Current Electricity

DISCLAIMER

Current Electrcity

ELECTRIC CURRENT (I)

Rate of flow of charge or amount of charge flows in per unit time represents electric current.

$$\boxed{i = \frac{dq}{dt}}; \quad \boxed{i = \frac{q}{t}}$$

If electric current 'i' = constant, amount of charge (q = it).

If electric current is i = f(t), amount of charge

$$q = \int_{t_1}^{t_2} i\,dt .$$

Unit of electric current(i) = amp or C/sec (SI unit)

Dimension = $[M^0L^0T^0A^1]$

q = integration of electric current w.r.t. time or area of electric current time graph gives amount of charge flow in given time interval.

Although it has direction, but it does not obey vector addition law.

Average current <i>:

$$\langle i \rangle = \frac{\Delta q}{\Delta t} = \frac{\text{total amount of flow of charge}}{\text{time taken in flow of charge}}$$

If i = f(t)

$$\left[\langle i \rangle = \int_{t_1}^{t_2} i\,dt \right]$$

Direction of flow of electric current.

Flow of electric current is due to potential difference.

Direction of flow of electric current in circuit is from high potential to low potential.

Direction of electric current is in opposite direction of flow of free electrons.

Direction of flow of electric current is in direction of flow of positive charge.

Charge carriers:

In conductors; free e^-.

In electrolytes $\rightarrow$ ions.

KEY POINTS

- Electric current
- Average current
- Charge carriers

Concept Reminder

- Current is defined as $i = \frac{dq}{dt}$ and average current as $i_{av} = \frac{\Delta Q}{\Delta t}$.

In discharge tube → ions.
In semiconductors → free e^-s and holes.

Types of Electric Current:

1. Direct Current (D.C.)

If direction of current with time remain unchanged.
DC is unidirectional.
Source: electric cell, battery, DC generator.

2. Alternating Current (A.C.)

If direction of current changes periodically with time.
Source: AC generator.

$$AC \xrightarrow[\text{Invertor}]{\text{Rectifier}} DC$$

Ex. Electric current of 8 ampere is flowing through a conductor. Calculate number of electrons flowing through cross-section of conductor in 1 minute.

Sol. $i = \frac{q}{t}$

$$8 = \frac{n \times e}{1 \times 60}$$

$$n = \frac{480 \times 10^{19}}{1.6} = 3 \times 10^{21} \text{ electrons.}$$

Ex. In a hollow tube 6.25 × 10^{17} α-particles per sec are flowing from A to B and 12.5 × 10^{17} electrons per sec are flowing from B to A. Calculate electric current through this tube.

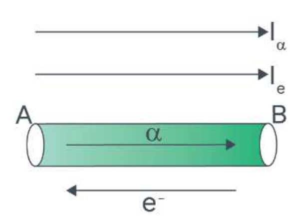
Sol.

$$I = I_\alpha + I_e$$

$$= \frac{6.25 \times 10^{17} \times 2 \times 1.6 \times 10^{-19}}{1} + 12.5 \times 10^{17} \times 1.6 \times 10^{-19}$$

$$I = 10^{-2}(1.6)[6.25 \times 2 + 12.5]$$

$$I = 1.6 \times 10^{-2} \times 25 = 0.4 \text{ amp} (A \text{ to } B)$$

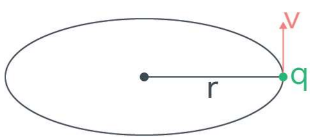


Concept Reminder

- Current is assumed to be a fundamental quantity in physics with unit ampere and dimension [A].

Ex. A particle of charge 'q' revolves in circular orbit of radius 'r' with orbiting speed of v. Calculate electric current corresponding to this option.

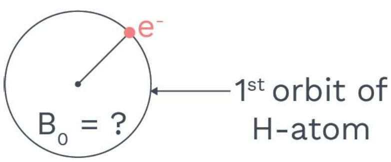
Sol. $I = \frac{q}{t} = \frac{qv}{2\pi r} = \frac{q\omega}{2\pi}$ Amp



$I = \frac{qv}{2\pi r}$

Ex. Calculate orbital current corresponding to motion of electron in 1st orbit of H-atom.

Sol. $I = \frac{qv}{2\pi r} = \frac{e \times 2.186 \times 10^{6}}{2\pi \times 0.529 \times 10^{-10}}$



$I = \frac{1.6 \times 10^{-19} \times 2.186 \times 10^{6}}{2 \times 3.14 \times 0.529 \times 10^{-10}}$

$I \simeq 1$ mA

Ex. An α-particle revolves with 6.25×10^{15} cycles/sec. Calculate corresponding orbital current.

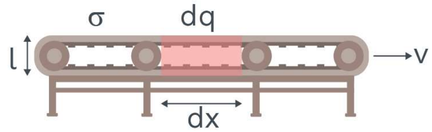
Sol. $I = qf = 2ef = 2 \times 1.6 \times 10^{-19} \times 6.25 \times 10^{15}$

$= \frac{3.2 \times 25 \times 10^{-4}}{4}$

$I = 20 \times 10^{-4}$

$I = 2$ mA

Ex. A charged belt with surface charge density σ is made to move with speed v. Width of belt is l. Calculate corresponding electric current.



Sol. $i = \frac{dq}{dt} = \frac{\sigma l dx}{dt}$

$i = \sigma l v$

Ex. Electric current through a conducting wire is as i = (2t + 3) amperes. Calculate amount of charge passing through cross-section of wire in 1^{st} 10 sec.

Sol. $0 \rightarrow 10$ sec,

$$q = \int i dt = \int_0^{10} (2t+3)dt$$

$$q = [t^2 + 3t]_0^{10}$$

q = 100 + 30

q = 130 C

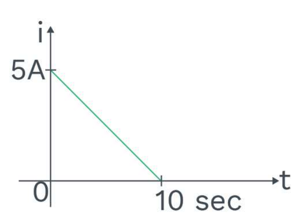
Ex. In the above question, calculate time at which inst. current is equal as average current of 1^{st} 5 sec.

Sol. $q = \int_0^5 (2t+3)dt = [t^2 + 3t]_0^5$

$q = 25 + 15 = 40$ C

or $\quad i_{avg} = \frac{\int_0^5 (2t+3)dt}{\int_0^5 dt} = \frac{[t^2+3t]_0^5}{[t]_0^5} = \frac{40}{5} = 8\,A$

Ex. If electric current passing through a wire decreases linearly from 5 A to 0 A in 10 sec. Calculate amount of charge flow through the wire.



Sol. $q = \int i dt$

$q = $ Area of 'i – t' curve

$q = \frac{1}{2} \times 10 \times 5 = 25\,Cb$

Ex. Charge passing through a wire is given as $q = at - bt^2$ where a and b are positive constants then:

(i) Find the expression of instant current.

(ii) Draw the variation of current with time.

(iii) Amount of charge flow through the wire.

Sol. $i = \frac{dq}{dt} = a - 2bt$

at t = 0, i = a

Charge flows till i = 0

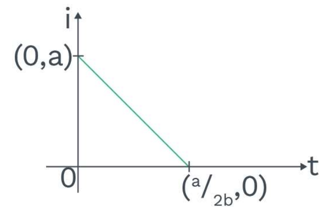
$a - 2bt = 0$

$t = -\frac{a}{2b}$

$q = \frac{aa}{2b} - \frac{ba^2}{4b^2} = \frac{a^2}{2b} - \frac{a^2}{4b} = \frac{a^2}{4b}$

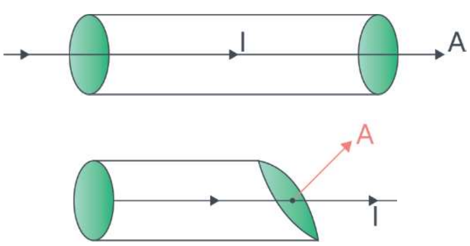
or $q = \frac{1}{2} \times \frac{q}{2b} \times a$

$q = \frac{a^2}{4b}$



CURRENT DENSITY (J)

- Current passing through per unit cross-sectional area of proportional cross-section is named current density.



$J = \frac{I}{A}$ amp/m^2

$J = \frac{I}{A\cos\theta}$

$[I = JA\cos\theta = \vec{J}\cdot\vec{A}]$

Current density is a vector with direction in direction of flow of electric current.

Definitions

Current passing through per unit cross-sectional area of proportional cross-section is named current density.

$J = \frac{I}{A}$ amp/m^2

KEY POINTS

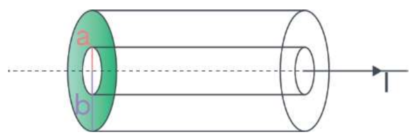
- Current density

Ex. Calculate current passing through cross-section of area $\vec{S} = (2\hat{i} + 3\,\hat{j}) \times 10^3\ \text{cm}^2$ according to current density of $\vec{J} = 4\,\hat{i}\ \text{A}/\text{m}^2$.

Sol. $I = \vec{J} \cdot \vec{A} = (4\hat{i}) \cdot (2\hat{i} + 3\hat{j}) \times 10^3 \times 10^{-4}$

$I = 0.8$ Amp

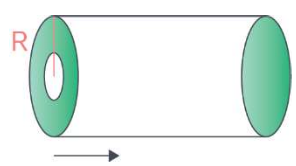
Ex. Current of I is passing through hollow cylindrical conductor along its length. Inner and outer radii of its cross-section a and b respectively. Calculate current density.



Sol. $J = \dfrac{I}{A} = \dfrac{I}{\pi(b^2 - a^2)}$

Ex. There is a solid cylindrical wire of radius of cross-section 'R'. Current through the wire is according to current density of $J = J_0\left(1 - \dfrac{r}{R}\right)$.

Where I_0 = constant and r = radial distance from axis of wire. Calculate current through wire.



Sol. Current through cross-section of small element:

dI = IdA = $\Rightarrow$

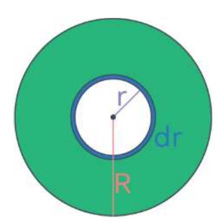
$$= J_0 r 2\pi\left(1 - \frac{r}{R}\right)dr$$

$$I = \int dI = J_0 \times 2\pi\int_0^R \left[r - \frac{r^2}{R}\right]dr$$

$$= 2\pi J_0\left[\frac{r^2}{2} - \frac{r^3}{3R}\right]$$

$$I = 2\pi J_0 r^2\left[\frac{1}{2} - \frac{r}{3R}\right]_0^R$$

$$I = 2\pi J_0 R^2\left[\frac{1}{6}\right] = \frac{\pi J_0 R^2}{3}$$



THERMAL SPEED (V_T)

- When there is no potential difference across the conductor then free electrons inside the conductor move randomly with thermal speed. This speed is due to thermal energy which is due to surrounding temperature.
- Expression of thermal speed is as:

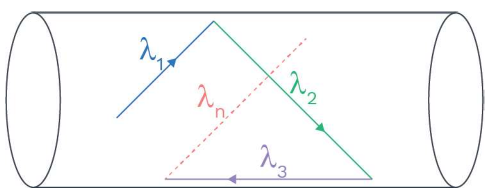
$$\left[V_T = \sqrt{\frac{3KT}{m}}\right]$$

At room temperature, order of thermal speed is 10^5 m/s. During random motion free electron collide with lattice and with fixed positive ions in lattice. Due to collisions direction of motion changes frequently so for electron → thermal speed = same motion of free electrons is in all possible direction so during this random motion, average thermal velocity is zero.

- During random motion, net flow of charge in particular direction through cross-section of conductor is zero so there is no flow of electric current in once of potential difference across conductor.

Mean Free Path ($\bar{\lambda}$):

- Average distance travelled by free electron between 2 successive collisions is named mean free path.



$$\bar{\lambda} = \frac{\text{Total distance}}{\text{Number of collisions}}$$

$$\boxed{\bar{\lambda} = \frac{\lambda_1 + \lambda_2 + \lambda_3 + \ldots.. + \lambda_n}{N}}$$

Mean free path is of 'A' order.

Definitions

Thermal speed is given as-

$$V_T = \sqrt{\frac{3KT}{m}}$$

Concept Reminder

According to modern view, a metal consists of a lattice of fixed positively charged ions in which billions and billions of free electrons are moving randomly at speed which at room temperature.

Time of Relaxation (τ):

Average time spend by free electron between 2 successive collisions is named time of relaxation.

$$\tau = \frac{\text{Total time}}{\text{Number of collisions}}$$

$$\tau = \frac{t_1 + t_2 + t_3 + ... + t_N}{N}$$

$$\tau = \frac{\frac{\lambda_1}{V_T} + \frac{\lambda_2}{V_T} + \frac{\lambda_3}{V_T} + ... \frac{\lambda_N}{V_T}}{N}$$

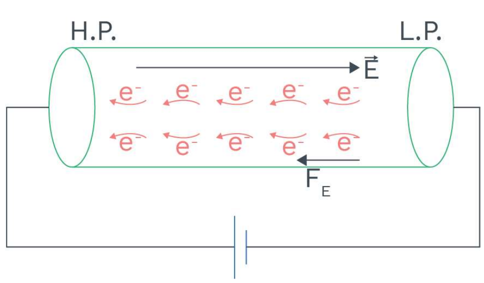
$$\tau = \frac{1}{V_T}\left[\frac{\lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_N}{N}\right]$$

$$\boxed{\tau = \frac{\bar{\lambda}}{V_T}}$$

Time of relaxation (τ) depends on temperature.
temperature increases $\rightarrow \tau$ decreases
At room temperature order of time of relaxation is 10^{-15} sec.

Drift Velocity (υ_d):

- If it is low, then also it is not possible to measure it because actual speed of E is very high and random.



- When potential difference is applied across the conductor then electric field is established inside the conductor, so electrons experience electric force due to this force, random motion of electrons get converted into regulated motion.

KEY POINTS

- Thermal velocity
- Mean free path
- Time of relaxation
- Drift velocity

Definitions

Drift velocity is the average uniform velocity acquired by free electrons inside a metal by the application of an electric field which is responsible for current through it $\vec{v}_d = -\frac{e\vec{E}\tau}{m}$.

Rack your Brain

The velocity of charge carriers of current (about 1 A) in a metal under normal conditions is of what order?

- Under the combined effect of acceleration equation retardation, free electrons move with constant average velocity named drift velocity.

$$\vec{\upsilon} = \vec{u} + \vec{a}t$$

$$< \vec{\upsilon} > = < \vec{u} > + < \vec{a}t >$$

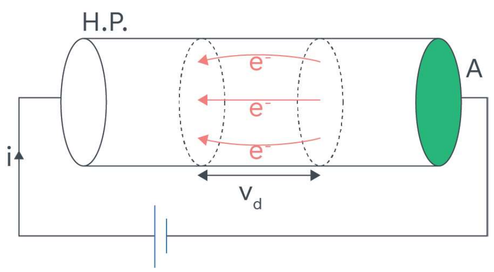
$$< \vec{\upsilon} > = 0 + < \vec{a}t >$$

$$\boxed{\vec{\upsilon}_d = -\frac{e\vec{E}(\tau)}{m}}$$

$\boxed{\upsilon_d = \frac{eE\tau}{m}}$ $\rightarrow$ Drift velocity depends upon temperature.

At room temperature, drift velocity is of mm/sec or 10^{-3}m/sec.

Relation between electric current and drift velocity:



$n \rightarrow$ Charge carrier density

$$n = \frac{\text{Total number of charge carries}}{\text{Volume}}$$

Number of free electrons passing through cross-section in 1 sec. = $n(A\upsilon_d)$

Charge passing through cross section i.e., electric current in 1 sec = $nA\upsilon_d e$.

$$\boxed{i = nA\upsilon_d e}$$

or in time dt

Concept Reminder

The two processes collision and acceleration result in a dynamic equilibrium in which a uniform average drift velocity is achieved by electrons.

$$dq = N(e)$$

$$dq = n(Adx)e$$

$$i = \frac{dq}{dt} = \frac{nAdxe}{dt}$$

$$= nAe\upsilon_d$$

Note: If electric current of 'i' is passing through conducting wire of length 'l' then total momentum of free electron in conducting wire is as:

Total momentum

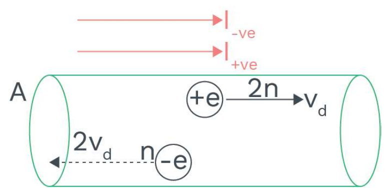
$$P = m\upsilon_d = Nm_e\upsilon_d = nAlm_e\upsilon_d$$

$$P = (nA\upsilon_d)m_e l$$

$$P = \frac{im_e l}{e}$$

$$\boxed{P = \frac{m_e il}{e}}$$

Ex. Charge carriers are flowing in a discharge tube. Calculate the electric current.



Sol. $I_T = I_+ + I_- = 2neA\upsilon_d + neA2\upsilon_d$

$I_T = 4\,neA\upsilon_d$

Ex. There is a cylindrical conductor of cross-section area 10^{-6} m^2 with its density 5×10^3 kg/m^3 at weight is 60. If current through conductor is 2 amp then calculate drift velocity of free electrons. (Given: 1 atom provides 1 e^-)

Sol. $i = neA\upsilon_d$

$$\upsilon_d = \frac{i}{neA}$$

Concept Reminder

- Current can be defined as-

  $i = nAev_d$
- Total momentum of free electrons in conducting wire is

  $$P = \frac{m_e il}{e}$$

$$n = \frac{N}{V} = \frac{60 \times 6 \times 10^{23}}{5}$$

$$= \frac{N_A \times \text{number of moles } \rho}{m}$$

$$= N_A \frac{\rho}{m} \frac{m}{M_w} = \frac{N_A \rho}{M_w}$$

$$= \frac{6 \times 10^{23} \times 5 \times 10^{23}}{60 \times 10^{-3}} = 5 \times 10^{28}$$

$$\upsilon_d = \frac{2}{5 \times 10^{28} \times 10^{-6} \times 1.6 \times 10^{-19}} = 2.5 \times 10^{-4} \text{ m/s}$$

Relation Between Current Density and Conductivity:

$$i = neA\upsilon_d$$

$$\frac{i}{A} = ne\upsilon_d$$

$$J = ne\left(\frac{eE\tau}{m}\right)$$

$$J = \frac{ne^2\tau}{m}E$$

$$J = \sigma E \qquad \left[\because \sigma = \frac{ne^2\tau}{m}\right]$$

$$\vec{J} = \sigma\vec{E}$$

Direction of current density of flow of electric current is in direction of established electric field. (due to applied potential difference)

$$\sigma = \frac{ne^2\tau}{m}$$

'n' characteristic feature of conductor.

- Electrical conductivity depends on temperature and nature of conductor. At given temperature, order of conductivity is as:

$$\sigma_{Ag} > \sigma_{Cu} > \sigma_{Au} > \sigma_{Al}$$

Concept Reminder

- $\vec{J} = \sigma\vec{E}$
- $\sigma = \frac{ne^2\tau}{m}$

Rack your Brain

Find out mobility of charged particles having drift velocity of 7.5×10^{-4} ms^{-1} in an electric field of 3×10^{-10} Vm^{-1}.

Mobility of Free e^-s:

- Drift velocity of free electrons corresponding to per unit applied electric field is mobility of free electrons.

Mobility,

$$\boxed{\mu = \frac{\upsilon_d}{E}} \Rightarrow \upsilon_d = \mu E$$

$$\upsilon_d = \frac{eE\tau}{m}$$

$$\Rightarrow \mu = \frac{e\tau}{m}$$

Ex. A steady current is passing through conductor of non-uniform cross-section as shown; then compare following physical quantities at both ends of conductor.

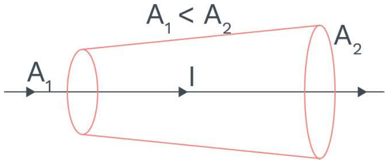
(i) Drift velocity

(ii) Current density

(iii) Conductivity

(iv) Mobility

(v) Electric field.



Sol. $\upsilon_{d_1} > \upsilon_{d_2}$, $J_1 > J_2$, $\sigma_1 = \sigma_2$,

$\mu_1 = \mu_2$, $E_1 = E_2$

(i) $i = neA\upsilon_d \Rightarrow neA\upsilon_d = \text{constant}$

$A\upsilon_d = \text{constant}$

$A_2 > A_1$; $\upsilon_{d_2} < \upsilon_{d_1}$

(ii) $J = \frac{I}{A} \Rightarrow J \propto \frac{1}{A}$

$A_2 > A_1$; $J_1 > J_2$

(iii) $\sigma = \frac{ne^2\tau}{m} \Rightarrow \sigma = \text{constant}$

$\sigma_1 = \sigma_2$

(iv) $\left[\mu = \frac{e\tau}{m}\right] \Rightarrow \mu_1 = \mu_2$

(v) $J = E\sigma$

$J \propto E \Rightarrow J_1 > J_2 \Rightarrow E_1 > E_2$

Concept Reminder

The direction of drift velocity for electrons in a metal is opposite to that of applied field $\vec{E}$.

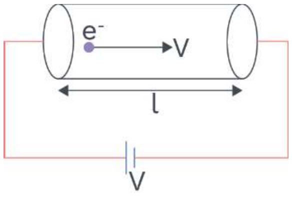
- During flow of electric current, number of free electrons of conductor remains unchanged. So current carrying conductor is electrically neutral. So, current carrying conductor does not produce electric field.
  Electric field outside the current carrying conductor is zero and inside the current carrying conductor is non-zero. (it is due to applied potential difference)
- When potential difference is applied across the conductor then establishment of electric field inside the conductor is with light speed and charge carriers (free e^-) are abundant in conductor so there is no time delay between applying the pot difference and flow of electric current (so bulb glows instantly)
  $\Delta V \rightarrow$ characteristic of source
  Current (I) $\rightarrow$ characteristic of ckt.

Ohm's Law:
This law states that if physical conditions of conductor (geometry and temperature of conductor) are unchanged then flow of electric current through conductor is directly proportional to applied potential difference.

$i \propto V$ or $V \propto i$

$\boxed{V = iR}$

R $\rightarrow$ proportional constant or resistance.



$$i = neAv_d = neA\left(\frac{eE\tau}{m}\right)$$

$$= \frac{ne^2AE\tau}{m} = \frac{ne^2\tau}{m}A\left(\frac{V}{\ell}\right)$$

KEY POINTS
- Conductivity
- Mobility
- Ohm's law

Definitions

The property of a substance due to which it opposes the flow of current through it is called its "electrical resistance".

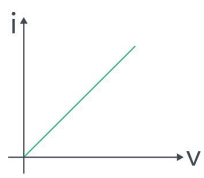
$$i = \frac{\sigma AV}{\ell} = \frac{A}{\rho \ell}V$$

$$\boxed{i = \frac{V}{R} \Rightarrow (i \propto V)}$$

Ohm's Law is not universal:

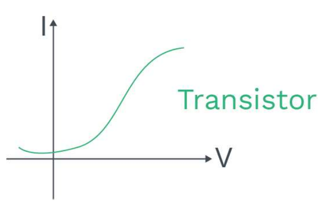
Ohmic substance $\rightarrow$ substance which follow ohm's law is named ohmic substance and corresponding ckt is ohmic ckt. (linear substance because $i \propto V$)

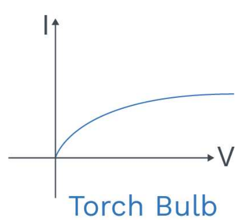
e.g., metallic conductors.

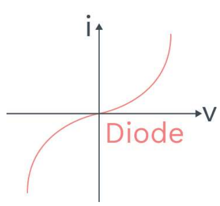


Non-ohmic substance $\rightarrow$ substance which doesn't follows ohm's law named non-ohmic substance and corresponding ckt is named non-ohmic ckt.

e.g., semiconductor, discharge tube, electrolyte.







Resistance (R):

- Opposition of flow of electric current or opposition of flow of charge carriers is named resistance.
- Resistance in conductors arise mainly due to collisions.

Rack your Brain

The resistance of discharge tube is:
(1) Non-ohmic
(2) Ohmic
(3) Zero
(4) Both (2) and (3)

Concept Reminder

- $R = \frac{V}{i}$
- $R = \frac{\rho L}{A}$
- $R = \frac{mL}{ne^2 \tau A}$

T↑ → R↑ in conductors (because collisions increases)

$$R = \frac{V}{i}$$

$$R = \frac{mL}{ne^2\tau A} \Rightarrow \left[R = \frac{\rho L}{A}\right] \Rightarrow \left[R = \frac{L}{\sigma A}\right]$$

[V = iR], macroform of ohm's law

$I = \frac{E}{\rho}$, microform of ohm's law

$\rho = \frac{m}{ne^2\tau}$, resistivity or specific resistance

$\rho = \frac{1}{\sigma}$, resistivity is reciprocal of conductivity.

- Resistivity depends on temperature and nature of conductor. It does not depend on geometry of conductor. (It is property of material)
- Resistance depends on temperature and nature of conductor along with geometry. It does not depend on current passing through it and applied potential difference.

$$\frac{\text{Joule}}{\text{Cb amp}} \Rightarrow \frac{\text{Joule}}{\text{Amp}^2\ \text{sec}}$$

$$= \frac{M^1L^2T^{-2}}{A^2T} = [M^1L^2T^{-3}A^{-2}]$$

Conductance (G):

- Reciprocal of resistance is named conductance.

$$\left[G = \frac{1}{R}\right]$$

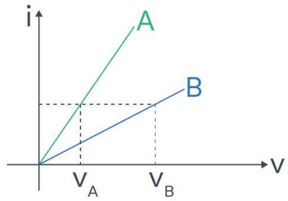
'Ω^{-1}' or 'mho' or 'Seimen'

$[M^{-1}L^{-2}T^{3}A^{2}]$

KEY POINTS

- Resistance
- Conductance
- Resistivity
- Conductivity

Ex. Current passing through 2 conductors A and B varies with applied potential difference across their edge as shown. Compare their resistances R_A and R_B.



Sol. V = iR

$$\frac{i}{V} = \frac{1}{R} = (\text{Slope})$$

$(\text{Slope})_A > (\text{Slope})_B \Rightarrow R_B > R_A$.

Ex. Three Cu-wires are with ratio of length 1 : 2 : 3 and ratio of cross sectional area 3 : 2 : 1. Find ratio of their resistances same material.

Sol. $R = \rho\frac{\ell}{A}$

$$R \propto \frac{\ell}{A}$$

$$R_1 : R_2 : R_3 :: \frac{1}{3} : \frac{2}{2} : \frac{3}{1}$$

$$R_1 : R_2 : R_3 :: 1 : 3 : 9$$

Ex. There are 3 Cu-wires with ratio of length 1 : 2 : 3 and ratio of their mass 2 : 3 : 4 find ratio of their resistances.

Sol. $R = \rho\frac{\ell}{A}$

$$R = \rho\frac{\ell\ell d}{m} = \rho\frac{\ell^2 d}{m}$$

$$R \propto \frac{\ell^2}{m}$$

$$R_1 : R_2 : R_3 :: \frac{1}{2} : \frac{4}{3} : \frac{9}{4}$$

$$R_1 : R_2 : R_3 :: 6 : 16 : 27$$

Ex. An aluminium wire of diameter 0.80 mm carrying current. Its resistivity is 2.75×10^{-8} Ω mt. If electric field inside the aluminium wire 0.55 volt/m then calculate:

(i) Electric current through Al-wire.

(ii) Potential difference between points of 12 m apart in Al wire.

(iii) Resistance of this wire of length 12 m.

Sol. (i) $J = \frac{E}{\rho}$

$$\frac{i}{A} = \frac{E}{\rho} = \frac{0.55 \times \pi \times 0.64 \times 10^{-6}}{2.75 \times 10^{-8}}$$

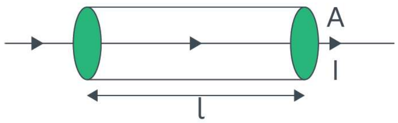
$$= i = \frac{16\,\pi}{5} = 10\,\text{amp} = 10.048\,\text{amp}$$

(ii) $E = \frac{\Delta V}{d} \Rightarrow V = E \times d$

$$0.55 \times 12 = \frac{11 \times 12}{20} = 6.6\,V$$

(iii) $R = \frac{V}{i} = \frac{6.6}{10.048} = 0.65\,\Omega$

Dependence of resistance on geometry of conductor



$$R = \rho \frac{\ell}{A}$$

$\ell \rightarrow$ Along flow of current

$A \rightarrow$ Area of direction proportional cross section

Ex. There is a conducting block with ratio of dimensions 1 : 2 : 4. Find ratio of maximum resistance and minimum resistance of conducting block.

Sol. $R = \frac{\rho \ell}{A} \Rightarrow R_{max} \propto \frac{\ell_{max}}{A_{min}}$

$$\Rightarrow \quad R_{max} = \frac{4}{2 \times 1}$$

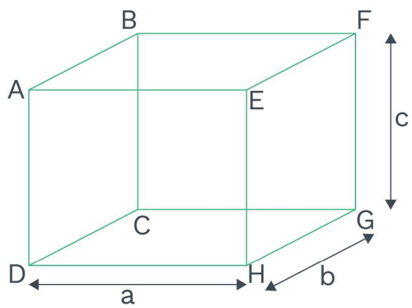
$$\frac{R_{max}}{R_{min}} = \frac{16}{1}$$

Ex. There is a cuboidal block with dimensions as shown. Resistivity of material of block is ρ. Find resistance of block between given faces:

(i) Between ABCD and EFGH

(ii) Between BFGC and AEHD

(iii) Between ABFE and DCGH



Sol. (i) $R = \frac{\rho a}{bc}$

(ii) $R = \frac{\rho b}{ac}$

(iii) $R = \frac{\rho c}{ab}$

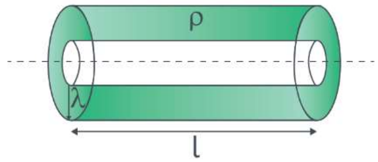
Ex. There is a hollow cylindrical conductor of length l. Resistivity of its material is ρ. Its inner and outer radii of cross-section are a and b. Then calculate resistance of this conductor.

(i) Between circular faces.

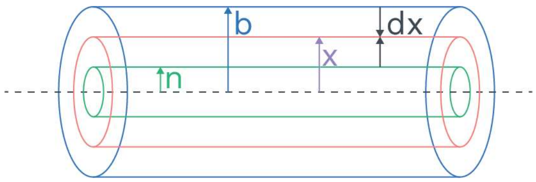
(ii) Between inner and outer curved surfaces.

Sol. (i) $R_1 = \frac{\rho l}{A} = \frac{\rho l}{\pi b^2 - \pi a^2}$

$R_1 = \frac{\rho l}{\pi(b^2 - a^2)}$



(ii) $R_2 = \frac{\rho l}{A} = \frac{\rho(b-a)}{A}$



Resistance of cylindrical shell small element

$$dR = \frac{\rho dx}{2\pi x l}$$

$$R = \int dR = \frac{\rho}{2\pi l}\int_a^b dx$$

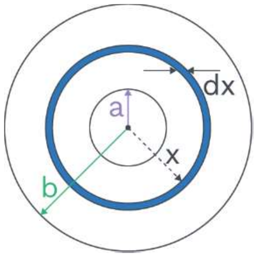
$$= \frac{\rho}{2\pi l}[\log_e x]_a^b$$

$$R = \frac{\rho}{2\pi l}\log_e\left(\frac{b}{a}\right)$$

Ex. There is a hollow conducting sphere with inner and outer radii 'a' and 'b'. Resistivity of material is ρ. Calculate resistance between inner and outer spherical surfaces.

