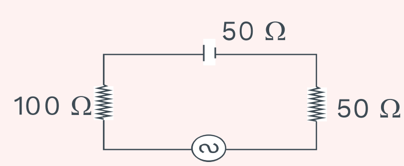


Concept Reminder

$\boxed{Q = \dfrac{\omega_0 L}{R}}$ or $\boxed{Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}}$

Rack your Brain

Find out power factor of given circuit.



KEY POINTS

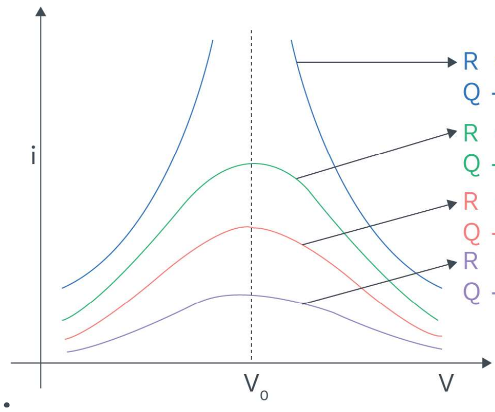
- Resonance
- Q-factor
- Sharpness of resonance
- Magnification

Magnification

At resonance V_L or V_C= Q × (supplied voltage)

Hence at resonance magnification factor is the Q-factor

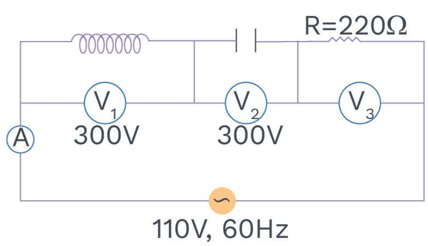
$$\boxed{Q - \text{factor} \propto \text{Magnification factor} \propto \text{Sharpness}}$$



According to figure, if 'R' is decreasing means Q-factor or sharpness will increase in the circuit.

TABLE FOR VALUES OF DIFFERENT PARAMETERS FOR DIFFERENT COMPONENTS APPLIED TO AC					
S.No.	**PARAMETER**	**R-L CIRCUIT**	**R-C CIRCUIT**	**L-C CIRCUIT**	**L-C-R CIRCUIT**
1.	Input emf	$E = E_0 \sin \omega t$	$E = E_0 \sin \omega t$	$E = E_0 \sin \omega t$	$E = E_0 \sin \omega t$
2.	Resulting current	$I = I_0 \sin(\omega t - \phi)$	$I = I_0 \sin(\omega t + \phi)$	$I = I_0 \sin\left(\omega t \pm \frac{\pi}{2}\right)$	$I = I_0 \sin(\omega t \pm \phi)$
3.	Resistance	R	R	0	R
4.	Net reactance	$X = X_L = \omega L$	$X = X_C = \frac{1}{\omega C}$	$X = \omega L - \frac{1}{\omega C}$	$X = \omega L - \frac{1}{\omega C}$
5.	Impedance	$Z = \sqrt{R^2 + (\omega L)^2}$	$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$	$Z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2}$	$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$
6.	Peak value of current	$I_0 = \frac{E_0}{Z}$	$I_0 = \frac{E_0}{Z}$	$I_0 = \frac{E_0}{Z}$	$I_0 = \frac{E_0}{Z}$
7.	Phase diff. between E & I	$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$	$\phi = \tan^{-1}\left(\frac{-1}{\omega RC}\right)$	$\phi = 90^0$	$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$
8.	Lead / Lag	I lags E by ϕ	I leads E by ϕ	If $X_L > X_C$, I Lags E by 90^0 If $X_L < X_C$, I Leads E by 90^0 If $X_L = X_C$, E and I are in phase	If $X_L > X_C$, I Lags E by ϕ If $X_L < X_C$, I Leads E by ϕ If $X_L = X_C$, I and E are in phase

Ex. A capacitor, a resistor and an inductor are connected in series to an ac-source of 110 V and frequency 60Hz. Find reading of voltmeter V_3 and ammeter in the given LCR-series circuit.



Sol. $\because$ $V_1 = V_2$

therefore $V_3 = V_R = 110$ volt

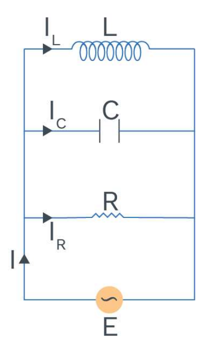
and $I = \frac{V}{R} = \frac{110}{220} = 0.5$ A

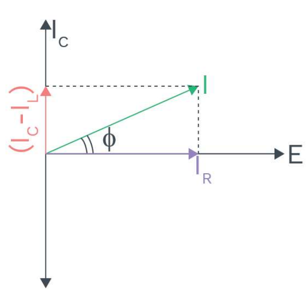
LCR-Parallel Circuit

$I_L = \frac{E}{X_L}$

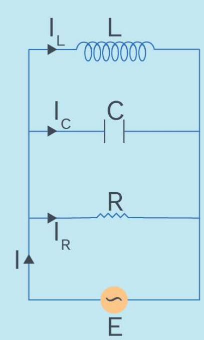
$I_C = \frac{E}{X_C}$

$I_R = \frac{E}{R}$





Concept Reminder



$I = \sqrt{I_R^2 + (I_C - I_L)^2}$

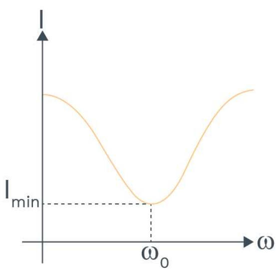
Let $I_c > I_L$

$$I = \sqrt{I_R^2 + (I_C - I_L)^2} \quad \text{... (1)}$$

Again $$Z = \frac{E}{I} = \frac{1}{\sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}} \quad \text{... (2)}$$

Condition of Resonance-

At $\omega = \omega_0; X_L = X_C$ or $I_L = I_C$

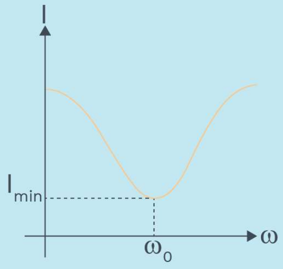


then $\boxed{Z_{max} = R}$

but $\boxed{I_{min} = \frac{E}{R}}$

Concept Reminder

For parallel LCR circuit, variation of I with ω is shown below.



