Circles

Exercise 10.1

Question no 1. Fill in the blanks:

(i) **The centre of a circle lies in ____________ of the circle. (exterior/ interior)**
(ii) **A point, whose distance from the centre of a circle is greater than its radius lies in __________ of the circle. (exterior/ interior)**
(iii) **The longest chord of a circle is a _____________ of the circle.**
(iv) **An arc is a ___________ when its ends are the ends of a diameter.**
(v) **Segment of a circle is the region between an arc and _____________ of the circle.**
(vi) **A circle divides the plane, on which it lies, in _____________ parts.**

Answer:

(i) The centre of a circle lies in **interior** of the circle.
(ii) A point, whose distance from the centre of a circle is greater than its radius lies in **exterior** of the circle.
(iii) The longest chord of a circle is a **diameter** of the circle.
(iv) An arc is a **semicircle** when its ends are the ends of a diameter.
(v) Segment of a circle is the region between an arc and **chord** of the circle.
(vi) A circle divides the plane, on which it lies, in **3 (three)** parts.

Question 2. Write True or False: Give reasons for your Solutions.

(i) **Line segment joining the centre to any point on the circle is a radius of the circle.**
(ii) **A circle has only finite number of equal chords.**
(iii) **If a circle is divided into three equal arcs, each is a major arc.**
(iv) **A chord of a circle, which is twice as long as its radius, is a diameter of the circle.**
(v) **Sector is the region between the chord and its corresponding arc.**
(vi) **A circle is a plane figure**

Answer:

(i) **True.** Any line segment drawn from the centre of the circle to any point on it is the radius of the circle and will be of equal length.
(ii) **False.** There can be infinite numbers of equal chords of a circle.

(iii) **False.** For unequal arcs, there can be major and minor arcs. So, equal arcs on a circle cannot be said as a major arc or a minor arc.

(iv) **True.** Any chord whose length is twice as long as the radius of the circle always passes through the centre of the circle and thus, it is known as the diameter of the circle.

(v) **False.** A sector is a region of a circle between the arc and the two radii of the circle.

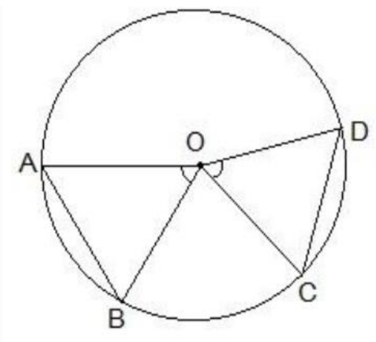
(vi) **True.** A circle is a 2d figure and it can be drawn on a plane

Exercise 10.2

Question no 1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Solution

A circle is a collection of points, each of which is equidistant from the centre. So, two circles can only be congruent if every point on both circles is at the same distance from the centre.



The answer to the second portion of the question is AB = CD, which means two equal chords. It is now necessary to demonstrate that angle AOB equals angle COD.

Proof:

Consider the triangles ΔAOB and ΔCOD,

OA = OC and OB = OD (Since they are the radii of the circle)

AB = CD (As given in the question)

$\Delta AOB \cong \Delta COD$ by SSS Congruency.

∴ By CPCT we have,

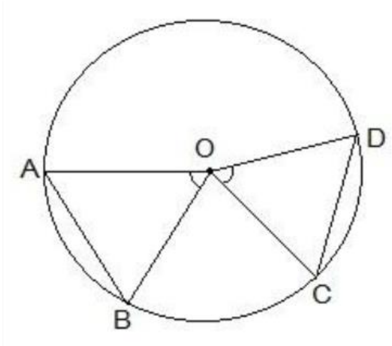
$\angle AOB = \angle COD.$

(Hence proved).

Question no.2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Answer:

Consider the following diagram:



Here, it is given that $\angle AOB = \angle COD$ i.e. they are equal angles.

Now, we will have to prove that the line segments AB and CD are equal i.e. AB = CD.

Proof:

In the triangles AOB and COD,

$\angle AOB = \angle COD$ (as given in the question)

OA = OC and OB = OD (these are the radii of the circle)

So, by SAS congruency, $\Delta AOB \cong \Delta COD$.

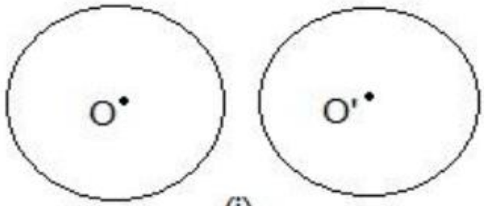
∴ By the rule of CPCT, we have

AB = CD. (Hence proved).

Exercise 10.3

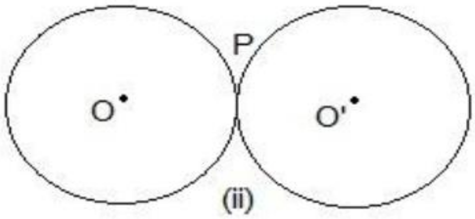
Question 1. Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

Solution:



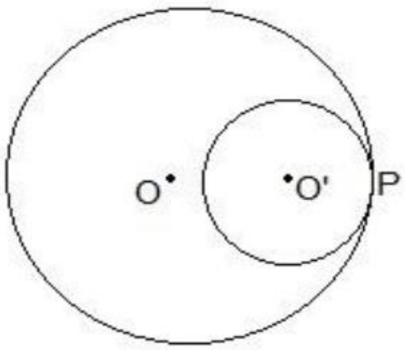
(i)

In these two circles there are no common points.



