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JEE MAIN 2021

ONLINE
22nd July
2nd Shift

PHYSICS

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. Match List-I with List-II.

List-I	List-II
(A) $\omega L > \frac{1}{\omega C}$	(i) Current is in phase with emf
(B) $\omega L = \frac{1}{\omega C}$	(ii) Current lags behind the applied emf
(C) $\omega L < \frac{1}{\omega C}$	(iii) Maximum current occurs
(D) Resonant frequency	(iv) Current leads the emf

Choose the correct answer from the options given below:

(a) (A) - (iv), (B) - (iii), (C) - (ii), (D) - (i)
(b) (A) - (ii), (B) - (i), (C) - (iv), (D) - (iii)
(c) (A) - (iii), (B) - (i), (C) - (iv), (D) - (ii)
(d) (A) - (ii), (B) - (i), (C) - (iii), (D) - (iv)

2. **Statement-I :** The ferromagnetic property depends on temperature. At high temperature, ferromagnet becomes paramagnet.

Statement-II : At high temperature, the domain wall area of a ferromagnetic substance increases.

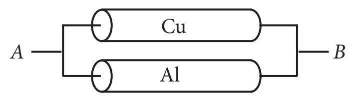
In the light of the above statements, choose the most appropriate answer from the options given below.

(a) Both Statement-I and Statement-II are true.
(b) Both Statement-I and Statement-II are false.
(c) Statement-I is true but Statement-II is false.
(d) Statement-I is false but Statement-II is true.

3. Consider a situation in which reverse biased current of a particular *P-N* junction increases when it is exposed to a light of wavelength ≤ 621 nm. During this process, enhancement in carrier concentration take place due to generation of hole -electron pairs. The value of band gap is

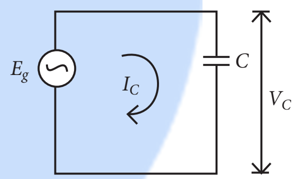
(a) 4 eV (b) 1 eV
(c) 0.5 eV (d) 2 eV

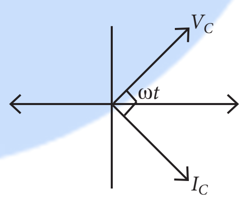
4. A copper (Cu) rod of length 25 cm and cross-sectional area 3 mm^2 is joined with a similar aluminium (Al) rod as shown in figure. Find the resistance of the combination between the ends *A* and *B*. (Take resistivity of copper = 1.7×10^{-8} Ω m, Resistivity of aluminium = 2.6×10^{-8} Ω m).

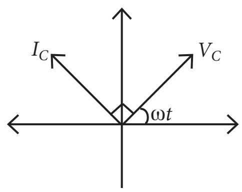


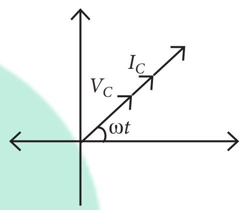
(a) 2.170 mΩ (b) 0.858 mΩ
(c) 0.0858 mΩ (d) 1.420 mΩ

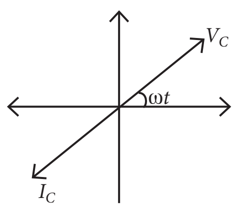
5. In a circuit consisting of a capacitance and a generator with alternating emf $E_g = E_{g0} \sin\omega t$, V_C and I_C are the voltage and current. Correct phasor diagram for such circuit is



(a)


(b)


(c)


(d)


6. What will be the projection of vector $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ on vector $\vec{B} = \hat{i} + \hat{j}$?

(a) $\sqrt{2}(\hat{i}+\hat{j})$ (b) $(\hat{i}+\hat{j})$
(c) $\sqrt{2}\,(\hat{i}+\hat{j}+\hat{k})$ (d) $2\,(\hat{i}+\hat{j}+\hat{k})$

7. Consider a situation in which a ring, a solid cylinder and a solid sphere roll down on the same inclined plane without slipping. Assume that they start rolling from rest and having identical diameter. The correct statement for this situation is

(a) The ring has the greatest and the cylinder has the least velocity of the centre of mass at the bottom of the inclined plane.
(b) The cylinder has the greatest and the sphere has the least velocity of the centre of mass at the bottom of the inclined plane.

(c) The sphere has the greatest and the ring has the least velocity of the centre of mass at the bottom of the inclined plane.
(d) All of them will have same velocity.

8. What should be the height of transmitting antenna and the population covered if the television telecast is to cover a radius of 150 km? The average population density around the tower is 2000/km^2 and the value of $R_e = 6.5 \times 10^6$ m.
(a) Height = 1800 m
Population covered = 1413×10^8
(b) Height = 1600 m
Population covered = 2×10^5
(c) Height = 1731 m
Population covered = 1413×10^5
(d) Height = 1241 m
Population covered = 7×10^5

9. Intensity of sunlight is observed as 0.092 Wm^{-2} at a point in free space. What will be the peak value of magnetic field at that point? ($\varepsilon_0 = 8.85 \times 10^{-12}$ C^{2}N^{-1} m^{-2})
(a) 1.96×10^{-8} T (b) 2.77×10^{-8} T
(c) 5.88 T (d) 8.31 T

10. An electric dipole is placed on x-axis in proximity to a line charge of linear charge density 3.0×10^{-6} C/m. Line charge is placed on z-axis and positive and negative charge of dipole is at a distance of 10 mm and 12 mm from the origin respectively. If total force of 4 N is exerted on the dipole, find out the amount of positive or negative charge of the dipole.
(a) 4.44 μC (b) 8.8 μC
(c) 0.485 mC (d) 815.1 nC

11. A nucleus with mass number 184 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 MeV, calculate the kinetic energy of the α-particle.
(a) 5.38 MeV (b) 0.12 MeV
(c) 5.0 MeV (d) 5.5 MeV

12. A body is projected vertically upwards from the surface of earth with a velocity sufficient enough to carry it to infinity. The time taken by it to reach height h is ______ s.

(a) $\sqrt{\dfrac{2R_e}{g}}\left[\left(1+\dfrac{h}{R_e}\right)^{3/2}-1\right]$

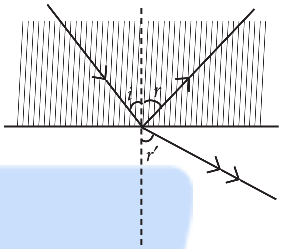
(b) $\dfrac{1}{3}\sqrt{\dfrac{R_e}{2g}}\left[\left(1+\dfrac{h}{R_e}\right)^{3/2}-1\right]$

(c) $\sqrt{\dfrac{R_e}{2g}}\left[\left(1+\dfrac{h}{R_e}\right)^{3/2}-1\right]$

(d) $\dfrac{1}{3}\sqrt{\dfrac{2R_e}{g}}\left[\left(1+\dfrac{h}{R_e}\right)^{3/2}-1\right]$

13. Choose the correct option.
(a) True dip is always greater than the apparent dip.
(b) True dip is less than the apparent dip.
(c) True dip is not mathematically related to apparent dip.
(d) True dip is always equal to apparent dip.

14. A ray of light passes from a denser medium to a rarer medium at an angle of incidence i. The reflected and refracted rays make an angle of 90° with each other. The angle of reflection and refraction are respectively r and r'. The critical angle is given by

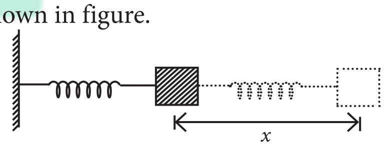


(a) $\sin^{-1}(\tan r')$ (b) $\tan^{-1}(\sin i)$
(c) $\sin^{-1}(\tan r)$ (d) $\sin^{-1}(\cot r)$

15. T_0 is the time period of a simple pendulum at a place. If the length of the pendulum is reduced to $\dfrac{1}{16}$ times of its initial value, the modified time period is
(a) $8\pi T_0$ (b) $\dfrac{1}{4}T_0$
(c) T_0 (d) $4T_0$

16. A bullet of 4 g mass is fired from a gun of mass 4 kg. If the bullet moves with the muzzle speed of 50 ms^{-1}, the impulse imparted to the gun and velocity of recoil of gun are
(a) 0.4 kg m s^{-1}, 0.1 m s^{-1} (b) 0.4 kg m s^{-1}, 0.05 m s^{-1}
(c) 0.2 kg m s^{-1}, 0.05 m s^{-1} (d) 0.2 kg m s^{-1}, 0.1 m s^{-1}

17. The motion of a mass on a spring with spring constant K is as shown in figure.



The equation of motion is given by $x(t) = A\sin\omega t + B\cos\omega t$ with $\omega = \sqrt{\dfrac{K}{m}}$. Suppose that at time $t = 0$, the position of mass is $x(0)$ and velocity $v(0)$, then its displacement can also be represented as $x(t) = C\cos(\omega t - \phi)$, where C and ϕ are

