Pairs of Linear Equations In Two Variables

Exercise 3.1

Q 1. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Solution.

Let Aftab's current age be 'x'.

And his daughter's current age is 'y'.

Seven years ago, we may write,

Aftab's age $= x - 7$

Age of daughter of Aftab $= y - 7$

Now, according to the question, we have;

$x - 7 = 7(y - 7)$

$x - 7 = 7y - 49$

$x - 7y = -42 - - - - - - - - - - (1)$

And also, 3 years from now or after 3 years,

Aftab's age will be $= x + 3$

Age of daughter of Aftab will be $= y + 3$

Now, as per given situation

$x + 3 = 3(y + 3)$

$x + 3 = 3y + 9$

$x - 3y = 6 - - - - - - - - -(2)$

Subtract the equation (1) from the equation (2) then we get

$(x - 3y) - (x - 7y) = 6 - (-42)$

$-3y + 7y = 6 + 42$

$4y = 48$

$y = 12$

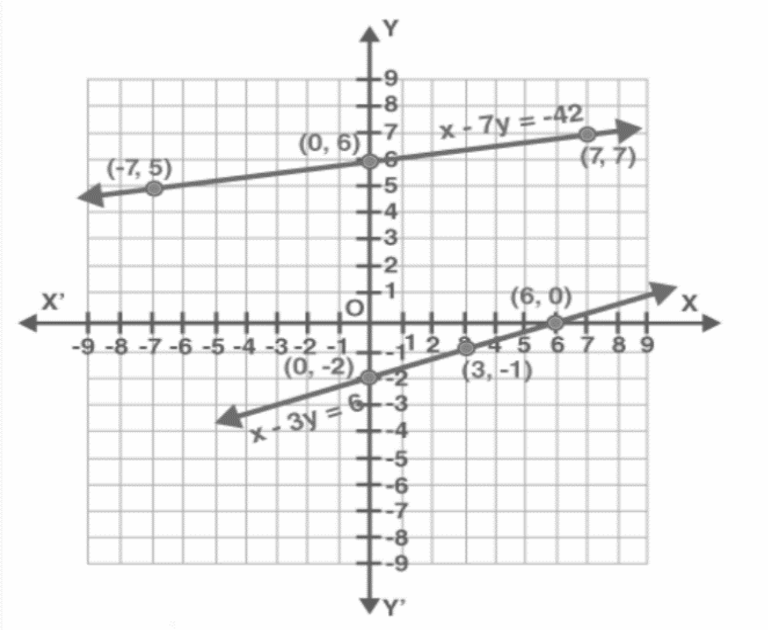
for equations, $x - 7y = -42$, or $x = -42 + 7y$

$x = -7$ when $y = 5$

$x = 0$ when $y = 6$

$x = 7$ when $y = 7$

for equations, $x - 3y = 6$, or $x = 6 + 3y$



Q 2. The coach of a cricket team buys 3 bats and 6 balls for Rs.3900. Later, she buys another bat and 3 more balls of the same kind for Rs.1300. Represent this situation algebraically and geometrically.

Solution.

Assume that the price of a bat is 'Rs x'.

And a ball should cost 'Rs y'.

The algebraic representation, according to the question is

$3x + 6y = 3900$

$x + 3y = 1300$

for equation $3x + 6y = 3900$

or $x = \frac{3900-6y}{3}$

Solution table is given as

X	300	100	700
Y	500	600	300

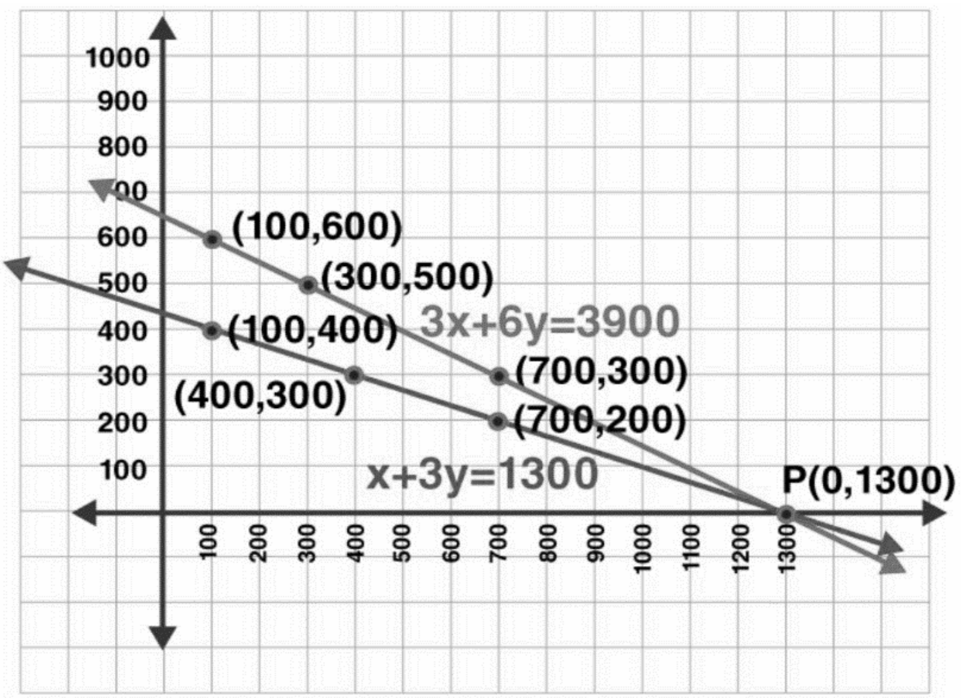
for equation $x + 3y = 1300$

or, $x = 1300 - 3y$

Solution table is given as

X	400	100	700
Y	300	400	200

Graphical representation is



Q3. The cost of 2 kg of apples and 1kg of grapes on a day was found to be Rs.160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs.300. Represent the situation algebraically and geometrically.

Solution: Let us consider the cost of 1 kg of apples is Rs. x.

Cost of 1 kg of Grapes is Rs. y.

The algebraic representation, according to the question is

$2x + y = 160$

$4x + 2y = 300$

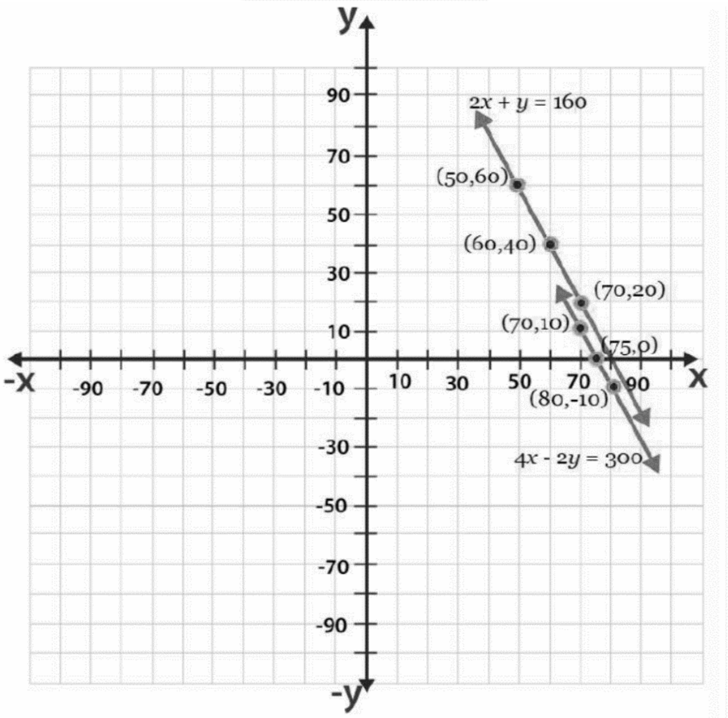
Solution table for equation $2x + y = 160$ or, $y = 160 - 2x$

x	50	60	70
y	60	40	20

Solution table for equation $4x + 2y = 300$ or $y = \frac{300-4x}{2}$

x	70	80	75
y	10	-10	0

Graphical Representation:



Exercise 3.2

Q1. Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost 50, whereas 7 pencils and 5 pens together cost 46. Find the cost of one pencil and that of one pen.

Solution :

(i) Let's say there are x girls and y boys in the class. The algebraic expression in the given question can be expressed as follows.