Sol. Resistance of spherical shell small element

$$dR = \frac{\rho dx}{4\pi x^2}$$



$$R = \int dR = \int \frac{\rho dx}{4\pi x^2} = \frac{\rho}{4\pi}\left[-\frac{1}{x}\right]_a^b$$

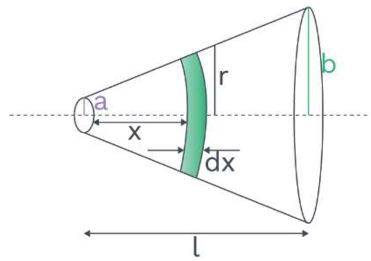
$$R = \frac{\rho}{4\pi}\left[\frac{1}{x}\right]_b^a$$

$$R = \frac{\rho}{4\pi}\left[\frac{1}{a}-\frac{1}{b}\right] = \frac{\rho}{4\pi}\left[\frac{b-a}{ab}\right]$$

$$R \propto \left(\frac{b-a}{ab}\right)$$

$$i \propto \left(\frac{ab}{b-a}\right)$$

Ex. There is a solid cylinder of non-uniform cross-section as shown, length of cylinder is l and resistivity of its material is ρ then calculate resistance along its length.



Sol. $y = mx + c$

$$r = a + \left(\frac{b+a}{l}\right)x$$

$$\frac{dr}{dx} = \left(\frac{b-a}{l}\right) \Rightarrow dx = \frac{l\,dr}{(b-a)}$$

Resistance of small element

$$dR = \frac{\rho dx}{A} = \frac{\rho dx}{\pi r^2}$$

$$\Rightarrow \quad dR = \frac{\rho l}{\pi r^2}\frac{dr}{(b-a)}$$

$$R = \int dR = \frac{\rho l}{\pi(b-a)}\left[-\frac{1}{r}\right]_a^b$$

$$= \frac{\rho l}{\pi(b-a)}\left[\frac{1}{r}\right]_b^a = \frac{\rho l}{\pi(b-a)}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$= \frac{\rho l}{\pi(b-a)}\left(\frac{b-a}{ab}\right)$$

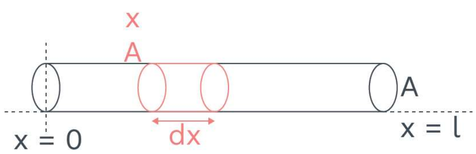
$$R = \frac{\rho l}{\pi ab}$$

If $a = b \Rightarrow a$

$$\Rightarrow \quad R = \frac{\rho l}{\pi a^2}$$

Ex. A cylindrical rod of length 'l' with cross-section area 'A' is placed between x = 0 to x = l. Resistivity of its material is $\rho = \rho_0 x$. Where ρ_0 = constant and x is distance from origin. Calculate resistance along its length.

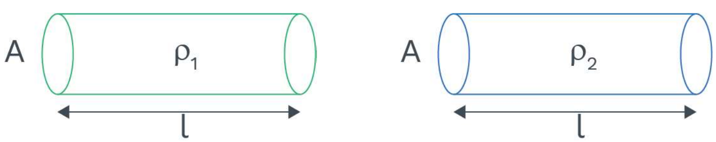
Sol. $R = \dfrac{\rho l}{A}$



Resistance of small element

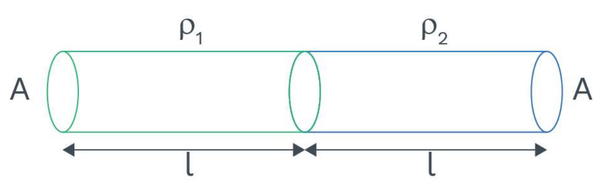
$$R = \int dR = \frac{\rho_0 x dx}{A} = \frac{\rho_0 x^2}{A} = \frac{\rho_0 l^2}{2A}$$

Ex. There are 2 cylindrical wires of same length is equal cross-sectional area. Resistivity of their material is ρ_1 and ρ_2. Calculate equivalent resistivity of their series I-combination and their II-combination.



(i) $R_1 = \dfrac{\rho_1 l}{2}$ (ii) $R_2 = \dfrac{\rho_2 l}{2}$

Sol. (i) $R_{eq} = R_1 + R_2$

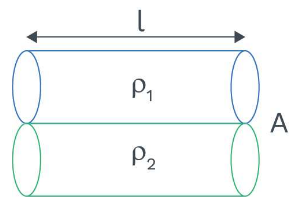


$$\frac{\rho_{eq} 2l}{A} = \frac{\rho_1 l}{A} + \frac{\rho_2 l}{A}$$

$$\rho_{eq} = \frac{\rho_1 + \rho_2}{2} \Rightarrow \sigma_{eq} = \frac{2\sigma_1\sigma_2}{\sigma_1 + \sigma_2}$$

(ii) $R_{eq} = \frac{R_1R_2}{R_1 + R_2}$

$$\frac{\rho_{eq}l}{2A} = \frac{\frac{\rho_1 l}{A} \times \frac{\rho_2 l}{A}}{\frac{\rho_1 l}{A} + \frac{\rho_2 l}{A}} = \frac{\rho_1\rho_2}{\rho_1 + \rho_2}\left(\frac{l}{A}\right)$$



$$\rho_{eq} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} \Rightarrow \sigma_{eq} = \frac{\sigma_1 + \sigma_2}{2}$$

Ex. In the above question, if conductivity of wires is σ_1 and σ_2 then calculate equivalent conductivity of series and II-combination.

Sol. Series,

$$\Rightarrow \quad \sigma_{eq} = \frac{2\sigma_1\sigma_2}{\sigma_1 + \sigma_2}$$

Parallel,

$$\sigma_{eq} = \frac{\sigma_1 + \sigma_2}{2}$$

Effect on resistance of wire due to change in its geometry by stretching it

$$R = \frac{\rho l}{A} \Rightarrow R \propto \frac{l}{A}$$

$$R \propto l^2 \propto \frac{1}{A^2} \propto \frac{1}{r^4}$$

For given wire:

mass = constant

volume × density = constant

volume = constant

Al = constant.

$$A \propto \frac{1}{l} \text{ or } l \propto \frac{1}{A}$$

r ⇒ radius of cross-section.

Ex. Resistance of a conducting wire is 'R'. Calculate its new resistance in case of n-times the length by stretching it.

Sol. $R' \propto l^2$

$R' \propto (nl)^2$

$$\frac{R}{R'} = \frac{1}{n^2} \Rightarrow R' = n^2R$$

Ex. Resistance of a wire is 10 Ω. Its length is increased 50% by stretching it. Calculate its new resistance.

Sol. $R' \propto l^2$

$$\frac{10}{R'} = \left(\frac{100}{150}\right)^2 = \frac{100}{225}$$

R' = 22.5 W

$\Delta R = R' - R = 22.5 - 10 = 12.5\,\Omega$

Ex. Length of a wire is increased 30% by stretching it. Calculate percentage change in resistance of wire.

Sol. $R \propto l^2$

$$\frac{100}{R'} = \left(\frac{100}{130}\right)^2 = \frac{100}{169}$$

R' = 169

Percentage change = $\frac{R' - R}{R} \times 100$

$= \frac{169 - 100}{100} \times 100 = 69\%$

In case of small percentage changes.

$y \propto x^n$

$$\frac{\Delta y}{y} = n\left(\frac{\Delta x}{x}\right)$$

e.g., in case of stretching the wire.

$$R \propto l^2 \propto \frac{1}{A^2} \propto \frac{1}{r^4}$$

$$\frac{\Delta R}{R} = 2\left(\frac{\Delta l}{l}\right) = -2\left(\frac{\Delta A}{A}\right) = -4\left(\frac{\Delta r}{r}\right)$$

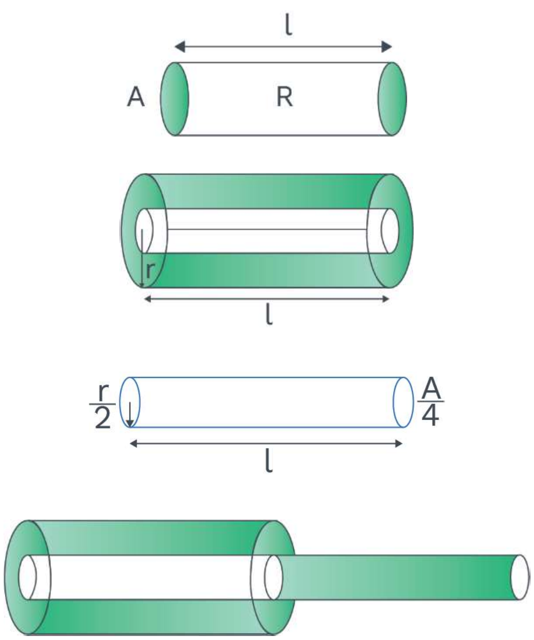
Ex. When a wire is stretched resulting decreasing in radius of cross-section is 0.3% then calculate percentage in resistance of wire.

Sol. $\frac{\Delta R}{R} = 4\left(-\frac{\Delta r}{r}\right)$

$$\frac{\Delta R}{R} = 4(-0.3)$$

$$\frac{\Delta R}{R} = 1.2$$

Ex. Resistance of a solid cylindrical conductor is 'R'. Its axial part of half of radius of cross-section is pulled out along length. Calculate total resistance of both the parts after pulling net.



Sol. $R_T = \frac{4R}{3} + 4R$

$R_T = \frac{16R}{3}$

Dependency of resistance on temperature:

$\rho_\theta = \rho_0(1+\alpha\theta)$

$R_\theta = R_0(1+\alpha\theta+\beta\theta)$

$R_\theta = R_0(1+\alpha\theta)$

R_0 = resistance at 0°C
R_θ = resistance at θ°C
α = temperature coefficient of resistance $\left[\text{in } \frac{1}{°C}\right]$

θ = temperature (in °C)

Concept Reminder

- Resistivity and resistance of alloy like manganin and constant are almost independent of temperature variation.

$$R_\theta = R_0 + R_0\alpha\theta$$

$$\frac{R_\theta - R_0}{R_0} = \alpha\theta \Rightarrow \alpha = \frac{R_\theta - R_0}{R_0(\theta)}$$

Fraction of change in resistance corresponding to unit change in temperature represents temperature coefficient of resistance.

For conductors: $\alpha = \oplus ve$ and $T \propto \frac{1}{R}$;

when $T\uparrow \Rightarrow R\downarrow$ and $T\downarrow \Rightarrow R\uparrow$

For semi-conductors: $\alpha = -ve$ and $T \propto \frac{1}{R}$;

When $T\uparrow \Rightarrow R\downarrow$ and $T\downarrow \Rightarrow R\uparrow$

For insulators: $\alpha = -ve$ [practically $R_{eff} = \infty$]

[theoretically $\rightarrow$ negligible]

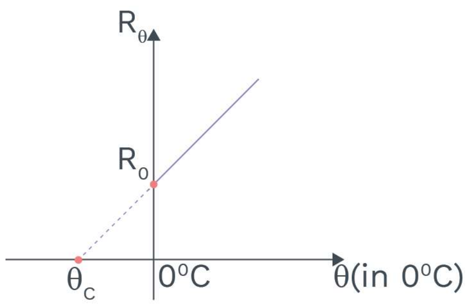
$$R = \frac{mL}{ne^2\tau A}$$

Temperature increase $\rightarrow$ n increases, τ decreases
For conductors $\rightarrow$ τ decreases $\rightarrow$ dominant
For semiconductors $\rightarrow$ n increases $\rightarrow$ dominant
For conductors

$$R_\theta = R_0(1 + \alpha\theta)$$

$$R_\theta = R_0 + R_0\alpha\theta$$

$$y = c + mx$$



Rack your Brain

The solids which have the negative temperature coefficient of resistance are:

(1) Metals
(2) Insulators only
(3) Semi-conductors only
(4) Insulators and semi-conductors

Definitions

At particular temperature, resistance and resistivity of conductors becomes zero. This temperature is named critical temperature.

Resistance of conductors varies τ linearly with variation in temperature so, when temperature of conductors decreases then their resistance decreases linearly. At particular temperature, resistance and resistivity of conductors becomes zero. This temperature is named critical temperature.

- When resistance of conductors becomes zero then they get convert into superconductors.

A-There is no need of potential difference to maintain the flow of electric current in superconductor.

B-Resistance of superconductors is zero.

Superconductivity was 1st observed by 'Kamerlingh Onnes' in mercury (Hg) in 1911.

For Hg $\rightarrow \theta_c$ = 3.2 K (1st time)
For Pb $\rightarrow \theta_c$ = 7.5 K
For Cu $\rightarrow \theta_c$ = 38.5 K

Ex. Temperature coefficient of resistance of a wire 0.004 °C^{-1}. Calculate temperature at which its resistance becomes 2 times of resistance at 0°C.

Sol. $R_\theta = R_0(1+\alpha\theta)$

$$2R_\theta = R_0(1+\alpha\theta)$$

$$\alpha\theta = 1$$

$$\theta = \frac{1000}{4} = 250°C$$

$$\theta = 250°C = 523\,K$$

Ex. Temperature coefficient of resistance of a wire is 0.00125 °C^{-1}. Resistance of this wire at 27°C is 1 Ω. Calculate temperature at which its resistance become 2 Ω.

Sol. $R_\theta = R_0(1+\alpha\theta)$

$$1 = R_0(1+\alpha\,27) \quad [R_0 \neq 1\,\Omega]$$

$$2 = R_0(1+\alpha\,\theta)$$

R_0 is resistance at 0°C while 1 Ω resistance is at 27°C.

$$\frac{1}{2} = \frac{1+27\alpha}{1+\alpha\theta}$$

$$1+\alpha\theta = 2+54\,\alpha$$

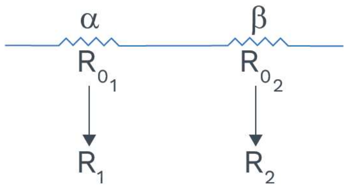
$$\alpha\theta = 1+54\,\alpha$$

$$\theta = \frac{1}{2}+54 = \frac{10^5}{125}+54 = \frac{4000}{5}+54$$

$$\theta = 854\,°C = 1127\,K$$

Ex. Temperature coefficient of resistance for 2 resistors are α and β. It equivalent resistance of their series combination does not depend on temperature. Then find ratio of their resistances at 0°C.

Sol. $R_{eq} = R_1 + R_2$



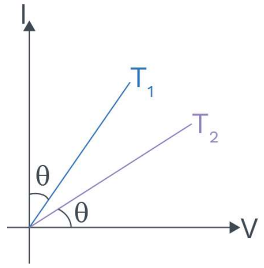
$R_{\theta_1} + R_{\theta_2} = R_{\theta_1} + R_{\theta_1}\alpha\theta + R_{\theta_2} + R_{\theta_2}\beta\theta$

$\dfrac{R_{\theta_1}}{R_{\theta_2}} = -\dfrac{\beta}{\alpha}$

or $R_\theta = R_0 + R_0\alpha\theta$

$R_{\theta_1}\alpha\theta + R_{\theta_2}\beta\theta = 0$

Ex. I-V curve for a conductor at temperatures T_1 and T_2 is as shown then compare temperatures:



Sol. Slope = $\dfrac{I}{V} = \dfrac{1}{R}$

$(\text{slope})_1 > (\text{slope})_2$

$R_2 > R_1$

$\therefore \quad T_2 > T_1$

Ex. In the above question $(T_2 - T_1)$ is proportional to.

Sol. Slope = $\tan\theta = \dfrac{1}{R}$

R_1 at T_1

$R_1 = R_0(1 + \alpha T_1)$

$= \dfrac{1}{\tan(90-\theta)} = \dfrac{1}{\cot\theta} = \tan\theta$ ...(i)

$R_2 = R_0(1 + \alpha T_2) = \dfrac{1}{\tan\theta} = \cot\theta$ ...(ii)

Equation (i) and (ii)

$R_2 - R_1 = R_0\alpha(T_2 - T_1)$

$$\cot\theta - \tan\theta = R_0\alpha(T_2 - T_1)$$

$$\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = R_0\alpha(T_2 - T_1)$$

$$\frac{2(\cos^2\theta - \sin^2\theta)}{2\sin\theta\,\cos\theta} = R_0\alpha(T_2 - T_1)$$

$$\because \quad \cos^2\theta - \sin^2\theta = \cos(2\theta)$$

$$\frac{2(\cos 2\theta)}{\sin 2\theta} = R_0\alpha(T_2 - T_1)$$

$$T_2 - T_1 = \frac{2\cot(2\theta)}{R_0\alpha}$$

$$\therefore \quad (T_2 - T_1) \propto \cot 2\theta$$

Special Point:

1. Bulb Filament:

- Material of filament of bulb should be of low resistivity and high melting point.
- Generally, bulb filament is made of tungsten.
- At high temperature, tungsten reacts with air and form oxide of avoid it, inert medium is used in bulb of tungsten filament instead of air.

2. Coil of Heating Element

- Material of coil of heating element (heater, electric geyser, electric press etc) should be of high resistivity and high mpt.
- Generally, coil of these elements is made of Nichrome.

3. Fused Wire:

- When current exceed in ckt, then fuse wire get melt and ckt remains safe.
- Material of fuse wire should be of low resistivity and low mpt.
- Generally fused wire is made of Sn-Pb alloy length of fused wire has no significance in its functioning.

Concept Reminder

- Resistivity of a conductor is independent of the geometry (i.e., length and area of cross-section) of conductor but depends on type of material and temperature.

- Let 'I' is the maximum safe current through fused wire and r is radius of cross-section of fused wire then

  $I^2 \propto r^3$

  $I^2 \propto A^{3/4}$

4. Wire of meter bride and potentiometer:

- Resistance of wire of potentiometer and meter bridge should remain unchanged with variation in temperature so material of wire of potentiometer and meter bridge should be of negligible temperature coefficient of resistance.
- Generally, this wire is made of 'magnon' and 'constantan/eureka'.

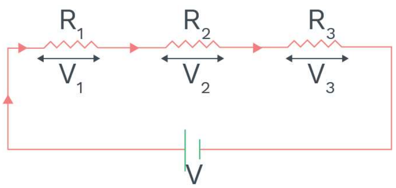
5. Thermistor (Thermal + Resistor):

- This type of material is highly sensitive with variation in temperature.
- Temperature coefficient of resistance for thermistor is high.
- Generally, copper made wires are used as connecting wires in ckt because of high conductivity and low resistivity of copper.

COMBINATION OF RESISTORS

Series Combination:

If no alternative path is available to flow the current in ckt then equal current is passing through all resistors and these resistors are in series.



Potential of resistors = potential difference between ends of resistor

Concept Reminder

- Metals have positive temperature coefficient of resistance, whereas semiconductors and insulators have negative temperature coefficient of resistance.

Rack your Brain

The resistance of a wire is R ohm. If it is melted and stretched to 'n' times its original length then find out value of new resistance.

$$V = V_1 + V_2 + V_3$$

$$IR = I_1R_1 + I_2R_2 + I_3R_3$$

$$R_{eq} = R_1 + R_2 + R_3......$$

If 'n' identical resistors, each of resistance R connected in series then

$$R_{eq} = nR$$

In series combination, potential drop across resistors is in ratio of their resistances

$$V = IR$$

$$V \propto R$$

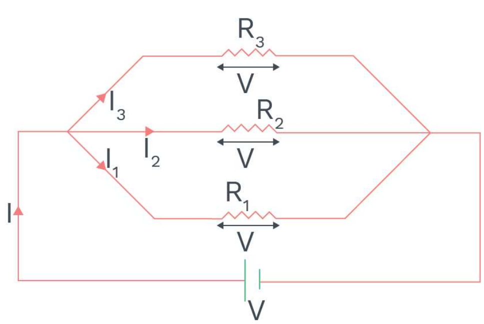
$$V_1 : V_2 : V_3 :: R_1 : R_2 : R_3$$

Equivalent resistance obtained in series combination is greater than greatest resistance.

Parallel Combination:

If alternative path is available to flow the current, then distribution of current in different branches at junction takes place. These branches are in II-combination.

Potential drop across II-branches is same



$$I = I_1 + I_2 + I_3 \text{ (Charge conservation)}$$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}.....$$

If R_1 and R_2 are in parallel

Concept Reminder

- In series combination
  $R_S = R_1 + R_2 + R_3$
- In parallel combination
  $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Concept Reminder

- To get minimum resistance, resistance must be connected in parallel and in parallel the resultant resistance is lesser than the smallest individual.

$$R_{eq} = \frac{R_1R_2}{(R_1 + R_2)}$$

If 'n' identical resistors, resistance of each R are connected in parallel then,

$$R_{eq} = \frac{R}{n}$$

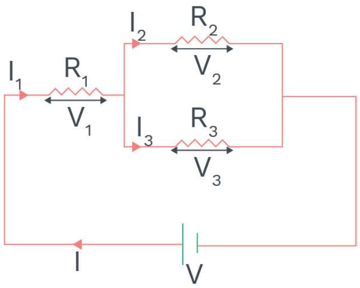
Distribution of current in II-branches at any junction is in inverse ratio of resistances

$$I = \frac{V}{R} \Rightarrow I \propto \frac{1}{R}$$

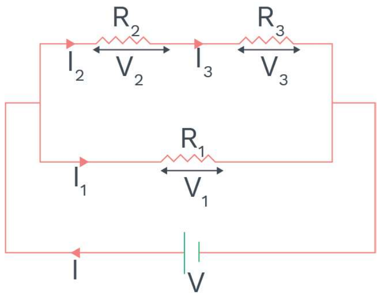
$$I_1 : I_2 : I_3 :: \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3}$$

Equivalent resistance obtained in II-combination is less than least resistance.

Mixed Combination:



$$R_{eq} = R_1 + \frac{R_2R_3}{R_2 + R_3}$$



$$R_{eq} = \frac{(R_2 + R_3)R_1}{R_1 + (R_2 + R_3)}$$

Ex. If 3 resistors are in parallel with a battery, current through these resistors is I, 2I and 3I. If these resistors are connected in series with same battery, then calculate current through these resistors.

Sol. $R_1 = \frac{V}{I}, R_2 = \frac{V}{2I}, R_3 = \frac{V}{3I}$

$$6I = \frac{V}{R_{eq}}$$

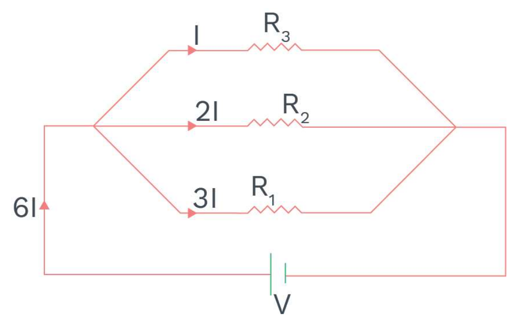
$$I' = \frac{V}{R_{eq}}$$

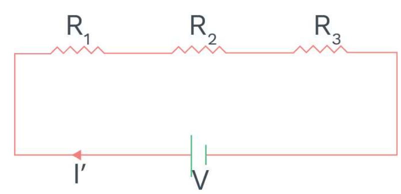
$$I' = \frac{V}{R_1 + R_2 + R_3} = \frac{V}{\frac{V}{I} + \frac{V}{2I} + \frac{V}{3I}}$$

$$I' = \frac{1}{\frac{1}{I} + \frac{1}{2I} + \frac{1}{3I}} = \frac{1}{\frac{6+3+2}{6I}} = \frac{6I}{11}$$

or $$\frac{1}{I'} = \frac{1}{I_1} + \frac{1}{I_2} + \frac{1}{I_3}$$

$$\frac{1}{I'} = \frac{1}{I} + \frac{1}{2I} + \frac{1}{3I}$$

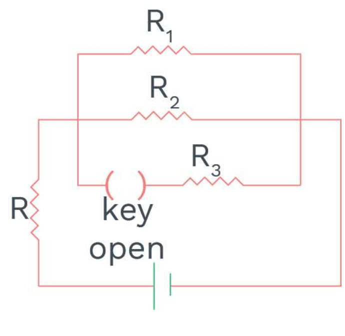




Open circuited branch:

A branch through which there is no current in circuit is named open circuited branch.

If any resistance is connected in open circuited branch, then it is meaningless, because resistor, through which current is not passing; is not used to calculate equivalent resistance.



Definitions

- A branch through which there is no current in circuit is named open circuited branch.
- A branch without resistance in circuit is named short -circuited branch.

Examples:

(a) Branch of inductor is initially open circuited.

(b) Branch of capacitor is open at steady state.

(c) If ideal voltmeter is connected in series with any resistor, then this branch will be open circuited.

Short -circuited branch

A branch without resistance in circuit is named short -circuited branch.

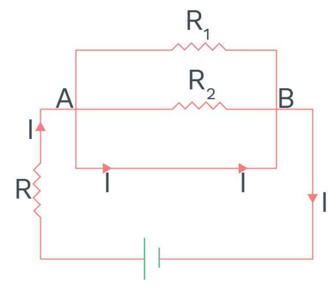
Branches which are II to short -circuited branches are meaningless.

For examples:

(a) At steady state, inductor behaves as short circuited.

(b) Initially, capacitor behaves as short circuited.

(c) If ideal ammeter is connected in parallel (by mistake) with any branch, then branch will be short circuited. (only this element will be short circuited, not whole branch)



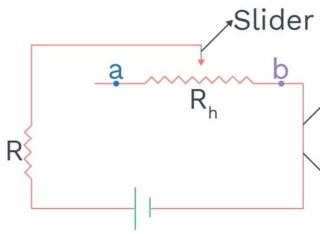
Rheostat (R_h)

It is a resistor of variable resistance which is used to control the current (current controlling unit) in circuit.

Rheostat can be used as potential divider.

Definitions

- It is a resistor of variable resistance which is used to control the current (current controlling unit) in circuit.

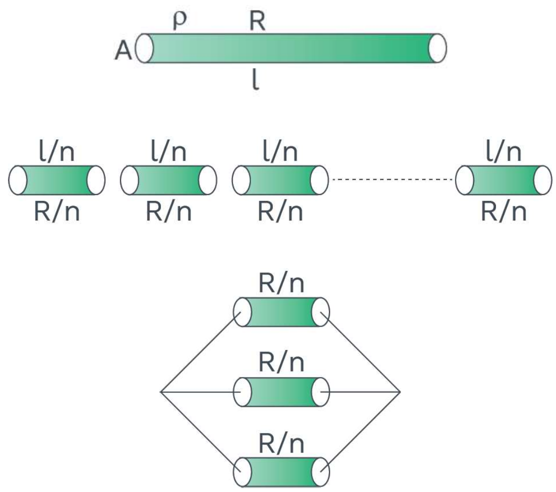


When slider is at point "a" $\Rightarrow R_h = \max$

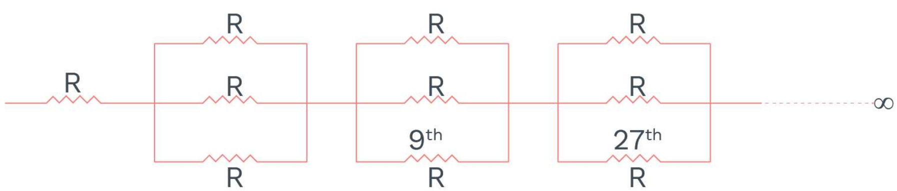
When slider is at point "b" $\Rightarrow R_h = 0$

Ex. A resistor of resistance R is cut into n equal parts and these parts are connected in parallel then calculate resistance of combination.

Sol. $R_{eq} = \dfrac{R}{n(n)} = \dfrac{R}{n^2}$



Ex. '∞' resistors, resistance of each R is connected as shown calculate R_{eq} of combination.



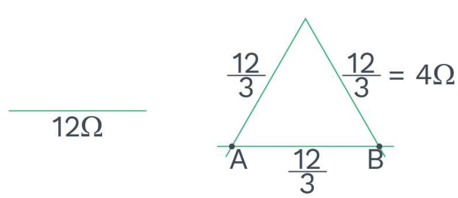
Sol. $R_{eq} = R + \frac{R}{3} + \frac{R}{9} + \frac{R}{27} + \ldots.. + \infty$

$$R_{eq} = R\left[1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots\ldots + \infty\right]$$

$$R_{eq} = R\left(\frac{1}{1 - \frac{1}{3}}\right) = \frac{3R}{2}$$

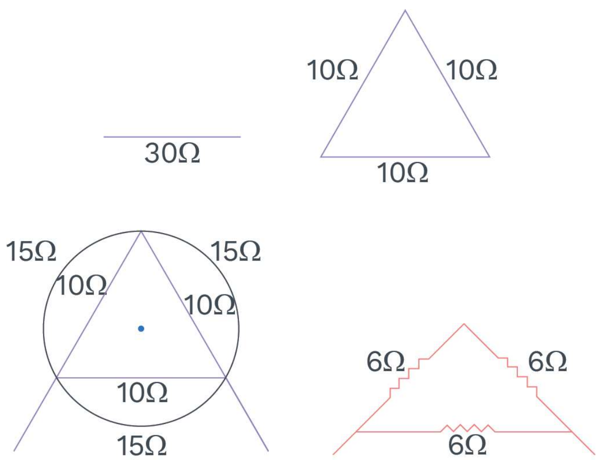
Ex. A wire of resistance 12 Ω is bent in equilateral triangle. Calculate equivalent resistance between any 2 vertices of Δ.

Sol. $R_{AB} = \frac{8 \times 4}{12} = \frac{8}{3}\Omega$

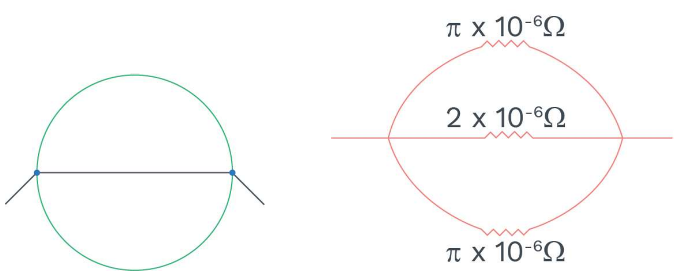


Ex. A uniform wire of resistance 30 Ω is bent in equivalent Δ and placed concentrically in a circle such that is vertices are in contact with periphery of circle. Total resistance of periphery of circle is 45 Ω. Calculate R_{eq} of combination between any two 2 vertices of Δ.

Sol. $R_{eq} = \frac{12 \times 6}{18} = 4\,\Omega$



Ex. A circle of radius 1 metre is made of uniform wire resistance 10^{-6} Ω/m. Same wire is connected along diameter of circle. Calculate equivalent resistance between diametric opposite points of this diameter.

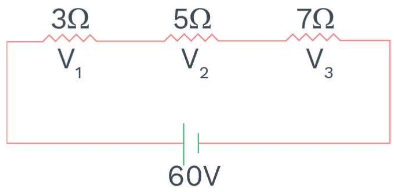


Sol. $\frac{1}{R_{eq}} = \frac{2}{\pi \times 10^{-6}} + \frac{1}{2 \times 10^{-6}} = \frac{4 + \pi}{2\pi \times 10^{-6}}$

$R_{eq} = \frac{2\pi \times 10^{-6}}{7.14} = \frac{6.28}{7.14} \times 10^{-6}$

$R_{eq} \approx 8.8 \times 10^{-7}\,\Omega$

Ex. In the given circuit, calculate potential drop across each resistor.



