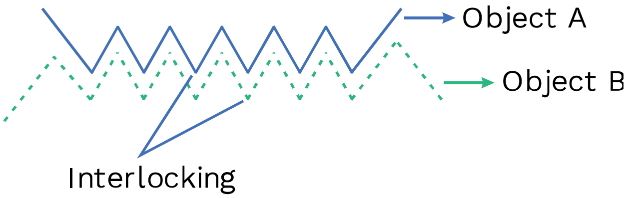
Friction

DISCLAIMER

Friction

FRICTION:

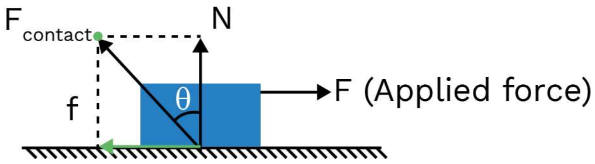
- It is that component of total contact force which acts parallel to the contact surface.
- The friction force always opposes the relative motion between the two contact surfaces.



> **Concept Reminder**
>
> Frictional force is a parallel component with surface of total contact force.

Friction depends on:

- Friction is proportional to the normal reaction acting on the body.
- The area covered by contact particles of contact surface (actual contact area), it does not depend on the area covered by the body (apparent contact area).
- Normal reaction (N); $f \propto N$



$F_{contact} = \sqrt{N^2 + f^2}$;

θ is angle of friction.

- Friction force is component of $F_{contact}$, which is parallel to surface.
- Friction angle (θ) is angle between normal force and net contact force.

$f \propto N$, $\boxed{\tan\theta = \frac{f}{N}}$ = angle of friction

$\boxed{f = \mu N}$

- μ = coefficient of friction which depends on nature of surface. If surface is more rough then value of μ is more.

  minimum value of friction, f = 0 (when applied force is zero)

  maximum value of friction, $f = \mu N$

> **Concept Reminder**
>
> Frictional force is not always equal to μN.
> Static friction has self adjusting value.

> **Key Points**
>
> - Contact force
> - Normal reaction
> - Friction angle
> - Static friction
> - Coefficient of friction

> **Concept Reminder**
>
> Limiting friction > Kinetic friction > Rolling friction

Therefore

$$N \le F_{contact} \le N\sqrt{1+\mu^2}$$

Their are 4 types of friction:

Static friction: Oppose tendency of relative motion. $f_s = \mu_s N$, μ_s = coefficient of static friction.

Kinetic/dynamic/sliding friction: Oppose relative sliding and has a constant value.

$f_k = \mu_k N$, μ_k = coefficient of kinetic friction.

Rolling friction:-

When an object rolls on the surface of another object friction developed is called as rolling friction.

$$f_r = \mu_R N$$

μ_R = Coefficient of rolling friction

$$\boxed{\mu_s > \mu_k > \mu_R}$$

- The values of μ_s, μ_k and μ_R depend on the nature of both the surfaces in contact and is not dependent of the area of contact.
- The values of μ depend on material of the surfaces in contact.
- μ_R, μ_s and μ_k are dimensionless and unitless.

Static friction:

- It is the force which is effective before relative motion. Static friction force is self adjusting force (in direction and magnitude)
Magnitude of static friction is

$$0 \le f_s \le \mu_s N$$

The maximum value of static friction is limiting friction.

Limiting friction:

- Maximum value of static friction is limiting friction.

$$f_l \propto N; \;\; f_l = \mu_s N$$

- Direction of the limiting friction force is always

Rack your Brain

Which one of the following statements is incorrect?

(1) Rolling friction is smaller than sliding friction

(2) Limiting value of static friction is directly proportional to normal reaction

(3) Frictional force opposes the relative motion

(4) Coefficient of sliding friction has dimensions of length

Key Points

- Rolling friction
- Limiting friction
- Kinetic friction

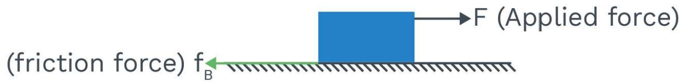
opposite to the direction in which the body is moving over the other.

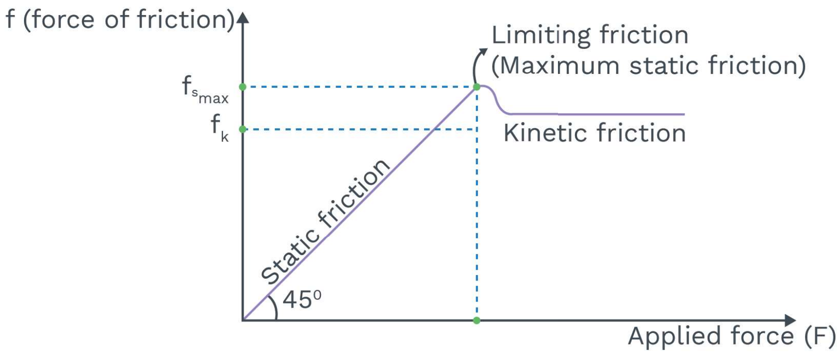
Kinetic Friction:

- If the body is in relative motion, the friction opposing its relative motion is called dynamic or kinetic friction. Its magnitude is slightly less than the limiting friction.
- Kinetic friction does not depend on type of motion of body such as accelerated motion, retarded motion or moving with constant velocity because it is a constant friction.

$f_k = \mu_k N$

Graph between applied force and force of friction:





Concept Reminder

Direction of friction force is such that it opposes the relative motion between surface and object.

Case-1	Case-2	Case-3
If $f_{lim} > F$	If $f_{lim} = F$	If $f_{lim} < F$
body at rest (a = 0)	body at rest (a = 0)	body in motion ($a \neq 0$)
$f_s = F$	$f_s = F = \mu_s N$	$f_k = \mu_k N$

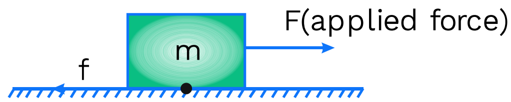
Note:

- If you are walking due east, then the friction on the feet is due east and the friction on the surface is due west.
- Car engine is connected to rear wheels. When the car is accelerated motion, direction of frictional force on the rear wheels will be in the direction of motion of car and on the front wheels in the opposite direction of motion.
- In cycling, the force exerted by rear wheels on the ground makes the friction force of friction to act on it in the forward direction on front wheel moving by itself experience force of friction in backward direction.

- If the pedalling cycle is accelerating on the horizontal surface, then the forward friction on the rear wheel is greater than the backward friction on the front wheel.
- When pedalling is stopped, the frictional force is in backward direction for both the wheels.

Motion on a horizontal rough surface:

- Consider a block of mass 'm' placed on a horizontal surface with normal reaction N.



Case-I: If applied force F = 0, then the force of friction is also zero.

Case-II: If applied force $F < (f_s)_{max}$, the block does not move and the force of friction is $f_s = F$.

Case-III: If applied force $F = (f_s)_{max}$ block just ready to slide and frictional force $(f_s)_{max} = F_l = \mu_s N$.

$F = \mu_s mg \ (\because N = mg)$; (at time t = 0)

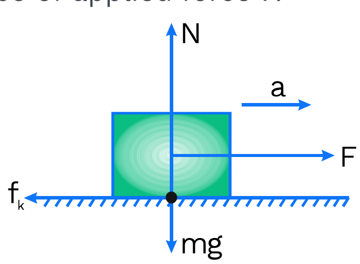
Case-IV: If the above applied force continues to act (t > 0) the block gets motion then static friction converts as in kinetic friction and block possesses acceleration.

$$a = \frac{F_{ext} - f_k}{m} = \frac{F_l - f_k}{m} = (\mu_s - \mu_k)g$$

Case-V: If the applied force is greater than limiting friction the body starts moving and gets acceleration.

$$a = \frac{F_{ext} - f_k}{m} \qquad \text{Here } F_{ext} > F_l$$

- If the block slides with an acceleration 'a' under the influence of applied force F.



$F_{net} = F - f_k$; $ma = F - f_k$

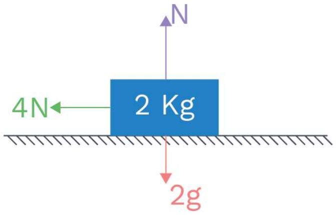
Concept Reminder

Value of static friction is not constant but value of kinetic friction is always constant.

$\therefore \quad a = \frac{F - f_k}{m} = \frac{F - \mu_k mg}{m} \qquad (f_k = \mu_k N = \mu_k mg)$

Ex. In the following condition find of friction force and acceleration if μ_s = 0.3 and μ_k = 0.2.