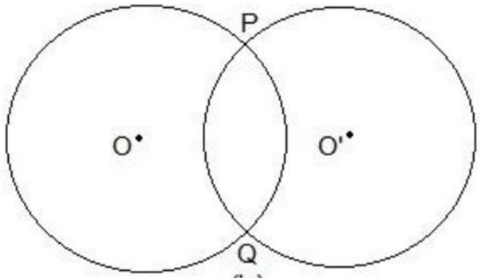
(ii)

Here, only the point ‘‘P’’ is Common.



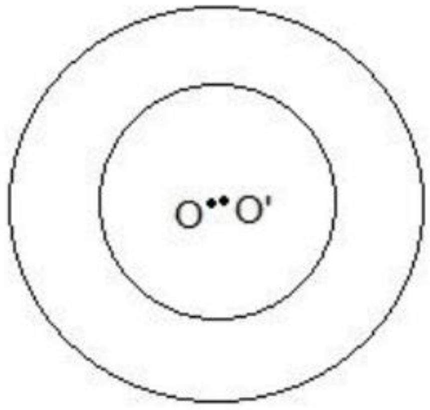
(iii)

Here, P is also the common point.



(iv)

Here, Common points are P and Q.

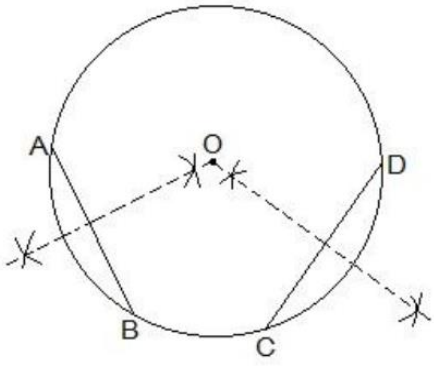


(v)

There is no common point in the above Circle.

Question 2. Suppose you are given a circle. Give a construction to find its centre.

Solution:



The steps of construction are as follows to find the center of the circle:

Step I: Draw a circle first.

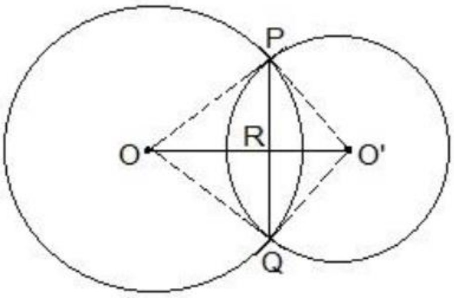
Step II: Draw 2 chords AB and CD in the circle.

Step III: Draw the perpendicular bisectors of AB and CD.

Step IV: Connect the two perpendicular bisectors at a point. This intersection point of the two perpendicular bisectors is the center of the circle.

Question 3. If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.

Solution:



Given in the question that two circles intersect each other at P and Q.

To Prove:

OO' is perpendicular bisector of PQ.

Proof:

Triangle $\Delta POO'$ and $\Delta QOO'$are similar by SSS congruency since

OP = OQ and O'P = OQ (Since they are also the radii)

OO' = OO' (It is the common side)

So, It can be said that $\Delta POO' \cong \Delta QOO'$

$\therefore \angle POO' = \angle QOO' — (i)$

Even triangles ΔPOR and ΔQOR are similar by SAS congruency as

OP = OQ (Radii)

$\angle POR = \angle QOR$ (As $\angle POO' = \angle QOO'$)

OR = OR (Common arm)

So, $\Delta POR \cong \Delta QOR$

$\therefore \angle PRO = \angle QRO$

Also, we know that

$\angle PRO + \angle QRO = 180°$

$\angle PRO = \angle QRO = \frac{180°}{2} = 90°$

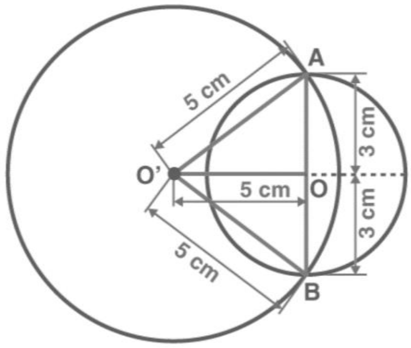
So, OO' is the perpendicular bisector of PQ as we see in the figure.

Exercise 10.4

Question no 1.

Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centers is 4 cm. Find the length of the common chord.

Answer:



The common chord's perpendicular bisector crosses across the centers of both circles.

We can make the above figure because the circles overlap at two spots. Consider the common chord AB, and the circles' centres O and O'.

O'A = 5 cm

OA = 3 cm

OO' = 4 cm [Distance between centres is 4 cm]

We know that the center of the smaller circle is inside the bigger circle because the radius of the bigger circle is greater than the distance between two centers.

The perpendicular bisector of AB is OO'

OA = OB = 3 cm

As O is the midpoint of AB

AB = 3 cm + 3 cm = 6 cm

Length of common chord is 6 cm

It is clear that common chord is the diameter of the smaller circle

Question 2.

If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Solution:

Assume that AB and CD are two equal chords (AB = CD). In the previous question, it is assumed that AB and CD cross at position E.

The line segments AE = DE and CE = BE must now be proven.

