Electrostatics

DISCLAIMER

Electrostatics

Definition

The branch of physics in which we study about charge at rest & its effects is known Electrostatics.

Charge

Charge is the property of any material by virtue of which, it can produce electric field, magnetic field & electromagnetic waves.

Unit of Charge

S.I unit = Coulomb

Electrostatic unit = Stat coulomb or Franklin.

Electromagnetic unit = Absolute coulomb

1 Faraday = 96500 C

1 Coulomb = 3×10^9 Stat coulomb = $\frac{1}{10}$ Absolute

1 Coulomb = $\frac{1}{96500}$ Faraday

Specific Properties of Charge

(i) Charge is a scalar quantity: It represents an excess or deficiency of electrons.

(ii) Charge is transferable: If a charged body is put in contact with another body, then charge can be transferred to another body.

(iii) Charge is always linked with mass

Charge can never exist without mass even though mass can exist without charge.

(a) So, the presence of charge itself is a convincing proof of existence of mass.

(b) In charging, the mass of a body changes.

(c) When body is given positive charge its mass decreases.

(d) When body is given negative charge, its mass increases.

(iv) Charge is quantized

The quantization of charge is the property due to which value of all free charges are integral

Definition

Charge is property of matter which is responsible for all electric and magnetic phenomenon in nature.

Concept Reminder

Transfer of proton is not reason for charge. Only excess or deficiency of electrons give rise to charge on body.

Key Points

- Electrostatics
- Charge
- Coulomb

multiple of a a charged of single electron. Thus, charge 'Q' of a body is always given by- Q is equal to ne i.e., Q = ne
n is +ve integer or –ve integer
A quantum of charge is the value of single electron or proton carries.
Note : Charge on a proton = (–ve) charge on an electron = 1.6×10^{-19} C

Concept Reminder

At relativistic speed, mass of body increases but charge remains constant.

(v) Charge is conserved

In an isolated system, net charge never changes with time, though individual charge may change i.e. charge can neither be created nor be destroyed. Conservation of charge can be used in all types of reactions either chemical (atomic) or nuclear. No exceptions have ever been found in this law.

Key Points

- Quantization
- Invariant

(vi) Charge is an invariant quantity

Charge is not dependent of frame of reference. i.e. charge on a body does not very whatever be its speed.

Concept Reminder

Gold leaf electroscope can be used to detect charge on body.

(vii) Accelerated charge radiates energy

v = 0 (i.e. at rest)- $\oplus$ Q produces only $\vec{E}$ (electric field)	v = constant $\oplus\rightarrow$ Q produces both $\vec{E}$ and $\vec{B}$ (magnetic field) but no radiation	v $\neq$ constant (i.e. time varying) $\oplus\rightarrow$ Q produces $\vec{E},\vec{B}$ and radiates energy

(viii) Attraction – Repulsion between charges

Charges of same nature repel each other while dissimilar charges attract.

Methods of Charging

(i) Friction

Consider we rub one body with other body, electrons are transferred from one body to the other. Transfer of electrons take places from lower work function body to higher work function body.

Positive Charge	Negative charge
Glass rod	Silk cloth
Woollen cloth	Rubber shoes, Amber, Plastic objects
Dry hair	Comb
Flannel or cat skin	Ebonite rod
Note: Clouds become charged by friction	

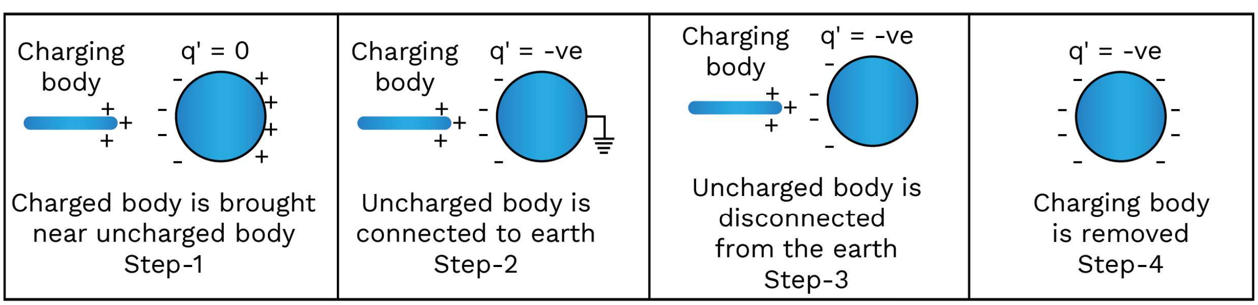
(ii) Electrostatic Induction

Suppose a charged body is brought near a metallic neutral body then the charged body will attract dissimilar charges and repel similar charges that are present in the neutral body. Because of this one side of the neutral body becomes negative while the other positive, this process is called "Electrostatic Induction".

Concept Reminder

In induction, contact between two bodies is not required.

Charging a body by induction process (in four successive steps)



- In electrostatic induction charge of inducer remains unchanged [value wise] but its distribution may change.
- Charge produced in electrostatic induction is of opposite nature as nature of inducer. This induced charge is given as

$$\boxed{q' = -q\left[1 - \frac{1}{\varepsilon_r}\right]}$$

q' = induced charge
q = charge of inducer
ε_r = dielectric constant of material of induced object.

- If $\varepsilon_r = 1$ [for vacuum]

  then q' = 0 [no induction]
- If $\varepsilon_r = \infty$ [for conductor]

  q' = −q [maximum induction]
- If $1 < \varepsilon_r < \infty$

  $0 < q' < |-q|$

Key Points

- Friction
- Induction
- Conduction

Some important facts related with induction-

(i) The inducing body neither gains nor loses charge in this process.

(ii) Nature of induced charge is always opposite to that of inducing charge or body.

(iii) Induction takes place only in bodies (either conducting or non-conducting) and not in particles.

Concept Reminder

In conductor, charge distribute itself over the entire surface. But in insulator charge stays at same place.

(iii) Conduction

The process of transfer of charge due to contact of two bodies is called conduction. Suppose a charged body is put in contact with uncharged body, then the uncharged body becomes charged because of transfer of electrons from one body to the other.

(a) The charged body loses some of its charge (which is equal to the charge gained by the uncharged body)

(b) The charge gained by the uncharged body is always lesser than initial charge present on the charged body.
(c) Flow of charge depends upon the P.D. between of both bodies.

It means "No potential difference $\Rightarrow$ No conduction".

+ve charge flows from higher potential to lower potential, while –ve charge flows from lower to higher potential.

eg.

Before connection

After connection

$$\left[\frac{q_1'}{q_2'} = \frac{R_1}{R_2}\right]$$

$$q_1' = \frac{R_1}{(R_1 + R_2)} \times (q_1 + q_2)$$

$$q_2' = \frac{R_2}{(R_2 + R_1)} \times (q_2 + q_1)$$

If $R_1 = R_2$

$$q_1' = \frac{q_1 + q_2}{2}$$

$$q_2' = \frac{q_1 + q_2}{2}$$

Important Points

1. Charge differs from mass in the following sense.

(a) In SI units, charge is a derived physical quantity while mass is fundamental quantity.
(b) Charge is always conserved but mass is not
Note: Mass can be converted into energy
$E = mc^2$

Concept Reminder

If two spheres are of same size then after connecting them charge will distribute equally on both.

Concept Reminder

Two similar charged bodies may attract each other because of induction process.

(c) The quanta of charge is electronic charge while that of mass it is yet not clear.

(d) For a moving charged body mass increases while charge remains constant.

2. True test of presence of charge is repulsion and not attraction since attraction can present between a charged and an uncharged object and also between two similarly Charged objects.

3. For a non-relativistic (i.e. v << c) charged particle, specific charge $\frac{q}{m}$ = constant.

4. For a relativistic charged particle $\frac{q}{m}$ decreases as value of v increases, where v is speed of charged object.

Ex. When a polythene is rubbed with wool, then a charge of value -2×10^{-7} C is evolved on polythene. Find the amount of mass, which is transferred to polythene?

Sol. Using Q = ne, So, the number of electrons transferred $n = \frac{Q}{e} = \frac{2 \times 10^{-7}}{1.6 \times 10^{-19}} = 1.25 \times 10^{12}$

So, mass of transferred electrons

= n × mass of one e^-

$= 1.25 \times 10^{12} \times 9.1 \times 10^{-31} = 11.375 \times 10^{-19}$ kg

Ex. 10^{12} α -particles (Nuclei of helium) per second falls on a neutral sphere, calculate time in which sphere gets charged by 2μC.

Sol. Number of α -particles falling in t second = 10^{12}t.

Charge on α - particle = + 2e,

So, charge incident in time t = $(10^{12}t).(2e)$

∵ Given charge is 2 μC

$$\therefore\ 2 \times 10^{-6} = (10^{12}t).(2e) \Rightarrow t = \frac{10^{-18}}{1.6 \times 10^{-19}} = 6.25\,s$$

Coulomb's Law

- The electrostatic force of interaction between any two point electric charges fixed at some points

Key Points

- Relativistic
- Coulomb's law
- α-particles

is directly proportional to the product of both charges, inversely proportional to the square of the distance between them and this force acts along the straight line that joins the two charges.

- If two points charges q_1 and q_2 which are separated by a distance r. Let F be the electrostatic force of interaction between these two charges. According to Coulomb's law.

$F \propto q_1 q_1$ and $F \propto \dfrac{1}{r^2}$

$$F_e = \frac{kq_1q_2}{r^2} \text{ where } \left[k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}\right] k$$

= coulomb's constant or electrostatic force constant.

$$k_{vaccum} = \frac{1}{4\pi\varepsilon_0}$$

Electric permittivity

1. Electric permittivity of vacuum (ε_0)

$$\varepsilon_0 = \frac{1}{4\pi k_{vaccum}}$$

$$= 8.85 \times 10^{-12} \frac{C^2}{N \times m^2}$$

2. Electric permittivity of medium (ε)

$$\boxed{\varepsilon = \varepsilon_0 \, \varepsilon_r}$$

3. Relative electric permittivity or dielectric constant of medium (ε_r) :

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = \frac{\text{electric permittivity of medium}}{\text{electric permittivity of vaccum}}$$

This is a unitless quantity.

Key Points

- Electric permittivity
- Relative permittivity
- Dielectric constant

Rack your Brain

Two point charges A and B, having charges +Q and –Q respectively, are placed at certain distance apart and force acting between them is F. If 25% charge of A is transferred to B, then find out force between the charges.

Ex. A charge Q is to be distributed on two conducting spheres. Find out values of charges on spheres so, that the repulsive force between them is maximum at fixed distance.

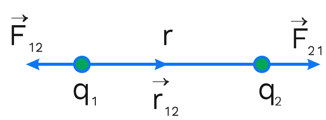
Sol. Let charge on 1st sphere = q, then charge on 2nd sphere = Q − q.

Force between them $F = \frac{kq(Q-q)}{r^2}$

For maximum value $\frac{\partial F}{\partial q} = 0 \Rightarrow q = \frac{Q}{2}$

Charge on both spheres should be $\frac{Q}{2}$.

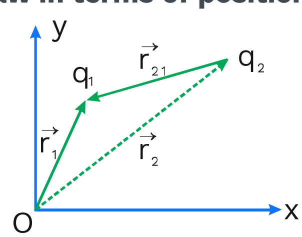
Coulomb's Law in Vector Form



$\vec{F}_{12}$ = force on q_1 due to q_2 $= \frac{kq_1q_2}{r^2}\hat{r}_{21}$

$\vec{F}_{21} = \frac{kq_1q_2}{r^2}\hat{r}_{12}$ (here $\hat{r}_{12}$ is unit vector from q_1 to q_2)

Coulomb's Law in terms of position vector



$$\vec{F}_{12} = \frac{kq_1q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$$

Principle of superposition

- The force is a two-body interaction, i.e., electrical force between two-point charges does not depend on presence or absence of other charges and therefore, the principle of superposition is applicable, i.e., force on a charged particle because of no. of point charges is the resultant of all forces due to individual point charges, i.e.,

Concept Reminder

$\vec{F}_{12} = -\vec{F}_{21}$

Coulomb's law follow newton's third law of motion. e.g., they form an action-reaction pair.

Rack your Brain

Two positive ions, each carrying a charge q, are separated by a distance d. If F is the force of repulsion between the ions. Find out the number of electrons missing from each ion.

$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + ...$

- **Note:** Principle of superposition is not valid in c case of nuclear force because nuclear force is many body interaction.
- When many charges are interacting, the net force on a given charge is vector sum of all the forces applied on it by all other charges individually.

$$F = \frac{kq_0q_1}{r_1^2} + \frac{kq_0q_2}{r_2^2} + ... + \frac{kq_0q_i}{r_i^2} + ... \frac{kq_0q_n}{r_n^2}$$

in vector form $\vec{F} = kq_0 \sum_{i=1}^{n} \frac{q_i}{r_i^2} \hat{r}_i$

Some Important Points Related To Coulomb's Law and Electric Force

(i) This law is based on physical observations and is not logically derivable from any other concept. Till now, experiments reveal its universal nature.

The law is analogous to Newton's gravitation law: $F = G\frac{m_1m_2}{r^2}$ but:

(a) Electric force between any charged particles is very stronger than gravitational force, i.e., $F_E >> F_G$. This is why, we neglect F_G in presence of F_E.

(b) Coulomb's force can be attractive or repulsive while force of gravitational is always attractive in nature.

(c) Electric force does depend on the nature of medium present between the charges while gravitational force does not depend on media.

(d) This force has an action-reaction pair, i.e., the amount of force which one charge applies on the other is equal and opposite to the force which the other charge applies on the first.

(ii) The electrostatic force is conservative in nature, i.e., work done to move a point charge once round a closed path under this force is always zero.

Rack your Brain

Suppose the charge of a proton and an electron differ slightly. One of them is $-e$, the other is $(e + \Delta e)$. If the net electrostatic force and gravitational force between two hydrogen atoms placed at a distance d(much greater than atomic size) apart is zero. Then find order of Δe.

Concept Reminder

Electric force is much stronger than gravitational force but gravitational force is more dominant in nature.

(iii) The net Coulomb's force on two charged particles in free space and in a medium filled up-to infinity are

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} \text{ and } F' = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2}.$$

So $\frac{F}{F'} = \frac{\varepsilon}{\varepsilon_0} = K$,

(iv) Dielectric constant (K) of a medium is numerically equal to the ratio of the force on two-point charges in free space to that in the medium filled up to infinity.

(v) The law expresses the force between two-point charges at rest. While applying this law to the case objects having extends bodies of finite size, care should be taken in considering the complete charge of a body to be concentrated at its centre as this is true only for spherical charged body, that too for external point.

(vi) Electric force between two charges does not depend on neighbouring charges.

Ex. Consider the distance between two equal point charges is made two times and their magnitude of their individual charges are also doubled, find what would happen to the force between them?

Sol. $F = \frac{1}{4\pi\varepsilon_0} \times \frac{q \times q}{r^2}$ ... (1)

Again, $F' = \frac{1}{4\pi\varepsilon_0}\frac{(2q)(2q)}{(2r)^2}$

or $F' = \frac{1}{4\pi\varepsilon_0}\frac{4q^2}{4r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = F$

So, the force will remain the same.

Ex. A particle of mass m carrying charge $+q_1$ and is revolving around a fixed charge $-q_2$ in a circle of radius r. Find the period of revolution.

Concept Reminder

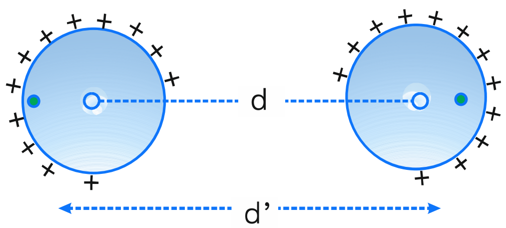
In this problem, electrostatic force between $+q_1$ and $-q_2$ is providing centripetal force for circular motion.

Sol. $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = mr\omega^2 = \frac{4\pi^2 mr}{T^2}$

$$T^2 = \frac{(4\pi\varepsilon_0)r^2(4\pi^2 mr)}{q_1q_2} \quad \text{or} \quad T = 4\pi r\sqrt{\frac{\pi\varepsilon_0 mr}{q_1q_2}}$$

Ex. The force of repulsion between two-point charges is F, when these are at a separation of 1 m. Suppose point charges are replaced by spheres of radii 25 cm having the charge equal as that of point charges. The distance between their centres is 1 m, then compare the force of repulsion in 2 cases.

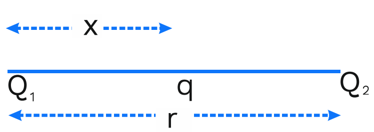
Sol. In 2nd case due to mutual repulsion, the effective distance between their centre of charges will be increased (d' > 0) so force of repulsion decreases as $F \propto \frac{1}{d^2}$.



Equilibrium of System of Charged Particles

In equilibrium net electric force on every charged particle is zero. The equilibrium of two charged particles, under the action of Colombian forces alone can never be stable.

(1) Equilibrium to Three Point Charges



(a) Two charges must be of like nature as

$$F_q = \frac{KQ_1q}{x^2} + \frac{KQ_2q}{(r-x)^2} = 0$$

Concept Reminder

For equilibrium

(i) If both charges are of same nature then third charge should be put between them.

(ii) If both charges are of opposite nature, then third charge should be put outside and close to less magnitude charge.

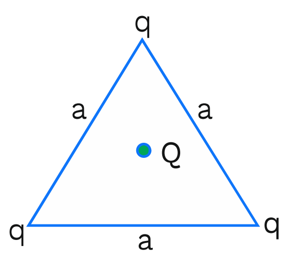
(b) Third charge should be of unlike nature as

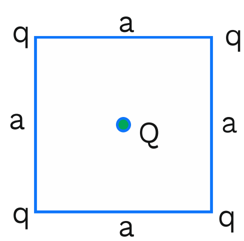
$$F_{Q_1} = \frac{KQ_1q}{r^2} + \frac{KQ_1q}{x^2} = 0$$

Therefore $$x = \frac{\sqrt{Q_1}}{\sqrt{Q_1} + \sqrt{Q_2}} r$$

and $$q = \frac{-Q_1Q_2}{(\sqrt{Q_1} + \sqrt{Q_2})^2}$$

(2) Equilibrium of Symmetric Geometrical Point Charged Particle system





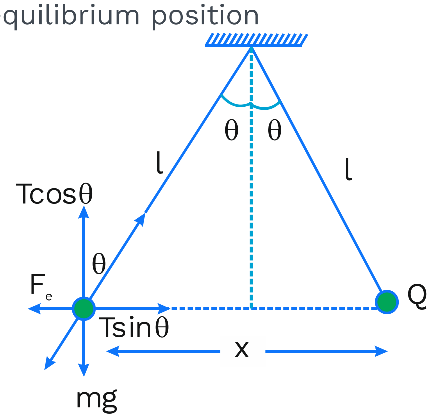
Value of Q at centre for which system to be in state of equilibrium.

(i) For equilateral triangle $Q = \frac{-q}{\sqrt{3}}$

(ii) For square $Q = \frac{-q(2\sqrt{2} + 1)}{4}$

(3) Equilibrium of Suspended Point Charge System

For equilibrium position



$$T\cos\theta = mg \text{ and } F_e = \frac{kQ^2}{x^2} = T\sin\theta$$

Rack your Brain

A charge q is placed at the centre of the line joining two equal charges Q. Find the value of q such that system of the three charges will be in equilibrium.

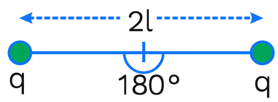
$$\Rightarrow \quad \tan\theta = \frac{F_e}{mg} = \frac{kQ^2}{x^2 mg}$$

(a) If θ is small then $\tan\theta \approx \sin\theta = \frac{x}{2l} = \frac{kQ^2}{x^2 mg}$

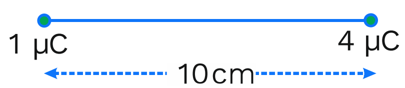
$$\Rightarrow x^3 = \frac{2kQ^2 l}{mg}$$

$$\Rightarrow x = \left[\frac{Q^2 l}{2\pi \epsilon_0 mg}\right]^{\frac{1}{3}}$$

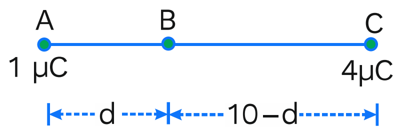
(b) If whole set up is taken into an artificial satellite ($g_{eff} \cong 0$) then $T = F_e = \frac{kq^2}{4l^2}$



Ex. Find out position of point from 1µC at which when +q charge placed experiences no force.



Sol. In this case B is equilibrium point.



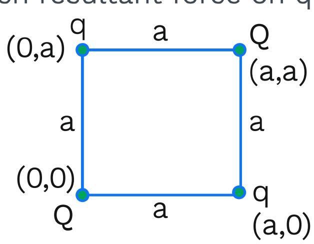
Net force on +q = 0

$F_{BA} = F_{BC}$

$$\frac{k(1)q}{d^2} = \frac{k(4)q}{(10-d)^2} \Rightarrow \left(\frac{10-d}{d}\right)^2 = 4$$

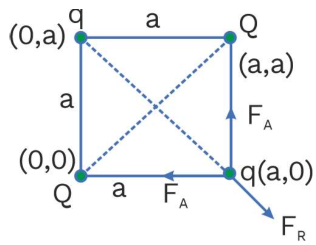
$$\frac{10-d}{d} = 2 \Rightarrow d = \frac{10}{3}\text{cm}$$

Ex. For the system shown in figure, find value of Q for which resultant force on q is 0.



Sol. For force on q to be 0, charges q and Q should be of opposite nature.

Total attraction force on q due to charges Q = Repulsion force on q due to q



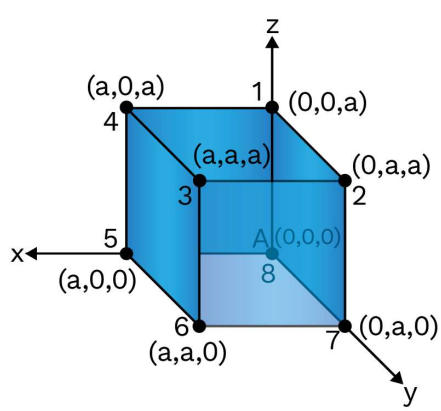
$$\sqrt{2}F_A = F_R \Rightarrow \frac{\sqrt{2}kQq}{a^2} = \frac{kq^2}{(\sqrt{2}a)^2}$$

$$\Rightarrow \sqrt{2}Q = \frac{q}{2} \Rightarrow q = 2\sqrt{2}Q$$

since q and Q must be of opposite nature, therefore $q = -2\sqrt{2}Q$.

Ex. Consider a cube with point charges q on each of its vertices. Find the force exerted on any of the charges because of rest of the seven charges.