Power Consumption In AC-Circuit

Instantaneous power

As $P_{inst} = VI$

$$= (V_0 \sin \omega t)(I_0 \sin(\omega t + \phi))$$

$$= V_0 I_0 \sin \omega t \sin(\omega t + \phi) = \frac{V_0 I_0}{2}[2 \sin \omega t \sin(\omega t + \phi)]$$

Hence $$P_{inst} = \frac{V_0 I_0}{2}[\cos \phi - \cos(2\omega t + \phi)]$$

$$\{\text{Since } 2 \sin A \sin B = \cos(A - B) - \cos(A + B)\}$$

Note: Therefore, frequency of power fluctuation is twice the frequency of applied ac-source.

Concept Reminder

In series LCR circuit, resonance occurs when $X_L = X_C$. Then $\omega_0 = \frac{1}{\sqrt{LC}}$.

For resonance to occur, presence of both L and C in the circuit is must.

Average Power

$$P_{av} = \frac{\int_0^T (V_0 \sin\omega t)(I_0 \sin(\omega t + \phi)dt}{\int_0^T dt}$$

$$= \frac{V_0 I_0}{T}\left[\cos\phi \int_0^T \sin^2 \omega t dt + \frac{\sin\phi}{2}\int_0^T \sin^2 \omega t\, dt\right]$$

$$= V_0 I_0 \left[\cos\phi \frac{\int_0^T \sin^2 \omega t dt}{T} + \frac{\sin\phi}{2}\frac{\int_0^T \sin^2 \omega t dt}{T}\right]$$

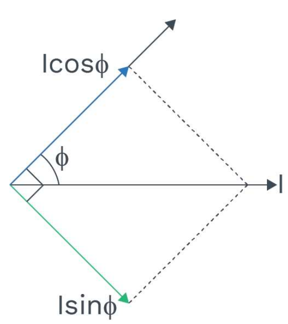
$$= V_0 I_0 \left[\cos\phi \times \frac{1}{2} + 0\right]$$

$$\boxed{P_{av} = \frac{1}{2} V_0 I_0 \cos\phi} \text{ or } \boxed{P_{av} = V_{rms} I_{rms} \cos\phi}$$

Note: Hence $\cos\phi = \frac{R}{Z}$ = Power factor of ac-circuit.

Wattless current

That component of current in ac-circuit which is not active.



Hence $I\cos\phi$ is the activity component of current because it is in phase with applied voltage.

But $I\sin\phi$ is the component, which is inactive, called as wattless current because it is in $\frac{\pi}{2}$ phase with applied voltage.

Rack your Brain

In an ac circuit the emf (ε) and current I at any instant are given respectively by

$e = E_0 \sin\omega t$ and

$I = I_0 \sin(\omega t - \phi)$

Find average power in the circuit

Concept Reminder

The voltage in series LCR AC circuit is given by

$V = \sqrt{V_R^2 + (V_L - V_C)^2}$

and not

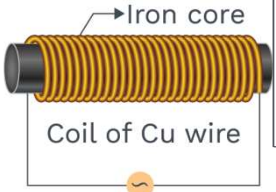
$V = V_R + V_L + V_C$

KEY POINTS

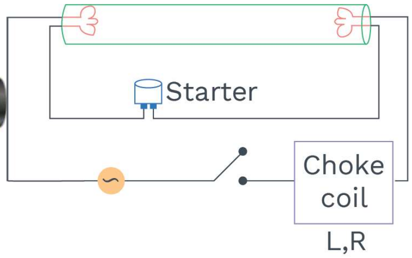
- Wattless current
- Choke coil
- Average power

Choke Coil:

- Choke coil is a device having high inductance and negligible resistance.
- It is used to control current in ac circuits and is used in fluorescent tubes.
- The power loss in a circuit containing choke coil is least.
- In a dc circuit current is reduced by means of a rheostat. This results in a loss of electrical energy I^2R per sec.



Choke coil



Application of choke coil

- It consists of a copper coil wound over a soft iron laminated core. This coil is put in series with the circuit in which current is to be reduced.
- Soft iron is used to improve inductance (L) of the circuit.
- The inductive reactance or effective opposition of the choke coil is given by $X_L = \omega L = 2\pi\nu L$
- For an ideal choke coil R = 0, no electric energy is wasted, i.e., average power P = 0.
- In actual practice choke coil is equivalent to a R–L circuit.
- Choke coil for different frequencies is made by using different substances in their core.
- For low frequency, L should be large thus iron core choke coil is used. For high frequency ac circuit, L should be small, so air cored choke coil is used.
- In electric circuit choke coil can be used only in ac circuits but not in dc circuits, because for dc frequency $\nu = 0$. Hence $X_L = 2\pi\nu L = 0$.
- Choke coil is work on the principle of wattless

Definitions

Choke coil (or ballast) is a device having high inductance and negligible resistance.

Rack your Brain

A transistor-oscillator using a resonant circuit with an inductor L and a capacitor C in series produces oscillation of frequency f. If L is doubled and C is changed to 4C, the frequency will be

(1) f/2 (2) 8f

(3) f/4 (4) $\frac{f}{2\sqrt{2}}$

current.

- The current in the circuit $I = \frac{E}{Z}$ with $Z = \sqrt{(R)^2 + (\omega L)^2}$
- The power loss in the coke

$P_{av} = V_{rms} I_{rms} \cos\phi \rightarrow 0$

as
$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$
$$= \frac{R}{\omega L} \rightarrow 0 \{\text{since } \omega L >> R\}$$

Ex. An ideal choke coil takes a current of 8 amperes when connected to an AC supply of 100 volt and 50 Hz. A pure resistance under the same conditions takes a current of 10 amperes. If now are connected to an AC supply of 150 volts and 40 Hz. then the current in a series combination of the above resistor and inductor is

Sol. For pure inductor,

$$X_L = \frac{E_0}{I_V} = \frac{100}{8} = \frac{25}{2}\Omega$$

$$\omega L = \frac{25}{2}; L = \frac{25}{2\omega} = \frac{25}{2 \times 2\pi \times 50} = \frac{1}{8\pi} H$$

$$R = \frac{V}{I} = \frac{100}{10} = 10\,\Omega$$

For the combination, the supply is 150 v, 40 Hz.

$$\therefore X_L = \omega L = 2\pi \times 40 \times \frac{1}{8\pi} = 10\,\Omega$$

$$Z = \sqrt{X_L^2 + R^2} = \sqrt{10^2 + 10^2} = 10\sqrt{2} \text{ ohm}$$

$$I_V = \frac{E_V}{Z} = \frac{150}{10\sqrt{2}} A = \frac{15}{\sqrt{2}} A$$

Ex. An electric bulb has a rated power of 50W at 100 V. If it is used on an AC source of 200 V, 50 Hz, a choke has to be used in series with it. This choke should have an inductance of