$x + y = 10$

$x - y = 4$

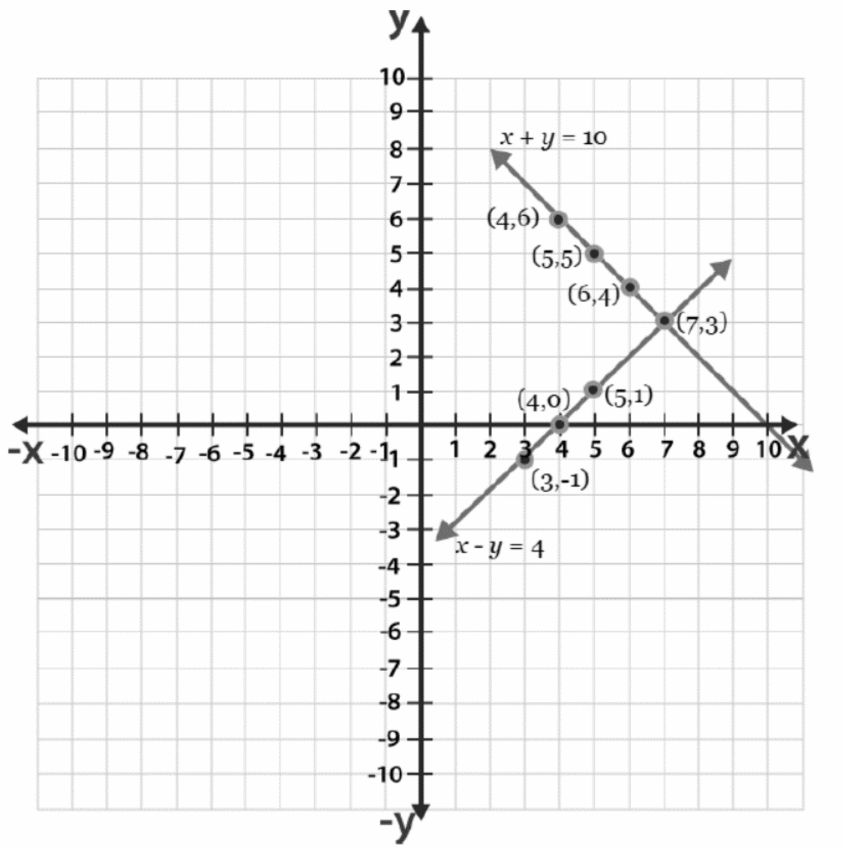
Solution table for the equation $x + y = 10$ or, $x = 10 - y$

X	5	4	6
Y	5	6	4

Solution table for the equation $x - y = 4$ or $x = 4 + y$

X	4	5	3
Y	0	1	-1

Graphical Representation :



The given lines intersect each other at position on the graph, as can be observed (7, 3). As a result, the class consists of 7 girls & 3 boys.

(ii) Let's pretend that cost of one pencil is Rs. x and cost of one pen is Rs. y.

The algebraic expression can be represented as in the question.

$5x + 7y = 50$

$7x + 5y = 46$

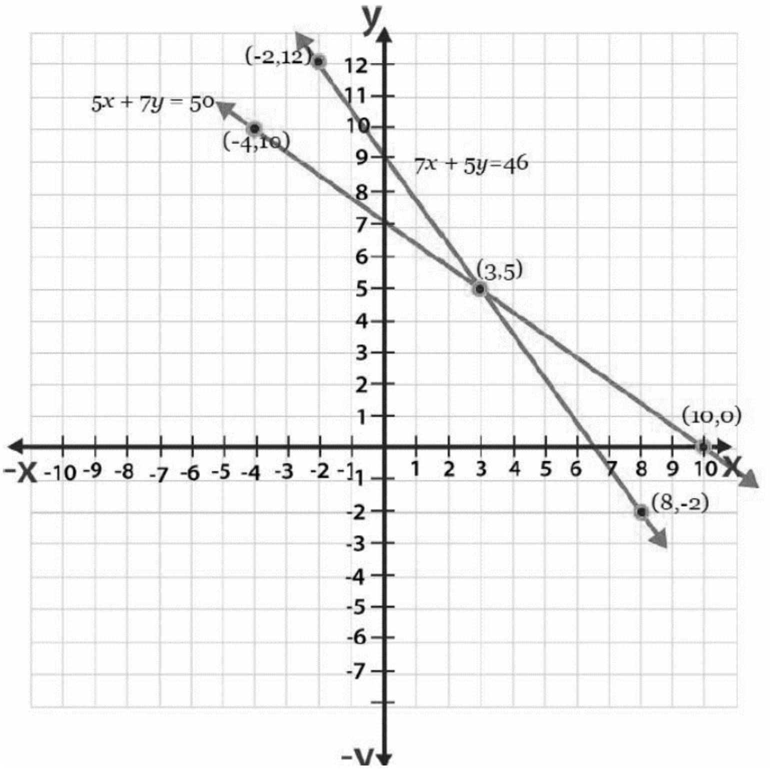
Solution table for the equation $5x + 7y = 50$ or $x = \frac{50-7y}{5}$

X	3	10	-4
Y	5	0	10

Solution table for the equation $7x + 5y = 46$ or $x = \frac{46-5y}{7}$

X	8	3	-2
Y	-2	5	12

Graphical Representation



Q2. On comparing the ratios $\frac{a1}{a2}, \frac{b1}{b2}, \frac{c1}{c2}$ find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

(i) $\mathbf{5x - 4y + 8 = 0}$
$\mathbf{7x + 6y - 9 = 0}$

(ii) $\mathbf{9x + 3y + 12 = 0}$
$\mathbf{18x + 6y + 24 = 0}$

(iii) $\mathbf{6x - 3y + 10 = 0}$
$\mathbf{2x - y + 9 = 0}$

Solution :

(i) $5x - 4y + 8 = 0$
$7x + 6y - 9 = 0$
by equating these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
then we get,
$a_1 = 5$
$b_1 = -4$
$c_1 = 8$
$a_2 = 7$
$b_2 = 6$
$c_2 = -9$
$\frac{a_1}{a_2} = \frac{5}{7}$
$\frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$

$\frac{c_1}{c_2} = \frac{8}{-9}$

as $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

As a result, each of the pairs of equations in the question has a unique solution, and the lines cross at exactly one location.

$9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

by equating these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ then we get,

$a_1 = 9$

$b_1 = 3$

$c_1 = 12$

$a_2 = 18$

$b_2 = 6$

$c_2 = 24$

$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$

as $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

As a result, the question's pairs of equations have a unique solution, and the lines intersect at precisely one location.

(i) $6x - 3y + 10 = 0$

$2x - y + 9 = 0$

by equating these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ then we get,

$a_1 = 6$

$b_1 = -3$

$c_1 = 10$

$a_2 = 2$

$b_2 = -1$

$c_2 = 9$

$\frac{a_1}{a_2} = \frac{6}{2} = \frac{3}{1}$

$\frac{b_1}{b_2} = \frac{-3}{-1} = \frac{3}{1}$

$\frac{c_1}{c_2} = \frac{10}{9}$

as $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

As a result, the pairs of equations which are given in the question are parallel to one another, the lines never overlap at any point, and the given pair of equations has no solution.

Q3. On comparing the ratio, $\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}$ find out whether the following pair of linear equations are consistent, or inconsistent.

(i) $\mathbf{3x + 2y = 5}$
$\mathbf{2x - 3y = 7}$

(ii) $\mathbf{2x - 3y = 8}$
$\mathbf{4x - 6y = 9}$

(iii) $\mathbf{\frac{3}{2}x + \frac{5}{3}y = 7}$
$\mathbf{9x - 10y = 14}$

(iv) $\mathbf{5x - 3y = 11}$
$\mathbf{-10x + 6y = -22}$

(v) $\mathbf{\frac{4}{3}x + 2y = 8}$
$\mathbf{2x - 3y = 12}$

Solution.

(i) $3x + 2y = 5$ or $3x + 2y - 5 = 0$
$2x - 3y = 7$ or $2x - 3y - 7 = 0$
by equating these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
then we get,
$a_1 = 3$
$b_1 = -2$
$c_1 = -5$
$a_2 = 2$
$b_2 = -3$
$c_2 = -7$
$\frac{a_1}{a_2} = \frac{3}{2}$
$\frac{b_1}{b_2} = \frac{2}{-3}$
$\frac{c_1}{c_2} = \frac{-5}{-7} = \frac{5}{7}$
as $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
As a result, the supplied equations meet at one point & there is only one solution. The formulas are correct.

(ii) $2x - 3y = 8$
$4x - 6y = 9$
by equating these equations with $a_1x + b_1y + c_1 = 0$and $a_2x + b_2y + c_2 = 0$
then we get,
$a_1 = 2$

$b_1 = -3$
$c_1 = -8$
$a_2 = 4$
$b_2 = -6$
$c_2 = -9$
$\frac{a_1}{a_2} = \frac{1}{2}$
$\frac{b_1}{b_2} = \frac{1}{2}$
$\frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9}$
as $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
As a result, the equations are parallel to one another and have no solution. As a result, the equations are incompatible.

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$
$9x - 10y = 14$
by equating these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ then we get,
$a_1 = \frac{3}{2}$
$b_1 = \frac{5}{3}$
$c_1 = -7$
$a_2 = 9$
$b_2 = -10$
$c_2 = -14$
$\frac{a_1}{a_2} = \frac{1}{6}$
$\frac{b_1}{b_2} = \frac{-1}{6}$
$\frac{c_1}{c_2} = \frac{-7}{-14} = \frac{1}{2}$
as $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
As a result, the equations meet at one point & there is only one feasible solution. As a result, the equations are consistent.

(iv) $5x - 3y = 11$
$-10x + 6y = -22$
by equating these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ then we get,
$a_1 = 5$
$b_1 = -3$
$c_1 = -11$
$a_2 = -10$
$b_2 = 6$
$c_2 = 22$

$\frac{a_1}{a_2} = \frac{-1}{2}$

$\frac{b_1}{b_2} = \frac{-1}{2}$

$\frac{c_1}{c_2} = \frac{-1}{2}$

As, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

These linear equations have an endless number of solutions since they are coincident lines. As a result, the equations are consistent.

