

# SIMPLE HARMONIC MOTION



## CHARACTERISTICS OF LINEAR SHM

- Differential Equation of S.H.M

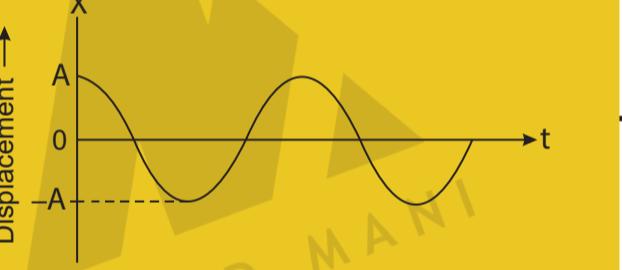
$$\frac{d^2x}{dt^2} + \omega_x^2 = 0$$

- Displacement -  $x = A \sin(\omega t + \phi)$

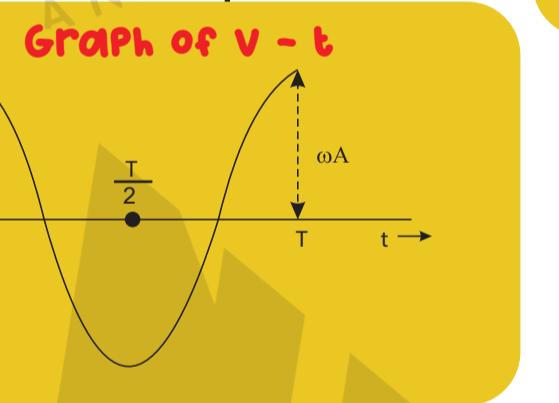
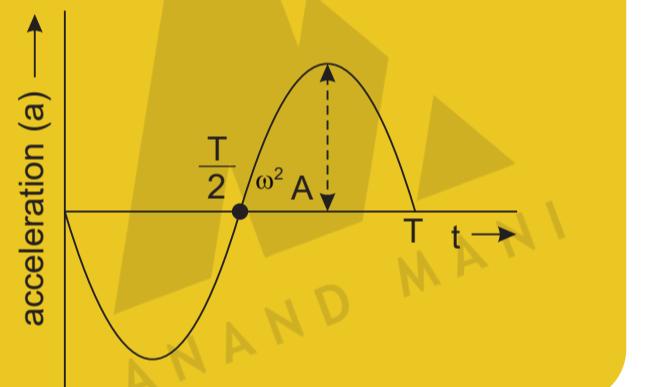
- Velocity -  $v = \frac{dx}{dt} = \omega A \cos(\omega t + \phi)$

- Acceleration -  $a = A \sin(\omega t + \phi) = -\omega^2 x$

### Graph of X - t



### Graph of a - t



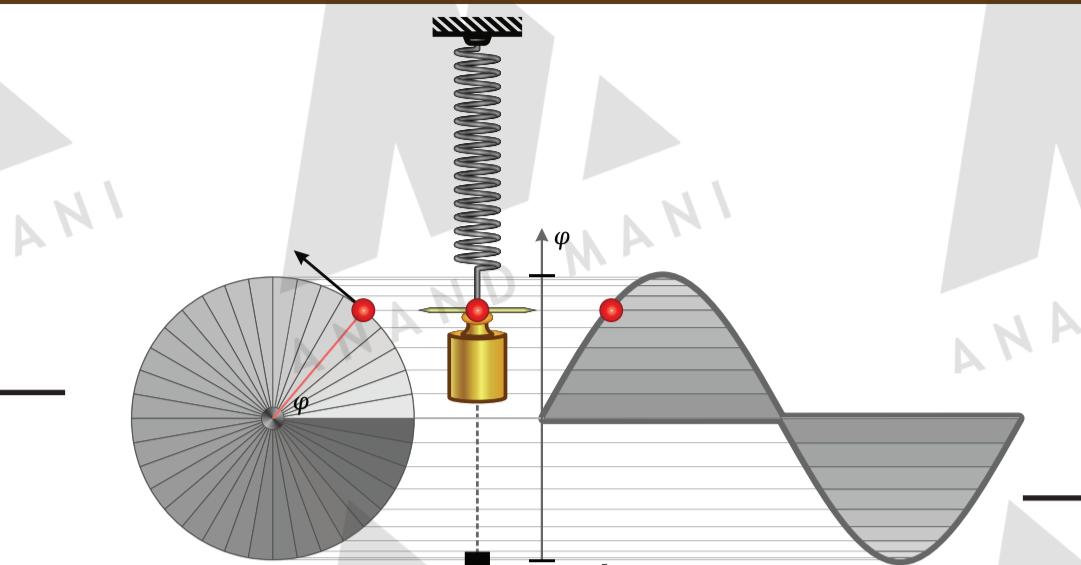
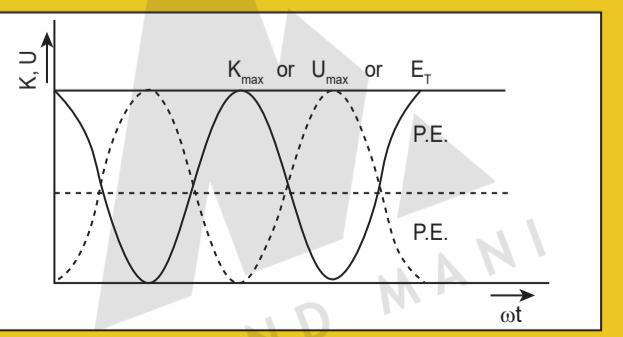
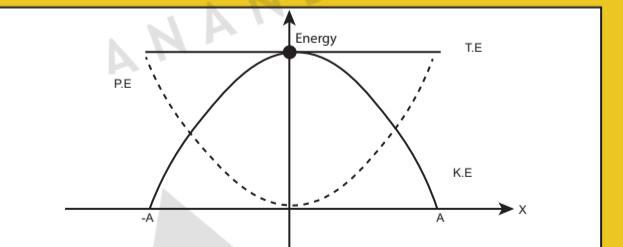
## ENERGY OF LINEAR S.H.M