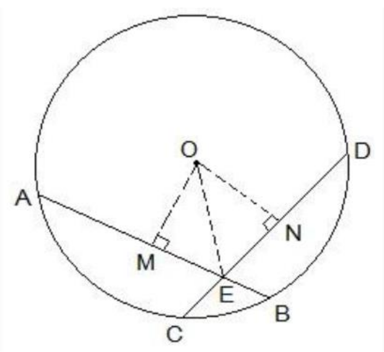
Step of Construction:

Step 1: From the center of the circle, draw a perpendicular to AB i.e. OM ⊥ AB

Step 2: Similarly, draw ON ⊥ CD.

Step 3: Join OE.

Now, the diagram is as follows-



Proof:

From the diagram, it is seen that OM bisects AB and so, OM ⊥ AB

Similarly, ON bisects CD and so, ON ⊥ CD

It is known that AB = CD. So,

AM = ND — (i)

and MB = CN — (ii)

Now, triangles ΔOME and ΔONE are similar by RHS congruency since

$\angle OME = \angle ONE$ (They are perpendiculars)

OE = OE (It is the common side)

OM = ON (AB and CD are equal and so, they are equidistant from the centre)

$\therefore \Delta OME \cong \Delta ONE$

ME = EN (by CPCT) — (iii)

Now, obtaining from equations (i) and (ii) we get,

AM+ME = ND+EN

So, AE = ED

Now from equations (ii) and (iii) we get,

MB-ME = CN-EN

So, EB = CE (Hence proved)

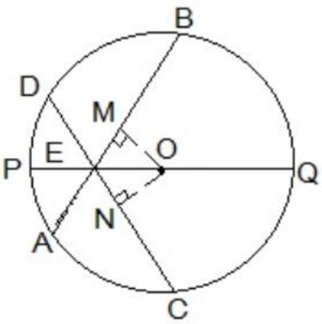
Question 3.

If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the center makes equal angles with the chords.

Solution:

From the question Given:

(i) AB and CD are 2 chords which are intersecting at point E.
(ii) PQ is the diameter of the circle.
(iii) AB = CD.



Now, we will have to prove that $\angle BEQ = \angle CEQ$

For this, the following construction has to be done:

Now, consider the triangles ΔOEM and ΔOEN.

Here,

(i) OM = ON
(ii) OE = OE [It is the common side]
(iii) $\angle OME = \angle ONE$ So, by RHS congruency criterion, $\Delta OEM \cong \Delta OEN$.
(iv) Hence, by CPCT rule, $\angle MEO = \angle NEO$

$\therefore \angle BEQ = \angle CEQ$ (Hence proved).

Question 4.

If a line intersects two concentric circles (circles with the same center) with center O at A, B, C and D, prove that AB = CD (see Fig. 10.25).

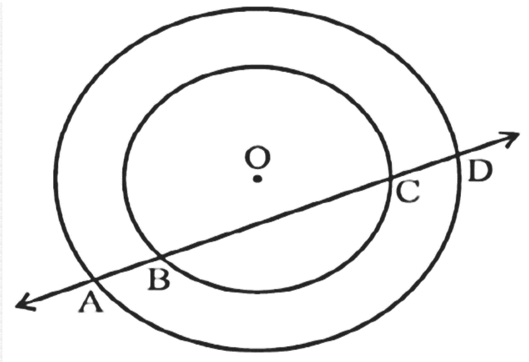


Fig. 10.25

Solution: The Image is:

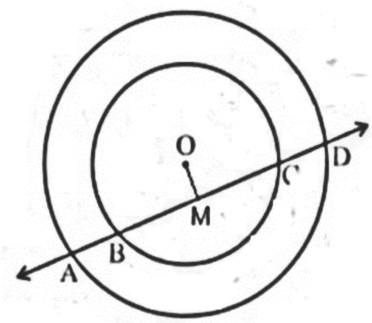


Fig. 10.25

First, draw a line segment from O to AD such that OM ⊥ AD.

So, now OM is bisecting AD since OM ⊥ AD.

Therefore, AM = MD — (i)

Also, since OM ⊥ BC, OM bisects BC.

Therefore, BM = MC — (ii)

From equation (i) and equation (ii),

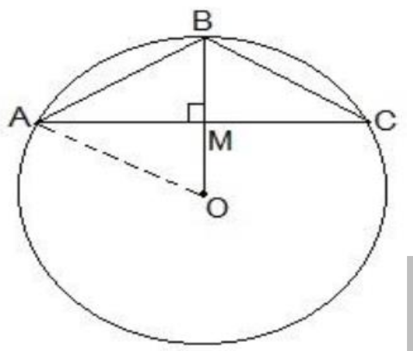
AM-BM = MD-MC

∴ AB = CD

Question 5.

Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?

Solution:



Let the positions of Reshma, Salma and Mandip be represented as A, B and C respectively.

From the question, we know that AB = BC = 6cm.

So, the radius of the circle i.e. OA = 5cm

Now, draw a perpendicular BM ⊥ AC.

ABC can be called an isosceles triangle since AB = BC. AC's midpoint is M. Since BM is the perpendicular bisector of AC, it goes through the circle's center.

Now,

let AM = y and

OM = x

So, BM will be = $(5-x)$.