(A)



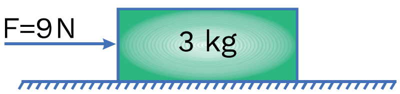
Sol. $f_{s_{max}} = \mu_s N \Rightarrow 0.3 \times 20 = 6\,N$

$F_{applied} < f_{s_{max}}$ (block does not move)

$\Rightarrow f_s = F_{applied} = 4N$

(ii) $a = 0$

(B)

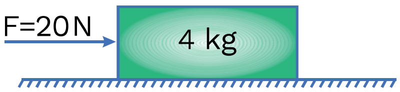


Sol. $f_{s_{max}} = \mu_s \times 3g = 0.3 \times 3g = 9N$

$\therefore$ $F_{applied} = f_{s_{max}}$ (block not accelerated)

(i) $F_{applied} = f_{s_{max}}$ (ii) $a = 0$

(C)



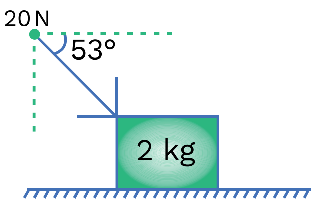
Sol. $f_{s_{max}} = \mu_s N = 0.3 \times 4g = 12\,N$

$\therefore$ $F_{applied} > f_{s_{max}}$ (block accelerated)

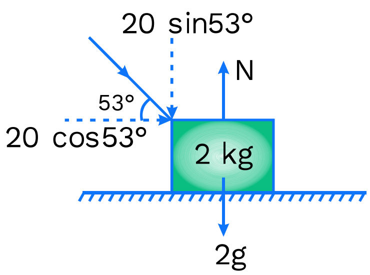
(i) $f_k = \mu_k N \Rightarrow 0.2 \times 4g = 8\,N$ (relative motion start than kinetic friction act)

(ii) $a = \frac{20 - 8}{4} = \frac{12}{4} = 3\,m/s^2$

(D)



Sol.



$$N = 2g + 20\sin 53^\circ$$

$$N = 2g + 20 \times \frac{4}{5}$$

$$N = 20 + 16 = 36\,N$$

$$f_{s_{max}} = \mu_s N = 0.3 \times 36 = 10.8\,N$$

$$F_{applied} = 20\cos 53^\circ = 20 \times \frac{3}{5} = 12\,N$$

$F_{applied} > f_{s_{max}}$ (block starts sliding then kinetic friction act)

$$f_k = \mu_k N = 0.2 \times 36 = 7.2\,N$$

$$a = \frac{F_{applied} - f_k}{m} = \frac{12 - 7.2}{2} = \frac{4.8}{2} = 2.4\ m/s^2.$$

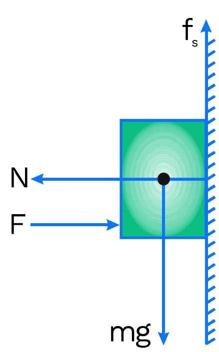
Concept Reminder

To move the block
$F_{applied}$ (parallel to surface) > $(f_s)_{max}$

Bodies in contact with vertical surfaces:-

- An object of mass 'm' is pressed against a wall without falling, by applying minimum horizontal force F. Then

$$\boxed{F = \frac{mg}{\mu_s}}$$



Concept Reminder

Limiting value of friction is $\mu_s N$, not $\mu_s mg$.
N is not always equal to mg.

As the body is in vertical equilibrium

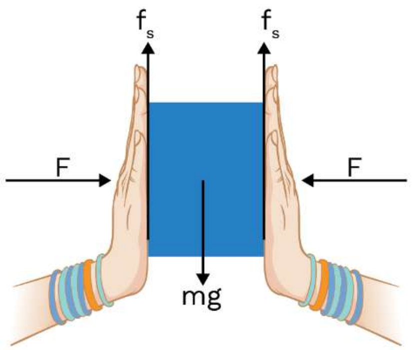
$f_s = mg; \quad \mu_s N = mg$

$\mu_s N = mg \ (\because \ N = F)$

$$\Rightarrow \quad F = \frac{mg}{\mu_s}$$

- An object is pressed between two hands without falling, by applying minimum horizontal force 'F' by each hand. Then

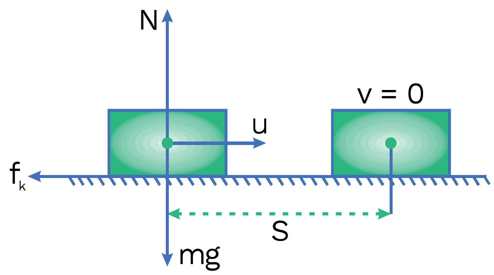
$$\boxed{F = \frac{mg}{2\mu_s}}$$



$W = 2f_s; \ mg = 2\mu_s F \qquad (f_s = \mu_s F)$

$$\Rightarrow \quad F = \frac{mg}{2\mu_s}$$

Sliding object on a horizontal rough surface coming to rest:-



$$f_k = \mu_k mg \quad \Rightarrow a = \frac{\mu_k mg}{m} = \mu_k g$$

Concept Reminder

For minimum force:

Net downward force should be equal to total value of limiting friction in upward direction.

(a) The retardation of the object is $a = \mu_k g$.

(b) Distance travelled by the object before coming to rest (Velocity = 0) is $S = \frac{u^2}{2\mu_k g}$.

(c) Time taken by the object to come to rest is $t = \frac{u}{\mu_k g}$.

Ex. Block of mass 'm' move with initial velocity 20 m/s on a rough horizontal surface, which coefficient of friction is 0.2. Then find out
(i) Retardation of block
(ii) Time after which block comes at rest
(iii) Distance travelled the block.

Sol. (i) $f_s = \mu mg = ma$

$$a = \frac{\mu mg}{m} = \mu g = 0.2 \times 10 = 2 \text{ m/s}^2$$

(ii) $v = u + at$

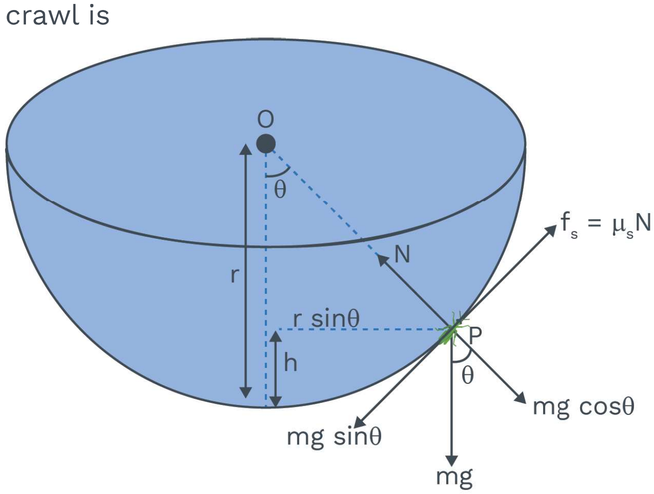
$$= u - \mu g \cdot t \Rightarrow t = \frac{u}{\mu g} = \frac{20}{0.2 \times 10} = 10 \text{ sec}$$

(iii) $v^2 = u^2 + 2as \Rightarrow 0 = u^2 - 2(\mu g) \cdot s$

$$\Rightarrow s = \frac{u^2}{2\mu g} = \frac{20 \times 20}{2 \times 0.2 \times 10} = 100\,m$$

Important Cases:-

- An insect is crawling in a bowl (hemispherical) of radius 'r'. Maximum height upto which it can crawl is



Definition

Distance travelled by object before coming to rest is known as **stopping distance**.
Time taken by block to come to rest is **stopping time**.

Key Points

- Stopping distance
- Stopping time

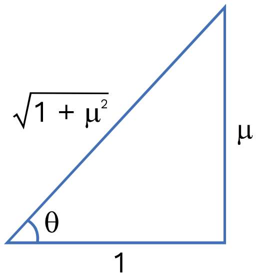
$$mg\sin\theta = \mu_s N \quad ...(i)$$

$$mg\cos\theta = N \quad ...(ii)$$

Solving equation (i) and (ii)

$$\tan\theta = \mu_s$$

$$\cos\theta = \frac{1}{\sqrt{\mu_s^2 + 1}}$$

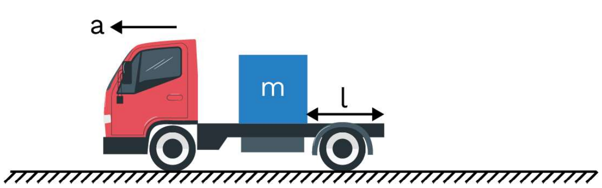


$$h = r - r\cos\theta$$

$$h = r(1 - \cos\theta) = r\left(1 - \frac{1}{\sqrt{\mu_s^2 + 1}}\right)$$

Maximum angular displacement upto which it can crawl is θ. Then $\mu_s = \tan\theta$.

- An object of mass 'm' is placed on rear horizontal surface of a truck moving along the horizontal with an acceleration 'a'. Then



$$F = \mu N \Rightarrow ma = \mu \cdot mg \Rightarrow a = \mu g$$

(1) The maximum acceleration 'a' of the truck for which block does not slide on the floor of the truck is $a = \mu g$.

(2) If $a < \mu_s g$ block does not slide and frictional force on the block is $f = \mu a$.

(3) If $a > \mu_s g$ block slips or slides on the floor the acceleration of the block relative to the truck is $a_1 = a - \mu_k g$.

(4) If 'l' is the distance of the block from rear side of the truck, time taken by the block to cover a distance 'l'.

$$t = \sqrt{\frac{2l}{a - \mu_k g}}$$

(5) Acceleration 'a' of the block relative to ground is $a = \mu_k g$.

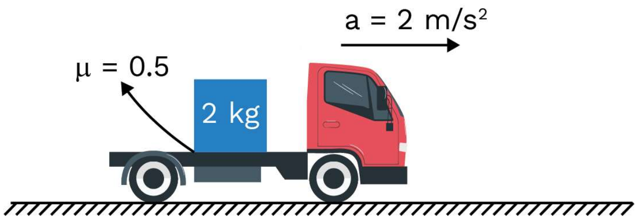
Concept Reminder

- Pseudo force = (mass of object) × (acceleration of system)
- If acts opposite to direction of motion of system.