$$\rightarrow P.E \rightarrow U = \frac{1}{2} Kx^2$$

$$K.E \rightarrow K = \frac{1}{2} K(A^2 - x^2)$$

$$\rightarrow P.E \rightarrow U = \frac{1}{2} K A^2 \sin^2(\omega t + \phi)$$

$$K.E \rightarrow K = \frac{1}{2} K A^2 \cos^2(\omega t + \phi)$$



## TIME PERIOD CALCULATION

$$(1) \text{ Force} \rightarrow \vec{F} = -m\omega_x^2 \text{ or } \vec{F} = -k\vec{x}; \left( \omega_x = \sqrt{\frac{k}{m}} \right) \text{ Time period } T = \frac{2\pi}{\omega_x} = 2\pi\sqrt{\frac{m}{k}}$$

K → spring Constant

## SPRING BLOCK SYSTEM

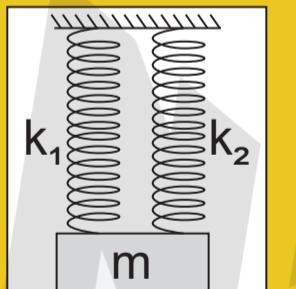
$$\text{Time Period} \rightarrow T = 2\pi\sqrt{\frac{m}{k_{eq}}}$$

$$(i) k_{eq} = K_1 + K_2$$

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$$

$$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

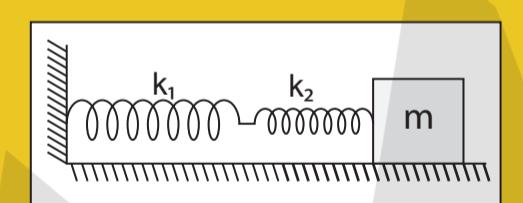
$$T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{K_1 K_2}}$$



$$(ii) k_{eq} = \frac{K_1 K_2}{K_1 + K_2},$$

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$$

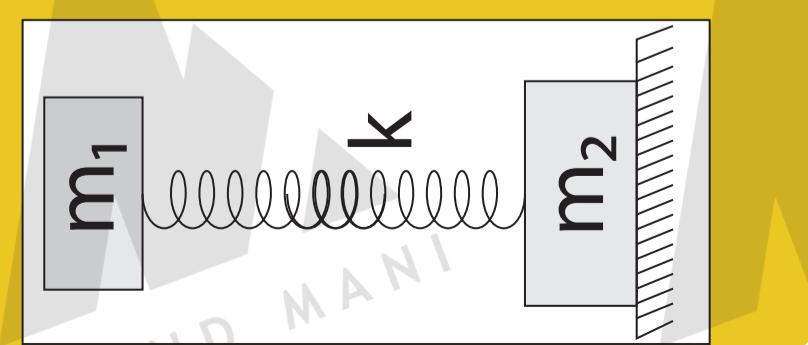
$$T = 2\pi\sqrt{\frac{m}{K_1 K_2}}$$



## TWO BLOCKS SPRING SYSTEM

$$\text{Reduced Mass: } \mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$T = 2\pi\sqrt{\frac{m_1 m_2}{K(m_1 + m_2)}} = 2\pi\sqrt{\frac{\mu}{k}}$$