(a) $C = \sqrt{\dfrac{2v(0)^2}{\omega^2} + x(0)^2}$, $\phi = \tan^{-1}\left(\dfrac{x(0)\omega}{2v(0)}\right)$

(b) $C = \sqrt{\dfrac{v(0)^2}{\omega^2} + x(0)^2}$, $\phi = \tan^{-1}\left(\dfrac{x(0)\omega}{v(0)}\right)$

(c) $C=\sqrt{\frac{2v(0)^2}{\omega^2}+x(0)^2}$, $\phi=\tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$

(d) $C=\sqrt{\frac{v(0)^2}{\omega^2}+x(0)^2}$, $\phi=\tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$

18. An electron of mass m_e and a proton of mass m_p are accelerated through the same potential difference. The ratio of the de-Broglie wavelength associated with the electron to that with the proton is

(a) 1 (b) $\frac{m_p}{m_e}$ (c) $\frac{m_e}{m_p}$ (d) $\sqrt{\frac{m_p}{m_e}}$

19. A porter lifts a heavy suitcase of mass 80 kg and at the destination lowers it down by a distance of 80 cm with a constant velocity. Calculate the work done by the porter in lowering the suitcase. (Take $g = 9.8\ ms^{-2}$)

(a) –62720.0 J (b) +627.2 J
(c) –627.2 J (d) 784.0 J

20. What will be the average value of energy for a monoatomic gas in thermal equilibrium at temperature T?

(a) k_BT (b) $\frac{3}{2}k_BT$
(c) $\frac{1}{2}k_BT$ (d) $\frac{2}{3}k_BT$

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

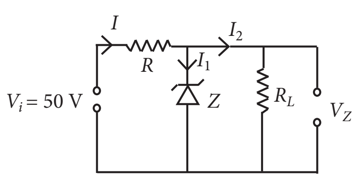
21. The total charge enclosed in an incremental volume of $2 \times 10^{-9}\ m^3$ located at the origin is _____ nC, if eletric flux density of its field is found as

$$D = e^{-x}\sin y\,\hat{i} - e^{-x}\cos y\,\hat{j} + 2z\hat{k}\ \ C/m^2.$$

22. The position of the centre of mass of a uniform semi-circular wire of radius R placed in x-y plane with its centre at the origin and the line joining its ends as x-axis is given by $\left(0, \frac{xR}{\pi}\right)$. Then the value of $|x|$ is _____.

23. In an electric circuit, a cell of certain emf provides a potential difference of 1.25 V across a load resistance of 5 Ω. However, it provides a potential difference of 1 V across a load resistance of 2 Ω. The emf of the cell is given by $\frac{x}{10}$ V. Then the value of x is _____.

24. In a given circuit diagram, a 5 V zener diode along with a series resistance is connected across a 50 V power supply. The minimum value of the resistance required, if the maximum zener current is 90 mA will be ____ Ω.



25. Three students S_1, S_2 and S_3 perform an experiment for determining the acceleration due to gravity (g) using a simple pendulum. They use different lengths of pendulum and record time for different number of oscillations. The observations are as shown in the table.

Student no.	Length of pendulum (cm)	No. of oscillations (n)	Total time for n oscillations	Time period (s)
1.	64.0	8	128.0	16.0
2.	64.0	4	64.0	16.0
3.	20.0	4	36.0	9.0

(Least count of length = 0.1 cm, least count for time = 0.1 s)

If E_1, E_2 and E_3 are the percentage errors in 'g' for students 1, 2 and 3 respectively, then the minimum percentage error is obtained by student no. _____.

26. Three particles P, Q and R are moving along the vectors $\vec{A}=\hat{i}+\hat{j}$, $\vec{B}=\hat{j}+\hat{k}$ and $\vec{C}=-\hat{i}+\hat{j}$ respectively. They strike on a point and start to move in different directions. Now, particle P is moving normal to the plane which contains vector $\vec{A}$ and $\vec{B}$. Similarly particle Q is moving normal to the plane which contains vector $\vec{A}$ and $\vec{B}$. Similarly particle Q is moving normal to the plane which contains vector $\vec{A}$ and $\vec{C}$. The angle between the direction of motion of P and Q is $\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)$. Then the value of x is ______.

27. In 5 minutes, a body cools from 75°C to 65°C at room temperature of body at the end of next 5 minutes is ____°C.

28. A ray of light passing through a prism $(\mu=\sqrt{3})$ suffers minimum deviation. It is found that the angle of incidence is double the angle of refraction within the prism. Then, angle of prism is _____ (in degrees).

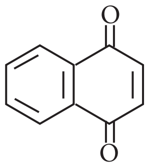
29. The centre of a wheel rolling on a plane surface moves with a speed v_0. A particle on the rim of the wheel at the same level as the centre will be moving at a speed $\sqrt{x}\,v_0$. Then the value of x is ______.

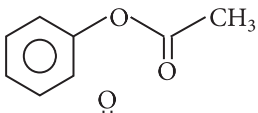
30. The area of cross-section of a railway track is 0.01 m^2. The temperature variation is 10°C. Coefficient of linear expansion of material of track is 10^{-5}/°C. The energy stored per metre in the track is _____ J/m.
(Young's modulus of material of track is $10^{11}\ Nm^{-2}$)

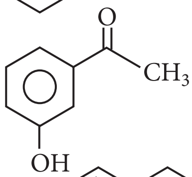
CHEMISTRY

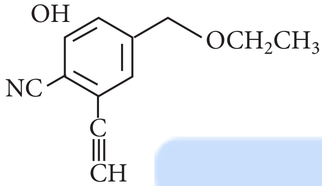
SECTION - A (MULTIPLE CHOICE QUESTIONS)

31. Which one of the following compounds will provide a tertiary alcohol on reaction with excess of CH_3MgBr followed by hydrolysis?

(a) 

(b) 

(c) 

(d) 

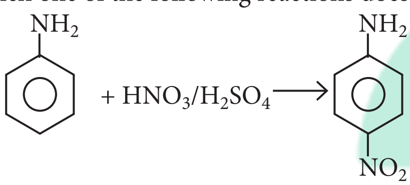
32. Match List-I with List-II.

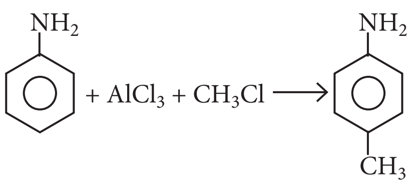
List-I (Species)	**List-II (Hybrid Orbitals)**
(A) SF_4	(i) sp^3d^2
(B) IF_5	(ii) d^2sp^3
(C) NO_2^+	(iii) sp^3d
(D) NH_4^+	(iv) sp^3
	(v) sp

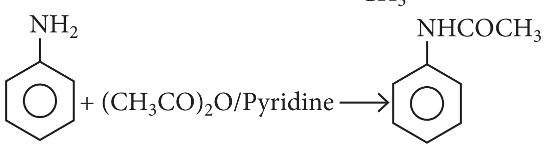
Choose the correct answer from the options given below.

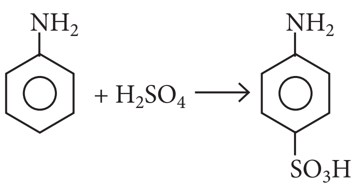
(a) (A) – (iii), (B) – (i), (C) – (v) and (D) – (iv)
(b) (A) – (ii), (B) – (i), (C) – (iv) and (D) – (v)
(c) (A) – (iv), (B) – (iii), (C) – (ii) and (D) – (v)
(d) (A) – (i), (B) – (ii), (C) – (v) and (D) – (iii)

33. Which one of the following reactions does not occur?

(a) 

(b) 

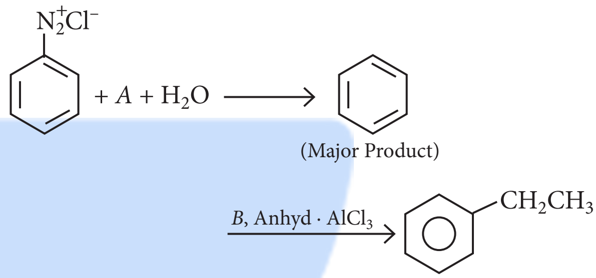
(c) 

(d) 

34. Thiamine and pyridoxine are also known respectively as
(a) Vitamin B_1 and Vitamin B_6
(b) Vitamin E and Vitamin B_2
(c) Vitamin B_2 and Vitamin E
(d) Vitamin B_6 and Vitamin B_2.

35. Which one of the following 0.06 M aqueous solutions has lowest freezing point?
(a) KI
(b) K_2SO_4
(c) $Al_2(SO_4)_3$
(d) $C_6H_{12}O_6$

36. Which one of the following group-15 hydride is the strongest reducing agent?
(a) SbH_3
(b) BiH_3
(c) PH_3
(d) AsH_3

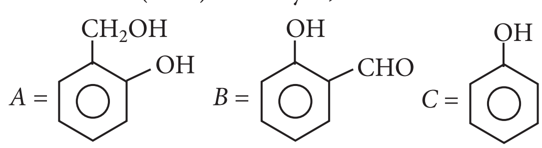
37. 