Sol. Here, P = 50 W, V = 100 volt

$$I = \frac{P}{V} = \frac{50}{100} = 0.5A, R = \frac{V}{I} = \frac{100}{0.5} = 200\,\Omega$$

Let L be the inductance of the choke coil.

$$\therefore I_V = \frac{E_V}{Z} = \text{or } Z = \frac{E_V}{I_V} = \frac{200}{0.5} = 400\,\Omega$$

Now $X_L = \sqrt{Z^2 - R^2} = \sqrt{400^2 - 200^2}$

$$\omega L = 100 \times 2\sqrt{3}$$

$$L = \frac{200\sqrt{3}}{\omega} = \frac{200\sqrt{3}}{2\pi\nu}$$

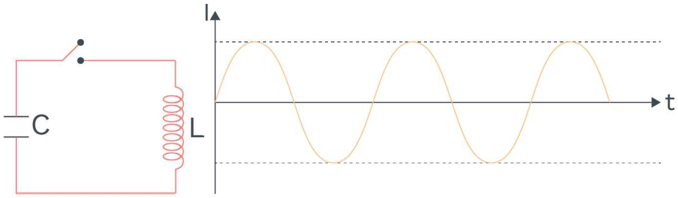
$$= \frac{200\sqrt{3}}{100\pi} = \frac{2 \times 1.732}{3.14} = 1.1H$$

LC Oscillation

The oscillation of energy between capacitor (electric field energy) and inductor (magnetic field energy) is called LC Oscillation.

1. Undamped oscillation

When the circuit has no resistance, the energy taken once from the source and given to capacitor keeps on oscillating between C and L then the oscillation produced will be of constant amplitude. These are called undamped oscillation.



After switch is closed

$$\frac{Q}{C} + L\frac{dI}{dt} = 0 \Rightarrow \frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$$

Definitions

LC Oscillation

The oscillation of energy between capacitor (electric field energy) and inductor (magnetic field energy) is called LC Oscillation

Concept Reminder

Frequency of LC oscillation is

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$\Rightarrow \frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$$

By comparing with standard equation of free oscillation $\left[\frac{d^2x}{dt^2} + \omega^2 x = 0\right]$

$$\omega^2 = \frac{1}{LC}$$

Frequency of oscillation $f = \frac{1}{2\pi\sqrt{LC}}$

Charge varies sinusoidally with time $q = q_m \cos \omega t$

current also varies periodically with t

$$I = \frac{dq}{dt} = q_m \omega \cos\left(\omega t + \frac{\pi}{2}\right)$$

If initial charge on capacitor is q_m then electrical energy stored in capacitor is

$$U_E = \frac{1}{2}\frac{q_m^2}{C}$$

At t = 0 switch is closed, capacitor starts to discharge.

As the capacitor is fully discharged, the total electrical energy is stored in the inductor in the form of magnetic energy.

$U_B = \frac{1}{2}LI_m^2$ where I_m = max. current

$$(U_{max})_{EPE} = (U_{max})_{MPE}$$

$$\Rightarrow \frac{1}{2}\frac{q_m^2}{C} = \frac{1}{2}LI_m^2$$

2. Damped oscillation

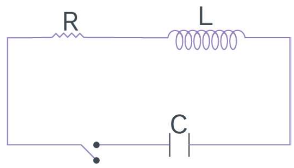
Practically, a circuit cannot be entirely resistance less, so some part of energy is lost in resistance and amplitude of oscillation goes on decreasing. These are called damped oscillation.

KEY POINTS

- LC oscillation
- Damped oscillation
- Undamped oscillation

Rack your Brain

A condenser of capacity C is charged to a potential V_1. The plates of condenser are then connected to an ideal inductor of inductance L. Find current through inductor when potential across condenser reduces to V_2.



Angular frequency of oscillation $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$

frequency of oscillation $f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$

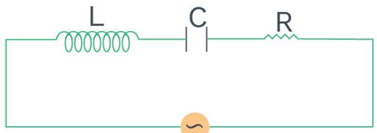
oscillation to be real if $\frac{1}{LC} - \frac{R^2}{4L^2} > 0$

Hence for oscillation to be real $\frac{1}{LC} > \frac{R^2}{4L^2}$

Important (Special) Key Points

- In damped oscillation amplitude of oscillation decreases exponentially with time.
- At $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}$ = energy stored is completely magnetic.
- At $t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}$ = energy is shared equally between L and C
- Phase difference between charge and current is $\frac{\pi}{2}$ (when charge is maximum, current is minimum and vice versa).
- Comparison of Damped Mechanical & electrical systems

(I) Series LCR circuit:



Concept Reminder

- At $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}$ = energy stored is completely magnetic.
- At $t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}$ = energy is shared equally between L and C

Concept Reminder

Every inductor has some resistance. The effect of this resistance is to introduce damping effect on charge.

$$\frac{d^2q}{dt^2}+\frac{R}{L}\frac{dq}{dt}+\frac{1}{LC}q=\frac{V}{L}$$

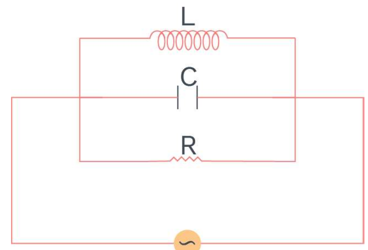
compare with mechanical damped system equation

$$\frac{d^2x}{dt^2}+\frac{b}{m}\frac{dx}{dt}+\frac{k}{m}x=\frac{F}{m}$$

where b = damping coefficient.