(v) $\frac{4}{3}x + 2y = 8$

$2x - 3y = 12$

$a_1 = \frac{4}{3}$

$b_1 = 2$

$c_1 = -8$

$a_2 = 2$

$b_2 = 3$

$c_2 = -12$

$\frac{a_1}{a_2} = \frac{4}{3\times2} = \frac{2}{3}$

$\frac{b_1}{b_2} = \frac{2}{3}$

$\frac{c_1}{c_2} = \frac{-8}{-12} = \frac{2}{3}$

As, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

These linear equations have an endless number of solutions since they are coincident lines. As a result, the equations are consistent.

Q4. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

(i) $\boldsymbol{x + y = 5}$

$\boldsymbol{2x + 2y = 10}$

(ii) $\boldsymbol{x - y = 8}$

$\boldsymbol{3x - 3y = 16}$

(iii) $\boldsymbol{2x + y - 6 = 0}$

$\boldsymbol{4x - 2y - 4 = 0}$

(iv) $\boldsymbol{2x - 2y - 2 = 0}$

$\boldsymbol{4x - 4y - 5 = 0}$

Solution.

(i) $x + y = 5$

$2x + 2y = 10$

$\frac{a_1}{a_2} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{1}{2}$

As, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

The equations are symmetric (or coincident) and have an endless number of solutions.

Therefore, equations are consistent.

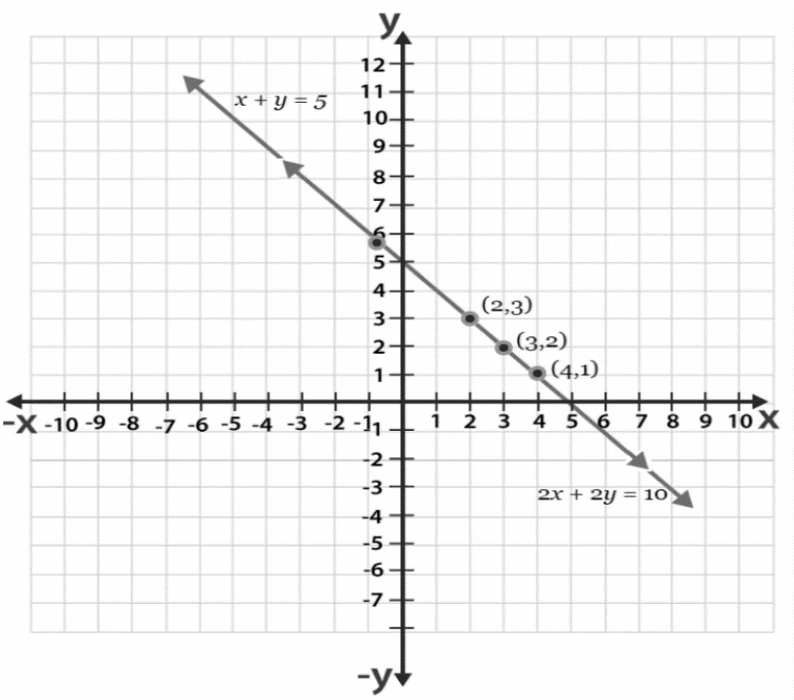
solution table for the equation $x + y = 5$ or, $x = 5 - y$

X	4	3	2
Y	1	2	3

Solution table for the equation, $2x + 2y = 10$

X	4	3	2
Y	1	2	3

Graphical Representation



(ii) $x - y = 8$

$3x - 3y = 16$

$\frac{a_1}{a_2} = \frac{1}{3}$

$\frac{b_1}{b_2} = \frac{1}{3}$

$\frac{c_1}{c_2} = \frac{1}{2}$

As, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

There are no solutions to the equations because they are parallel to each other. As a result, pairs of linear equations is inconsistent.

(iii) $2x + y - 6 = 0$

$4x - 2y - 4 = 0$

$\frac{a_1}{a_2} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{1}{-2}$

$\frac{c_1}{c_2} = \frac{3}{2}$

As, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

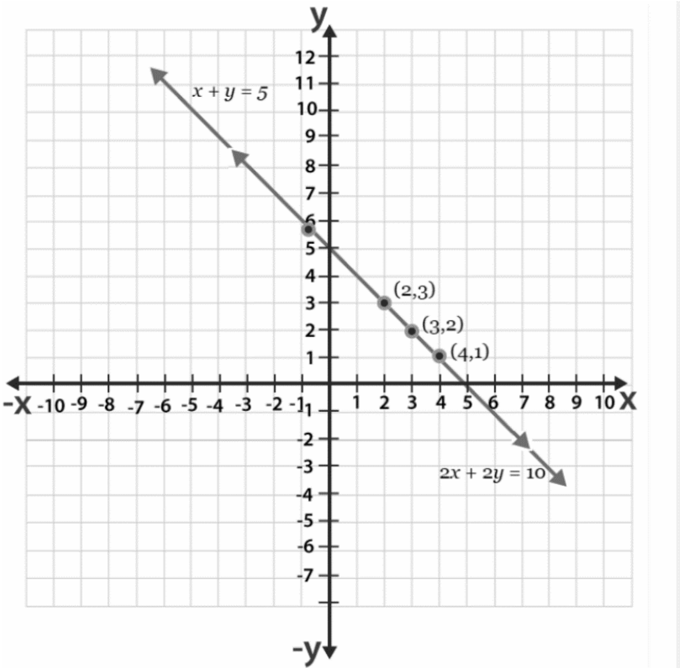
There is only one solution to the above linear equations since they cross at one place. As a result, the pair of linear equations is stable or consistent.

Solution table for the equation $2x + y - 6 = 0$

X	0	1	2
Y	6	4	2

Solution table for the equation $4x - 2y - 4 = 0$

X	1	2	3
Y	0	2	4



(iv) $2x - 2y - 2 = 0$

$4x - 4y - 5 = 0$

$\frac{a_1}{a_2} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{2}{5}$

As, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

There are no solutions to the equations because they are parallel to each other. As a result, pairs of linear equations are inconsistent.

Q5. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Solution.

Let the length of the rectangular garden be x and the width be y.

$y - x = 4$

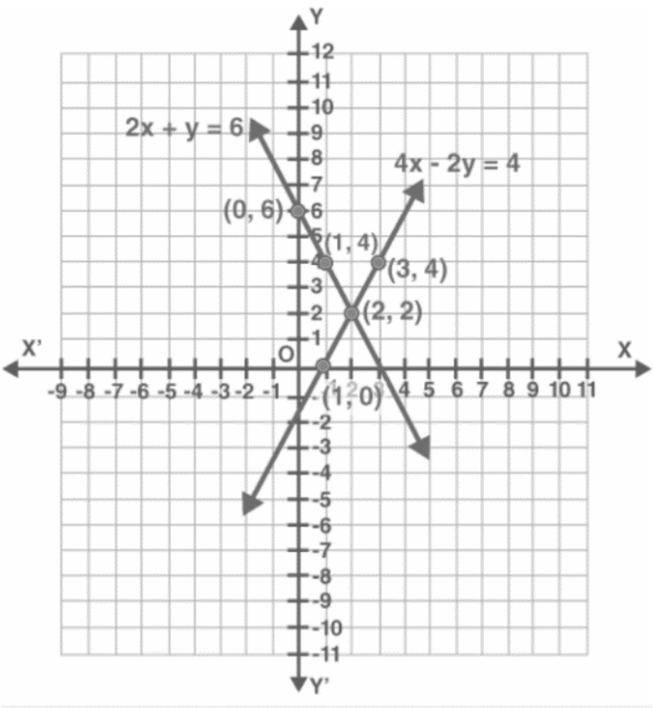
$y + x = 36$

Solution table for the equation $y - x = 4$

X	0	8	12
Y	4	12	16

Solution table for the equation $y + x = 36$

X	0	36	16
Y	36	0	20



The lines intersect at a location in the graph, as you can see (16, 20). As a result, the garden's width is 16 and its length is 20.

Q6. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:

(i) Intersecting lines

(ii) Parallel lines

(iii) Coincident lines

Solution.

(i) To discover another linear equation in 2 variables that has geometrical representation of intersecting lines, it must meet the following criteria:

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Therefore, other equation can be $2x - 7y + 9 = 0$

So that,

$\frac{a_1}{a_2} = \frac{2}{2} = 1$

$\frac{b_1}{b_2} = \frac{3}{-7}$

Hence, other equation satisfies the condition.

(ii) $2x + 3y - 8 = 0$

To find another two-variable linear equation that has the geometrical representation of parallel lines, it must satisfy the following conditions:

As, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, other equation can be $6x + 9y + 9 = 0$

So that,

$\frac{a_1}{a_2} = \frac{1}{3}$

$\frac{b_1}{b_2} = \frac{1}{3}$

$\frac{c_1}{c_2} = \frac{-8}{9}$

Hence, other equation satisfies the condition.