Sol. $V \propto R$

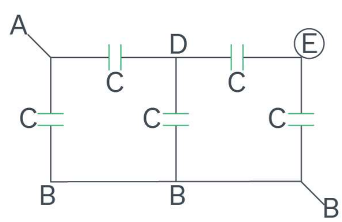
$V_1 : V_2 : V_3 :: 3 : 5 : 7$

$V_1 = \frac{5}{15} \times 60 = 12\ V$

$V_2 = \frac{5}{15} \times 60 = 20\ V$

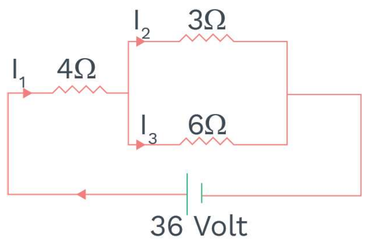
$V_3 = \frac{7}{15} \times 60 = 28\ V$

Ex. In the given circuit equivalent cap between A and B.



Sol. $C_{eq} = \frac{3C + C}{5} = \frac{8C}{5}$

Ex. Calculate current and potential across each resistor in given circuit.



Sol. $R_{eq} = 4 + 2 = 6\Omega$

$$I = \frac{36}{R_{eq}} = \frac{36}{6} = 6\,A$$

$$I_1 = 6\,A \quad \Rightarrow V_1 = 6 \times 4 = 24\,V$$

$$I_2 = \frac{6}{9} \times 6 = 4\,A \quad \Rightarrow V_2 = 3 \times 4 = 12\,V$$

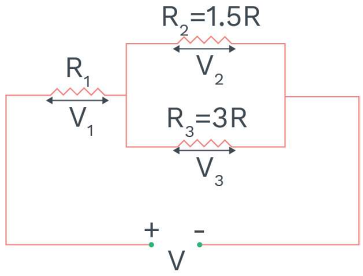
$$I_3 = 2\,A \quad \Rightarrow V_3 = 6 \times 2 = 12\,V$$

Ex. In given combination, if potential drop across R_1, R_2 and R_3 are V_1, V_2 and V_3 respectively, then calculate.

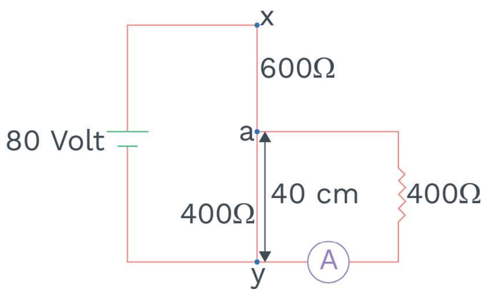
Sol. $V_1 : V_2 :: 1 : 1$

$V_1 : V_3 :: 1 : 1$

$V_1 = V_2 = V_3$



Ex. In given circuit, uniform wire xy is of length 1 metre and resistance of 1 KΩ. Calculate reading of ammeter.



Sol. $R_{xy} = 1000$ W
$R_{ay} + R_{ax} = 1000$ W
$R_{ax} = 600$ W
$R_{ay} = 400$ W
Total current in circuit,

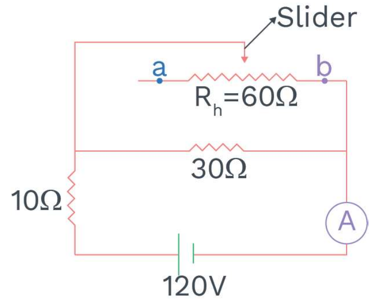
$$I = \frac{80}{R_{eq}} = \frac{80}{200 + 600}$$

$I = 0.1\ A$

Reading of ammeter

$$I_A = \frac{400}{800} \times 0.1 = 0.05\ A$$

Ex. In the given circuit, rheostat of 60 Ω is connected as shown. Calculate maximum and minimum possible reading of ammeter.



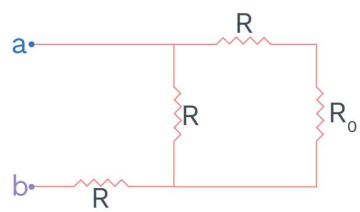
Sol. Reading of ammeter = total current in circuit

$$I_{max} = \frac{V}{(R_{eq})_{min}} = \frac{120}{10} = 12\ A$$

Because branch 'ah' is short circuited so R = 30 Ω also becomes meaningless.

$$I_{min} = \frac{V}{(R_{eq})_{max}} = \frac{120}{10 + \frac{30 \times 60}{90}} = \frac{120}{30} = 4\ A$$

Ex. In given circuit, if equivalent resistance between points a and b is R_0 then calculate resistance R.



Sol. $$R_{eq} = R + \frac{(R + R_0)R}{2R + R_0}$$

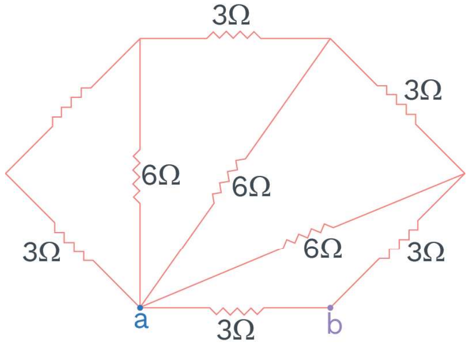
$$R_0 = R + \frac{R^2 + R_0 R}{2R + R_0}$$

$$R_0^2 + 2RR_0 = 2R^2 + RR_0 + R^2 + R_0R$$

$$R_0^2 = 3R^2$$

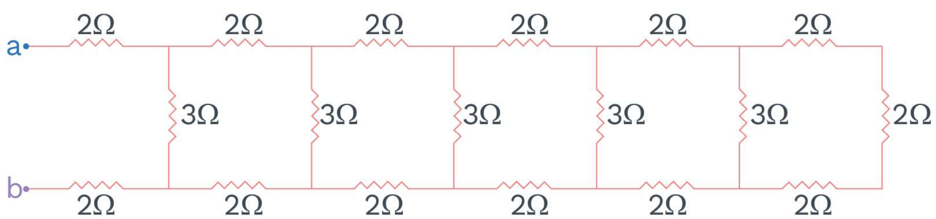
$$\Rightarrow \quad R = \frac{R_0}{\sqrt{3}}$$

Ex. Calculate equivalent resistance between points a and b in given network.

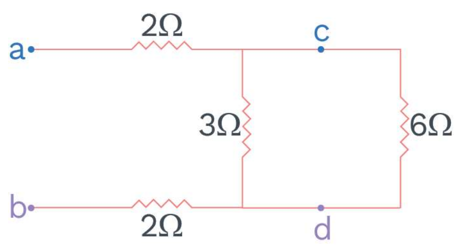


(a)

Sol. $R_{ab} = \dfrac{3 \times 6}{9} = 2\,\Omega$

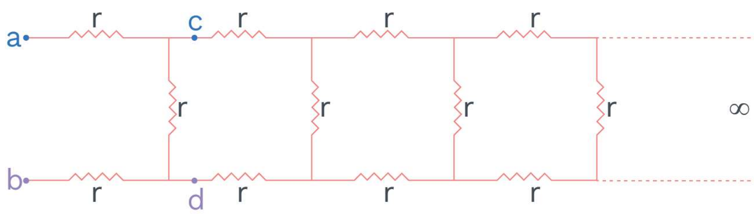


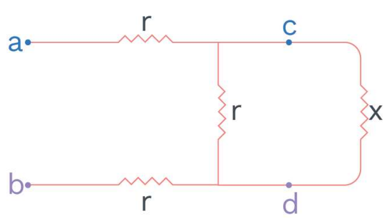
(b)



Sol. R_{ab} = 6 Ω (does not depend on number of units)

Ex. Calculate equivalent resistance between a and b in given ∞ ladder.





Sol. $R_{ab} = x$

$$r + r + \frac{rx}{r + x} = x$$

$$2r + \frac{rx}{r + x} = x$$

$$2r^2 + 2rx + rx = x^2 + rx$$

$$x^2 - 2rx - 2r^2 = 0$$

$$x = \frac{2r \pm \sqrt{4r^2 + 8r^2}}{2}$$

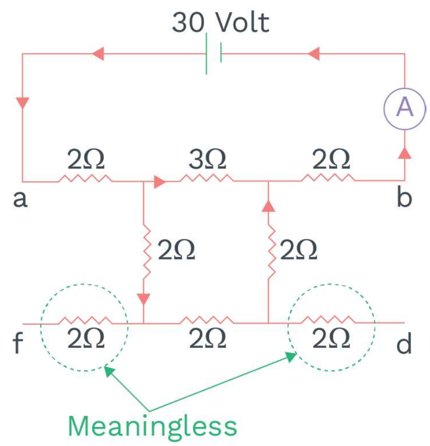
$$= \frac{2r + r\sqrt{12}}{2} = r + r\sqrt{3}$$

$$x = (\sqrt{3} + 1)r$$

so, $R_{ab} = (\sqrt{3} + 1)r$

$R_{ab} = 2.73r$

Ex. Calculate reading of ammeter in given circuit.



Sol. $R_{eq} = 4 + \frac{3 \times 6}{9} = 6\,\Omega$

$I = \frac{V}{R_{eq}} = \frac{30}{6} = 5\,\text{Amp}$ (Reading of ammeter)

- If there is no current through any resistor then this resistor is not used to calculate equivalent resistance.
- As per requirement, points of same potential can be separated or can be connected at same position.
- If movement in circuit is without crossing circuit element then potential remains unchanged during this movement.

Same potential point method:

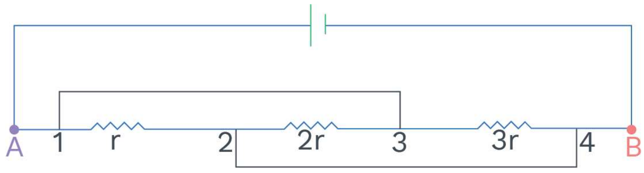
(i) Given numbering for each and every junction.

(ii) Collect the number of same potentials.

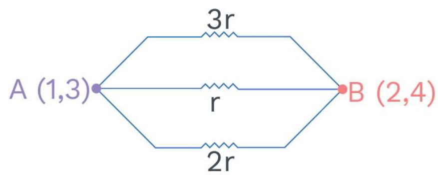
(iii) Rearrange the resistors between corresponding numbers.

Ex. Calculate equivalent resistance between points A and B in given circuits

(a)



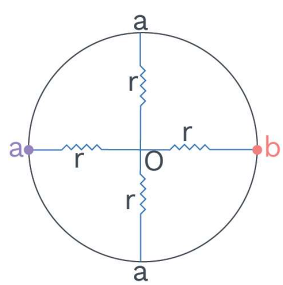
Sol. 'I' through L, 2 r – right to left



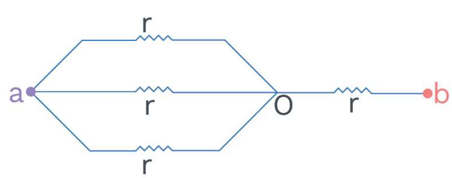
$$\frac{1}{R_{eq}} = \frac{1}{3r} + \frac{1}{2r} + \frac{1}{r} = \frac{2+3+6}{6r}$$

$$R_{eq} = \frac{6r}{11}$$

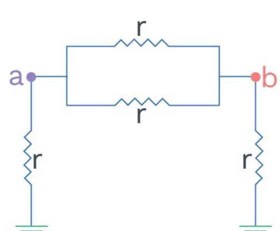
(b)



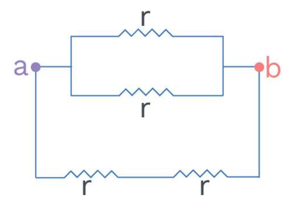
Sol. $R_{ab} = \frac{r}{3} + r = \frac{4r}{3}$

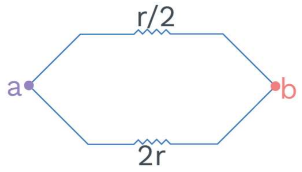


(c)

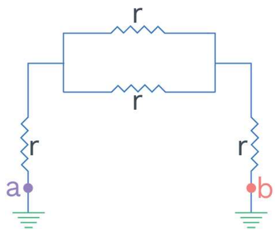


Sol. $R_{ab} = \frac{\frac{r}{2} \times 2r}{\frac{5r}{2}} = \frac{2r}{5}$

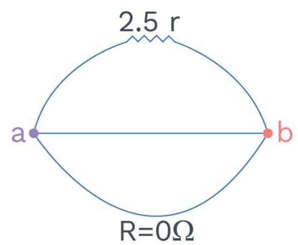




(d)



Sol. If capacitance is given



$$C_{ab} = \frac{5C}{2}$$

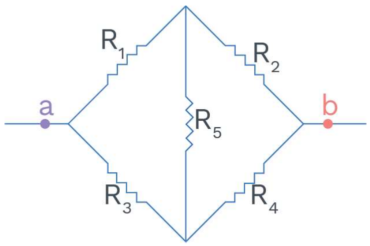
$V_a = V_b$

$R_{ab} = 0$

- Equivalent resistance between points of same potential is zero.
- Equivalent capacitance between points of same potential is infinite.

Method based on balanced Wheat stone Bridge:

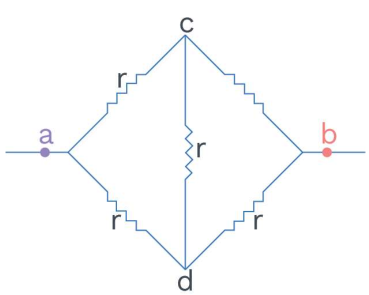
Ratio of resistances of upper branch = ratio of resistances in lower branch



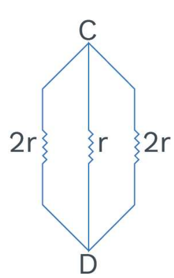
If $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

In balanced wheat stone bridge, there is no current through bridge resistor (R_5) so, Bridge resistor can be removed.

Ex. Find ratio of equivalent resistance $\frac{R_{ab}}{R_{cd}}$ in given circuit.



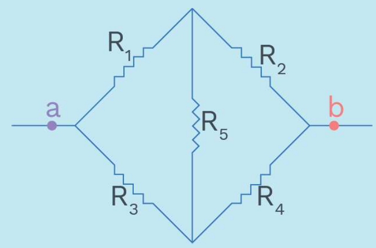
Sol. $\frac{R_{ab}}{R_{cd}} = \frac{r}{r/2} = \frac{2}{1}$



$R_{CD} = \frac{r}{2}$

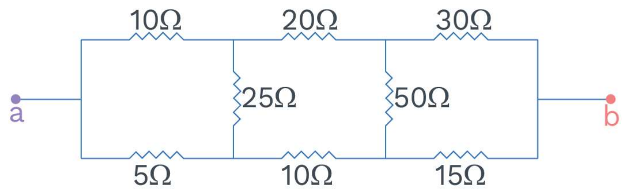
Concept Reminder

- Balanced condition of Wheat stone bridge is:

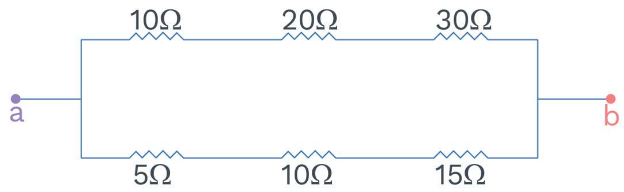


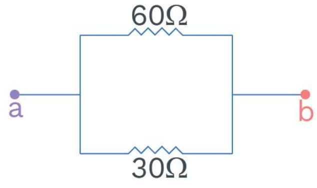
$\frac{R_1}{R_2} = \frac{R_3}{R_4}$

Ex. Calculate equivalent resistance between points a and b in given network.



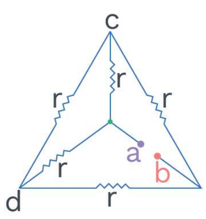
Sol. 1 : 2 : 3, ratio of resistance of upper branches
1 : 2 : 3, ratio of resistance of lower branches
These are 2 ad-jointed balanced wheat stone bridge



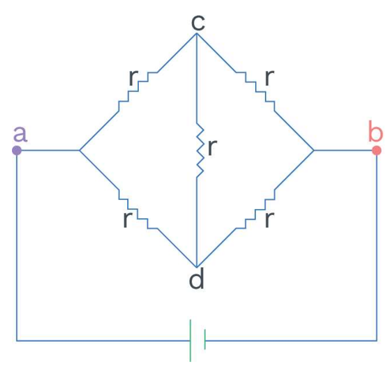


$$R_{ab} = \frac{30 \times 60}{90} = 20\,\Omega$$

Ex. When potential difference of 'V' is applied between points a and b then calculate current in branch acb.

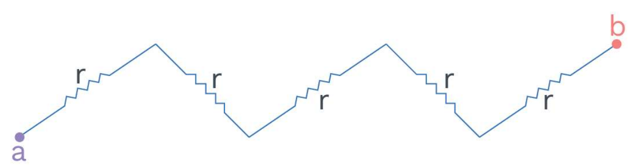


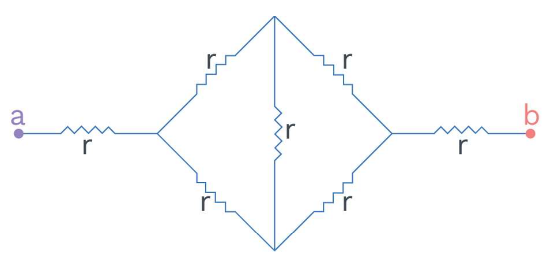
Sol. $I = \frac{V}{R_{eq}} = \frac{V}{r}$



$$I_{acb} = \frac{V}{2r} = \frac{I}{2}$$

Ex. If 2 identical wires, resistance of each r are connected along dotted lines then find ratio of equivalent resistance between a and b before and after the connection of wires.

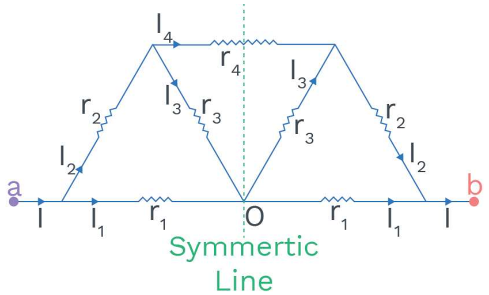


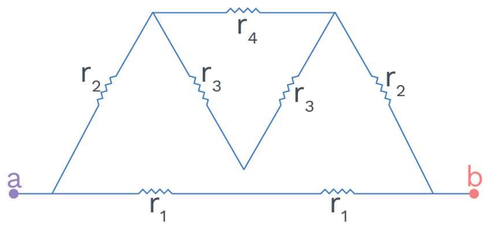


Sol. $\frac{R_{before}}{R_{after}} = \frac{5r}{3r} = \frac{5}{3}$

Percentage change in resistance = – 40%

Pseudo-junction method and symmetric line method:





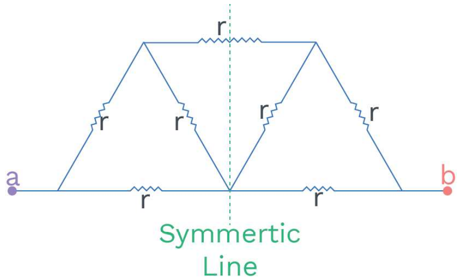
- A junction in circuit at which there is no distribution of current is named Pseudo-junction. Pseudo-junction can be removed.

Symmetric line

- A line which is proportional to line joining the a and b which divides the circuit into 2 equal parts is named symmetric line.
- Potential of points at symmetric line is equal and junction lies on symmetric line is pseudo-junction. Removal of pseudo-junction should be along symmetrical line.

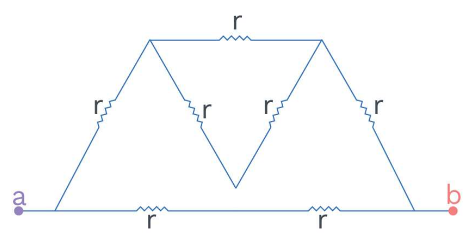
Ex. Calculate equivalent resistance between a and b in given network.

(a)

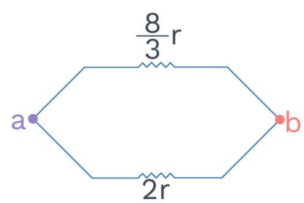


Rack your Brain

A wire of resistance 12 ohms per meter is bent to form a complete circle of radius 10 cm. Find out resistance between its two diametrically opposite points.



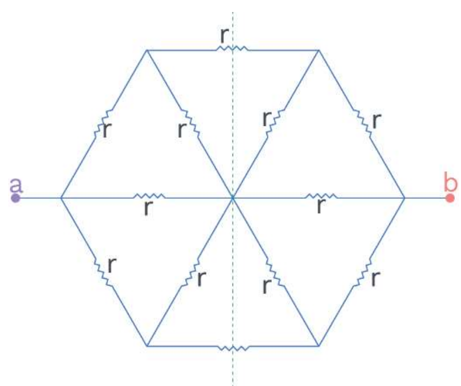
Sol. $R_{ab} = \dfrac{\frac{8}{3}\times 2}{\frac{8}{3}+2} = \dfrac{\frac{8}{3}\times 2}{\frac{14}{3}} = \dfrac{8r}{7}$

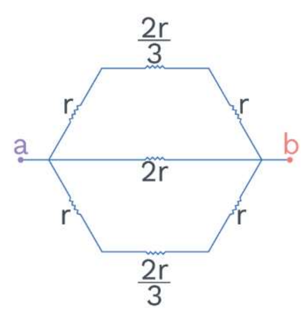


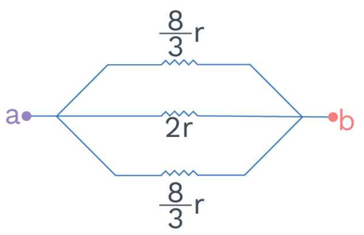
In case of capacitance

$C_{ab} = \dfrac{7}{8}C$

(b)

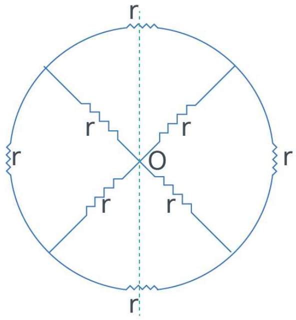






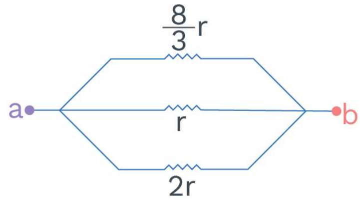
Sol. $\frac{1}{R_{eq}} = \frac{1}{2r} + \frac{3}{8r} + \frac{3}{8r} = \frac{4+3+3}{8r}$

R_{eq} = 0.8 r

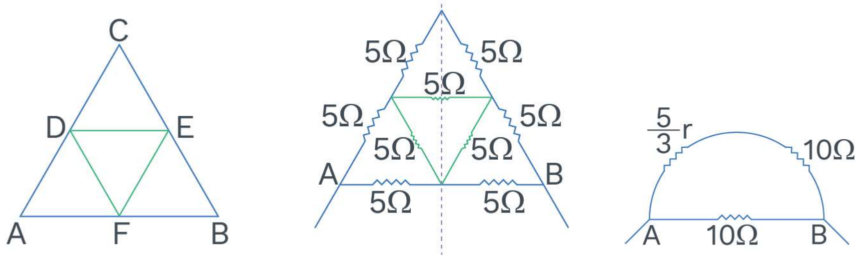
(c) 

Sol. $\frac{1}{R_{eq}} = \frac{1}{r} + \frac{1}{2r} + \frac{3}{8r}$

$R_{eq} = \frac{8+4+3}{8r} = \frac{8r}{15}$



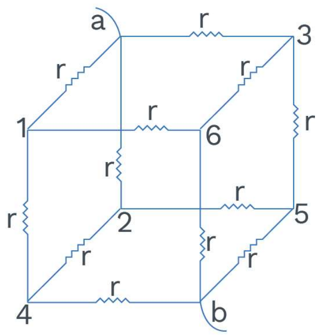
Ex. Two equilateral triangles ABC and DEF have same centroid. The ratio of their sides are 4 : 2. The resistance per unit length is constant. The resistance in AB is 10 Ω. Find equivalent resistance between A and B.



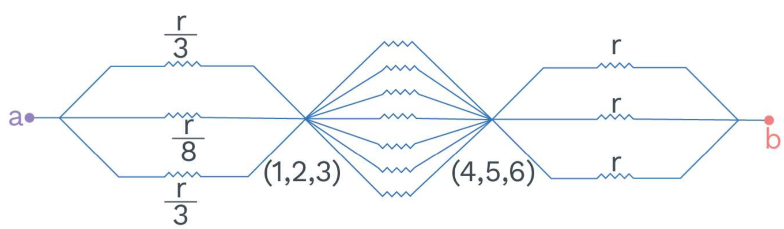
Sol. R_{eq} = 5.56 W

Ex. If each side of cube represents resistor of resistance 'r' then resistance between given points is as:

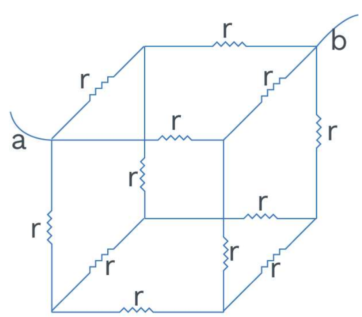
Sol.



$$R_{ab} = \frac{5r}{6};\ C_{ab} = \frac{6}{5}C$$



Body diagonal-



$$R_{ab} = \frac{5r}{6};\ C_{ab} = \frac{6}{5}C$$

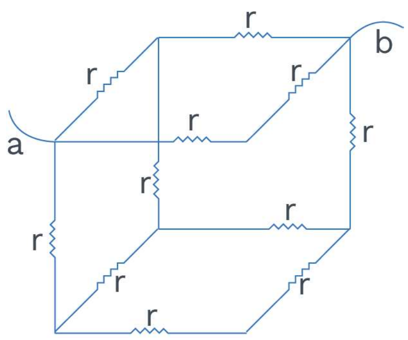
$$R_{ab} = \frac{r}{3} + \frac{r}{6} + \frac{r}{3} = \frac{2r}{3} + \frac{r}{6} = \frac{4r + r}{6}$$

$$R_{ab} = \frac{5r}{6}$$

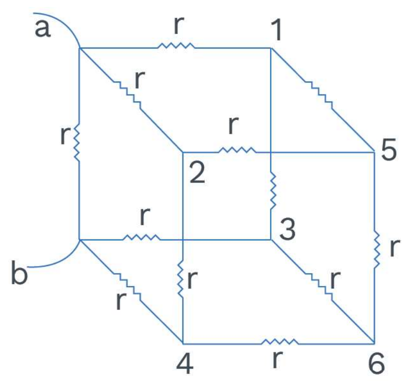
$$R_{ab} = \frac{3r}{4}$$

$$C_{ab} = \frac{4C}{3}$$

Face diagonal-

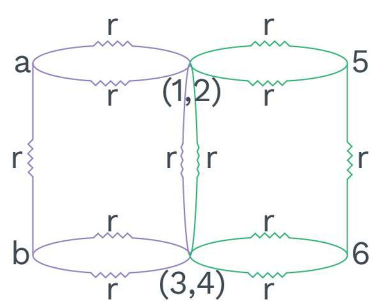


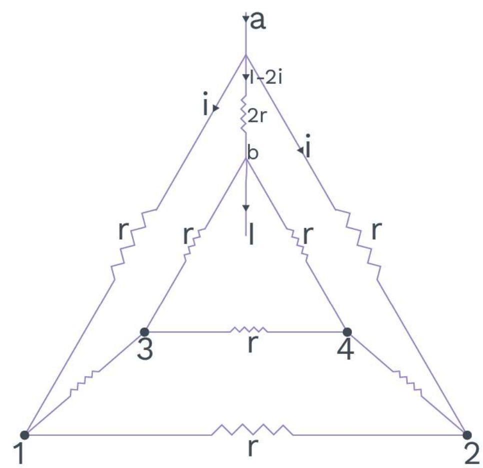
$$\frac{r \times 3r}{4r} = \frac{3r}{4}$$



$$R_{ab} = \frac{7r}{12}, C_{ab} = \frac{12C}{7}$$

Adjacent Point-

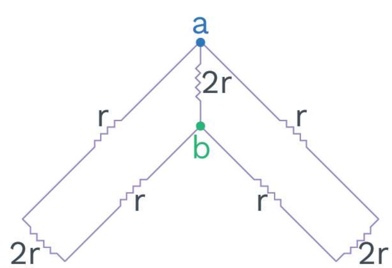




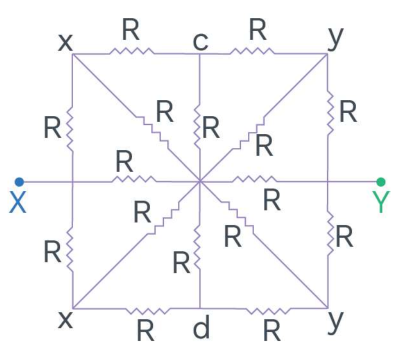
$R_{ab} = ?$

$V_1 = V_2$

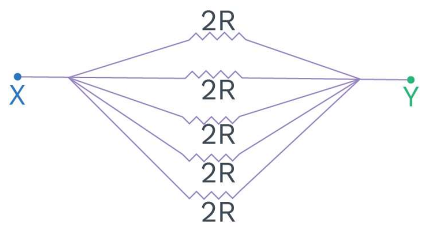
$V_3 = V_4$ [although $V_2 \neq V_3$]



$R_{ab} = r$



Branch cd will remove out ($\because V_c = V_d$)

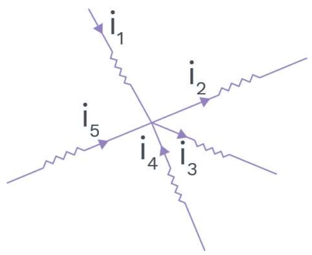


$$R_{xy} = \frac{2R}{5}$$

KIRCHOFF LOW

Kirchhoff Current Law/Junction law/First law:

- This law states that algebraic sum of electric current at any junction in circuit is zero that means current towards junction is equal as current away from junction (because there is not any source or reservoir at junction point to increases or decreases current respectively)



Concept Reminder

- Kirchoff's junction rule is based on conservation of charge.
- According to this rule, at any junction of circuit $\Sigma i = 0$.

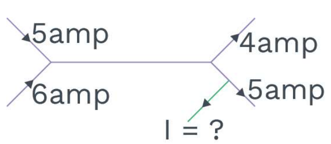
$$i_1 + i_2 + i_3 + i_4 + i_5 = 0$$

$$i_1 + i_4 + i_5 = -(i_2 + i_3)$$

towards junction = away from junction

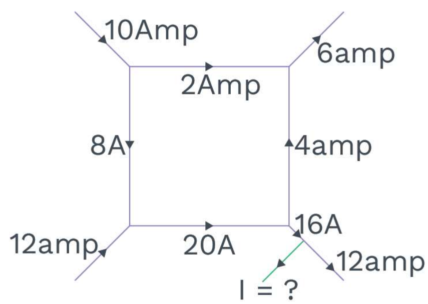
Kirchhoff law is based on conservation of charge. Kirchhoff law (conservation of charge) is the consequence of continuity equation.

Ex. Calculate current I in given network.