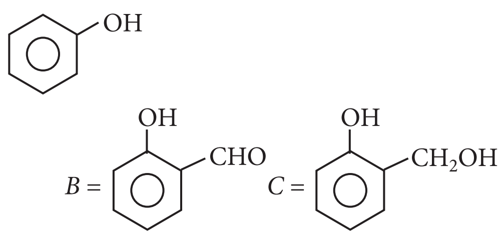
In the chemical reactions given above *A* and *B* respectively are
(a) H_3PO_2 and CH_3CH_2Cl
(b) CH_3CH_2Cl and H_3PO_2
(c) H_3PO_2 and CH_3CH_2OH
(d) CH_3CH_2OH and H_3PO_2.

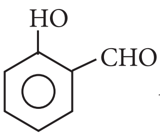
38. The set having ions which are coloured and paramagnetic both is
(a) Cu^+, Zn^{2+}, Mn^{4+}
(b) Ni^{2+}, Mn^{7+}, Hg^{2+}
(c) Cu^{2+}, Cr^{3+}, Sc^+
(d) Sc^{3+}, V^{5+}, Ti^{4+}

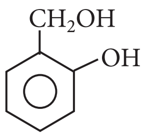
39. Which one of the following molecules does not show stereo isomerism?
(a) 3-Ethylhex-3-ene
(b) 3, 4-Dimethylhex-3-ene
(c) 3-Methylhex-1-ene
(d) 4-Methylhex-1-ene

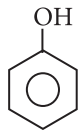
40. An organic compound $A(C_6H_6O)$ gives dark green colouration with ferric chloride. On treatment with $CHCl_3$ and KOH, followed by acidification gives compound *B*. Compound *B* can also be obtained from compound *C* on reaction with pyridinium chlorochromate (PCC). Identify *A*, *B* and *C*.

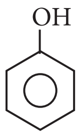
(a) 

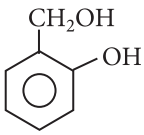
(b) 

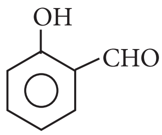
(c) $A =$ 

$B =$ 

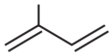
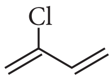
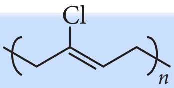
$C =$ 

(d) $A =$ 

$B =$ 

$C =$ 

41. Match List I with List-II.

List-I	**List-II**
(A) Chloroprene	(i) 
(B) Neoprene	(ii)  Cl
(C) Acrylonitrile	(iii)  Cl, n
(D) Isoprene	(iv) $CH_2=CH-CN$

Choose the correct answer from the options given below.

(a) (A) – (iii), (B) – (iv), (C) – (ii), (D) – (i)
(b) (A) – (ii), (B) – (i), (C) – (iv), (D) – (iii)
(c) (A) – (iii), (B) – (i), (C) – (iv), (D) – (ii)
(d) (A) – (ii), (B) – (iii), (C) – (iv), (D) – (i)

42. Which purification technique is used for high boiling organic liquid compound (decomposes near its boiling point)?

(a) Simple distillation (b) Fractional distillation
(c) Reduced pressure distillation
(d) Steam distillation

43. Given below are the statements about diborane.

(A) Diborane is prepared by the oxidation of $NaBH_4$ with I_2.
(B) Each boron atom is in sp^2 hybridised state.
(C) Diborane has one bridged 3 centre-2-electron bond.
(D) Diborane is a planar molecule.

The option with correct statement(s) is

(a) (C) only (b) (A) and (B) only
(c) (C) and (D) only (d) (A) only

44. Sulphide ion is soft base and its ores are common for metals

(A) Pb (B) Al (C) Ag (D) Mg

Choose the correct answer from the options given below.

(a) (A) and (C) only (b) (A) and (B) only
(c) (A) and D) only (d) (C) and (D) only

45. Match List-I with List-II.

List-I (Elements)	**List-II (Properties)**
(A) Ba	(i) Organic solvent soluble compounds
(B) Ca	(ii) Outer electronic configuration $6s^2$
(C) Li	(iii) Oxalate insoluble in water
(D) Na	(iv) Formation of very strong monoacidic base

Choose the correct answer from the options given below.

(a) (A) – (i), (B)-(iv), (C)-(ii) and (D)-(iii)
(b) (A) – (ii), (B)-(iii), (C)-(i) and (D)-(iv)
(c) (A) – (iv), (B)-(i), (C)-(ii) and (D)-(iii)
(d) (A) – (iii), (B)-(ii), (C)-(iv) and (D)-(i)

46. Which one of the following statements for D.I. Mendeleev, is incorrect?

(a) Element with atomic number 101 is named after him.
(b) He authored the textbook-Principles of Chemistry.
(c) He invented accurate barometer.
(d) At the time, he proposed Periodic Table of elements, structure of atom was known.

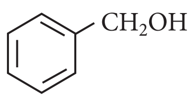
47. The water having more dissolved O_2 is

(a) polluted water (b) water at 80°C
(c) boiling water (d) water at 4°C.

48. When silver nitrate solution is added to potassium iodide solution then the sol produced is

(a) KI/NO_3^- (b) AgI/Ag^+
(c) $AgNO_3/NO_3^-$ (d) AgI/I^-

49. Which one of the following compounds does not exhibit resonance?

(a) $CH_3CH_2OCH=CH_2$ (b) 

(c) $CH_3CH_2CH=CHCH_2NH_2$
(d) $CH_3CH_2CH_2CONH_2$

50. Isotopes(s) of hydrogen which emits low energy β^- particles with $t_{1/2}$ value > 12 years is/are

(a) protium (b) tritium
(c) deuterium (d) deuterium and tritium.

SECTION - B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

51. If the concentration of glucose ($C_6H_{12}O_6$) in blood is 0.72 g L^{-1}, the molarity of glucose in blood is ________ $\times 10^{-3}$ M.(Nearest Integer)
(Given : Atomic mass of C = 12, H = 1, O = 16 u)

52. The total number of unpaired electrons present in $[Co(NH_3)_6]Cl_2$ and $[Co(NH_3)_6]Cl_3$ is ________.

53. A copper complex crystallising in a *ccp* lattice with a cell edge of 0.4518 nm has been revealed by employing X-ray diffraction studies. The density of a copper complex is found to be 7.62 g cm^{-3}. The molar mass of copper complex is ________ g mol^{-1}. (Nearest Integer) [Given : $N_A = 6.022 \times 10^{23}\ mol^{-1}$]

54. $N_2O_{5(g)} \longrightarrow 2NO_{2(g)} + \frac{1}{2}O_{2(g)}$

In the above first order reaction the initial concentration of N_2O_5 is 2.40×10^{-2} mol L^{-1} at 318 K. The concentration of N_2O_5 after 1 hour was 1.60×10^{-2} mol L^{-1}. The rate constant of the reaction at 318 K is ________ $\times 10^{-3}$ min^{-1}. (Nearest Integer)
[Given : log3 = 0.477, log5 = 0.699]

55. Methylation of 10 g of benzene gave 9.2 g of toluene. Calculate the percentage yield of toluene ________ . (Nearest Integer)

56. Assume a cell with the following reaction
$Cu_{(s)} + 2Ag^+(1 \times 10^{-3}\ M) \rightarrow Cu^{2+}(0.250\ M) + 2\ Ag_{(s)}$
$E^\circ_{cell} = 2.97$ V
E_{cell} for the above reaction is ________ V. (Nearest Integer)
[Given : log 2.5 = 0.3979, T = 298 K]

57. If the standard molar enthalpy change for combustion of graphite powder is -2.48×10^2 kJ mol^{-1}, the amount of heat generated on combustion of 1 g of graphite powder is ________ kJ. (Nearest Integer)

58. Number of electrons that Vanadium (Z = 23) has in p-orbitals is equal to ________.

59. The number of acyclic structural isomers (including, geometrical isomers) for pentene are ________.

60. Value of K_p for the equilibrium reaction $N_2O_{4(g)} \rightleftharpoons 2NO_{2(g)}$ at 288 K is 47.9. The K_c for this reaction at same temperature is ________. (Nearest Integer) (R = 0.083 L bar K^{-1} mol^{-1})

MATHEMATICS

SECTION - A (MULTIPLE CHOICE QUESTIONS)

61. Let $A = [a_{ij}]$ be a real matrix of order 3 × 3, such that $a_{i1} + a_{i2} + a_{i3} = 1$, for i = 1, 2, 3. Then, the sum of all the entries of the matrix A^3 is equal to
(a) 1 (b) 9 (c) 3 (d) 2