Mechanical System	Electrical Systems (Series RLC)
Displacement (x)	Charge (q)
Driving force (F)	Driving voltage (V)
Kinetic energy $\left(\frac{1}{2}mv^2\right)$	Electromagnetic energy of moving charge $\frac{1}{2}L\left(\frac{dq}{dt}\right)^2=\frac{1}{2}Li^2$
Potential energy $\frac{1}{2}kx^2$	Energy of static charge $\frac{q^2}{2C}$
Mass (m)	L
Power P =Fv	Power P=VI
Damping (b)	Resistance (R)
Spring constant (k)	1/C
Velocity (v)	Current (i)
Acceleration $\left(\frac{d^2x}{dt^2}\right)$	Rate of change of current $\left(\frac{di}{dt}\right)$

(II) Parallel LCR circuit: In this case



$V=V_0\sin\omega t$

$$I=I_L+I_C+I_R=\frac{\phi}{L}+\frac{d}{dt}C\left(\frac{d\phi}{dt}\right)+\frac{1}{R}\frac{d\phi}{dt}$$

$$\Rightarrow \frac{d^2\phi}{dt^2} + \frac{1}{RC}\frac{d\phi}{dt} + \frac{1}{LC}\phi = \frac{I}{C}$$

Displacement (x)	Flux linkage (ϕ)
Velocity $\left(\frac{dx}{dt}\right)$	Voltage $\left(V = \frac{d\phi}{dt}\right)$
Mass (m)	Capacitance (C)
Spring constant (k)	Reciprocal Inductance (1/L)
Damping coefficient (b)	Reciprocal resistance (1/R)
Driving force (F)	Driving force (F)
Potential energy $\frac{1}{2}kx^2$	$\frac{1}{2}\frac{\phi^2}{L}$
Kinetic energy $\frac{1}{2}mv^2$	$\frac{1}{2}CV^2$

Ex. In LCR oscillation circuit resistance is $10\,\Omega$ and inductive reactance at resonance condition is $1k\Omega$. After how many oscillations peak value of current will fall to $\left(\frac{1}{e}\right)$ times maximum value of peak current.

Sol. From equation damped oscillator

$$x = \left[Ae^{\frac{-b}{2m}t}\right]\sin\omega t$$

From comparison

$$I = \left[I_0 e^{\frac{-R}{2L}t}\right]\cos\omega t \Rightarrow I_0 e^{\frac{-R}{2L}t} = \frac{I_0}{e} \Rightarrow t = \frac{2L}{R}$$

Phase displacement $\phi = \omega t = \frac{2\omega L}{R} = \frac{2x_L}{R}$

$$2\pi n = \frac{2x_L}{R}$$

$$\boxed{n = \frac{x_L}{\pi R} = \frac{100}{\pi}}$$

Ex. An LC circuit contains a 20mH inductor and a 50 F capacitor with an initial charge of 10mC. The resistance of the circuit is negligible. Let the instant the circuit is closed to be t = 0 time.

(a) Calculate is the total energy stored initially.

(b) Find out the natural frequency of the circuit.

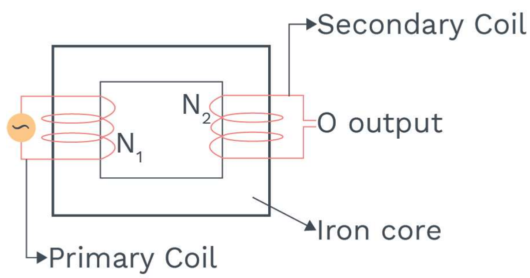
Sol. (a) $U_E = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2} \times \frac{(10 \times 10^{-3})^2}{50 \times 10^{-6}} = 1.0\,J$

(b) $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20 \times 10^{-3} \times 50 \times 10^{-6}}}$

$= 10^3\,\text{rad / sec}$

$\Rightarrow \quad f = 159\,\text{Hz}$

Transformer



- A transformer works on the principle of mutual induction.
- It is a static device that is used to increase or decrease the voltage in an AC circuit.
- On a laminated iron core two insulated copper coils called primary and secondary are wound.
- Primary is connected to an alternating source of emf, by mutual induction, an emf is induced in the secondary.

Concept Reminder

Transformers are based upon mutual induction which transform an alternating voltage from one to another of greater or smaller value.

Voltage Ratio:

- If V_1 and V_2 are the primary and secondary voltages in a transformer, N_1 and N_2 are the number of turns in the primary and secondary coils of the transformer, then $\frac{V_1}{V_2} = \frac{N_1}{N_2}$.

- In a transformer the voltage per turn is the same in primary and secondary coils.
- The ratio N_2/N_1 is called transformation ratio.
- The voltage ratio is the same as the ratio of the number of turns on the two coils.

Current RATIO:

- If the primary and secondary currents are I_1 and I_2 respectively, then for ideal transformer $\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1}$.
- In an ideal transformer the ampere turns are the same in primary and secondary coils.
- If $N_S > N_P$ voltage is stepped up, then the transformer is called step - up transformer.
- If $N_S < N_P$ voltage is stepped down, then the transformer is called step - down transformer.
- In step - up transformer, $V_S > V_P$ and $I_S < I_P$.
- In step - down transformer, $V_S < V_P$ and $I_S > I_P$.
- Frequency of input a.c is equal to frequency of output a.c.
- Transformation of voltage, is not possible with d.c.

Efficiency Of Transformer (η):-

Efficiency is explained by the ratio of output power to input power.

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}}$$

$$\text{i.e., } \eta\% = \frac{P_{out}}{P_{in}} \times 100 = \frac{V_s i_s}{V_P i_P} \times 100$$

- For an ideal transformer $P_{out} = P_{in}$ so $\eta = 100\%$

(But efficiency of practical transformer lies between 70% - 90 %)

KEY POINTS

- Transformer
- Voltage ratio
- Current ratio
- Step-up transformer
- Step-down transformer

Concept Reminder

- In step - up transformer, $V_S > V_P$ and $I_S < I_P$
- In step - down transformer, $V_S < V_P$ and $I_S > I_P$

Definitions

Efficiency is defined as the ratio of output power and input power.

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}}$$

For practical transformer $P_{in} = P_{out} + P_{losses}$

So $\eta = \frac{P_{out}}{P_{in}} \times 100$

$$= \frac{P_{out}}{(P_{out} + P_L)} \times 100 = \frac{(P_{in} - P_L)}{P_{in}} \times 100$$

- In an ideal transformer the input power is equal to th

The efficiency of an ideal transformer is 100%.

Rack your Brain

A 220 Volt input is supplied to transformer. The output circuit draws a current of 2 amperes at 440 volts. If the efficiency of transformer is 80%, find current drawn by primary winding of the transformer.

Losses In A Transformer

- The losses in a transformer are divided into two types. They are copper losses and iron losses.
- The loss of energy that occurs in the copper coils of the transformer (i.e., primary and secondary coils) is called 'copper losses'. These are nothing but joule heating losses where electrical energy is converted into heat energy.
  The loss of energy that occurs in the iron core of the transformer (i.e., hysteresis loss and eddy current loss) is called 'iron losses'.