Sol. I + 5 = 7

I = 2 Amp

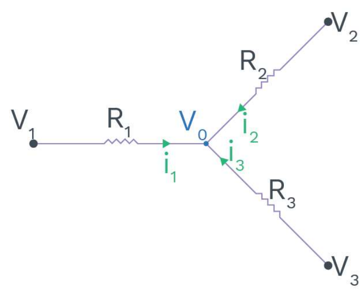


16 = 19 + I

I = – 3 Amp

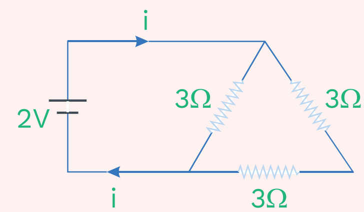
It means current is shown in wrong direction current I will be inward.

Calculation of potential at junction:



Rack your Brain

Find out correct in the following circuit



$V_0 \neq$ Lesser than all (V_1, V_2, V_3)

$V_0 \neq$ Greater than all (V_1, V_2, V_3)

[least < V_0 < greatest]

According to Kirchhoff law

$$i_1 + i_2 + i_3 = 0$$

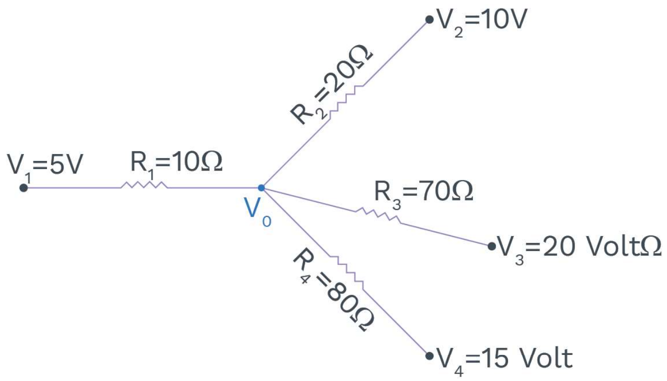
$$\frac{V_1 - V_0}{R_1} + \frac{V_2 - V_0}{R_2} + \frac{V_3 - V_0}{R_3} = 0$$

$$\frac{V_1}{R_1} - \frac{V_0}{R_1} + \frac{V_2}{R_2} - \frac{V_0}{R_2} + \frac{V_3}{R_3} - \frac{V_0}{R_3} = 0$$

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = \frac{V_0}{R_1} + \frac{V_0}{R_2} + \frac{V_0}{R_3}$$

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = V_0\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Ex. In the given network, calculate current through resistor of resistance 10 Ω.



Sol. Current in 1st branch L, away from junction current in 3rd branch towards junction

∵ least < V_0 < greatest

$$V_0\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)=\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}+\frac{V_4}{R_4}$$

$$V_0\left(\frac{1}{10}+\frac{1}{20}+\frac{1}{40}+\frac{1}{80}\right)=\frac{5}{10}+\frac{10}{20}+\frac{20}{40}+\frac{15}{80}$$

$$V_0\left(\frac{8+4+2+1}{80}\right)=\frac{4.0+40+4.0+15}{80}$$

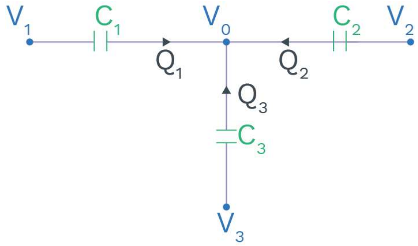
$$V_0=\frac{135}{15}=9\,V$$

$$I=\frac{V_0-V_1}{R_1}=\frac{9-5}{10}=\frac{4}{10}=0.4\text{ Amp}$$

(away from junction)

Kirchhoff law in network of capacitors:

In network of capacitors, algebraic sum of charge at any junction is zero.



At junction,

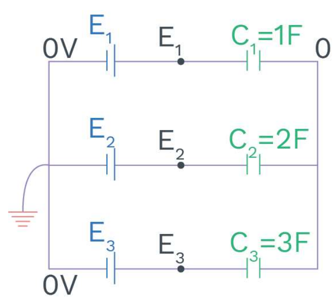
$$Q_1 + Q_2 + Q_3 = 0$$

$$C_1(V_1 - V_0) + C_2(V_2 - V_0) + C_3(V_3 - V_0) = 0$$

$$C_1V_1 + C_2V_2 + C_3V_3 = (C_1 + C_2 + C_3)V_0$$

$$\boxed{V_0 = \frac{C_1V_1 + C_2V_2 + C_3V_3}{C_1 + C_2 + C_3}}$$

Ex. Calculate potential at point O in given network.

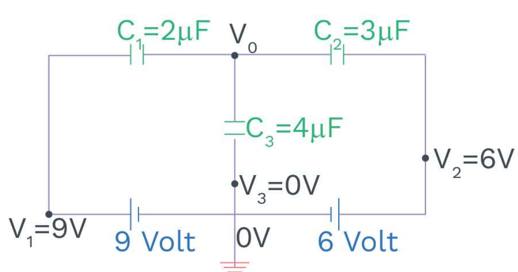


Sol. At junction point O

$$V_0 = \frac{C_1V_1 + C_2V_2 + C_3V_3}{C_1 + C_2 + C_3}$$

$$V_0 = \frac{E_1 + 2E_2 + 3E_3}{6}$$

Ex. Calculate charge on capacitance 4 μF in given network.



Sol. $V_0 = \frac{C_1V_1 + C_2V_2 + C_3V_3}{C_1 + C_2 + C_3}$

$$= \frac{18 + 18 + 0}{9} = 4\,\text{Volt}$$

Charge on C_3;

$$Q_3 = C_3(V_0 - V_3) = (4\,\mu F)(4 - 0)$$

Q_3 = 16 μC (towards earthened point)

Kirchhoff's Voltage Law/Loop Law/2nd Law:

- This law states that algebraic sum of potential drops in closed loop (including battery) across all circuit elements is zero.

  In close loop,

$$\pm IR \pm \frac{q}{C} \pm E = 0$$

- Kirchhoff 2nd law is based on conservation of energy (overall loop). Although it may increase or decreases for a particular element.

Concept Reminder

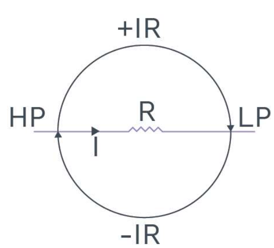
- Kirchhoff's loop rule is based on conservation of energy.

Sign Convention:

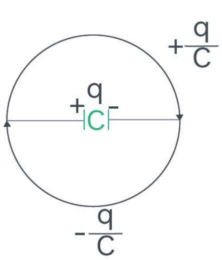
Potential decreases = negative

Potential increases = positive

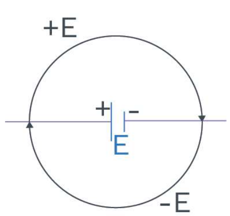
For resistor



For capacitor

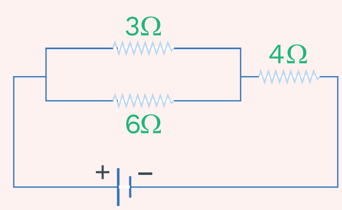


For battery



Rack your Brain

Current through 3Ω is 0.8 A, then find out potential difference across 4Ω resistor.



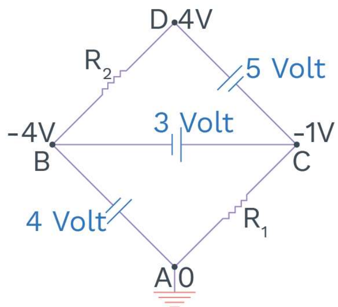
Meaning of $V_A - V_B = 10$ volt is that potential at point A(positive terminal) is greater than potential at point B(negative terminal) of 10 volt.

e.g. $V_A = 7$ V, $V_B = -3$ V

$V_A = 0$ V, $V_B = -10$ V

$V_A = 10$ V, $V_B = 0$ V

Ex. Mark potential at points B, C and D.



Sol. $V_B = -4$ V (least)

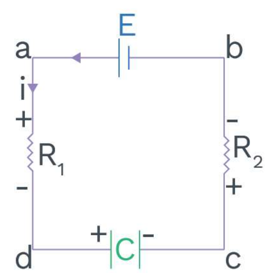
$V_C = -1$ V

$V_D = 4$ V (greatest)

Current in $R_1 \Rightarrow A \rightarrow C$

Current in $R_2 \Rightarrow D \rightarrow B$

e.g.

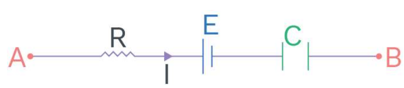


KVL in closed loop; adcba

$$-iR_1 - \frac{q}{C} - iR_2 + E = 0$$

$$E = iR_1 + iR_2 + \frac{q}{C}$$

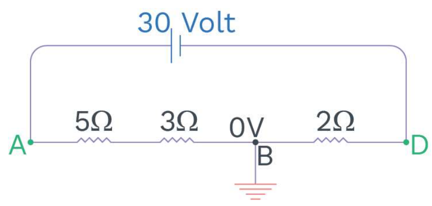
KVL in branch



$$V_A - IR + E - \frac{q}{C} = V_B$$

Current I is not given in behalf of battery. It is just assumed. It may be in opposite direction.

Ex. In the given network, calculate potential at point A and at point D.



Sol. $V_B = 0\,V$

$0 - 2I = V_0$

$V_0 = 6\,volt$

By KVL,

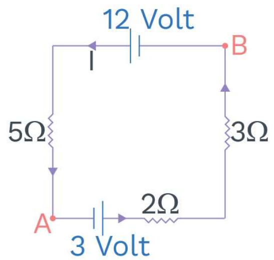
$+30 - 10I = 0$

I = 3 Amp

$V_A - 8I = V_B$

$V_A = 8I + V_B = 24 + 0 = 24\,V$

Ex. In the given circuit, calculate potential difference between point A and B.



Sol. Using KVL,

$+12 - 5I + 3 - 2I - 3I = 0$

$10I = 15 \Rightarrow I = 1.5$ Amp

KVL from A $\rightarrow$ B,

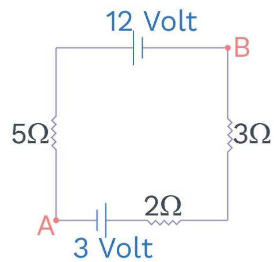
$V_A + 3 - 1.5 \times 2 - 3 \times 1.5 = V_B$

$V_A - V_B = 3 + 4.5 - 3 = 7.5 - 3 = 4.5\,volt$

Ex. In the above question, if battery of 3 volt is used with reversed polarity, then calculate potential difference between points A and B.

Sol. $+12 - 5I - 3 - 5I = 0$ s

$10I = 9 \quad \Rightarrow I = 0.9$ Amp

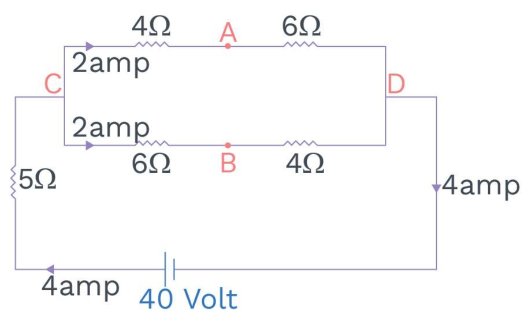


$V_A - 3 - 5 \times 0.9 = V_B$

$V_A - V_B = 3 + 4.5 = 7.5$ volt

Value of current will confirm changed when polarity of lither battery is reversed but direction current will change or not, it depends on polarity of which battery is reversed.

Ex. In the given circuit, calculate potential difference between points A and B.



Sol. I = 4 Amp

KVL from A → B via C,

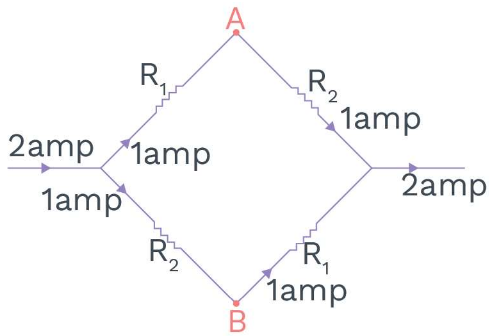
$V_A + 4 \times 2 - 2 \times 6 = V_B$

$V_A - V_B = 12 - 8 = 4$ volt

$V_C - V_D = 20$ volt

If wire is connected between point A and B then direction of current will be from A to B.

Ex. In the given circuit, calculate potential difference between points A and B.



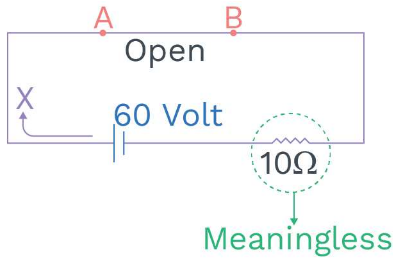
Sol. Remember, it is not balanced wheat stone bridge

$V_0 - 1 \times R_1 = V_A$

$V_0 - 1 \times R_2 = V_B$

$V_A - V_B = (R_2 - R_1)$ volt

Ex. In the given circuit; calculate potential difference between point A and B.



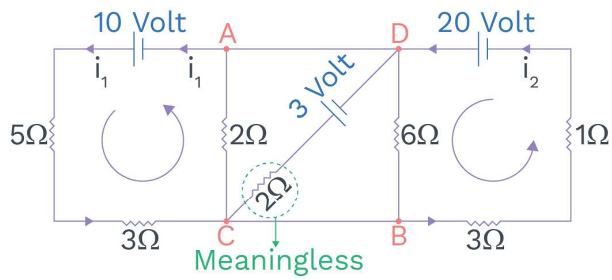
Sol. $V_A - V_B = 80$ volt

Current drawn from positive terminal of battery should be enter at negative terminal.

DC is unidirectional so it does not retrace its path.

It there is no complete circuit from positive to negative terminal of battery then there is no withdrawal of current from battery.

Ex. Calculate potential difference between points A and B in given circuit.



Sol. $i_1 = \frac{10}{10} = 1 \text{Amp}$

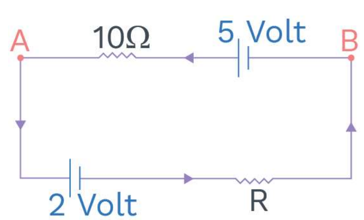
$i_2 = \frac{20}{10} = 2 \text{ Amp}$

Using KVL for ACDB branch

$V_A + 2i_1 + 3 - 6i_2 = V_B$

$V_A + V_B = 6i_2 - 3 - 2i_2 = 12 - 3 - 2 = 7 \text{ volt}$

Ex. In the given circuit, if potential difference between points A and B is $V_A - V_B = 4$ volt then calculate resistance R.



Sol. $V_A + 10I - 5 = V_B$

$\Rightarrow$ $4 - 5 = -10I \Rightarrow I = 0.1 \text{ Amp}$

$10I - 5 = -2 - IR$

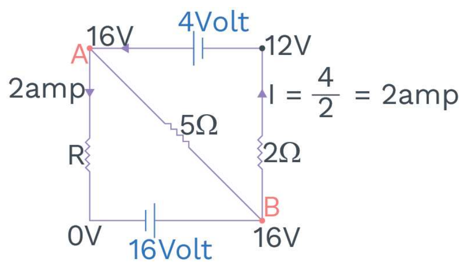
$1 - 5 = -2 - 0.1R$

$0.1 R = -2 + 4$

$0.1 R = -2 + 4$

$R = 20 \ \Omega$

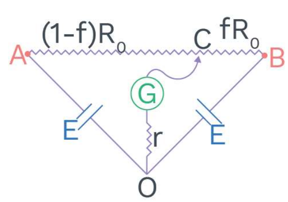
Ex. In given circuit, if there is no current through resistor of 5 Ω then calculate resistance R.



Sol. Current through 5 Ω ⇒ Zero

i.e., $V_A - V_B = 0$

Ex. Consider the arrangement shown in figure AB is a wire having uniform resistance with total resistance R_0. The sliding contact at C, divides the wire into resistance fR_0 and $(1 - f)R_0$. Assume that the batteries are identical and have negligible internal resistances. For what value of 'f', the galvanometer shows no deflection.



Sol. $E_{CO} = \dfrac{E_1 r_2 - E_2 r_1}{r_1 + r_2} = 0$

$E_1 r_2 = E_2 r_1$

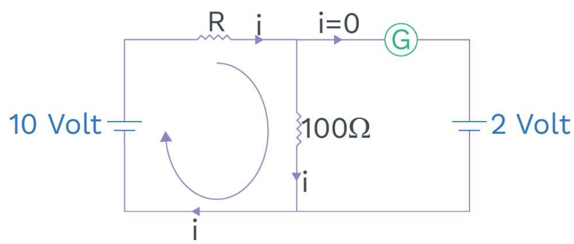
$E(fR_0) = E(1-f)R_0$

$f = 1 - f$

$2f = 1$

$f = \dfrac{1}{2}$

Ex. In given circuit, if reading of galvanometer is zero then calculate resistance R.



Sol. $i = \dfrac{10}{R + 100}$

$\dfrac{2}{100} = \dfrac{10}{R + 100}$

$500 = R + 100 \Rightarrow R = 400\,\Omega$

Using KVl through 2 volt battery

$2 - 100\,i = 0$

$100\,i = 2 \Rightarrow i = \dfrac{1}{50}$ Amp

or voltage about 100 Ω = 2 volt

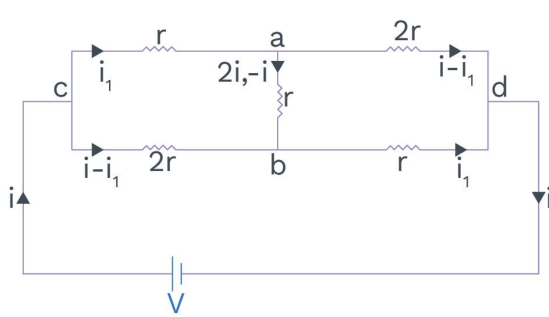
$t = \dfrac{2}{100}$

voltage across resistance R = 8 volt

$8 = \dfrac{2}{100}R$

R = 400 Ω

Ex. In given circuit, calculate current in branch 'ab':



Sol. KV in close loop cabc

$-i_1 r - (2i_1 - i)r + (i - i_1)2r = 0$

$$-i_1 r = 2i_1 r + ir + 2ir - 2i_1 r = 0$$

$$3ir = 5i_1 r$$

$$3i = 5i_1 \quad \Rightarrow i_1 = \frac{3i}{5} \qquad \text{...(i)}$$

KLV in loop cadc

$$-i_1 r - (i - i_1)2r + V = 0$$

$$-i_1 r - 2ir + 2i_1 r + V = 0$$

$$-2ir + i_1 r + V = 0$$

$$V = 2ir - i_1 r = 2ir - \frac{3i}{5} r$$

$$\frac{V}{i} = \frac{10r - 3r}{5} = \frac{7r}{5} \quad \Rightarrow R_{eq} = \frac{7r}{5}$$

$$R_{eq} = \frac{7r}{5} = 1.4r \ \text{(between 1.5 r and 1.33 r)}$$

Using KVL from a to b via c

$$V_a + i_1 r - (i - i_1)2r = V_b$$

$$V_a - V_b = (i - i_1)2r - i_1 r$$

$$= \frac{2i(2r)}{5} - \frac{3ir}{5} = \frac{ir}{5}$$

$$\Rightarrow \quad \boxed{V_a - V_b = \frac{V}{7}}$$

$$I_{ab} = \frac{V_a - V_b}{r} = \frac{ir}{5r} = \frac{i}{5}$$

$$I_{ab} = \frac{V}{5R_{eq}} = \frac{V}{57r} = \frac{V}{7r}$$

$$\boxed{I_{ab} = \frac{V}{7r}}$$

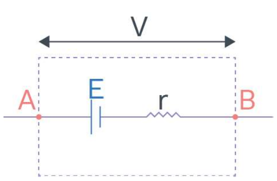
Electric Cell:

- Electric cell is the device provider constant potential difference.
- In electric cell, chemical energy converts into electrical energy

  $E \rightarrow$ Electromotive force (emf)

  $r \rightarrow$ Internal resistance

  $V_A - V_B = V \rightarrow$ Terminal voltage or terminal potential difference



Electromotive Force of Cell (E):

If cell is in open circuit or if no current associate with cell, then potential difference between terminals of cell represents emf of cell

$$V_A - E + (0)r = V_B$$

$$(V_A - V_B) = E$$

- emf of cell depends on concentration of electrolyte and nature of electrode (by Nernst equation)

Internal Resistance (r):

- Opposition of flow of ions inside the cell is named internal resistance.
- Internal resistance depends on-

$r \propto$ distance between electrodes

$$r \propto \frac{1}{\text{Surface area of electrode dipped in electrolyte}}$$

$$(r)_{\text{dry cell}} > (r)_{\text{electrolyte cell}}$$

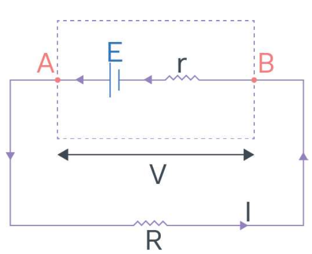
$r \propto$ concentration of electrolyte.

$$r \propto \frac{1}{\text{temperature of electrolyte}}$$

Terminal Voltage (V):

- High potential electrode of cell is named the terminal and low potential electrode of cell is named negative terminal of cell.
- Potential difference between positive and negative terminal of cell is named terminal voltage.

Case-I: When current is drawn from cell (discharging of cell)



Definitions

If cell is in open circuit or if no current associate with cell, then potential difference between terminals of cell represents emf of cell.

KEY POINTS

- Electric cell
- EMF of cell

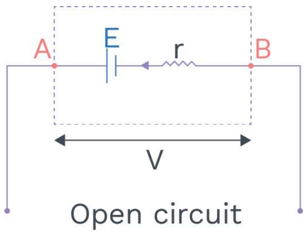
Concept Reminder

Internal resistance of a cell depends on the surface area of electrodes, the separation between them and nature, concentration and temperature of electrolyte.

$$V_A - E + Ir = V_B$$

$$V_A - V_B = E - Ir$$

Case-II: When cell is in open circuit.



Open circuit
(R = ∞)

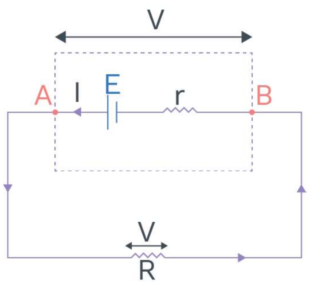
$$I = \frac{E}{R + r} = \frac{E}{0 + r} = \frac{E}{r}$$

$$V_A - E + Ir = V_B$$

$$V_A - V_B = E - Ir = E - \frac{IE}{r} = 0$$

V = 0

When current is drawn from cell (discharging of cell):



$$V = E - Ir$$

$$V = IR$$

$$V = \frac{ER}{R + r}$$

$$\frac{V}{E} = \frac{R}{R + r}$$

Concept Reminder

Potential difference across cell:

(a) While discharging,

V = E – Ir

(b) While charging, V = E + Ir

Rack your Brain

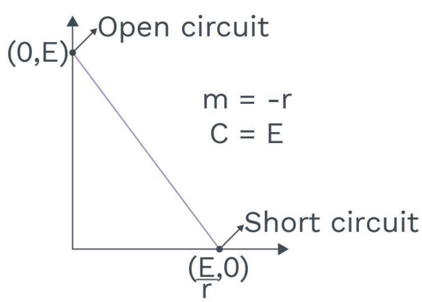
The internal resistance of a cell of emf 2 V is 0.1 Ω. It is connected to a resistance of 0.9 Ω. Find voltage across the cell.

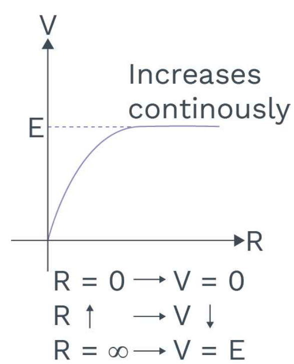
EXTERNAL RESISTANCE (R)	$I = \frac{E}{R + r}$	$V = \frac{ER}{R + r}$
If R = 0 (Short circuit)	$I = \frac{E}{r}$	V = 0
If R increases	I Decreases	Increases
R = ∞ (Open circuit)	I = 0	V = E

- Equation between terminal voltage and drawn current from cell is called cell equation.

$V = E - Ir$ (cell equation)

$y = c + mx$





Ex. Emf of a cell is E and its terminal voltage is V. When external resistance of R is connected across it. Calculate its internal resistance.

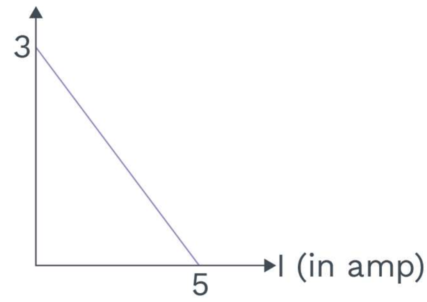
Sol. $V = \frac{ER}{R + r}$

$\Rightarrow \quad VR + Vr = ER$

$r = \frac{R(E - V)}{V}$

$r = R\left(\frac{E}{V} - 1\right)$

Ex. When current is drawn from cell then its terminal voltage varies with drawn current as shown. Calculate emf and internal resistance of cell. (V in volt)



Sol. V = E – Ir

E = 3 volt

$-r = \frac{-3}{5} \Rightarrow r = 0.6\,\Omega$

Ex. When external resistance of 18 Ω is connected across cell then potential drop across external resistance is 90% of emf of cell. Calculate internal resistance of cell.

Sol. $V = \frac{90\,E}{100}$

$$r = \frac{R(E-V)}{V} = \frac{R\left(E - \frac{9\,E}{10}\right)}{\frac{9\,E}{10}} = \frac{R\left(\frac{E}{10}\right)}{\frac{9\,E}{10}}$$

$$r = \frac{R}{9} = \frac{18}{9} = 2\,\Omega$$

Ex. When current of 4 amp is drawn from a cell then its terminal voltage is 20 volt. If current of 1 amp is given to same cell then terminal voltage of cell is 25 volt. Calculate internal resistance and emf of cell.

Sol. $V = E - Ir \quad \Rightarrow 20 = E - 4\,r$

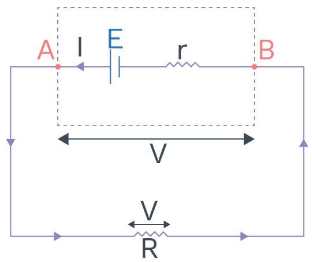
$V = E + Ir \quad \Rightarrow 25 = E + R$

–5 = – 5r

r = 1 Ω

E = 20 + 4 = 24 volt

Electric power delivered by cell during withdrawal of current from cell:



Load Resistance:

- An external resistance across which electric power is obtained from cell is named load resistance.

Drawn current, $I = \frac{F}{R + r}$

Electric power delivers by cell

$$P = VI = I^2R = \frac{V^2}{R}$$

$$\boxed{P = \frac{E^2R}{(R+r)^2}}$$

P = f(R)

- For maximum power $\frac{dP}{dR} = 0$

$$\frac{d}{dR}\left(\frac{E^2R}{(R+r)^2}\right) = 0$$

R = r

- R = r, i.e., when load resistance is equal as internal resistance of cell then cell delivers maximum power. This maximum power is as-

$$P_{max} = \frac{E^2}{4r}$$

Power in external circuit or dissipated power in internal resistance.

Total power supplied by cell = $\frac{E^2}{2r}$

Concept Reminder

Power developed in a resistor R

(a) At constant potential difference V is

$$P = \frac{V^2}{R}\left(P \propto \frac{1}{R}\right)$$

(b) At constant current i through it is

$$P = i^2R \ (P \propto R)$$

Load or external resistance (R):

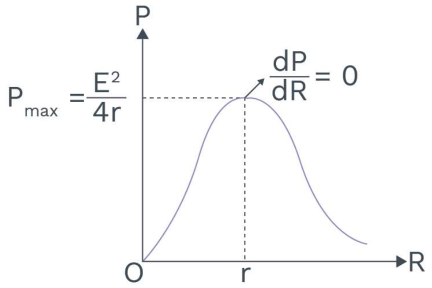
R = 0 (short circuit)

R increases (r > R > 0)

R = r

R increases (∞ > R > r)

R = ∞ (open circuit)



- For maximum power across any external resistance R
R = Equivalent resistance of circuit excluding R

Ex. When an electric cell is used separately with external resistance of R_1 and R_2 then it delivers same power across these resistances. Calculate internal resistance of cell.

Sol. $P = \frac{E^2R}{(R+r)^2} \Rightarrow P_1 = P_2$

$$\frac{E^2R_1}{(R_1+r)^2} = \frac{E^2R_2}{(R_2+r)^2}$$

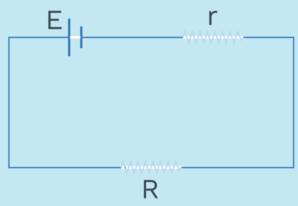
$$\frac{\sqrt{R_1}}{R_1+r} = \frac{\sqrt{R_2}}{R_2+r}$$

$$R_2\sqrt{R_1} + r\sqrt{R_1} = r\sqrt{R_2} + R_1\sqrt{R_2}$$

$$r = \frac{R_1\sqrt{R_2} - \sqrt{R_1}(R_2)}{\sqrt{R_1} - \sqrt{R_2}} = \frac{\sqrt{R_1R_2}(\sqrt{R_1} - \sqrt{R_2})}{\sqrt{R_1} - \sqrt{R_2}}$$

$$\boxed{r = \sqrt{R_1R_2}}$$

Concept Reminder



(a) Condition of maximum power across R is that R = r.

(b) Value of maximum power

$$P_{max} = \frac{E^2}{4r}$$

Efficiency of Electric Cell:

- Watt efficiency $n\% = \dfrac{\text{Output power}}{\text{Input power}} \times 100\%$

$$n\% = \frac{VI}{EI} \times 100$$

$$n\% = \frac{V}{E} \times 100 = \left(\frac{R}{R+r}\right) \times 100$$

$$n\% = \frac{P_{out}}{P_{out} + P_{loss}} \times 100 = \frac{I^2R}{I^2R + I^2r} \times 100$$

$$= \frac{P_{in} - P_{loss}}{P_{in}} \times 100$$

For maximum power,
Efficiency = 50% (because R = r)

- If cell is ideal

$r = 0 \quad \Rightarrow \quad \%n = 100\%$

Watt hour efficiency

$$n\% = \frac{(\text{voltage} \times \text{current} \times \text{time})_{output}}{(\text{voltage} \times \text{current} \times \text{time})_{input}} \times 100\%$$

Current Capacity of Cell:

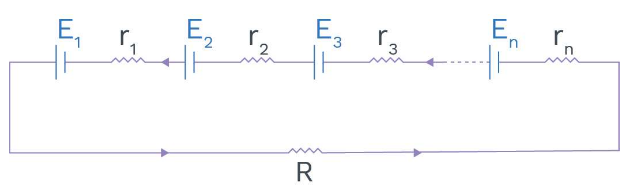
Product of drawn current and maximum possible time for which that current can be drawn represents current capacity of cell

e.g., Current capacity of cell,

20 A-h →
- 2 Amp for 10 hr
- 5 Amp for 4 hr
- 1 Amp for 6 hr + 2 Amp for 7 hr

COMBINATION OF ELECTRIC CELLS:

1. Series combination:



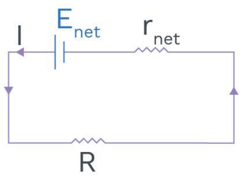
Concept Reminder

Efficiency of electric cell is

$$\eta = \frac{\text{Output power}}{\text{Input power}}$$

KEY POINTS

- Efficiency of cell
- Current capacity of cell



$$E_{net} = E_1 + E_2 + E_3 + ... + E_n$$

$$r_{net} = r_1 + r_2 + r_3 + ... + r_n$$

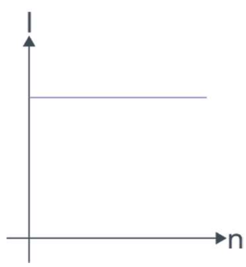
$$\boxed{I = \frac{E_{net}}{(R + r_{net})}}$$

- If n identical electric cells emf of each E and internal resistance of each r are connected in series,

$$E_{net} = nE$$

$$r_{net} = nr$$

$$I = \frac{E_{net}}{R + r_{net}} = \frac{nE}{(nr + R)}$$



$$\boxed{I = \frac{nE}{R + nr}}$$

If R << nr

$$\Rightarrow \quad I = \frac{E}{r} \quad [I \propto n^o]$$

Ir R >> nr

$$\Rightarrow \quad I = \frac{nE}{R}$$

- If total internal resistance of cells is negligible as compare load resistance, then series combination of electric cell is preferred.