By applying Pythagorean Theorem in ΔOAM we get,

$$OA^2 = OM^2 + AM^2$$

$$\Rightarrow 5^2 = x^2 + y^2 — (i)$$

Again, by applying Pythagorean Theorem in ΔAMB,

$$AB^2 = BM^2 + AM^2$$

$\Rightarrow 6^2 = (5-x)^2 + y^2 — (ii)$

Subtracting equation (i) from equation (ii), we get

$$36 - 25 = (5 - x)^2 + y^2 - x^2 - y^2$$

Now, solving this equation we get the value of x as

$$x = \frac{7}{5}$$

Substituting the value of x in equation (i), we get

$$y^2 + \left(\frac{49}{25}\right) = 25$$

$$\Rightarrow y^2 = 25 - \left(\frac{49}{25}\right)$$

Solving it we get the value of y as

$$y = \frac{24}{5}$$

Thus,

$$AC = 2 \times AM$$

$$= 2 \times y$$

$$= 2 \times \left(\frac{24}{5}\right) m$$

$$AC = 9.6\,m$$

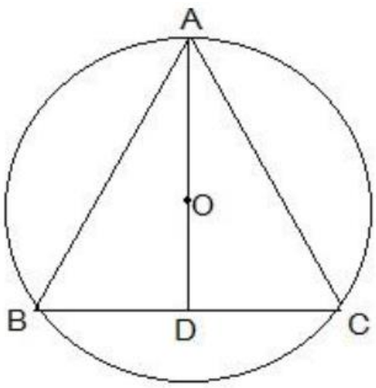
So, the distance between Reshma and Mandip is 9.6 m.

Question 6.

A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary, each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Answer:

First, create a diagram based on the statements provided. The diagram will look like this:



Ankur, Syed, and David's positions are depicted as A, B, and C, respectively. The triangle ABC will form an equilateral triangle since they are sitting at equal distances.

AD ⊥ BC is drawn. Now, AD is the median of ΔABC and it passes through the centre O.

Also, O is the centroid of the ΔABC . OA is the radius of the triangle.

$OA = \frac{2}{3} AD.$

Let the side of a triangle a meter then $BD = \frac{a}{2}\ m.$

Applying Pythagoras theorem in ΔABD,

$$AB^2 = BD^2 + AD^2$$

$$\Rightarrow AD^2 = AB^2 - BD^2$$

$$\Rightarrow AD^2 = a^2 - \left(\frac{a}{2}\right)^2$$

$$\Rightarrow AD^2 = \frac{3a^2}{4}$$

$$\Rightarrow AD = \frac{\sqrt{3}a}{2}$$

$$OA = \frac{2}{3} AD$$

$$20\ m = \frac{2}{3} \times \frac{\sqrt{3}a}{2}$$

$$a = 20\sqrt{3}\ m.$$

So, the length of the string of the toy is $20\sqrt{3}\ m.$

Exercise 10.5

Question 1.

In Fig. 10.36, A,B and C are three points on a circle with centre O such that $\angle BOC = 30°$ and $\angle AOB = 60°$. If D is a point on the circle other than the arc ABC, find ∠ADC.

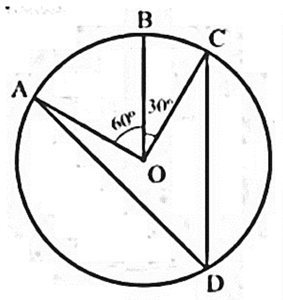


Fig. 10.36

Solution:

It is given that,

∠AOC = ∠AOB + ∠BOC

So, ∠AOC = 60°+30°

∴ ∠AOC = 90°

It is well known that the angle subtended by an arc at the centre of the circle is twice the angle subtended by that arc at any other point on the circle.

So,

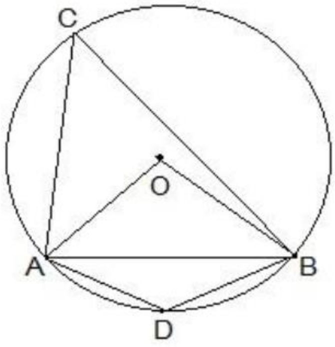
$$\angle ADC = \left(\frac{1}{2}\right)\angle AOC$$

$$= \left(\frac{1}{2}\right) \times 90° = 45°$$

Question 2.

A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Solution:



The radius of the circle is equal to the chord AB. The two radii of the circle in the diagram above are OA and OB.

Now, consider the ΔOAB. Here,

AB = OA = OB = radius of the circle.

So, it can be said that ΔOAB has all equal sides and thus, it is an equilateral triangle.

∴ ∠AOC = 60°

And, $\angle ACB = \frac{1}{2} \angle AOB$

So, $\angle ACB = \frac{1}{2} \times 60° = 30°$

Now, since ACBD is a cyclic quadrilateral,

∠ADB +∠ACB = 180° (Since they are the opposite angles of a cyclic quadrilateral)

So, ∠ADB = 180°-30° = 150°

As a result, the chord's angle subtended at a point on the minor arc and a position on the major arc is $150°$ and $30°$, respectively.

Question no 3.

In Fig. 10.37, ∠PQR = 100°, where P, Q and R are points on a circle with center O. Find ∠OPR.

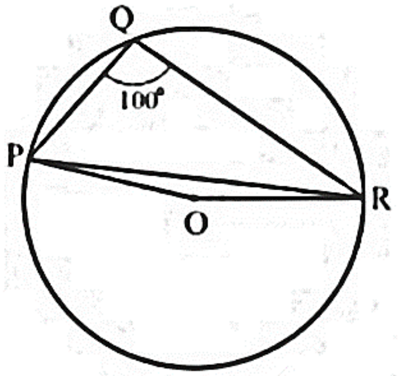


Fig. 10.37

Solution:

So because the angle subtended by an arc in the center of the circle is twice the angle subtended by that arc at any other point on the circle.