Minimizing The Losses In A Transformer:

- The core of a transformer is laminated and each lamination is coated with a paint of insulation to reduce the 'eddy current' losses.
- By choosing a material with narrow 'hysteresis loop' for the core, the hysteresis losses are minimized.

Uses of Transformer:-

A transformer is used in ac operations, e.g

1. In voltage regulators for refrigerator, TV, computer, air conditioner etc.
2. In the induction furnaces.
3. For welding purposes step down transformer is used.
4. In the transmission line of ac over long distance.
5. Step down and step-up transformers are used in electrical power distribution.
6. Audio Frequency transformers are used in television, radiography, radio, telephone etc.
7. Radio frequency transformers are used in commonly radio communication.

Ex. A transformer having efficiency 90% is working on 100 V and at 2.0 kW power. If the value of current in the secondary coil is 5A, calculate(i) the current in the primary coil and (ii) voltage across the secondary coil.

Sol. Here $\eta = 90\% = \frac{9}{10}, I_s = 5A, E_p = 100\ V$

(i) $E_pI_p = 2kW = 2000\ W$

$$I_p = \frac{2000}{E_p} \text{ or } I_p = \frac{2000}{100} = 20\ A$$

(ii) $$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{E_sI_s}{E_pI_p}$$

or $E_sI_s = \eta \times E_pI_p$

$$= \frac{9}{10} \times 2000 = 1800\ W$$

$$\therefore \quad E_s = \frac{1800}{I_s} = \frac{1800}{5} = 360 \text{ volt}$$

Concept Reminder

- A step-up transformer converts low ac voltage to high ac voltage but reduces the current.
- A step-down transformer converts high ac voltage to a low ac voltage but increases the current.

Ex. A step-up transformer operates on a 230V line and a load current of 2 ampere. The ratio of the primary and secondary windings is 1 : 25. Find out the current in the primary coil?

Sol. Using the relation $\frac{N_P}{N_S} = \frac{I_S}{I_P}; I_P = \frac{N_SI_S}{N_P}$

Here $\frac{N_p}{N_s} = 1/25$ (or) $\frac{N_s}{N_p} = 25/1 = 25$ and $I_S = 2A$

Current in primary, $I_P = 25 \times 2 = 50A$

Skin Effect:

- A direct current flows uniformly throughout the cross section of the conductor.
- An alternating current, on the other hand, flows mainly along the surface of the conductor. This effect is known as skin effect.
- When alternating current flows through a electric conductor, the flux changes in the inner part of the conductor are higher.
- Therefore, the inductance of the inner part is higher than that of the outer part. Higher the frequency of alternating current, more is the skin effect.
- The depth upto which ac current flows through a wire is called skin depth (δ)

$$R = \frac{V_R^2}{P_R} \Rightarrow R \propto V_R^2$$

(V_R = rated voltage, P_R = rated power)

Concept Reminder

- A direct flow uniformly throughout the cross section of the conductor.
- An alternating current, on the other hand, flows mainly along the surface of the conductor. This effect is known as skin effect.

Examples

Q1 **A circuit containing a 0.1 H inductor and a 500 μF capacitor in series is connected to a 230 volt, 100/π Hz supply. The value of resistance of the electric circuit is negligible.**

(a) Find out the current amplitude and rms values.

(b) Obtain the rms values of potential drops across each element.

(c) What is the average power transferred to the inductor?

(d) Calculate the average power transferred to the capacitor?

(e) What is the total average power absorbed by the circuit? ['Average' implies average over one cycle.]

Sol: Given that

$L = 0.1\ H$

$C = 500 \times 10^{-6}\ F$

$$V_{rms} = 230 volt, f = \frac{100}{\pi} Hz$$

(a) $I_{rms} = \frac{V_{rms}}{Z} = \frac{230}{Z}$

Where $Z = \omega L - \frac{1}{\omega c} = 2\pi f L - \frac{1}{2\pi f c} = 20 - 10 = 10$

and $I_0 = \sqrt{2} \times I_{rms} = 23\sqrt{2}$

(b) $V_L = I_{rms}(\omega L)\ V_C = I_{rms}\left(\frac{1}{\omega c}\right)$

(c) $< P_L > = I_{rms} V_{rms} \cos\phi$ here $\phi = 90°$

So $< P > = 0$

(d) $< P_C > = 0$

(e) $< P_{Net} > = 0.$

Q2 **A series LCR circuit with L = 0.125/π H, C = 500/π nF, R = 23 Ω is connected to a 230 V variable frequency supply.**

(a) What is the source frequency for which current amplitude is maximum? Obtain this maximum value.

(b) What is the source frequency for which average power absorbed by the circuit is maximum? Obtain the value of this maximum power.

(c) For what reactance of the circuit, the power transferred to the circuit is half the power at resonance? What is the current amplitude at this reactance?

(d) If ω is the angular frequency at which the power consumed in the circuit is half the power at resonance, write an expression for ω.

(e) What is the Q-factor (Quality factor) of the given circuit?

Sol: Given that

$$L = 0.125/\pi, C = \frac{500}{\pi} \times 10^{-9} F, R = 23, V_{rms} = 230 \text{ volt}$$

(a) $f_R = \frac{1}{2\pi}\frac{1}{\sqrt{CL}} = 2000\text{Hz}, \quad I_{rms} = \left(\frac{230}{R}\right) = 10 \text{ Amp.}$

(b) at Resonance $f_R = 2000, P = (10)^2(23) = 2300$ watt.

(c) $$\frac{P_{max}}{2^2_{rms}\dfrac{I^2_{max}}{2^2_{rms}\dfrac{I_{rms}}{\sqrt{2}}_{max}}}$$

$$\frac{V_{rms}}{\sqrt{R^2 + x^2}} = \frac{I_{rms}}{\sqrt{2}} = \frac{230}{\sqrt{(23)^2 + x^2}} = \frac{10}{\sqrt{2}} x = 23\Omega$$

$$I_0 = I'_{rms} \times \sqrt{2} = 10 \text{ Amp.}$$

(d) $\omega L - \frac{1}{\omega C} = +2\Omega$

(e) $Q = \frac{\omega_r L}{R} = \frac{2 \times \pi \times 2000 \times \frac{0.125}{\pi}}{23} = \frac{500}{23}.$

Q3 **An LCR circuit has L = 10 mH, R = 150 Ω and C = 1 μF connected in series to a source of 150 $\sqrt{2}$ cos ωt volt. At a frequency that is 50% of the resonant frequency, calculate.**

(a) the net reactance of the circuit.