(iii) $2x + 3y - 8 = 0$

To find another two-variable linear equation that has the geometrical representation of coincident lines, it must satisfy the following conditions:

As, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore, other equation can be $4x + 6y - 16 = 0$

So that,

$\frac{a_1}{a_2} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{1}{2}$

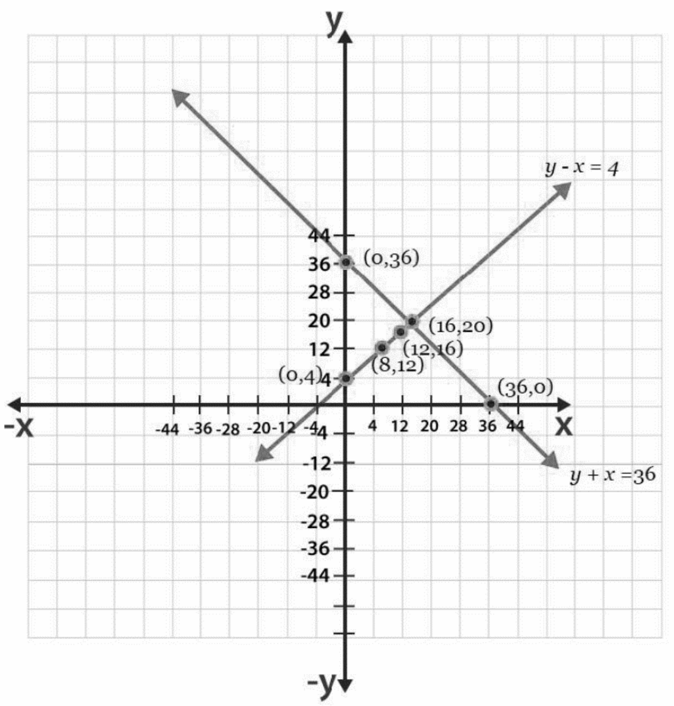
Hence, other equation satisfies the condition.

Q7. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Solution. Equations: $x - y + 1 = 0$ and $3x + 2y - 12 = 0$

Solution table for the equation $x - y + 1 = 0$

X	0	1	2
Y	1	2	3



These lines intersect each other at point (2, 3) & the x-axis at (1, 0), as can be seen in the diagram (4, 0). As a result, the triangle's vertices are (2, 3), (1, 0), & (4, 0).

Exercise 3.3

Q1. Solve the following pair of linear equations by the substitution method

(i) $x + y = 14$
$x - y = 4$

(ii) $s - t = 3$
$(s/3) + (t/2) = 6$

(iii) $3x - y = 3$
$9x - 3y = 9$

(iv) $0.2x + 0.3y = 1.3$
$0.4x + 0.5y = 2.3$

(v) $\sqrt{2}x + \sqrt{3}y = 0$
$\sqrt{3}x - \sqrt{8}y = 0$

(vi) $\frac{3x}{2} - \frac{5y}{3} = -2$
$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

Solution.

(i) Given that

$x + y = 14$ and $x - y = 4$

now,

$x = 14 - y$

put it in 2nd equation

$(14 - y) - y = 4$

$2y = 10$

$y = 5$

therefore,

$x = 14 - y$

$x = 14 - 5$

$x = 9$

(ii) Given that

$s - t = 3$

$(s \div 3) + (t \div 2) = 6$

$s = 3 + t$

put s in other equation

$\frac{3+t}{3} + \frac{t}{2} = 6$

$\frac{6+2t+3t}{6} = 6$

$\frac{6+5t}{6} = 6$

$5t = 30$

$t = 6$

therefore,

$s = 3 + 6$

$s = 9$

(iii) Given that

$3x - y = 3$

$9x - 3y = 9$

Now,

$x = \frac{3+y}{3}$

put it in 2nd equation

$9\left(\frac{3+y}{3}\right) - 3y = 9$

$9 + 3y - 3y = 9$

$9 = 9$

Therefore, y has infinite values.

(iv) $0.2x + 0.3y = 1.3$

$0.4x + 0.5y = 2.3$

$x = (1.3 - 0.3y) \div 0.2$

put it in other equation

$0.4\left(\frac{1.3-0.3y}{0.2}\right) + 0.5 = 2.3$

$2.6 - 0.6y + 0.5y = 2.3$

$0.1y = 0.3$

$y = 1$

Now,

$x = \frac{1.3-0.3(3)}{0.2}$

$x = \frac{0.4}{0.2}$

$x = 2$

(v) $\sqrt{2}x + \sqrt{3}y = 0$

$\sqrt{3}x - \sqrt{8}y = 0$

$x = -\left(\frac{\sqrt{3}}{\sqrt{2}}\right) y$

Put it in other equation

$\sqrt{3}\left(-\frac{\sqrt{3}}{\sqrt{2}}\right) y - \sqrt{8}y = 0$

$y = 0$

Now,

$x = 0$

(vi) $\frac{3x}{2} - \frac{5y}{3} = -2$

$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

from 1st equation

$x = 2\left(\frac{-6+5y}{9}\right)$

$x = \left(\frac{-12+10y}{9}\right) - - - - - - -(1)$

put it in other equation

$\frac{(-12+10y)\div 9}{3} + \frac{y}{2} = \frac{13}{6}$

$\frac{-12+10y}{27} + \frac{y}{2} = \frac{13}{6}$

$\frac{y}{2} = \frac{117+24-20y}{54}$

$y = 3$

now,

$\frac{3x}{2} - 3\frac{5}{3} = -2$

$x = 2$

Q2. Solve $2x + 3y = 11$ and $2z - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$

Solution.

$2x + 3y = 11 - - - - - (1)$

$2x - 4y = -24 - - - - - - -(2)$

from equation 2

$x = \frac{11-3y}{2} - - - - - - (3)$

put x in equation 2

$2\frac{11-3y}{2} - 4y = 24$

$-7y = -35$

$y = 5$

Now,

$x = \frac{11-3\times5}{2}$

$x = \ 2$

Also,

$y = mx + 3$

$5 = -2m + 3$

$m = -1$

Thus, the value of m is equal to -1.

Q3. Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

Solution.

Let the 2 numbers be x & y, in such a way that $y < \ x$.

As per the question,

$y = 3x - - - - - - (1)$

$y - x = 26 - - - - - - (2)$

Put (1) in (2)

$3x - x = 26$

$x = 13$

Now,

$y = 39(\text{ from equation } (1))$

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

Solution.

Let us consider the larger angle be x^o and smaller angle be y^o.

As we know, the sum of 2 supplementary angles is always 180^o .

According to question,

$x + y = 180° - - - -(1)$

$x - y = 18° - - - -(2)$

From equation (1), we have

$x = 180° - y - - - -(3)$

Put equation (3) in equation (2), then we get

$180° - y - y = 18°$

$180° - 2y = 18°$

$y = 81°$

Put the value of y in equation (3), then we get

$x = 99°$

Hence, the angles are 99\degree and 81°.

(iii) The coach of a cricket team buys 7 bats and 6 balls for Rs.3800. Later, she buys 3 bats and 5 balls for Rs.1750. Find the cost of each bat and each ball.

Solution.

Let us consider the cost of a bat be x and the cost of a ball be y.

Now, as per the question,

$7x + 6y = 3800 - - - - - (1)$

$3x + 5y = 1750 - - - - - (2)$

From equation (1),

$y = \frac{3800-7x}{6} - - - -(3)$

Put equation (3) in equation (2)

$3x + 5\left(\frac{3800-7x}{6}\right) = 1700$

$3x + \frac{9500}{3} - \frac{35x}{6} = 1750$

$\frac{18x-35x}{6} = \frac{5250-9500}{3}$

$\frac{-17x}{6} = \frac{-4250}{3}$

$x = 500$

Therefore,

$y = \frac{3800-(7\times500)}{6}$

$y = 50$

Therefore, the cost of one bat is Rs 500 and the cost of one ball is Rs 50.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs 105 and for a journey of 15 km, the charge paid is Rs 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

Solution.

Let us considered the fixed charge = Rs x

And per km charge = Rs y.

Now, according to question,

$x + 10y = 105 - - - - - (1)$

$x + 15y = 155 - - - - - -(2)$

From equation 1, we have

$x = 105 - 10y$

Put x in equation 2

$105 - 10y + 15y = 155$

$y = 10$

Now,

$x = 105 - (10 \times 10)$

$x = 5$

Therefore, the fixed charge is equals to Rs 5 and per km charge is equal to Rs 10.

(v) **A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.**

Solution.

Let us consider, the fraction is $\frac{x}{y}$

According to question,

$\frac{x+2}{y+2} = \frac{9}{11}$

$11x + 22 = 9y + 18$

$11x - 9y = -4 ------(1)$

$\frac{x+3}{y+3} = \frac{5}{6}$

$6x + 18 = 5y + 15$

$6x - 5y = -3 ------(2)$

from equation(1), we get

$x = \frac{-4+9y}{11} ------(3)$

put x in equation (2)

$6\left(\frac{-4+9y}{11}\right) - 5y = -3$

$-24 + 54y - 55y = -33$

$y = 9$

now, put y in equation 3, we get

$x = 7$

(vi) **Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?**

Solution.

Let us consider the age of Jacob & his son to be x & y.

According to question,

$(x + 5) = 3(y + 5)$

$x - 3y = 10 ------(1)$

$(x - 5) = 7(y - 5)$

$x - 7y = 30 ------(2)$

from equation (1)

$x = 3y + 10$

Put x in equation (2))

$3y + 10 - 7y = -30$

$y = 10$

put y in equation (3), then we get

$x = 40$

Therefore, present age of Jacob & his son is 40 years & 10 years.