Sol. The resultant force on particle A can be given by vector sum of force experienced by this particle because of all the other charges on vertices of this cube. In this we use vector form of coulomb's law $\vec{F} = \frac{kq_1q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$



From given figure the different forces acting on A are given as $\vec{F}_{A_1} = \frac{kq^2\left(-a\hat{k}\right)}{a^3}$

$$\vec{F}_{A_2} = \frac{kq^2\left(-a\hat{j} - a\hat{k}\right)}{\left(\sqrt{2}a\right)^3},\ \vec{F}_{A_3} = \frac{kq^2\left(-a\hat{i} - a\hat{j} - a\hat{k}\right)}{\left(\sqrt{3}a\right)^3};$$

$$\vec{F}_{A_4} = \frac{kq^2\left(-a\hat{i} - a\hat{k}\right)}{\left(\sqrt{2}a\right)^3}$$

$$\vec{F}_{A_5} = \frac{kq^2\left(-a\hat{i}\right)}{a^3}, \vec{F}_{A_6} = \frac{kq^2\left(-a\hat{i} - a\hat{j}\right)}{(\sqrt{2}a)^3},\ \vec{F}_{A_7} = \frac{kq^2\left(-a\hat{i}\right)}{a^3}$$

$\Rightarrow$ Net force on A

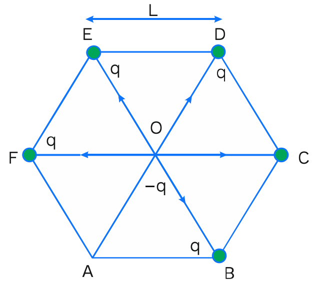
$$\Rightarrow \vec{F} = \vec{F}_{A_1} + \vec{F}_{A_2} + \vec{F}_{A_3} + \vec{F}_{A_4} + \vec{F}_{A_5} + \vec{F}_{A_6} + \vec{F}_{A_7}$$

$$\Rightarrow \vec{F} = \frac{-kq^2}{a^2}\left[\left(\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{2}} + 1\right)\left(\hat{i} + \hat{j} + \hat{k}\right)\right]$$

Ex. Five-point charges, each of value +q are placed on 5 vertices of a regular hexagon of side L m. Determine the magnitude of the force on a point charge of value -q C placed at the centre of the hexagon?

Sol. Suppose there had been a 6th charge +q at the remaining vertex of hexagon, force due to all 6th charges on −q at O will be zero (as the forces due to individual charges will balance each other).

Now if $\vec{f}$ is the force due to sixth charge and $\vec{F}$ due to remaining give charges.



$$\vec{F} + \vec{f} = 0 \Rightarrow \vec{F} = -\vec{f} \Rightarrow F = f = \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{L^2} = \frac{q^2}{4\pi\varepsilon_0 L^2}$$

Rack your Brain

Two identical charged spheres suspended from a common point by two massless strings of length l, are initially at a distance d(d << l) apart because of their mutual repulsion. The charges begin to leak from both the spheres at a constant rate. As a result, the spheres approach each other with a velocity v. Then find out how v varies as a function of the distance x between the spheres.

Electric Field

Electric field is that region around any charged particle or charged body in which if another charge is placed, it experiences electrostatic force.

Electric Field Intensity $\vec{E}$:

An electric field intensity at a point is the electrostatic force experienced by a unit positive point charge both in magnitude and direction.

If a test charge q_0 is placed at a point in an electric field and experiences a force $\vec{F}$ due to some charges (called source charges), the electric field intensity at that point due to source is given by $\vec{E} = \dfrac{\vec{F}}{q_0}$.

If the $\vec{E}$ is to be determined practically then the test charge q_0 must be small in magnitude otherwise it will affect the distribution of charge on the source which is responsible to produce the electric field and hence modify the quantity which is measured.

Concept Reminder

If we know value of electric field in a region then we can find electrostatic force on any charge q.

$\vec{F} = q\vec{E}$

Properties of Electric Field Intensity $\vec{E}$:

(i) Electric field intensity is a vector quantity. Its direction is same as the force experienced by +ve charge.

(ii) Direction of electric field due to +ve charge is always away from it while due to –ve charge, always coming towards it.

(iii) Its S.I. unit of an electric field is N/C.

(iv) It has dimensional formula $[MLT^{-3}A^{-1}]$

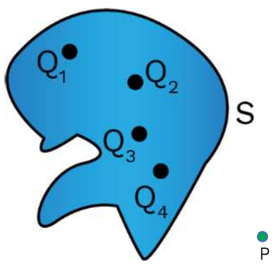
(v) Electric force experienced by charge q placed in a region of electric field at a point where the electric field intensity $\vec{E}$ is given by $\vec{F} = q\vec{E}$.

Electric force on a charge is in the same direction of electric field on +ve charge and in opposite direction on a –ve charge.

(vi) This field obeys the superposition principle, i.e., the field intensity at a point due to a system of charges is vector sum of the field intensities due to individual point charges.

i.e. $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$

(vii) It is produced by source charges. The electric field will be a fixed value at a point unless we change the distribution of source charges.



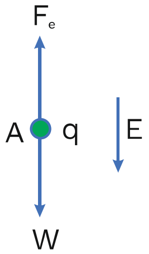
Ex. Electrostatic force on a –3μC charge placed at point P due to a system S of fixed-point charges as shown in figure is $\vec{F} = (21\hat{i} + 9\hat{j})\ \mu N$.

Then find out electric field intensity at point P due to S.

Sol. $\vec{F} = q\vec{E} \Rightarrow (21\hat{i} + 9\hat{j})\mu N = -3\mu C \times \vec{E}$

$$\Rightarrow \vec{E} = -7\hat{i} - 3\hat{j}\frac{N}{C}$$

Ex. Find the electric field intensity which would be just sufficient to balance the weight of a particle of charge –10 μC and mass 10 mg. (given g = 10 ms^{-2})



Sol. The force experienced by point charge q in an electric field $\vec{E}$ is

$$\vec{F}_e = q\vec{E}$$

So, according to given problem:
[W = weight of particle]

$$|\vec{F}_e| = |\vec{W}|$$

i.e., $|q|\ E = mg$

i.e., $E = \frac{mg}{|q|} = 10N/C.,$ in downward direction.

Ex. Determine electric field intensity at point A (0, 1m, 2m) due to a point charge –20μC situated at point $B(\sqrt{2}m, 0, 1m)$.

Sol. $\vec{E} = \frac{KQ}{|\vec{r}|^3}\vec{r} = \frac{KQ}{|\vec{r}|^2}\hat{r}$

$\because \vec{r} = \vec{r}_A - \vec{r}_B$

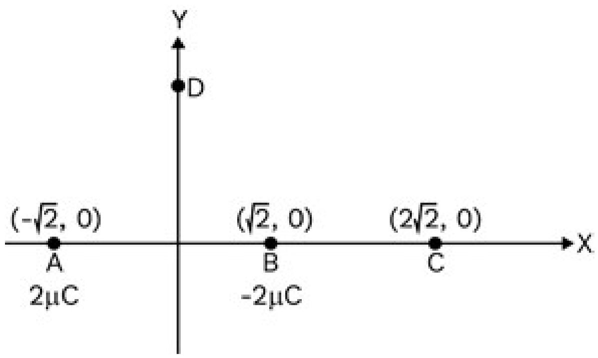
$\vec{r} = (-\sqrt{2}\hat{i} + \hat{j} + \hat{k})$

$|\vec{r}| = \sqrt{(\sqrt{2})^2 + (1)^2 + (1)^2} = 2$

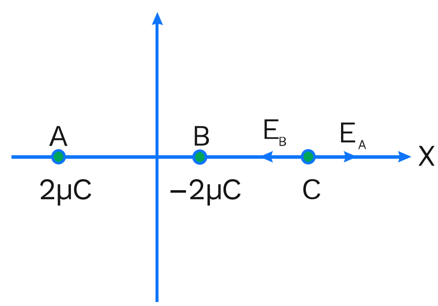
$$\vec{E} = \frac{9\times10^{9}\times(-20\times10^{-6})}{8}\,(-\sqrt{2}\hat{i}+\hat{j}+\hat{k})$$

$$= -22.5\times10^{3}\,(-\sqrt{2}\hat{i}+\hat{j}+\hat{k})\text{N / C}$$

Ex. Two-point charges 2μC and –2μC are placed at points A and B as shown in given figure. Find out electric field intensity at points C and D. [All the distance are measured in meter].



Sol. Electric field at point C:



(E_A, E_B are magnitudes only and arrows represent directions)

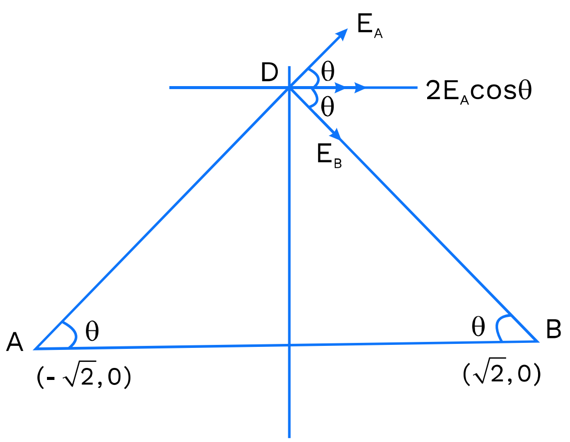
Electric field due to positive charges is away from it while due to negative charge, it is towards the charge. It is clear then $E_B > E_A$.

$\therefore \quad E_{Net} = (E_B - E_A)$ towards negative X-axis

$= \dfrac{K(2\mu C)}{(\sqrt{2})^2} - \dfrac{K(2\mu C)}{(3\sqrt{2})^2}$ towards negative X-axis

$= 8000\ (-\hat{i})\text{N / C}$

The electric field at point 'D' :



Here magnitude of both charges are same and also AD = BD

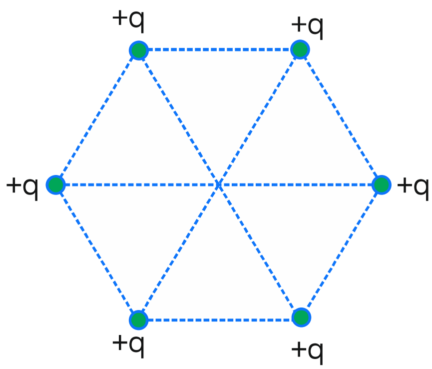
So, $E_A = E_B$

Vertical components of $\vec{E}_A$ and $\vec{E}_B$ cancel each other while horizontal components are in the same direction.

So, $E_{net} = 2E_A \cos\theta = \dfrac{2K(2\mu C)}{2^2}\cos 45°$

$$= \frac{K \times 10^{-6}}{\sqrt{2}} = \frac{9000}{\sqrt{2}}\ N/C\ .$$

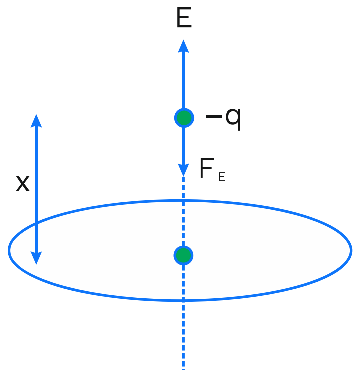
Ex. Six equal point charges are placed at corners of a regular hexagon of side a. What is electric field intensity at the centre of hexagon?



Sol. Zero (By symmetry)

Ex. A +ve charge Q is distributed uniformly over a circular ring of radius a. A point particle having a mass m and a negative charge –q, is placed fixed on its axis at a distance y from its centre. Find the force on the particle. Assuming y<<a, find the time period of oscillation of the particle if it is released from there. (Neglect gravity)

Sol. When the negative charge is shifted at a distance x from the centre of the ring along its axis then force acting on the point charge due to the ring:



$F_E = qE$ (towards centre)

$$= q\left[\frac{KQx}{(a^2 + x^2)^{3/2}}\right]$$

If a >> x then

$a^2 + x^2 \simeq a^2$

$$\therefore\ F_E = \frac{1}{4\pi\varepsilon_0}\frac{Qqx}{a^3} \text{ (Towards centre)}$$

Since, restoring force $F_E \propto x$, therefore motion of charge the particle will be S.H.M.

Time period of SHM :

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\left(\frac{Qq}{4\pi\varepsilon_0 a^3}\right)}} = \left[\frac{16\pi^3\varepsilon_0 ma^3}{Qq}\right]^{1/2}$$

Charge Distribution

1. Linear charge density (λ) **:**

- Charge per unit length represents linear charge density.

$$\lambda = \frac{q}{L}\ C/m$$

- Charge on any small element of length dx is as $dq = \lambda dx$

Concept Reminder

Linear charge density (λ) concept is used for infinitely long charged wire and ring.

2. Surface charge density (σ)**:**

- Charge per unit surface area represents surface charge density.

$$\sigma = \frac{q}{A}\ C/m^2$$

- Charge on any small element of surface area dA is as

$$dq = \sigma dA$$

3. Volume charge density (ρ):

Charge per unit volume represents volume charge density.

$$\rho = \frac{Q}{V}\ C/m^3$$

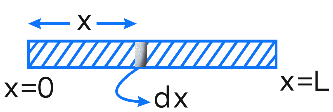
- Charge in volumetric small element of volume dV is as

$$dq = \rho dV$$

Note : Equal amount of charge [upto maximum limit] can be given to solid conducting sphere and hollow conducting sphere of same radii.

Ex. A thin rod of length L is from x = 0 to x = L with linear charge density of $\lambda = \lambda_0 x$ where λ_0 is a constant and x is the distance from origin. Calculate total charge distributed on this rod.

Sol.



Charge on small element =

$$dq = \lambda dx$$

$$dq = \lambda_0 x dx$$

Total charge

$$Q = \lambda_0 \int_0^L x dx = \lambda_0 \left[\frac{x^2}{2}\right]_0^L$$

$$\Rightarrow \quad Q = \frac{\lambda_0 L^2}{2}$$

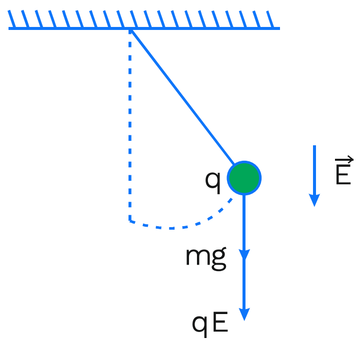
Concept Reminder

- Surface charge density concept is used for charged plate, spherical shell, conducting sphere etc.
- Volume charge density concept is used for charged solid non-conducting sphere.

Time period of Simple pendulum in uniform Electric field:

Case-I

If uniform electric field is directed vertically downwards.

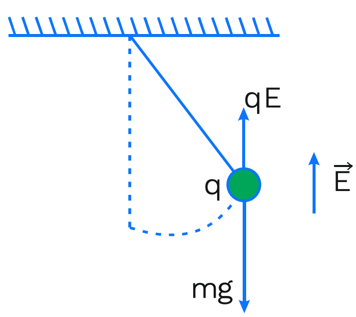


$$g_{eff} = g + \frac{qE}{m}$$

$$T' = 2\pi\sqrt{\frac{l}{\left(g + \frac{qE}{m}\right)}}$$

Case-II:

If uniform electric field is directed vertically upwards



$$g_{eff} = g - \frac{qE}{m}$$

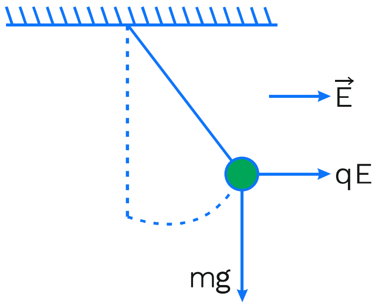
$$T' = 2\pi\sqrt{\frac{l}{\left(g - \frac{qE}{m}\right)}}$$

Concept Reminder

- If $\vec{E}$ is applied vertically downward or horizontally then time period of simple pendulum decreases.
- If $\vec{E}$ is applied vertically upward, then time period of simple pendulum increases.

Case-III:

If uniform electric field is directed horizontally.

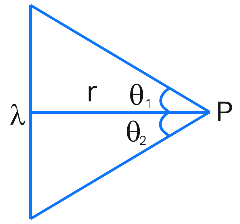


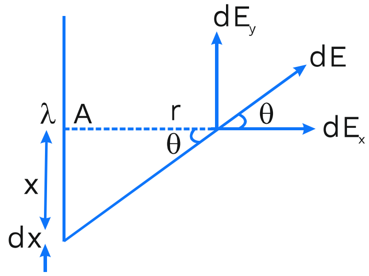
$$g_{eff} = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}$$

$$T' = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

Electric Field Due to Uniformly Charged Wire

(i) Line Charge of Finite Length: Derivation of expression for electric field intensity at a point because of line charge of finite size of uniform linear charge density λ. The normal distance of the point from the line charge is r and lines joining ends of line charge distribution make angle θ_1 and θ_2 with the perpendicular line.





Consider a small element dx on line charge distribution at distance x from point A (see

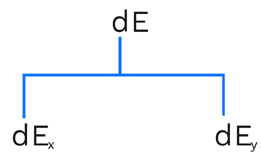
Concept Reminder

- If we choose a point such that $\theta_1 = \theta_2$ then, y-components will cancel each other and net electric field will be in x-direction only.

figure). The charge of this element will be $dq = \lambda dx$. Due to this charge (dq), the intensity of electric field at the point P is dE.

Then $dE = \dfrac{K(dq)}{r^2 + x^2} = \dfrac{K(\lambda dx)}{r^2 + x^2}$

There will be two components of this field :



$$E_x = \int dE_x = \int dE\cos\theta = \int \frac{K\lambda dx}{r^2 + x^2}.\cos\theta$$

Assuming, $x = r\tan\theta \Rightarrow dx = r\sec^2\theta.d\theta$

So, $$E_x = \int_{-\theta_2}^{+\theta_1} \frac{K\lambda r\sec^2\theta.\cos\theta.d\theta}{r^2 + r^2\tan^2\theta}$$

$$= \frac{E\lambda}{r}\int_{-\theta_2}^{+\theta_1}\cos\theta.d\theta = \frac{K\lambda}{r}[\sin\theta_1 + \sin\theta_2]$$

Similarly y-component.

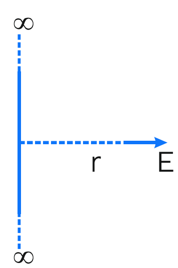
$$E_y = \frac{K\lambda}{r}\int_{-\theta_2}^{+\theta_1}\sin\theta.d\theta = \frac{K\lambda}{r}[\cos\theta_2 - \cos\theta_1]$$

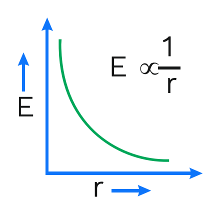
Net electric field at the point :

$$E_{net} = \sqrt{E_x^2 + E_y^2}$$

(ii) We can derive a result for infinitely long line charge :

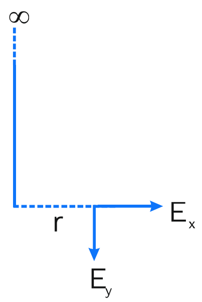
If we put $\theta_1 = \theta_2 = 90°$ we can get required result.





$$E_{net} = E_x = \frac{2K\lambda}{r}$$

(iii) For Semi-infinite wire:

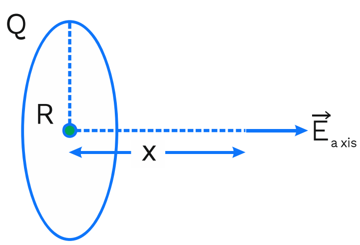


$\theta_1 = 90°$ and $\theta_2 = 0°$,

so, $E_x = \frac{K\lambda}{r}, E_y = \frac{K\lambda}{r}$

Electric field on the axis of a uniformly charged Ring.

- Ring of radius 'R' is charged uniformly with charge of 'Q' then electric field on its axis at distance x from centre is a following.



$$\boxed{E_{axis} = \frac{kQx}{(R^2 + x^2)^{3/2}}}$$

- Direction of electric field on axis of uniformly charged ring is along axis of ring.
  For positively charged ring is along axis away from centre.
  For negatively charged ring along axis towards centre.

1. At centre of ring.

$(x = 0)$

$E_{centre} = 0$

2. At nearby axial point (x << R)

$$E_{axis} = \frac{kQx}{(R^2 + x^2)^{3/2}} \simeq \frac{kQx}{R^3}$$

Concept Reminder

- For semi-infinite wire-
  $\vec{E} = E_x\hat{i} - E_y\hat{j}$
  $\vec{E} = \frac{K\lambda}{r}\hat{i} - \frac{K\lambda}{r}\hat{j}$
- Along the axis of charged ring, all components of electric field perpendicular to axis cancel each other.

3. For distant axial point (x >> R)

$$E_{axis} = \frac{kQx}{(R^2 + x^2)^{3/2}} \simeq E_{axis} = \frac{kQ}{x^2}$$

(Ring behaves as point charge)

4. At x = ∞

$$E_{\infty} = 0$$

5. Maximum electric field on the axis of uniformly charged Ring.

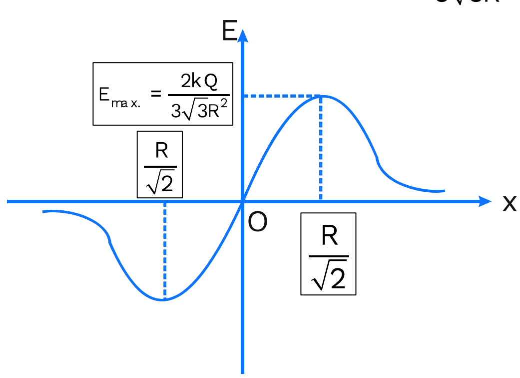
$$E_{axis} = \frac{kQx}{(R^2 + x^2)^{3/2}}$$

For maximum electric field $\frac{dE}{dx} = 0$

$$\frac{d}{dx}\left[\frac{Kqx}{(R^2 + x^2)^{3/2}}\right] = 0$$

$$\Rightarrow x = \pm\frac{R}{\sqrt{2}}$$

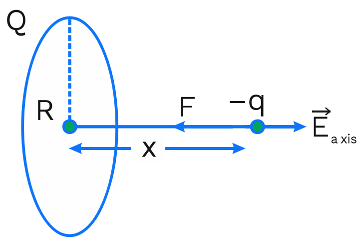
Maximum electric field is $E_{max.} = \frac{2kQ}{3\sqrt{3}R^2}$



- Centre of uniformly charged ring is the mean position of stable equilibrium for unlike charged particle [along axial displacement]
- When unlike charged particle placed at centre of uniformly charged ring is displaced along axis and released then charged particle will execute oscillatory motion and may execute SHM about centre of ring if displacement is small.

Concept Reminder

- As we move away from center of charged ring along axis, $\vec{E}$ increases till $\frac{R}{\sqrt{2}}$ distance then after this $\vec{E}$ decreases.