So, the reflex ∠POR = 2×∠PQR

We know the values of angle PQR as 100°

So, ∠POR = 2×100° = 200°

∴ ∠POR = 360°-200° = 160°

Now, in ΔOPR,

OP and OR are the radii of the circle

So, OP = OR

Also, ∠OPR = ∠ORP

Now, we know the sum of the angles in a triangle is equal to 180 degrees

So,

∠POR+ ∠OPR+ ∠ORP = 180°

∠OPR+∠OPR = 180°-160°

As ∠OPR = ∠ORP

2∠OPR = 20°

Thus, ∠OPR = 10°

Question 4.

In Fig. 10.38, ∠ABC = 69°, ∠ACB = 31°, find ∠BDC.

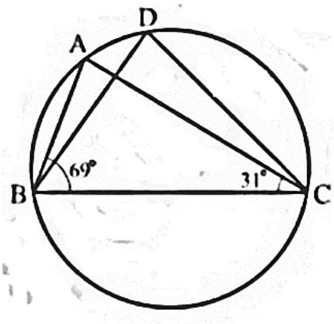


Fig. 10.38

Solution:

We know that angles in the segment of the circle are equal so,

∠BAC = ∠BDC

Now in ΔABC, the sum of all the interior angles will be 180°

So, ∠ABC+∠BAC+∠ACB = 180°

Now, by putting the values,

∠BAC = 180°-69°-31°

So, ∠BAC = 80°

∴ ∠BDC = 80°

Question 5:

In Fig. 10.39, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠ BEC = 130° and ∠ ECD = 20°. Find BAC.

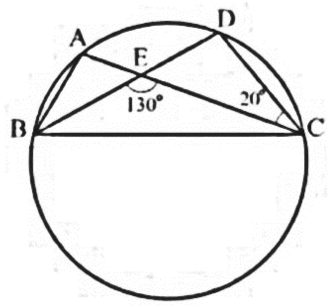


Fig. 10.39

Solution:

Since the angles in the segment of the circle are equal.

So,

∠ BAC = ∠ CDE

Now, by using the exterior angles property of the triangle

In ΔCDE we get,

∠ CEB = ∠ CDE+∠ DCE

We know that ∠ DCE is equal to 20°

So, ∠ CDE = 110°

∠ BAC and ∠ CDE are equal

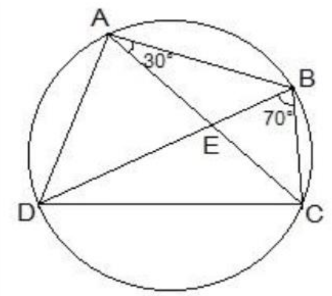
∴ ∠ BAC = 110°

Question 6.

$ABCD$ is a cyclic quadrilateral whose diagonals intersect at a point E. If ∠ DBC = 70°, ∠ BAC is 30°, find ∠ BCD. Further, if AB = BC, find ∠ ECD.

Solution:

Consider the following diagram.



Consider the chordCD,

We know that angles in the same segment are equal.

So, ∠ CBD = ∠ CAD

∴ ∠ CAD = 70°

Now, ∠ BAD will be equal to the sum of angles BAC and CAD.

So, ∠ BAD = ∠ BAC+∠ CAD

= 30°+70°

∴ ∠ BAD = 100°

We know that the opposite angles of a cyclic quadrilateral sums up to 180 degrees.

So,

∠ BCD+∠ BAD = 180°

It is known that ∠ BAD = 100°

So, ∠ BCD = 80°

Now consider the ΔABC.

Here, it is given that AB = BC

Also, ∠ BCA = ∠ CAB (They are the angles opposite to equal sides of a triangle)

∠ BCA = 30°

also, ∠ BCD = 80°

∠ BCA +∠ ACD = 80°

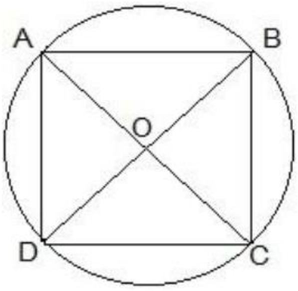
Thus, ∠ ACD = 50° and ∠ ECD = 50°

Question 7.

If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Solution:

Draw a cyclic quadrilateral $ABCD$ inside a circle with a centre O and diagonal AC and BD equal to two circle diameters.



We know that the angles in the semi-circle are equal.

So, ∠ ABC = ∠ BCD = ∠ CDA = ∠ DAB = 90°

Since each internal angle is 90 degrees, the quadrilateral ABCD can be described as a rectangle.

Question 8.

If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Solution:

Construction:

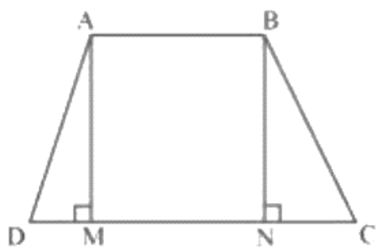
Consider a trapezium ABCD with AB||CD and BC=AD.

Draw:

AM⊥CD and BN⊥CD

In $\triangle AMD \; and \; \triangle BNC,$

The Diagram will look as follows:



In △AMD and △BNC,

AD = BC (Given)

<AMD = <BNC (By construction, each is 90°)

AM = BM (Perpendicular distance between two parallel lines is same)

$\triangle AMD \cong \triangle BNC$ (RHS congruence rule)

<ADC = <BCD (CPCT) ... (1)

<BAD and <ADC are on the same side of transversal AD.

$$< BAD + < ADC = 180^0 \ldots(2)$$

$< BAD + < BCD = 180^o$

[Using equation (1)]

This equation shows that the opposite angles are supplementary.

