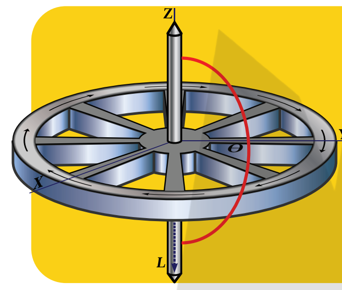
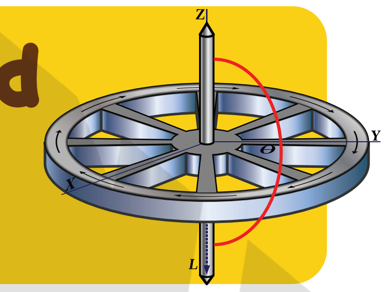
# System of Particles and Rotational Motion





1. Position of centre of mass depends upon shaped, size and distribution of mass of body
2. Position of centre of mass of an object changes in translation motion.
3. For bodies of normal dimensions centre of mass & center of gravity coincide.
4. Centre of mass of rigid bodies is independent of the state i.e rest or motion of the body.

## CENTRE OF MASS

The point where whole mass of system is supposed to be concentrated

## Position of Centre of mass of system

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

## Velocity of centre of mass of system

$$\vec{V}_{cm} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$$

## Acceleration of Centre of mass of system

$$\vec{a}_{cm} = \frac{\sum m_i \vec{a}_i}{\sum m_i}$$

## Rotational Equilibrium

$$\vec{\tau}_{ext} = \varepsilon \vec{r} \times \vec{F}_{ext} = O$$

## RIGID BODY

A body with perfectly definite and unchanging shape.

## Translational Equilibrium

$$\sum \vec{F}_{ext} = O$$

## PRINCIPLE OF MOMENTS FOR A LEVER

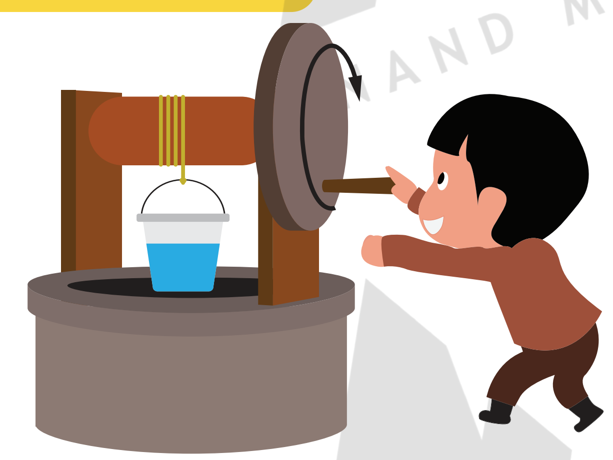
According to this principle:-

Load × load arm = effort × effort arm

## Factors & radius of gyration depends

(1) Position & configuration of the axis of rotation

(2) distribution of mass about the axis of Rotation.



## MOMENT OF INERTIA

### Inertia of Rotational motion

$$\text{M.I. } I = \sum_{i=1} m_i r_i^2 = MK^2$$

where r is distance perpendicular to the axis of Rotation.