Restoring force $F = -q \times E_{axis}$

$$F = \frac{-kQqx}{(R^2 + x^2)^{3/2}}$$

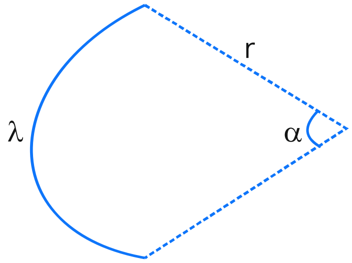
if axial displacement is small

$F \propto -x$ [SHM]

if axial displacement is not small force is not proportional to $-x$ then only oscillatory motion (not Simple harmonic motion)

ELECTRIC FIELD DUE TO UNIFORMLY CHARGED ARC

An arc of radius r is charged uniformly with linear charge density of λ and subtends angle of α at centre then electric field of centre of arc is as following.



$$E = \frac{2k\lambda}{r}\sin\left(\frac{\alpha}{2}\right) \text{ where } \lambda = \frac{q}{\text{length of arc}} = \frac{q}{\alpha r}$$

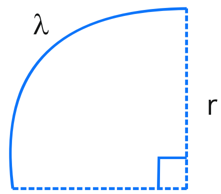
if charge on arc is q then

$$E = \frac{2kq}{\alpha r^2}\sin\left(\frac{\alpha}{2}\right)$$

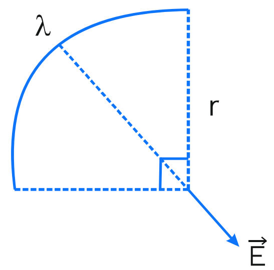
Concept Reminder

- Electric field due to semi-circular charged ring of density λ is $E = \frac{2k\lambda}{r}$.

Ex. Calculate electric field at centre of uniformly charged arc as shown in figure.



Sol.



Using the formula of electric field due to an arc.

$$E = \frac{2k\lambda}{r}\sin\left(\frac{\alpha}{2}\right)$$

Given that

$$E = \frac{2k\lambda}{r}\sin 45^\circ$$

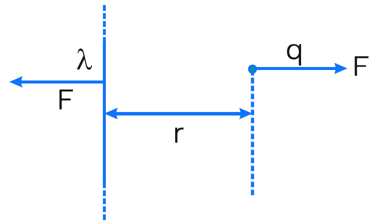
$$E = \frac{2k\lambda}{r}\times\frac{1}{\sqrt{2}}$$

In vector form $\vec{E} = \frac{k\lambda}{r}\left(\hat{i} - \hat{j}\right)$

Ex. A point charge 'q' is placed at a separation r from a very long charged thread of uniform linear charge density λ. What is total electric force experienced by the line charge due to the point charge (Neglect gravity).

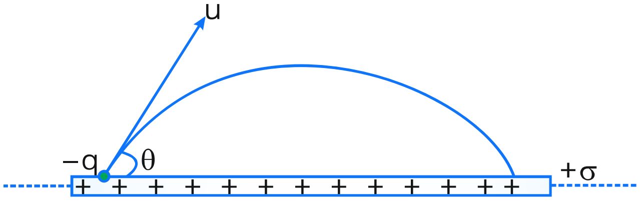
Sol. Force on charge q due to the thread,

$$F = \left(\frac{2K\lambda}{r}\right).q$$



By Newton's 3rd law, every action has equal and opposite reaction, so force on the thread = $\frac{2K\lambda}{r}.q$ (away from point charge)

Ex. An infinitely large plate of surface charge density $+\sigma$ is lying in horizontal xy plane. A particle having charges $-q_0$ and mass m is projected from the plate with velocity u making an angle θ with sheet. Find :

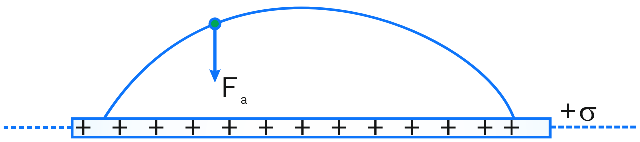


(i) The time taken by the particle to return on the plate.

(ii) Maximum height achieved by the particle.

(iii) At what distance will it strike the plate (Neglect gravitational force on the particle)

Sol.



Electric force acting on the particle

$$F_e = q_0E \Rightarrow F_e = (q_0)\left(\frac{\sigma}{2\varepsilon_0}\right) \text{ (downward)}$$

So, acceleration of the particle :

$$a = \frac{F_e}{m} = \frac{q_0\sigma}{2\varepsilon_0 m} \text{ (uniform)}$$

This acceleration will like 'g' (acceleration due to gravity)
So, the particle will perform projectile motion.

(i) $T = \dfrac{2u\sin\theta}{g} = \dfrac{2u\sin\theta}{\left(\dfrac{q_0\sigma}{2\varepsilon_0 m}\right)}$

(ii) $H = \dfrac{u^2\sin^2\theta}{2g} = \dfrac{u^2\sin^2\theta}{2\left(\dfrac{q_0\sigma}{2\varepsilon_0 m}\right)}$

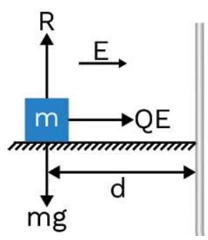
(iii) $R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{u^2\sin 2\theta}{\left(\dfrac{q_0\sigma}{2\varepsilon_0 m}\right)}$

Ex. A block of mass 'm' and charge Q is resting on a frictionless plane at a distance 'd' from fixed large non-conducting infinite sheet of uniform charge density (σ). suppose that collision of the block with the sheet is perfectly elastic, find the time period of oscillatory motion of the block. Is it SHM?

Sol. The situation is shown in Figure. Electric force due to sheet will accelerate this block towards the sheet giving it an acceleration. Acceleration will be uniform since, electric field E due to the sheet is uniform.

$$a = \frac{F}{m} = \frac{QE}{m}, \text{ where } E = \sigma / 2\varepsilon_0$$

As initially this block is placed at rest and acceleration is uniform, from 2nd equation of motion, time taken by this block to reach the wall can be calculated as



$$d = \frac{1}{2}at^2$$

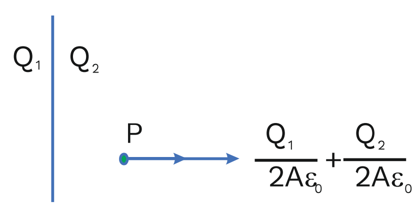
$$\Rightarrow \quad t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2md}{QE}} = \sqrt{\frac{4md\varepsilon_0}{Q\sigma}}$$

Since collision with the wall is perfectly elastic process, the block will come back with same speed and as now its motion is opposite to the acceleration, it will come to rest after travelling same distance d in same time t. After getting stop, it will again be accelerated towards the wall and so the block will execute oscillatory motion with 'span' d and time period.

However, as the restoring force F = QE is constant and not proportional to displacement x, the motion is not simple harmonic.

Ex. Consider an isolated infinite sheet having charge Q_1 on its one surface and charge Q_2 on its other surface. Prove that intensity of electric field at a point in front of sheet will be $\frac{Q}{2A\varepsilon_0}$, where $Q = Q_1 + Q_2$

Sol. Electric field at point P :

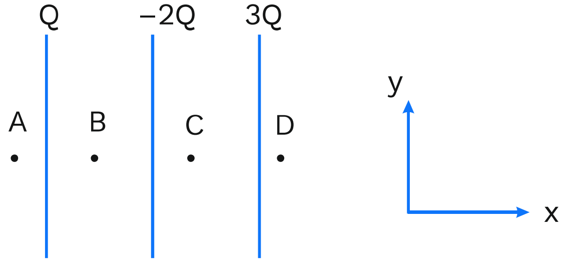


$$\vec{E} = \vec{E}_{Q_1} + \vec{E}_{Q_2}$$

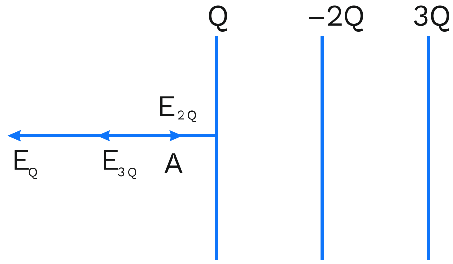
$$= \frac{Q_1}{2A\varepsilon_0}\hat{n} + \frac{Q_2}{2A\varepsilon_0}\hat{n} = \frac{Q_1 + Q_2}{2A\varepsilon_0}\hat{n} = \frac{Q}{2A\varepsilon_0}\hat{n}$$

[This result tells us that the net field due to a sheet depends only on the net charge of the sheet and does not depend on the distribution of charge on individual surfaces.]

Ex. Three large conducting parallel sheets are placed at a finite distance from each other as shown in figure. Find out electric field intensity at points A, B, C & D.



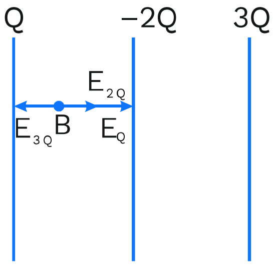
Sol. For point A



$$\vec{E}_{net} = \vec{E}_Q + \vec{E}_{3Q} + \vec{E}_{-2Q}$$

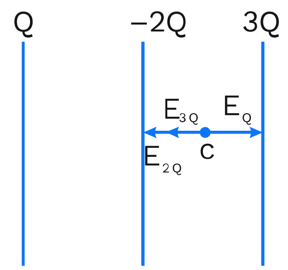
$$= -\frac{Q}{2A\varepsilon_0}\hat{i} - \frac{3Q}{2A\varepsilon_0}\hat{i} + \frac{2Q}{2A\varepsilon_0}\hat{i} = -\frac{Q}{A\varepsilon_0}\hat{i}$$

For point B :



$$\vec{E}_{net} = \vec{E}_{3Q} + \vec{E}_{-2Q} + \vec{E}_Q = -\frac{3Q}{2A\varepsilon_0}\hat{i} + \frac{2Q}{2A\varepsilon_0}\hat{i} + \frac{Q}{A\varepsilon_0}\hat{i} = 0$$

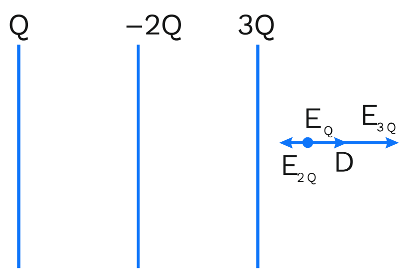
For point C :



$$\vec{E}_{net} = \vec{E}_Q + \vec{E}_{3Q} + \vec{E}_{-2Q}$$

$$= +\frac{Q}{2A\varepsilon_0}\hat{i} - \frac{3Q}{2A\varepsilon_0}\hat{i} - \frac{2Q}{2A\varepsilon_0}\hat{i} = -\frac{2Q}{A\varepsilon_0}\hat{i}$$

for point D :



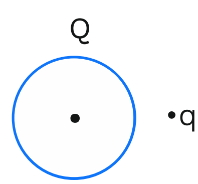
$$\vec{E}_{net} = \vec{E}_Q + \vec{E}_{3Q} + \vec{E}_{-2Q}$$

$$= +\frac{Q}{2A\varepsilon_0}\hat{i} + \frac{3Q}{2A\varepsilon_0}\hat{i} - \frac{2Q}{2A\varepsilon_0}\hat{i} = \frac{Q}{A\varepsilon_0}\hat{i}$$

Concept Reminder

- For two large conducting parallel sheets having equal charge density, electric field in between plates will be zero.
- For two large conducting parallel sheets having equal and opposite charge density, electric field outside plates will be zero.

Ex. Given figure shows a uniformly charged sphere of radius 'R' and total charge 'Q'. A point charge 'q' is placed outside the sphere at a distance r from centre of sphere. Determined below forces:



(i) Force acting on the point charge q due to the sphere.

(ii) Force acting on the sphere due to the point charge.

Sol. (i) Electric field at the position of point charge

$$\vec{E} = \frac{KQ}{r^2}\hat{r}$$

So, $\vec{F} = \frac{KqQ}{r^2}\hat{r}$

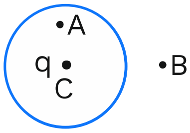
$\Rightarrow$ $|\vec{F}| = \frac{KqQ}{r^2}$

(ii) Since we know that every action has equal and opposite reaction so

$$\vec{F}_{sphere} = -\frac{KqQ}{r^2}\hat{r}$$

$$\Rightarrow |\vec{F}_{sphere}| = \frac{KqQ}{r^2}$$

Ex. Figure shows a uniformly charged thin sphere of total charge Q and radius R. A point charge q is also situated at the centre of the sphere. Find out the following:



(i) Force on point charge q

(ii) Electric field intensity at 'A'.

(iii) Electric field intensity at 'B'.

Sol. (i) Electric field at the centre of the uniformly charged hollow sphere is zero

So, force on charge q = 0

(ii) $\vec{E}_A = \vec{E}_{sphere} + \vec{E}_q = 0 + \frac{Kq}{r^2}; r = CA$

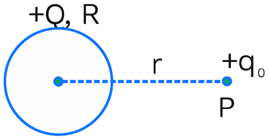
An electric field due to sphere is zero, because point lies inside the charged hollow sphere.

(iii) Electric field $\vec{E}_B$ at point $B = \vec{E}_{Sphere} + \vec{E}_q$

$$= \frac{KQ}{r^2}.\hat{r} + \frac{Kq}{r^2}.\hat{r} = \frac{K(Q+q)}{r^2}.\hat{r} \quad ; r = CB$$

Note: In this case, we can also suppose that the net charge of sphere is concentrated at the centre, to calculate of electric field at B.

Ex. A spherical shell having charge +Q (uniformly distributed) and a point charge $+q_0$ are placed as shown.



Find the force between shell and the point charge (r >>R).

Sol. Force on the point charge $+q_0$ due to the shell

$$= q_0\vec{E}_{shell} = (q_0)\left(\frac{KQ}{r^2}\right)\hat{r} = \frac{KQq_0}{r^2}\hat{r}$$

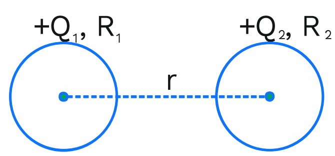
where $\hat{r}$, is unit vector along OP.

From action – reaction principle, force on the shell due to the point charge will

also be : $\vec{F}_{shell} = \frac{KQq_0}{r^2}(-\hat{r})$

Conclusion : To find the force on a hollow sphere due to outside charges, we can replace the sphere by a point charge kept at centre.

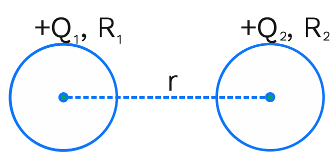
Ex. Find force acting between two shells of radius R_1 & R_2 which have uniformly distributed charges Q_1 and Q_2 respectively and distance between their centres is r.

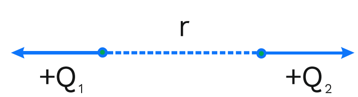


Sol. These shells can be replaced by point charges kept at centre,

So, force between them

$F = \frac{KQ_1Q_2}{r^2}$;



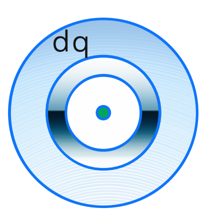


Electric field due to a uniformly charged solid sphere

Derivation of an expression for electric field due to solid sphere of radius ‘R’ and total charge ‘Q’ which is uniformly distributed in the volume, at a point which is at a distance ‘r’ from centre for given two cases.

(i) $r \geq R$ (ii) $r \leq R$

Assume an elementary concentric shell of charge dq. Due to this shell, the electric field at the point (r > R) will be :



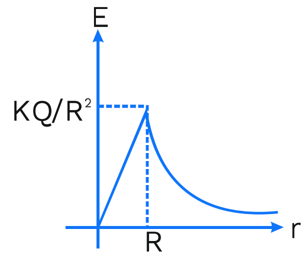
Concept Reminder

- Electric field at outside point due to any charged sphere will be similar to electric field of a point charge having same charge placed at center of sphere.

$$dE = \frac{Kdq}{r^2} \text{[from above result of hollow sphere]}$$

$$E_{net} = \int dE = \frac{KQ}{r^2}$$

For r < R, there will be no electric field due to shell of radius greater than r, so electric field at the point will be present only due to shell having radius less than r.



$$E'_{net} = \frac{KQ'}{r^2}$$

Here, $Q' = \frac{Q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3 = \frac{Qr^3}{R^3}$

$\therefore \quad E'_{net} = \frac{KQ'}{r^2} = \frac{KQr}{R^3}$; away from the centre.

Note : The electric field inside and outside the sphere is always in radius direction.

Ex. A non conducting solid sphere of radius R has uniform volume charge density has its centre at origin. What will be electric field intensity in vector form at following positions:

(i) $(R/\sqrt{2}, 0, 0)$ (ii) $\left(\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}}, 0\right)$

(iii) (R, R, 0)

Sol. (i) At $(R/\sqrt{2}, 0, 0)$: Distance of point from horizon $= \sqrt{(R/\sqrt{2})^2 + 0^2 + 0^2} = R/\sqrt{2} < R$, so point lies at the sphere, so

$$\vec{E} = \frac{\rho \vec{r}}{3\varepsilon_0} = \frac{\rho}{3\varepsilon_0}\left[\frac{R}{\sqrt{2}}\hat{i}\right]$$

(ii) At $\left(\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}}, 0\right)$; distance of point from centre is $\sqrt{(R/\sqrt{2})^2 + (R/\sqrt{2})^2 + 0^2} = R$

The point lies at the surface of sphere, therefore

$$\vec{E} = \frac{KQ}{R^3}\vec{r} = \frac{K\frac{4}{3}\pi R^3 \rho}{R^3}\left[\frac{R}{\sqrt{2}}\hat{i} + \frac{R}{\sqrt{2}}\hat{j}\right]$$

Rack your Brain

The electric field at a distance $\frac{3R}{2}$ from the centre of a charged conducting spherical shell of radius R is E. Find out the electric field at a distance $\frac{R}{2}$ from the the centre of the sphere.

$$= \frac{\rho}{3\varepsilon_0}\left[\frac{R}{\sqrt{2}}\hat{i} + \frac{R}{\sqrt{2}}\hat{j}\right]$$

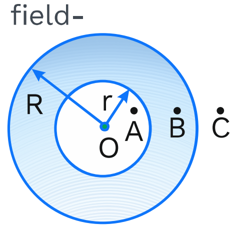
(iii) At (R, R, 0); distance of the point from centre

$$= \sqrt{R^2 + R^2 + 0^2} = \sqrt{2}R$$

So, point is outside the sphere

$$\vec{E} = \frac{KQ}{R^3}\vec{r} = \frac{K\frac{4}{3}\pi R^3\rho}{(\sqrt{2}R)^3}\left[R\hat{i} + R\hat{j}\right] = \frac{\rho}{6\sqrt{6}\varepsilon_0}\left[R\hat{i} + R\hat{j}\right]$$

Ex. A uniformly charged non-conducting solid sphere of uniform volume charge density and radius R is having a concentric spherical cavity of radius r. Find out electric field-



(i) Point A (ii) Point B

(iii) Point C

Sol. (i) For point 'A' :

Here, consider the solid part of sphere to be made of large no. of spherical shells which have charge uniformly distributed on its surface. Since point 'A' lies inside all spherical shells so electric field intensity due to all shells will be zero. $\vec{E_A} = 0$

(ii) For point 'B' :

All the spherical shells for which point 'B' lies inside will make electric field zero at point 'B'. So electric field is because of charge present from radius r to OB.

$$\text{So, } \vec{E_B} = \frac{K\frac{4}{3}\pi(OB^3 - r^3)\rho}{OB^3}\overrightarrow{OB} = \frac{\rho}{3\varepsilon_0}\frac{[OB^3 - r^3]}{OB^3}\overrightarrow{OB}$$

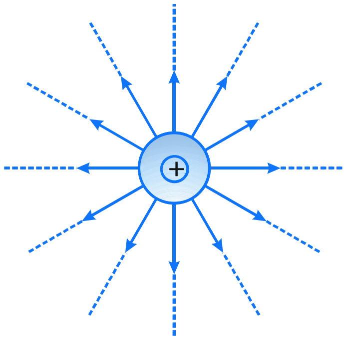
(iii) For point C,

similarly we can say that for all the shell point C lies outside the shell

$$\vec{E_C} = \frac{K\frac{4}{3}\pi(R^3 - r^3)}{[OC]^3}\overrightarrow{OC} = \frac{\rho}{3\varepsilon_0}\frac{R^3 - r^3}{[OC]^3}\overrightarrow{OC}$$

Properties of Electric Field Lines

1. **(a)** The field-lines of an isolated positive charge are straight lines emerging from the charge radially outward to infinity. (See in figure a).

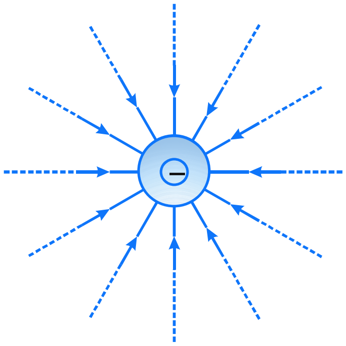


[a]

Concept Reminder

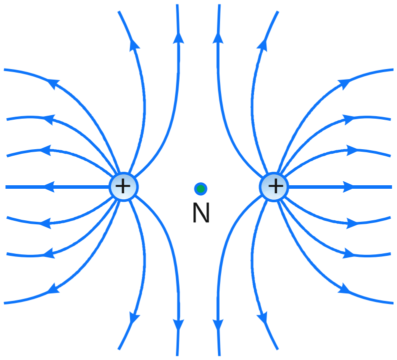
- Electric field lines are pictorial representation of electric field in a region.

(b) The field-lines of an isolated negative point charge are straight-lines ending at the charge radially inward from infinity (See figure b).



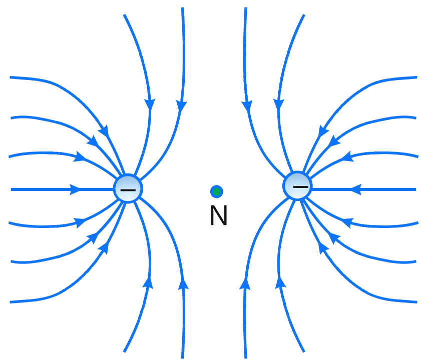
[b]

(c) The field—lines for the two equal positive point charges is shown in figure c.



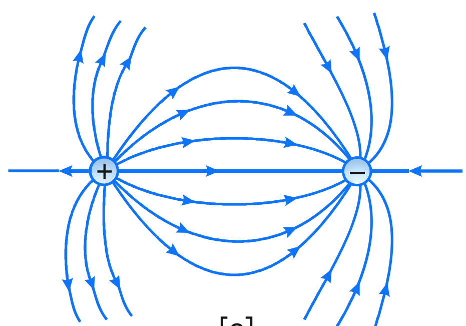
[c]

(d) The field—lines for the two equal negative point charges is shown in figure d.



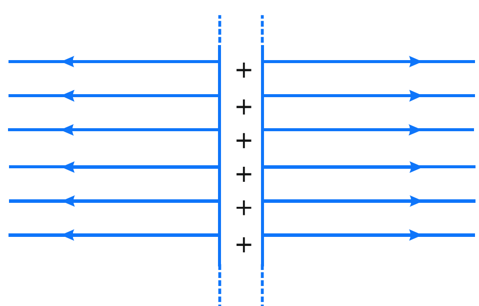
[d]

(e) The field lines for a dipole (two equal and opposite charges) shown in figure e.