Therefore, ABCD is a cyclic quadrilateral.

Question 9.

Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig. 10.40). Prove that ∠ ACP = ∠ QCD.

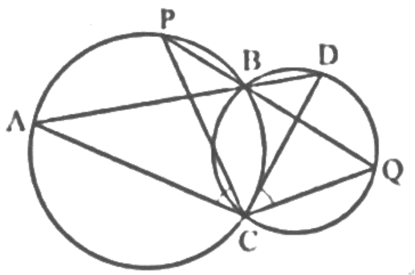


Fig. 10.40

Solution:

Construction:

Join the chords AP and DQ.

For chordAP, we know that angles in the same segment are equal.

So, ∠ PBA = ∠ ACP — (i)

Similarly for chord DQ,

∠ DBQ = ∠ QCD — (ii)

It is known that ABD and PBQ are two line segments which are intersecting at B.

At B, the vertically opposite angles will be equal.

∴ ∠ PBA = ∠ DBQ — (iii)

From equation (i), equation (ii) and equation (iii) we get,

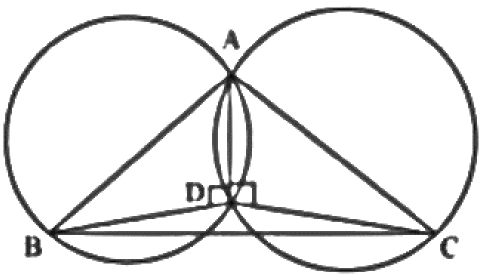
∠ ACP = ∠ QCD

Question 10.

If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Solution:

Draw a triangle ABC first, followed by two circles with diameters of AB and AC, respectively. Now we must show that D is on BC and that BDC is a straight line.



Proof:

We know that angles in the semi-circle are equal

So, ∠ ADB = ∠ ADC = 90°

Hence, ∠ ADB+∠ ADC = 180°

∴ ∠ BDC is straight line.

So, it can be said that D lies on the line BC.

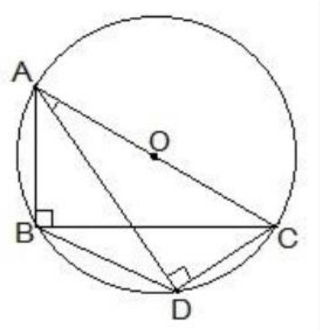
Question 11.

ABC and ADC are two right triangles with common hypotenuse AC. Prove that ∠ CAD = ∠CBD.

Solution:

AC is the common hypotenuse and ∠ B = ∠ D = 90°.

Now, it has we have to prove that ∠ CAD = ∠ CBD



Since here, ∠ ABC and ∠ ADC are 90°, it can be said that they lie in the semi-circle.

So, triangles ABC and ADC are in the semi-circle and the points A, B, C and D are concyclic.

Therefore, CD is the chord of the circle with center O.

We know that the angles which are in the same segment of the circle are equal.

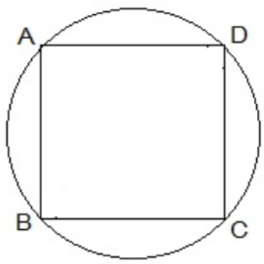
Hence, ∠ CAD = ∠ CBD

Question 12.

Prove that a cyclic parallelogram is a rectangle.

Solution:

ABCD is a cyclic parallelogram, and we must show that ABCD is a rectangle.



Proof:

Let $ABCD$ be a cyclic parallelogram.

∠A+ ∠C = 180° (Opposite angle of cyclic quadrilateral)... (1)

We know that opposite angles of a parallelogram are equal

∠A = ∠C and ∠ B = ∠D

From equation (1) we get,

∠A+ ∠C = 180°

∠A+ ∠A = 180°

2 ∠A = 180°

∠A = 90°

Hence, Parallelogram ABCD has one of its interior angles as 90^0.

Thus, ABCD is a Rectangle.

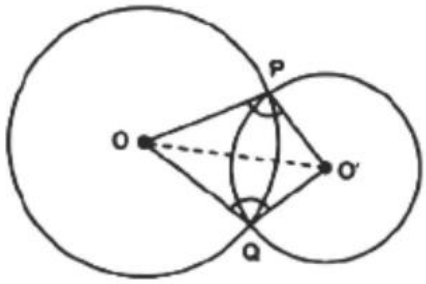
Exercise 10.6

Question 1.

Prove that the line of centers of two intersecting circles subtends equal angles at the two points of intersection.

Solution:

Let us consider the following Diagram,



Now, In ΔPOO’ and ΔQOO’

OP = OQ (Radius of circle 1)

O’P = O’Q (Radius of circle 2)

OO’ = OO’ (Common arm)

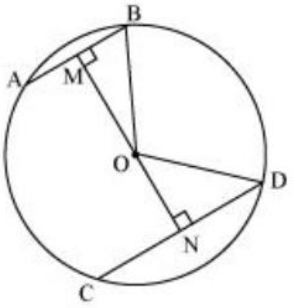
So, by SSS congruency $\Delta POO' \cong \Delta QOO'$

Thus, ∠OPO’ = ∠OQO’ (proved).

Question 2.

Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its center. If the distance between AB and CD is 6, find the radius of the circle.