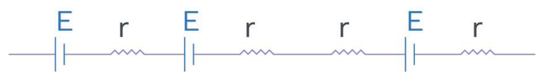
Concept Reminder

In series combination of electric cell, current is

$$I = \frac{E_{net}}{r_{net} + R}$$

Rack your Brain

A set of n equal resistors, of value R each, are connected in series to a battery of emf E and internal resistance R. The current drawn is I. Now, the n resistors are connected in parallel to same battery. Then the current drawn from battery becomes 10 I. Find the value of n.



E_{net} = E + E – E + E = 2 E

E_{net} = 2 E (depends on polarity)

r_{net} = 4 r (does not depends on polarity)

- If n identical cell are in series such that P cells out of them are with reversed polarity then;

  n > 2 P

  E_{net} = (n – 2 P)E

  r_{net} = nr

  E – emf of each cell

  r – internal resistance of each cell

Concept Reminder

If n identical cell are in series such that P cells out of them are with reversed polarity then;

n > 2 P

E_{net} = (n – 2 P)E

r_{net} = nr

Ex. 100 identical cells, emf of each 4 volt are kept in box in series combination. If net emf across the box is 360 volt then calculate number of cells with reversed polarity in the box.

Sol. E_{net} = (n – 2 P)E

360 = (100 – 2 P)4

2 P = 10

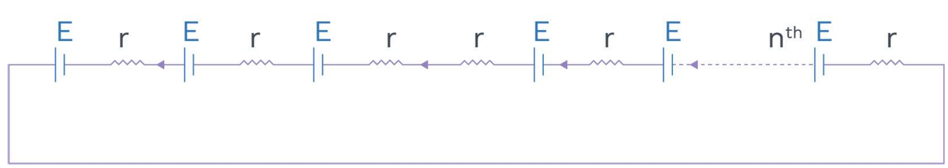
P = 5

i.e., 5 cells are with reversed polarity

Ex. n identical cells emf of each E and internal resistance of each r are connected in series such that 2 cell out of them are with reversed polarity. Calculate terminal voltage of either cell with reversed polarity (n > 4).

Sol. V = E + Ir

$$= E + \frac{nEr}{nr} = 2E$$



$$I = \frac{E_{net}}{nr} = \frac{E(n-4)}{nr}$$

If all cells are connected in series with same polarity,

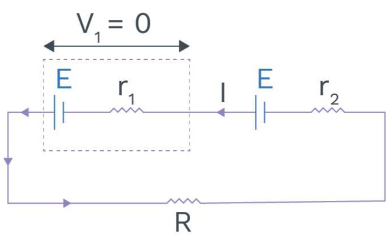
V = E + IR

$$V = E + \frac{E(n-4)r}{nr} = \frac{nE + nE - 4E}{n}$$

$$V = \frac{2nE - 4E}{n} \Rightarrow V = \frac{2nE}{n}\left(1 - \frac{2}{n}\right)$$

$$\Rightarrow \quad V = 2E\left(1 - \frac{2}{n}\right)$$

Ex. 2 electric cells emf of each E but internal resistance r_1 and r_2 ($r_1 > r_2$) are connected in series with external resistance of R. If terminal voltage of 1st cell is zero, then correct relation.



Sol. $V_1 = E - Ir_1 = 0$

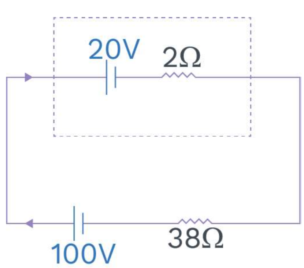
$E = Ir_1$

$$I = \frac{E_{net}}{r_1 + r_2 + R} = \frac{2E}{r_1 + r_2 + R}$$

$$E = \frac{2Er_1}{r_1 + r_2 + R} \Rightarrow r_1 + r_2 + R = 2r_1$$

$R = r_1 - r_2$

Ex. An electric cell of emf 20 volt and internal resistance 2 Ω is being charged by source of 100 volt using resistance of 38 Ω in series. Calculate terminal voltage of cell.



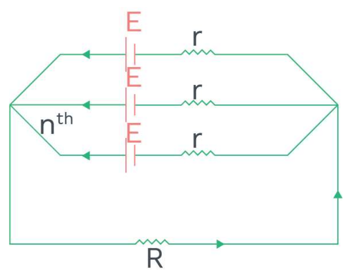
Sol. $V = E + IR = 20 + I(2)$

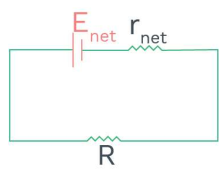
$$I = \frac{100 - 20}{38 + 2} = 2\,\text{Amp}$$

$V = 20 + 4 = 24$ volt

2. Parallel Combination:

- For identical cells:





> **Concept Reminder**
>
> If n identical cells are connected in parallel combination then
>
> $E_{net} = E$ and
>
> $r_{net} = \frac{r}{n}$

$E_{net} = E$ (same polarity of battery)

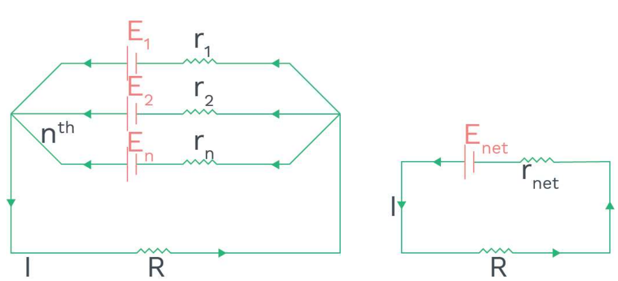
$$r_{net} = \frac{r}{n}$$

$$I = \frac{E_{net}}{r_{net} + R} \Rightarrow I = \frac{E}{\frac{r}{n} + R}$$

If $R >> \frac{r}{n}, \quad I = \frac{E}{R}$

If $R << \frac{r}{n}, \quad I = \frac{nE}{r}$

- If load resistance is negligible as compare total internal resistance, then parallel combination of electric cell is preferred.
- For non-identical cells

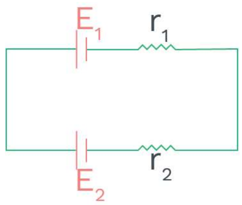


> **Concept Reminder**
>
> If n non-identical cells are connected in parallel combination then
>
> $$E_{net} = \left(\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} + ... + \frac{E_n}{r_n}\right)$$
>
> $$r_{net} = \left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + ... + \frac{1}{r_n}\right)$$

Series,

$E_{net} = E_1 - E_2$

$r_{net} = r_1 + r_2$

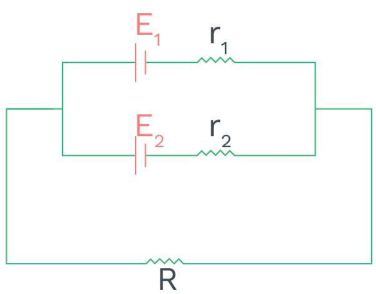


$$E_{net} = \frac{\left(\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} + ... + \frac{E_n}{r_n}\right)}{\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + ... + \frac{1}{r_n}\right)}$$

$$\frac{1}{r_{net}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + ... + \frac{1}{r_n}$$

$$\boxed{I = \frac{E_{net}}{r_{net} + R}}$$

- If 2 non-identical electric cells are connected in parallel with same polarity



$$E_{net} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

$$r_{net} = \frac{r_1 r_2}{r_1 + r_2}$$

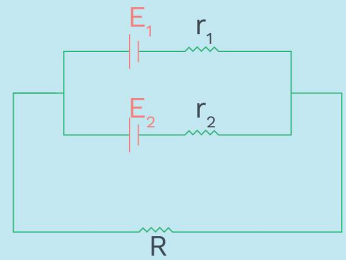
Concept Reminder

If n non-identical cells are connected in parallel combination then

$$E_{net} = \left(\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} + ... + \frac{E_n}{r_n}\right)$$

$$r_{net} = \left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + ... + \frac{1}{r_n}\right)$$

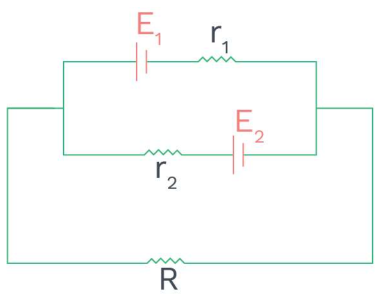
Concept Reminder



$$E_{net} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

$$r_{net} = \frac{r_1 r_2}{r_1 + r_2}$$

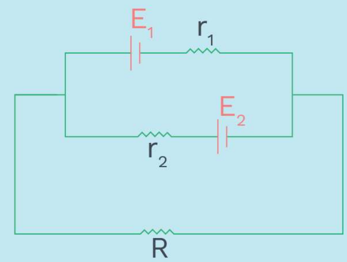
- If 2 non-identical electric cells are connected in parallel with reverse polarity, then;



$$E_{net} = \frac{E_1 r_2 - E_2 r_1}{r_1 + r_2}$$

$$r_{net} = \frac{r_1 r_2}{r_1 + r_2}$$

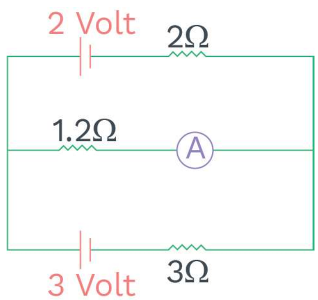
Concept Reminder



$$E_{net} = \frac{E_1 r_2 - E_2 r_1}{r_1 + r_2}$$

$$r_{net} = \frac{r_1 r_2}{r_1 + r_2}$$

Ex. Calculate reading of Ammeter in given circuit.



Sol. $I = \frac{E_{net}}{r_{net} + R} = \frac{12}{5(9.4)}$

$$E_{net} = \frac{6+6}{5} = \frac{12}{5} = 2.4 \text{ volt}$$

$$r_{net} = \frac{6}{5} = 1.2$$

Ex. n identical electric cells, internal resistance of each r either connected in series or connected in parallel with external resistance of R. If current through external resistance in both the cases s same, then correct relation.

Sol. In series, $I_S = \dfrac{nE}{nr + R}$

According to ....

$I_S = I_P$

$$\frac{nE}{nr + R} = \frac{nE}{r + nR}$$

nr + R = r + nR

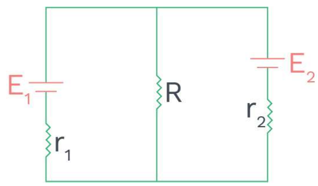
r(n – 1) = (n – 1)R

r = R

In parallel,

$$I_P = \frac{E}{\frac{r}{n} + R}$$

Ex. In the given circuit, calculate maximum power across resistor of resistance R.



Sol. For maximum power

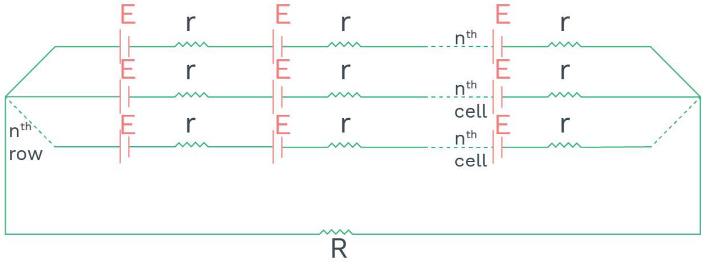
$R = R_{eq}$ excluding R

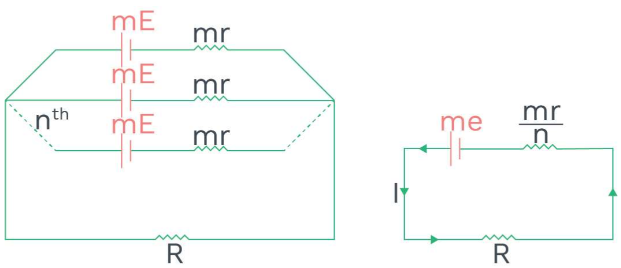
$$R = \frac{r_1 r_2}{r_1 + r_2}$$

$$P_{max} = \frac{E_{net}^2}{4 r_{net}}$$

$$P_{max} = \left(\frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}\right)^2 \frac{r_1 + r_2}{4 r_1 r_2} = \frac{(E_1 r_2 + E_2 r_1)^2}{4 r_1 r_2 (r_1 + r_2)}$$

3. Mixed Combination:





Drawn current, $I = \dfrac{mE}{\left(\dfrac{mr}{n} + R\right)}$

Power delivered by mixed combination,

$$P = I^2R \Rightarrow P = \frac{m^2E^2R}{\left(\dfrac{mr}{n} + R\right)^2}$$

Total number of cell in this combination = nm.
For maximum current/maximum power across R,

$$R = \frac{mr}{n}$$

Rack your Brain

For a cell terminal potential difference is 2.2 V when circuit is open and reduces to 1.8 V when cell is connected to a resistance of R = 5 Ω. Determine internal resistance of cell.

Ex. 24 identical electrical cells, internal resistance of each 0.5 Ω are used in mixed combination to obtain maximum current through external resistance of 3 Ω. Calculate number of rows and number of cells in each row.

Sol. n × m = 24

$$R = \frac{mr}{n}$$

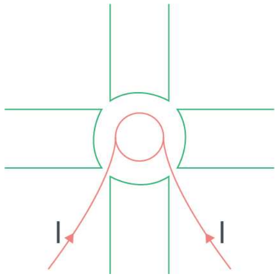
$$3 = \frac{m}{n} \times 0.5 \quad \Rightarrow \frac{m}{n} = 6$$

n × 6n = 24

$n = \sqrt{4} = 2 \Rightarrow$ Number of rows

m = 12 ⇒ 12 cells in each row.

MEASURING DEVICES:



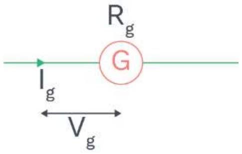
Galvanometer:

- It is used to detect the direction of current in circuit (prime requirement).
- It can be used for measurement of small current and small potential difference.
- **Principle:** When current is passing through a coil placed in magnetic field (magnetic poles are horse-shoe shaped to obtain radial) then coil experiences a torque and get deflect. Magnetic field so that on the basis of deflection, direction of current is detected and its valve also can be detected.

  Moving magnet galvanometer,

  $\tau = MB\sin\theta$ (Coil is fixed and magnet experience torque)

  Current in galvanometer coil $\propto$ deflection $\propto$ number of divisions.



R_g – Resistance of galvanometer coil.

I_g – Current range of galvanometer or maximum safe current for galvanometer or full scale deflection current.

V_g – Voltage range of galvanometer

$$\boxed{V_g = I_g R_g}$$

Concept Reminder

A galvanometer is a device to detect small currents and the direction of current in circuit.

KEY POINTS

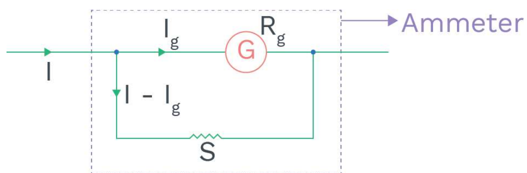
- Galvanometer
- Ammeter
- Shunt resistance

Concept Reminder

In galvanometer, amount of deflection is proportional to current passes through it.

Ammeter:

- It is used to measure large current.
- **Shunting:** Process in which small resistance is connected in parallel with high resistance named shunting and connected small resistance is named shunt resistance.
- When a small resistance is connected in parallel with galvanometer or when shunt resistance is connected with galvanometer then galvanometer gets convert into ammeter.



I – Range of ammeter.

Potential difference across shunt = Potential difference across galvanometer

$(I - I_g)S = I_gR_g$

$$S = \frac{I_gR_g}{I - I_g}$$

In ideal ammeter, no reading is possible because whole current will pass through branch which is parallel to galvanometer.

- Resistance of ammeter,

$$R_A = \frac{SR_g}{S + R_g}$$

- Resistance of ammeter in this case,

$$R_A = \frac{R_g}{n}$$

- Resistance of ammeter is to be decreased for R_1 increase in its range and to R_2 its accuracy.
- Ammeter is connected in series with circuit element.
- Ammeter measures less current as compare actual.
- Resistance of ideal ammeter should be zero so potential drop across ideal ammeter is zero.

Definitions

Shunt resistance: Process in which small resistance is connected in parallel with high resistance named shunting and connected small resistance is named shunt resistance.

Concept Reminder

Value of shunt resistance required to make ammeter

$$S = \frac{I_gR_g}{I - I_g}$$

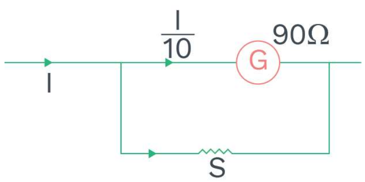
Rack your Brain

A galvanometer having 30 divisions has a current sensitivity of 20 mA/division. It has a resistance of 5Ω. How will you convert it into an ammeter measuring upto 1 A?

Ex. Resistance of galvanometer coil is 90 Ω when it is converted into ammeter then current in galvanometer is 10% of main current. Calculate used shunt.

Sol. $I_g = \frac{10}{100} I \Rightarrow I = 10 I_g \quad (n = 10)$

$$S = \frac{R_g}{n-1} = \frac{90}{10-1} = 10\,\Omega$$



Ex. Resistance of galvanometer coil is 75 Ω and its full-scale deflection current is 2 amp. Calculate required shunt to convert it into ammeter of range of 32 amp.

Sol. $V_g = V_s$

$I_g R_g = (I - I_g)s$

$$s = \frac{2 \times 75}{30} = 5\,\Omega$$

I = 32 Amp, I_g = 2 Amp

$I = nI_g \quad \Rightarrow 32 = ng$

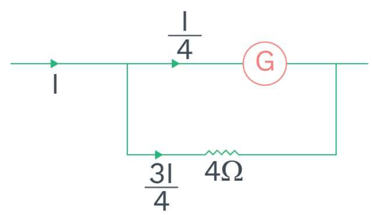
$$R_A = \frac{R_g}{n} = \frac{75}{16}\Omega$$

Note: If additional shunt is to be used then it should be connected in parallel with previous shunt.

Ex. When shunt of 4 Ω is connected with galvanometer then current passing through galvanometer is $\frac{1}{4}$ of main current. If addition shunt of 2 Ω is used, then current passing through galvanometer is $\frac{1}{n}$ times of main current. Calculate n.

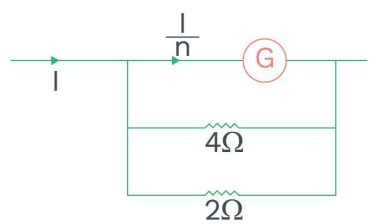
Sol. $V_g = V_s \Rightarrow R_g \frac{I}{4} = \frac{3}{4} \times 4 \times I$

$R_g = 12\,\Omega$



$$V_g = V_s \Rightarrow \frac{I}{n}R_g = \left(I-\frac{I}{n}\right)\frac{8}{6}$$

$$\frac{12}{n} = \frac{8}{6} - \frac{8}{6n} \Rightarrow \frac{12}{n} + \frac{8}{6n} = \frac{8}{6} \Rightarrow \frac{72+8}{6n} = \frac{8}{6} \Rightarrow n = 10$$



Ex. A galvanometer of resistance 50 Ω is connected to a battery of 3 V along with resistance of 2950 W in series. A full-scale deflection of 30 divisions is obtained in the galvanometer. In order to reduce this deflection to 20 divisions, the resistance in series should be.

Sol. 30 division, $\frac{3}{3000}$

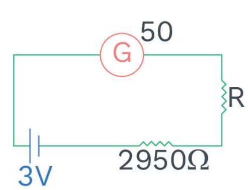
20 division, $\frac{3}{3000} \times \frac{20}{30}$

$$I' = \frac{3}{50+2950+R} \Rightarrow \frac{3}{3000} \times \frac{2}{3} = \frac{3}{3000+R}$$

$$9000 = 6000 + 2R \Rightarrow R = 1500\,\Omega$$

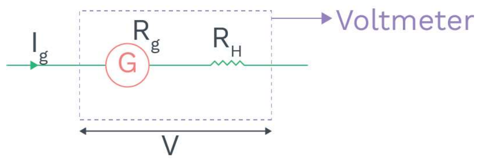
Resistance in series,

(1500 + 2950) Ω = 4450 Ω



Voltmeter:

- Galvanometer of high resistance.
- It is used to measure large potential difference.
- When high resistance is connected in series with galvanometer then it converts into voltmeter.



V – range of voltmeter

$$V = I_g(R_g + R_H)$$

$$\frac{V}{I_g} = R_g + R_H$$

$$\boxed{R_H = \frac{V}{I_g} - R_g}$$

Resistance of voltmeter = R_g + R_H

$$R_V = \frac{V}{I_g}$$

$R_V > R_g$

$R_V > R_H$

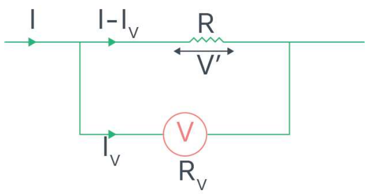
- When a galvanometer is converted into ammeter and voltmeter then

  $R_V > R_G > R_A$

- If range of voltmeter is n times the voltage range of galvanometer, i.e., $V = nV_g$

$$R_H = \frac{V}{I_g} - R_g = \frac{nV_g}{I_g} - R_g$$

$\boxed{R_H = R_g(n-1)}$ In this case: $R_V = nR_g$



Concept Reminder

- To convert galvanometer into voltmeter, a very high resistance is connected in series with galvanometer.
- Value of that resistance is

$$R = \frac{V}{I_g} - R_g$$

KEY POINTS

- Voltmeter

$$V' = \frac{(I - I_V)R}{I} = I_V R_V$$

$$V' < V$$

- Voltmeter measures less potential difference as compare actual.
- Ideal voltmeter should not draw any current from parallel branch so resistance of ideal voltmeter should be infinite but practically it is not possible.

Concept Reminder

- An ideal ammeter should have zero resistance.
- An ideal voltmeter should have infinite resistance.

Ex. Resistance of galvanometer coil is 100 Ω. Its full-scale deflection current is 1 mA. Calculate required higher resistance to convert into voltmeter of range 5 V.

Sol. $R_H = \frac{V}{I_g} - R_g = \frac{5}{10^{-3}} - 100$

$R_H = 5000 - 100 = 4900\,\Omega$

Ex. Resistance of galvanometer coil is 150 Ω and deflection current is 20 μA per division. Number of divisions on its full scale is 100. Calculate required higher resistance to convert it into voltmeter of range 8 volt.

Sol. 1 division = 20 mA

For 100 division,

$I_g = 20 \times 10^{-16} \times 100$

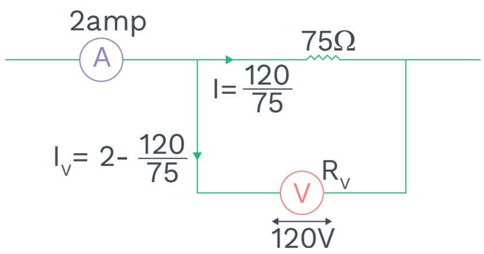
$I_g = 2 \times 10^{-3}$ Ampere

$R_H = \frac{V}{I_g} - R_g = \frac{8000}{2} - 150 = 4000 - 150$

$R_H = 3850\,\Omega$

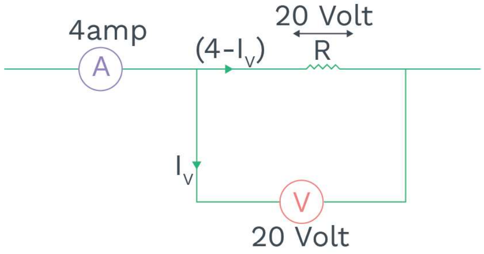
Ex. Reading of ammeter and voltmeter in given circuit are 2 Amp and 120 volt respectively. Calculate resistance of voltmeter.

Sol. $R_V = \frac{V}{I_V} = \frac{120 \times 75}{150 - 120}$



$R_V = 4 \times 75 = 300\,\Omega$

Ex. In given circuit, ammeter and voltmeter are non-ideal. If their readings are 4 A and 20 V respectively. Then resistance R is:

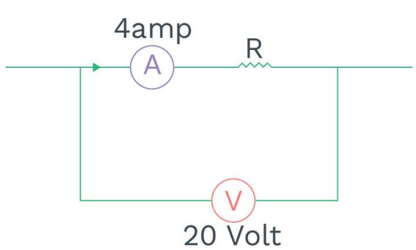


(1) R = 5 Ω for this, voltmeter should be ideal
(2) R is slightly greater than 5 Ω
(3) R is slightly less than 5 Ω
(4) None of these

Sol. As voltmeter is non-ideal

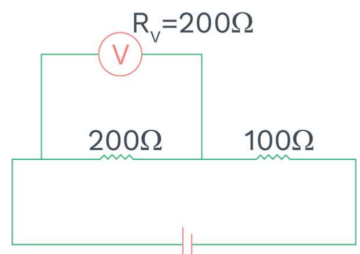
$(4 - I_V)R = 20$

$R = \dfrac{20}{4 - I_V} \Rightarrow R > 5\,\Omega$



R < 52 Ω; for R = 5 Ω; ammeter should be ideal

Ex. In given circuit, calculate percentage change in potential drop across resistor of resistance 200 Ω. In +nce of voltmeter.



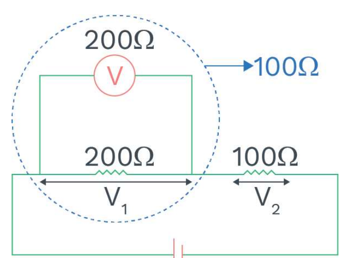
Measured value – in +nce of voltmeter or ammeter.
Actual value – In –nce of voltmeter and ammeter.

Sol. In –nce of voltmeter (actual potential difference)

$V_1 : V_2 : : 2 : 1$

$$V_1 = \frac{2}{3} \times 12 = 8 \text{ volt}$$

In +nce of voltmeter (measured potential difference)



$V_1 : V_2 : : 1 : 1$

$$V_1 = \frac{1}{2} \times 12 = 6 \text{ volt}$$

Percentage change

$$= \left[\frac{\text{Measured P.D.} - \text{Actual P.D.}}{\text{Actual potential difference}}\right] \times 100$$

$= \frac{6-8}{8} \times 100 = -25\%$ (does not depends on value of V)

Note: If ammeter and voltmeter are to be change in another ammeter and voltmeter then previous meters should be considered as galvanometer.

Ex. The resistance of an ammeter is 13 Ω and its scale is graduated for a current upto 100 A. After an additional shunt has been connected to this ammeter it becomes possible to measure currents upto 750 A by this meter. The value of shunt resistance is.

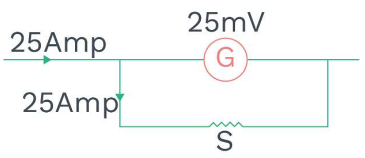
Sol. $I_g = 100$ A, $R_g = 13\ \Omega$

$I = nI_g \quad \Rightarrow 750 = n(100) \Rightarrow n = 7.5$

$$R_S = \frac{R_g}{n-1} = \frac{13}{6.5} = \frac{130}{65} = 2\,\Omega$$

Ex. A milli voltmeter of 25 mV range is to be converted into an ammeter of 25 A range. The value of necessary shunt will be.

Sol. $V_g = 25$ mV



$V_S = IS$

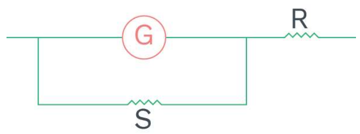
$$S = \frac{25 \times 10^{-3}}{25} = 0.001\ \Omega$$

Ex. A galvanometer of resistance G is shunted by resistance S Ω. To keep the main current in circuit unchanged, the resistance to be put m series with galvanometer is.

Sol. To keep unchanged the main current,

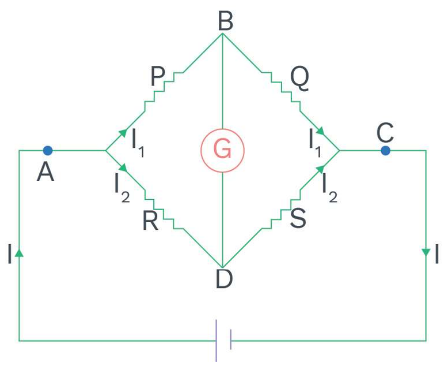
$R_{eq} = G$

$$\frac{GS}{G+S} + R = G$$



$$R = \frac{G - GS}{G+S} = \frac{G^2 + GS - GS}{G+S} = \frac{G^2}{G+S}$$

Wheatstone Bridge:



- In experimental arrangement of Wheatstone Bridge, P and Q are ratio arm, R is the branch of resistance box and S is the branch of unknown resistance.
- Balanced Wheatstone bridge is used to measure unknown resistance (S). (Prime requirement of balanced Wheatstone bridge)
- Calculation of unknown resistance is done when galvanometer shows 'null deflection' i.e., balancing of Wheatstone bridge is bases on null deflection method.

In balanced Wheatstone bridge:

$$I_g = 0;\ V_B = V_0;\ \frac{P}{Q} = \frac{R}{S}$$

- KVL is closed loop ABDA

$$-I_1P + I_2R = 0$$

$$I_2R = I_1P$$

$$\frac{P}{R} = \frac{I_2}{I_1} \qquad \text{...(i)}$$

- KVL in closed loop BCDB

$$-I_1Q + I_2S = 0$$

$$I_2S = I_1Q$$

KEY POINTS

- Wheatstone bridge
- Balanced Wheatstone bridge

Concept Reminder

- In balanced Wheatstone bridge, we can interchange galvanometer and cell without affecting the circuit condition.

$$\frac{Q}{S} = \frac{I_2}{I_1} \quad \text{...(ii)}$$

$$\frac{P}{R} = \frac{Q}{S} \Rightarrow \left[\frac{P}{Q} = \frac{R}{S}\right]$$

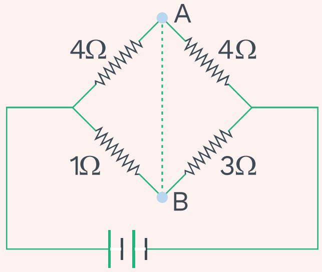
- Balancing of Wheatstone bridge is depend only on P, Q, R and S. It does not depend on resistance of galvanometer and emf of cell.
- In balanced Wheatstone bridge, galvanometer and cell can be interchanged.
- If P, A, R and S are of same order then Wheatstone is most sensitive.
- In balanced Wheatstone bridge, element in bridge branch (BD) can be removed.