[e]

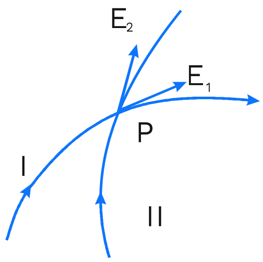
(f) The field lines for a large plane sheet is shown in figure f.



[f]

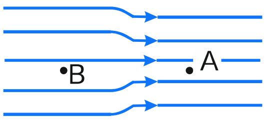
2. The number of lines originating or terminating on a charge is proportional to the magnitude of charge. In SI units it has been taken that $\left(\frac{1}{\varepsilon_0}\right)$ electric lines are associated with one coulomb of charge.

3. No two field-lines can intersect each other, because if they do so, at the point of intersection of field lines two tangents can be drawn which would mean two directions of the force at the point, which is impossible.



[g]

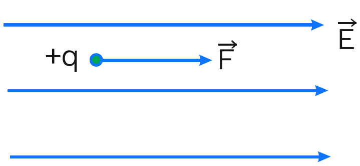
4. Due to conservative nature of electric field, the electrostatic field lines will not form any closed loop.
5. Field lines have tendency to contract in length (longitudinal contraction) like a stretched elastic string. This explains attraction between two unlike charges. The lateral pressure between field-lines explains the mutual repulsion between like charges.
6. Field-lines are always perpendicular at the surface of a conductor, but they never enter inside the conductor because there is no electrostatic field inside the material of a conductor.
7. The number of field-lines crossing through per unit area normal to the lines, is proportional to the magnitude of the electric field at the point. Hence crowded field lines in a region indicate the large magnitude of the field in the region. In the given figure, the magnitude of the field at A is greater than that at B.



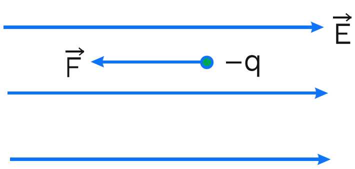
8. The tangent drawn at any point of a field-lines gives the direction of the force on a positive charge at that point.

Motion of charged particle in uniform electric field.

1. if charge particle is at rest.



[+ve charge]



[−ve charge]

Concept Reminder

- If charged particle is initially at rest then it will move along direction of electric field if positive or it will move opposite to electric field if negative.

In both cases force F = ma = qE

$\Rightarrow \; a = \dfrac{qE}{m}$

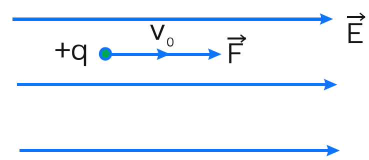
$\therefore$ Charge particle will accelerate in straight line.

2. If charge is moving along straight line parallel to uniform electric field.

(a) For positive charge

Velocity after time t

$$v = v_0 + \frac{qE}{m}t$$

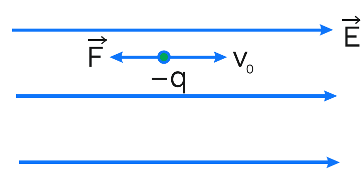


[+ve charge]

(b) For negative charge

Velocity after time t

$$v = v_0 - \frac{qE}{m}t$$



[−ve charge]

3. If charge particle is initially moving perpendicular to uniform Electric field.

Along x-axis,

$u_x = v_0, a_x = 0, S_x = x, v_x = v_0$

$$S = ut + \frac{1}{2}at^2 \Rightarrow x = v_0 t$$

Along y-axis,

$$u_y = 0, a_y = \frac{qE}{m}, y = \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

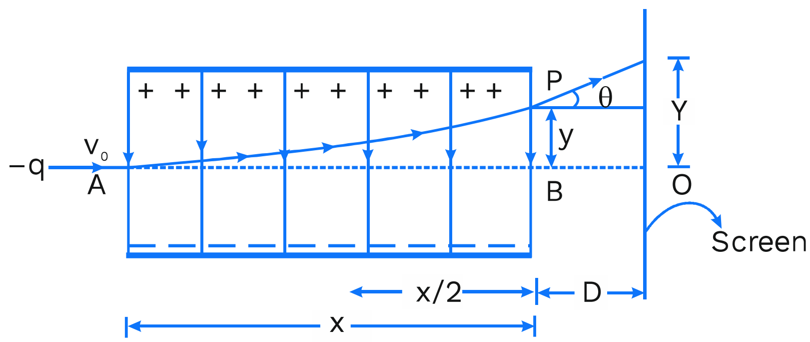
$$y = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x}{v_0}\right)^2 \Rightarrow y = \frac{1}{2}\frac{qEx^2}{mv_0^2}$$

$\therefore \quad y \propto X^2$ i.e., path parabolic

The deflection on the screen Y = $(y + D\tan\theta)$

Concept Reminder

- The path of charged particle in uniform electric field may be straight line or parabolic depending upon initial velocity of charged particle.



$$\tan\theta = \frac{v_y}{v_x} = \frac{qEt}{mv_0} = \frac{qEx}{mv_0^2}$$

So, $$\tan\theta = \frac{qEx^2}{mv_0^2} \times \frac{2}{x}$$

$\Rightarrow$ $$\tan\theta = \frac{y}{\left(\frac{x}{2}\right)}$$

It means that the tangent at point P intersect the line AB at its mid-point i.e. at distance $\frac{x}{2}$ from B.

$\Rightarrow$ $$Y = \left(\frac{x}{2} + D\right)\tan\theta$$

Gauss's Law:

It relates the total electric flux of an electric field through a closed surface to the total charge enclosed by that surface. According to Gauss law, the total flux linked with a closed surface is $1/\varepsilon_0$ times the charge enclosed by the closed surface,

Mathematically $\oint_s \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0}$

Definition of Gaussian Surface:

A Gaussian surface is defined an imaginary surface which enclose a charge, such that the electric flux is perpendicular to each point on the surface and the electric potential at every point is equal.

Rack your Brain

A charge Q is enclosed by a Gaussian spherical surface of radius R. If the radius is doubled, then what will be the effect on electric flux through surface.

Definition

- Electric flux is defined as total number of electric field line passing through an area.

Some Important Points regarding Gaussian Surface

(i) Gauss's law is valid for any closed surface, no matter what its shape or size.

(ii) The term charge in the Gauss's law includes the total charge enclosed by the surface. The charges may be situated anywhere inside the surface.

(iii) In the situation when the surface is chosen such that there are some charges inside and some outside the surface. The term charge in Gauss's law represents only the total charge inside the surface.

(iv) You can select any Gaussian surface and apply Gauss's law. However, don't care about that the Gaussian surface pass through any discrete charge.

(v) Gauss's law is generally useful towards a much easier calculation of the electric field when the system has some symmetry. This is facilitated by the choice of a appropriate Gaussian surface.

(vi) This law is based on the inverse square dependence on distance contained in the Coulomb's law. Any breaking of Gauss's law will indicate departure from the inverse square law.

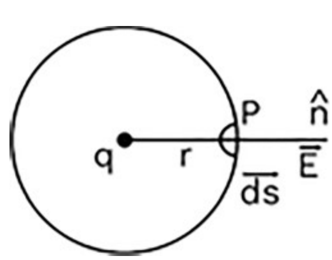
Key Points

- Gauss's law
- Gaussian surface
- Electric flux

Gauss's law and Coulomb's law are equivalent:

We can prove Coulomb's law and Gauss's law using each other by assuming one obtained to prove another.

To prove Gauss's law by using Coulomb's law assume a hypothetical spherical surface known as Gaussian surface of radius 'r' with point charge 'q' at its centre. According to Coulomb's law intensity at a point P on the surface will be equal to $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r}$



Electric flux linked with area $\vec{ds}$

$$\vec{E} \cdot \overrightarrow{ds} = \frac{q}{4\pi\varepsilon_0 r^3} \vec{r} \cdot \overrightarrow{ds}$$

direction of $\vec{r}$ and $\overrightarrow{ds}$ are same i.e.,

$$\vec{r} \cdot \overrightarrow{ds} = r\, ds \cos 0° = r\, ds$$

So, $\vec{E} \cdot \overrightarrow{ds} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q}{r^2} ds$

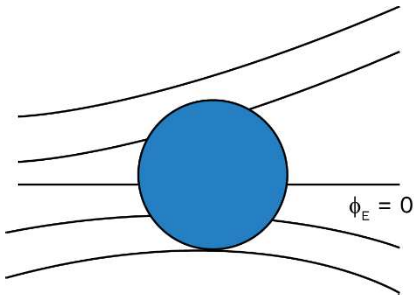
or $\oint_s \vec{E} \cdot \overrightarrow{ds} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q}{r^2} ds$

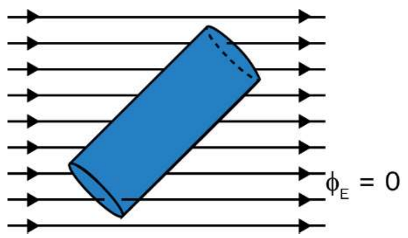
For all points on sphere r = constant

$$\oint \vec{E} \cdot \overrightarrow{ds} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \oint ds = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} (4\pi r^2) = \frac{q}{\varepsilon_0}$$

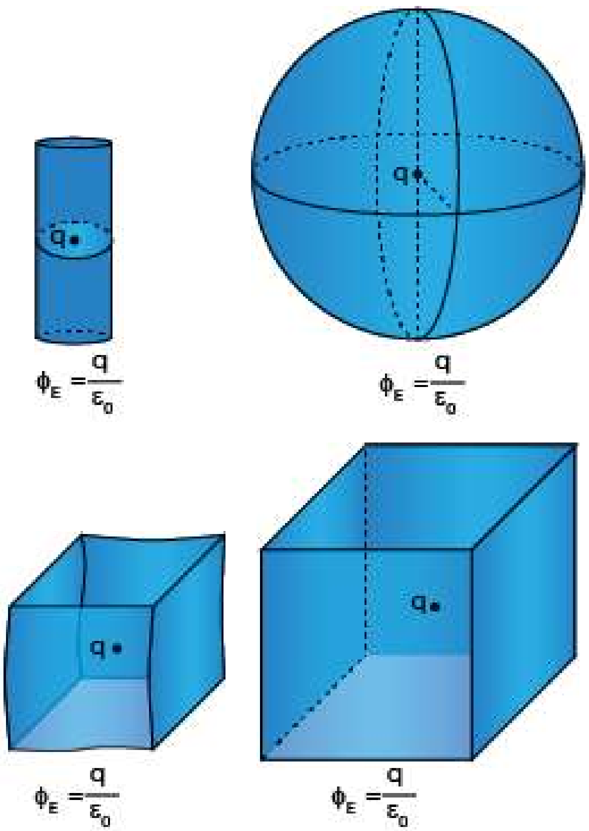
$$\left[\text{as} \oint ds = 4\pi r^2\right]$$

- The results are true for any random surface provided the surface is closed.
- It relates the total flux linked with a closed surface to the charge covered by the closed surface.
- If a closed body which doesn't enclose any charge is placed in either uniform on non-uniform electric field then total flux linked with it will be zero.





- If a closed body encloses a charge q, total flux linked with the body doesn't depend on the shape and size of the body and position of charge inside it.



Ex. If a point charge 'q' is placed at the centre of a cube, then find the flux linked (a) with the cube? (b) with each face of the cube?

Sol. (a) By to Gauss's law total flux linked with a closed surface is $(1/\varepsilon_0)$ times the charge enclosed and for this condition the closed body cube is enclosing a charge 'q' so, total flux $\phi_T = \dfrac{q}{\varepsilon_0}$

(b) Because cube is a symmetrical body with six faces and the point charge is placed at its centre, so flux linked with each face will be $\phi_F = \dfrac{1}{6}(\phi_T) = \dfrac{q}{6\varepsilon_0}$

Note : (i) Here flux linked with cube or one of its faces doesn't depend on the side of cube.

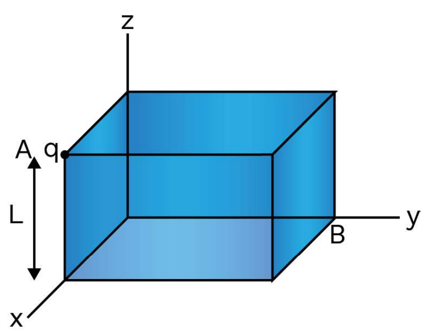
(ii) If charge is not placed at the centre of cube (but anywhere inside it), then total flux will remain same, but the flux linked with different faces will be different.

Ex. If a point charge 'q' is at one corner of a cube, what is the flux linked with the cube?

Sol. For this case by placing 3 cubes at three sides of given cube and four cubes above, the charge will be at the centre. Therefore, the flux linked with each cube will be $(1/8)^{th}$ of the flux $\frac{q}{\varepsilon_0}$

$\therefore$ Flux associated with given cube = $\frac{q}{8\varepsilon_0}$

Ex. A point charge 'q' is placed at one corner of a cube of edge 'a'. Calculate the amount of the flux through each face of the cube?



Sol. At a corner, eight cubes can be placed symmetrically, flux linked with each cube due to a charge 'q' at the corner will be $\frac{q}{8\varepsilon_0}$

.

For the faces passing through the edge 'A', electric field E will be parallel to area of face and so electric flux through these three faces will be 0.

Since, the cube has six faces and flux linked with three faces (through A) is zero, so flux linked with other three face will be $\left(\frac{q}{8\varepsilon_0}\right)$.

The other three faces are symmetrical so flux linked with each of the three faces passing through B will be, $\frac{1}{3}\times\left[\frac{1}{8}\left(\frac{q}{\varepsilon_0}\right)\right]=\frac{1}{24}\frac{Q}{\varepsilon_0}$.

Rack your Brain

A charge Q μC is placed at the centre of a cube. Find out the flux coming out from each face.

Ex If charges $\frac{q}{2}$ and 2q are placed at the centre of face and at the corner, of a cube. Then find the total flux through cube.

Sol. Electric flux through cube, when $\frac{q}{2}$ is placed at the centre face, is

$$\phi = \frac{q/2}{2\varepsilon_0} = \frac{q}{4\varepsilon_0}$$

Flux through cube, when 2q is placed at the corner of cube, is $\phi_2 = \frac{2q}{8\varepsilon_0} = \frac{q}{4\varepsilon_0}$

Total flux = $\phi_1 + \phi_2 = \frac{q}{4\varepsilon_0} + \frac{q}{4\varepsilon_0} = \frac{1}{2}\frac{q}{\varepsilon_0}$

Rack your Brain

A hollow cylinder has a charge q coulomb within it. If ϕ is the electric flux in units of volt meter associated with the curved surface, then find the flux linked with the plane surface in units of V-m.

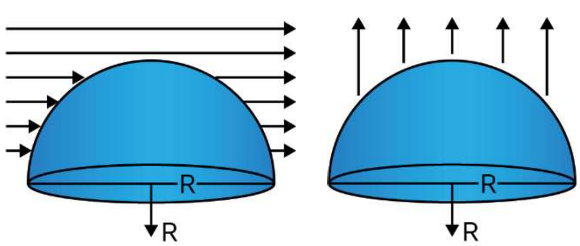
Ex. Electric flux entering a closed surface is 2000 V-m. Flux leaving that surface is 8000 V-m. Find the charge inside surface.

Sol. Net flux = (8000 – 2000) = 6000 V–m

$$\phi = \frac{q}{\varepsilon}$$

$\Rightarrow$ q = (6000) (8.85×10^{-12}) = 0.053 μC.

Ex. A hemispherical shaped body is placed in a uniform electric field E. What is the flux linked with the curved surface, if field is (a) parallel to base figure A (b) perpendicular to base figure B.



Concept Reminder

- The flux entering a surface is negative and flux leaving surface is positive.

Sol. Considering the hemispherical body as a closed body with a curved surface and a plane base the flux linked with the body will be zero as it does not enclose any charge, i.e.,

$\phi = \phi_{CS} + \phi_{PS} = 0$ ...(i)

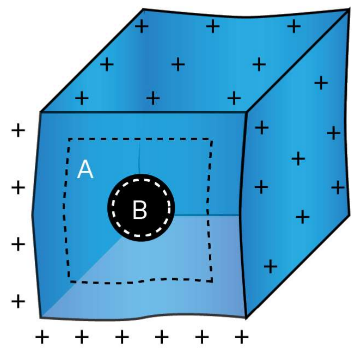
(a) When field is parallel to base, the flux linked with base

$\phi_{PS} \times E \times \pi R^2 \cos 90° = 0$ Substituting this value of ϕ_{PS} in equation (i), we get $\phi_{CS} = 0$

(b) When field is perpendicular to base, the flux linked with base

$$\phi_{PS} = E \times \pi R^2 \cos 180^\circ = -\pi R^2 E.$$

Ex. Find the field in the cavity, if a conductor having a cavity is charged? Does the result depend on the shape and size of cavity of conductor?



Sol. When a conductor is charged, charge resides on its outer surface, so for Gaussian surface A in the conductor or B in the cavity from Gauss's law we have,

$$\oint \vec{E} \cdot \vec{ds} = \frac{1}{\varepsilon_0}(0) \text{ i.e., } E = 0.$$

i.e., inside a conductor or in a cavity in a conductor electric field vanishes. This effect is called **'electrostatic shielding'** and is independent of shape and size of cavity of conductor.

Ex. A square of side 20 cm is enclosed by a spherical surface having 80 cm radius. Square and sphere have the same centre. Four charges $+2 \times 10^{-6}$ C, -5×10^{-6} C, -3×10^{-6} C, $+6 \times 10^{-6}$ C are placed at the 4 corners of a square, then out going total flux from spherical surface in $N\text{–}m^2/C$ will be

Sol. Since, $\phi_{enc.} = (2 \times 10^{-6} - 5 \times 10^{-6} - 3 \times 10^{-6} + 6 \times 10^{-6}) = 0$

Charge enclosed is zero, so total flux is zero.

Concept Reminder

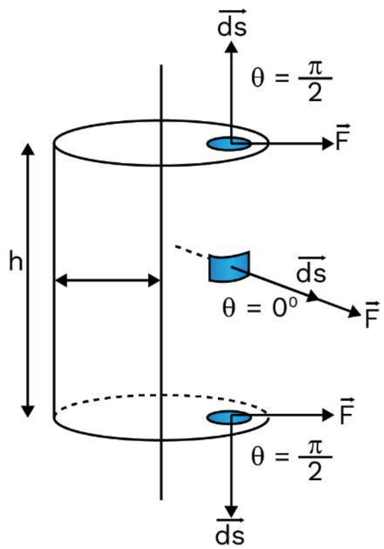
- Gauss law is applicable for any closed surface but to find electric field using gauss law, surface should be symmetrical about charge distribution.

Applications of Gauss Law:

Gauss law is used to find electric field of symmetrical charge distributions. Few of the cases are discussed here.

(a) Electric field due to a uniformly charged line:

Gauss law is useful in finding electric field intensity because of symmetrical charge distributions. We take a gaussian surface which is cylindrical in safe of radius r which encloses a line charge of length h with line charge density λ.

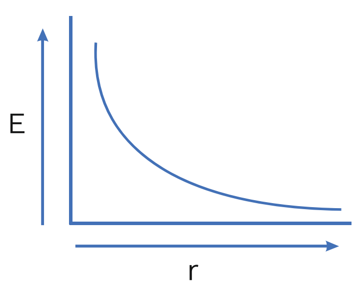


According to Gauss Law

$$\oint \vec{E} \cdot \vec{ds} = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow \int_{\text{Cylindrical Surface}} \vec{E}.\vec{ds} + \int_{\text{I Circular Surface}} \vec{E}.\vec{ds} + \int_{\text{II Circular Surface}} \vec{E}.\vec{ds} = \frac{\lambda h}{\varepsilon_0}$$

$$\Rightarrow \int_{\text{Cylindrical Surface}} Eds\cos 0^0 + \int_{\text{I Circular Surface}} Eds\cos\frac{\pi}{2} + \int_{\text{II Circular Surface}} Eds\cos\frac{\pi}{2} = \frac{\lambda h}{\varepsilon_0}$$



$$\Rightarrow E(2\pi rh) = \frac{\lambda h}{\varepsilon_0} \quad \text{So,} \quad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Rack your Brain

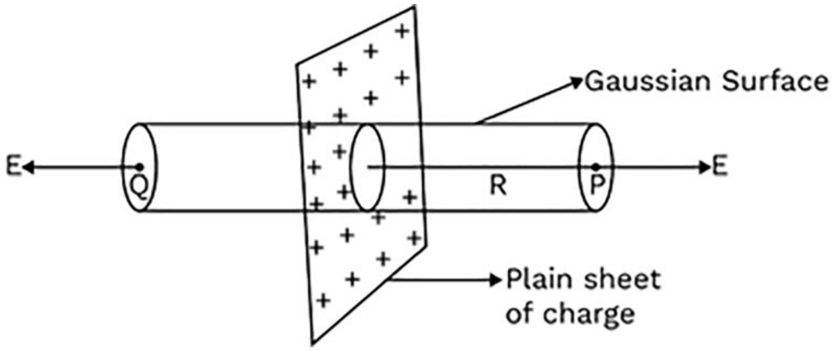
Two parallel infinite line charges with linear charge densities $+\lambda$ C/m and $-\lambda$ C/m are placed at a distance of 2R in free space. What is the electric field mid-way between the two line charges?

(b) Electric field due to a thin infinite plane sheet of charge:

To determine electric field due to the plane sheet of charge at any point 'P' distant 'R' from it, select a cylinder of area of cross-section A through the point 'P' as the Gaussian surface. The electric flux due to the electric field of the uniformly charged plane sheet passes only through the two circular parts of the cylinder.

Suppose surface charge density = σ

> **Concept Reminder**
>
> - Electric field due to conducting sheet of charge density σ is $\frac{\sigma}{\varepsilon_0}$ and due to non-conducting sheet is $\frac{\sigma}{2\varepsilon_0}$.



According to gauss law

$$\oint \vec{E} \cdot \vec{ds} = q_{in} / \varepsilon_0$$

$$\int_{\substack{\text{I circular} \\ \text{surface}}} E\,ds\cos\theta + \int_{\substack{\text{II circular} \\ \text{surface}}} E\,ds\cos\theta + \int_{\substack{\text{cylindrical} \\ \text{surface}}} E\,ds\cos\theta = \frac{\sigma A}{\varepsilon_0}$$

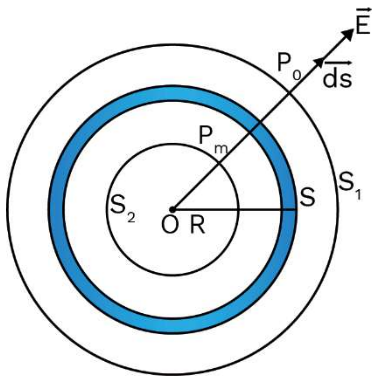
or $\quad EA + EA + 0 = \frac{\sigma A}{\varepsilon_0} \quad \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$

The magnitude of the electric field intensity due to an infinite plane sheet of charge doesn't depend upon the distance from the sheet. Consider the sheet is +vely charged, the direction of the field is perpendicular to the sheet and directed away from sheep on both sides. But for a –vely charged sheet, the field is directed normally inwards on both sides of the sheet.