Exercise 3.4

Q1. Solve the following pair of linear equations by the elimination method and the substitution method:

(i) $\boldsymbol{x + y = 5}$ **and** $\boldsymbol{2x - 3y = 4}$

(ii) $\boldsymbol{3x + 4y = 10}$ **and** $\boldsymbol{2x - 2y = 2}$

(iii) $\boldsymbol{3x - 5y - 4 = 0}$ **and** $\boldsymbol{9x = 2y + 7}$

(iv) $\boldsymbol{\frac{x}{2} + \frac{2y}{3} = -1}$ **and** $\boldsymbol{x - \frac{y}{3} = 3}$

Solutions:

(i) $x + y = 5$ and $2x - 3y = 4$

Elimination Method

$x + y = 5 - - - - - - (1)$

$2x - 3y = 4 - - - - -(2)$

multiply the equation (1) with 2, then we get,

$2x + 2y = 10$

subtract the equation (2) from equation (3), then we get

$5y = 6$

$y = \frac{6}{5} - - - - - - -(4)$

Therefore,

$x = \frac{19}{5}$

Substitution Method

From equation (1), we get;

$x = 5 - y - - - - - - -(5)$

put x in equation (2), then

$2(5 - y) - 3y = 4$

$y = \frac{6}{5}$

Put y in equation (5), then

$x = \frac{19}{5}$

$(ii) \quad 3x + 4y = 10$ and $2x - 2y = 2$

Elimination Method

$3x + 4y = 10 - - - - - - (1)$

$2x - 2y = 2 - - - - - - -(2)$

multiply equation (1) and equation (2) with 2, then

$4x - 4y = 4 - - - - - - -(3)$

add equation (1) and (3), then

$7x = 14$

$x = 2$

put x in equation (1), then

$6 + 4y = 10$

$y = 1$

Substitution Method

From the equation (2) we get,

$x = 1 + y - - - - - - (5)$

put equation (5) in equation (1), then

$3(1 + y) + 4y = 10$

$y = 1$

put y in equation (5), then

$x = 1 + 1$

$x = 2$

$(iii)\ 3x - 5y - 4 = 0$ and $9x = 2y + 7$

Elimination Method

$3x - 5y - 4 = 0$

$9x = 2y + 7$

$9x - 2y - 7 = 0$

from equation (1) and equation (2), we get

$9x - 15 - y12 = 0 - - - - - - (3)$

subtract equation (3) from equation (2), then

$13y = -5$

$y = \frac{-5}{13} - - - - - (4)$

put equation (4) in equation (1), then

$3x + \frac{25}{13} - 4 = 0$

$x = \frac{9}{13}$

Substitution Method:

From equation (1) we get,

$x = \frac{5y+4}{3} - - - - (5)$

put x in equation (2), then

$9\left(\frac{5y+4}{3}\right) - 2y - 7 = 0$

$y = \frac{-5}{13}$

put y in equation (5), then we get

$x = \frac{5\left(-\frac{5}{13}\right)+4}{3}$

$x = \frac{9}{13}$

$(iv)\ \frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$

Elimination Method

$3x + 4y = -6 - - - - - (1)$

$3x - y = 9 - - - - - (2)$

Subtract equation(2)from equation(1), *then*

$5y = -15$

$y = -3 - - - - - (3)$

put equation (3) in equation (1)

$3x - 12 = -6$

$3x = 6$

$x = 2$

Substitution Method

From equation (2) we get;

$x = \frac{y+9}{3}$

put x in equation (1), then

$\frac{3(y+9)}{3} + 4y = -6$

$y = -3$

put y in equation (5), then

$x = 2$

Q2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method:

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes if we only add 1 to the denominator. What is the fraction?

Solution.

let us consider the fraction be $\frac{a}{b}$

according to question

$\frac{a+1}{b-1} = 1$

Therefore

$a - b = -2 - - - - (1)$

$\frac{a}{b+1} = \frac{1}{2}$

Therefore

$2a - b = 1 - - - - - - - (2)$

subtract equation (1) from equation (3), then

$a = 3 - - - - - - (3)$

put a in equation (1), then

$3 - b = -2$

$b = 5$

$b = 5$

(ii) **Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?**

Solution.

Let the present age of Nuri is x

And the present age of Sonu is y.

Now, according to the question;

$x - 5 = 3(y - 5)$

$x - 3y = -10 - - - - (1)$

$x + 10 = 2(y + 10)$

$x - 2y = 10 - - - - - - (2)$

subtract equation (1) from (2), then

$y = 20 - - - - - - -(3)$

put y in equation (1), then

$x - 3(20) = -10$

$x = 50$

Therefore,

The age of Nuri is 50 years

And the age of Sonu is 20 years.

(iii) **The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.**

Solution.

Let us consider the unit digit & tens digit of a number be A & B.

Then,

Number $(n) = 10B + A$

N after reversing the order of digits $= 10A + B$

According to question,

$A + B = 9 - - - - - - (1)$

$9(10B + A) = 2(10A + B)$

$88\,B\ - 11\,A\ =\ 0$

$-A + 8B = 0 - - - - - - (2)$

add equations (1) and (2), then

$9B\ =\ 9$

$B = 1 - - - - - - -(3)$

put B in equation (1), then

$A = 8$

(iv) Meena went to a bank to withdraw Rs.2000. She asked the cashier to give her Rs.50 and Rs.100 notes only. Meena got 25 notes in all. Find how many notes of Rs.50 and Rs.100 she received.

Solution.

Let us consider, number of Rs.50 notes be A and number of Rs.100 notes be B

According to the question,

$A\ +\ B\ =\ 25$

$50A\ +\ 100B\ =\ 2000$

Multiply equation (1) is with (2) we get;

$50A + 50B = 1250 - - - - - (3)$

Subtract equation (3) from equation (2) we get,

$50B\ =\ 750$

$B\ =\ 15$

put B in equation (1), then

$A = 10$

Therefore, Meena has 10 notes of Rs.50 & 15 notes of Rs.100.

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs.27 for a book kept for seven days, while Susy paid Rs.21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Solution.

Let us consider the fixed charge for first 3 days be Rs. A & the charge for each extra day be Rs. B.

According to the question,

$A + 4B = 27 - - - - - (1)$

$A + 2B = 21 - - - - - (2)$

Subtract equation (2) from (1)

$2B = 6$

$B = 3$

Put B in equation (1)

$A + 12 = 27$

$A = 15$

Therefore, fixed charge is Rs.15.

And Charge per day is Rs.3.

Exercise 3.5

1. **Which of the following pairs of linear equations has unique solution, no solution, or infinitely many solutions. In case there is a unique solution, find it by using cross multiplication method.**

(i) $\boldsymbol{x - 3y - 3 = 0}$ **and** $\boldsymbol{3x - 9y - 2 = 0}$

(ii) $\boldsymbol{2x + y = 5}$ **and** $\boldsymbol{3x + 2y = 8}$

(iii) $\boldsymbol{3x - 5y = 20}$ **and** $\boldsymbol{6x - 10y = 40}$

(iv) $\boldsymbol{x - 3y - 7 = 0}$ **and** $\boldsymbol{3x - 3y - 15 = 0}$

Solution.

(i) $x - 3y - 3 = 0$ and $3x - 9y - 2 = 0$

$\frac{a_1}{a_2} = \frac{1}{3}$

$\frac{b_1}{b_2} = \frac{1}{3}$

$\frac{c_1}{c_2} = \frac{3}{2}$

As, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Because the specified set of lines is parallel to each other, they will not intersect, and hence these equations have no solution.

(ii) $2x + y = 5$ and $3x + 2y = 8$

$\frac{a_1}{a_2} = \frac{2}{3}$

$\frac{b_1}{b_2} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{-5}{-8}$

as $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

By using the cross-multiplication approach, these equations will have unique solution because they intersect at a unique point:

$$\frac{x}{b_1c_2-c_1b_2}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-b_1a_2}$$

$$\frac{x}{(-8-(-10))}=\frac{y}{(-15-(-16))}=\frac{1}{4-3}$$

$$\frac{x}{2}=\frac{y}{1}=1$$

$$\therefore x=2 \text{ and } y=1$$

(iii) $3x-5y=20$ and $6x-10y=40$

$$\frac{a_1}{a_2}=\frac{3}{6}=\frac{1}{2}$$

$$\frac{b_1}{b_2}=\frac{-5}{-10}=\frac{1}{2}$$

$$\frac{c_1}{c_2}=\frac{20}{40}=\frac{1}{2}$$

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

There will be an endless number of solutions for these equations because the supplied sets of lines overlap.

(iv) $x-3y-7=0$ and $3x-3y-15=0$

$$\frac{a_1}{a_2}=\frac{1}{3}$$

$$\frac{b_1}{b_2}=\frac{-3}{-3}=1$$

$$\frac{c_1}{c_2}=\frac{-7}{-15}$$

as $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

Since these lines intersect each other at a unique point, so there should be a unique solution.