In unbalanced Wheatstone Bridge:

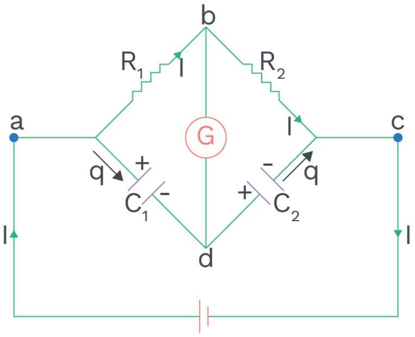
- If $\frac{P}{Q} < \frac{R}{S}$, current in bridge branch – from B to D ($V_B > V_D$)
- If $\frac{P}{Q} > \frac{R}{S}$, current in bridge branch – from D to B ($V_D > V_B$)

Rack your Brain

In circuit below if a conducting wire is connected between A an B, then in which direction will current flow?



Ex. In given circuit, if galvanometer shows null deflection, then correct option:



(1) $\frac{R_1}{R_2} = \frac{C_1}{C_2}$ (2) $\frac{R_1}{R_2} = \frac{C_2}{C_1}$

(3) $\frac{R_1}{R_2} = \frac{C_1^2}{C_2^2}$ (4) $\frac{R_1}{R_2} = \frac{C_2^2}{C_1^2}$

Sol. KVL in closed loop 'abda'

$$-IR_1 + \frac{q}{C_1} = 0$$

$$IR_1 = \frac{q}{C_1} \quad \text{...(i)}$$

KVL in closed loop 'bcdb'

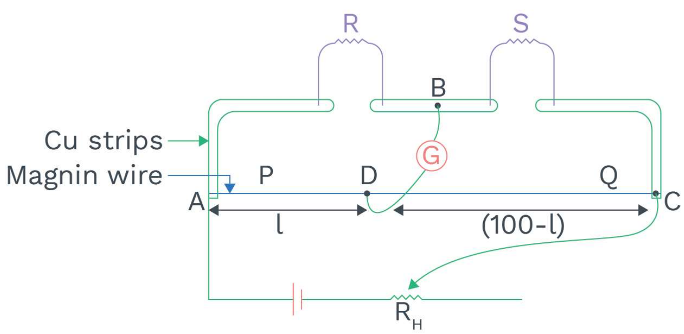
$$-IR_2 + \frac{q}{C_2} = 0$$

$$IR_2 = \frac{q}{C_2} \quad \text{...(ii)}$$

From equation (i)/(ii),

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{C_2}{C_1}$$

Meter Bridge:



- In experimental arrangement of meter bridge, known resistance (R) is connected in left gap and unknown resistance (S) is connected in right gap.
- Balanced meter bridge is used to calculate unknown resistance.
- Balanced meter bridge is same as balanced Wheatstone bridge, i.e., it is based on null deflection method.
- In balanced meter bridge, resistance of wire AD and DC are same as P and Q of balanced Wheatstone bridge respectively.
- Wire of length 1 m made of Magnin and constantan is connected between points A and C as shown.

KEY POINTS

- Meter bridge
- Balanced meter bridge

Concept Reminder

- Meter bridge is based on principle of Wheatstone bridge.
- In balanced meter bridge

$$\left[\frac{R}{S} = \frac{l}{100 - l}\right]$$

In balanced meter bridge:

$$I_g = 0; \ \frac{P}{Q} = \frac{R}{S}$$

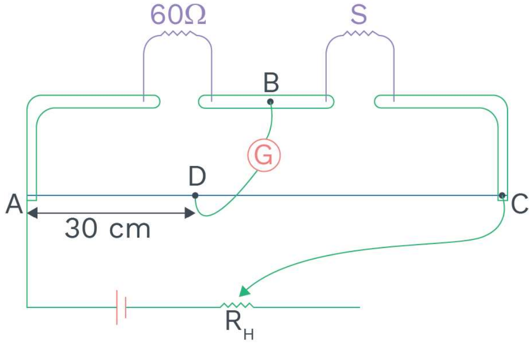
$$\frac{R_{AD}}{R_{DC}} = \frac{R}{S} \Rightarrow \frac{\frac{\rho l}{A}}{\frac{\rho(100-l)}{A}} = \frac{R}{S}$$

$$\frac{l}{100-l} = \frac{R}{S}$$

$$S = \frac{R(100-l)}{l} \Rightarrow \boxed{\frac{R}{l} = \frac{S}{100-l}} \quad (l \text{ in cm})$$

- Meter bridge wire should be uniform.
- Distance of null point from high potential end (A) is named balancing length, i.e., balancing length is measured from high potential end of meter bridge wire.
- By default, balancing length si considered for resistance in left gap of meter bridged.

Ex. In given meter bridge galvanometer shows null deflection. Calculate unknown resistance S.



Sol. $\frac{R}{l} = \frac{S}{100-l} \Rightarrow \frac{60}{30} = \frac{S}{70}$

$S = 140 \ \Omega$

Ex. In meter bridge arrangement, resistances x and y are used in 2 gaps of bridge. Null point is at distance of 25 cm from one of the end of wire (y > x). If resistance of 2 x is used instead of x then calculate distance of null point from same end of wire.

Sol. **Case-I:**

$$\frac{x}{25}=\frac{y}{75} \Rightarrow x=\frac{y}{3}$$

Case-II:

$$\frac{2x}{l'}=\frac{y}{100-l'} \Rightarrow \frac{2y}{3l'}=\frac{y}{100-l'}$$

$$200-2l'=3l' \Rightarrow l'=40\,\text{cm}$$

Ex. Resistances R_1 and R_2 are used in left gap and right gap of meter bridge respectively. Null point is at distance of 1/3 m. If resistance of 6 Ω is connected in series with smaller resistance, then null point is at distance of 2/3 m from same end. Calculate R_1 and R_2.

Sol. $$\frac{R}{l}=\frac{S}{100-l}$$

Case-I:

$$\frac{R_1}{1}\times 3=\frac{R_2}{2}\times 3 \Rightarrow R_2=2R_1$$

Case-II:

$$\frac{(R_1+6)}{2}\times 3=\frac{R_2}{1}\times 3 \Rightarrow R_1+6=2R_2$$

$R_1 + 6 = 4\ R_1$

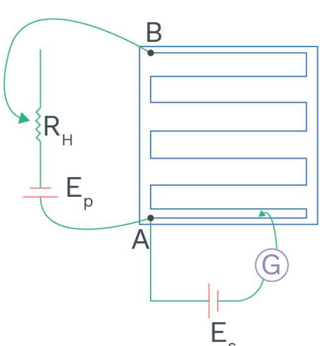
$3\ R_1 = 6$

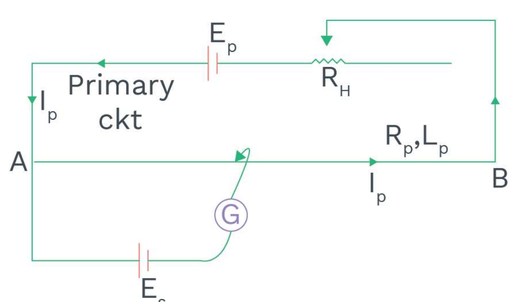
$R_1 = 2\ \Omega$

$R_2 = 4\ \Omega$

Potentiometer:

- Practically possible voltmeter draws some current from parallel branch and measures less potential difference so to calculate accurate potential difference, potentiometer is used.





E_p – Source of known emf in 1 circuit.
I_p – Current in 1 circuit.
R_p – Resistance of potentiometer wire.
L_p – Length of potentiometer wire.
E_s – Unknown potential difference connected in 2 circuits.

- In experimental arrangement of potentiometer, long length potentiometer wire made of magnin and constantan is is connected in between points A and B as shown.
- Source of known emf (E_p), Rheostat and potentiometer wire are connected in series named primary circuit.
- Potentiometer remains unaffected by internal resistance of E_s.
- Source of unknown potentiometer difference is used in secondary circuit which is connected parallel with primary circuit.
- Calculation of unknown potentiometer difference connected in 2 circuit is being done by comparison with uniformly distributed known potentiometer difference on potentiometer wire at balancing condition.
- Balancing condition means galvanometer of 2 circuit shows null deflection.
- Primary circuit of potentiometer does not draw any current from 2 circuit at balancing condition, so potentiometer behaves as ideal voltmeter.

Potential Gradient (x):

- Potential drops on per unit length of potentiometer wire is named potential gradient (x).

$$\boxed{x = \frac{V_{AB}}{L_p}} \ \frac{\text{volt}}{\text{metre}}$$

L_p – Length on which there is potential drop of V_{AB}.

$$\boxed{x = \frac{V_{AB}}{L_p} = \frac{I_p R_p}{L_p} = \frac{I_p \rho}{A}}$$

Concept Reminder

- A potentiometer is an accurate and versatile device to make electrical measurements of emf because if involves a condition of no current flow through the galvanometer.

Rack your Brain

A resistance wire connected in the left gap of a meter bridge balances a 10Ω resistance in the right gap at a point which divides the bridge wire in ratio of 3 : 2. If length of wire is 1.5 m then, find the length of 1Ω resistance wire.

$$x = \frac{I_p R_p}{L_p} = \frac{E_p \times R_p}{(R_h + R_p)L_p}$$

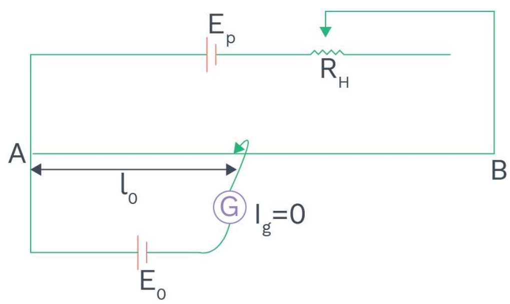
$R_h = 0 \Rightarrow x$ – maximum.

R_h = Maximum $\Rightarrow x$ – minimum

- As potential gradient (x) is the parameter of 1 circuit, so it does not depend on 2 circuit.

Standardization of Potentiometer:

A practical process using standard cell of known emf in 2 circuit to calculate potential gradient accurately is named standardization.



Potential difference across length l_0 is E_0

Potential difference across unit length = $\frac{E_0}{l_0}$

i.e., potential gradient (x) = $\frac{E_0}{l_0}$

- High potential end of 1 circuit and 2 circuit should be connected at same point.
- Balancing length is measured from high potential end of potentiometer wire.

$$\frac{E_0}{E_s} \le E_p$$

Strength of Potentiometer:

- Maximum possible potential difference which can be measured from potentiometer is named strength of potentiometer.

KEY POINTS

- Potentiometer
- Potential gradient

Concept Reminder

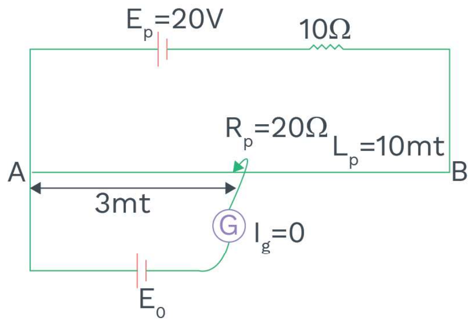
- The length of potentiometer wire is kept large because if wire long then potential gradient will be smaller. Hence balancing length will be increased.

- Potential drop across entire length of potentiometer wire represents strength of potentiometer.
  Strength of potentiometer = $V_{AB} = xL_p$

Sensitivity of Potentiometer:

- Balancing length for per unit potential difference is sensitivity of potentiometer.
  Sensitivity of potentiometer
  $$= \frac{\text{Balancing length}}{\text{Potential difference}} = \frac{l}{V} = \frac{1}{x}$$
- If potential gradient on potentiometer wire is x and balancing length for unknown potential difference connected in 2 circuit is l then this unknown potential difference is as
  $$E_s = xl$$

Ex. In given potentiometer circuit, galvanometer of 2 circuit show null deflection. Calculate potential difference connected in 2 circuit.



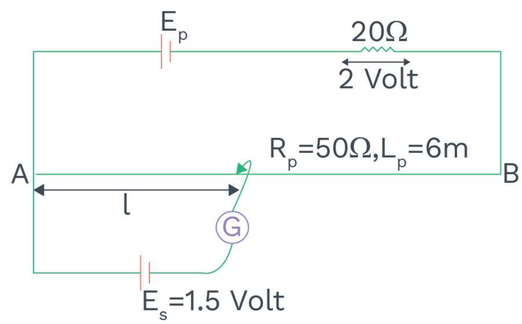
Sol. From 1 circuit,

$$x = \frac{E_p R_p}{(R_p + R_H)L_p} = \frac{20 \times 20}{30 \times 10} = \frac{4}{3} \text{ volt/metre}$$

From 2 circuit,

$$E_s = xl \Rightarrow E_s = \frac{4}{3} \times 3 = 4 \text{ volt}$$

Ex. In given potentiometer circuit, potential drop across resistance of 20 Ω is 2 volt. Calculate balancing length for potential difference connected in 2 circuit.

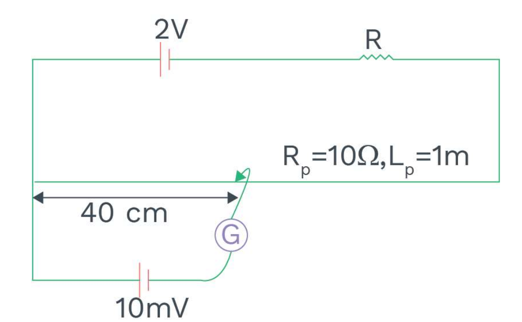


Sol. $I_p = \frac{2}{20} = \frac{1}{10}$

$E_s = xl = \frac{I_p R_p}{L_p} \times l$ 3

$l = \frac{1.5 \times 6 \times 10}{1 \times 50} = 1.8$ metre

Ex. There is a potentiometer wire of length 1 m and resistance 10 Ω. This potentiometer is connected in series with resistor of resistance R and source of emf 2 volt. If Balancing length for potential difference of 10 mV is 40 cm, then calculate resistance R.



Sol. From 1 circuit,

$x = \frac{I_p R_p}{L_p} = \frac{E_p R_p}{(R_p + R) L_p}$

$\frac{1}{40} = \frac{2}{(R+10)} \times \frac{10}{1} \Rightarrow R + 10 = 800$

R = 790 Ω

From 2 circuit,

$$E_s = xl \Rightarrow \frac{10 \times 10^{-3}}{40 \times 10^{-2}} = x$$

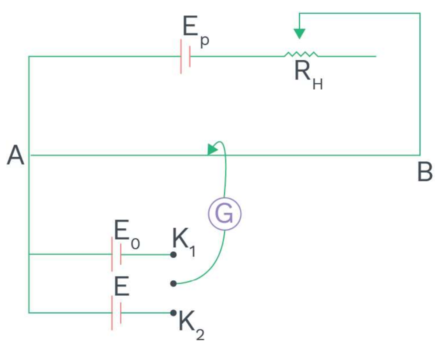
$$x = \frac{10^{-1}}{4} = \frac{1}{40} \text{ volt/metre}$$

Note: If radius of wire is doubled then potential gradient (x) will remains same.

$$x = \frac{I_P R_P}{L_p}$$

APPLICATIONS OF POTENTIOMETER:

1. Measurement of unknown emf of cell:



- When key K_1 is used then standardization emf E_0 will be balanced. Let l_0 be balancing length

$$E_0 = xl_0$$

$$x = \frac{E_0}{l_0} \Rightarrow \text{standardization}$$

- Key K_1 is removed and K_2 is used then unknown emf E is balanced, let l be balancing length

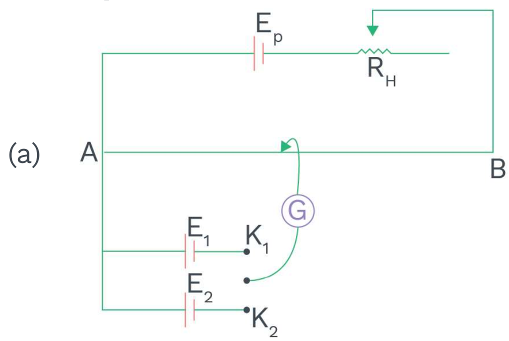
$$E = xl$$

$$E = \left(\frac{E_0}{l_0}\right) l$$

Concept Reminder

- In potentiometer, larger is the length of wire, move is the accuracy. Thus for a sensitive potentiometer, potential gradient should be small.

2. Comparison of emf of 2 cells:



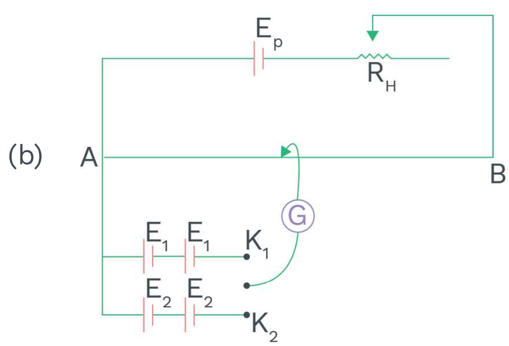
- One of the cell is chosen as a standardization cell whose emf is known to a high degree of accuracy.
- No standardization is used since x is cancelled finally, so accurate value of x.
- When key K_1 is used then emf E_1 will be balanced. Let l_1 is balancing length

  $E_1 = xl_1$ ...(i)
- Key K_1 is removed and K_2 is used then emf E_2 will be balanced. Let l_2 is balancing length

  $E_2 = xl_2$ ...(ii)

  From equation (i)/(ii)

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$



$(E_1 > E_2)$

- When key K_1 is used then combinedly $(E_1 + E_2)$ will be balanced, let l_1 is balancing length

  $E_1 + E_2 = xl_1$ ...(i)
- Key K_1 is removed and K_2 is used then combinedly $(E_1 - E_2)$ will be balanced, let l_2 is balancing length

  $E_1 - E_2 = xl_2$ ...(ii)

Rack your Brain

A potentiometer wire is 100 cm long and a constant P.d is maintained across it. Two cells are connected in series first to support one another an then in opposite direction. The balance point is 50 cm and 10 cm from positive end of wire in two cases. Find the ratio of emf's.

From equation (i)/(ii)

$$\frac{(E_1+E_2)}{(E_1-E_2)}=\frac{l_1}{l_2} \Rightarrow \frac{E_1}{E_2}=\frac{l_1+l_2}{l_1-l_2}$$

Ex. Two cell of emf E_1 and E_2 ($E_1 > E_2$) are connected in series in 2 circuit then balancing length is 3 metre. If these cells are connected in series with another pattern such that balancing length becomes 5 metre. Find ratio $\frac{E_1}{E_2}$.

Sol. $\frac{E_1}{E_2}=\left(\frac{l_1+l_2}{l_1-l_2}\right)=\frac{5+3}{5-3}=\frac{4}{1}$

Ex. Two cells are to be balanced separately on potentiometer wire then difference between corresponding balancing length is 20 cm. If difference in emf of these cells is 0.5 volt then calculate potential gradient on potentiometer wire. If length of potentiometer wire is 10 m then calculate strength of potentiometer also.

Sol. $E_1 = xl_1$

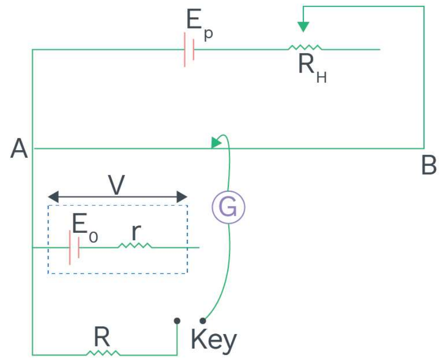
$E_2 = xl_2$

$E_1 - E_2 = x(l_1 - l_2)$

$0.5 = x\frac{20}{100} \Rightarrow x = 2.5$ volt/metre

Strength = $V_{AB} = xl_p = 2.5 \times 10 = 25$ volt

3. Measurement of internal resistance of given cell:



When key is opened in sub-circuit of 2 circuit then balancing will be emf of cell; let l_0 is balancing length

$$E = xl_0 \quad \text{...(i)}$$

- When key is closed in sub-circuit of 2 circuit then balancing will be terminal voltage of cell; let l_c is balancing length

$$V = xl_c \quad \text{...(ii)}$$

When current is drawn from cell

$$V = \frac{ER}{R + r} \Rightarrow r = \frac{(E - V)R}{V}$$

$$r = \left(\frac{xl_0 - xl_c}{xl_c}\right)R$$

$$\left[r = \left(\frac{l_0 - l_c}{l_c}\right)R\right] \quad \text{(Where } l_0 > l_c\text{)}$$

Concept Reminder

- Internal resistance

$$r = \left(\frac{l_0}{l_c} - 1\right)R$$

Ex. A cell is connected in 2 circuit to calculate its internal resistance. Balancing of cell is across length of 6 m. When external resistance of 10 Ω is connected across cell then balancing length becomes 5 m. Calculate internal resistance of cell.

Sol. $r = \left(\frac{l_0 - l_c}{l_c}\right)R = \left(\frac{6 - 5}{5}\right)10 = 2\Omega$

Ex. An electric cell connected in 2 circuit is being balanced across length of 90 cm. When external resistance of 5 Ω is connected with cell then balancing length becomes 75 cm. If resistance of 4 Ω instead of 5 Ω is connected with cell then calculate balancing length.

Sol. **Case-I:**

$$r = \left(\frac{l_0 - l_c}{l_c}\right) \times R = \left(\frac{90 - 75}{75}\right) \times 5 = \frac{15}{15}$$

$r = 1\ \Omega$

Case-II:

$$r = \left(\frac{l_0 - l'_c}{l'_c}\right)R = \left(\frac{90 - l'_c}{l'_c}\right)4$$

$$l'_c = 360 - 4l'_c \Rightarrow 5l'_c = 360$$

$$l'_c = 72 \text{ cm}$$

4. Comparison of resistances of 2 resistors:

- Key K_1 is used then potential drop across R_1 will be balanced, let l_1 is balancing length

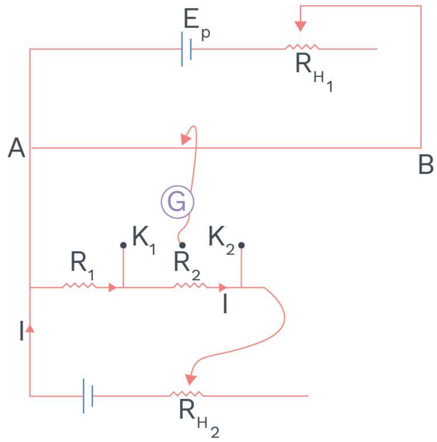
  $IR_1 = xl_1$ ...(i)
- Key K_1 is removed and K_2 is used then combinedly potential drop across R_1 and R_2 is balanced, let l_2 is balancing length

  $$I(R_1 + R_2) = xl_2$$

  $$IR_1 + IR_2 = xl_2$$

  $$xl_1 + IR_2 = xl_2$$

  $$IR_2 = x(l_2 - l_1) \quad \text{...(ii)}$$



From equation (i)/(ii)

$$\frac{R_1}{R_2} = \frac{l_1}{l_2 - l_1}$$

5. Measurement of current in given circuit:

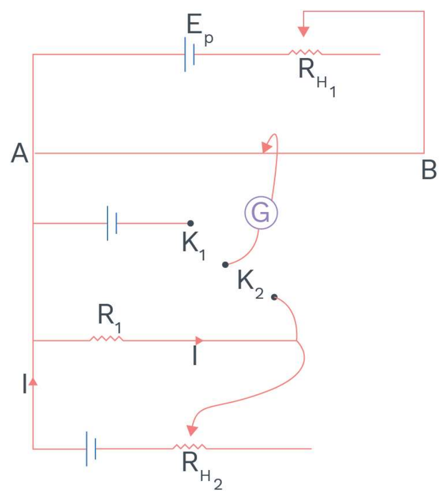
- When key K_1 is used then standardization emf E_0 will be balanced, let l_0 is balancing length

  $E_0 = xl_0$

  $$x = \frac{E_0}{l_0} \text{ (standardization)}$$
- Key K_1 is removed and key K_2 is used then potential drop across resistance R is balanced, let l is balancing length

  $IR = xl$

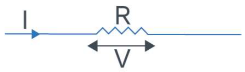
  $$IR = \left(\frac{E_0}{l_0}\right)l \Rightarrow I = \left(\frac{E_0}{l_0}\right)\frac{l}{R}$$



Heating Effect of Current (Joule's law of heating):

- When current is passing through any conductor or resistor then heat is produced across that conductor or resistor.
- Heating effect of current does not depend on direction of current.

  Amount of heat produced (H):



$$H = I^2Rt = \frac{V^2t}{R} = VIt$$

I – In amp, V – In volt, R – in Ohm, t – n sec and H – Joule.

1 cal = 4.18 joule

$1\text{ cal} \simeq 4.2\text{ joule}$

ELECTRIC POWER (P):

- Rate of heat produced, or amount of heat produced in per unit time in electric circuit is named electric power.

$$P = \frac{dH}{dt};\ P = \frac{H}{t}$$

$$P = I^2R = \frac{V^2}{R} = VI \text{ Joule/sec or watt}$$

1 KWh = 3600 × 10^3 watt-sec

1 KWh = 36 × 10^5 Joule

1 KWH = 3.6 × 10^6 Joule

If P = constant

H = Pt

If $I = f(t) \Rightarrow P = f(t)$

$$\Rightarrow H = \int_{t_1}^{t_2} P\,dt$$

Note: Integration of electric power with time or area of electric power-time graph gives amount of heat produced in given time interval.

Definitions

Rate of heat produced, or amount of heat produced in per unit time in electric circuit is named electric power.

$$P = \frac{dH}{dt};\ P = \frac{H}{t}$$

$$P = I^2R = \frac{V^2}{R} = VI \quad \text{Joule/sec or watt}$$

Ex. The charge flowing through a resistance R varies with time t as Q = at – bt^2, where a and b are positive constants. The total heat produced in R is.

Sol. Q = at – bt^2

I = a – 2bt

$I = f(t) \Rightarrow P = f(t)$

$$H = \int_{t_1}^{t_2} P\,dt = \int_{t_1}^{t_2} I^2R\,dt = \int_0^{a/2b} (a - 2bt)^2 R\,dt$$

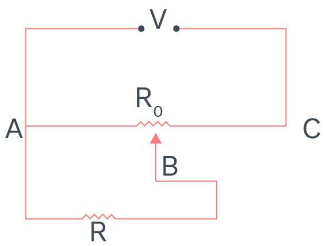
$$= \int_0^{a/2b} R(a^2 + 4b^2t^2 - 4abt)\,dt$$

$$= R\left[a^2t + \frac{4b^2t^3}{3} - \frac{4abt^2}{2}\right]_0^{a/2b}$$

$$H = R\left[\frac{a^3}{2b} + \frac{4b^2}{3}\frac{a^3}{8b^3} - \frac{2aba^2}{4b^2}\right]$$

$$H = R\left[\frac{a^3}{2b} + \frac{a^3}{6b} - \frac{a^3}{2b}\right] = \frac{a^3R}{6b}$$

Ex. A resistance of R Ω draws current from a potentiometer. The potentiometer has a total resistance R_0 Ω. A voltage V is supplied to potentiometer. What is voltage across R when sliding contact is at middle of potentiometer.



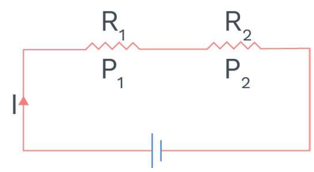
Sol. $R_{net} = \frac{R_0}{2} + \frac{\frac{R_0R}{2}}{\frac{R_0}{2}+R}$

$$I = \frac{V}{\frac{\frac{R_0}{2}+\frac{R_0}{2}R}{\frac{R_0}{2}+R}} = \frac{V}{\frac{R_0}{2}+\frac{R_0R}{R_0+2R}} = \frac{V}{\frac{R_0}{2}+\frac{R_0R}{R_0+2R}}$$

$$I = \frac{\frac{IR_0R}{2}}{\left(\frac{R_0}{2}+R\right)} = \frac{2V(R_0+2R)\frac{R_0R}{2}}{\left(\frac{R_0+2R}{2}\right)R_0(R_0+4R)} = \frac{2VR}{R_0+4R}$$

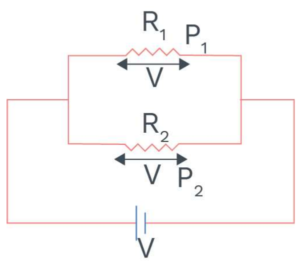
$$P = I^2R = \frac{V^2}{R} = VI$$

e.g., $R_1 > R_2$



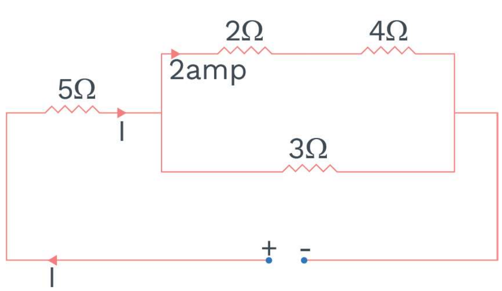
$P = I^2R$

$P \propto R \Rightarrow P_1 > P_2$



$$P = \frac{V^2}{R} \Rightarrow P \propto \frac{1}{R} \Rightarrow P_1 < P_2$$

Ex. In the given circuit if power dissipation across resistance of 2 Ω is 8 watt, then calculate power dissipation across resistance of 5 Ω.



Sol. $2 = \frac{3}{9} \times I \Rightarrow I = 6 \text{ Amp}$

For 2 Ω;

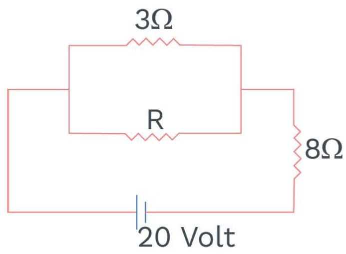
$P = I_1^2R \Rightarrow 8 = I_1^2 2 \Rightarrow I_1 = 2 \text{ Amp}$

Across 5 Ω,

$P = I^2R \Rightarrow P = 36 \times 5 = 180$ watt

Voltage applied = $IR = 6 \times 7 = 42$ volt

Ex. In the given circuit, total power dissipation is 40 watt then calculate resistance R.

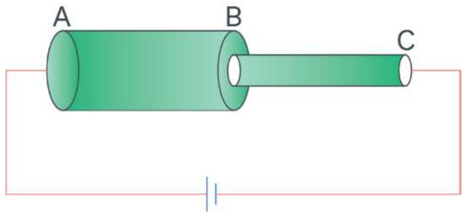


Sol. $P = \dfrac{V^2}{R_{eq}} \Rightarrow 40 = \dfrac{400}{R_{eq}} \Rightarrow R_{eq} = 10\,\Omega$

$$8 + \frac{3R}{3+R} = 10$$

$$\frac{3R}{3+R} = 2 \Rightarrow 3R = 6 + 2R \Rightarrow R = 6\,\Omega$$

Ex. Two copper wire AB and BC of equal length are connected in circuit as shown. If radius of cross-section of AB is twice the radius of cross-section of BC. Find ratio of power dissipation across AB and BC.