(c) Electric field intensity due to an uniformly charged spherical shell:

Let us consider a thin shell of radius R carrying a charge Q on its surface.

(i) at a point P_0 outside the shell (r > R)



According to gauss law

$$\oint_{S_1} \vec{E}_0.\overrightarrow{ds} = \frac{Q}{\varepsilon_0} \text{ or } E_0\,(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$E_0 = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{\sigma}{\varepsilon_0}\frac{R^2}{r^2}$$

Where the surface charges density

$$\sigma = \frac{\text{total charge}}{\text{surface area}} = \frac{Q}{4\pi R^2}$$

The value of electric field at any point outside the shell is similar as if the entire charge is concentrated at centre of shell.

(ii) at a point P_s on surface of shell (r = R)

Field at surface is obtained by substituting r = R in above result,

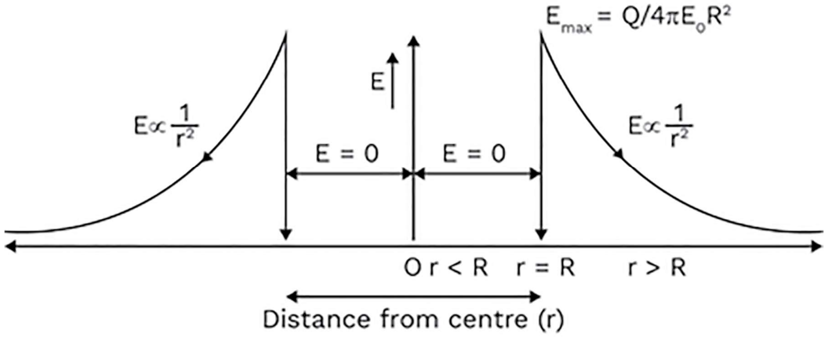
$$E_S = \frac{Q}{4\pi\varepsilon_0 R^2} = \frac{\sigma}{\varepsilon_0}.$$

(iii) at a point P_{in} inside the shell (r < R)

According to gauss law $\oint_{S_2} \vec{E}.\overrightarrow{ds} = \frac{q_{in}}{\varepsilon_0}$ Since, enclosed charge $q_{in} = 0$

Therefore, E_{in} is zero.

i.e., the electric field inside the spherical shell is always zero.



(d) Intensity of electric field due to a spherical an uniform spherical charge distribution:

Let us consider a spherical uniformly charge distribution of radius R in which net charge Q is uniformly distributed throughout the volume.

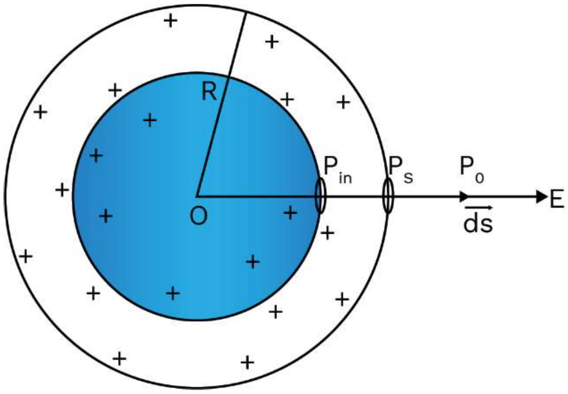
The charge density

$$\rho = \frac{\text{total charge}}{\text{total volume}} = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{3Q}{4\pi R^3}$$

(i) At a point P_0 outside the sphere (r > R)

according to gauss law

$$\oint \vec{E}_0.\vec{ds} = \frac{Q}{\varepsilon_0} \text{ or } E_0(4\pi r^2) = \frac{Q}{\varepsilon_0}$$



or $E_0 = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{\rho}{3\varepsilon_0}\left(\frac{R^3}{r^2}\right)$

(ii) At a point P_S on surface of sphere (r = R)

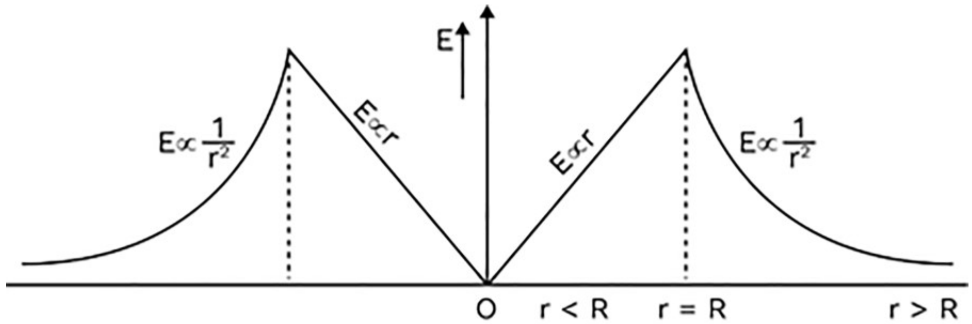
$$E_S = \frac{Q}{4\pi\varepsilon_0 R^2} = \frac{\rho}{3\varepsilon_0}R$$

(iii) At a point P_{in} inside the sphere (r < R)

According to gauss law

$$\oint \vec{E}_{in}.d\vec{s} = \frac{q_{in}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\rho.\frac{4}{3}\pi r^3 = \frac{Qr^3}{\varepsilon_0 R^3}$$

$$E_{in}(4\pi r^2) = \frac{Qr^3}{\varepsilon_0 R^3} \quad \text{or} \quad E_{in} = \frac{Qr}{4\varepsilon_0 R^3} = \frac{\rho}{3\varepsilon_0}r$$



Ex. The charge density of a metallic plate which is uniformly charged is given by $\sigma = -2\mu\frac{C}{m^2}$. What is the distance from which a 50eV electron must be projected so that it does not strike the plate?

Sol. The electric field due to metallic sheet $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n} = K(4\pi\sigma)\hat{n}$

where $\hat{n}$ is unit vector perpendicular to plate

$$\vec{E} = 9\times10^9 \times 4\times\pi(-2\times10^{-6})\hat{n}$$

$$= 72\pi\times10^3(-\hat{n})\,N/C$$

Force on electron due to this field

$$= q\vec{E} = 72\pi\times10^{+3}(-1.6\times10^{-19})(-\hat{n})$$

$$= 72\times1.6\times\pi\times10^{-16}\hat{n}\,N/C$$

Suppose electron be left from a distance d then,

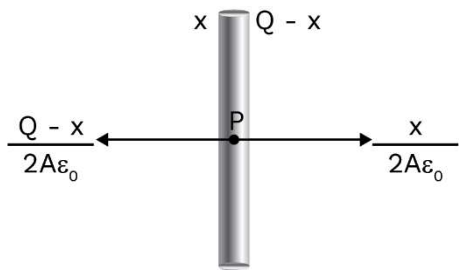
F.d = energy of electron

or $\quad 72\times1.6\times\pi\times10^{-16}\times d = 50\times1.6\times10^{-19}$

or $\quad d = 0.221\times10^{-3}\,m$

Ex. Prove that if an isolated large conducting sheet is given a charge then the charge distributes equally on its two surfaces.

Sol. Suppose there is x charge on left side of sheet and Q−x charge on right side of sheet. Therefore, point P lies inside the conductor so $E_P = 0$



$$\frac{x}{2A\varepsilon_0} - \frac{Q-x}{2A\varepsilon_0} = 0 \Rightarrow \frac{2x}{2A\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

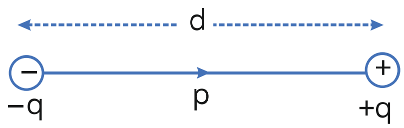
$$\Rightarrow x = \frac{Q}{2}$$

$$Q - x = \frac{Q}{2}$$

Hence, charge in equally distributed on both sides.

Electric Dipole & Dipole Moment

A system that consists of two equal and opposite charges separated by a small distance is termed an electric dipole.



For Example : $Na^+ Cl^-$, $H^+ Cl^-$etc.

An isolated atom can not be a dipole because centre of +ve charge coincides with centre of –ve charge. But if atom is placed in an electric field, then the positive and negative centres are displaced relative to each other and atom become a dipole.

Dipole Moment:

The product of the magnitude of charge and distance between them is called the dipole moment.

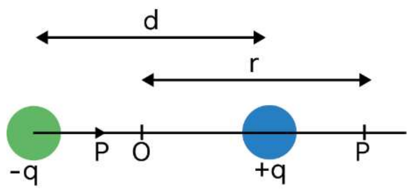
- It is a vector quantity and its directed from negative to positive charge.
- Unit: Coulomb – metre (C-m)
- Dimension: $[M^0L^1T^1A^1]$
- It is denoted by $\vec{p}$ and $\vec{p} = q\vec{d}$

Electric Field of a Dipole:

Electric field of a dipole is a superposition of field of positive and negative charges of dipole.

(a) At an Axial Point

Consider a point P on axis of dipole at a distance r from centre of dipole as show in figure.



Electric field at P due to negative charge

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(r+\frac{d}{2}\right)^2}(-\hat{r}),\quad E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-d/2)^2}\hat{r}$$

Total electric field at axial point P is $\vec{E}_a = \vec{E}_1 + \vec{E}_2$

$$\vec{E}_a = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(r-d/2)^2} - \frac{1}{(r+d/2)^2}\right)\hat{r}$$

$$\text{or}\quad \vec{E}_a = \frac{q}{4\pi\varepsilon_0}\frac{2.r.d}{(r^2-d^2/4)^2}\hat{r} = \frac{2pr}{4\pi\varepsilon_0(r^2-d^2/4)^2}\hat{r}$$

When r >> d, $\vec{E}_a = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}\hat{r}$

The axial field is parallel to dipole moment.

Ex. A point charge which is placed at any point on the axis of an electric dipole at a distance very large distance experiences a force F. Find the force experienced by this point charge when its distance gets doubled from the dipole.

Sol. Force acting on a point charge in dipole field varies as $F \propto \frac{1}{r^3}$ where r is the distance of point charge from the centre of dipole. Hence if r makes double so new force $F' \propto \frac{F}{8}$.

Concept Reminder

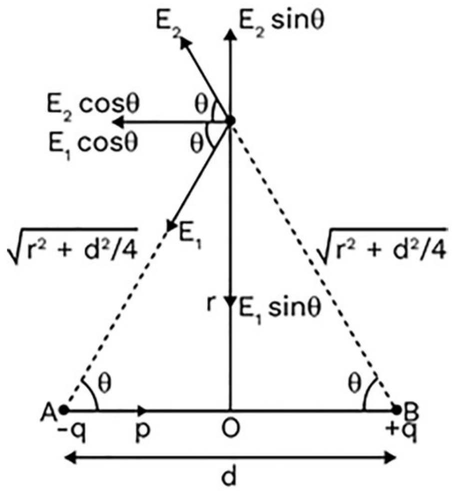
- Net charge of dipole is zero but electric field and potential due to dipole is non-zero.

Key Points

- Dipole
- Axial point
- Equatorial point

(b) At an Equatorial Point

Consider a point P distant r from centre of a dipole on its equatorial axis,



Electric field at P due to negative charge $E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2 + d^2/4)}$

Electric field at P due to positive charge $E_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r^2 + d^2/4}\right)$

Fields E_1 and E_2 are equal in magnitude.

Resolving E_1 and E_2 into two components one along OP and other perpendicular to OP we find $E_1 \sin\theta = E_2 \sin\theta$

Total field

$$E_{Eq} = E_1\cos\theta + E_2\cos\theta = 2E_1\cos\theta = 2E_2\cos\theta$$

$$E_{Eq} = \frac{q}{4\pi\varepsilon_0}\frac{1}{(r^2 + d^2/4)}\frac{d/2}{\sqrt{r^2 + d^2/4}}$$

$$= \frac{q.d}{4\pi\varepsilon_0(r^2 + d^2/4)^{3/2}} = \frac{p}{4\pi\varepsilon_0(r^2 + d^2/4)^{3/2}}$$

If r >> d, $\vec{E}_{Eq} = \frac{1}{4\pi \in_0}\frac{p}{r^3}(-\hat{r})$ i.e. field at equatorial point is antiparallel to dipole moment.

The electric field at axial point is parallel to dipole moment vector.

The electric field at equatorial point is antiparallel to dipole moment vector.

The ratio of field at axial point to field at equatorial point is $E_a : E_{Eq} = 2:1$.

The dipole field $E \propto \frac{1}{r^3}$ decreases more rapidly as compared to field of a point charge $E \propto \frac{1}{r^2}$.

Ex. An the electric field due to a short dipole at a distance r, on an axial line, from its mid-point is the same as that of electric field at a distance r', on the equatorial line, from its mid-point. Determine the ratio $\frac{r}{r'}$.

Sol. Given ; $\frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r'^3}$

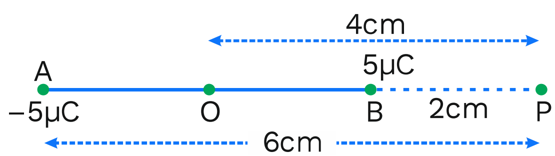
or $\frac{2}{r^3} = \frac{1}{r'^3}$ or $\frac{r^3}{r'^3} = 2$

or $\frac{r}{r'} = 2^{1/3}$

Ex. Two charges each having magnitude 5 μC but opposite in nature, are placed 4 cm apart. Find the electric field intensity of a point that is at a distance 4 cm from the mid-point on an axial line of the dipole.

Sol. We cannot use formula of short dipole here because distance of the point is comparable to the distance between the two-point charges.

$q = 5 \times 10^{-6}C$, $a = 4 \times 10^{-2}$ m, $r = 4 \times 10^{-2}$ m



$$E_{res} = E_+ + E_- = \frac{K(5\mu C)}{(2cm)^2} - \frac{K(5\mu C)}{(6\,cm)^2}$$

$$= 10^8\ NC^{-1}$$

(c) Electric field at an Arbitrary Point

We resolve dipole moment p in two components one along r and another perpendicular to r. The radial component of electric field

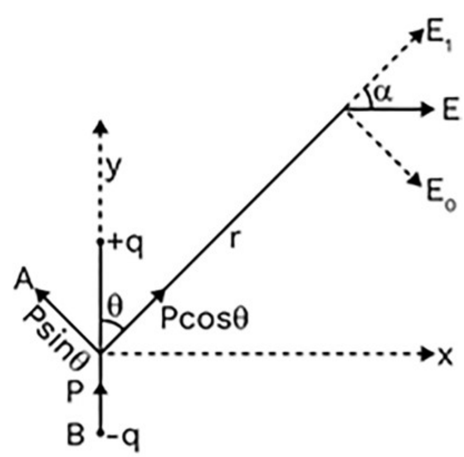
$$E_r = \frac{1}{4\pi\epsilon_0}\frac{2p\cos\theta}{r^3}$$

The magnitude of resultant field is

$$E_\theta = \frac{1}{4\pi\epsilon_0}\frac{p\sin\theta}{r^3}$$

The magnitude of resultant field is

$$E = \sqrt{E_r^2 + E_\theta^2} = \frac{1}{4\pi\epsilon_0}\frac{p}{r^3}\sqrt{1+3\cos^2\theta}$$



The direction of resultant field is

$$\tan\ \alpha = \frac{E_\theta}{E_r} = \frac{1}{2}\tan\theta$$

Case I at axial point $\theta = 0°$ so

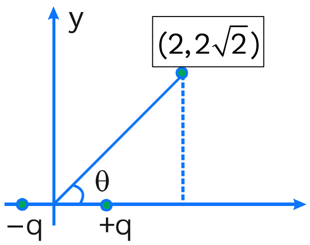
$$E = \frac{1}{4\pi \in_0}\frac{p}{r^3}\sqrt{1+3\cos^2 0^0} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

Case II at equatorial point $\theta = \pi/2$

$$\text{So, } E = \frac{1}{4\pi \in_0}\frac{p}{r^3}\sqrt{1+3\cos^2 \pi/2} = \frac{1}{4\pi \in_0}\frac{p}{r^3}$$

Ex. A short electric dipole is placed at origin and it is directed along positive x-axis. Find out the direction of electric field at point $(2,\ 2\sqrt{2})$ is-

Sol.



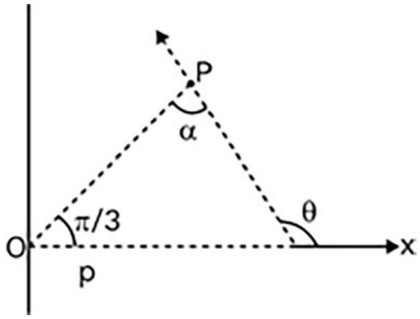
$$\tan\theta = \frac{y}{x} = \sqrt{2}; \cot\theta = \frac{1}{\sqrt{2}}$$

Also $\tan\alpha = \frac{\tan\theta}{2} = \frac{1}{\sqrt{2}} = \cot\theta \Rightarrow \theta + \alpha = 90^0$

i.e., $\vec{E}$ is along positive y-axis.

Ex. An electric dipole is placed along the x-axis at the origin O. A point P is at a distance of 20 cm from this origin such that OP makes an angle $\frac{\pi}{3}$ with the x-axis. If the electric field at P makes an angle θ with x-axis, then find the value of θ?

Sol. According to question we can draw following figure.



As we have discussed earlier in theory

$$\tan\alpha = \frac{1}{2}\tan\frac{\pi}{3} \Rightarrow \alpha = \tan^{-1}\frac{\sqrt{3}}{2}$$

So, $\theta = \frac{\pi}{3} + \alpha$ gives $\theta = \frac{\pi}{3} + \tan^{-1}\frac{\sqrt{3}}{2}$

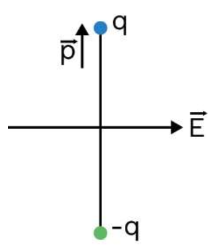
Behaviour of Dipole in an Electric Field:

(a) Uniform Electric Field:

Whenever a dipole is placed in a uniform electric field the two charges experience a force. These forces have equal in magnitude but opposite in direction and do not have same line of action. They constitute a couple of forces which produces a torque. The torque rotates the dipole so as to align it parallel to direction of field.

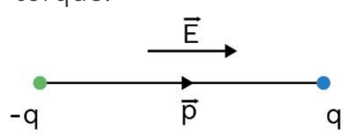
Net force $\vec{F} = \vec{F}_1 + \vec{F}_2 = qE(\hat{i}) + qE(-\hat{i}) = 0$

Case I-: $\tau = \tau_{max} = pE$ when $\theta = \pi/2$. When dipole is perpendicular to the field it experiences maximum torque.

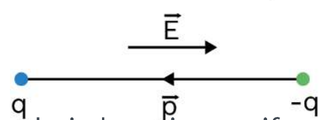


Case II: $\tau = \tau_{min} = 0$ when $\theta = 0°$ or π. When dipole is parallel or anti-parallel to field it experiences minimum torque.

(Parallel)



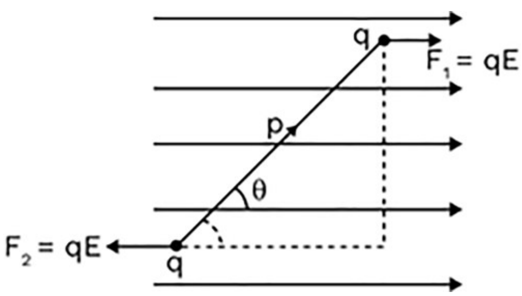
(Anti-parallel)



Concept Reminder

- In uniform electric field net force on dipole is always zero, but net torque may or may not be zero.

1. When a dipole is kept in a uniform electric field then it experiences no force.
2. It experiences a torque given by $\vec{\tau} = \vec{p} \times \vec{E}$.
3. The direction of torque is perpendicular to plane containing $\vec{p}$ and $\vec{E}$.
4. Only rotational motion of dipole takes place.



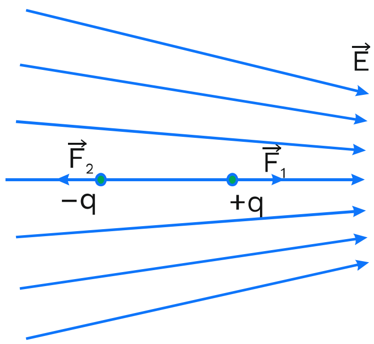
Torque = force × perpendicular distance between forces.

$\tau = qE \times d\sin\theta = pE\sin\theta$

or torque = $\vec{\tau} = -\vec{p} \times \vec{E}$

(b) Dipole in non-uniform external electric field.

(i) Electric field increases form left to right



Concept Reminder

- When dipole is parallel to external uniform electric field then potential is minimum and dipole is in stable equilibrium.

In this case $\vec{F}_1 > \vec{F}_2$

$\therefore \quad F_{net} \neq 0$

But, $\tau_{net} = 0$

(ii) Electric field increasing from left to right.

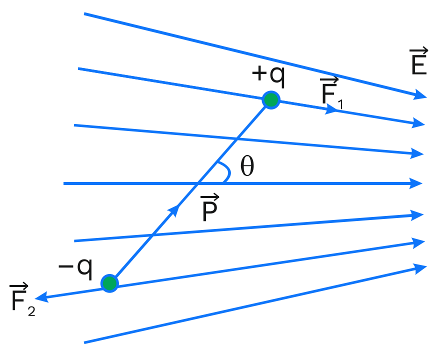
$\vec{P}$ makes an angle θ with $\vec{E}$

$F_1 \neq F_2$

In this case $F_{net} \neq 0$

$\tau_{net} \neq 0$

$\tau_{net} = pE\sin\theta$



Ex. There is a certain region in which electric field is along the z direction through-out. The magnitude of electric field is not constant but increases uniformly along the positive z-direction, at the rate of 10^5 N C^{-1} per metre. Find out the force and torque experienced by a system having a total dipole moment equal to 10^{-7} Cm in the negative z-direction?

Sol. Dipole moment of the system is p = q × dl = -10^{-7} Cm.

The rate of increase of electric field per unit length,

$$\frac{dE}{dl} = 10^{+5}\,NC^{-1}.$$

Force (F) experienced by this system is given by the equation, F = qE

$$F = q\frac{dE}{dl} \times dl = p \times \frac{dE}{dl}$$

So, F = $-10^{-7} \times 10^{5} = -10^{-2}$ N

The force acting on dipole is 10^{-2} N in the –ve z-direction i.e., opposite to the direction of electric field. therefore, the angle between electric field and dipole moment is 180°.

Torque (τ) acting on dipole is given by

τ = pE sin180° = 0

Hence, the torque experienced by the system is 0.