Key Points

- Pseudo force

Ex. A block of mass 2 kg is placed on a horizontal surface of a truck, which is moving horizontally with acceleration 2 m/s². If coefficient of friction is 0.5, then find out friction acting on block.



Sol. $F_{pseudo} = ma = 2 \times 2 = 4\,N$

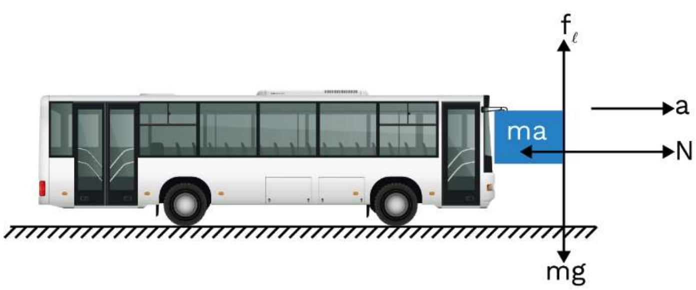
$f_{lim} = \mu_s N = 0.5 \times 20 = 10\,N$

$f_{lim} > F_{pseudo} \rightarrow$ block is at rest w.r.t. truck.

$f_r = 4\,N$

When body placed in contact with the front surface of accelerated vehicle:

- When a body of mass m' is placed in contact with the front face of the bus moving with acceleration 'a' then a pseudo force 'F_{pf}' acts on the body in a direction opposite to the direction of bus.



Under equilibrium,
$f_l = mg$; $N = ma$

$\mu_s N = mg \Rightarrow \mu_s ma = mg \Rightarrow a_{min} = \dfrac{g}{\mu_s}$

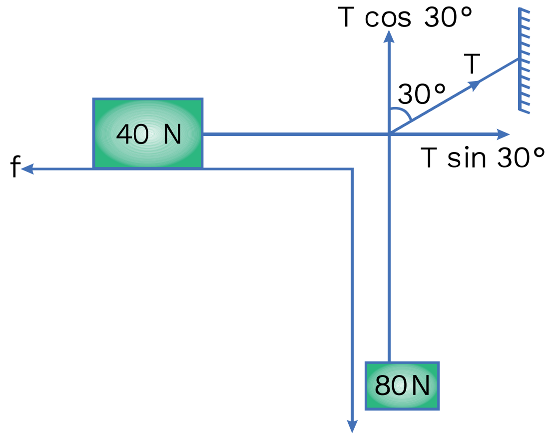
Rack your Brain

A block B is pushed momentarily along a horizontal surface with an initial velocity V. If μ is the coefficient of sliding friction between B and the surface, calculate the time after which block B will come to rest.

Concept Reminder

- $a_{min} = \dfrac{g}{\mu_s}$
- Minimum acceleration of vehicle so that block remains stationary with front face of vehicle.

Ex. A block on table shown in figure is just on the edge of slipping. Find the coefficient of static friction between the block and table.



Sol. $f = T\sin 30°$

$\mu mg = T\sin 30°$ ...(i)

$80 = T\cos 30°$ ...(ii)

$$\frac{T\sin 30°}{T\cos 30°} = \frac{\mu mg}{80};$$

$$\Rightarrow \tan 30° = \frac{\mu 40}{80}; \quad \frac{1}{\sqrt{3}} = \frac{\mu}{2}$$

$$\Rightarrow \mu = \frac{2}{\sqrt{3}} = 1.15$$

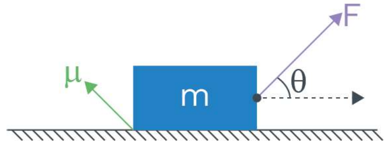
Ex. When a bus of mass 1000 kg is moving with a velocity of 20 ms^{-1} on a rough horizontal road, surface its engine is switched off then, how far does the bus move before it comes to rest (velocity = 0) if the coefficient of kinetic friction between the road surface and tyres of the bus is 0.75?

Sol. Here $v = 20\,ms^{-1}$; $\mu_k = 0.75$, $g = 10\,ms^{-2}$

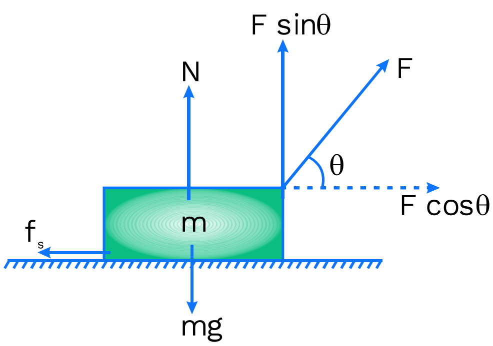
Stopping distance

$$S = \frac{v^2}{2\mu_k g} = 26.67\,m$$

Ex. In the following diagram find out minimum value of F for that block will just start its motion. (coefficient of friction is μ)



Sol.



$$N + F\sin\theta = mg$$

$$\Rightarrow \quad N = mg - F\sin\theta \qquad ...(i)$$

$$F\cos\theta = f_s \qquad ...(ii)$$

$$f_s = \mu N = \mu(mg - F\sin\theta)$$

For minimum value of F for motion

$$F\cos\theta \geq \mu N \Rightarrow F\cos\theta \geq \mu mg - \mu F\sin\theta$$

$$F\cos\theta + \mu F\sin\theta \geq \mu mg$$

$$\boxed{F \geq \frac{\mu mg}{\cos\theta + \mu\sin\theta}}$$

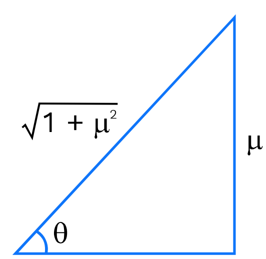
If F is minimum, means (cos θ + μ sin θ) is maximum.

$$\frac{d}{d\theta}(\cos\theta + \mu\sin\theta) = 0$$

$$-\sin\theta + \mu\cos\theta = 0$$

$$\Rightarrow \quad \mu = \tan\theta$$

$$\sin\theta = \frac{\mu}{\sqrt{1+\mu^2}}; \quad \cos\theta = \frac{1}{\sqrt{1+\mu^2}}$$



$$\Rightarrow \quad F \geq \frac{\mu mg}{\cos\theta + \mu\sin\theta}$$

$$\Rightarrow \quad F \geq \frac{\mu mg}{\frac{1}{\sqrt{1+\mu^2}} + \mu \cdot \frac{\mu}{\sqrt{1+\mu^2}}}$$

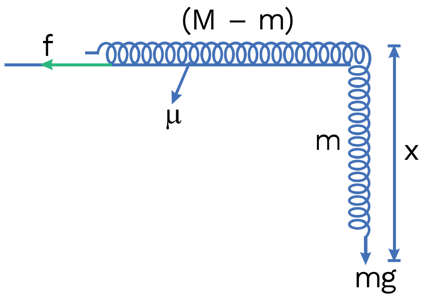
$$F \geq \frac{\mu mg}{\sqrt{1+\mu^2}}$$

$$\boxed{F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}}} \quad \text{with } \theta = \tan^{-1}(\mu)$$

Concept Reminder

- $\boxed{F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}}}$ is minimum value of force required to move a block on rough surface when applied at an angle $\theta = \tan^{-1}(\mu)$ from horizontal.

Ex. A chain of mass 'M' and length 'L' is placed on a horizontal table such that, some part of it is hanging outside the table. If coefficient of friction between chain and table is 'μ'. Then find out length of a chain which can be hanged outside the table so that chain does not slide or just begin to slide.



> **Concept Reminder**
>
> - For does not slide or just slide condition, net force on chain should be zero.

Sol. For balancing

$f = \mu N = mg\,,\qquad N = (M - m)g$

$\mu(M - m)g = mg$

$\mu M = m(1 + \mu) \qquad \left[\because\ m = \frac{M}{L}\cdot x\right]$

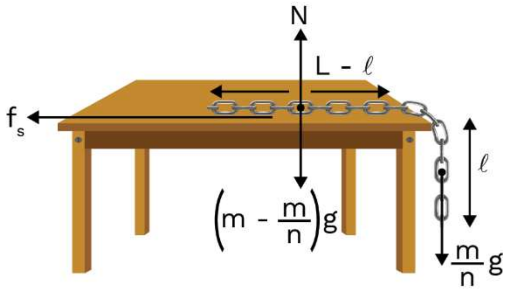
$\mu M = \frac{M}{L}(1 + \mu)\cdot x$

$$\boxed{x = \frac{\mu L}{1 + \mu}}$$

Sliding of a chain on a rough horizontal table:-

Consider a uniform chain of mass 'm' and length 'L' lying on a horizontal table of coefficient of friction 'ms' When $1/n^{th}$ of its length is hanging from the edge of the rough table, the chain is found to be about to slide from the rough table.

Weight of the drop part of the chain = $\frac{mg}{n}$.



> **Concept Reminder**
>
> - In equilibrium condition, maximum fractional length of chain hanging from edge is $\frac{\mu_s}{\mu_s + 1}$.