62. Four dice are thrown simultaneously and the numbers shown on these dice are recorded in 2 × 2 matrices. The probability that such formed matrices have all different entries and are non-singular, is
(a) $\frac{45}{162}$ (b) $\frac{23}{81}$ (c) $\frac{43}{162}$ (d) $\frac{22}{81}$

63. The number of solutions of $\sin^7 x + \cos^7 x = 1$, $x \in [0, 4\pi]$ is equal to
(a) 9 (b) 5 (c) 11 (d) 7

64. Let three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ be such that $\vec{a} \times \vec{b} = \vec{c}$, $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{a}| = 2$. Then, which one of the following is not true?
(a) $[\vec{a}\ \vec{b}\ \vec{c}] + [\vec{c}\ \vec{a}\ \vec{b}] = 8$
(b) Projection of $\vec{a}$ on $(\vec{b} \times \vec{c})$ is 2.
(c) $|3\vec{a} + \vec{b} - 2\vec{c}|^2 = 51$
(d) $\vec{a} \times ((\vec{b} + \vec{c}) \times (\vec{b} - \vec{c})) = \vec{0}$

65. Let n denote the number of solutions of the equation $z^2 + 3\bar{z} = 0$, where z is a complex number. Then the value of $\sum_{k=0}^{\infty} \frac{1}{n^k}$ is equal to
(a) $\frac{3}{2}$ (b) 2 (c) 1 (d) $\frac{4}{3}$

66. Let L be the line of intersection of planes $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 2$ and $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) = 2$. If $P(\alpha, \beta, \gamma)$ is the foot of perpendicular on L from the point (1, 2, 0), then the value of $35(\alpha + \beta + \gamma)$ is equal to
(a) 134 (b) 101 (c) 143 (d) 119

67. Which of the following Boolean expressions is not a tautology?
(a) $(p \Rightarrow {\sim}q) \vee ({\sim}q \Rightarrow p)$ (b) $(p \Rightarrow q) \vee ({\sim}q \Rightarrow p)$
(c) $(q \Rightarrow p) \vee ({\sim}q \Rightarrow p)$ (d) $({\sim}p \Rightarrow q) \vee ({\sim}q \Rightarrow p)$

68. Let $[x]$ denote the greatest integer less than or equal to x. Then, the values of $x \in R$ satisfying the equation $[e^x]^2 + [e^x + 1] - 3 = 0$ lie in the interval
(a) $[0, 1/e)$ (b) $[0, \log_e 2)$
(c) $[\log_e 2, \log_e 3)$ (d) $[1, e)$

69. Let $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$. Let E_2 be another ellipse such that it touches the end points of major axis of E_1 and the foci of E_2 are the end points of minor axis of E_1. If E_1 and E_2 have same eccentricities, then its value is
(a) $\frac{-1+\sqrt{6}}{2}$ (b) $\frac{-1+\sqrt{3}}{2}$
(c) $\frac{-1+\sqrt{8}}{2}$ (d) $\frac{-1+\sqrt{5}}{2}$

70. If the domain of the function $f(x) = \frac{\cos^{-1}\sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ is the interval $(\alpha, \beta]$, then $\alpha + \beta$ is equal to
(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{3}{2}$

71. Let $f : R \rightarrow R$ be defined as

$$f(x) = \begin{cases} \frac{x^3}{(1-\cos 2x)^2} \log_e\left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right), & x \neq 0 \\ \alpha, & x = 0 \end{cases}.$$

If f is continuous at $x = 0$, then α is equal to
(a) 2 (b) 0 (c) 3 (d) 1

72. If $\int_{0}^{100\pi} \frac{\sin^2 x}{e^{\left(\frac{x}{\pi}-\left[\frac{x}{\pi}\right]\right)}} dx = \frac{\alpha\pi^3}{1+4\pi^2}, \alpha \in R,$ where $[x]$ is the greatest integer less than or equal to x, then the value of α is
(a) $200(1 - e^{-1})$ (b) $150(e^{-1} - 1)$
(c) $50(e - 1)$ (d) $100(1 - e)$

73. Let a line $L: 2x + y = k, k > 0$ be a tangent to the hyperbola $x^2 - y^2 = 3$. If L is also a tangent to the parabola $y^2 = \alpha x$, then α is equal to
(a) -12 (b) 24 (c) 12 (d) -24

74. Let the circle $S: 36x^2 + 36y^2 - 108x + 120y + C = 0$ be such that it neither intersects nor touches the co-ordinate axes. If the point of intersection of the lines, $x - 2y = 4$ and $2x - y = 5$ lies inside the circle S, then
(a) $81 < C < 156$ (b) $100 < C < 165$
(c) $100 < C < 156$ (d) $\frac{25}{9} < C < \frac{13}{3}$

75. If the shortest distance between the straight lines $3(x-1) = 6(y-2) = 2(z-1)$ and $4(x-2) = 2(y-\lambda) = (z-3)$, $\lambda \in R$ is $\frac{1}{\sqrt{38}}$, then the integral value of λ is equal to
(a) 3 (b) 5 (c) 2 (d) -1

76. Let a vector $\vec{a}$ be coplanar with vectors $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$. If $\vec{a}$ is perpendicular to $\vec{d} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, and $|\vec{a}| = \sqrt{10}$. Then, a possible value of $[\vec{a}\,\vec{b}\,\vec{c}] + [\vec{a}\,\vec{b}\,\vec{d}] + [\vec{a}\,\vec{c}\,\vec{d}]$ is equal to
(a) -38 (b) -29 (c) -40 (d) -42

77. Let $y = y(x)$ be the solution of the differential equation $\operatorname{cosec}^2 x\,dy + 2dx = (1 + y\cos 2x)\operatorname{cosec}^2 x\,dx$, with $y\left(\frac{\pi}{4}\right) = 0$. Then, the value of $(y(0) + 1)^2$ is equal to
(a) $e^{-\frac{1}{2}}$ (b) e (c) $e^{\frac{1}{2}}$ (d) e^{-1}

78. Let $f: R \to R$ be defined as
$$f(x) = \begin{cases} -\frac{4}{3}x^3 + 2x^2 + 3x, & x > 0 \\ 3xe^x, & x \le 0 \end{cases}.$$
Then f is increasing function in the interval
(a) $\left(-1, \frac{3}{2}\right)$ (b) $\left(-\frac{1}{2}, 2\right)$
(c) $(0, 2)$ (d) $(-3, -1)$

79. The values of λ and μ such that the system of equations $x + y + z = 6$, $3x + 5y + 5z = 26$, $x + 2y + \lambda z = \mu$ has no solution, are
(a) $\lambda = 2, \mu \ne 10$ (b) $\lambda \ne 2, \mu = 10$
(c) $\lambda = 3, \mu = 5$ (d) $\lambda = 3, \mu \ne 10$

80. Let S_n denote the sum of first n-terms of an arithmetic progression. If $S_{10} = 530$, $S_5 = 140$, then $S_{20} - S_6$ is equal to
(a) 1862 (b) 1852 (c) 1842 (d) 1872

SECTION - B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

81. The sum of all the elements in the set $\{n \in \{1, 2, \ldots\ldots, 100\} \mid$ H.C.F. of n and 2040 is $1\}$ is equal to ________.

82. Let $f: R \to R$ be a function defined as
$$f(x) = \begin{cases} 3\left(1 - \frac{|x|}{2}\right), & \text{if } |x| \le 2 \\ 0, & \text{if } |x| > 2 \end{cases}.$$
Let $g: R \to R$ be given by $g(x) = f(x + 2) - f(x - 2)$. If n and m denote the number of points in R, where g is not continuous and not differentiable, respectively, then $n + m$ is equal to ________.

83. The area (in sq. units) of the region bounded by the curves $x^2 + 2y - 1 = 0$, $y^2 + 4x - 4 = 0$ and $y^2 - 4x - 4 = 0$, in the upper half plane is ________.

84. If the constant term, in binomial expansion of $\left(2x^r + \frac{1}{x^2}\right)^{10}$ is 180, then r is equal to ________.

85. Let $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$. Then the number of bijective functions $f: A \to A$ such that $f(1) + f(2) = 3 - f(3)$ is equal to ________.

86. Let $y = y(x)$ be the solution of the differential equation
$$\left((x+2)e^{\left(\frac{y+1}{x+2}\right)} + (y+1)\right)dx = (x+2)dy, \; y(1) = 1.$$
If the domain of $y = y(x)$ is an open interval (α, β), then $|\alpha + \beta|$ is equal to ________.

87. If the digits are not allowed to repeat in any number formed by using the digits 0, 2, 4, 6, 8, then the number of all numbers greater than 10,000 is equal to ________.

88. The number of elements in the set $\{n \in \{1, 2, 3, \ldots\ldots, 100\} \mid (11)^n > (10)^n + (9)^n\}$ is ________.