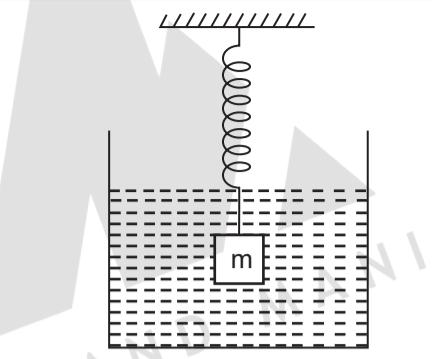


## DAMPED AND FORCE OSCILLATIONS

$$(1) \text{ Amplitude} \rightarrow A^1 = Ae^{-bt/2m}$$

$$(2) \text{ Angular Frequency} \rightarrow \omega^1 = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}},$$

where - b = damping Constant



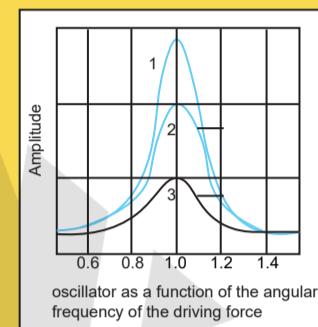
## FORCED OSCILLATION

$$(1) \text{ Amplitude (For } \omega_d \gg \omega \rightarrow A^1 = \frac{F_0}{m(\omega^2 - \omega_d^2)}$$

$$(2) \text{ Amplitude} \rightarrow A^1 = F_0 / w_d b$$

$\omega_d \rightarrow$  Driving Frequency

$\omega \rightarrow$  Natural Frequency



## ANGULAR S.H.M

$$(i) \text{ Different Equation} \rightarrow \frac{d^2\theta}{dt^2} + \omega_0^2 \theta = 0$$

⇒ Displacement →  $\theta = \theta_0 \sin(\omega t + S)$

⇒ Torque →  $T = K\theta$

⇒ Angular Velocity →  $W = \sqrt{\frac{K}{I}}$ ; Angular acceleration →  $\alpha = -\frac{K\theta}{I}$

$$\Rightarrow \text{Time period} - T = 2\pi\sqrt{\frac{I}{K}}$$

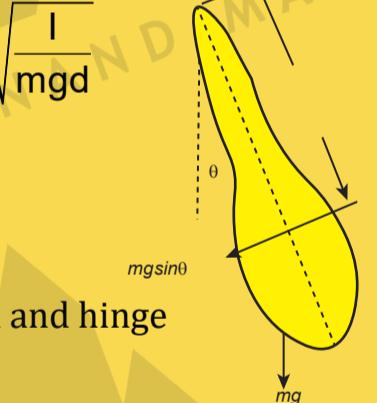
## PHYSICAL PENDULUM

$$\text{: Time period} \rightarrow T = 2\pi\sqrt{\frac{I}{mgd}}$$

I : MoI of system

M : Mass of System

d: distance between com and hinge



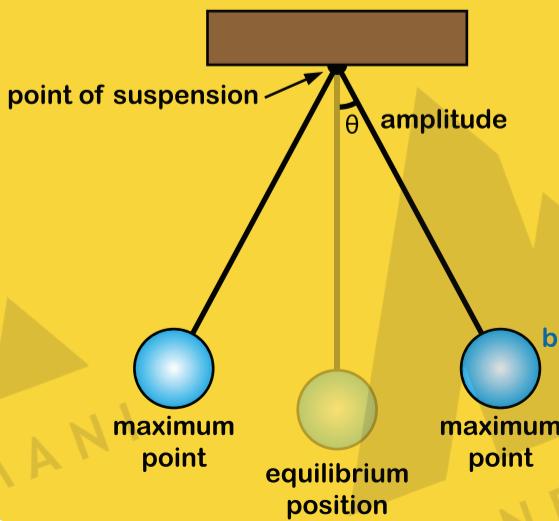
## PENDULUM

## SIMPLE PENDULUM

$$F \propto -\theta$$

$$F = -K\theta$$

$$\text{Time period} \rightarrow T = 2\pi\sqrt{\frac{l}{g}}$$



## TORSIONAL PENDULUM

$$T \propto \theta$$

$$T = -C\theta \quad [C = \text{Torsional Constant}]$$

$$\text{Time period} - T = 2\pi\sqrt{\frac{l}{C}}$$

I : Moment of Inertia