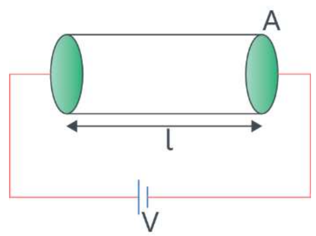


Sol. $P = I^2R = \dfrac{I^2\rho l}{A} \Rightarrow P \propto \dfrac{1}{r^2}$

$$\frac{P_{AB}}{P_{BC}} = \left(\frac{r_{BC}}{r_{AB}}\right)^2 = \frac{1}{4}$$

Ex. There is a solid cylindrical conductor of resistivity ρ. When potential difference is applied across the conductor then electric field inside the conductor is E. Calculate rate of heat produced in per unit volume of conductor.

Sol. Rate of heat produced, i.e.



$$\text{Power} = \frac{V^2}{R} = \frac{E^2 l^2 A}{\rho l} = \frac{E^2 A l}{\rho} = \frac{E^2 V'}{\rho}$$

$$\frac{\text{Power}}{\text{Volume}} = \frac{E^2 V'}{\rho V'} = \frac{E^2}{\rho}$$

Ex. Current passing through resistor of resistance R decreases linearly from I_0 to 0 in time of t_0. Calculate total amount of heat produced in resistor.

Sol. $H = \int I^2 R\, dt$

$$H = \int_0^{t_0} I_0^2 \left(1 - \frac{t}{t_0}\right)^2 R\, dt = I_0^2 R \int_0^{t_0} \left(1 - \frac{t}{t_0}\right)^2 dt$$

$$= I_0^2 R \int_0^{t_0} \left(1 + \frac{t^2}{t_0^2} - \frac{2t}{t_0}\right) dt = I_0^2 R \left[t + \frac{t^3}{3t_0^2} - \frac{t^2}{t_0}\right]_0^{t_0}$$

$$H = I_0^2 R \left[t_0 + \frac{t_0}{3} - t_0\right] = \frac{I_0^2 R t_0}{3}$$

$$H = \frac{I_0^2 R t_0}{3}$$

$$\text{or } H = \left(\frac{I_0}{\sqrt{3}}\right)^2 R t_0$$

Power Rating:

- Power rating of electrical appliance gives the information about maximum possible supply voltage across the electrical appliance and maximum possible consumed power by the appliance.

  e.g.,

  Power rating (100 W, 200 V)

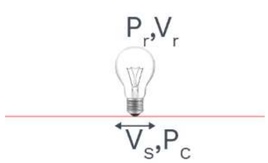
  $(V_S)_{max} = V_r = 200\text{ V}$ $\quad$ $(P_c)_{max} = P_r = 100\text{ W}$

  Power rating of electrical appliance gives the information about the resistance of appliance also.

  $$R = \frac{V_r^2}{P_r} = \frac{(\text{Rated voltage})}{\text{Rated power}}$$

- If rated voltage is not mentioned in power rating, then rated voltage is considered 220 volt.

BULB:



V_r – rated voltage, $\quad$ P_r – rated power,

V_s – supplied voltage, P_c – consumed power

- Resistance of bulb

  $$\left[R_b = \frac{V_r^2}{P_r}\right]$$

- Generally, V_r – same

  $$\left[R_b \propto \frac{1}{P_r}\right]$$

  e.g., $R_{60W} > R_{100W}$

Concept Reminder

- Rated power P_r and rated voltage V_r are used to find resistance of appliance.

  $$R = \frac{V_r^2}{P_r}$$

KEY POINTS

- Rated power
- Rated voltage

Rack your Brain

The charge flowing through a resistor R varies with time t as $Q = at - bt^2$ where a and b are positive constants. Then find total heat produced in R.

- Consumed power

$$P_C = \frac{V_S^2}{R} \Rightarrow P_C = \frac{V_S^2}{\frac{V_r^2}{P_r}} \Rightarrow P_C = \left(\frac{V_S}{V_r}\right)^2 P_r$$

Brightness of bulb $\propto P_C$

- If $V_S < V_r \Rightarrow P_C < P_r \Rightarrow$ Bulb, glows with less brightness.
  $V_S = V_r \Rightarrow P_C = P_r \Rightarrow$ Bulb, glows with maximum brightness
  $V_S > V_r \Rightarrow P_C > P_r \Rightarrow$ Bulb, get fuse.

Ex. A bulb of power rating (100 W, 200 volt) is used with supply of 110 V. Calculate amount of heat produced in 1 minute.

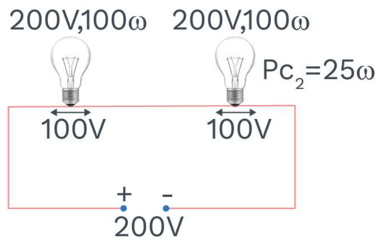
Sol. $H = P_C t = \left(\frac{V_S}{V_r}\right)^2 P_r t$

$$= \left(\frac{110}{220}\right)^2 \times 100 \times 60$$

$$H = \frac{6000}{4} = 1500 \text{ watt sec}$$

H = 1500 Joule

Ex. Calculate total power dissipation in given combination of bulb.



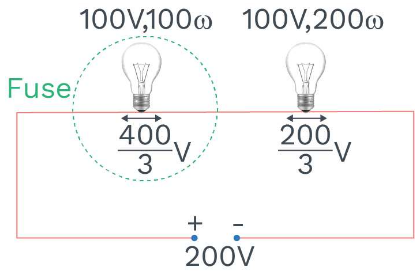
Sol. $R_1 = \frac{V_r^2}{P_r} = \frac{200 \times 200}{100} = 400\,\Omega$

$R_2 = 400\,\Omega$

$(P_C)_{total} = \frac{V_C^2}{R_{eq}} = \frac{V_C^2}{R_1 + R_2} = \frac{200 \times 200}{4800} = 50 \text{ watt}$

or $(P_C)_{total} = P_{C_1} + P_{C_2} = (25 + 25)W = 50 \text{ watt}$

Ex. Calculate total power dissipation in given combination of bulbs.



Sol. $R_1 = 100\ \Omega$

$$R_2 = \frac{100 \times 100}{200} = 50\ \Omega$$

$V_1 : V_2 :: 2 : 1$

$$V_1 = \frac{2}{3} \times 200 = 133\ W$$

Combination of Bulbs:

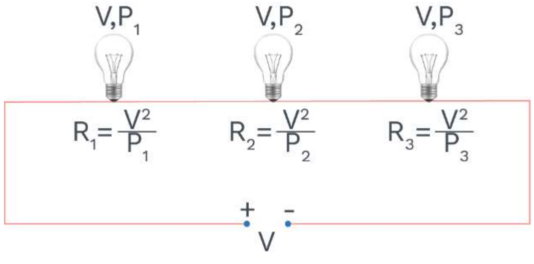
1. Series Combination:

$P_C = I_S^2 R$ (in series, supplied current I_S)

$$\left[P_C \propto R \propto \frac{1}{P_r}\right]$$

Brightness $\propto P_C$

- In series combination, bulb of less rated power glows with greater brightness.
- If bulb of same rated voltage but different rated power are connected in series with supply equal as rated, then;



$$R_T = R_1 + R_2 + R_3$$

$$\frac{V^2}{(P_C)_{total}} = \frac{V^2}{P_1} + \frac{V^2}{P_2} + \frac{V^2}{P_3}$$

$$\frac{1}{(P_C)_{total}} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3}$$

Concept Reminder

In series,

Effective power is

$$P_s = \left(\frac{1}{P_1} + \frac{1}{P_2}\right)^{-1}$$

In parallel,

Effective power is

$P_p = P_1 + P_2$

$(P_C)_{total} = P_{C_1} + P_{C_2} + P_{C_3}$ (in series, parallel or mixed combination)

- In the above combination, if n identical bulbs, rated power of each P, are used then; consumed for each

$$(P_C)_{total} = \frac{P}{n}$$

$$(P_C)_{each} = \frac{P}{n^2}$$

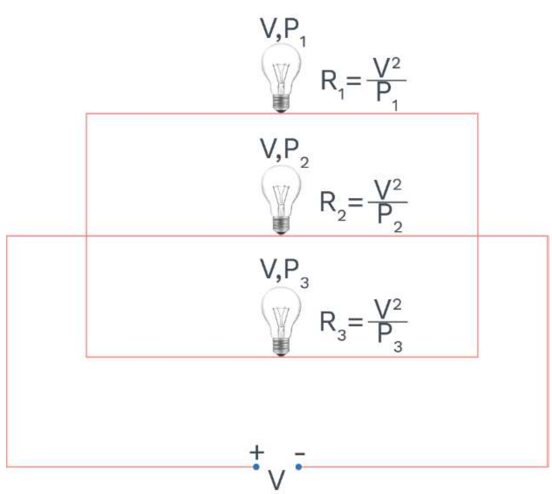
2. Parallel Combination:

$$P_C = \frac{V_S^2}{R} \text{ (in parallel, supplied voltage } V_S)$$

$$P_C \propto \frac{1}{R} \propto P_r$$

Brightness µ P_C

- IN parallel combination, bulb of greater rated power glows with greater brightness.
- If bulbs of same rated voltage but different rated power are used in parallel with supply equal as rated voltage, then;



$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{(P_C)_{total}}{V^2} = \frac{P_1}{V_2} + \frac{P_2}{V_2} + \frac{P_3}{V_2}$$

$$(P_C)_{total} = P_1 + P_2 + P_3$$

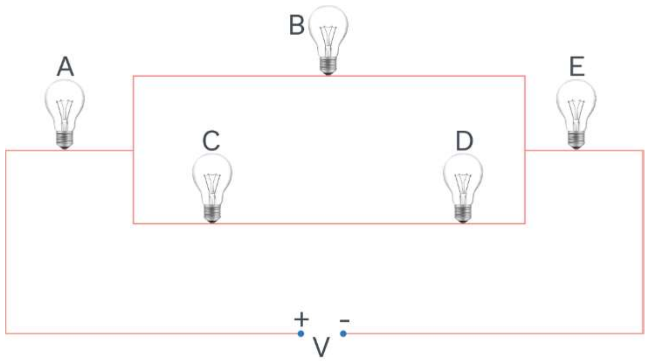
- If n identical bulb rated power of each P are used in above combinations:

  $(P_C)_{total} = nP$

Consumed for each:

  $(P_C)_{each} = P$

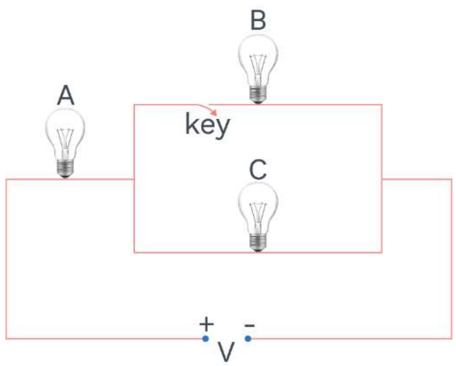
Ex. In the given combination, all bulbs are identical compare their brightness.



Sol. $P_C = I^2 R \Rightarrow P_C \propto I_S^2$

$\Rightarrow \quad (B)_A = (B)_E > (B)_B > (B)_C = (B)_D$

Ex. In the given combination, if key is opened then predict effect on brightness of bulb A and C. (all bulbs are identical)



Sol. When key is not open,

$$R_{eq} = \frac{3R}{2} \Rightarrow I_A = \frac{2V}{3R} = \frac{0.66V}{R}$$

$$I_C = \frac{V}{3R} = \frac{0.33\,V}{R}$$

When key is opened:

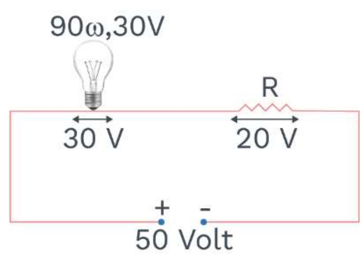
$R_{eq} = 2R$

$$\Rightarrow \quad I_A = \frac{V}{2R} = \frac{0.5\,V}{R} \Rightarrow \text{Brightness of A decreases}$$

$$I_C = \frac{V}{2R} = \frac{0.5\,V}{R} \Rightarrow \text{Brightness of C increases}$$

Although, final brightness of A and C are same.

Ex. A bulb of power rating 90 W, 30 V is to be used with supply of 50 volt. Calculate resistance which should be connected in series of bulb.



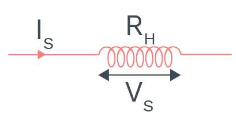
Sol. $R = \frac{20}{I} = \frac{20}{3}\Omega$

or $\quad R_{bulb} = \frac{30 \times 30}{90} = 10\,\Omega$

$30 : 20 :: 10 : R$

$$\frac{3}{2} = \frac{10}{R} \Rightarrow R = \frac{20}{3}\Omega$$

Heater:



Concept:

Required heat = Heat produced by heater

$$mS\Delta Q \text{ or } mL = \frac{V_S^2}{R_H}t \text{ or } I_S^2R_Ht \text{ or } V_SI_St$$

For water,

Concept Reminder

Time taken by 2 heaters to produce equal amount of heat are t_1 and t_2. If these heaters are used combinedly then time taken is as;

In series,

$\Rightarrow t_{series} = (t_1 + t_2)$

In parallel,

$$\Rightarrow t_{parallel} = \frac{t_1t_2}{(t_1 + t_2)}$$

$$S_w = \frac{1\text{ cal}}{\text{gm}\,°\text{C}} = \frac{418\text{ J}}{\text{gm}\,°\text{C}} \simeq \frac{4.2\text{ J}}{\text{gm}\,°\text{C}}$$

$$S_w = 1000\frac{\text{cal}}{\text{kg}\,°\text{C}} = \frac{4180\text{ J}}{\text{kg}\,°\text{C}} \simeq 4200\frac{\text{I}}{\text{kg}\,°\text{C}}$$

- To produce equal amount of required heat, time taken by different heaters is directly proportional to their resistances (in case of same supply voltage or by default)

$$mS\Delta Q = \frac{V_S^2}{R_H}t \Rightarrow \frac{t}{R_H} = \text{same}$$

$t \propto R_H \propto$ length of coil of heater.

- Time taken by 2 heaters to produce equal amount of heat are t_1 and t_2. If these heaters are used combinedly then time taken is as;
In series $\Rightarrow t_{series} = (t_1 + t_2)$

In parallel $\Rightarrow t_{parallel} = \dfrac{t_1t_2}{(t_1 + t_2)}$

Ex. Amount of heat produced by a heater in a time t is H. If coil of heater is cut into n equal parts and these parts are used in parallel with same source, then calculate amount of heat produced by the combination in time t.

Sol. $H = \dfrac{V^2t}{R}$

$$H' = \frac{V^2t}{R'} = \frac{V^2n^2t}{R} = Hn^2$$

$$H' = n^2H$$

Ex. Resistance of heater coil is 60 W current passing through heater is 7 ampere. Calculate change in temperature of water of 42 litre in per minute.

Sol. $ms\Delta Q = I_s^2R_Ht$

$$42 \times 4200\,\Delta Q = 49 \times 60 \times 60$$

$$\Delta Q = \frac{49 \times 6 \times 6}{42 \times 42} = 1°\text{C or } 1\text{ K}$$

Ex. A heater of power 836 watt is used to raise the temperature of H_2O of 1 lt. from 10°C to 40°C. Calculate time taken.

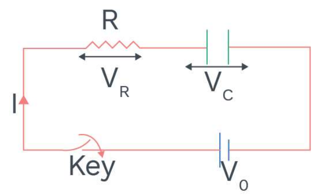
Sol. $ms\Delta Q = Pt$

$$1 \times 4180 \times 30 = 836 \times t$$

$$t = \frac{4280 \times 30}{836} = 150 \text{ sec}$$

RC-DC Circuit (Circuit of charging and discharging of capacitor):

1. During charging of capacitor:



Concept Reminder

During charging of a capacitor, charge varies with time as

$$q = Q_0(1 - e^{-t/RC})$$

At t = 0 or just after closing the key

$$V_C = 0 \Rightarrow V_R = V_0 = I_0 R$$

As $t \uparrow \Rightarrow q \uparrow \Rightarrow V_C \uparrow \Rightarrow V_R \downarrow \Rightarrow I \downarrow$

$$t = \infty;\ q = Q_0 = CV_0$$

$$V_C = V_0 = \frac{Q_0}{C};\ V_R = 0$$

At any instant current I and charge q

$$V_R + V_C = V_0$$

$$IR + \frac{q}{C} = V_0$$

$$\Rightarrow IR = V_0 - \frac{q}{C} \Rightarrow I = \frac{CV_0 - q}{R}$$

$$\frac{dq}{dt} = \frac{(Q_0 - q)}{RC}$$

$$\int_0^q \frac{dq}{Q_0 - q} = \int_0^t \frac{dt}{RC}$$

$q = Q_0(1 - e^{-t/RC})$ (Exponential increasing)

Definitions

Capacitive time constant is the time in which charging and discharging, or capacitor get completed 63%.

$$q = Q_0(1 - e^{-t/\tau})$$

$\tau = R_C \rightarrow$ capacitive time constants

At $t = \tau$

$$q = Q_0(1 - e^{-t/\tau}) = Q_0(1 - e^{-1})$$

$$q = Q_0(1 - 0.37)$$

$$q = 0.63\,Q_0$$

i.e., capacitive time constant is the time in which charging and discharging, or capacitor get completed 63%

- Capacitive time constant (τ) s named mean life of RC = DC circuit.
- **Half lifetime ($t_{1/2}$):**
  Time in which charging and discharging of capacitor get completed 50% is named half lifetime of RC-DC circuit.

$$t_{\frac{1}{2}} = \ln(2)\,\tau = 0.693\,\tau$$

Potential drop across capacitor

$$V_C = \frac{q}{C} = \frac{Q_0}{C}(1 - e^{-t/\tau})$$

$V_C = V_0(1 - e^{-t/\tau})$ (Exponential increasing)

Potential drop across resistor

$$V_R = V_0 - V_C$$

$$V_R = V_0 - V_0(1 - e^{-t/\tau})$$

$\boxed{V_R = V_0 e^{-t/\tau}}$ (Exponential decreasing)

Current through resistor

$$I = \frac{V_R}{R} = \frac{V_0 e^{-t/\tau}}{R}$$

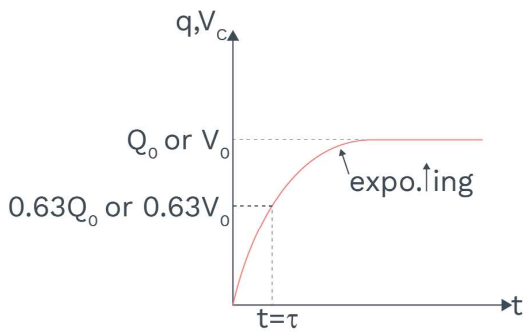
$I = I_0 e^{-t/\tau}$ (Exponential decreasing)

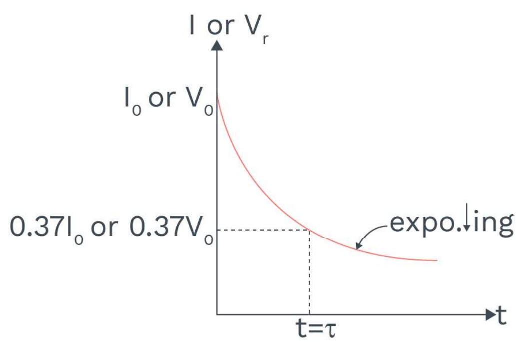
At $t = \tau$

$$I = I_0 e^{-1} \Rightarrow I = 0.37\,I_0$$

Definitions

Time in which charging and discharging of capacitor get completed 50% is named half lifetime of RC-DC circuit.





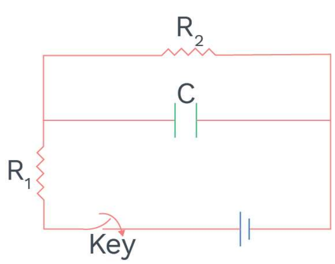
Behaviour of Capacitor in DC circuit:

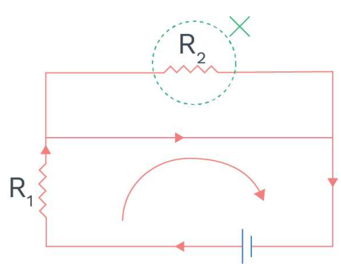
Case-I: At t = 0 (just after key closing)

$$I = I_0e^{-t/RC} = I_0e^{-0}$$

$$I = I_0$$

Initially just after key closing, capacitor does not offer any opposition in flow of electric current i.e.., behaves as short circuit element.



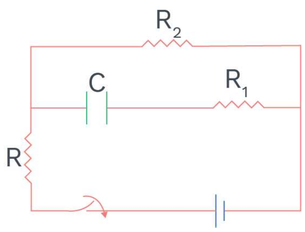


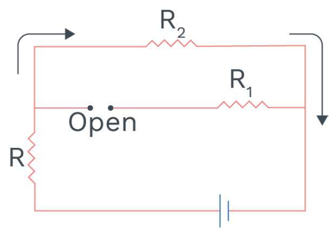
Case-II: At t = ∞ (practically after some time key closing or at steady state)

$$I = I_0e^{-t/RC} = I_0e^{-\infty} = 0$$

- Practically, after some time key closing, capacitor is considered fully charged (steady state).

At steady state capacitor does not allow current flow, i.e., behaves as open circuit.
At steady state, resistance n capacitor containing branch is meaningless (by default steady state is considered)

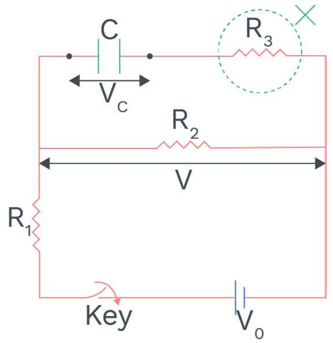




Concept Reminder

In RC circuit:
- At t = 0, capacitor behaves short circuited.
- At t = ∞, capacitor behaves open circuited.

Ex. In the given circuit, calculate charge of capacitor at steady state.

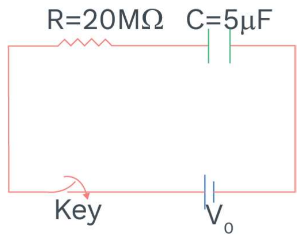


Sol. $V = V_C$

$$V_C = V = \frac{V_0R^2}{R_1 + R_2}$$

$$\text{Charge on capacitor} = CV_C = \frac{CV_0R^2}{R_1 + R_2}$$

Ex. In given circuit, key is closed at t = 0. Calculate time at which potential drop across resistor and capacitor.

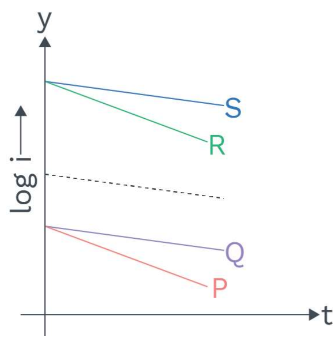


Sol. $V_R = V_C = \frac{V_0}{2}$, i.e.; half life

$$t_{1/2} = 0.693\,\tau = 0.693 \times 20 \times 10 \times 5 \times 10^{-6}$$

$$t_{1/2} = 69.3 \text{ sec}$$

Ex. In an RC circuit while charging, the graph of log i v/s time is as shown by the dotted line, where i is the current. When the value of R is doubled, which of the solid curve best represents the variation of log i.



Sol. $I = I_0 e^{-t/RC}$

$$\log i = \log i_0 - \frac{t}{RC}$$

R increasing,

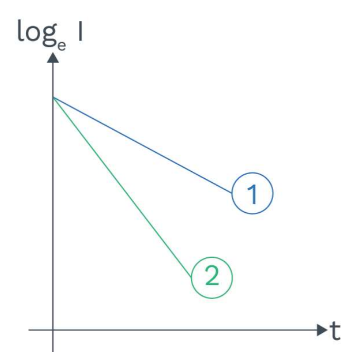
$$i_0 = \frac{V_0}{R} \Rightarrow I_0 \text{ decreasing}$$

$$m = \frac{1}{RC} \Rightarrow m \text{ decreasing}$$

Ex. In RC-DC circuit during charging of capacitor, current varies with time then variation of $\log_e I$ with t is as line (1). If one of the parameters out of V_0, R and C is changed keeping another two constant then variation of $\log_e I$ with t is as line (2) then predict this change.

(1) R increasing (2) R decreasing

(3) C increasing (3) C decreasing



Sol. $I = I_0 e^{-t/RC}$

$$\log_e I = \log_e I_0 - \frac{t}{RC}$$

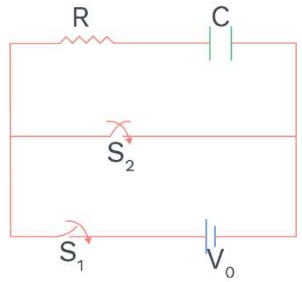
$$y = C - mx$$

$$m = \frac{1}{RC} \Rightarrow m \text{ increasing } \rightarrow C \text{ decreasing}$$

Intercept is constant because

$$I_0 \Rightarrow \text{constant},\ I_0 = \frac{V_0}{R_0}$$

2. During Discharging of capacitor:



Concept Reminder

- In discharging of capacitor, the charge varies with time as

$$q = Q_0 e^{-t/RC}$$

- When switch S_1 is closed keeping S_2 opened then capacitor is being charged.
- After steady state switch S_1 is opened and S_2 is closed then in new circuit capacitor gets discharged through resistor.
- During discharging, capacitor behaves as source so potential drop across resistor is equal as potential of capacitor at every moment.

During discharging:

At $t = 0$

$$Q_0 = CV_0 \Rightarrow V_C = V_0 = Q_0$$

$$V_R = V_0 = I_0 R$$

As $t\uparrow;\ q\downarrow;\ V_C\downarrow;\ V_R\downarrow;$ i.e.; $I\downarrow$

$$t = \infty;\ q = 0;\ V_C = 0;\ V_R = 0;\ I = 0$$

At any instant current I and charge q

$$V_R = V_C$$

$$IR = \frac{q}{C} \Rightarrow I = \frac{q}{RC}$$

$$\frac{-dq}{dt} = \frac{q}{RC}$$

$$\int_{Q_0}^{q} \frac{dq}{q} = -\int_{0}^{t} \frac{dt}{RC}$$

$q = Q_0 e^{-t/RC}$ (Exponential decreasing)

$q = Q_0 e^{-t/\tau}$

At $t = \tau$

$$q = Q_0 e^{-1} \Rightarrow q = 0.37\,Q_0 = \frac{Q_0}{e}$$

$$V_C = V_0 e^{-t/RC};\ V_R = V_0 e^{-t/RC};\ I = I_0 e^{-t/RC}$$

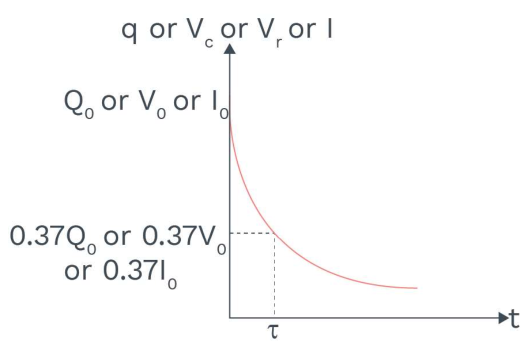
At $t = 0$

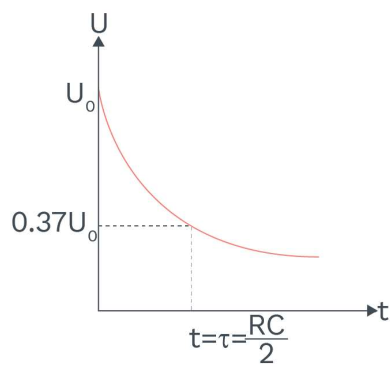
$$\Rightarrow U_0 = \frac{Q_0^2}{RC} \Rightarrow U = U_0 = \frac{Q_0^2}{2C} e^{2t/RC}$$

$$U = U_0 e^{-2t/RC} = U_0 e^{-t/\tau}$$

Time constant $\tau = \frac{RC}{2}$(mean life)

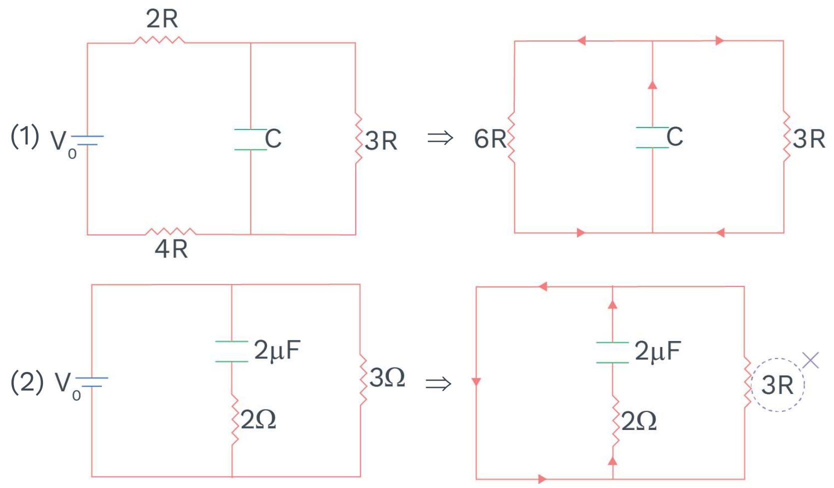
$$t_{1/2} = 0.693\left(\frac{RC}{2}\right)$$





- During discharging of capacitor, amount of heat developed through the resistor through which capacitor get discharged completely is equal as energy stored in capacitor
- **Calculation of equivalent time constant:**
  Consider the battery. short-circuited.
  Calculate equivalent resistance, equivalent capacitance and then equivalent time constant.

Ex. Calculate equivalent time constant for given circuit.



Sol. (1) $R_{eq} = 2R$

When charging,

$\tau = RC = 2RC$

When discharging,

$\tau_{eq} = 3RC$

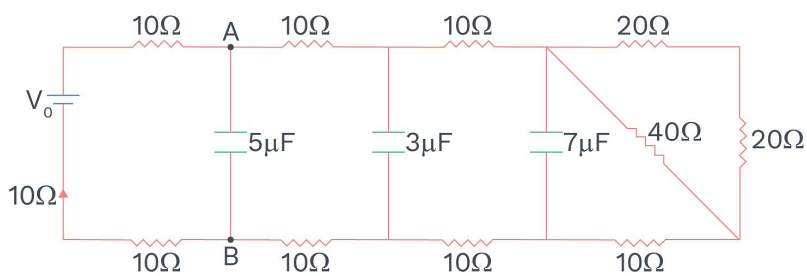
(2) $R_{eq} = 2\ \Omega \Rightarrow \tau_{eq} = R_{eq}C = 2 \times 2 \times 10^{-6}$

When charging, $\tau_{eq} = 4\,\mu\,\text{sec}$

When discharging,

$\tau_{eq} = R_{eq}C = 5 \times 2\,\mu\,\text{sec} = 10\,\mu\,\text{sec}$

Ex. Find charge on 5 μF capacitor.