Work Done in Rotating the Dipole

The work done in rotating dipole by small angle $d\theta$ is $dW = \tau d\theta$

Work done in rotating form angle θ_1 to θ_2 is

$$W = \int dw = \int_{\theta_1}^{\theta_2} pE\ \sin\theta\, d\theta$$

or W = pE $[\cos\theta_1 - \cos\theta_2]$

If dipole is rotated from field direction

Then θ_1 = 0°, $\theta_2 = \theta$

So, W = $pE[1-\cos\theta]$

Concept Reminder

- Work done by external agency is equal to change in potential energy.
- For small angle $\sin\theta \simeq \theta$

Angular SHM of a dipole in an Uniform Electric Field:

If a dipole is slightly displaced from its stable equilibrium position in a uniform electric field, it executes angular SHM having some period of oscillation. If I = moment of inertia of dipole about the axis passing through it's centre and perpendicular to it's length.

$\tau = I\alpha = -pE\sin\theta$

when θ is very small $\sin\theta \simeq \theta$

$$\Rightarrow I\frac{d^2\theta}{dt^2} = -pE\theta$$

For electric dipole: $T = 2\pi\sqrt{\frac{I}{pE}}$

Ex. An electric dipole having dipole moment 4×10^{-9} Cm makes an angle 30° with the direction of a uniform electric field of magnitude 5×10^4 NC^{-1}. Calculate the magnitude of the torque acting on the dipole. Also find out work required to rotate the dipole to make angle 90° with the direction of the electric field.

Sol. $\tau = PE\sin 30° = 10^{-4}$ N m, $W_{req} = \Delta U = \sqrt{3} \times 10^{-4}$ J.

Rack your Brain

An electric dipole is placed at an angle of 30° with an electric field intensity 2×10^5 N C^{-1}. It experiences a torque equal to 4 N m. Find the charge on the dipole, if the dipole length is 2 cm.

Dipole Interaction:

(a) Dipole-point charge interaction:

If a point charge is placed in dipole field at a distance r from the mid-point of dipole then force experienced by point charge varies according to the relation $F \propto \frac{1}{r^3}$.

Ex. A point charge placed at any point on the axis of an electric dipole at some large distance experiences a force F. Find the force acting on the point charge when it's distance from the dipole is doubled.

Sol. Force acting on a point charge in dipole field varies as $F \propto \frac{1}{r^3}$ where r is the distance of point charge from the centre of dipole. Hence if r makes double so new force $F' = \frac{F}{8}$.

(b) Dipole-dipole Interaction:

When two dipoles placed closed to each other, they experiences a force due to each other. If suppose two dipoles (1) and (2) are placed then both the dipoles are placed in the field of one another hence potential energy dipole (2) is

$$U_2 = -p_2E_1\cos 0 = -p_2E_1 = -p_2 \times \frac{1}{4\pi\varepsilon_0}.\frac{2p_1}{r^3}$$

then by using $F = \frac{dU}{dr}$,

Force on dipole (2) is $F_2 = \frac{dU_2}{dr}$

$$\Rightarrow F_2 = \frac{d}{dr}\left\{\frac{1}{4\pi\varepsilon_0}.\frac{2p_1p_2}{r^3}\right\} = -\frac{1}{4\pi\varepsilon_0}.\frac{6p_1p_2}{r^4}$$

Similarly force experienced by dipole (1)

$$F_1 = -\frac{1}{4\pi\varepsilon_0}.\frac{6p_1p_2}{r^4}$$

SO, $F_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{6p_1p_2}{r^4}$

Negative sign indicates that force is attractive.

$$|F_1| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{6p_1p_2}{r^4} \text{ and } F_1 \propto \frac{1}{r^4}.$$

Ex. An electric dipole of moment $\vec{p}$ is placed normal to the lines of force of electric intensity , then find out the work done in deflecting it through an angle of 180°.

Sol. Work done $= \int_{90}^{270} pE\sin\theta\, d\theta = [-pE\cos\theta]_{90}^{270} = 0$

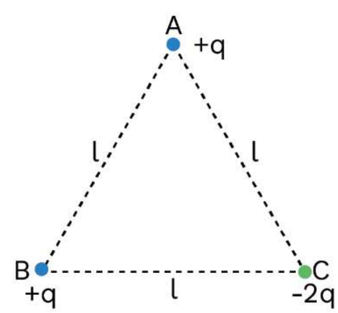
Ex. The distance between the two charges +q and –q of a dipole is r. On the axial line at a distance d from the centre of dipole, find the intensity.

Sol. Field along the axis of the dipole,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{d^3} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2(q \times r)}{d^3}$$

$$\Rightarrow \quad E \propto \frac{qr}{d^3}$$

Ex. Electric charges of value q, q, –2q are placed at the corners of an equilateral triangle ABC of side l. Find the magnitude of electric dipole moment of the system.



Sol. $p_{net} = \sqrt{p^2 + p^2 + 2pp\cos 60^0} = \sqrt{3}\,p = \sqrt{3}\,ql$

Ex. A molecule having a dipole moment p is placed in an electric field of magnitude E. In starting the dipole is aligned parallel to the field. Now, if the dipole is to be rotated to be anti-parallel to the field, find out the work immediate to be done by an external agency.

Sol. W = PE(1 – cos θ),

here θ = 180°

$\therefore$ PE = (1 – cos180°) = 2PE.

ELECTRIC POTENTIAL:

In electrostatic field the electric potential (because of some source charges) at any point P is described as the work done by external agent in taking a point unit +ve charge from a reference point (which is generally taken at infinity) to that point P without any change in its kinetic energy.

1. Mathematical Representation:

If $(W_{\infty \to P})_{ext}$ is the work needed in moving a point charge q from infinity to any point P, the electric potential of the point P is

$$V_P = \left.\frac{W_{\infty \to P})_{ext}}{q}\right]_{\Delta K=0} = \frac{-W_{elc})_{\infty \to P}}{q}$$

Note that:

(i) $(W_{\infty \to P})_{ext}$ is also known as the work required to be done by some external agent against an electric force on a unit +ve charge because of the source charge.

(ii) Always write both W and q with proper sign.

2. Properties:

(i) Potential is a scalar quantity, it may be positive, negative or zero.

(ii) Its S.I. Unit of potential is volt = $\frac{\text{joule}}{\text{columb}}$ and its dimensional formula is $[M^1L^2T^{-3}A^{-1}]$.

(iii) An electric potential at a point is also equal to the –ve of the work needed to be done by the electric field in taking a point charge from reference point (i.e. infinity) to that particular point.

(iv) Electric potential due to a positive charge is always +ve and due to –ve charge is always negative. Only exception is at infinity. (taking $V_\infty = 0$).

(v) Value of potential decreases along the direction of electric field.

(vi) $V = V_1 + V_2 + V_3 + ...$

3. Use of potential:

If we know the value of potential at some fixed point (in terms of numerical value or formula) then we can find out the work done by electric force when charge moves from point 'P' to ∞ by the formula $W_{el})_{P \to \infty} = qV_P$

Ex. A point charge of magnitude 2μC is moved from infinity to a point in an electric field, without change in its velocity. If given that work done against electrostatic forces is –40μJ, then find out the potential at that point.

Sol. $V = \frac{W_{ext}}{q} = \frac{-40\mu J}{2\mu C} = -20V$

Ex. When charge of magnitude 10 μC is shifted from infinity to a point in an electric field, then it was found that work done by electrostatic forces is $-10\ \mu J$. If this charge is doubled and moved again from infinity to the same point without changing its velocity, then find out the work done by electric field and against this electric field.

Sol. Given

$W_{ext})_{\infty \to P} = -W_{el})_{\infty \to P} = W_{el})_{P \to \infty} = 10\ \mu J$

Since, $\Delta KE = 0$

$$V_P = \frac{W_{ext})_{\infty \to P}}{q} = \frac{10\mu J}{10\mu C} = 1V$$

Therefore, if the charge is doubled and taken from infinity then

$$1 = \frac{W_{ext})_{\infty \to P}}{20\mu C}$$

$\Rightarrow \quad W_{ext})_{\infty \to P} = 20\ \mu J$

$\Rightarrow \quad W_{el})_{\infty \to P} = -20\ \mu J$

Ex. A charge $3\mu C$ is released at rest from a point P. At this point electric potential is 20 V then its kinetic energy when it reaches to infinite is:

Sol. $W_{el} = \Delta K = K_f - 0$

$W_{el})_{P \to \infty} = qV_P = 60\ \mu J$

so, $K_f = 60\ \mu J$

4. Potential due to a point charge:

The electrostatic potential at a point in an electric field due to the point charge equal to the amount of work done per unit +ve test charge in moving it from infinity to that point (without acceleration) against the electrostatic force due to the electric field of point charge. It is a scalar quantity.

> **Concept Reminder**
>
> - Potential due to a point charge is not defined at its own location.

Consider a point charge +q placed at point O. Suppose that V_A is electric potential at point A, whose distance from the source charge +q is r_A. If $W_{\infty A}$ is work done in moving a vanishingly small positive test charge q_0 from infinity to point A, then

$$V_A = \frac{W_{\infty A}}{q_0}$$

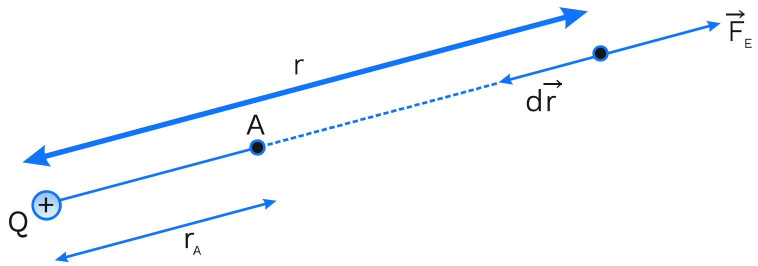
Derivation:

(i) Consider a positive point charge Q at the origin. We wish to calculate the potential at any point A with position vector r from the origin.

(ii) Work done in bringing a unit positive test charge from infinity to the point A. The work done against the repulsive force on this test charge is +ve.

(iii) Since work done does not depend on the path, we must choose a possible path - along the radial direction from infinity to the point A.

(iv) At some intermediate point A' on the path, the electrostatic force on a unit positive charge is $\vec{F}_E = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ where $\hat{r}$ is the unit vector along OP'. Work done by electric field on test charge for small displacement $(d\vec{r})$ as shown in figure.



$\vec{F}_{ext} = -\vec{F}_E$

$dw = \vec{F}_{ext}.d\vec{r}$

$dw = (-\vec{F}_E).(d\vec{r})$

$dw = F_E(-dr)$

(Here $\vec{r}$ is decreasing so we will take dr negative)

$$W_{\infty A} = -\int_{\infty}^{r_A} dw = -\int_{\infty}^{r_A} \frac{Q}{4\pi\varepsilon_0 r^2} dr$$

$$= V_A - V_\infty = \frac{Q}{4\pi\varepsilon_0 r^2} (V_\infty = 0)$$ (reference point is taken at infinity)

$$V_A = \frac{Q}{4\pi\varepsilon_0 r_A}$$

In case, the distance of point A is from the charge +Q is denoted by r

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

Electric potential due to a system of charges:

Let us now find electrostatic potential at a point P due to a group of point charges q_1, q_2, q_3 ... q_n lying at distances r_1, r_2, r_3 .... r_n from point P (fig.). The electrostatic potential at point P due to these charges is found by calculating electrostatic potential P due to each individual charge, considering the other charges to be absent and then adding up these electrostatic potentials algebraically. The electrostatic potential at point P due to charge q_1, when other charges are considered absent,

$$V_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r_1}$$

Similarly, electrostatic potentials at point P due to the individual charges q_2, q_3....q_n (when other charges are absent) are given by

$$V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{r_2};\ V_3 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_3}{r_3};$$

$$.......;\ V_n = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_n}{r_n}$$

Hence, electrostatic potential at point P due to the group of n point charges,

$$V = V_1 + V_2 + V_3 + ... + V_n$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{r_2} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_3}{r_3} .... + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_n}{r_n}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + ... + \frac{q_n}{r_n}\right)$$

$$\Rightarrow V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$

Ex. If infinite charges are placed at 1m, 2m, 4m, 8m.... and so on and the alternative charges are unlike, then find the potential at origin?

Sol. Then, $\frac{1}{4\pi \in_0}\left(\frac{q}{1} - \frac{q}{2} + \frac{q}{4} - \frac{q}{8} + ...\infty\right)$

$$= \frac{1}{4\pi \in_0}\left[\left(\frac{q}{1} + \frac{q}{4} + \frac{q}{16} + ...\infty\right) - \left(\frac{q}{2} + \frac{q}{8} + ...\infty\right)\right]$$

$$= \frac{1}{4\pi \in_0}\left[\frac{1}{1-\frac{1}{4}} - \frac{1}{2}\left(\frac{1}{1-\frac{1}{4}}\right)\right] = \frac{1}{4\pi \in_0}\frac{2q}{3}$$

Potential Difference:

Work required to be done in moving a unit charge from one point to the other in an electric field is known as the potential difference between these two points.

Thus, if W be work done in moving a charge q_0 form B to A then the potential difference is given by-

$$V_A - V_B = \frac{W}{q_0}$$

(i) Its SI units is volt.

(ii) This is a scalar quantity

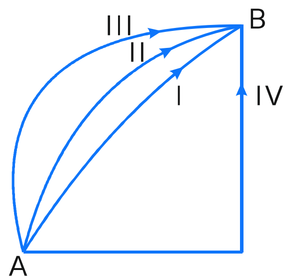
Concept Reminder

- It is not the actual value of potential but the potential difference that is physically significant.

(iii) Potential difference does not depend upon Co-ordinate system

(iv) Potential difference does not depend upon the path followed. This is, because electric field is a conservative force field and work done is conservative force field does not depend upon path followed.

Ex. In following figure, along which path the work done will have maximum value in moving a charge from A to B in the presence of other charges.



Sol. Work done will be same for all the paths. [Since, the work done doesn't depend upon the path]

Ex. A charge 20mC is situated at the origin of X-Y plane. What will be potential difference between points (5a, 0) and (–3a, 4a)

Sol. Distance between (0, 0) & (5a, 0)

$$r_1 = \sqrt{25a^2 + 0} = 5a$$

$$\therefore \quad V_1 = \frac{kq}{5a}$$

Distance between (0, 0) & (–3a, 4a)

$$r_2 = \sqrt{9a^2 + 16a^2} = 5a$$

$$V_2 = \frac{kq}{5a}$$

$$\therefore \quad V_1 - V_2 = 0$$

Potential due to an Electric dipole:

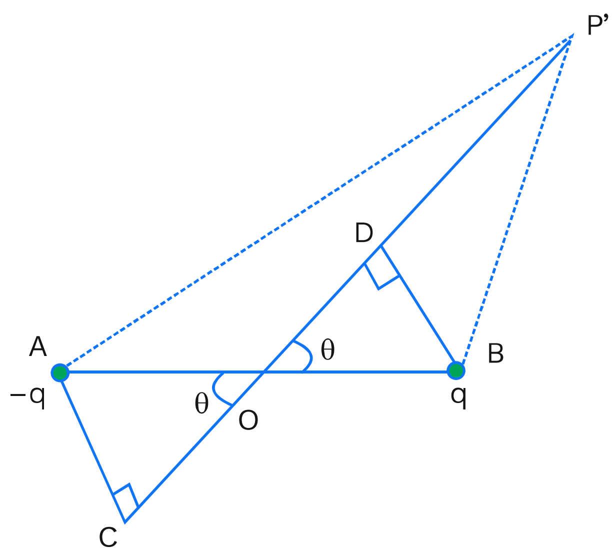
Consider AB be an electric dipole of length 2a and P' be any point (OP' = r) where we have to find potential due to dipole.

Rack your Brain

Four point charges –Q, –q, 2q and 2Q are placed, one at each corner of the square. Find the relation between Q and q for which the potential at the centre of the square is zero.

Key Points

- Potential
- Potential difference



Potential at P’ will be $V = \left(\frac{kq}{BP'} - \frac{kq}{AP'}\right)$ ...(1)

Find : AP’ and BP’

Given AB = 2a

OP’ = r

AO = OB = a

Draw perpendicular from A and B on OP’ line.

$AC \perp CP'$ & $BD \perp OP'$

OC = OD = $a\cos\theta$

if r >> a

OP’ + OC = AP’

AP’ = $r + a\cos\theta$

Also OP’ – OD = BP’ = $r - a\cos\theta$

Putting values in equation (1)

$$V = \frac{kq}{(r - a\cos\theta)} - \frac{kq}{(r + a\cos\theta)}$$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\left(\frac{2aq\cos\theta}{r^2 - a^2\cos^2\theta}\right)$$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\left(\frac{P\cos\theta}{r^2 - a^2\cos^2\theta}\right)$$

P = 2aq = dipole moment

for short dipole $V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

Note : The electric dipole potential falls off as $\frac{1}{r^2}\left(\text{not as}\frac{1}{r}\right)$

Concept Reminder

- The electric dipole potential falls off, at large distance, as $1/r^2$, not as 1/r.

Relationship between electric potential and intensity of electric field:

(i) $V_A = -\int_{\infty}^{A} \vec{E}.\,\vec{dr}$,

V_A is an electric potential at any point A.

(ii) Value of potential difference between two points in an electric field is given by –ve value of line integral of electric field intensity i.e.

$$V_B - V_A = -\int_{A}^{B} \vec{E}.\,\vec{dr}$$

(iii) $\vec{E} = -\vec{\nabla} V$

$$\nabla = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial x}\hat{k}\right)$$

$$E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$

(iv) If V is a function of position r only, then $E = -\frac{dV}{dr}$

(v) For a uniform electric field, $E = -\frac{\Delta V}{\Delta r}$ and it's direction is along the decrease in the value of V.

Ex. Electric potential for a point (x, y, z)m is given by $V = 4x^2$ volt. Electric field at point (1, 0, 2)m is

Sol. $E = -\frac{dV}{dx} = -8x$

at (1, 0 ,2) E = – 8 V/m

$\therefore$ Magnitude of $\vec{E}$ = 8V/m direction along-x axis.

Ex. Electric field is given by $E = \frac{100}{x^2}$ potential difference between x = 10 and x = 20 m.

Sol. $E = -\frac{dV}{dx} \Rightarrow dV = -Edx$

$$\Rightarrow \int_{A}^{B} dV = -\int_{A}^{B} E.dx$$

$$\Rightarrow V_B - V_A = \int_{10}^{20} \frac{100}{x^2} = 5 \text{ volts}$$

Concept Reminder

- If electric field equals zero at a given point then potential V at that point may or may not be zero and vice-versa.

Rack your Brain

In a region, the potential is represented by $V(x, y, z) = 6x - 8xy - 8y + 6yz$, where V is in volts and x, y, z are in metres. The electric force experienced by a charge of 2 coulomb situated at point (1, 1, 1).

Potential difference = 5 volt.

Ex. The potential at a point (x, 0, 0) is given as $V = \left(\frac{1000}{x} + \frac{1500}{x^2} + \frac{500}{x^3}\right)$.

Find out the electric field intensity at x = 1 m?

Sol. $\therefore\ \vec{E} = -\vec{\nabla} V = -\hat{i}\frac{\partial V}{\partial x} - \hat{j}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z}$

or $\quad iE_x + jE_y + kE_z = -\hat{i}\frac{\partial V}{\partial x} - \hat{j}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z}$

$$= -\frac{\partial V}{\partial x}\left[\because \frac{\partial V}{\partial y} = 0 = \frac{\partial V}{\partial z}\right]$$

Comparing both sides

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left[\frac{1000}{x} + \frac{1500}{x^2} + \frac{5000}{x^3}\right]$$

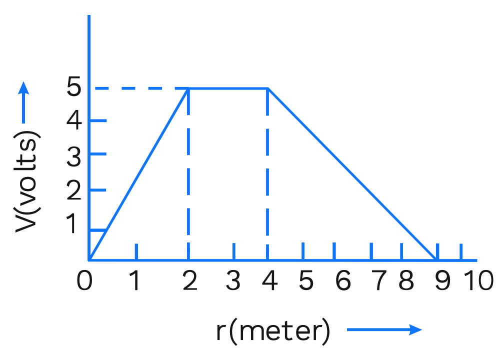
$$= -\left[-\frac{1000}{x^2} - \frac{2 \times 1500}{x^3} - \frac{3 \times 5000}{x^4}\right]$$

For x = 1, (E_x) = 5500 V/m

Ex. In the following fig, what will be the electric field intensity at r = 3.

For 2 < r < 4, V = 5 volts

$\therefore \quad E = -\frac{dV}{dr} = 0$



Note:

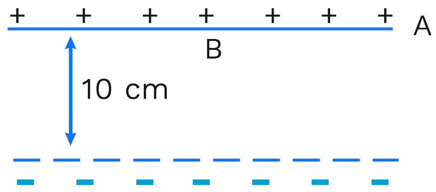
In the above problem, find out value of E at r = 6 ?

at $\quad r_2$ = 7m $\quad\quad V_2$ = 2 volt

at $\quad r_1$ = 5m $\quad\quad V_1$ = 4 volt

$\therefore \quad E = -\left(\frac{V_2 - V_1}{r_2 - r_1}\right) = -\left(\frac{2-4}{7-5}\right) = 1$ volt/metre

Ex. An oil drop 'B' has charge 1.6×10^{-19}C and mass 1.6×10^{-14} kg. If the drop is in equilibrium position, then what will be the potential diff. between the plates. [The distance between the plates is 10mm]



Sol. For equilibrium, electric force = weight of drop

$\Rightarrow$ qE = mg

$$\Rightarrow q.\frac{V}{d} = mg$$

$$V = \frac{mgd}{q} = \frac{1.6 \times 10^{-14} \times 9.8 \times 10 \times 10^{-3}}{1.6 \times 10^{-19}}$$

$V = 1 \times 10^4$ volt

[Formula qE = mg is often used when a charged particle is in equilibrium in electric field].

Potential due to a uniformly charged shell:

Consider a uniformly charged shell. The radius of shell is R and its total charge is q, which is uniformly distributed over the surface. The electric field for such arrangement has already been calculated in previous chapter.

(i) For any point inside the shell, E = 0

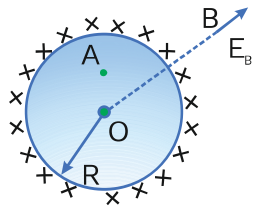
(ii) For any point outside the shell, $E = \frac{q}{4\pi\varepsilon_0 r^2}$,

where 'r' is the distance of concerned point from the centre of the shell. The electric field is similar to that of a point charge. To calculate potential, we use the relation between field and potential, $dV = -\vec{E}.\vec{dr}$

Rack your Brain

There is a uniformly charged non-conducting solid sphere made of material of dielectric constant one. If electric potential at infinity be zero, then the potential at its surface is V. If we take electric potential at its surface to be zero, then find the potential at the centre.