By cross multiplication

$$\frac{x}{45-21}=\frac{y}{-21+15}=\frac{1}{-3+9}$$

$$\frac{x}{24}=\frac{y}{-6}=\frac{1}{6}$$

$$\therefore x=4, y=1$$

2. (i) For which values of a and b does the following pair of linear equations have an infinite number of solutions?

$$\mathbf{2x+3y=7}$$

$$\mathbf{(a-b)x+(a+b)y=3a+b-2}$$

(i) For which value of k will the following pair of linear equations have no solution?

$$\mathbf{3x+y=1}$$

$$\mathbf{(2k-1)x+(k-1)y=2k+1}$$

Solution.

(i) $2x + 3y = 7$

$(a-b)x + (a+b)y = 3a + b - 2$

$\frac{a_1}{a_2} = \frac{2}{a-b}$

$\frac{b_1}{b_2} = \frac{3}{a+b}$

$\frac{c_1}{c_2} = \frac{-7}{-(3a+b-2)}$

for infinitely many solutions,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore \frac{2}{a-b} = \frac{7}{3a+b-2}$

$6a + 2b - 4 = 7a - 7b$

$a - 9b = -4 - - - - - - (1)$

$\frac{2}{a-b} = \frac{3}{a+b}$

$2a + 2b = 3a - 3b$

$a - 5b = 0$

put this equation in equation (1), then

$a - 5 x 1 = 0$

$a = 5$

Thus, at $a = 5$ & $b = 1$, the given equations have infinite solutions.

(ii) $3x + y = 1$

$(2k-1)x + (k-1)y = 2k + 1$

$\frac{a_1}{a_2} = \frac{3}{2k-1}$

$\frac{b_1}{b_2} = \frac{1}{k-1}$

$\frac{c_1}{c_2} = -\frac{1}{-2k-1} = \frac{1}{2k+1}$

for no solution

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$

$\frac{3}{2k-1} = \frac{1}{k-1}$

$k = 2$

Thus, for k = 2 the pair of linear equations have no solution.

3. Solve the following pair of linear equations by the substitution and cross-multiplication methods:

$\mathbf{8x + 5y = 9}$

$\mathbf{3x + 2y = 4}$

Solution.

$8x + 5y = 9$

$3x + 2y = 4$

$8x + 5y = 9 ------(1)$

$3x + 2y = 4 ----- (2)$

from equation (2), we get

$x = \frac{4-2y}{3}$

put x in equation (1)

$\frac{8(4-2y)}{3} + 5y = 9$

$32 - 16y + 15y = 27$

$-y = -5$

$y = 5$

put x in equation (2)

$3x + 10 = 4$

$x = -2$

Cross Multiplication

$8x + 5y - 9 = 0$

$3x + 2y - 4 = 0$

$\frac{x}{-20+18} = \frac{y}{-27+32} = \frac{1}{16-15}$

$\frac{-x}{2} = \frac{y}{5} = 1$

$\therefore x = -2, y = 5$

4. Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

(i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs.1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs.1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii) A fraction becomes 1/3 when 1 is subtracted from the numerator and it becomes 1/4 when 8 is added to its denominator. Find the fraction.

(iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks

been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Solution.

(i) Let us consider x be the fixed charge & y be the charge of per day food.
As per the question,

$x = 20y = 1000 - - - - - (1)$

$x + 26y = 1180 - - - - - (2)$

subtract equation (1) from equation (2, then)

$6y = 180$

$y = Rs.30$

put y in equation 2, then

$x = 1180 - 26 \times 30$

$x = Rs.400$

Thus, the fixed charges is Rs.400 & the charge per day is Rs.30.

(ii) Let us consider the fraction be $\frac{x}{y}$

So, according to the given the question,

$\frac{x-1}{y} = \frac{1}{3}$

$\therefore 3x - y = 3 - - - - - (1)$

$\frac{x}{y+8} = \frac{1}{4}$

$\therefore 4x - y = 8 - - - - - (2)$

subtract equation (1) from equation (1), then we get

$x = 5 - - - - - (3)$

put x in equation (2), then we get

$(4 \times 5) - y = 8$

$y = 12$

(iii) Let us consider the number of correct answers is x & number of wrong answers is y

Now, as per the given question;

$3x - y = 40 ------(1)$

$4x - 2y = 50$

$\therefore 2x - y = 25 ----- (2)$

subtract equation (2) from equation (1), then we get

$x = 15$

put x in equation (2), then we get

$30 - y = 25$

$y = 5$

Thus, number of correct answers is 15 & number of wrong answers is 5

Therefore, total number of questions are 20.

(iv) Let us consider the speed of car from point A be x km/h & the speed of car from point B is y km/h.

When the car travels in same direction,

$5x - 5y = 100$

$x - y = 20 ----- (1)$

When the car travels in opposite direction, then

$x + y = 100 ----(2)$

solving the equations (1) and (2), then we get

$x = 60\ km/h$

put x in equation (1), then we get

$60 - y = 20$

$y = 40\ km/h$

Thus, the speed of car from point A is 60 km/h and the Speed of car from point B is 40 km/h.

(v) Let us consider

Length of rectangle = x unit

Breadth of rectangle = y unit

According to the given question;

$(x-5)(y+3) = xy - 9$

$3x - 5y - 6 = 0 - - - - - - (1)$

$(x+3)(y+2) = xy + 67$

$2x + 3y - 61 = 0 - - - - - - (2)$

use cross multiplication method, then

$\frac{x}{305+18} = \frac{y}{-12+183} = \frac{1}{9+10}$

$\frac{x}{323} = \frac{y}{1713} = \frac{1}{19}$

$\therefore x = 17, y = 9$

Thus, the length of the rectangle is 17 units and the breadth of rectangle is 9 units.

Exercise 3.6

1. **Solve the following pairs of equations by reducing them to a pair of linear equations:**

(i) $\mathbf{\frac{1}{2x} + \frac{1}{3y} = 2}$

And

$\mathbf{\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}}$

Solution.

Let $\frac{1}{x} = m$ and $\frac{1}{y} = n$ then the equation will change as follows.

$\frac{m}{2} + \frac{n}{3} = 2$

$\therefore 3m + 2n - 12 = 0 - - - - (1)$

$\frac{m}{3} + \frac{n}{2} = \frac{13}{6}$

$\therefore 2m + 3n - 13 = 0 - - - - - - (2)$

Use cross multiplication method, then

$\frac{m}{-26-(-36)} = \frac{n}{-24-(-39)} = \frac{1}{9-4}$

$\frac{m}{10} = \frac{n}{15} = \frac{1}{5}$

$\therefore m = 2, n = 3$

$\frac{1}{x} = 2$

$\therefore x = \frac{1}{2}$

$\frac{1}{y} = 3$

$\therefore x = \frac{1}{3}$

(ii) $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$

$\frac{4}{\sqrt{x}} + \frac{9}{\sqrt{y}} = 1$

Solution.

put $\frac{1}{\sqrt{x}} = m$ and $\frac{1}{\sqrt{y}} = n$

$2m + 3n = 2 ------(1)$

$4m - 9n = -1 ----- (2)$

multiply the equation (1) with 3, then we get

$6m + 9n = 6 ----- (3)$

add equation (2) and (3), then we get

$10m = 5$

$m = \frac{1}{2}$

put m in equation (1) then

$2 \times \frac{1}{2} + 3n = 2$

$3n = 1$

$n = \frac{1}{3}$

$\frac{1}{\sqrt{x}} = m$

$\frac{1}{\sqrt{x}} = \frac{1}{2}$

$x = 4$

$\frac{1}{\sqrt{y}} = n$

$\frac{1}{\sqrt{y}} = \frac{1}{3}$

$y = 9$

(iii) $\frac{4}{x} + 3y = 14$

$\frac{3}{x} - 4y = 23$

Solution.

Let $\frac{1}{x} = m$ then the equation will change as follows.

$4m + 3y = 14$

$\therefore 4m + 3y - 14 = 0 - - - - - - -(1)$

$3m - 4y = 23$

$\therefore 3m - 4y - 23 = 0 - - - - - - (2)$

using cross multiplication method

$\frac{m}{-69-56} = \frac{y}{-42-(-92)} = \frac{1}{-16-9}$

$\frac{-m}{125} = \frac{y}{50} = \frac{-1}{25}$

$\therefore m = 5$

$y = -2$

$\therefore x = \frac{1}{5}$

(iv) $\mathbf{\frac{5}{x-1} + \frac{1}{y-2} = 2}$

$\mathbf{\frac{6}{x-1} - \frac{3}{y-2} = 1}$

Solution :

Let $\frac{1}{x} = m$ and $\frac{1}{y} = n$ then the equation will change as follows.

$5m + n = 2 - - - - - - -(1)$

$6m - 3m = 1 - - - - - -(2)$

Equation (1) by 3, then

$15m + 3n = 6 - - - - - - -(3)$

Add equation (2) and (3), then

$21m = 7$

$m = \frac{1}{3}$

Put m in equation (1), then

$5 \times 3 + n = 2$

$n = \frac{1}{3}$

Now,

$m = \frac{1}{x-1}$

$\therefore x = 4$

and

$n = \frac{1}{y-2}$

$\therefore y = 5$

(v) $\frac{7x-2y}{xy} = 5$

$\frac{8x+7y}{xy} = 15$

Solution.