Solution:



Here, OM ⊥ AB and ON ⊥ CD. is drawn and OB and OD are joined.

We know that AB bisects BM as the perpendicular from the centre bisects chord.

Since AB = 5 so,

$$BM = \frac{AB}{2} = \frac{5}{2}$$

Similarly,

$ND = \frac{CD}{2} = \frac{11}{2}$

Now, let ON be x.

So,

$OM = 6 - x.$

Consider ΔMOB,

$$OB^2 = OM^2 + MB^2$$

Or,

$$OB^2 = 36 + x^2 - 12x + \frac{25}{4} \dots \dots (1)$$

Now Consider ΔNOD,

$OD^2 = ON^2 + ND^2$

Or,

$OD^2 = X^2 + \frac{121}{4} \dots \dots (2)$

We know, $OB = OD(radii)$,

Now, from Equation 1 and 2 we get

$$36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$$

$$12x = 36 + \frac{25}{4} - \frac{121}{4}$$

$$x = \frac{144 + 25 - 121}{4}$$

$$x = 12x = \frac{48}{4} = 12$$

$$x = 1$$

Now, from equation (2) we have,

$$OD^2 = 1^2 + \left(\frac{121}{4}\right)$$

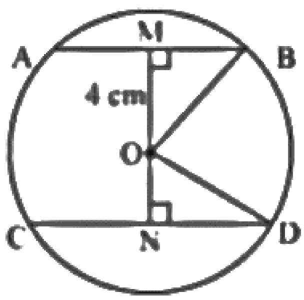
$$\text{Or, } OD = \left(\frac{5}{2}\right) \times \sqrt{5}\ Cm$$

Question 3.

The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at a distance 4 cm from the center, what is the distance of the other chord from the center?

Solution:

Let us consider the following figure,



Here AB and CD are 2 parallel chords. Join OB and OD.

Distance of smaller chord AB from the center of the circle = 4 cm

So, OM = 4 cm

$$MB = \frac{AB}{2} = 3\ cm$$

Consider ΔOMB

$$OB^2 = OM^2 + MB^2$$

Or, OB = 5cm

Consider ΔOND,

OB = OD = 5 (since they are the radii)

$$ND = \frac{CD}{2} = 4\ cm$$

Now, $OD^2 = ON^2 + ND^2$

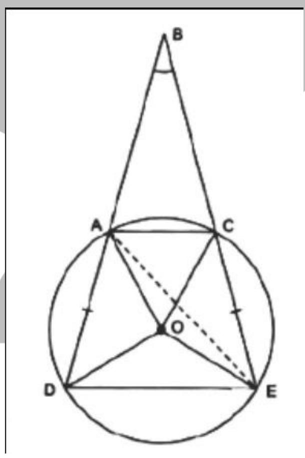
Or, $ON = 3\ cm.$

Question 4.

Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that ∠ABC is equal to half the difference of the angles subtended by the chords AC and DE at the center.

Solution:

Let us consider the following diagram:



Here AD = CE

We know, any exterior angle of a triangle is equal to the sum of interior opposite angles.

So,

∠DAE = ∠ABC+∠AEC (in ΔBAE) ———————-(i)

DE subtends $\angle DOE$ at the centre and $\angle DAE$ in the remaining part of the circle.

So,

$$\angle DAE = (½)\angle DOE ————————(ii)$$

Similarly,

$\angle AEC = \left(\frac{1}{2}\right)\angle AOC$ ————————(iii)

Now, from equation (i), (ii), and (iii) we get,

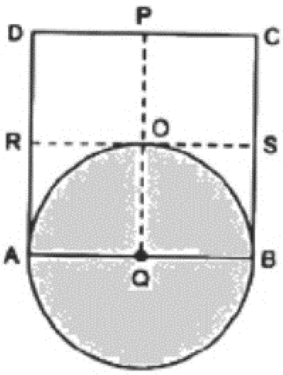
$$\left(\frac{1}{2}\right)\angle DOE = \angle ABC + \left(\frac{1}{2}\right)\angle AOC$$

Or, $\angle ABC = \left(\frac{1}{2}\right)[\angle DOE - \angle AOC]$ (hence proved).

Question 5.

Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.

Solution:



To Prove:

A circle drawn with Q as centre, will pass through A, B and O (i.e. QA = QB = QO)

As all sides of a rhombus are equal,

AB = DC

Now, multiply $\left(\frac{1}{2}\right)$ on both sides

$\left(\frac{1}{2}\right)AB = \left(\frac{1}{2}\right)DC$

So, AQ = DP

BQ = DP

As Q is the midpoint of AB,

AQ= BQ

Similarly,

RA = SB

Again, as PQ is drawn parallel to AD,

RA = QO

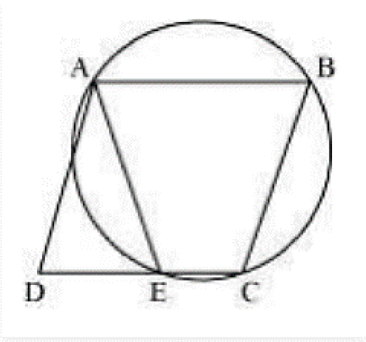
Now, as AQ = BQ and RA = QO we get,

QA = QB = QO (Hence proved).

Question 6.

ABCD is a parallelogram. The circle through A, B and C intersects CD (produced if necessary) at E. Prove that AE, = AD.