Weight of the chain lying on the rough table

$$= mg - \frac{mg}{n} = mg\left(1 - \frac{1}{n}\right)$$

When the chain is about to slide from edge of the rough table.
The weight of the drop part of the chain = frictional force between the chain and the table surface.

$$\frac{mg}{n} = \mu_s mg\left(1 - \frac{1}{n}\right)$$

$$\Rightarrow \quad \frac{mg}{n} = \mu_s mg\left(\frac{n-1}{n}\right)$$

$$\therefore \quad \boxed{\mu_s = \frac{1}{(n-1)}}$$

If l is the length of the drop part, then $n = \frac{L}{l}$. Substituting this equation in the above expression we get,

$$\boxed{\mu_s = \frac{l}{L - l}} \text{ or } \boxed{n = \frac{L}{l} = \frac{\mu_s + 1}{\mu_s}}$$

The maximum fractional length of chain drop from the edge of the table in equilibrium is

$$\boxed{\frac{l}{L} = \frac{\mu_s}{\mu_s + 1}}$$

- Fractional length of chain on the table

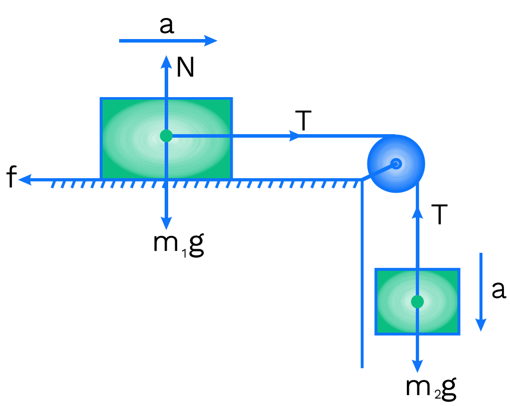
$$\boxed{\frac{L - l}{L} = \frac{1}{\mu_s + 1}}$$

Connected bodies:-

- A block of mass 'm_1' placed on a rough horizontal surface, is connected to a block of mass 'm_2' by a string which passes over a smooth pulley. The coefficient of friction between m_1 and the table is μ_k.

Concept Reminder

- For inextensible string, acceleration of both block will be same.



For body of mass m_2

$$m_2g - T = m_2a \quad \text{...(i)}$$

For body of mass m_1

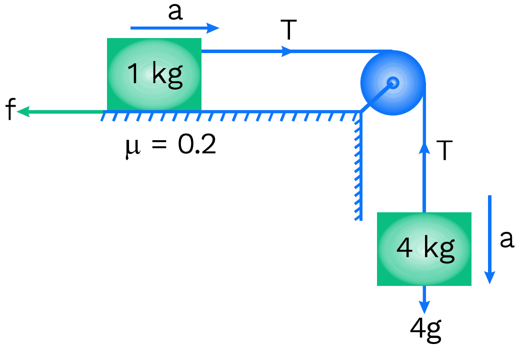
$$T - f_k = m_1a \quad \Rightarrow T - \mu_k N = m_1a$$

$$T - \mu_k m_1 g = m_1 a \quad \text{...(ii)}$$

Solving equations (i) and (ii), we get

$$a = \left(\frac{m_2 - \mu_k m_1}{m_1 + m_2}\right)g; \quad T = \frac{m_1 m_2 g}{m_1 + m_2}(1 + \mu_k)$$

Ex. Find out acceleration and tension in the string.



Sol. $4g - T = 4a$ ...(i)

$T - \mu g = 1a$ ...(ii)

By equation (i) and (ii)

$$a = \frac{4g - 0.2 \times g}{4 + 1} = \frac{40 - 2}{5} = \frac{38}{5}\ m/s^2$$

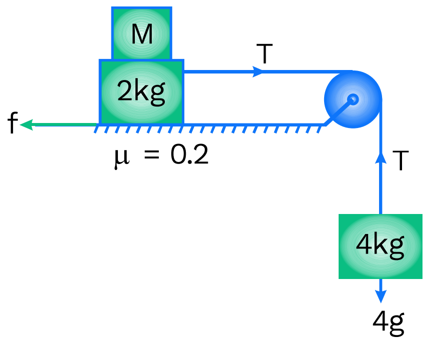
$$4g - T = 4a$$

$$\Rightarrow \quad T = 40 - 4 \times \frac{38}{5} = \frac{200 - 152}{5} = \frac{48}{5}N$$

Rack your Brain

A system consists of three masses m_1, m_2 and m_3 connected by a string passing over a pulley P. The mass m_1 hangs freely and m_2 and m_3 are on a rough horizontal table (the coefficient of friction = μ). The pulley is frictionless and of negligible mass. Then calculate downward acceleration of mass m_1.

Ex. Find out minimum value of M for that system is in equilibrium.



Sol. System is equilibrium, a = 0

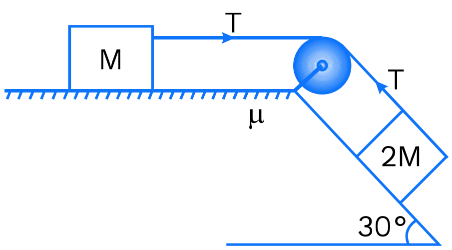
$$4g - T = 4 \times 0 \quad \Rightarrow T = 4g$$

$$T - f = 0, \quad (f = \mu(M+2)g)$$

$$4g = \mu(M+2)g \quad \Rightarrow 4 = 0.2(M+2)$$

$$M + 2 = \frac{40}{2} = 20 \quad \Rightarrow M = 18\,kg$$

Ex.



Sol. (i) $2Mg\sin 30° - T = 2M \times a$

$$Mg - T = 2Ma \qquad ...(i)$$

$$T - \mu Mg = Ma \quad (f = \mu Mg) \qquad ...(ii)$$

Solving equation (i) and (ii)

$$a = \frac{Mg - \mu Mg}{3M} = \frac{g(1-\mu)}{3} \ m/s^2$$

(ii) Mg – T = 2Ma

$$Mg - 2Mg\left(\frac{1-\mu}{3}\right) = T$$

$$T = Mg\left[1 - \frac{2}{3}(1-\mu)\right]$$

$$T = \frac{mg}{3}[1 + 2\mu]\,N$$

Rack your Brain

A plank with a box on it at one end is gradually raised about the other end. As the angle of inclination with the horizontal reaches 30°, the box starts to slip and slides 4.0 m down the plank is 4.0s. Calculate the coefficients of station and kinetic friction between the box and the plank.

Ex. A block of mass 10 kilogram is pushed by a force 'F' on a horizontal rough plane is moving with acceleration '5 ms^{-2}'. When force is doubled, its acceleration becomes '18 ms^{-2}'. Find out the coefficient of friction between the block and rough horizontal surface. ($g = 10\ ms^{-2}$)

Sol. On a rough horizontal surface, acceleration 'a' of a block of mass 'm' is given by

$$ma = F - \mu_k mg \quad (\because f_k = \mu_k mg)$$

$$a = \frac{F}{m} - \mu_k g \qquad \text{...(i)}$$

Initially, $a = 5\ ms^{-2}$

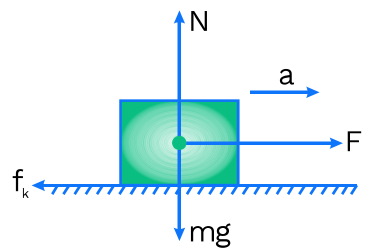
$$5 = \frac{F}{10} - \mu_k(10) \qquad \text{...(ii)}$$

When force is doubled $a = 18\ ms^{-2}$.

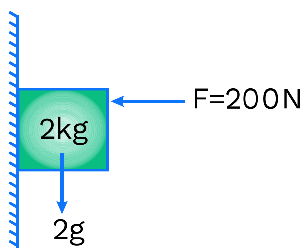
$$18 = \frac{2F}{10} - \mu_k(10) \qquad \text{...(iii)}$$

Multiplying equation (ii) with 2 and subtracting from equation (iii)

$$8 = \mu_k(10) \quad \Rightarrow \mu_k = \frac{8}{10} = 0.8$$



Ex. If coefficient of friction between wall and block is $\mu_s = 0.3$ and $\mu_k = 0.2$ then find out value of friction force and acceleration of block?



Sol. $f_{s_{max}} = \mu_s N = 0.3 \times 200 = 60\,N$

$f_k = \mu_k N = 0.2 \times 200 = 40\,N$

$F = 2 \times 10 = 20\,N$

Concept Reminder

- Since block is moving, therefore this is a coefficient of kinetic friction.

Key Points

- Coefficient of kinetic friction.

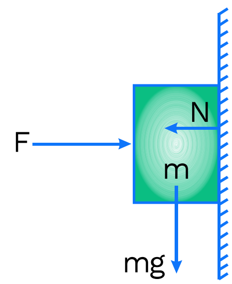
(i) $F < f_{s_{max}}$

than block at rest than

$f_s = F = 20\,N$

(ii) $a = 0$

Ex. Find out minimum value of horizontal force required to keep the given block of mass m' at rest against the vertical wall having coefficient of friction μ.

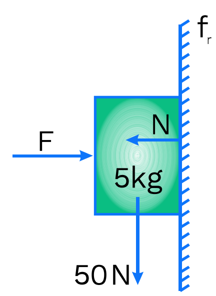


Sol. For rest

$f_{lim} = mg \quad \Rightarrow \quad \mu N = mg$

$\mu F = mg \quad \Rightarrow \quad F_{min} = \dfrac{mg}{\mu}$

Ex. Find the minimum value of force (F) for which it remains stationary ($\mu = 0.4$).