Sol. $Q = C(V_A - V_B)$

$$I = \frac{100}{60 + 40} = 1 \text{ Amp}$$

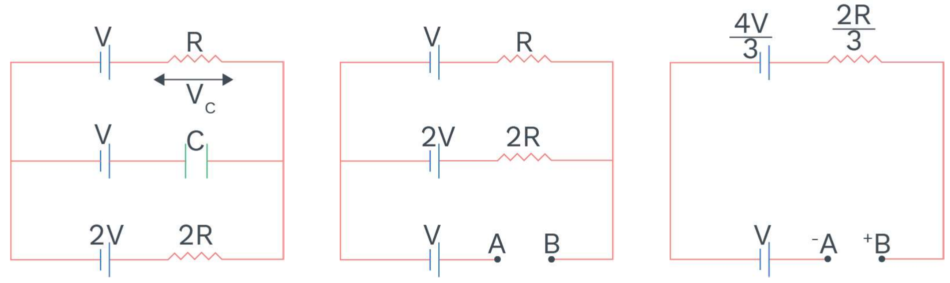
By using KVL,

$100 - 10 = V_A \Rightarrow V_A = 90$ V

$10 + 10 = V_B \Rightarrow V_B = 20$ V

$Q = 5(90 - 20) = 350\ \mu$Cb

Ex. In given circuit calculate potential drop across capacitor at steady state.

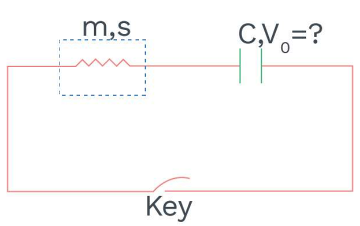


Sol. $V_C = V_B - V_A = \frac{V}{3}$

$V_A = V + \frac{4V}{3} = V_B$

$V_A - V_B = V - \frac{4V}{3} = -\frac{V}{3}$

Ex. A capacitor of capacitance C is fully charged. This capacitor is being discharged through a resistor coil which is embedded in thermally insulated block of mass m and specific heat of S. If temperature of block is increased by ΔQ till complete discharging of capacitor. Calculate initial potential of capacitor.



Sol. Required heat = heat produced across resistor coil during discharging of capacitor.

$$ms\Delta Q = \frac{1}{2}CV_0^2$$

$$V_0 = \sqrt{\frac{2ms\Delta Q}{C}}$$

Van-de Graaff Generator:

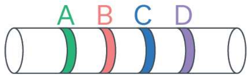
- **Application:** To produce high potential difference and to accelerate charge particles. (Electrons, protons, ions)
- **Principle:**

1. When a hollow conducting body is given charge then charge get distributed uniformly on outer surface.

2. Corona discharge: Due to high charge density at sharp ends, charge leaks out and ionized surrounding, which is named as corona discharge.

Colour Coding of Carbon resistor:

Colour code for resistors: A resistor is a current device made of specific value of resistance. The value of resistance used in electrical and electronic circuits vary over a very wide range. A colour code is used to indicate the value of resistance. A resistor has usually four concentric rings or bands A, B, C and D of different colours. The colours of first two bands A and B indicate the first two significant figures of the resistance in ohm, while the colour of third band C indicate the decimal multiplier. The colour of fourth ring or D (which is either silver or gold) tells the tolerance. Sometimes, only three colour bands A, B and C are marked.



COLOR NAME	VALUE AS FIGURE	AS DECIMAL MULTIPLIER	TOLERANCE ±
Black	0	1	± 20%
Brown	1	x 10^1	± 1%
Red	2	x 10^2	± 2%
Orange	3	x 10^3	-
Yellow	4	x 10^4	± 5%
Green	5	x 10^5	± 0.5%
Blue	6	x 10^6	± 0.25%
Violet	7	x 10^7	± 0.1%
Grey	8	x 10^8	± 0.05%
White	9	x 10^9	± 10%
Golden	-	x 10^{-1}	± 5%
Silver	-	x 10^{-2}	± 10%

Ex. Colour code for carbon resistance is as Red, Orange, Yellow, Gold then value of resistance is.

Sol. $R = 23 \times 10^4\ \Omega \pm 5\%$

Ex. Colour code for carbon resistance is as Blue, Black, Brown, Silver then resistance is.

Sol. $R = 60 \times 10^1\ \Omega \pm 10\% = 600\ \Omega \pm 10\%$

Ex. Carbon resistance is $R = 4700\ \Omega \pm 5\%$ then its colour code is.

Sol. $R = 47 \times 10^2 \pm 5\%$

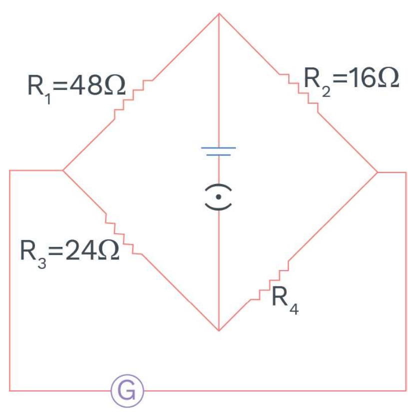
Yellow violet red gold.

Ex. Carbon resistance is $R = 950\ \Omega \pm 10\%$ then its colour code is.

Sol. $R = 95 \times 10^1\ \Omega \pm 10\%$

Code → White green, brown, silver.

Ex. A Wheatstone bridge circuit has been set up as shown. The resistor R_4 is an ideal carbon resistance (tolerance = 0%) having bands of colours black, yellow and brown marked on it. The galvanometer, in this circuit, would show a null point when another ideal carbon resistor X is connected across R_4, having bands of colours:



(1) Black, brown, black, is put in parallel with R_4

(2) Black, brown, brown, is put in parallel with R_4

(3) Brown, black, brown, is put in series with R_4

(4) Black, brown, black, is put in series with R_4

Sol. Resistance R_4 = black, yellow, brown = $40\ \Omega$

Required resistance X with R_4 to balance bridge $X = 10\ \Omega$

$\therefore$ X = black, brown, brown (in parallel)

EXAMPLES

Q1 **The current through a wire depends on time as $i = i_0 + \alpha \sin \pi t$, where $i_0 = 10A$ and $\alpha = \frac{\pi}{2}A$. Find the charge crossed through a section of the wire in 3 seconds, and average current for that interval.**

Sol: $i = i_0 + \alpha \sin \pi t.$

$$\frac{dq}{dt} = 10 + \frac{\pi}{2}\sin \pi t$$

$$q = \int_0^3 \left(10 + \frac{\pi}{2}\sin \pi t\right)dt = 10 \times 3 + \frac{\pi}{2} \times \frac{1}{\pi} \times 2 = 31C$$

Average current $= \frac{q}{\Delta t} = \frac{31}{3}A$

Q2 **A cylindrical conducting wire of radius 0.2 mm is carrying a current of 20 mA. (a) How many electrons are transferred per second between the supply and the wire at one end? (b) Write down the current density in the wire.**

Sol: (a) $\frac{20 \times 10^{-3}}{e} \rightarrow$ no of electrons passing per second

$$= \frac{20 \times 10^{-3}}{1.6 \times 10^{-19}} = \frac{2 \times 10^{17}}{1.6} = 1.25 \times 10^{17}.$$

(b) $$j = \frac{20 \times 10^{-3}}{\pi\left(0.2 \times 10^{-3}\right)^2} = \frac{1}{2\pi} \times 10^6 \ A/m^2.$$

Q3 **If copper wire is stretched to make it 0.1% longer, what is the percentage change in its resistance?**

Sol: Resistance $R = \dfrac{\rho\ell}{A}$

By partial differentiation

$\Rightarrow \quad \dfrac{\Delta R}{R} = \dfrac{\Delta \ell}{\ell} - \dfrac{\Delta A}{A}$ ...(1)

$\because \quad \rho$ = Constant

$\because$ Volume of wire remains constant

$A \times \ell$ = Constant

By partial differentiation

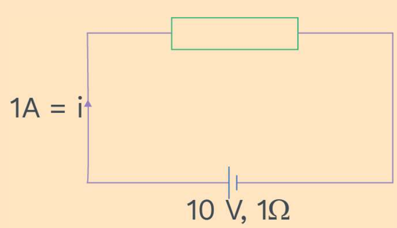
$\Rightarrow \quad \dfrac{\Delta A}{A} + \dfrac{\Delta \ell}{\ell} = 0$ ...(2)

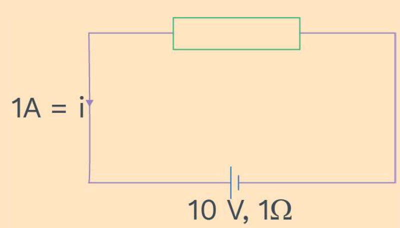
By equation (1) and (2)

We get $\dfrac{\Delta R}{R} = 2\dfrac{\Delta \ell}{\ell}$

(% change in R) = 2 (% change in length) = 0.2%

Q4 **In following diagram boxes may contain resistor or battery or any other element then determine in each case**





(a) E.m.f. of battery
(b) Battery is acting as a source or load
(c) Potential difference across each battery
(d) Power input to the battery or output by the battery.
(e) The rate at which heat is generated inside the battery.

(f) The rate at which the chemical energy of the cell is consumed or increased.
(g) Potential difference across box
(h) Electric power output across box.

Sol:
(a) In each case E.M.F. = 10 V

(b) For case (A), battery is providing current to the circuit hence acting as source.
For case (B) battery is taking current from external source, hence acting as a load.

(c) For case (A) $V_A = E - ir = 10 - 1 \times 1 = 9V$
For case (B) $V_B = E + ir = 10 + 1 \times 1 = 11V$

(d) For case (A) $P_A = V_A i = 9 \times 1 = 9W$
For case (B) $P_B = V_B i = 11 \times 1 = 11W$

(e) For case (A) $\frac{H_A}{t} = i_A^2 r_A = (1)^2 \times 1 = 1W$

(f) For case (A) $P_A = E_A i = 10 \times 1 = 10\,W$
For case (B) $P_B = E_B i = 10 \times 1 = 10\,W$

(g) For case (A) $V_{Box} = V_A = 9\,V$
For case (B) $V_{Box} = V_B = 11\,V$

(h) For case (A) $P_{Box} = -9 \times 1 = -9\,W$
For case (B) $P_{Box} = 11 \times 1 = 11\,W$

Q5 **1 kW, 220 V electric heater is to be used with 220 V D.C. supply.**
(a) What is the current in the heater.
(b) What is its resistance.
(c) What is the power dissipated in the heater.
(d) How much heat in calories is produced per second.
(e) How many grams of water at 100°C will be converted per minute into steam at 100°C with the heater. (Latent heat of vaporization of water = 540 cal/g)] [J = 4.2 J/cal].

Sol:
(a) $i = \frac{P}{V} = \frac{1000}{220} = \frac{50}{11} = 4.55A$

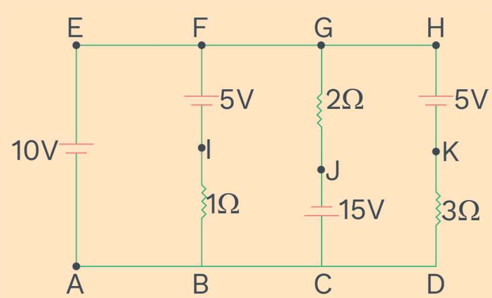
(b) $R = \frac{V^2}{P} = \frac{(220)^2}{1000} = \frac{22 \times 11}{5} = 48.4\Omega$

(c) P = 1 W

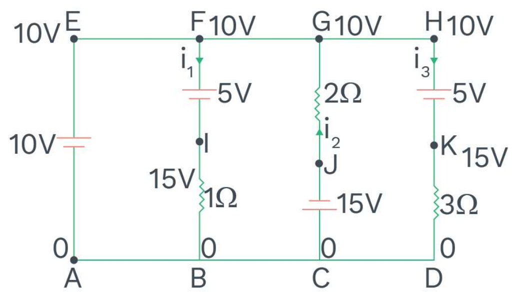
(d) $H = \frac{Q}{J} = \frac{1000}{4.2} = 240 \text{ cal / sec}$

(e) $tH = mL \Rightarrow m = \frac{Ht}{L} = \frac{240 \times 60}{540} = \frac{80}{3}$ gm.

Q6 **In following circuit potential at point 'A' is zero then determine**



(a) Potential at each point
(b) Potential difference across each resistance
(c) Identify the batteries which act as a source
(d) Current in each battery
(e) Which resistance consumes maximum power
(f) Which battery consume or gives maximum power.



(a) $V_A = V_B = V_C = V_D = 0 \quad V_E = V_F = V_G = V_H = 10V \quad V_1 = 10 + 5 = 15V$

$V_J = 15V \quad V_K = 10 + 5 = 15V$

(b) $V_{BI} = 15V, V_{JG} = 5V, V_{KD} = 15V$

(c) Each battery is supplying the current hence each battery is acting as a source.

(d) Let current through BF, CG, HP is respectively i_1, i_2, i_3

$$i_1 = \frac{15}{1} = 15\,\text{amp}\downarrow, \quad i_2 = \frac{5}{2} = 2.5\,\text{amp}\uparrow, \quad i_3 = \frac{15}{3} = 5\,\text{amp}\downarrow$$

For 10 V Battery, current = $i_1 - i_2 + i_3 \quad = 15 - 2.5 + 5 \quad = 17.5\,\text{amp}\uparrow$

(e) $$P_{1\Omega} = \frac{V^2}{R} = \frac{(15)^2}{1} = 225\,W, \quad P_{2\Omega} = \frac{(5)^2}{2} = 12.5\,W$$

$$P_{3\Omega} = \frac{(15)^2}{3} = \frac{225}{3} = 75\,W$$

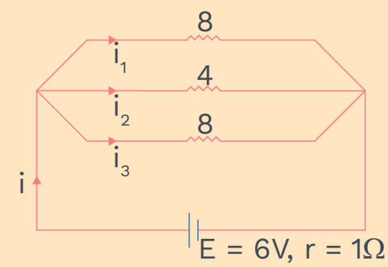
Hence, 1Ω resistance consumes the maximum power.

(f) $P_I = E_1 i_1 = 10 \times 17.5 = 175\,W, \quad P_{II} = E_2 i_1 = 5 \times 15 = 75\,W$

$P_{III} = E_3 i_2 = 15 \times 2.5 = 37.5\,W, \quad P_{IV} = E_4 i_3 = 5 \times 5 = 25\,W$

Hence left most battery consume maximum power.

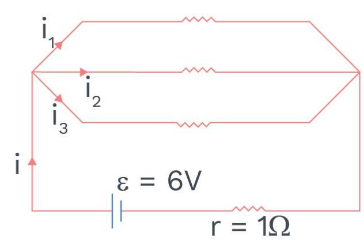
Q7 **In given circuit determine**



(a) Equivalent resistance (Including internal resistance).
(b) Current I, i_1, i_2 and i_3
(c) Potential difference across battery and each resistance
(d) The rate at which the chemical energy of the cell is consumed
(e) The rate at which heat is generated inside the battery
(f) Electric power output
(g) Which resistance consumes maximum power?
(h) Power dissipated across 4Ω resistance

Sol: (a) $R_{eq} = 2 + 1 = 3\Omega$

(b) $i = \frac{\varepsilon}{R_{eq}} = \frac{6}{3} = 2A$



$i_1 + i_2 + i_3 = 2$, $i_1 : i_2 : i_3 = \frac{1}{8} : \frac{1}{4} : \frac{1}{8} = 1 : 2 : 1$, $i_1 = i_3 = \frac{1}{2}A$, $i_2 = 1A$

(c) V_{across} battery $= \varepsilon - ir = 6 - 2 \times 1 = 4V$, V_{across} each cell $= 4V$

(d) P of the cell consumed = εI = 12 W

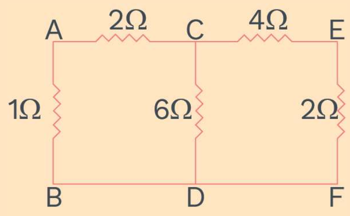
(e) P heat generated in cell P = i^2r = 4 W

(f) $P_{output} = \varepsilon i - i^2 r = 8\,W$

(g) In parallel, $P \propto \frac{1}{R}$ 4Ω consumes Max power

(h) $P_{4\Omega} = \frac{v^2}{R} = \frac{4 \times 4}{4} = 4\,W$

Q8 **Find the equivalent resistance of the circuit given in figure between the following point:**



(i) A and B **(ii) C and D** **(iii) E and F**
(iv) A and F **(v) A and C**

Sol:

(i) $R_{AB} = \frac{5 \times 1}{5+1} = \frac{5}{6}\Omega$

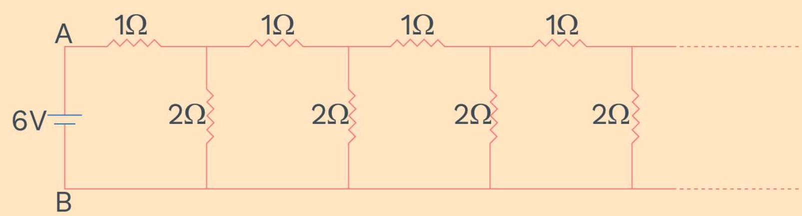
(ii) $R_{CD} = \frac{\left(\frac{3 \times 6}{3+6}\right) \times 6}{\left(\frac{3 \times 6}{3+6}\right) \times 6} = \frac{2 \times 6}{2+6} = 1.5\,\Omega$

(iii) $R_{EF} = \frac{\left[\left(\frac{3 \times 6}{3+6}\right) + 4\right] \times 2}{\left(\frac{3 \times 6}{3+6}\right) + 4 + 2} = \frac{(2+4)}{2+4+2} \times 2 = \frac{3}{2} = 1.5\,\Omega$

(iv) $R_{AF} = R_{AB} = \frac{5}{6}\Omega$

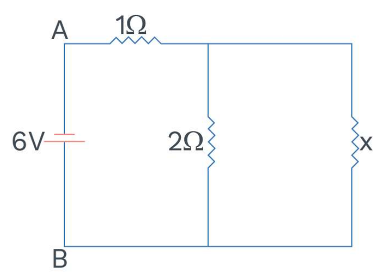
(v) $R_{AC} = \frac{\left(\frac{6 \times 6}{6+6} + 1\right) 2}{\left(\frac{6 \times 6}{6+6} + 1\right) + 2} = \frac{(3+1)2}{(3+1)+2} = \frac{4}{3}\Omega$

Q9 **An infinite ladder network of resistance is constructed with 1Ω and 2Ω resistance, as shown in figure.**



(i) Show that the effective resistance between A and B is 2Ω.
(ii) What is the current that passes through the 2Ω resistance nearest to the battery?

Sol: (i) Let R_{AB} = x. Then, we can break one chain and connect a resistance of magnitude x in place of it.
Thus, the circuit remains as shown in figure.



Now, 2 Ω and x are in parallel. So, their combined resistance is $\frac{2x}{2+x}$.

or $R_{AB} = 1 + \frac{2x}{2+x}$

But R_{AB} is a assumed to x. Therefore,

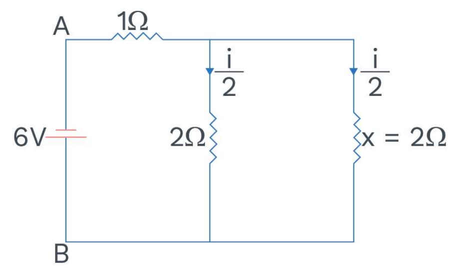
$$x = 1 + \frac{2x}{2+x}$$

Solving this equation, we get

X = 2 Ω

Hence proved.

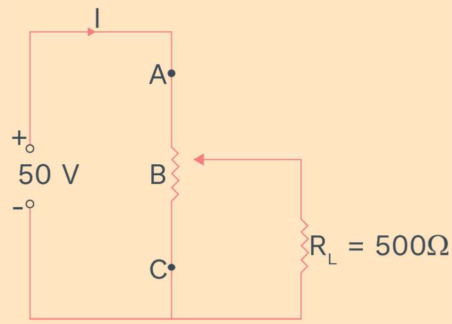
(ii) Net resistance of circuit $R = 1 + \frac{2 \times 2}{2+2} = 2\Omega$



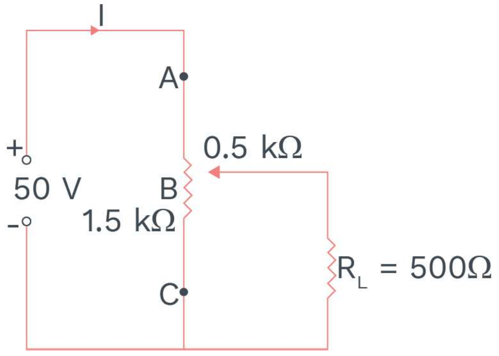
$\therefore$ Current through battery $i = \frac{6}{2} = 3A$

This current is equally distributed in 2 Ω and 2 Ω resistances. Therefore, the desired current is $\frac{i}{2}$ or 1.5 A.

Q10 **As shown in figure a variable rheostat of 2 kΩ is used to control the potential difference across 500 ohm load. (i) If the resistance AB is 500 Ω, what is the potential difference across the load? (ii) If the load is removed, what should be the resistance at BC to get 40 volt between B and C?**



Sol: (i)

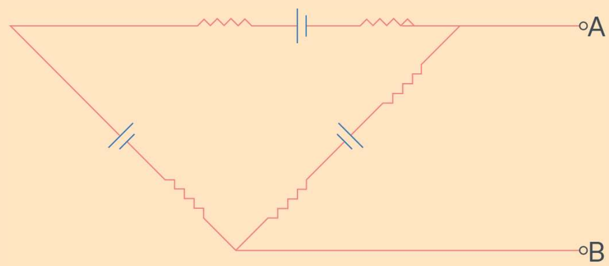


As $V \propto R$, $V_2 = V_{BC} = \left(\frac{R_{BC}}{R_{AB} + R_{BC}}\right)V = \left(\frac{\frac{1.5 \times 0.5}{1.5 + 0.5}}{0.5 + \frac{1.5 \times 0.5}{1.5 + 0.5}}\right)50$

$$= \left(\frac{0.75}{1 + 0.75}\right)50 = \frac{150}{7} = 21.43\,V$$

(ii) $V_{BC} = 40\,V \Rightarrow V_{BC} = \left(\frac{R_{BC}}{R_{BC} + R_{AC}}\right)V \Rightarrow 40 = \frac{R_{BC}}{2000}50 \Rightarrow R_{BC} = 1600\,\Omega$

Q11 **In the circuit shown all five resistors have the same value 200 ohms and each cell has an emf 3 volts. Find the open circuit voltage and the short circuit current for the terminals A and B.**



Sol:

$$E_{eq} = \frac{\frac{6}{600} + \frac{3}{400}}{\frac{1}{600} + \frac{1}{400}} = \frac{12 + 9}{2 + 3} = \frac{21}{5} = 4.2\,V$$

$$\frac{1}{r_{eq}} = \frac{1}{600} + \frac{1}{400} = \frac{2 + 3}{1200} = \frac{5}{1200} \Rightarrow r_{eq} = 240\,\Omega$$

Short circuit current in AB

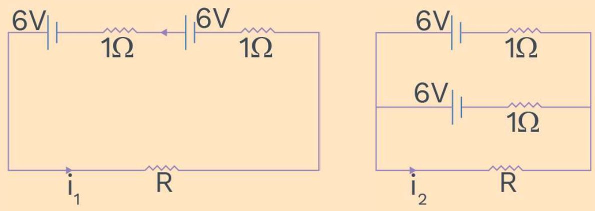
$$i = \frac{E_{eq}}{r_{eq}} = \frac{\frac{21}{5}}{240} = \frac{21}{5 \times 240} = 17.5 \times 10^{-3}\text{ amp}$$

$\Rightarrow$ i = 17.5 mA (from B to A)

Q12 **Find the value of i_1 / i_2 in figure if**

(a) R = 0.1 Ω **(b) R = 1 Ω** **(c) R = 10 Ω**

Note from your answer that in order to get more current from a combination of two batteries they should be joined in parallel if the external resistance is small and in series if the external resistance is large as compared to the internal resistances.



Sol: (a) $i_1 = \frac{12}{2+0.1} = \frac{12}{2.1}$, $\quad i_2 = \frac{6}{0.5+0.1} = \frac{6}{0.6} = 10\text{ A}$

$\Rightarrow \quad \frac{i_1}{i_2} = \frac{12}{2.1 \times 10} = 0.57$

(b) $i_1 = \frac{12}{2+1} = 4\text{ A}$, $\quad i_2 = \frac{6}{0.5+1} = 4\text{ A}$

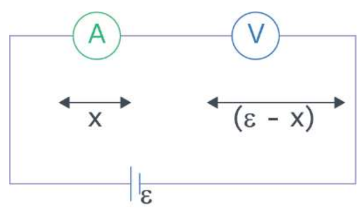
$\Rightarrow \quad \frac{i_1}{i_2} = 1$

(c) $i_1 = \frac{12}{2+10} = \frac{12}{12} = 1\text{A}$ & $i_2 = \frac{6}{0.5+10} = \frac{6}{10.5}$

$\Rightarrow \quad \frac{i_1}{i_2} = \frac{1 \times 10.5}{6} = 1.75.$

Q13 **An ammeter and a voltmeter are connected in series to a battery with an emf ε = 6.0 V. When a certain resistance is connected in parallel with the voltmeter, the reading of the voltmeter decrease η = 2.0 times, whereas the reading of the ammeter increases the same number of times. Find the voltmeter reading after the connection of the resistance.**

Sol:



$$\frac{(\varepsilon - x)}{\eta} + \eta x = \varepsilon$$

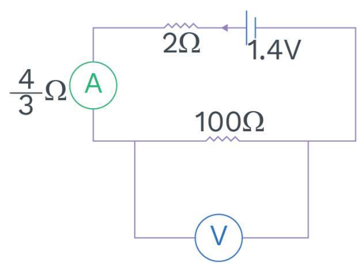
$\Rightarrow$ $$x = \frac{\varepsilon}{\eta + 1}$$

Reading of voltmeter after connection of resistance is $\frac{\varepsilon - x}{\eta}$

$$= \frac{\varepsilon}{(\eta + 1)} = \frac{6}{2+1} = 2V$$

Q14 **A battery of emf 1.4 V and internal resistance 2 Ω is connected to a resistor of 100 Ω through an ammeter. The resistance of the ammeter is 4/3 Ω. A voltmeter has also been connected to find the potential difference across the resistor.**
(i) Draw the circuit diagram.
(ii) The voltmeter reads 1.10 V, what is the zero error in the voltmeter?

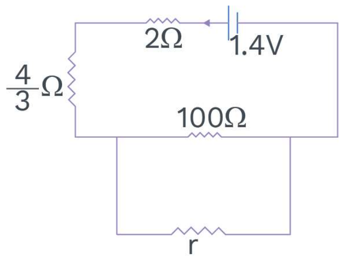
Sol: (i)



(ii) 200 Ω

(iii) $1.1 - \frac{4}{3} = -0.23\,V$

(i) $\frac{4}{3}\Omega$



(ii) $R_{eq} = 2 + \frac{4}{3} + \frac{100r}{100 + r} = \frac{600 + 6r + 400 + 4r + 300r}{3(100 + r)} = \frac{310r + 1000}{3(100 + r)}$

$i = \frac{V}{R}$

$0.02 = \frac{1.40}{310r + 1000} \times (300 + 3r)$

$310r + 1000 = 21000 + 210r$

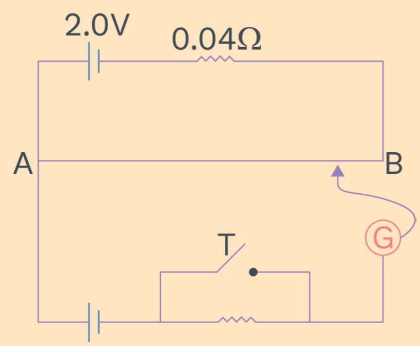
$10\,r = 2000$

$r = 200\ \Omega$

(iii) $V = ir = 0.02 \times \frac{200 \times 100}{200 + 100} = \frac{4}{3} = 1.33$

Zero error = 1.1 – 1.33 = – 0.23 V

Q15 **Figure shows a potentiometer with a cell of emf 2.0 V and internal resistance 0.04 Ω maintaining a potential drop across the potentiometer wire AB. A standard cell which maintains a constant emf of 1.02 V (for very moderate currents up to a few ampere) gives a balance point of 67.3 cm length of the wire. To ensure very low currents drawn from the standard cell, a very high resistance of 600 kΩ is put in series with it which is shorted close to the balance point. The standard cell is then replaced by a cell of unknown emf E and the balance point found similarly turns out to be at 82.3 cm length of the wire.**



(a) What is the value of E?
(b) What purpose does the high resistance of 600 kΩ have?
(c) is the balance point affected by this high resistance?
(d) Is the balance point affected by the internal resistance of the driver cell?
(e) Would the method work in the above situation if the driver cell of the potentiometer had an emf of 1.0 V instead of 2.0 V?
(f) Would the circuit work well for determining extremely small emf, say, of the order of few mV (such typical emf of thermocouple)?

Sol:
(a) $\frac{82.3}{67.3} \times 1.02 = 1.25\,\text{V}$

(b) The high resistance to keep the initial current low when null point is being located. This saves the standard cell from damage.

(c) This high resistance does not affect the balance point because then there is no flow of current through the standard cell branch.

(d) The internal resistance of driver cell affects the current through the potentiometer wire. Since potential gradient is changed, therefore, the balance point must be affected.

(e) No, it is necessary that the emf of the driver cell is more than the emf of the cells.

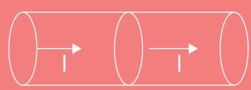
(f) This circuit will not work well for measurement of small emf (mV) because the balance point will be very near to end A, and percentage error in EMF measured due to length measurement would be very large $E = \frac{V}{100}\ell \Rightarrow \frac{dE}{E} = \frac{d\ell}{\ell}$ will be large if ℓ is very small.

Mind Map

CURRENT ELECTRICITY

Electric Current

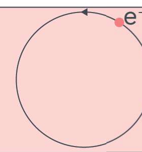
- $I = \frac{dQ}{dt} = \oint \vec{J}.d\vec{S}$



- $J = \frac{I}{A_{\perp}}$

Current in an orbit of a moving e^- with speed v.

$I = \frac{ev}{2\pi r}$



Flow of Charge

$\Delta Q = I\Delta T = \int Idt$
= Area under I-t graph

Drift Speed

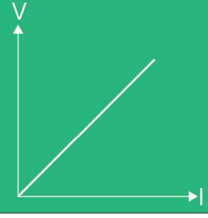
- $v_d = \frac{eE}{m}\tau$
- $I = neAv_d$

Mobility

$\mu_e = \frac{v_d}{E} = \frac{e\tau}{m}$

Ohm's Law

V = IR



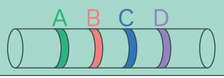
Dependency of R on temperature (T)

$R = R_0[1 + \alpha (T - T_0)]$

Resistance

$R = \frac{\rho l}{A} = \frac{ml}{ne^2\tau A}$

Resistance of a resistor with color code

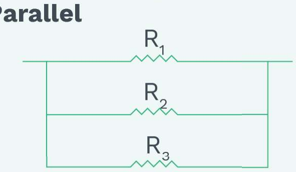


Combination of resistors

Series

$R_S = R_1 + R_2 + R_3 ......$

Parallel



$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$