89. Consider the following frequency distribution:

Class	0-6	6-12	12-18	18-24	24-30
Frequency	a	b	12	9	5

If mean $= \frac{309}{22}$ and median $= 14$, then the value $(a - b)^2$ is equal to ________.

90. Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then the number of 3×3 matrices B with entries from the set $\{1, 2, 3, 4, 5\}$ and satisfying $AB = BA$ is ________.

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HINTS & EXPLANATIONS

1. (b) : (A) If $\omega L > \frac{1}{\omega C}$, means $X_L > X_C$, ϕ is positive, so current lags behind the applied emf.

(B) If $\omega L = \frac{1}{\omega C}$, $X_L = X_C$, so current is in phase with emf.

(C) If $\omega L < \frac{1}{\omega C}$, $X_L < X_C$, ϕ is negative, so current leads the emf.

(D) At resonant frequency, maximum current occurs.

(A)→(ii), (B)→(i), (C)→(iv), (D)→(iii)

2. (c) : In ferromagnetic material, all the molecular magnetic dipoles are pointed in the same direction. When the ferromagnetic material heated, all magnetic dipoles are distributed and get disoriented. Due to which the net magnetic dipole moment becomes very less, so that they behave as paramagnetic material. So, statement I is true.

3. (d) : The band gap energy is

$$\Delta E = \frac{hc}{\lambda_0}$$

where, h is Planck's constant rod, c is speed of light and λ_0 is the threshold wavelength.

$$\therefore \quad \Delta E = \frac{6.63\times10^{-34}\times3\times10^{8}}{621\times10^{-9}}\text{ J}$$

$$\Delta E = \frac{6.63\times10^{-34}\times3\times10^{8}}{621\times10^{-9}\times1.6\times10^{-19}} = 2\text{ eV}$$

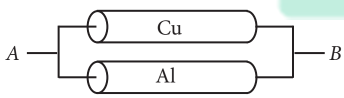
4. (b) : Length of Copper rod, $L_1 = 25$ cm, $\rho_{Cu} = 1.7\times10^{-8}\ \Omega$ m

Area of copper rod, $A_1 = 3$ mm^2,

Length of Aluminium rod, $L_2 = 25$ cm, $\rho_{Al} = 2.6\times10^{-8}$ W m

Area of Aluminium rod = 3 mm^2

Here both the wires are connected in parallel as shown in figure.



$$R_1 = \rho_1\frac{L_1}{A_1} \text{ and } R_2 = \rho_2\frac{L_2}{A_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1}+\frac{1}{R_2} = \frac{A_1}{\rho_1 L_1}+\frac{A_2}{\rho_2 L_2}$$

$$\frac{1}{R_{eq}} = \frac{A}{L}\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right) = \frac{3\times10^{-6}}{0.25}\left(\frac{10^8}{1.7}+\frac{10^8}{2.6}\right)$$

$$\frac{1}{R_{eq}} = \frac{3\times10^{-6}\times10^{8}}{0.25}\left[\frac{2.6+1.7}{2.6\times1.7}\right] = 1167.42$$

or $\quad R_{eq} = 0.858\times10^{-3}\ \Omega$

$\therefore \quad R_{eq} = 0.858$ mΩ

5. (b) : In case of capacitor the current lags by $\pi/2$ by emf. So, option (b) is correct.

6. (a) : Given, $\vec{A} = \hat{i}+\hat{j}+\hat{k}$, $\vec{B} = \hat{i}+\hat{j}$

Projection of vector A over vector $B = (A\cdot B)\hat{B}$

$$= (\hat{i}+\hat{j}+\hat{k})\cdot(\hat{i}+\hat{j})\frac{[\hat{i}+\hat{j}]}{\sqrt{2}}$$

$$= \frac{(1+1)(\hat{i}+\hat{j})}{\sqrt{2}} \qquad (\because\ \hat{i}\cdot\hat{i}=1, \hat{j}\cdot\hat{j}=1)$$

$$= \sqrt{2}(\hat{i}+\hat{j})$$

7. (c) : Moment of inertia of ring = mr^2

Moment of inertia of solid cylinder $= \frac{1}{2}mr^2$

Moment of inertia of solid sphere $= \frac{2}{5}mr^2$

The acceleration is given by, $a = \dfrac{g\sin\theta}{1+\dfrac{K^2}{r^2}}$ ...(i)

For ring, $K = r$

For solid cylinder, $K = \frac{r}{\sqrt{2}}$

For solid sphere, $K = \sqrt{\frac{2}{5}}\,r$

Now, using eq. (i),

$$a_{ring} = \frac{g\sin\theta}{2} = 0.5\,g\sin\theta$$

$$a_{solid\ cylinder} = \frac{g\sin\theta}{1+\frac{1}{2}} = \frac{2}{3}g\sin\theta = 0.66g\sin\theta$$

$$a_{solid\ sphere} = \frac{g\sin\theta}{1+\frac{2}{5}} = \frac{5}{7}g\sin\theta = 0.7g\sin\theta$$

So, the sphere has the greatest and ring has least velocity of centre of mass at the bottom of inclined plane.

8. (c) : Radius coverage, R = 150 km = 150 × 1000 m

Population density = 2000/km^2

Radius of earth, $R_e = 6.5\times10^6$ m

Let the height is H.

So, radius $R = \sqrt{2R_eH}$

$$150\times1000 = \sqrt{2\times6.5\times10^6\times H}$$

$(1000\times150)^2 = 2\times6.5\times10^6\,H$

$H \simeq 1731$ m.

Population covered = πR^2 × population density

= $3.14(150)^2\times2000 = 1413\times10^5$

9. **(b) :** Intensity of sunlight, $I = 0.092\ \text{W/m}^2$

$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$

Intensity is given by

$$I = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}\frac{B_0^2 c}{\mu_0}$$

$$\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c \Rightarrow \frac{1}{\mu_0} = \varepsilon_0 c^2$$

So, $I = \frac{1}{2} B_0^2 c\, \varepsilon_0 c^2 = \frac{1}{2} B_0^2\, \varepsilon_0 c^3$

$$B_0 = \sqrt{\frac{2I}{\varepsilon_0 c^3}} = \sqrt{\frac{2 \times 0.092}{8.85 \times 10^{-12} \times (3 \times 10^8)^3}}$$

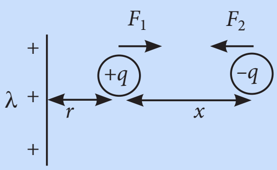
$B_0 = 2.77 \times 10^{-8}\ \text{T}$

10. **(a) :** Linear charge density, $\lambda = 3 \times 10^{-6}\ \text{C/m}$

Net force, $F = 4\ \text{N}$

$r = 10\ \text{mm}$, $x = 12\ \text{mm}$

Let the charge on dipole is q



$$F_1 = \frac{2k\lambda}{r} \cdot q$$

$$F_2 = \frac{2k\lambda q}{r + x}$$

$$F_{\text{net}} = F_1 - F_2 = \frac{2K\lambda q}{r} - \frac{2K\lambda q}{r + x}$$

$$F_{\text{net}} = \frac{2K\lambda q \cdot x}{r(r + x)}$$

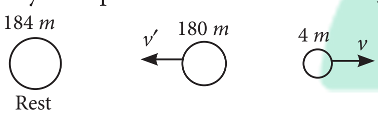
$$4 = \frac{2 \times 9 \times 10^9 \times 3 \times 10^{-6} \times 2 \times 10^{-3} \times q}{10 \times 10^{-3} \times 12 \times 10^{-3}}$$

$q = 4.44 \times 10^{-6}\ \text{C} = 4.44\ \mu\text{C}$

11. **(a) :** Mass number, $A = 184$

$Q_{\text{value}} = 5.5\ \text{MeV}$

Let the velocity of α-particle is v and for 180 m, it is v'.



Use conservation of momentum,

$184\ m \times 0 = 180\ mv' - 4mv$

$$v' = \frac{4v}{180} \qquad \text{...(i)}$$

Now using conservation of energy,

$$\frac{1}{2}(4m)v^2 + \frac{1}{2}(180m)v'^2 = 5.5\ \text{MeV}$$

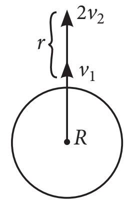
$$\frac{1}{2} \cdot 4mv^2 \left[1 + 45 \times \left(\frac{4}{180}\right)^2\right] = 5.5\ \text{MeV} \quad (\text{Using (i)})$$

Here $\text{K} \cdot \text{E}_\alpha = \frac{1}{2}(4mv^2)$

$$\text{K} \cdot \text{E}_\alpha \left(1 + 45 \times \left(\frac{4}{180}\right)^2\right) = 5.5\ \text{MeV}$$

$$\text{K} \cdot \text{E}_\alpha = \frac{5.5}{1 + 45\left(\frac{4}{180}\right)^2} = 5.38\ \text{MeV}$$

12. **(d) :** The minimum velocity of body is escape velocity as it goes to infinity



$$v_1 = \sqrt{\frac{2GM}{R_e}} \qquad \text{....(i)}$$

Let it reaches to distance r with velocity v_2 and time t.