Sol. $f_{lim} = mg$

$\Rightarrow \quad \mu N = mg$

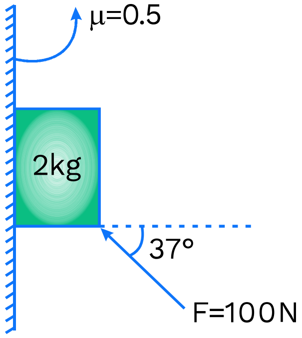
$0.4 \times F = 50$

$\Rightarrow \quad F = \dfrac{500}{4} = 125\,N$

Rack your Brain

A block A of mass m_1 rests on a horizontal table. A light string connected to it passes over a frictionless pulley at the edge of table and from its other end another block B of mass m_2 is suspended. The coefficient of kinetic friction between the block and the table is μ_k. When the block A is sliding on the table, then calculate tension the string.

Ex. For a given block, find out magnitude, direction of friction force acting on it.



Sol. $N = F\cos 37^\circ = 80\,N$

$(F_{net})_{up} = F\sin 37^\circ - 2g$

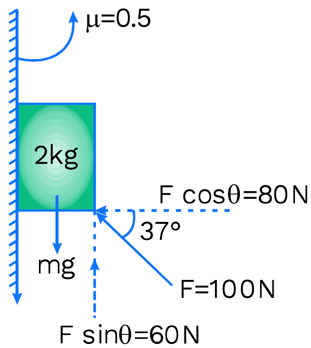
$(F_{net})_{up} = 60 - 20 = 40\,N$

$f_{lim} = \mu N = 0.5(80) = 40\,N$

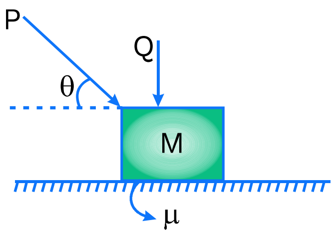
$\because$ $f_{lim} = (F_{net})_{up}$

$\therefore$ block is at rest

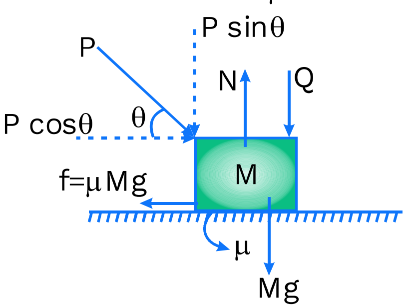
Friction = 40 N, downward.



Ex. In the following find out maximum value of μ for that body will just start its motion.



Sol.



Concept Reminder

- Pulling an object is easier than pushing for rough surfaces.
- On smooth horizontal surfaces both pushing and pulling will be same.

(i) $N = Mg + P\sin\theta + Q$

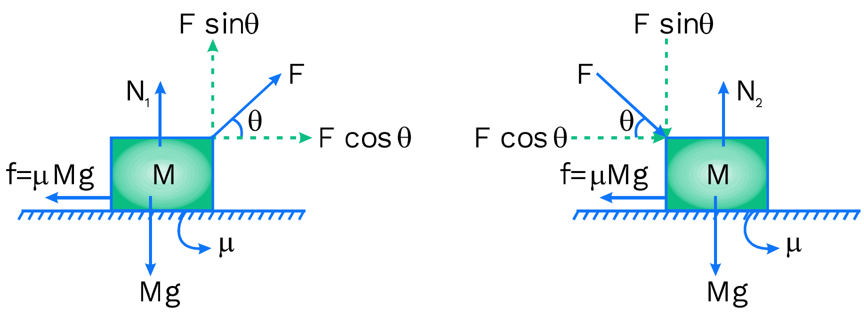
(ii) $f_s = \mu_s N$ (for body will just start when applied force is equal to friction force)

$\mu(Mg + P\sin\theta + Q) = P\cos\theta$

$$\mu = \frac{P\cos\theta}{Mg + P\sin\theta + Q}$$

Ex. According to following diagram which one is easier pushing or pulling.

Sol.



In pulling

$N_1 + F\sin\theta = Mg$,

$N_1 = Mg - F\sin\theta$

$N_2 > N_1 \ (f \propto N)$

$\mu N_1 < \mu N_2 \Rightarrow f_{r_1} < f_{r_2}$

$F_{pulling} < F_{pushing}$

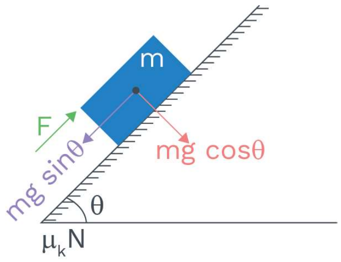
In pushing

$N_2 = Mg + F\sin\theta$

Note: Pulling an object is easier than pushing it.

Ex. A body is placed on rough incline plane if a force F, is applied upwards along the plane then for value of force F_1. It will just start slipping upwards and for value of force F_2 body will prevent its slipping downward find out ratio of F_1 and F_2 if tan θ = 3μ (θ = angle of inclination and μ = friction coefficient)

Sol.



Case-I: If block move upward

$$F_1 = mg\sin\theta + \mu mg\cos\theta$$

Case-II: If block move downward

$$F_2 = mg\sin\theta - \mu mg\cos\theta$$

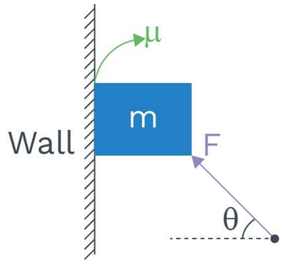
$$\frac{F_1}{F_2} = \frac{mg\sin\theta + \mu mg\cos\theta}{mgsn\theta - \mu mg\cos\theta}$$

Divide by mg cos θ

$$\Rightarrow \quad \frac{F_1}{F_2} = \frac{\tan\theta + \mu}{\tan\theta - \mu} = \frac{3\mu + \mu}{3\mu - \mu}$$

$$\Rightarrow \quad \frac{F_1}{F_2} = \frac{4\mu}{2\mu} = \frac{2}{1}$$

Ex. In the following diagram find out minimum value of F for that block will just prevent its slipping.



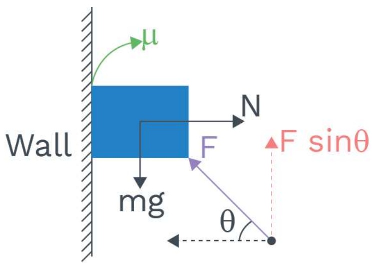
Rack your Brain

A conveyor belt is moving at a constant speed of $2ms^{-1}$. A box is gently dropped on it. The coefficient of friction between them is μ = 0.5. Find out the distance that the box will move relative to belt before coming to rest. (take g = 10 ms^{-2})

Key Points

- Pulling
- Pushing

Sol.



(i) $N = F\cos\theta$

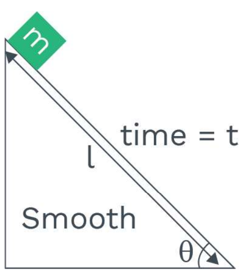
(ii) $F\sin\theta + \mu N = mg$

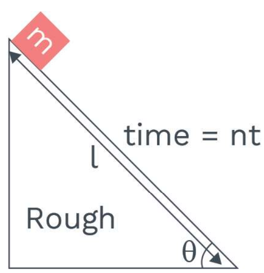
$$F(\sin\theta + \mu\cos\theta) = mg$$

$$F = \frac{mg}{\sin\theta + \mu\cos\theta}$$

Ex. A block takes 'n' times as much time to slide down on a rough inclined plane of inclination of θ as it takes to slide down on a perfectly smooth inclined plane of same inclination. Find coefficient of friction.

Sol.





If the time 't' is the time taken to travel the distance 'l' with initial velocity u = 0 at the top of the plane,

$$t_{rough} = \sqrt{\frac{2l}{a_1}} = \sqrt{\frac{2l}{g(\sin\theta - \mu\cos\theta)}}$$

$$t_{smooth} = \sqrt{\frac{2l}{a_2}} = \sqrt{\frac{2l}{g\sin\theta}}$$

The time taken by an object to slide down on a rough inclined surface is n times the time taken by object to slide down on a smooth (μ = 0) inclined surface of same inclination angle and length, then coefficient of friction is

$$t_{rough} = nt_{smooth}$$

$$n = \frac{t_{rough}}{t_{smooth}} = \frac{\sqrt{\frac{2l}{g(\sin\theta - \mu\cos\theta)}}}{\sqrt{\frac{2l}{g\sin\theta}}}$$

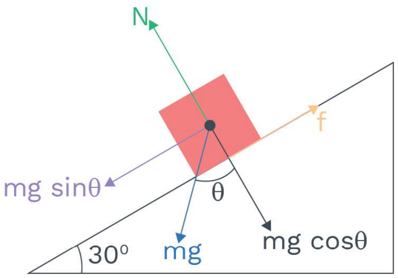
$$n^2 = \frac{\sin\theta}{\sin\theta - \mu\cos\theta}$$

$\Rightarrow$ $n^2 \sin\theta - n^2\mu\cos\theta = \sin\theta$

$\Rightarrow$ $\boxed{\mu = \tan\theta\left[1 - \frac{1}{n^2}\right]}$

Ex. An object is moving down a long inclined plane of angle of inclination θ for which the coefficient of friction depends with distance x as given by $\mu(x) = kx$; where k is constant and x is the distance travelled by the object down the plane. The net force on the object will be zero at a distance x_0 is given by.

Sol.