Solution:



ABCE is a cyclic quadrilateral. As we know that in a cyclic quadrilateral, the sum of the opposite angles is 180°.

So, ∠AEC+∠CBA = 180°

As ∠AEC and ∠AED are linear pair,

∠AEC+∠AED = 180°

Or, ∠AED = ∠CBA … (1)

As we know that in a parallelogram; opposite angles are equal.

So, ∠ADE = ∠CBA … (2)

Now, from equations (1) and (2) we get,

∠AED = ∠ADE

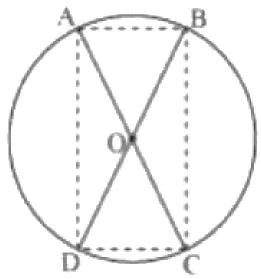
Now, AD and AE are angles opposite to equal sides of a triangle,

∴ AD = AE (proved).

Question 7.

AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters; (ii) $ABCD$ is a rectangle.

Solution:



Here Given the chords AB and CD intersect each other at O.

Consider ΔAOB and ΔCOD,

∠AOB = ∠COD (They are vertically opposite angles)

OB = OD (Given in the question)

OA = OC (Given in the question)

So, by SAS congruency, ΔAOB ≅ ΔCOD

Also, AB = CD (By CPCT)

Similarly, ΔAOD ≅ ΔCOB

Or, AD = CB (By CPCT)

In quadrilateral ACBD, opposite sides are equal.

So, ACBD is a parallelogram.

We know that opposite angles of a parallelogram are equal.

So, ∠A = ∠C

Also, as ABCD is a cyclic quadrilateral,

∠A+∠C = 180°

⇒∠A+∠A = 180°

Or, ∠A = 90°

As ACBD is a parallelogram and one of its interior angles is 90°, so it is a rectangle.

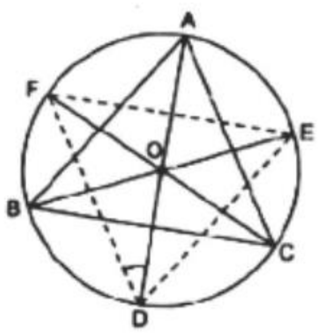
∠A is the angle subtended by chord BD. And as ∠A = 90°, therefore, BD should be the diameter of the circle. Similarly, AC is the diameter of the circle.

Question 8.

Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are 90°–(½)A, 90°–(½)B and 90°–(½)C.

Solution:

Let us consider the following diagram,



Here, ABC is inscribed in a circle with center O and the bisectors of ∠A, ∠B and ∠C intersect the circumcircle at D, E and F respectively.

Now, join DE, EF and FD

As angles in the same segment are equal, so,

∠EDA = ∠FCA ————-(i)

∠FDA = ∠EBA ————-(ii)

By adding equations (i) and (ii) we get,

$$\angle FDA + \angle EDA = \angle FCA + \angle EBA$$

$$Or, \angle FDE = \angle FCA + \angle EBA = \left(\frac{1}{2}\right)\angle C + \left(\frac{1}{2}\right)\angle B$$

We know that $\angle A + \angle B + \angle C = 180°$

So, $\angle FDE = \left(\frac{1}{2}\right)[\angle C + \angle B] = \left(\frac{1}{2}\right)[180° - \angle A]$

$\angle FDE = [90 - (\frac{\angle A}{2})]$

In a similar way,

$$\angle FED = [90° - \left(\frac{\angle B}{2}\right)]°$$

And,

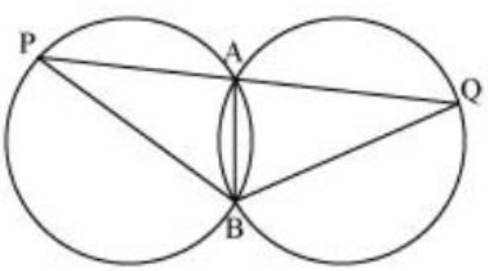
$\angle EFD = [90° - \left(\frac{\angle C}{2}\right)]°$

Question 9.

Two congruent circles intersect each other at points A and B. Through An any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

Solution:

Let us consider the diagram will be



Here, ∠APB = ∠AQB (as AB is the common chord in both the congruent circles.)

Now, consider ΔBPQ,

∠APB = ∠AQB

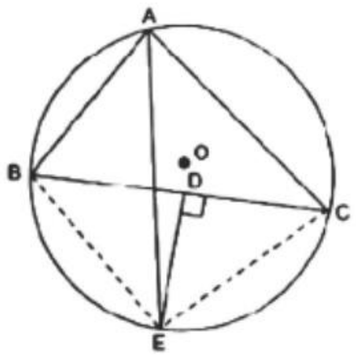
So, the angles are opposite to equal sides of a triangle.

∴ BQ = BP

Question 10.

In any triangle ABC, if the angle bisector of ∠A and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.

Solution:



Firstly, join BE and CE.

Now, As AE is the bisector of ∠BAC,

∠BAE = ∠CAE

Also,

∴arc BE = arc EC

This indicates, chord BE = chord EC

Now, consider triangles ΔBDE and ΔCDE,

DE = DE

BD = CD (It is given in the question)

BE = CE (Already proved)

So, by SSS congruency we say that, ΔBDE ≅ ΔCDE.

Thus, ∴∠BDE = ∠CDE

Since we know, ∠BDE = ∠CDE = 180°

Or, ∠BDE = ∠CDE = 90°

Therefore, DE ⊥ BC

(Hence proved).