Potential at B



Using $dV = -\vec{E}.\vec{dr}$, we have

Put $\left(-\vec{E}.\vec{dr}\right)$

$$\int_{\infty}^{V} \frac{q}{4\pi\varepsilon_0 r^2} dr = -\frac{q}{4\pi\varepsilon_0 r^2}\int_{\infty}^{r} r^{-2} dr \qquad \left\{ \because E_A = 0; \vec{E}_B = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} \right.$$

$\Rightarrow \quad V_B = \frac{q}{4\pi\varepsilon_0 r}$, For a point on surface, take r = R, so, $V_{surface} = \frac{q}{4\pi\varepsilon_0 r}$

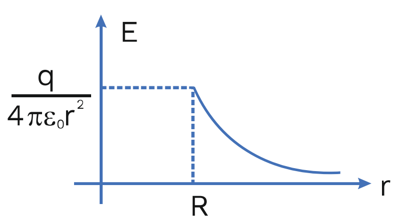
Potential at A

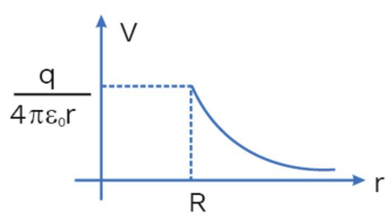
using $-\vec{E}.\vec{dr}$, we have

$dV = 0 \Rightarrow V = \text{Constant}$.

So, the potential at any point inside a uniformly charged shell is same everywhere. This will be equal to potential at surface. So, $V_A = \frac{q}{4\pi\varepsilon_0 r}$.

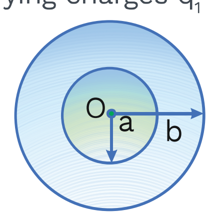
The above variation of electric field and potential can be represented graphically as shown.





Potential due to a group of concentric shells:

To calculate the potential due to a group of concentric shells, we can use superposition principle. For any point, we can simply add the potentials due to individual shells. Consider a pair of two uniformly charged concentric shells having radii ‘a’ and ‘b’ and carrying charges q_1 and q_2 respectively let a < b.



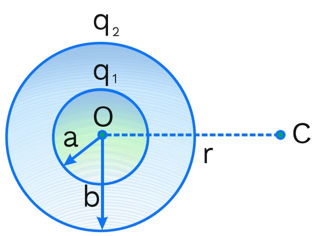
We will calculate potential at three different points A, B and C. A lies inside the inner shell B lies in the space between the two shells and C lies outside the outer shell.

Potential at point C

OC = r, where r > b.

As the point C, lies outside the outer shell potential at C due to outer shell is $V_2 = \frac{q_2}{4\pi\varepsilon_0 r}$

Similarly potential at C due to inner shell is $V_1 = \frac{q_1}{4\pi\varepsilon_0 r}$



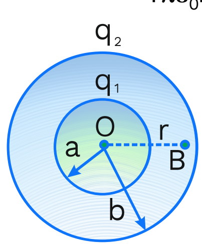
So, potential at C is $V_C = \frac{q_1 + q_2}{4\pi\varepsilon_0 r}$

Potential at point B

Let OB = r, where a < r < b.

As the point B, lies inside the outer shell,

Potential at B due to outer shell is $V_2 = \frac{q_2}{4\pi\varepsilon_0 b}$.



The point B lies outside the inner shell, the potential at B,

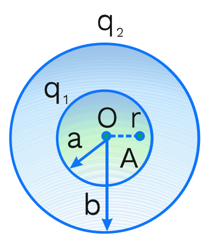
due to inner shell is $V_1 = \frac{q_1}{4\pi\varepsilon_0 r}$

So, potential at B is $V_B = \frac{q_1}{4\pi\varepsilon_0 r} + \frac{q_2}{4\pi\varepsilon_0 b}$

Potential at point A

Let OA = r, such that r < a.

The point A lies inside the outer shell,

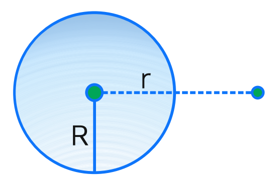


So, potential at A, due to outer shell is $V_2 = \frac{q_2}{4\pi\varepsilon_0 b}$

Similarly, potential at A, due to inner shell is $V_1 = \frac{q_1}{4\pi\varepsilon_0 a}$

So, potential at A is $V_A = \frac{q_1}{4\pi\varepsilon_0 a} + \frac{q_2}{4\pi\varepsilon_0 b}$.

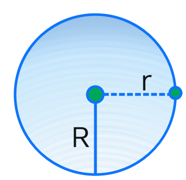
Potential due to uniformly charged non conducting sphere:



1. For point outside sphere (r > R)

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

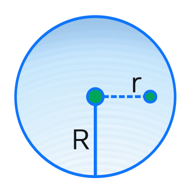
$$V = \frac{Q}{4\pi\varepsilon_0 r}$$



2. For surface point (r = R)

$$E = \frac{Q}{4\pi\varepsilon_0 R^2}$$

$$V_s = \frac{Q}{4\pi\varepsilon_0 R}$$

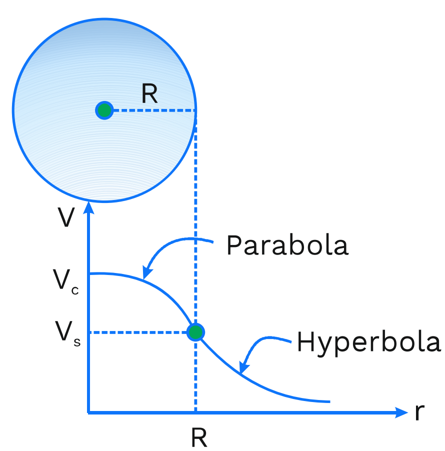


3. For inside point (r < R)

$$E = \frac{Qr}{4\pi\varepsilon_0 R^3}$$

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{3R^2 - r^2}{2R^3}\right]$$

$$V_{centre} = \frac{3}{2}\left(\frac{Q}{4\pi\varepsilon_0 R^2}\right) = \frac{3}{2}V_s$$



3. Equipotential Surface

(i) These are the imaginary surface (drawn in an electric field) where the potential at any point on the surface has the same value.

(ii) No two equipotential surfaces ever intersect.

(iii) Equipotential surfaces are perpendicular to the electric field lines

(iv) Work done in moving a charge from a one point to the other on an equipotential surface is zero irrespective of the path followed and hence there is no change in kinetic energy of the charge.

(v) Component of electric field parallel to equipotential surface is zero.

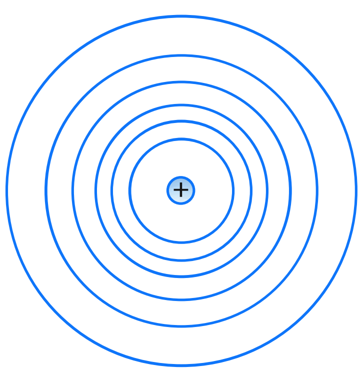
(iv) Nearer the equipotential surfaces, stronger the electric field intensity

Rack your Brain

Charge q_2 is at the centre of a circular path with radius r. Find out work done in carrying charge q_1, once around this equipotential path.

Equipotential Surfaces for Some cases:

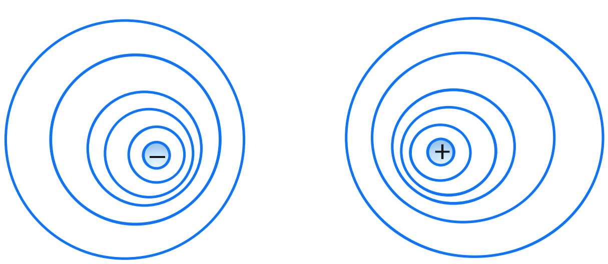
1. Point charge



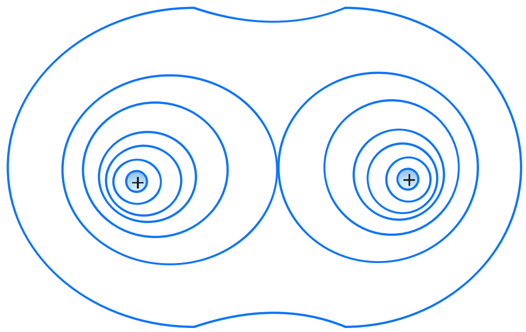
Concept Reminder

- Equatorial plane of a dipole is also a equipotential surface at which potential is zero.

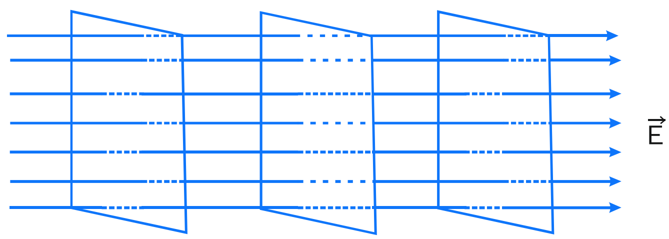
2. Dipole



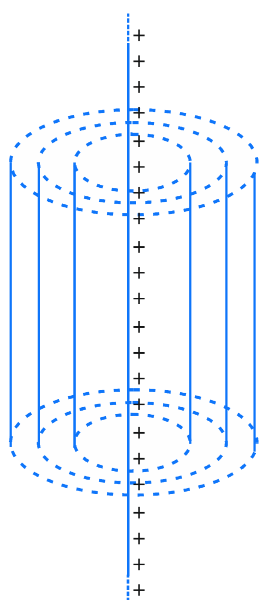
3. Two Equal Charges Separated by some distance



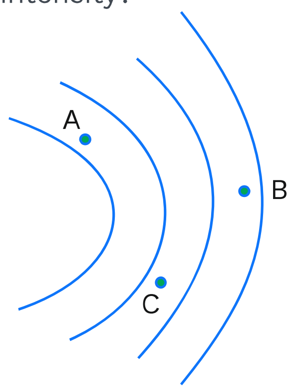
4. Uniform Electric Field



5. Uniformly charged infinitely long wire



Ex. Some equipotential surfaces are shown in figure. Find the correct order of electric field intensity?



Sol. $E_B > E_C > E_A$

Potential energy of charged particle in an electric field

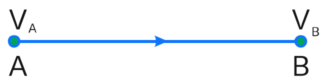
(i) Work done in moving a point charge from infinity to any point against the electric field is defined as the potential energy of that point charge.

(ii) Potential energy of a charge of a point is equal to the product of magnitude of charge and electric potential at that point i.e. P.E = qV.

(iii) Work done in moving a charge from one point to other in an electric field is equal to change in it's potential energy i.e. work done in moving Q from

A to B = $qV_B - qV_A$.

$= U_B - U_A$



(iv) Work done in moving a unit charge from one point to other is equal to potential difference between two points.

Note:

Circumference of the circle in above example can be considered as equipotential surface and hence work done will be zero.

POTENTIAL ENERGY OF SYSTEM

(i) The electric potential energy of a system of charges is the work that has been done in bringing those charges from infinity to near each other to form the system.

Concept Reminder

Self potential energy for-

(i) Uniformly charged conducting sphere

$$= \frac{Q^2}{8\pi\varepsilon_0 R}$$

(ii) Uniformly charged non-conducting sphere

$$= \frac{3}{5}\left(\frac{Q^2}{4\pi\varepsilon_0 R}\right)$$

(ii) If a system is given negative of its potential energy, then all charges will move to infinity. This negative value of total energy is called the binding energy.

(iii) (a) Energy of a system of two charges

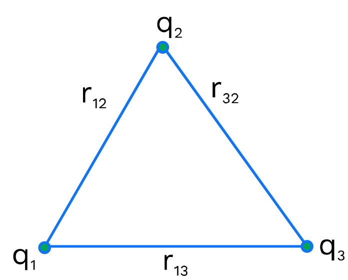
$$PE = \frac{1}{4\pi \epsilon_0}\frac{q_1q_2}{d}$$

(b) Potential Energy of a system of three charges

$$PE = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_1}{r_{31}}\right]$$

(c) Potential Energy of a system of n charges.

$$PE = \frac{1}{2}\cdot\frac{1}{4\pi\varepsilon_0}\left(\sum_{i=1}^{n} q_i\left(\sum_{\substack{j=1\\ i\neq j}}^{n}\frac{q_j}{r_{ij}}\right)\right)$$



Note :

Method to find energy of a system of n charges.

(a) Find the PE of each charge relative to all other charges.

(b) Add these all

(c) Divide the addition by 2 and resultant will be the potential energy of the system.

Work Done in an Electric Field:

(i) If electric potential at a point is V then potential energy (PE) of a charge placed at that point will be qv.

(ii) Work done in moving a charge from A to B is equal to change is PE of that charge W_{AB} = work done from A to B

$= PE_B - PE_A = q\,(V_B - V_A)$

(iii) Work done in moving a charge along a closed surface in an electric field is zero.

Rack your Brain

Four equal charges (q) are placed at corners of a square of side l. Find out potential energy of system.

(iv) Total energy remains constant in an electric field i.e. $KE_A + PE_A = KE_B + PE_B$

KE = Kinetic energy

PE = potential energy

(v) A free charge moves from higher PE to lower PE state in an electric field. Hence

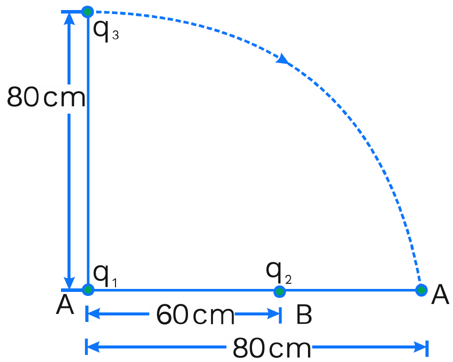
(a) a positive charge will move from higher potential to lower potential while,

(b) a negative charge will move from lower potential to higher potential

(c) Work done for displacement through $\vec{r}$ for a charge experiencing a force $\vec{F}$

$W = \vec{F}.\vec{r}$

Ex. What will be change in potential energy of q_3, in moving it along CD for the following fig.



Sol. Potential energy of q_3 at C

[where $q_1 = 2 \times 10^{-8}C$,

$q_2 = 0.4 \times 10^{-8}$ C, $q_3 = 0.2 \times 10^{-8}$ C]

$$U_c = k\left[\frac{q_1q_3}{0.8} + \frac{q_3q_2}{1}\right]$$

[$\therefore$ $BC = \sqrt{80^2 + 60^2}$ cm $= \sqrt{10^4}$ cm

$= 10^2$ cm = 1m]

Potential energy of q_3 at D,

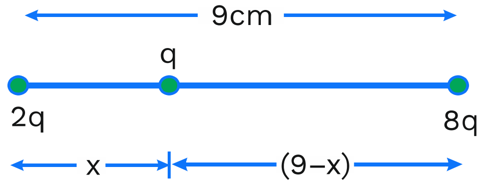
$$U_D = k\left[\frac{q_1q_3}{0.8} + \frac{q_3q_2}{0.2}\right]$$

$$\therefore \quad U_D - U_C = kq_2q_3\left[\frac{1}{0.2} - \frac{1}{1}\right]$$

$= 9 \times 10^9 \times 4 \times 10^{-9} \times 0.2 \times 10^{-8} \times 4$

$= 2.88 \times 10^{-7}$ Joule

Ex. In the following fig, where the charge 'q' must be kept, so that the potential energy of the system will be minimum.?



Sol. Suppose the charge q is placed at distance x from 2q. Potential energy of the system

$$U = k\left[\frac{2qq}{x \times 10^{-2}} + \frac{8qq}{(9-x)\times 10^{-2}} + \frac{2q \times 8q}{9 \times 10^{-2}}\right]$$

For U to be minimum $\frac{\partial U}{\partial X} = 0$ which gives X = 3cm.

Ex. The charges of 10mc each are kept at three corners of an equilateral triangle of 10cm side. What is the potential energy of the system?

Sol. PE of 1 = $U_1 = = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r} + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r}$

PE of 2 = $U_2 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r} + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r}$

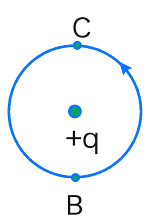
PE of 3 = $U_3 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r} + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r}$

PE of system = $\frac{1}{2}(U_1 + U_2 + U_3)$

$$= \frac{3}{4\pi\varepsilon_0}\cdot\frac{q^2}{r}$$

$$= \frac{3 \times 9 \times 10^9 \times (10 \times 10^{-6})^2}{10 \times 10^{-2}} = 27\ \text{J}.$$

Ex. A point charge +q is placed fixed at the centre of a circle. What will be the amount of work done in carrying a charge q' from B to C in the figure.



Concept Reminder

- Work done in moving a charged particle on equipotential surface is zero.

Sol. Zero, because circular path is a equipotential surface

Hence $V_B - V_C = 0$

$\therefore$ $W = q'(V_B - V_D) = 0$

Ex. An electron (mass m, charge e) is accelerated through a potential difference of V volt. Find the final velocity of electron.

Sol. $KE_i = 0$

$PE_i = eV_1$

$KE_f = 1/2\ mv^2$

$PE_f = eV_2$

$\because$ $KE_i + PE_i = KE_f + PE_f$

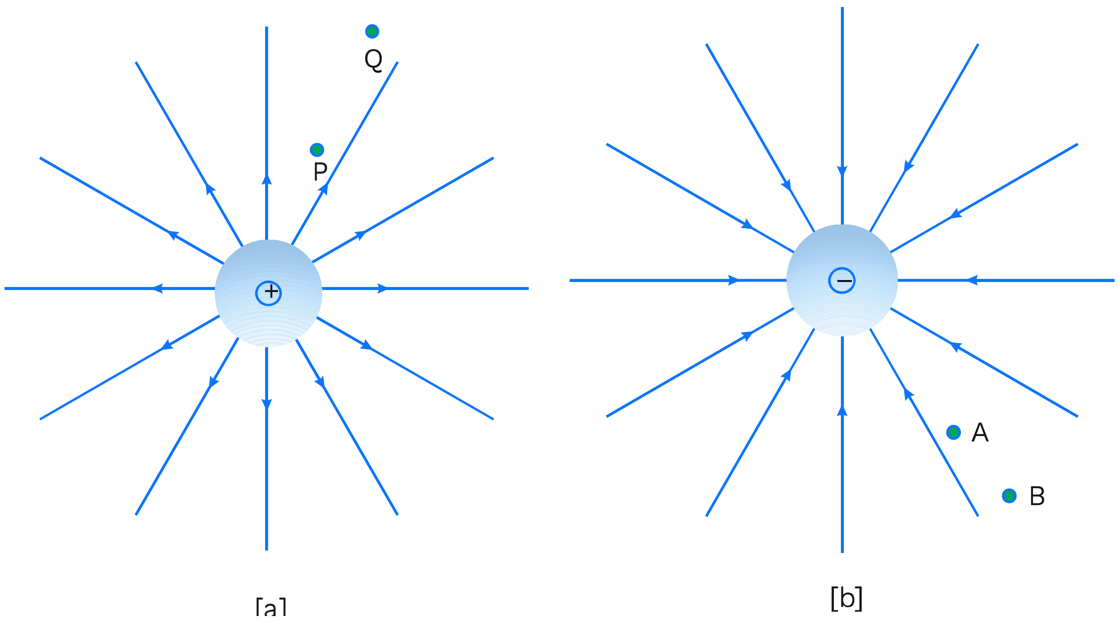
$\Rightarrow$ $0 + eV_1 = \frac{1}{2} mv^2 + eV_2$

$\Rightarrow$ $\frac{1}{2} mv^2 = e(V_2 - V_1) = eV$

$\Rightarrow$ $v = \sqrt{\frac{2eV}{m}}$

[When an electron is accelerated through potential difference 'V' the following formula is generally used $\frac{1}{2} mv^2 = eV$]

Ex. Figures (a) and (b) show the field lines of a positive and negative point charge respectively.



(a) Give the signs of the potential difference
$V_P - V_Q$; $V_B - V_A$.

(b) Give the sign of the potential energy difference of a small negative charge between the points Q and P; A and B.

(c) Give the sign of the work done by the field in moving a small positive charge from Q to P.

(d) Give the sign of the work done by the external agency in moving a small negative charge from B to A.

(e) Does the kinetic energy of a small negative charge increase or decrease in going from B to A?

Sol. (a) As $V \propto \frac{1}{r}$, $V_P > V_Q$. Thus, $(V_P - V_Q)$ is positive. Also V_B is less negative than V_A. Thus, $V_B > V_A$ or $(V_B - V_A)$ is positive.

(b) A small negative charge will be attracted towards positive charge. The negative charge moves from higher potential energy to lower potential energy. Therefore the sign of potential energy difference of a small negative charge between Q and P is positive. Similarly, $(P.E.)_A > (P.E.)_B$ and hence sign of potential energy differences is positive.

(c) In moving a small positive charge from Q to P, work has to be done by an external agency against the electric field. Therefore, work done by the field is negative.

(d) In moving a small negative charge from B to A work has to be done by the external agency. It is positive.

(e) Due to force of repulsion on the negative charge, velocity decreases and hence the kinetic energy decreases in going from B to A.

EXAMPLES

Q1 **Which of the following value of charge is not possible-**

(1) 3.2×10^{-19} C (2) 1.6×10^{-20} C (3) 14.4×10^{-19} C (4) None of these

Sol. 1.6×10^{-20} C is not possible, because this is 1/10 of electronic charge and hence not an integral multiple.

Q2 **A Cu sphere of mass 2.0 g contains almost 2×10^{22} atoms. Given that value of charge on the nucleus or each atom is 29e.**

(a) How many electrons must be removed from this sphere to give it a charge of +2C?

(b) Determine the fraction of electrons removed.

(c) Is there any change in mass of sphere when it is given positive charge?

Sol. (a) Number of electrons to be removed

(b) Total number of electrons in the sphere = $29 \times 2 \times 10^{22} = 5.8 \times 10^{23}$

Fraction of electrons removed.

Thus 2.16×10^{-9} % of electrons are to be removed to give the sphere a charge of 2C.

(c) Yes, mass decreases, when body is given a positive charge. decrease of mass m = $9 \times 10^{-31} \times 1.25 \times 10^{13} = 1.125 \times 10^{-17}$ kg

Q3 **A polythene piece when rubbed with wool can it was found to have a negative charge of value 3×10^{-7} C.**

(a) Find the number of electrons get transferred (from which to which?).

(b) Can we say that there a transfer of mass from wool to polythene?

Sol. When polythene piece is rubbed against wool, a number of electrons is transferred from wool to polythene. Therefore, wool becomes +vely charged and polythene becomes –vely charged.

Value of charge on the polythene piece, q = -3×10^{-7} C

Value of charge on an electron, e = -1.6×10^{-19} C

No. of electrons that get transferred from wool to polythene = n

we can calculate 'n' using the relation, q = ne

= 1.87×10^{12}

So, the no. of electrons get transferred from wool to polythene = 1.87×10^{12}.

(b) Yes. Mass will be also get transferred. This is because an e^- has mass, $m_e = 9.1 \times 10^{-3}$ kg

Total mass transferred to polythene from wool,

$m = m_e \times n = 9.1 \times 10^{-31} \times 1.87 \times 10^{12} = 1.702 \times 10^{-18}$ kg

Hence, a negligible amount of mass is transferred from wool to polythene.