$F = mg\sin\theta - f$

$N = mg\cos\theta$; $f = \mu N = \mu mg\cos\theta$

$F = mg\sin\theta - \mu mg\cos\theta$

$F = mg(\sin\theta - kx\cos\theta)$

If $F = 0$;

$\sin\theta - kx_0\cos\theta = 0$

$\Rightarrow$ $x_0 = \frac{\tan\theta}{k}$

Ex. A body is sliding down an inclined surface having coefficient of friction '0.5'. If the normal reaction force is twice that of resultant downward force along the inclined surface, then find the angle θ between the inclined surface and the horizontal.

Sol. $\mu = 0.5, \quad N = mg\cos\theta$

$N = 2F, \quad F = mg(\sin\theta - \mu\cos\theta)$

$N = 2mg(\sin\theta - \mu\cos\theta)$

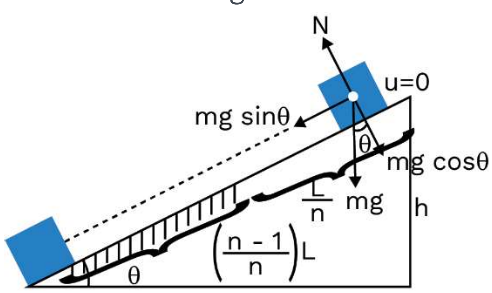
$mg\cos\theta = 2mg(\sin\theta - \mu\cos\theta)$

$\cos\theta = 2\cos\theta(\tan\theta - \mu)$

$\frac{1}{2} = \left(\tan\theta - \frac{1}{2}\right) \quad \Rightarrow \tan\theta = 1 \quad \Rightarrow \theta = 45°$

Ex. An object is released from rest from the top of an inclined plane of length 'L' and angle of inclination 'θ'. The top of plane of length $\frac{L}{n}(n > 1)$ is smooth and the remaining part is rough. If the object comes to rest on reaching the bottom of the plane then find the value of coefficient of friction of rough surface.

Sol.



For smooth part:

Using $v^2 - u^2 = 2as, \quad v^2 = 2a_1\frac{L}{n}$

$a_1 = g\sin\theta, \quad v^2 = 2g\sin\theta \cdot \frac{L}{n}$

For rough part:

$0 - v^2 = 2a_2\left(\frac{n-1}{n}\right)L$

$a_2 = g(\sin\theta - \mu\cos\theta)$

$2g\sin\theta \cdot \frac{L}{n} = -2g(\sin\theta - \mu\cos\theta)\left(\frac{n-1}{n}\right)L$

$\sin\theta = -[\sin\theta - \mu\cos\theta](n-1)$

$\sin\theta + \sin\theta(n-1) = \mu\cos\theta(n-1)$

$\sin\theta(1 + n - 1) = \mu\cos\theta(n-1)$

Rack your Brain

The upper half of an inclined plane of inclination θ is perfectly smooth while lower half is rough. A block starting from rest at the top of the plane will again come to rest at the bottom, derive the relation between coefficient of friction and inclination angle θ.

$$\tan\theta = \mu\left(\frac{n-1}{n}\right)$$

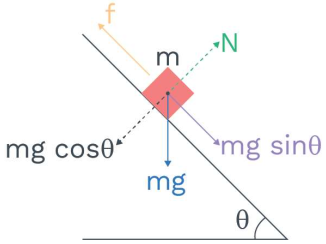
$$\boxed{\mu = \tan\theta\left[\frac{n}{n-1}\right]}$$

Angle of Repose:-

- Minimum angle 'θ' of inclination of a plane with the horizontal at which a body placed on it just begins to slide down.

  or

- Maximum angle of inclination of plane with the horizontal at which a body placed on it does not slide.



$N = mg\cos\theta$

$mg\sin\theta = f = \mu N$

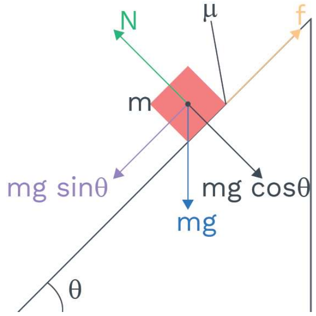
$mg\sin\theta = \mu \cdot mg\cos\theta$

$\mu = \tan\theta$

$\boxed{\theta = \tan^{-1}(\mu)}$; θ is angle of repose.

Motion on rough inclined plane:

Case-1: Downward sliding



$N = mg\cos\theta$

$f_s = \mu_s N = \mu_s\, mg\cos\theta$

Definition

- Maximum angle of inclination of plane with horizontal at which body does not slide is called angle of repose.
- $\theta = \tan^{-1}(\mu)$

Definition

- Angle of repose depends only on coefficient of friction.

(i) If mg sin $\theta < \mu_s$ mg cos θ then block does not slide and static friction act.

$f_s = mg \sin\theta$ (self adjusting)

a = 0

(ii) $mg\sin\theta = \mu_s mg\cos\theta$ then block just sliding condition, a = 0.

friction act, $f_s = \mu_s N = \mu_s mg\cos\theta = mg\sin\theta$

In this condition block move with constant velocity.

(iii) $mg\sin\theta > \mu_s mg\cos\theta$ then block

accelerate in downward direction,

$ma = mg\sin\theta - \mu mg\cos\theta$

$a = g(\sin\theta - \mu\cos\theta)$

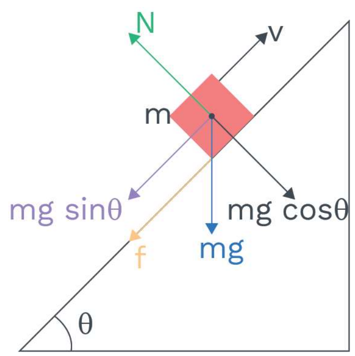
Friction force act as kinetic friction,

$f_k = \mu_k N, \quad a = g(\sin\theta - \mu_k \cos\theta)$

Key Points

- Angle of repose.
- Angle of friction.

Case-2: Upward sliding- when body is projected on rough inclined plane with some initial velocity, then its retardation.



Concept Reminder

- When body is projected upward on rough inclined plane then frictional force acts downward on plane.

N = mg cos θ

$f_s = \mu mg\cos\theta$

Retardation is

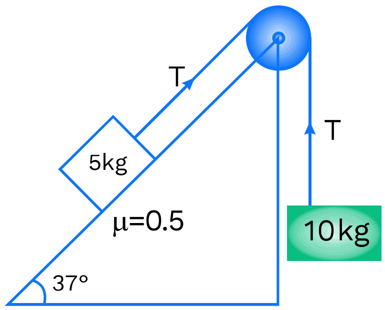
$$a = \frac{F_{net}}{m} = \frac{mg\sin\theta + \mu_k mg\cos\theta}{m}$$

$a = g(\sin\theta + \mu_k \cos\theta)$

Minimum required force to just move the block upwards.

$F_{upward} = mg\sin\theta + \mu_k mg\cos\theta$

Ex. For a given system find out tension in string.



Sol. $f_s = \mu_s N = \mu_s mg\cos 37°$

$$f_s = 0.5 \times 50 \times \frac{4}{5} = 20\,N$$

$$10g - T = 10\,a \qquad \text{...(i)}$$

$$T - (\mu_s mg\cos 37° + mg\sin 37°) = 5a \qquad \text{...(ii)}$$

Add equation (i) and (ii)

$$10g - (\mu_s mg\cos 37° + mg\sin 37°) = 15\,a$$

$$100 - \left(20 + 50 \times \frac{3}{5}\right) = 15\,a$$

$$\frac{100-50}{15} = a \quad \Rightarrow a = \frac{50}{15} = \frac{10}{3}\,m/s^2$$

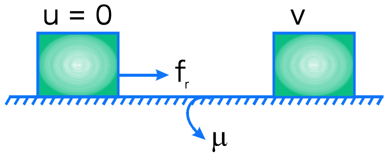
Value of a put in equation (i)

$$10g - T = 10 \times \frac{10}{3}$$

$$T = 100 - \frac{100}{3} = \frac{200}{3}\,N$$

Ex. Convert belt is moving horizontally with v m/s. Block gently placed on it find out time taken by it before it will be stationary with respect to belt (friction coefficient between both = μ)

Sol.



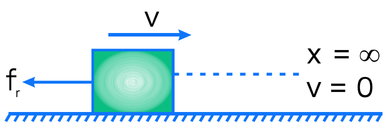
Initial velocity of block, u = 0
final velocity of block = v

$$v = 0 - a \cdot t \qquad \left(a = \frac{f_r}{m} = \frac{\mu mg}{m} = \mu g\right)$$

$$\Rightarrow \quad t = \frac{v}{a} = \frac{v}{\mu g} \text{ sec}$$

Ex. Find out minimum velocity given to the body at x = 1 on rough horizontal surface $\left(\mu = \frac{A}{x^2}\right)$ constant for that it will never stop.

Sol. It will stop at infinity



$$\therefore \quad a = \mu g$$

$$-\frac{vdv}{dx} = \frac{A}{x^2}g \qquad \Rightarrow \qquad -\int_v^0 vdv = Ag\int_1^\infty x^{-2}dx$$

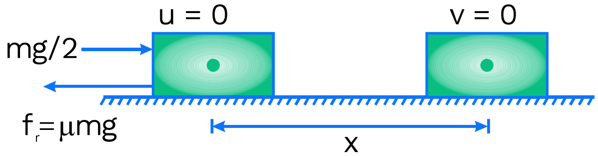
$$-\left[\frac{v^2}{2}\right]_v^0 = Ag\left(-\frac{1}{x}\right)_1^\infty$$

$$\Rightarrow \quad \frac{v^2}{2} = -Ag\left(\frac{1}{\infty} - \frac{1}{1}\right)$$

$$\frac{v^2}{2} = Ag \;\Rightarrow\; v^2 = 2Ag \;\Rightarrow\; v = \sqrt{2Ag}$$

Ex. A body of mass 'm' is placed on rough horizontal surface ($\mu = \mu_0 x$) and external horizontal force $\frac{mg}{2}$ is applied on body then find out distance travel by it before it will again stop.