$\frac{7x-2y}{xy} = 5$

$\frac{7}{y} - \frac{2}{x} = 5 - - - - - - (1)$

$\frac{8x+7y}{xy} = 15$

$\frac{8}{y} + \frac{7}{x} = 15 - - - - (2)$

Put $\frac{1}{x} = m$ and $\frac{1}{y} = n$

$-2m + 7n = 5$

$\therefore -2m + 7n - 5 = 0 - - - - (3)$

$7m + 8n = 15$

$7m + 8n - 15 = 0$

Use cross multiplication, the

$\frac{m}{-105-(-40)} = \frac{n}{-35-30} = \frac{1}{-16-49}$

$\frac{m}{-65} = \frac{n}{-65} = \frac{1}{-65}$

$\therefore m = 1$

$n = 1$

$\therefore x = 1$

$y = 1$

(vi) $6x + 3y = 6xy$

$2x + 4y = 5xy$

Solution.

$6x + 3y = 6xy$

$\frac{6}{y} + \frac{3}{x} = 6$

put $\frac{1}{x} = m$ and $\frac{1}{y} = n$

$\therefore \therefore 6n + 3m = 6$

$3m + 6n - 6 = 0 ----(1)$

now $2x + 4y = 5xy$

$\frac{2}{y} + \frac{4}{x} = 5$

$2n + 4m = 5$

$2n + 4m - 5 = 0$

Use cross multiplication method, then

$\frac{m}{-30-(-12)} = \frac{n}{-24-(15)} = \frac{1}{6-24}$

$\frac{m}{-18} = \frac{n}{-9} = \frac{1}{-18}$

$\therefore m = 1, n = \frac{1}{2}$

$Now,$

$m = \frac{1}{x}$

$\therefore x = 1$

and

$n = \frac{1}{y}$

$y = 2$

(vii) $\frac{10}{x+y} + \frac{2}{x-y} = 4$

$\frac{15}{x+y} - \frac{5}{x-y} = -2$

Solution.

put $\frac{1}{x+y} = m$ and $\frac{1}{x-y} = n$

$10m + 2n = 4$

$10m + 2n - 4 ----(1)$

$15m - 5n = -2$

$15m - 5n + 2 = 0 ----(2)$

Use cross multiplication method

$\frac{m}{4-20} = \frac{n}{-60-(20)} = \frac{1}{-50-30}$

$\frac{m}{-16} = \frac{n}{-80} = \frac{1}{-80}$

$\therefore m = \frac{1}{5}$

$n = 1$

$x + y = 5 ------ (3)$

$x - y = 1 ------ (4)$

add equation (1) and (2), then

$2x = 6$

$x = 3$

put x in equation (3), then

$y = 2$

(viii) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$

$\frac{1}{2(3x+y)} + \frac{1}{2(3x-y)} = \frac{-1}{8}$

Solution.

put $\frac{1}{3x+y} = m$ and $\frac{1}{3x-y} = n$

$m + n = \frac{3}{4} ------ (1)$

$\frac{m}{2} - \frac{n}{2} = \frac{-1}{8}$

$m - n = \frac{-1}{4} ----(2)$

Add equation (1) and (2), then

$2m = \frac{3}{4} - \frac{1}{4}$

$2m = \frac{1}{2}$

put m in equation (2)

$\frac{1}{4} - n = \frac{-1}{4}$

$\therefore n = \frac{1}{2}$

$m = \frac{1}{4}$

$3x + y = 4 - - - - (3)$

$n = \frac{1}{3x-y} = \frac{1}{2}$

$3x - y = 2 - - - - (4)$

Add equation (3) and (4), then

$6x = 6$

$x = 1$

put x in equation (3), then we get

$y = 1$

2. Formulate the following problems as a pair of equations, and hence find their solutions:

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Solution.

(i)

Let,

Ritu's speed in still water = x km/hr

Speed of Stream = y km/hr

Now, the speed of Ritu during,

Downstream $= x + y$ km/hr

Upstream $= x - y$ km/hr

According to question

$2(x + y) = 20$

$x + y = 10 - - - - - (1)$

$and,$

$2x = 12$

$x = 6$

put x in equation (1), then we get

$y = 4$

Therefore, the speed of Ritu rowing in the still water is 6 km/hr.

And the speed of Stream is 4 km/hr.

(ii) Let,

Total number of days taken by the women to finish the work is x

Total number of days taken by the men to finish the work is y

Work done by the women in 1 day = $\frac{1}{x}$

Work done by the women in 1 day = $\frac{1}{y}$

According to the given question,

$4\left(\frac{2}{x} + \frac{5}{y}\right) = 1$

$\left(\frac{2}{x} + \frac{5}{y}\right) = \frac{1}{4}$

$3\left(\frac{3}{x} + \frac{6}{y}\right) = 1$

$\left(\frac{3}{x} + \frac{6}{y}\right) = \frac{1}{3}$

put $m = \frac{1}{x}$ and $n = \frac{1}{y}$

$2m + 5n = \frac{1}{4}$

$\therefore 8m + 20n = 1 - - - - - - (1)$

$3m + 6n = \frac{1}{3}$

$\therefore 9m + 18n = 1$

use cross multiplication

$\frac{m}{20-18} = \frac{n}{9-8} = \frac{1}{180-144}$

$\frac{m}{2} = \frac{n}{1} = \frac{1}{36}$

$\therefore m = \frac{1}{18}$

$m = \frac{1}{x}$

$\therefore x = 18$

$n = \frac{1}{36}$

$n = \frac{1}{y}$

$y = 36$

Thus,

The number of days taken by the women to finish the work is 18.

And the number of days taken by the men to finish the work is 36.

(iii) Let,

The speed of train = x km/h

The speed of bus = y km/h

So, per the given question,

$\frac{60}{x} + \frac{240}{y} = 4 ------(1)$

$\frac{100}{x} + \frac{200}{y} = \frac{25}{6} ----(2)$

put $m = \frac{1}{x}$ and $n = \frac{1}{y}$

$60m + 240n = 4 -------(3)$

$100m + 200n = \frac{25}{6}$

$600m + 1200n = 25 ----(4)$

multiply equation (3) with 10, then we get

$600m + 2400n = 40 ------(5)$

subtract equation (4) from equation (5), then we get

$1200n = 15$

$n = \frac{1}{80}$

$y = 80$

put n in equation (3)

$60m + 3 = 4$

$m == \frac{1}{60}$

$m = \frac{1}{x}$

$\therefore x = 60$

$y = 80$

Thus,

The speed of train is 60 km/h

The speed of the bus is 80 km/h.

Exercise 3.7

Q1. The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.

Solution.

Ani and Biju are three years apart in age.

Biju is three years older than Ani, or Ani is three years older than Biju. We can deduct from both situations that Ani's father is 30 years older than Cathy's father.

Let Ani and Biju's ages be A and B, respectively.

Thus,

$\text{age of Dharam} = 2 \times A = 2A \text{ years}$

$\text{age of Biju sister Ani is } = \frac{B}{2} \text{ years}$

When Ani is older than Biju by 3 years then;

1st case:

$A - B = 3 - - - -(1)$

$2AA - \frac{B}{2} = 30$

$4A - B = 60 - - - - - (2)$

$\text{subtract equation (1) from equation (2)}$

$3A = 60 - 3$

$A = 19$

Thus, the age of Ani is 19 years

And the age of Biju is 19 – 3 equals to 16 years.

2nd Case:

$B - A = 3 - - - - (1)$

$2A - \frac{B}{2} = 30$

$4A - B = 60 - - - - (2)$

Add equation (1) and (2)

$3A = 63$

$A = 21$

Thus, the age of Ani is 21 years

And the age of Biju is 21 + 3 equals to 24 years.

Q2. One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital? [From the Bijaganita of Bhaskara II] [Hint : $x + 100 = 2(y - 100), y + 10 = 6(x - 10)$].

Solution.

Let us consider that Sangam have Rs A with him & Reuben have Rs B with him.

Now;

$A + 100 = 2(B - 100)$

$\therefore A + 100 = 2B - 200$

$A - 2B = -300 - - - - (1)$

And

$6(A - 10) = (B + 10)$

$6A - 60 = B + 10$

$6A - B = 70 - - - - (2)$

Multiply equation (2) with 2, then

$12A - 2B = 140$

subtract equation (1) from equation (2), then we get

$11A = 140 + 300$

$11A = 440$

$A = 40$

Put A in equation (1)

$40 - 2B = -300$

$40 + 300 = 2B$

$2B = 340$

$B = 170$

Thus, Sangam had Rs 40 with him & Reuben had Rs 170 with him.

Q3. A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Solution.

Let us consider the speed of train be A km/hr & time taken by the train to travel a distance be N hours and distance to travel be X hours.

Speed of train = Distance travelled by the train / Time taken to travel that distance by the train

$A = \frac{N(Distance)}{X(Time)}$

$N = AX - - - - (1)$

Now, we get

$A + 10 = \frac{X}{N-2}$

$(A + 10)(N - 2) = X$

$AN + 10N - 2A - 20 = X$

from equation (1)

$-2A + 10N = 20 - - - - - (2)$

$A - 10 = \frac{X}{N+3}$

$(A - 10)(N + 3) = X$

$AN - 10N + 3A - 30 = X$

from equation (1)

$3A - 10N = 30 - - - - - (3)$

add equation (2) and equation (3), then we get

$A = 50$

put A in equation (1), then we get

$(-2) \times (50) + 10N = 20$

$-100 + 10N = 20$

$N = 12\ hrs$

from equation (1), the distance travelled by the train is

$x = AN$

$X = 50 \times 12$

$X = 600km$

Q4. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Solution.