Q4 **Point out important differences between charge and mass?**

Sol. (i) The force between two masses is always attractive whereas that between two charges may be attractive or repulsive.

(ii) Charge on a body always remains constant and does not change with speed of the body. But the mass of a body changes with speed according to the following equation :

$$m = \frac{m_0}{\sqrt{1-\left(\frac{V^2}{C^2}\right)}}$$

Q5 **We know that mass changes with speed of body. Does motion of a body changes the charge over it? What will happen to specific charge of than particle as its speed increases?**

Sol. No, motion of a body does not increase amount of charge over it. But specific charge $\frac{e}{m}$ reduces.

Q6 **A soap bubble is given negative charge. What happens to its radius?**

Sol. Due to repulsive force radius increases.

Q7 **The electrostatic force of repulsion between two equal positively charged ions is 3.7×10^{-9}N, when they separation of 5 Å between them. How many electrons are missing from each ion?**

Sol. Given, $F = 3.7 \times 10^{-9}$ N; $r = 5$ Å $= 5 \times 10^{-10}$ m

Let number of electrons removed to form each positive ion = n.

So, charge on each positive ion $q_1 = q_2 = n \times 1.6 \times 10^{-19}$ C

Now, $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$ so, $3.7 \times 10{-9} = \frac{9 \times 10^9 \times (n \times 1.6 \times 10^{-19})^2}{(5 \times 10^{-10})^2}$

so, n = 2

Q8 **Force F is acting between two charges when they are in air. If we place a sheet of glass (e_r = 6) between these two charges completely fill space between them. What will be the force?**

Sol.

$$K = \frac{F_{air}}{F_{medium}}$$

$$F_{medium} = \frac{F_{air}}{F} = \frac{F}{6}$$

Q9 **Two-point charges q_1 and q_2 (both of same sign) and mass m are placed such that gravitation attraction between them balances the electrostatic repulsion. What type of equilibrium is present in this case?**

Sol. In given Illustration: $= \frac{K q_1 q_2}{r^2} = \frac{Gm^2}{r^2}$

As per the condition given, we can see that irrespective of distance between them charges will remain in equilibrium. If now distance is increased or decreased, then there is no effect in their equilibrium. Therefore, it is a neutral equilibrium.

Q10 **Two-point charges 1 unit and 5 unit are separated by a certain distance. What will be ratio of forces acting on these two.**

Sol. Both the charges will experience same force so ratio is 1:1 as $F_{15} = F_{51}$

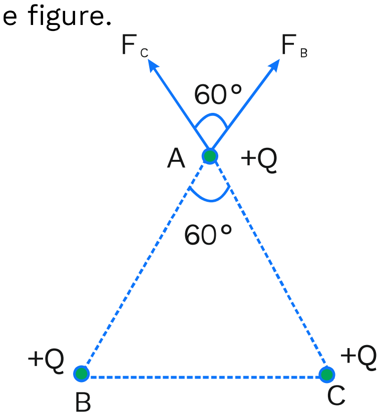
Q11 **Two small charged metallic spheres carrying charges of 40 μC and –20 μC and are placed at some distance apart. They are touched and kept at the same distance. Find the ratio of the initial to the final force between them.**

Sol. Since only magnitude of charges are changes that's why

$$F \propto q_1q_2 \Rightarrow \frac{F_1}{F_2} = \frac{q_1q_2}{q'_1\,q'_2} = \frac{40 \times 20}{10 \times 10} = \frac{8}{1}$$

Q12 **Three equal charges each +Q, placed at the corners of on equilateral triangle of side a what will be the force on any charge.**

Sol. Suppose net force is to be calculated on the charge which is kept at A. Two charges kept at B and C are applying force on that particular charge, with direction as shown in the figure.

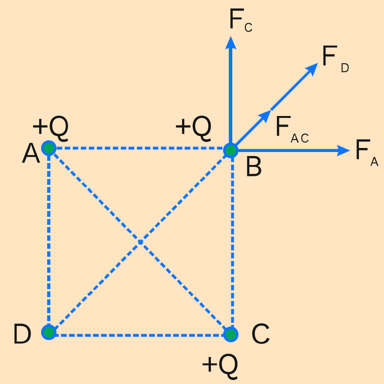


Since $F_b = F_c = F = k\dfrac{Q^2}{a^2}$

So, $F_{net} = \sqrt{F_B^2 + F_C^2 + 2F_BF_C \cos 60}$

$F_{net} = \sqrt{3}\,F$ (Vertically upward direction)

Q13 **Four equal charges Q are placed at the four corners A, B, C, D of a square of length a. Find the magnitude of the force on the charge at B.**



Sol. Resultant force on charge at B is-

$$F_{net} = F_{AC} + F_D = \sqrt{F_A^2 + F_C^2} + F_D$$

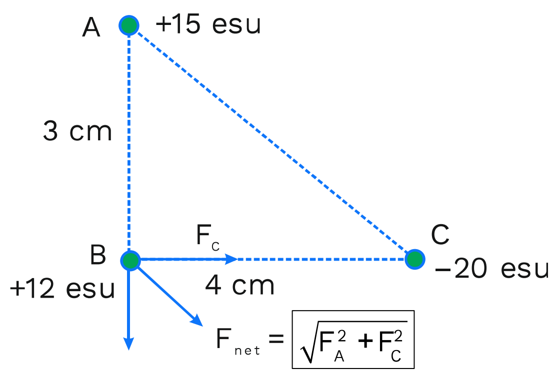
$$F_A = F_C = \frac{kQ^2}{a^2} \quad \text{Since and } F_D = \frac{kQ^2}{(a\sqrt{2})^2}$$

$$F_{net} = \frac{\sqrt{2}kQ^2}{a^2} + \frac{kQ^2}{2a^2} = \frac{kQ^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right) = \frac{Q^2}{4\pi\varepsilon_0 a^2}\left(\frac{1+2\sqrt{2}}{2}\right)$$

Q14 **ABC is a right-angle triangle in which AB = 3 cm, BC = 4 cm and** $\angle ABC = \frac{\pi}{2}$.

The three charges +15, +12 and – 20 e.s.u. are placed respectively on A, B and C. Find the force acting on B.

Sol. Net force on B $F_{net} = \sqrt{F_A^2 + F_C^2}$



$$F_A = \frac{15 \times 12}{(3)^2} = 20\,\text{dyne}$$

$$F_C = \frac{12 \times 20}{(4)^2} = 15\,\text{dyne}$$

$$F_{net} = 25 \text{ dyne}$$

Q15 How should we divide a charge 'Q' into two parts to get maximum force of repulsion between them?

Sol. Let q and Q – q be the two parts $F = \frac{1}{r\pi\varepsilon_0}\frac{q(Q-q)}{r^2}$

For maximum F, $\frac{dF}{dq} = 0$ or $\frac{1}{4\pi\varepsilon_0}\frac{Q-2q}{r^2} = 0$ or $q = \frac{Q}{2}$

Hence Q must be divided in two equal parts.

Q16 Force F is acting between two charges. If a sheet of glass (ε_r = 9) is placed between the two charges, what will be the force.

Sol. $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$ or $F' = \frac{1}{4\pi \in_0 \in_r}\frac{q_1q_2}{r^2} = \frac{F}{\in_r} = \frac{F}{9}$

Q17 Calculate the electric field at origin in (a) & (b), due to infinite number of charges as shown in figures below.

Fig (a)

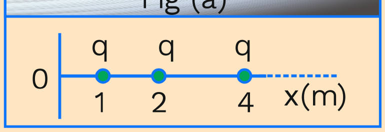
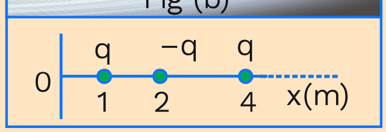


Fig (b)



Sol. (a) $E_0 = kq\left[\frac{1}{1} + \frac{1}{4} + \frac{1}{16} + - - - -\right] = \frac{kq.1}{(1-1/4)} = \frac{4kq}{3}$

[As, $S_\infty = \frac{a}{1-r}$, a = 1 and r = $\frac{1}{4}$]

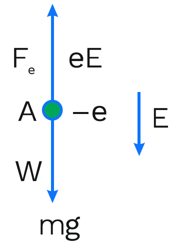
(b) $E_0 = kq\left[\frac{1}{1} - \frac{1}{4} + \frac{1}{16} - - - - -\right] = \frac{kq.1}{(1-(-1/4))} = \frac{4kq}{5}$

Q18 **A charged particle is in equilibrium between the plates of Millikan oil drop experiment in the electric field. If the direction of the electric field between the plates is reversed, then find acceleration of the charged particle.**

Sol. Let mass of the particle is m,
Charge on particle is q
Intensity of electric field in between plates is E,
Initially mg is equal to qE
When direction of field is reversed then $ma = mg + qE \Rightarrow ma = 2mg$
Hence, Acceleration of particle is 2g

Q19 **Find the value of electric field intensity E that would be just sufficient to balance the weight of an e^-. Consider that this electric field is due to a second e^- located below the first one then find the distance between them?**
[Given that: charge of $e^- = -1.6 \times 10^{-19}$ C, mass of $e^- = 9.1 \times 10^{-31}$ kg and $g = 9.8$ m/s^2]

Sol. Since, force on a charge e^- in an electric field E

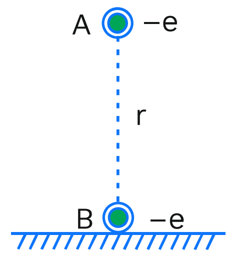


(A)

$F_e = eE$
So according to given problem,
$F_e = W \Rightarrow eE = mg$

$$\Rightarrow E = \frac{mg}{e} = \frac{9.1 \times 10^{-31} \times 9.8}{1.6 \times 10^{-19}} = 5.57 \times 10^{-11} \frac{V}{m}$$

Since, this intensity E is due to another electron B, located at a distance r below A.

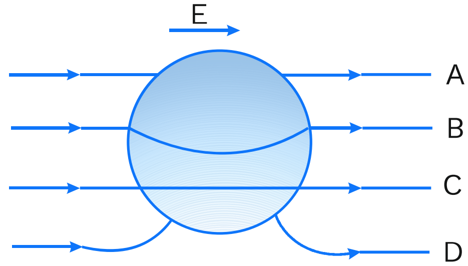


(B)

$$E = \frac{1}{4\pi\varepsilon_0}\frac{e}{r^2} \Rightarrow \sqrt{\frac{e}{4\pi\varepsilon_0 E}} \quad \text{So, } r = \left[\frac{9\times10^9\times1.6\times10^{-19}}{5.57\times10^{-11}}\right] = 5\,\text{m}$$

Q20 **A solid metallic sphere is placed in a uniform electric field. Then which of the lines A, B, C and D shows the correct path and why?**

Sol. Path (A) is incorrect as lines of force do not start or end normally on the surface of a conductor. Path (B) and (C) are incorrect as lines of force does not exist inside a conductor. Also lines of force are not perpendicular to the surface of conductor. Here, path (D) represents the correct situation as lines of force are normal to surface and does not exist inside the conductor.



Q21 **The electric field in a region is given by $\vec{E} = \frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}$ with $E_0 = 2.0 \times 10^3$ N/C. Determine the electric flux of this field through a rectangular surface of area 0.2 m^2 parallel to the Y–Z plane.**

Sol. $\text{Flux} = \vec{E}\cdot\vec{S} = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right).\left(0.2\,\hat{i}\right) = 240\frac{N-m^2}{C}$

Q22 **Suppose a point charge q is placed at the centre of a cube. Find the flux linked (a) with the cube? (b) with each face of the cube?**

Sol. (a) According to Gauss's law electric flux linked with any closed body is ($1/e_0$) times the total charge enclosed and here the closed body cube is enclosing a charge q so,

total flux $\phi_T = \dfrac{1}{\varepsilon_0} q$

(b) Since, cube is a symmetrical body with six faces and a point charge is placed at its centre, so flux linked with each face will be $\phi_F = \dfrac{1}{6}(\phi_T) = \dfrac{q}{6\varepsilon_0}$

Note:

(i) In this case, flux linked with cube or one of its faces is not dependent on the side of cube.

(ii) If charge is not placed at the exact centre of cube (but anywhere inside it), total electric flux will not change, but the flux linked with different faces will be different.

Q23 **If a point charge q is placed at one corner of a cube, what is the flux linked with the cube?**

Sol. Here, after placing three cubes at three sides of cube and four such cubes above, the charge will come in the centre. Therefore, the flux linked with each cube will be one–eight of the flux $\dfrac{q}{\varepsilon_0}$.

$\therefore$ Flux associated with given cube $= \dfrac{q}{8\varepsilon_0}$

Q24 **If charges $\dfrac{q}{2}$ and 2q are placed at the centre of face and at the corner, of a cube. Then find the total flux through cube.**

Sol. Flux through cube, when $\dfrac{q}{2}$ is placed at the centre face, is $\phi_1 = \dfrac{q/2}{2\varepsilon_0} = \dfrac{q}{4\varepsilon_0}$

Flux through cube, when 2q is placed at the corner of cube, is $\phi_2 = \dfrac{2q}{8\,\varepsilon_0} = \dfrac{q}{4\,\varepsilon_0}$

Total flux $= \phi_1 + \phi_2 = \dfrac{q}{4\,\varepsilon_0} + \dfrac{q}{4\,\varepsilon_0} = \dfrac{1}{2}\dfrac{q}{\varepsilon_0}$

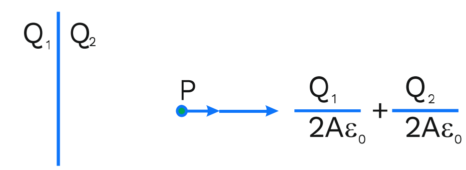
Q25 **A square of side 20 cm is enclosed by a surface of sphere of 80 cm radius. Square and sphere have the same centre. Four charges + 2 × 10^{-6} C, – 5 × 10^{-6} C, – 3 × 10^{-6} C, +6 × 10^{-6} C are placed at the 4 corners of a square, then out going total flux from spherical surface in N–m²/C will be**

Sol. Since, $\phi_{enc.} = (2 \times 10^{-6} - 5 \times 10^{-6} - 3 \times 10^{-6} + 6 \times 10^{-6}) = 0$

So, Charge, enclosed is zero.

Q26 **Consider an isolated infinite sheet contains charge Q_1 on its one surface and charge Q_2 on other surface then prove that electric field intensity at a point in front of sheet will be , where Q = Q_1 + Q_2**

Sol. Electric field at point P :



$$\vec{E} = \vec{E}_{Q_1} + \vec{E}_{Q_2} = \frac{Q_1}{2A\varepsilon_0}\hat{n} + \frac{Q_2}{2A\varepsilon_0}\hat{n}$$

$$= \frac{Q_1 + Q_2}{2A\varepsilon_0}\hat{n} = \frac{Q}{2A\varepsilon_0}\hat{n}$$

Q27 **An infinite line charge produces a field of 9 × 10^4 N/C at a distance of 2 cm. Calculate the linear charge density.**

Sol. Electric field due to the infinite line charges at a distance d having linear charge density l is $E = \frac{\lambda}{2\pi \in_0 d}; \lambda = 2\pi \in_0 dE$

where value of d is 2 cm or 0.02 m;
value of electric field is 9 × 10^4 N/C
ε_0 = permittivity of free space

$$\frac{1}{4\pi \in_0} = 9 \times 10^9 \text{ N m2 C–2}; \ \lambda = \frac{0.02 \times 9 \times 10^4}{2 \times 9 \times 10^9} = 10\,\mu C / m.$$

Q28 **A point charge placed at any point on the axis of an electric dipole at some large distance experiences a force F. Find the force acting on the point charge when it's distance from the dipole is doubled.**

Sol. Force acting on a point charge in dipole field varies as $F \propto \frac{1}{r^3}$ where r is the distance of point charge from the centre of dipole. Hence if r makes double so, new force $F' \propto \frac{F}{8}$.

Q29 **In certain region of space, electric field is directed along the z direction in whole space. The magnitude of electric field is not constant but increases uniformly along the positive z-direction, at the rate of 10^5 N C^{-1} per metre. Find the force and torque experienced by a system having a total dipole moment equal to 10^{-7} Cm in the negative z-direction?**

Sol. The dipole moment of the system is, $p = q \times dl = -10^{-7}$ C m.

The rate of increase of electric field per unit length, $\frac{dE}{dl} = 10^{+5}\,NC^{-1}$.

Then, force (F) experienced by this system is

$F = qE$

$$F = q\frac{dE}{dl} \times dl = p \times \frac{dE}{dl}$$

So, $F = -10^{-7} \times 10^{-5} = -10^{-2}$ N

The force is 10^{-2} N in –ve z-direction i.e., opposite to electric field direction. Therefore,

the angle between electric field and dipole moment is 180°.

Torque (τ) is $\tau = pE \sin 180° = 0$

Hence, the torque experienced by the system is zero.

Q30 **An electric dipole with dipole moment 4×10^{-9} C m makes an angle 30° with the direction of a uniform electric field having magnitude 5×10^4 NC^{-1}. Determine the magnitude of the torque acting on the dipole. Also find out work required to rotate the dipole to make angle 90º with the direction of the electric field.**

Sol. 10^{-4} N m, $W_{req} = \Delta U = \sqrt{3} \times 10^{-4}$J.

Q31 **An electric dipole of moment $\vec{p}$ is placed normal to the lines of force of electric intensity $\vec{E}$, then find the work done in deflecting it through an angle of 180°.**

Sol. Work done $= \int_{90}^{270} pE\sin\theta \, d\theta = [-pE\cos\theta]_{90}^{270} = 0$

Q32 **The distance between the two charges +q and -q of a dipole is r. On the axial line at a distance d from the centre of dipole, find the intensity. (d>>r)**

Sol. Field along the axis of the dipole,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{d^3} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2(q \times r)}{d^3}; \; E \propto \frac{qr}{d^3}$$

Q33 **An electron and a proton are at a distance of 1 Å. Find the moment of this dipole (cm).**

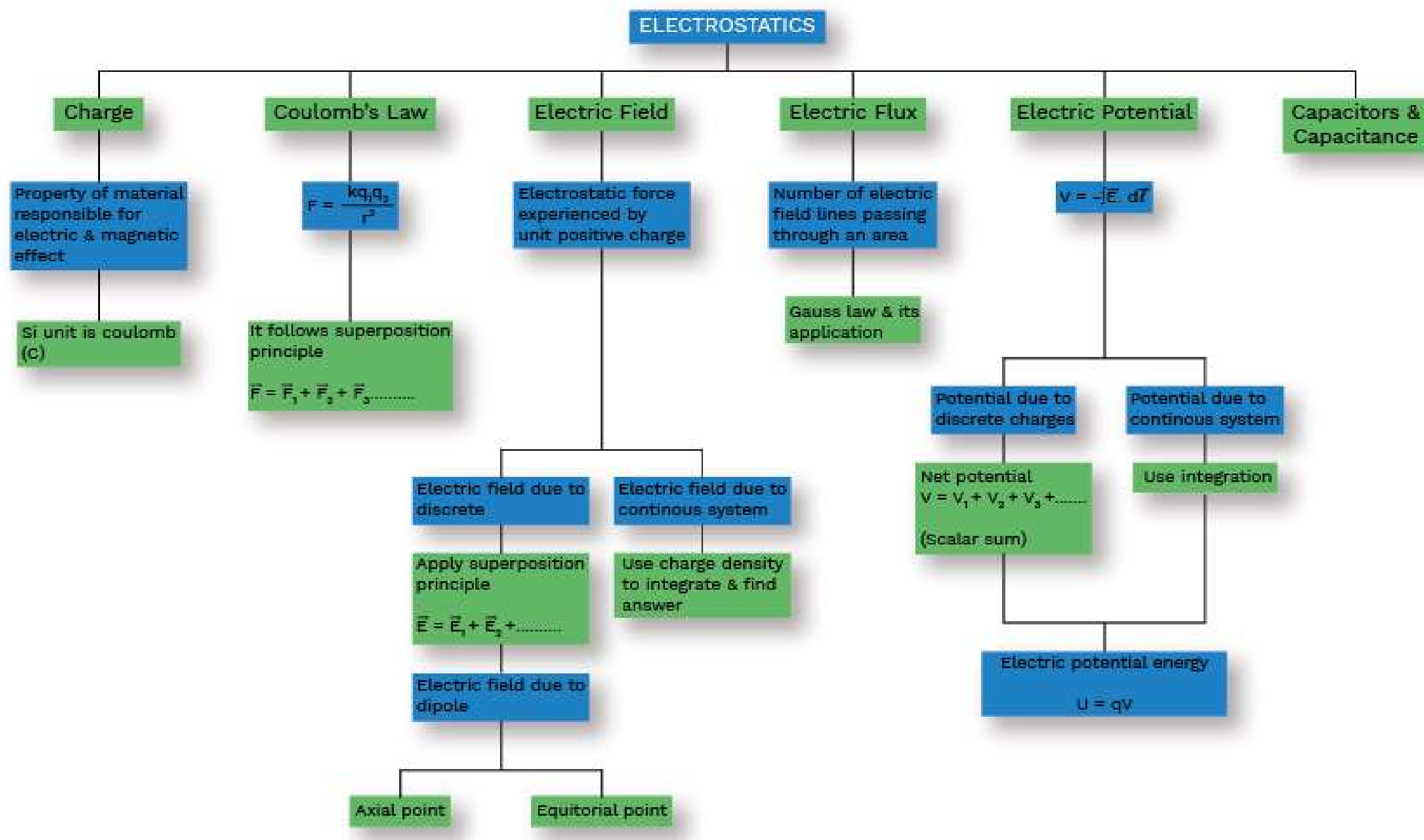
Sol. $p = q \times (2l) = 1.6 \times 10^{-19} \times 10^{-10} = 1.6 \times 10^{-29}$ C-m

Q34 **A molecule having a dipole moment p is placed in an electric field of strength E. At start the dipole is aligned parallel with the field. If the dipole has to be rotated to be in anti-parallel with the field, bodies the work has to be done by an external agency?**

Sol. $W = PE(1 - \cos\theta)$ here, $\theta = 180°$

$\therefore \quad PE = (1 - \cos 180^0) \quad = 2PE.$

Mind Map


ELECTROSTATICS
Charge
Property of material responsible for electric & magnetic effect
Si unit is coulomb (C)
Coulomb's Law
$F = \frac{kq_1q_2}{r^2}$
It follows superposition principle
$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$..........
Electric Field
Electrostatic force experienced by unit positive charge
Electric field due to discrete
Apply superposition principle
$\vec{E} = \vec{E}_1 + \vec{E}_2 +$..........
Electric field due to dipole
Axial point
Equitorial point
Electric field due to continous system
Use charge density to integrate & find answer
Electric Flux
Number of electric field lines passing through an area
Gauss law & its application
Electric Potential
$V = -\int \vec{E} \cdot d\vec{r}$
Potential due to discrete charges
Net potential
$V = V_1 + V_2 + V_3 +$.......
(Scalar sum)
Potential due to continous system
Use integration
Electric potential energy
$U = qV$
Capacitors & Capacitance