Sol.



$$f_{net} = \frac{mg}{2} - \mu mg \qquad \Rightarrow a = \frac{g}{2} - \mu_0 xg$$

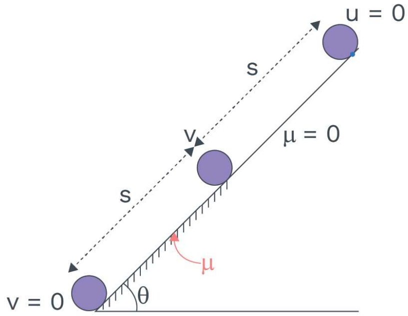
$$\frac{vdv}{dx} = \frac{g}{2} - \mu_0 gx \qquad \Rightarrow \int_0^0 vdv = \int_0^x \left(\frac{g}{2} - \mu_0 gx\right)dx$$

$$0 = \frac{g}{2}x - \mu_0 g\frac{x^2}{2} \qquad \Rightarrow 0 = x(1 - \mu_0 x)$$

$$\therefore \quad 1 - \mu_0 x = 0 \;\Rightarrow\; x = \frac{1}{\mu_0}$$

Ex. Upper half of incline plane is smooth and lower half is rough (μ). If a body is released from top of incline plane then it will again stop at the bottom of incline plane find out angle of inclination (θ).

Sol.



On smooth surface

$$v^2 = 0 + 2g\sin\theta\, s$$

On rough surface

$$0 = v^2 - 2(\mu g\cos\theta - g\sin\theta)s$$

$$2g\sin\theta\, s = 2(\mu g\cos\theta - g\sin\theta)\, s$$

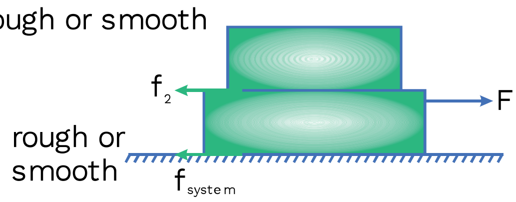
$$\sin\theta = -\sin\theta + \mu g\cos\theta$$

$$\mu\cos\theta = 2\sin\theta$$

$$\frac{\mu}{2} = \tan\theta$$

$$\theta = \tan^{-1}\left(\frac{\mu}{2}\right)$$

Two block system in friction:



This problem is solved by following steps.

Step-1: Draw the FBD of the combined block system. If friction appear in the FBD, then applied force > limiting friction between system and surface, then motion is possible then acceleration of system is

$$a_{system} = \frac{F - f_{system}}{M_{total}}$$

If applied force < limiting friction, then $a_{system} = 0$.

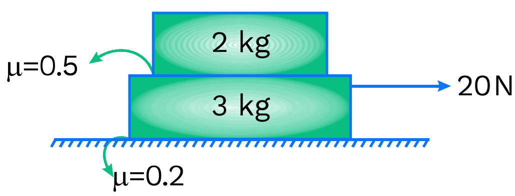
If motion occurs then, either the blocks move together or separately depends on frictional force able to support between objects.

Step-2: Find common acceleration (a_c) and condition check of the body on which external force is not applied. Find the frictional force f required to make it move combined by with the other block.

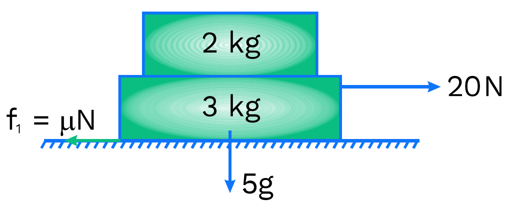
Compare the above calculated force with the limiting value f_l(maximum static friction). If $f \le f_l$, then both move together with common acceleration a_c, otherwise, they move separately.

Step-3: For separate motion draw individual FBD's of both blocks with kinetic friction forces acting wherever applicable. Find the individual acceleration of the two blocks.

Ex. Find out acceleration of both blocks and friction force?



Sol.



$f_1 = 0.2 \times 50 \Rightarrow \quad f_1 = 10$ N

$$a_{system} = \frac{F - f_1}{M_{total}} = \frac{20 - 10}{5} \Rightarrow a_{system} = 2\,m/s^2$$

Check condition on 2 kg block acceleration 2 m/s² required force = 2 × 2 = 4 N.

maximum friction force between 2 kg and 3 kg block.

$$f_2 = \mu_s N = 0.5 \times 20 = 10\,N$$

maximum possible acceleration of block 2 kg is $\frac{10}{2}$ = 5 m/s².

$F < f_2 \Rightarrow$ then both block move together with common acceleration 2 m/s².

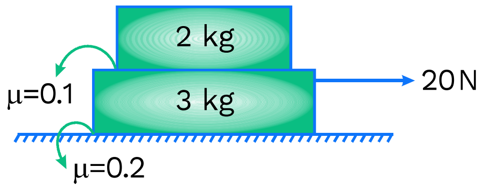
friction force act between both blocks is

$f_{2kg} = 4$ N

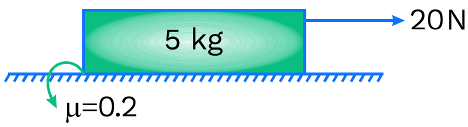
and friction force between block and surface is

$f_{2kg} = 10\,N$

Ex. Find out acceleration of both blocks and friction force.



Sol. We consider a system



$f_1 = 0.2 \times 50 = 10\,N$

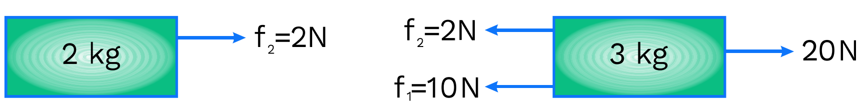
$a_{system} = \dfrac{20-10}{5} = 2\,m/s^2$

check condition on 2 kg block

$f_{2_{max}} = 0.1 \times 20 = 2\,N$

maximum possible acceleration of block 2 kg is $= \dfrac{2}{2} = 1\,m/s^2$.

$f > f_l$, then both block not move together means both block move different acceleration.

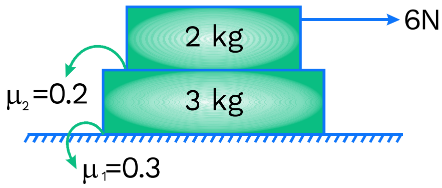


$a_{2kg} = \dfrac{2}{2} = 1\ m/s^2$; $\quad a_{3kg} = \dfrac{20-(10+2)}{3}$

$a_{3kg} = \dfrac{8}{3}\,m/s^2$

Note: If both blocks move with different acceleration then kinetic friction act between blocks because slipping is start.

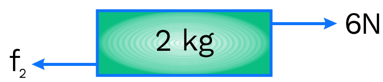
Ex. For a given system, find out acceleration of block.



Sol. $f_1 = 0.3 \times 50 = 15\,N$

$$a_{system} = \frac{F - f_1}{M_{total}}, \quad f_1 > F$$

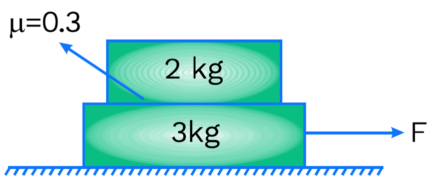
means 3 kg block is at rest
for 2 kg block



$f_2 = 0.2 \times 20 = 4\,N$

$$a_{2kg} = \frac{6-4}{2} = \frac{2}{2} = = 1\ m/s^2$$

Ex. In the following find out maximum value of F for that both will move together or minimum value of F for that relative motion between both will just start.



Sol. 2 kg block maximum acceleration is

$$a_{2kg} = \frac{f_{s_{max}}}{2} = \frac{0.3 \times 20}{2} = \frac{6}{2} = 3\ \ m/s^2$$

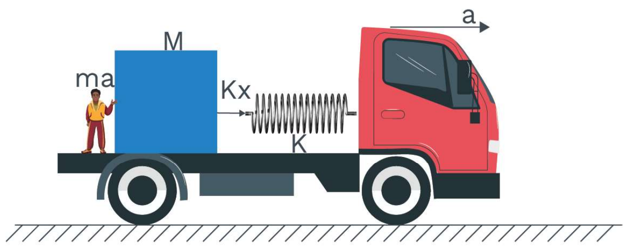
than maximum acceleration of system is 3 m/s^2.
minimum value of force

$F = ma = (3+2) \times 3 = 15\,N$

EXAMPLES

Q1 **In the following diagram block is in equilibrium w.r.t trolley find out extension in spring**

Sol. F.B.P of block w.r.t trolley



$$Kx = ma$$

$$x = \frac{ma}{K}$$

Q2 **Find the direction of kinetic friction force**

(a) on the block, exerted by the ground
(b) on the ground, exerted by the block

Sol. (a)

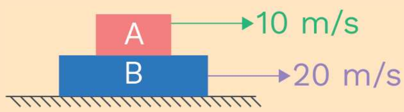
(b)

Friction of kinetic acts in such a way so as to reduce relative motion.