Let us consider the number of rows be A & the total number of students in a row is B.

The total Number of students = Total Number of rows x Total Number of students in a row

=AB

1st Condition:

total number of students $= (A-1)(B+3)$

$AB = (A-1)(B+3) = AB - B + 3A - 3$

$3A - B = 3 - - - - - - (1)$

2nd Condition:

total number of students $= (A+2)(B-3)$

$AB = AB + 2B - 3A - 6$

$3A - 2B = -6 - - - - - - (2)$

subtract equation (2) from equation (1), then we get

$(3A - B) - (3A - 2B) = 3 - (-6)$

$-B + 2B = 3 + 6$

$B = 9$

put B in equation (1), then we get

$3A - 9 = 3$

$A = 4$

total number of rows, $A = 4$

total number of students in a row, $B = 9$

number of total students in a class $= AB = 4 \times 9 = 36$

Q5. In a $\triangle ABC, \angle C = 3\angle B = 2(\angle B + \angle A)$. Find the three angles.

Solution.

$\angle C = 3\angle B = 2(\angle B + \angle A)$

$3\angle B = 2\angle A + 2\angle B$

$\angle B = 2\angle A$

$2\angle A - \angle B = 0$

we know, sum of all interior angles of a triangle is 180 degree

$\angle A + \angle B + \angle C = 180^o$

$\angle A + \angle B + 3\angle B = 180^o$

$\angle A + 4\angle B = 180^o - - - - - (2)$

multiply equation (1) with 4, then we get

$8\angle A - 4\angle B = 0$

Add equation (2) and (3), then we get

$9\angle A = 180^o$

$\angle A = 20^o$

put A in equation (2), then we get

$20^o + 4\angle B = 180^o$

$\angle B = 40^o$

$3\angle B = \angle C$

$\angle C = 120^o$

Q6. Draw the graphs of the equations $5x - y = 5$ and $3x - y = 3$. Determine the co-ordinates of the vertices of the triangle formed by these lines and the y axis.

Solution.

$5x - y = 5$

$y = 5x - 5$

Solution table is:

x	2	1	0
y	5	0	-5

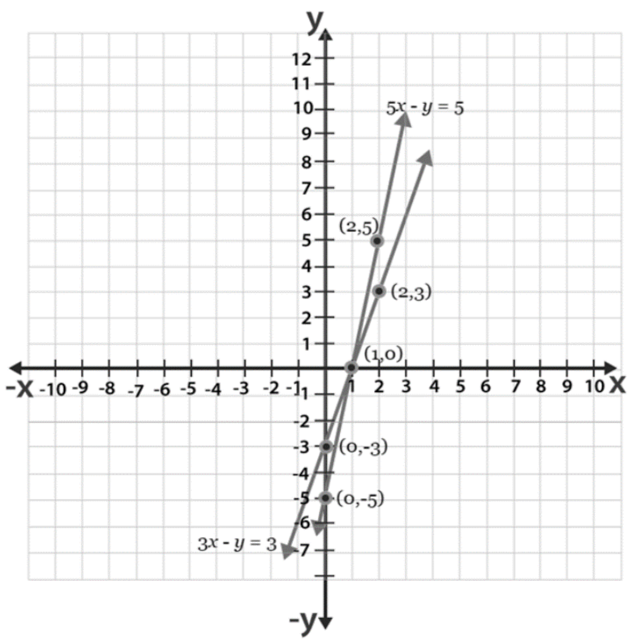
$3x - y = 3$

$y = 3x - 3$

Solution table is:

x	2	1	0
y	3	0	-3

Graphical Representation



The triangle created by the lines & the y axis is ABC, as shown in the graph above. A(1,0), C(0,-5) and B are the coordinates of vertices (0,-3).

Q7. Solve the following pair of linear equations:

(i) $px + qy = p - q$

$qx - py = p + q$

(ii) $ax + by = c$

$bx + ay = 1 + c$

(iii) $\boldsymbol{\frac{x}{a} - \frac{y}{b} = 0}$

$\boldsymbol{ax + by = a^2 + b^2}$

(iv) $\boldsymbol{(a-b)x + (a+b)y = a^2 - 2ab + b^2}$

$\boldsymbol{(a + b(x + y) = a^2 + b^2}$

(v) $\boldsymbol{152x - 378y = -74}$

$\boldsymbol{-378x + 152y = -604}$

Solution.

(i) $px + qy = p - q$

$qx - py = p + q$

multiply the equation (1) with p and equation (2) with q, then

$p^2x + pqy = p^2 - pq - - - - - - -(3)$

$q^2x - pqy = q^2 + pq - - - - - - (4)$

adding equation (3) and (4), then

$p^2x + q^2x = p^2 + q^2$

$(p^2 + q^2)x = p^2 + q^2$

$\therefore x = 1$

put x in equation (1), then

$p(1) + qy = p - q$

$qy = p - q - p$

$y = -1$

(ii) $ax + by = c$

$bx + ay = 1 + c$

multiply the equation (1) with a and equation (2) with b, then

$a^2x + aby = ac - - - - - - -(3)$

$b^2x + aby = b + bc - - - - - - (4)$

subtract equation (4) from equation (3), then

$(a^2 + b^2)x = ac - bc - b$

$x = \frac{(ac-bc-b)}{(a^2-b^2)}$

$x = \frac{(c(a-b)-b)}{(a^2-b^2)}$

from equation (1), we get

$ax + by = c$

$\frac{a(c(a-b)-b)}{(a^2-b^2)} + by = c$

$by = c - ac(a-b) - \frac{ab}{(a^2-b^2)}$

$y = c(a-b) + \frac{a}{(a^2-b^2)}$

(iii) $\frac{x}{a} - \frac{y}{b} = 0$

$ax + by = a^2 + b^2$

$\frac{x}{a} - \frac{y}{b} = 0$

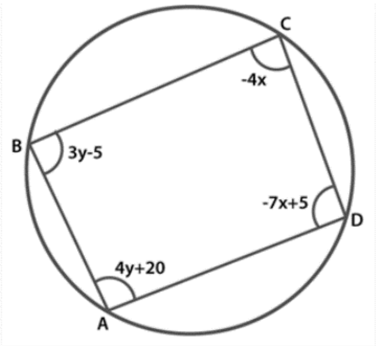
$bx - ay = 0 - - - -(1)$

$ax + by = a^2 + b^2 ------ (2)$
a multiply a and b to equation (1) and (2), then we get
$b^2x - aby = 0 ------ (3)$
$x(b^2 + a^2) = a(a^2 + b^2)x = a$
from equation (1)
$b(a) - ay = 0$
$ab - ay = 0$
$y = b$

(iv) $(a - b)x + (a + b)y = a^2 - 2ab + b^2$
$\left(a + b(x + y)\right) = a^2 + b^2$
$(a + b)y + (a - b)x = a^2 - 2ab + b^2 ------ (1)$
$(x + y)(a + b) = a^2 + b^2$
$(a + b)y + (a + b)x = a^2 + b^2 ------ (2)$
subtract equation (2) from equation (1), then
$(a - b)x - (a + b)x = (a^2 - 2ab + b^2) - (a^2 + b^2)$
$x(a - b - a - b) = -2ab - 2b^2$
$x = b + a$
put x in equation (1), then
$(a + b)y + (a - b)(a + b) = a^2 - 2ab + b^2$
$(a + b)y = -2ab$
$y = \frac{-2ab}{a+b}$

(v) $152x - 378y = -74$
$-378x + 152y = -604$
$x = \frac{189y-37}{76} ------ (1)$
$-378x + 152y = -604$
$-189x + 76y = -302 ------ (2)$
put x in equation (2)
$-189\left(\frac{189y-37}{76}\right) + 76y = -302$
$-(189)^2y + 189 \times 37 + (76)^2y = -302 \times 76$
$6993 + 22952 = (189 - 76)(189 + 76)y$
$29945 = (113)(265)y$
$y = 1$
put y in (1)
$x = \frac{189-37}{76}$
$x = 2$

Q8. ABCD is a cyclic quadrilateral. Find the angles of the cyclic quadrilateral.



Solution.

As we known that the sum of opposite angles of a cyclic quadrilateral is equals to $180°$.

Therefore,

$$\angle A + \angle C = 180°$$
$$4y + 20 - 4x = 180$$
$$4y - 4x = 160$$
$$y - x = 40 - - - - - - (1)$$
$$\angle B + \angle D = 180$$
$$3y - 5 - 7x + 5 = 180$$
$$3y - 7x = 180 - - - - - - (2)$$
multiply equation (1) with 3
$$3y - 3x = 120 - - - - - - (3)$$
add equation (2) and (3)
$$-7x + 3x = 180 - 120$$
$$\therefore x = -15$$
put x in equation (1), then
$$x - y = 40$$
$$-15 - y = 40$$
$$y = 25$$
$$\angle A = 4y + 20 = 20 + 4(25) = 120°$$
$$\angle B = 3y - 5 = 70°$$
$$\angle C = -4x = 60°$$
$$\angle D = 5 - 7x = 110°$$