(b) the current amplitude and the average power dissipated per cycle.

Sol: $f = \frac{50}{100} \times f_r \qquad f = \frac{1}{2} \times \frac{1}{2\pi\sqrt{LC}} \qquad \omega = \frac{1}{2\sqrt{LC}}$

(a) $X = \left|\omega L - \frac{1}{\omega C}\right| = 150$

(b) $I_0 = \frac{V_0}{Z} = \frac{150\sqrt{2}}{\sqrt{R^2 + x^2}}, \qquad P_{cw} = I_{rms}V_{rms}\cos\phi = \frac{I_0V_0}{2} \times \frac{R}{Z}$

Q4 **A 2000 Hz, 20 volt source is connected to a resistance of 20 ohm, an inductance of 0.125/π H and a capacitance of 500/π nF all in series combination. Calculate the time when the resistance (thermal capacity = 100 joule/°C) will get heated by 10° C. (Assume no loss of heat).**

Sol:

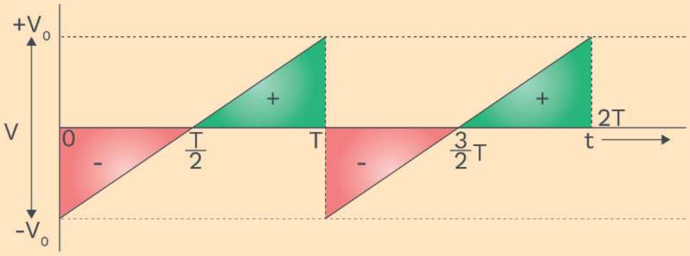
$$\text{Power }(P) = I_{rms}^2 RP = \left[\frac{20}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}}\right]^2 \times R$$

Where $\omega = 2 \times \pi \times 2000$

$$\Delta H = (ms)\Delta\theta = P(\Delta t) \Rightarrow \Delta t = \frac{(ms)\Delta\theta}{P} = \frac{100 \times 10}{P} = 50 \text{ sec.}$$

Q5

Find the rms value for the saw-tooth voltage of peak value V_0 from t = 0 to t = 2T as shown in figure.



Sol:

General equation of V

$$V = \frac{V_0}{T/2}t - V_0 = \frac{2V_0}{T}t - V_0$$

$$V_{rms} = \left[\frac{\int_0^T V^2 dt}{T}\right]^{\frac{1}{2}} = \left[\frac{\int_0^T \left(\frac{2V_0}{T}t - V_0\right)^2 dt}{T}\right]^{\frac{1}{2}} = \frac{V_0}{\sqrt{3}}.$$

Q6

A current of '4' A flows in a coil when connected to a 12 V d.c. source. If the same coil is connect to a 12 voltage, 50 rad/s, AC source, a current of 2.4 ampere flows in the circuit. Calculate the inductance of the coil. Also, find out the power developed in the circuit if a 2500 μF condenser is connected in

Sol:

series with coil.

Given that

$$R = \frac{V}{I} = \frac{12}{4} = 3\Omega$$

$$Z = \frac{12}{2.4} = 5\Omega$$

$$Z^2 = R^2 + (\omega L)^2$$

$$25 = 9 + (50 \times L)^2 \Rightarrow L = 0.08H$$

Now for LCR circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{3^2 + \left(50 \times 0.08 - \frac{1}{50 \times 2500 \times 10^{-6}}\right)^2} = 5\Omega$$

$$\text{So,power} = \left(\frac{12}{Z}\right)^2 \times R = \left(\frac{12}{5}\right)^2 \times 3 = 17.28 \text{ watt.}$$

Q7 **A series LCR circuit containing a resistance of 120 ohm has angular resonance frequency 4×10^5 rad s^{-1}. At resonance, the voltage across resistance and inductance are 60 V and 40 V respectively. Find the values of L and C. At what frequency the current in the circuit lags the voltage by 45°?**

Sol:

$$\omega = \frac{1}{\sqrt{LC}} \quad \text{...(i)}$$

$$\frac{V}{R} \times (\omega L) = 40 \quad \text{...(ii)}$$

$$\frac{V}{R} \times R = V = 60 \quad \text{...(iii)}$$

$$\omega L = \frac{40}{60} R \quad \text{...(iv)}$$

$$\frac{1}{\omega C} = \frac{40R}{60} \quad \text{...(v)}$$

From equation (i), (iv) & (v)

$$L = 2 \times 10^{-4} \text{H}; C = \frac{1}{32} \mu\text{F}$$

From phasor diagram, $V_L - V_C = V_R$

$$\Rightarrow X_L - V_C = R$$

$$\Rightarrow \omega L - \frac{1}{\omega_C} = R$$

$$\Rightarrow \omega^2 L - \omega R - \frac{1}{C} = 0$$

$$\Rightarrow \omega = \frac{R + \sqrt{R^2 + 4L/C}}{2L} = 8 \times 10^5 \text{ rad/sec.}$$

Q8 **A coil Q and a box P are connected in series combination with an AC source of variable frequency. The EMF of source is constant at '10' V. Box P contains a capacitance of 1 μF in series with a resistance of 32 Ω. Coil Q has a self-inductance 4.9 mH and a resistance of 68 Ω. The frequency is adjusted therefore the maximum current flows in Q and P. Find the impedance of P and Q at this frequency. Also find the voltage across P and Q respectively.**

Sol:

Here L = 4.9 mH

$C = 1\mu F$

$R = 68 + 32 = 100\ \Omega$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{7} \times 10^5 \text{ rad/sec}$$

$$Z_p = \sqrt{(32)^2 + (1/\omega C)^2} = 76.96\Omega$$

$$Z_Q = \sqrt{(68)^2 + (\omega L)^2} = 97.59\Omega$$

$$\text{Total impedance } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{100^2 + \left(\frac{10^5}{7} \times 4.9 \times 10^{-3} - \frac{7 \times 10^{-5}}{10^{-6}}\right)^2} = 100\Omega$$

$$I_{rms} = \frac{10}{100} A$$

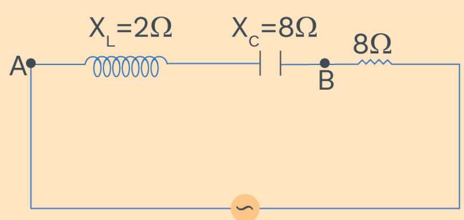
$$V_P = Z_P \times I_{rms} = 7.7V$$

$$V_Q = Z_Q \times I_{rms} = 9.8V.$$

Q9 **An inductor ($x_L = 2\Omega$) a capacitor ($x_c = 8\Omega$) and a resistance (8Ω) is connected in series with an ac source. The voltage output of AC source is given by $v = 10 \cos 100\pi t$.**

(a) Find the impedance of the circuit.