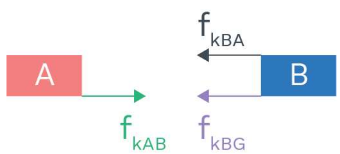
Q3 **In above example correct relation between magnitude of f_1 and f_2 is**

Sol. ($f_1 = f_2$) : By third law of Newton the above friction forces are action-reaction pair, equal but opposite to each other in direction.
Also note that the kinetic friction direction has nothing to do with applied force F.

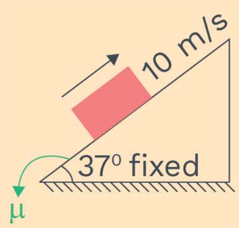
Q4 **All surfaces as shown in the figure are rough. Draw the friction force on A & B**



Sol. Kinetic friction acts in such a way that so as to reduce relative motion.



Q5 **Find out the distance covered by the block on incline before it stops. Initial speed of the block is u = 10 m/s and coefficient of friction between the block and incline is μ = 0.5.**



Sol.

$N = mg \cos 37°$

$\therefore$ $mg \sin 37° + \mu N = Ma$

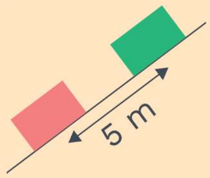
$a = 10\ m/s^2$ down the incline

Now $v^2 = u^2 + 2as$

$0 = 10^2 + 2(-10)S$

$\therefore$ $S = 5\ m$

Q6 **Find out the time taken in the above example by the block before it stops.**



Sol.

$a = g \sin 37° + \mu g \cos 37°$

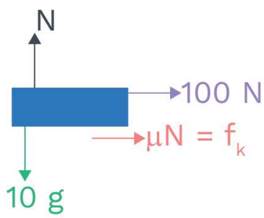
$\therefore$ $a = 10\ m/s^2$ down the incline

$\therefore$ $S = ut + \frac{1}{2}at^2$ $\Rightarrow$ $5 = \frac{1}{2} \times 10 \times t^2$

$\therefore$ $t = 1$ sec.

Q7 **A block is given a initial velocity of 10 m/s and a force of 100 N in addition to friction force is also acting on the block. Find out the retardation of the block?**

Sol. As there is relative motion between block and surface



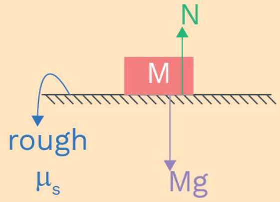
kinetic friction will act to reduce relative motion between surface

$f_k = \mu N = \mu mg = 0.1 \times 10 \times 10 = 10$ N

$100 + 10 = 10a$

$a = 11\ m/s^2$

Q8 **What is value of static friction force on the block?**

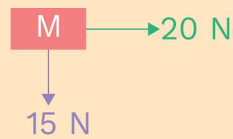


Sol. In horizontal direction as acceleration is zero.

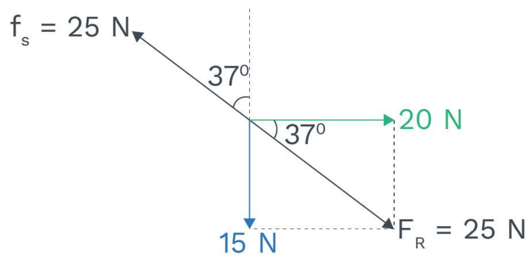
Therefore $\Sigma F = 0.$

$\therefore \quad f = 0$

Q9 **In the following diagram a body of mass M is kept on a rough table as seen. Forces are applied on it as shown figure. Find out the direction of static friction if the body does not move.**

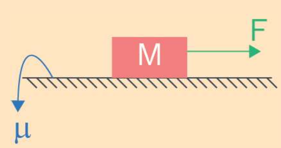


Sol. We first draw the F.B.D. to find the resultant force.



As the object does not move this is not a case of limiting friction. The direction of static friction 'f_s' is opposite to the direction of the resultant force 'F_R' as shown in figure by 'f_s'. Its magnitude is equal to 25 N

Q10 **Find out acceleration of the block for different ranges of F.**



Sol.

$0 \le F \le \mu_s N$

$\Rightarrow \quad 0 \le F \le \mu_s mg$

$a = 0$ if $F \le \mu_s mg$

$\Rightarrow \quad a = \dfrac{F - \mu Mg}{M}$ If $F > \mu Mg$

Q11 **Find out acceleration of the block. Initially the block is at rest.**

Sol. $0 \le f_s \le \mu_s N \quad \Rightarrow \quad 0 \le f_s \le 50$

Now $\quad F_{applied} > f_s$

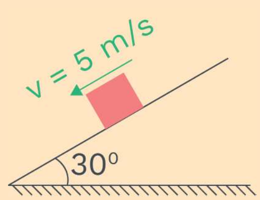
$\therefore$ Block will sliding condition but if the block starts sliding then kinetic friction

is involved.

$f_k = \mu_k N = 0.3 \times 100 = N$

$\therefore \quad 51 - 30 = 10\ A$

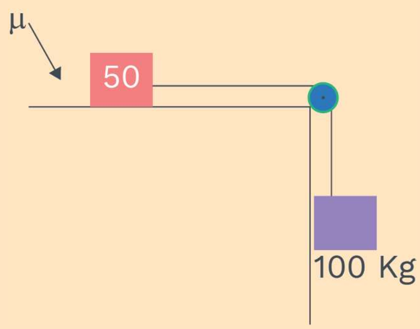
Q12 **A particle of mass 5 kg is moving on rough fixed inclined plane with constant speed of 5 m/s as shown in the figure. Find the friction force acting on a body by plane.**



Sol. $f_k = \mu_k N = \mu_k\ mg \cos 37° = mg \sin 30° = 5\ (10) \left(\frac{1}{2}\right)$

$\Rightarrow \ f_k = 25\ N$

Q13 **Find minimum μ so that the blocks remains stationary.**



Sol.

T = 100 g = 1000 N

f = 1000 to keep the block stationary

Now $f_{max} = 1000$

$\mu N = 1000$

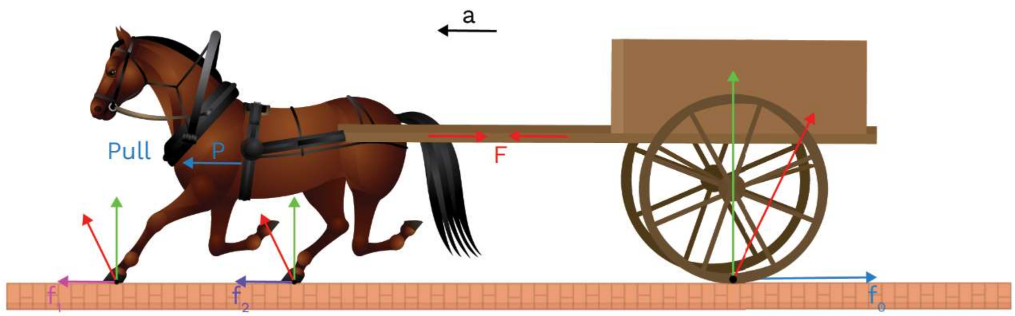
$\mu = 2$

Can μ be greater than 1?

Yes $0 < \mu \leq \infty$

Mind Map

Horse Cart



(Forward/Driving Friction)

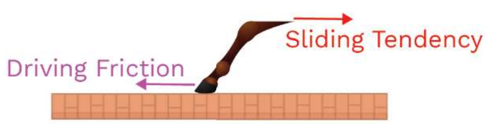
$f_d = f_1 + f_2$

(Backward/Opposing Friction)

Common acceleration of the cart/horse = a
M = mass of horse
M_0 = Mass of the cart

Horse

$f_d = F = Ma$



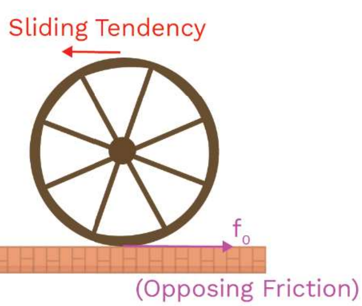
Cart

$F - f_0 = M_0 a$

As acceleration must be common

$$\frac{f_d - F}{M} = \frac{F - f_0}{M_0}$$

Friction forces are self adjusting



FRICTION

Static Friction

$f_s \propto R$

$(f_s) \leq \mu_s R$

μ_s = Static friction coefficient

Kinetic Friction

$f_K \propto R$

$f_K = \mu_K R$

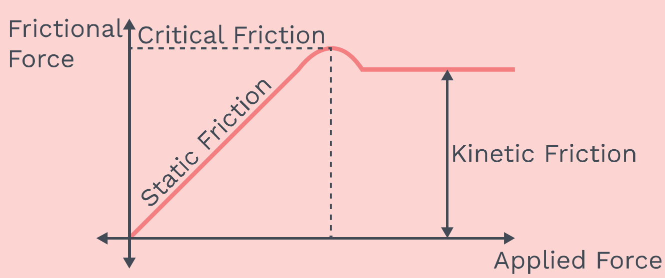
μ_K = Kinetic friction coefficient

Rolling Friction

$f_R \propto \frac{R}{r}$

$f_R = \frac{\mu_r R}{r}$

$\mu_s > \mu_k > \mu_r$



$(f_s)_{max} = \mu_s R$

$f_k = \mu_k R$