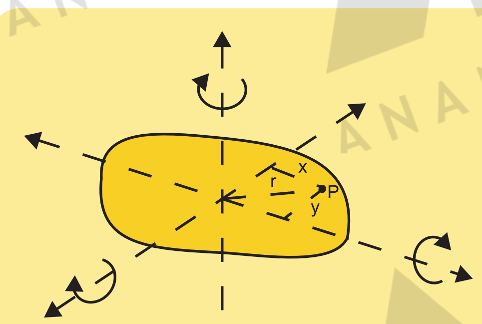
### Radius of gyration

$$K = \sqrt{\frac{r_1^2 + r_2^2 \ldots\ldots\ldots r_n^2}{n}}$$

$$K = \sqrt{\frac{I}{M}}$$

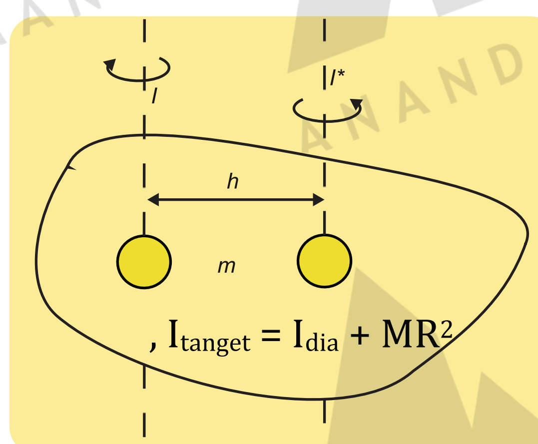
## Theorem of moment of Inertia

### Perpendicular axis theorem



$$I_Z = I_X + I_Y$$

### Parallel – axis theorem




$$I_{tanget} = I_{dia} + MR^2$$

| SHAPE OF AREA | | DISTANCE X | DISTANCE Y | AREA |
|---|---|---|---|---|
| Square | | $a/2$ | $a/2$ | $a^2$ |
| Rectangle | | $a/2$ | $b/2$ | $ab$ |
| Circle | | $r$ | $r$ | $\pi r^2$ |
| Semi-circle | | $4r/3\pi$ | $r$ | $\pi r^2/2$ |
| Right-angled triangle | | $b/3$ | $h/3$ | $bh/2$ |

## ANGULAR MOMENTUM CONSERVATION

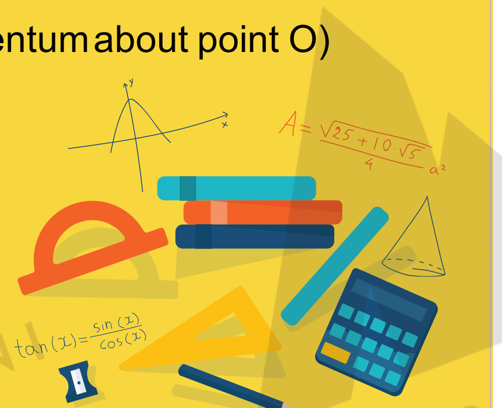
$$\vec{L}_O = \vec{r}_{OA} \times \vec{P} \text{ (angular momentum about point O)}$$

$$= \vec{r}_{OA} \times (m\vec{v})$$

$$= m\vec{r}_{OA} \times \vec{v}$$

$$\vec{L}_O = \vec{r}_{OA} \times \vec{P} = r_{OA} P \sin\theta$$

$$= r_{OA}\, mv \sin\theta$$



## ANGULAR MOMENTUM CONSERVATION

$$\vec{\tau}_{net} = \frac{d\tau}{dt} \quad \text{if } \vec{\tau}_{net} = O \Rightarrow \vec{L} = \text{constant}$$

$$\vec{L}_{system} = \sum_{i=1}^{n} L_i$$

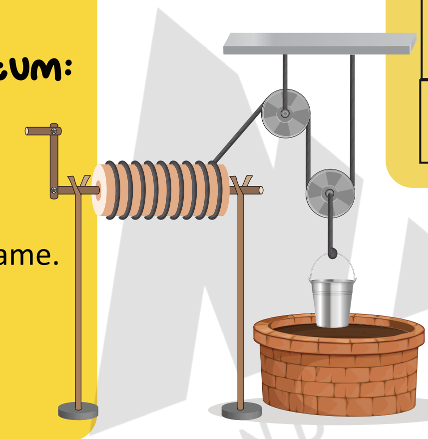
Angular momentum of rigid body performing pure rotation about fixed axis $(L_{sys})_{AOR} = I_{AOR}{}^{w}$

**Relation between Torque & Angular momentum:**

- $\vec{\tau}_{net} = \frac{d\vec{t}}{dt}$
- Unit of Torque = N.m
- Dimensional formula = $[m^1 L^2 T^{-2}]$ Valid in only inertial frame.

Angular Impulse:- $\vec{J} = \int \vec{\tau}.dt$, $\vec{J}_{net} = \vec{L}_f - \vec{L}_i$, $\vec{J} = \vec{r} \times \vec{I}$, Unit → NmS

Linear Impulse:- $\vec{I} = \int \vec{F}\, dt$, $\vec{I}_{net} = \vec{P}_f - \vec{P}_i$, Unit → N.S