Use conservation of energy

$$\frac{1}{2}m\left(\frac{2GM}{R_e}\right) - \frac{GMm}{R_e} = \frac{1}{2}mv_2^2 - \frac{GMm}{R + r}$$

$$\frac{1}{2}mv_2^2 = \frac{GMm}{R + r}$$

$$v_2 = \sqrt{\frac{2GM}{R + r}}$$

Velocity is given by

$$v = \frac{dr}{dt}$$

$$\sqrt{2GM}\int_0^t dt = \int_{R_e}^{R_e + h} \sqrt{R + r}\, dr$$

$$\sqrt{2GM} \times t = \frac{2}{3}\left[(R + r)^{3/2}\right]_{R_e}^{R_e + h}$$

$$t = \frac{2}{3}\sqrt{\frac{R_e^3}{2GM}}\left[\left(1 + \frac{h}{R_e}\right)^{3/2} - 1\right] \qquad \left(\because\ g = \frac{GM}{R_e^2}\right)$$

$$\therefore\quad t = \frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1 + \frac{h}{R_e}\right)^{3/2} - 1\right]$$

13. **(b) :** Let the apparent dip is at α with true dip

$$\tan\phi = \frac{B_V}{B_H} \qquad \text{...(i)}$$

and $\tan\phi' = \dfrac{B_V}{B_H \cos\alpha} = \dfrac{\tan\phi}{\cos\alpha}$ (From (i))

True dip circle as shown in figure (i).

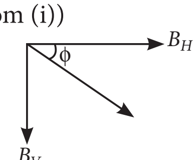


Figure (i)

Apparent dip circle as shown in figure (ii).

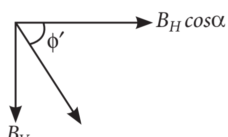


Figure (ii)

So, $\tan\phi = \tan\phi' \cos\alpha \qquad (0 < \cos\alpha < 1)$

$\therefore\quad \tan\phi < \tan\phi'$

Hence, $\phi < \phi'$

14. (c) : According to question, $r + r' = 90°$
and $r + r' + 90° = 180°$
$r' = 90° - r = 90° - i$ ...(i)
By Snell's law
$n_1 \sin i = n_2 \sin r'$,
From eq. (i), we have
$n_1 \sin i = n_2 \sin(90° - i)$
$n_1 \sin i = n_2 \cos i \Rightarrow \tan i = \frac{n_2}{n_1}$ ...(ii)
Also $\sin C = \frac{n_2}{n_1}$ where, C is critical angle.
From eq. (ii), we get
$\sin C = \tan i$
$C = \sin^{-1}(\tan i) = \sin^{-1}(\tan r)$

15. (b) : Initial time period of a simple pendulum, $T_1 = T_0$, and length $= L$

Final length, $L' = \frac{L}{16}$

Let the new time period is T'.
The time period of simple pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

So, $T \propto \sqrt{L}$

$$\therefore \quad \frac{T'}{T_0} = \sqrt{\frac{L}{16L}} = \frac{1}{4} \Rightarrow T' = \frac{T_0}{4}$$

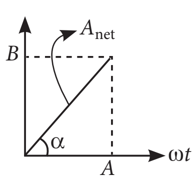
16. (c) : Mass of bullet, $m = 4$ g
Mass of gun, $M = 4$ kg
Muzzle speed of bullet, $v = 50$ m/s
Let the recoil velocity of gun is V.
Use conservation of momentum
$mv = MV$
$0.004 \times 50 = 4\ V$
$V = 0.05$ m/s
Also, Impulse = Change in momentum
$MV - mv$
$= MV - 0 = 4 \times 0.05 = 0.2$ kg m/s $(\because v = 0)$

17. (d) : Given, $x(t) = A\sin\omega t + B\cos\omega t$

As, $v = \frac{dx}{dt} = A\omega\cos\omega t - B\omega\sin\omega t$

At $t = 0$, $x = B$, and $v = A\omega$

From the graph, $A_{net} = \sqrt{A^2 + B^2}$



or $\tan\alpha = \frac{B}{A} \Rightarrow \alpha = \tan^{-1}\left(\frac{B}{A}\right)$

$$x(t) = \sqrt{A^2 + B^2}\left[\frac{A}{\sqrt{A^2+B^2}}\sin\omega t + \frac{B}{\sqrt{A^2+B^2}}\cos\omega t\right]$$

Let $\frac{A}{\sqrt{A^2+B^2}} = \sin\phi\,;\ \frac{B}{\sqrt{A^2+B^2}} = \cos\phi$

$\Rightarrow x(t) = C[\sin\phi \sin\omega t + \cos f \cos\omega t]$
$\Rightarrow x(t) = C\cos(\omega t - \phi)$

As, $C = \sqrt{A^2 + B^2} = \sqrt{\frac{[v(0)]^2}{\omega^2} + x(0)^2}$

$\phi = 90 - \alpha$,

$\tan\phi = \cot\alpha = \frac{A}{B} = \frac{v(0)}{x(0)\,\omega}$

$$\phi = \tan^{-1}\left[\frac{V(0)}{\omega x(0)}\right]$$

18. (d) : The de Broglie wavelength is

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

where, h is Planck's constant, m is mass and q is charge, V is potential difference.

So, $\lambda \propto \frac{1}{\sqrt{2mV}}$

As potential difference is same, so

$$\therefore \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}}$$

19. (c) : Mass of suitcase, $m = 80$ kg
Distance $= h = 80$ cm, $g = 9.8$ m/s^2
Let the work done by porter is W_{porter}.
According to the work energy theorem
$W_{porter} + W_{mg} = \Delta KE$
$W_{porter} = -W_{mg} = -mgh \quad (\because \Delta KE = 0)$
$= (80 \times 9.8 \times 0.8)$
$= -627.2$ J

20. (b) : The average value of energy for a monoatomic gas in thermal equilibrium at temperature T is

$$U = \frac{3}{2}k_B T$$

where, k_B is Boltzmann constant.

21. (4) : The electric flux density is

$$\vec{D} = \frac{\text{Charge}}{\text{Area}}\hat{r} = \frac{Q}{4\pi r^2}\hat{r} = \varepsilon_0\left[\frac{Q}{4\pi\varepsilon_0 r^2}\right]\hat{r}$$

Given, $D = e^{-x}\sin y\,\hat{i} - e^{-x}\cos y\,\hat{j} + 2z\hat{k}$ C/m^2

As, $E = \frac{\vec{D}}{\varepsilon_0} = \frac{e^{-x}\sin y\,\hat{i} - e^{-x}\cos y\,\hat{j} + 2z\hat{k}}{\varepsilon_0}$

According to Gauss's theorem

$$\frac{\rho}{\varepsilon_0} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\vec{E}$$

or $\dfrac{\rho}{\varepsilon_0} = \left(\dfrac{\partial}{\partial x}\hat{i} + \dfrac{\partial}{\partial y}\hat{j} + \dfrac{\partial}{\partial z}\hat{k}\right)\cdot\dfrac{\vec{D}}{\varepsilon_0}$

Also, $\rho = \dfrac{\partial}{\partial x}(e^{-x}\sin y) + \dfrac{\partial}{\partial y}(-e^{-x}\cos y) + \dfrac{\partial}{\partial z}(2z)$

or $\rho = -e^{-x}\sin y + e^{-x}\sin y + 2$

At origin $\rho = 2\ C/m^3$

The total charge enclosed, $q = \rho \times \text{Volume} = 2 \times 2 \times 10^{-9} = 4 \times 10^{-9}$ C

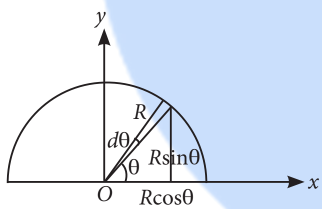
$\therefore \quad q = 4$ nC

22. (2) : Let M is the mass, R is the radius and O is the centre.

$\dfrac{\text{Mass}}{\text{Length}} = \dfrac{M}{\pi R}$

Consider an elementary portion, so mass of elementary

$$dm = \frac{M}{\pi R}\cdot R d\theta = \frac{M}{\pi}d\theta$$



The coordinates of centre of mass are

$$X = \frac{1}{M}\int x\, dm = \frac{1}{M}\int_0^{\pi}(R\cos\theta)\left(\frac{M}{\pi}\right)d\theta = \frac{R}{\pi}[\sin\theta]_0^{\pi}$$

So, $X = 0$

Similarly, $Y = \dfrac{1}{M}\int y\, dm = \dfrac{1}{M}\int_0^{\pi} R\sin\theta\dfrac{M}{\pi}\cdot d\theta = \dfrac{2R}{\pi}$

On comparing the given value with $\dfrac{xR}{\pi}$, we get, $x = 2$.