(b) Find the instantaneous p.d. between A and B, when it is half of the voltage output from source at that instant.

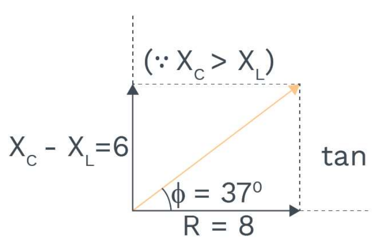


Sol: (a) impedance of circuit $Z = \sqrt{R^2 + (X_C - X_L)^2}$

$$Z = \sqrt{8^2 + (8-2)^2} = 10\Omega$$

(b) The current leads in phase by $\phi = 37°$

$$\therefore \quad i = \frac{10\cos(100\pi t + 37°)}{Z} = \cos(100\pi t + 37°)$$



$$\tan\phi = \frac{X_C - X_L}{R} = \frac{3}{4}$$

The instantaneous potential difference across AB is

$$= I_m (X_C - X_L)\cos(100\pi t + 37° - 90°)$$

$= 6 \cos (100 \pi t - 53°)$

The instantaneous potential difference across AB is half of source voltage.

$$\Rightarrow \quad 6\cos(100\pi t - 53°) = 5\cos 100\pi t$$

Solving we get $\cos 100\pi t = \frac{1}{\sqrt{1+(7/24)^2}} = \frac{24}{25}$

$\therefore$ Instantaneous potential difference = $5 \times \frac{24}{25} = \frac{24}{5}$ volts.

Q10 **In a series LCR circuit with an ac source of 50 V, R = 300 Ω, frequency $v = \frac{50}{\pi}$ Hz. The average field energy (electric) stored in the capacitor and average magnetic energy stored in the coil are 25 mJ and 5 mJ respectively. The RMS current value in the circuit is 0.10 A. then find:**

(a) Capacitance (c) of capacitor

(b) Inductance (L) of inductor.

(c) The sum of rms potential difference across the three elements.

Sol: Average electric field energy = $\left(\frac{1}{2}CV_{rms}^2\right) = 25 \times 10^{-3}$ J

$\therefore \frac{1}{2} \times c.I_{rms}^2 \times \frac{1}{2\pi^2 v^2 c^2} = 25 \times 10^{-3}$ J

$\therefore$ C = 20 μF

Average magnetic energy $\left(\frac{1}{2}LI_{rms}^2\right) = 5 \times 10^{-3}$

$\therefore L = \frac{2 \times 5 \times 10^{-3}}{(0.10)^2}$

$\Rightarrow$ L = 1

$V_R = I_{rms}R \qquad V_C = I_{rms}X_C \qquad V_L = I_{rms} \times \omega L$

$= 0.10 \times 300 = (0.10) \times \frac{1}{2\pi\left(\frac{50}{\pi}\right) \times 20 \times 10^{-6}} = 0.10 \times 2\pi \times \frac{50}{\pi}(1)$

$= V_R = 30$ V $\qquad V_C = 50$ V $\qquad V_L = 10$ V

Q11 **An inductor 20 × 10^{-3} Henry, a capacitor 100 μF and a resistor 50 Ω are connected in series across a source of emf V = 10 sin 314 t. Find out the energy dissipated in the circuit in '20' minutes. If resistance is removed from the circuit and the value of inductance is doubled, then find the variation of current with time (t in second) in the new circuit.**

Sol: Given that

$L = 20 \times 10^{-3}$ H

$C = 100 \times 10^{-6}$ F

$R = 50\ \Omega$ $\qquad \omega = 314$

$V = 10 \sin 314\, t$ $\qquad \Delta t = 20 \times 60$ second

(i) $\Delta H = I_{rms}^2 R(\Delta t) = \left(\dfrac{V_0}{\sqrt{2Z}}\right)^2 R \times \Delta t$

here $z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2} = \sqrt{(50)^2 + (6.28 - 31.8)^2} = 56.15\Omega$.

(ii) $I = I_0 \sin(314t + \pi/2)$ $\qquad$ where $I_0 = \dfrac{V_0}{\left(\omega(2L) - \dfrac{1}{\omega C}\right)}$.

Q12 **The electric current in an AC circuit is given by I = I_0 sin ωt. What is the time taken by the current to change from its maximum value to the rms value ?**

Sol: $I_1 = I_0 \sin \omega\, t_1$

$I_0 = I_0 \sin \omega\, t_1$

$t_1 = \dfrac{\pi/2}{2\pi/T} = \dfrac{T}{4}$ and $I_2 = \dfrac{I_0}{\sqrt{2}} = I_0 \sin(\omega t_2)$

$t_2 = \dfrac{\pi/4}{2\pi/T} = \dfrac{T}{8}$

$\Delta t = t_1 - t_2 = \dfrac{T}{4} - \dfrac{T}{8} = \dfrac{T}{8}$.

Q13 **In a LR circuit discharging current is given by $I = I_0 e^{-t/\tau}$ where τ is the time constant of the circuit find the rms current for the period t = 0 to t = τ.**

Sol:

$$I_{rms}^2 = \frac{\int_0^{\tau}\left(I_0 e^{\frac{-t}{\tau}}\right)^2 .dt}{\tau} \Rightarrow I_{rms} = \frac{I_0}{e}\sqrt{\left(e^2 - 1\right)/2}\,.$$

Q14 **20 volts 5-watt lamp (Lamp to be treated as a resistor) is used on AC mains of 200 volts and $\frac{50}{\pi}\sqrt{11}$ c.p.s. Calculate the**

(i) Capacitance of the capacitor, or inductance of the inductor, to be put in series to run the lamp.

(ii) How much pure resistance should be included in place of the above device so that the lamp can run on its rated voltage.

(iii) which is more economical (the capacitor, the inductor or the resistor).

Sol:

(i) $\left[\frac{1}{\omega C}\times I_{rms}\right]^2 + [20]^2 = 200^2 \quad \Rightarrow C = 3.78\ \mu F$

or $\left[(\omega L)I_{rms}\right]^2 + (20)^2 = 200^2 \quad \Rightarrow L = 2.4\ H$

(ii) $I_{rms}\times R + 20 = 200 \quad \Rightarrow R = 720\Omega$

(iii) It will be more economical to use inductance or capacitance in series with the lamp to run it as it. It consumes no power while there would be dissipation of power when resistance is inserted in series with the lamp.