## Analogy between linear & Rotational motion

| Linear motion | Rotational Motion |
|---|---|
| Velocity $V = \frac{ds}{dt}$ | Angular velocity $W = \frac{dQ}{dt}$ |
| acceleration $a = \frac{ds}{dt}$ | angular acceleration $\alpha = \frac{dw}{dt}$ |
| Force $F = ma = \frac{mdv}{dt}$ | torque $\tau = I\propto = \frac{d}{dt}(Iw)$ |
| work done $W = F.S$ | work – done $W = \tau.Q$ |
| linear K.E $\frac{1}{2}mv^2$ | rotational K.E $\frac{1}{2}Iw^2$ |
| Power $P = F.V,$ | Power $P = \tau.w,$ |
| Linear momentum $P = mv$ | angular momentum $L = Iw$ |
| Impulse $F\Delta t = mv - mu$ | angular Impulse $\tau pt = Iw_f - Iw_i$ |

| Z | Body | Axis | Figure | I |
|---|---|---|---|---|
| (1) | Thin circular ring, radius R | Perpendicular to plane, at centre | | $MR^2$ |
| (2) | Thin circular ring, radius R | Diameter | | $MR^2/2$ |
| (3) | Thin rod, lenght L | Perpendicular to rod, at mid point | | $ML^2/12$ |
| (4) | Circular disc, radius R | Perpendicular to disc at centre | | $MR^2/2$ |
| (5) | Circular disc, radius R | Diameter | | $MR^2/4$ |
| (6) | Hollow cylinder, radius R | Axis of cylinder | | $MR_2$ |
| (7) | Solid cylinder, radius R | Axis of cylinder | | $MR^2/2$ |
| (8) | Solid cylinder, radius R | Diameter | | $2MR^2/5$ |
| (9) | Hollow sphere, radius R | Diameter | | $\frac{2}{3}mR^2$ |

**(4) Time taken to reach the bottom of the inclined plane is.**

$$t = \frac{1}{\sin\theta}\sqrt{\frac{2n\left(1+\frac{K^2}{R^2}\right)}{g}}$$

## MOTION OF SYSTEM OF PARTICLES & RIGID BODY

### Pure Rotational Motion:-

(1) Since distance between two particles of a rigid body remains constant. So the relative motion of one particle w.r.t other particle is circular motion.

(2) Angular velocity of all the particles about a given point of a Rigid body is same

$S = RQ,\ |V| = Rw;$

(3) If $\alpha$ = Constant (angular acceleration).

$), W_f = w_i + \alpha t,$

$Q_f = w_i t + \frac{1}{2}\alpha t^2\ w_f^2 =$

$w_f^2 + 2\alpha\theta,\ \theta = \left(\frac{w_i + w_f}{2}\right)t$

$\theta = w_f t - \frac{1}{2}\alpha t^2 \to K.E_{rolling} = \frac{1}{2}mv^2 + \frac{1}{2}Iw^2,$

$\frac{1}{2}mv^2 + \frac{1}{2}mk^2\left(\frac{V^2}{r^2}\right)$

$\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)$

### Combined Rotation + translation Motion (CRTM):-

$\vec{V}_{CRTM} = \vec{V}_{pure\ rotation} + \vec{V}_{translational}$

$\vec{a}_{CRTM} = \vec{a}_{pure\ rotation} + \vec{a}_{translational}$

### Dynamics of CRTM

for analysing its motion we apply two equation

$\sum \vec{\tau}_{ext} = M\vec{a}_{cm}$

$\sum \vec{\tau}_{ext} = I\vec{\alpha} = \vec{r} \times \vec{F}_{ext}$

Newton's laws of motion is valid in inertial frame.

To apply Second equation of Newton about Non – inertial Point. Pseudo – force is applied at Com of body $\Sigma$ of pseudo force is also taken into account.

$\to K.E_{CRTM} = K.E_{rotation} + K.E_{translation};$

$K.E = \frac{1}{2}I_{cmw^2} + \frac{1}{2}MV_{cm}^2;$

$K.E = \frac{1}{2}MK^2w^2 + \frac{1}{2}MV_{cm}^2$

→ angular momentum of Rigid body per forming CRTM: Pure Rotational as a Rigid body about C.O.M: Translation as a particle

### (1) ROLLING ON INCLINED PLANE

$(E_k)_r$ = rotational K.E  $(E_k)_t$ = translation K.E

(a) for solid sphere, $(E_k)_r$ = 40% of $(E_k)_t$,

(b) For snell $(E_k)_r$ = 66% of $(E_k)_t$,

(c) For disc, $(E_k)_r$ = 50% of $(E_k)_t$ of $(E_k)_t$,

(d) For ring, $(E_k)_r = (E_k)_t$

### (2) VELOCITY AT LOWEST POINT

$$V = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}}$$

### (3) ACCELERATION ALONG INCLINED PLANE

$$a = \sqrt{\frac{g\sin\theta}{1+\frac{K^2}{R^2}}}$$