23. (15) : Let the emf is E and internal resistance is r.

Case I : Current, $I = \dfrac{V}{r} = \dfrac{1.25}{5}$

As, $\varepsilon - Ir = V$

$$\varepsilon - \frac{1.25}{5}r = 1.25 \qquad \text{...(i)}$$

Case II : Current, $I = \dfrac{V}{r} = \dfrac{1}{2}$

Also, $\varepsilon - Ir = V$ or $\varepsilon - \dfrac{1}{2}r = 1$ ...(ii)

By solving eqns. (i) and (ii), we get

$\varepsilon = 1.5$ V, $r = 1\ \Omega$

On comparing the given value, with $\dfrac{x}{10}$V, we get $x = 15$

$\left(\because \dfrac{15}{10}\text{V or } 1.5\text{V}\right)$

24. (500) : Given, $I_1 = 90$ mA, $I_2 = \dfrac{5}{R_L}$ ...(i)

Voltage across R, $V' = 50 - 5 = 45$ V

So, $R = \dfrac{45}{I_1 + I_2}$

$$R = \frac{45}{0.09 + \dfrac{5}{R_L}} \qquad \text{(Using (i))}$$

The current in zener is maximum when $R_L = \infty$

So, $I_2 = 0$, $I_1 = I$

$\therefore \quad R = \dfrac{45}{0.09} = 500\ \Omega$

25. (1) : The time period of simple pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where, L is length of pendulum and g is the acceleration due to gravity.

Also, $T^2 = 4\pi^2\dfrac{L}{g}$

$$g = 4\pi^2\frac{L}{T^2}$$

Percentage error, $\dfrac{\Delta g}{g}\times 100 = \dfrac{\Delta L}{L}\times 100 + \dfrac{2\Delta T}{nT}\times 100$

ΔL and ΔT is same for all so, $\dfrac{\Delta g}{g}$ is minimum for highest value of L, n and T.

So, the minimum percentage error is obtained by student 1.

26. (3) : Given, $\vec{A} = \hat{i} + \hat{j}$, $\vec{B} = \hat{j} + \hat{k}$, $\vec{C} = -\hat{i} + \hat{j}$

$$\hat{n}_1 = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$$

$$\hat{n}_2 = \frac{\vec{A}\times\vec{C}}{|\vec{A}\times\vec{C}|} = \frac{2\hat{k}}{2} = \hat{k}$$

Angle between $\hat{n}_1$ and $\hat{n}_2$ is

$$\cos\theta = \frac{\hat{n}_1\cdot\hat{n}_2}{|\hat{n}_1||\hat{n}_2|} = \frac{\frac{1}{\sqrt{3}}}{(1)\,(1)} = \frac{1}{\sqrt{3}} \text{ or } \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

On comparing the given value with $\cos^{-1}\left(\dfrac{1}{\sqrt{x}}\right)$,

We get, $x = 3$.

27. (57) : In 5 min, a body cools from temperature 75°C to 65°C.

Room temperature = 25°C

By Newton's law of cooling,

$$t = \frac{1}{K}\ln\left(\frac{\theta_2 - \theta_n}{\theta_1 - \theta_n}\right)$$

where, t is time, θ_2 is final temperature, θ_1 is initial temperature, θ_n = temperature of surroundings.

So $5=\frac{1}{K}\ln\left(\frac{65-25}{75-25}\right)$ ...(i)

Let the temperature is θ' after 5 minutes.

Now, $5=\frac{1}{K}\ln\left(\frac{\theta'-25}{65-25}\right)$ ...(ii)

From eq. (i) and (ii), we have

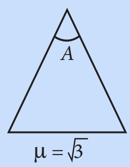
$$\ln\left(\frac{40}{50}\right)=\ln\left(\frac{\theta'-25}{40}\right)$$

$$\frac{4}{5}=\frac{\theta'-25}{40}$$

or $\theta' = 57°C$

28. (60) : Given, with in the prism, $i = 2r$

According to Snell's law,



$$\mu=\frac{\sin i}{\sin r}=\frac{\sin 2r}{\sin r}$$

$$\sqrt{3}=\frac{2\sin r\cos r}{\sin r}$$

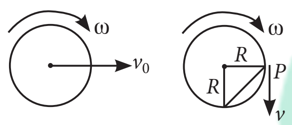
$$\cos r=\frac{\sqrt{3}}{2} \Rightarrow r=30°$$

For minimum deviation, angle of prism

$A = 2r = 2 \times 30° = 60°$

29. (2) : Speed at centre of a rolling wheel = v_0

$v_0 = R\omega$ ...(i)



At point P, $v=R\omega=\frac{v_0}{\omega}\cdot\omega=v_0$ (Using (i))

Resultant speed, $v=\sqrt{v_0^2+v_0^2}=\sqrt{2}v_0$

On comparing given value with $\sqrt{xv_0}$, we get $x = 2$.

30. (5) : Given, $A = 0.01\ m^2$, $\alpha = 10^{-5}\ °C$, $Y = 10^{11}\ N/m^2$

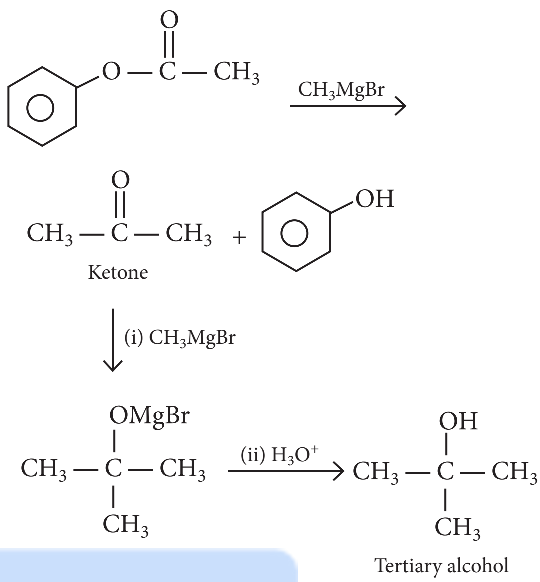
As, Energy $=\frac{Y}{2}\times(\text{Strain})^2\times\text{Area}\times\text{length}$

$$\frac{\text{Energy}}{\text{Length}}=\frac{Y}{2}\times\left(\frac{\Delta l}{l}\right)^2\times A=\frac{Y}{2}A\times(\alpha\cdot\Delta T)^2$$

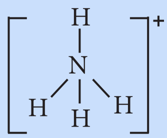
$$=\frac{10^{11}}{2}\times0.01(10^{-5}\times10)^2=5\ \text{J/m} \qquad (\because\ \Delta l = l\alpha\,\Delta T)$$

The energy stored per meter in the track is 5 J/m.

31. (b) :



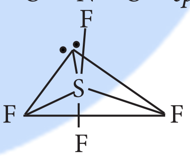
32. (a) : NH_4^+



sp^3

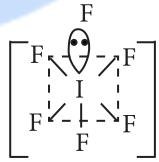
NO_2^+ $\quad O=\overset{+}{N}=O \quad$ sp

SF_4



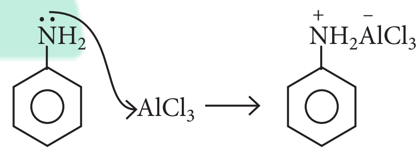
sp^3d

IF_5



sp^3d^2

33. (b) : $AlCl_3$ is used as catalyst in Friedel-Crafts reaction which is acidic in nature and aniline gives lone pair to $AlCl_3$ and converts to cation.



Thus, benzene ring is deactivated towards Friedel-Crafts reaction. Hence, aniline does not undergo Friedel-Crafts reaction.

34. (a) : Thiamine is known as vitamin B_1 while pyridoxine is vitamin B_6.

35. (c) : $\Delta T_f \propto i$

So, greater the value of i, greater will depression in freezing point and lower will be freezing point.

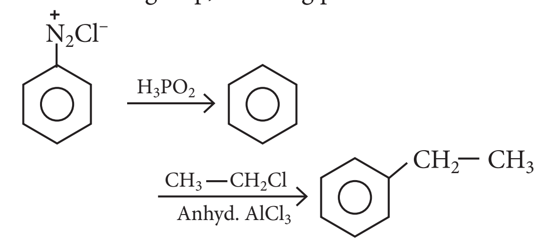
Solution	**(*i*)**
KI	2
K_2SO_4	3
$Al_2(SO_4)_3$	5
$C_6H_{12}O_6$	1(No ionization)

36. (b) : Group-15 hydrides:

NH_3, PH_3, AsH_3, SbH_3, BiH_3

As we move down the group, reducing power increases.