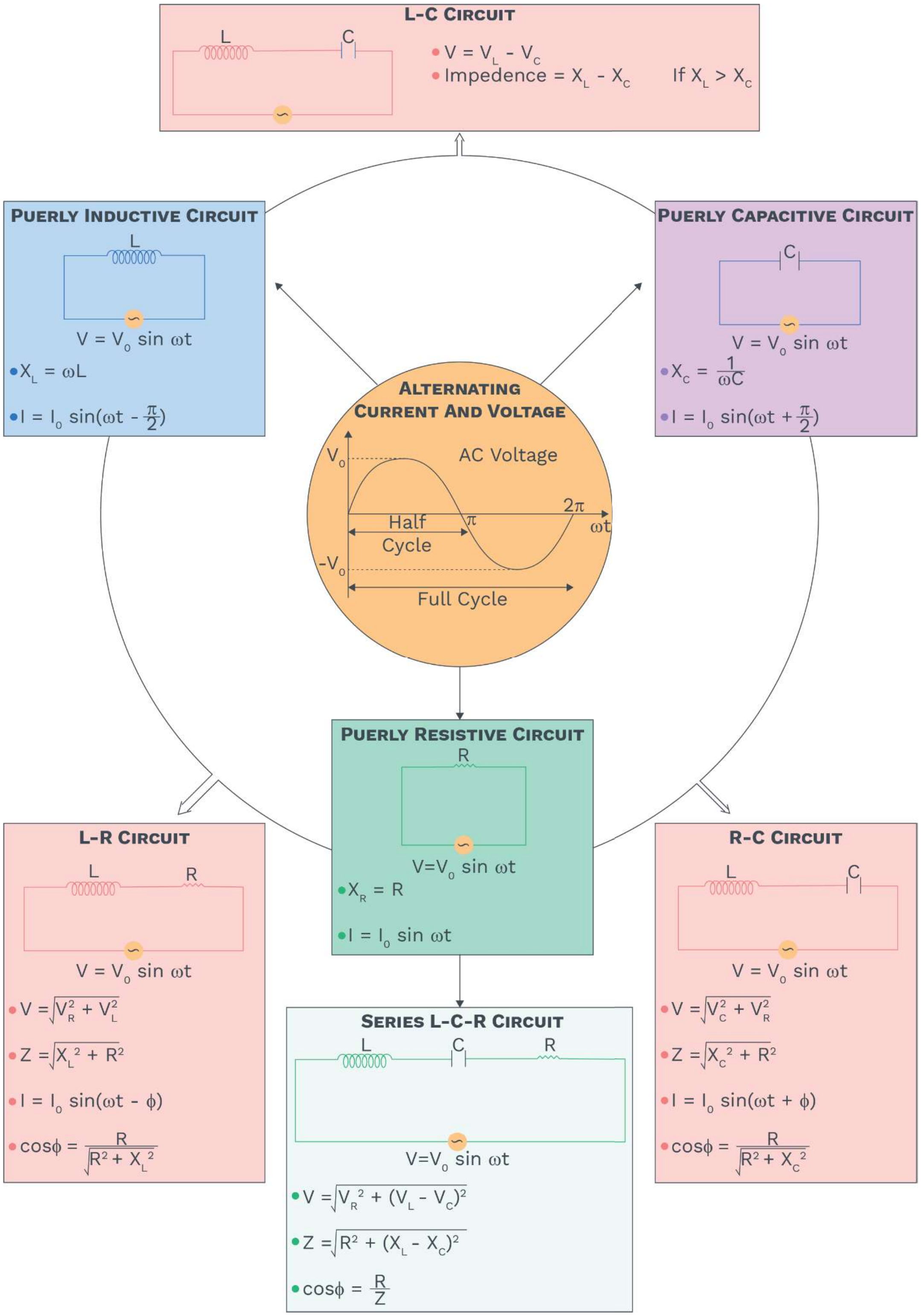
Q15 **A series circuit consists of a inductance, resistance and capacitance. The applied current and the voltage at any instant are given by**
E = 141.4 cos (5000 t – 10°)
and I = 5 cos (5000 t – 370°)
The inductance is 0.01 henry. Calculate the value of capacitance and resistance.

Sol: Here phase difference $\phi = 360°$ and $\omega = 5000$

at Resonance $C = \frac{1}{\omega^2 L} = \frac{1}{(5000)^2} \times \frac{1}{0.01} = 4$ microfarad

$R = \frac{V_0}{I_0} = \frac{141.4}{5} = 28.3$ ohm.

Mind Map


L-C Circuit
L
C
$V = V_L - V_C$
Impedence $= X_L - X_C$
If $X_L > X_C$
Purely Inductive Circuit
L
$V = V_0 \sin \omega t$
$X_L = \omega L$
$I = I_0 \sin(\omega t - \frac{\pi}{2})$
Purely Capacitive Circuit
C
$V = V_0 \sin \omega t$
$X_C = \frac{1}{\omega C}$
$I = I_0 \sin(\omega t + \frac{\pi}{2})$
Alternating Current And Voltage
V_0
AC Voltage
2π
Half Cycle
π
ωt
$-V_0$
Full Cycle
Purely Resistive Circuit
R
$V=V_0 \sin \omega t$
$X_R = R$
$I = I_0 \sin \omega t$
L-R Circuit
L
R
$V = V_0 \sin \omega t$
$V = \sqrt{V_R^2 + V_L^2}$
$Z = \sqrt{X_L^2 + R^2}$
$I = I_0 \sin(\omega t - \phi)$
$\cos\phi = \frac{R}{\sqrt{R^2 + X_L^2}}$
R-C Circuit
L
C
$V = V_0 \sin \omega t$
$V = \sqrt{V_C^2 + V_R^2}$
$Z = \sqrt{X_C^2 + R^2}$
$I = I_0 \sin(\omega t + \phi)$
$\cos\phi = \frac{R}{\sqrt{R^2 + X_C^2}}$
Series L-C-R Circuit
L
C
R
$V=V_0 \sin \omega t$
$V = \sqrt{V_R^2 + (V_L - V_C)^2}$
$Z = \sqrt{R^2 + (X_L - X_C)^2}$
$\cos\phi = \frac{R}{Z}$