37. (a) :



38. (c) :

Ion	**No.of unpaired e^-**
Cu^{2+}	1
Sc^+	2
Cr^{3+}	3

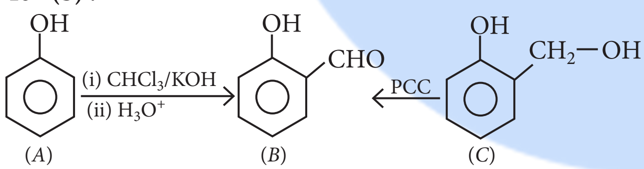
This set of ions is both paramagnetic and coloured.

39. (a) : $CH_3-CH_2-\underset{\large CH_2-CH_3}{\underset{|}{C}}=CH-CH_2-CH_3$

3–Ethylhex–3–ene

The compound neither shows geometrical (because both alkyl groups are same on one side of the carbon attached to double bond) nor optical isomerism (no asymmetric carbon is present).

40 (b) :



41. (d) : (A) – (ii), (B) – (iii), (C) – (iv), (D) – (i)

42. (c) : The compounds which have high boiling point and generally decompose before their boiling point is reached, can be purified by distillation under reduced pressure which lowers the boiling point of most liquids. So, these can be collected before decomposition.

43. (d) : The boron atom is sp^3-hybridised in diborane. Diborane is not planar in structure as 2 hydrogen atoms are present above and below the plane containing four terminal bonds between B and H. Two 3-centre-2-electron bonds or banana bonds are present in diborane.

$$2NaBH_4 + I_2 \longrightarrow B_2H_6 + 2NaI + H_2$$

44. (a) : Sulphide ores $\Rightarrow$ PbS, Ag_2S, $CuFeS_2$, ZnS.

45. (b) : Ba – $[Xe]6s^2$

Ca – CaC_2O_4 is highly insoluble in water.

Li – Organic solvent soluble compounds due to covalent nature

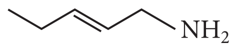
Na – Formation of very strong monoacidic base *eg.*, NaOH

46. (d) : Preliminary work for his great textbook "Principles of Chemistry" led Mendeleev to propose the periodic law and to construct his Periodic Table of elements. At that time, the structure of atom was unknown and Mendeleev's idea to consider that the properties of the elements were in someway related to their atomic masses was a very imaginative one.

The element with atomic number 101 is named as Mendelevium.

47. (d) : Solubility of oxygen increases with decrease in temperature.

48. (d) : $AgNO_3 + KI \longrightarrow AgI/I^-$

49. (c) :  has no conjugation between π-bond and lone-pair of N. Hence, there will be no resonance in this compound. Rest all will show resonance.

50. (b) : Only tritium is radioactive and emits low energy β particles ($t_{1/2}$ = 12.33 years)

51. (4) : $M = \dfrac{W_{solute}}{M_{solute} \times V_{soln}\,(\text{in L})} = \dfrac{0.72}{180}$

$= 0.004 = 4 \times 10^{-3}$ M

52. (1) : Complex

(i) $[Co(NH_3)_6]Cl_2 \Rightarrow Co^{2+} = 3d^7\ (t_{2g}^6 e_g^1)$,

unpaired electron = 1

(ii) $[Co(NH_3)_6]Cl_3 \Rightarrow Co^{3+} = 3d^6\ (t_{2g}^6 e_g^0)$,

unpaired electron = 0

Total unpaired electrons =1

53. (106) : $d = \left\{\dfrac{Z \times M}{N_A \times \text{Volume}}\right\}$

$$7.62 = \frac{4 \times M}{6.022 \times 10^{23} \times [0.4518 \times 10^{-7}]^3}$$

$$M = \frac{7.62 \times 6.022 \times 10^{23} \times [0.4518 \times 10^{-7}]^3}{4}$$

$= 1.057 \times 10^2 = 105.7$ gram/mole ≈ 106 g/mol

54. (7) :

	$N_2O_{5(g)}$	$\rightarrow$ $2NO_{2(g)}$	+ $\frac{1}{2}O_{2(g)}$
Initial	$a = 2.4 \times 10^{-2}$ M	0	0
After 1 hour	$(a-x) = 1.6 \times 10^{-2}$ M	x	x

$$K = \frac{1}{t}\ln\left(\frac{a}{a-x}\right)$$

$$k = \frac{2.303}{60}\log\left(\frac{2.4 \times 10^{-2}}{1.6 \times 10^{-2}}\right)$$

$$k = \frac{2.303}{60}\log\left(\frac{3}{2}\right)$$

$k = 0.0067 = 6.7 \times 10^{-3}\ \text{min}^{-1}$ or $k \approx 7 \times 10^{-3}\ \text{min}^{-1}$.

55. (78) : $C_6H_6 \xrightarrow{\text{Methylation}} C_6H_5CH_3$

1 mol of benzene gives 1 mol of toluene.

78 g of benzene gives 92 g of toluene.

$$(W_{\text{theoretical}}) = \frac{10}{78} \times 92$$

$$\% \text{ yield} = \frac{W_{\text{actual}}}{W_{\text{theoretical}}} \times 100 = \left[\frac{9.2}{10 \times 92} \times 78\right] \times 100 = 78\%$$

56. (3) : For the given cell:

$$E_{\text{cell}} = E^{\circ}_{\text{cell}} - \frac{0.059}{2} \log \frac{[Cu^{2+}]}{[Ag^{+}]^2}$$

$$= 2.97 - \frac{0.059}{2} \log\left\{\frac{0.250}{(10^{-3})^2}\right\}$$

$= 2.97 - 0.0295 \log[2.5 \times 10^5]$
$= 2.97 - 0.0295[5 + \log 2.5] = 2.8 \approx 3$ V

57. (21) : $\underset{12\text{ gram}}{C(\text{graphite})} + O_2 \longrightarrow CO_{2(g)}$ $\quad \Delta H = -2.48 \times 10^2$ kJ/mole

$$\text{Total heat released by 1 gram} = 2.48 \times \frac{1}{12} \times 10^2$$

$= 20.67$ kJ ≈ 21 kJ

58. (12) : $_{23}V = 1s^2 2s^2 2p^6 3s^2 3p^6 3d^3 4s^2$

59. (6) :

C—C—C—C=C (Pent-1-ene)

C—C(—C)—C=C (3-Methylbut-1-ene)

C—C—C(—C)=C (2-Methylbut-1-ene)

C—C(—C)=C—C (2-Methylbut-2-ene)

C—C=C—C—C (Pent-2-ene) $\rightarrow$ 2 Geometrical Isomers

60. (2) : $K_p = K_c(RT)^{\Delta n_g}$

$47.9 = K_c(0.083 \times 288)^1 \Rightarrow K_c = 2$

61. (c) : Given, $A = [a_{ij}]$ be a real matrix of order 3×3 such that $a_{i1} + a_{i2} + a_{i3} = 1$, for $i = 1, 2, 3$.

$$\therefore \quad A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ Sum of all the entries of the matrix A^3 is $1 + 1 + 1 = 3$.

62. (c) : Four dice are thrown simultaneously.

$\therefore$ Total number of possible matrices $= 6 \times 6 \times 6 \times 6$

Now, let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

If A is singular matrix $\Rightarrow |A| = 0$

$\Rightarrow ad - bc = 0 \Rightarrow ad = bc$

$\therefore$ $a = 1, d = 6$ and $b = 3, c = 2$ $\rightarrow$ 4 possible matrices
$a = 2, d = 6$ and $b = 4, c = 3$ $\rightarrow$ 4 possible matrices
$a = 2, d = 3$ and $b = 1, c = 6$ $\rightarrow$ 4 possible matrices
$a = 3, d = 4$ and $b = 2, c = 6$ $\rightarrow$ 4 possible matrices

Thus, total number of cases for singular matrix having different entries $= 4 + 4 + 4 + 4 = 16$

$\therefore$ Total number of non-singular matrices having different entries $= 6 \times 5 \times 4 \times 3 - 16 = 344$

$\therefore$ The probability that the matrices are non-singular and have different entries $= \frac{344}{1296} = \frac{43}{162}$.

63. (b) : We have, $\sin^7 x + \cos^7 x = 1, x \in [0, 4\pi]$

If $\sin x \neq 0$ and $\cos x \neq 0$, then

$\sin^7 x < \sin^2 x$ ...(i)

and $\cos^7 x < \cos^2 x$ ...(ii)

Adding both (i) and (ii), we get

$\sin^7 x + \cos^7 x < \sin^2 x + \cos^2 x$

$\Rightarrow \sin^7 x + \cos^7 x < 1$

Hence, $\sin^7 x + \cos^7 x = 1$ is the only possibility, when $\sin x = 1$ and $\cos x = 0$ or $\cos x = 1$ and $\sin x = 0$.

$\therefore \quad x = 0, \frac{\pi}{2}, \frac{5\pi}{2}, 2\pi, 4\pi$ [As $x \in [0, 4\pi]$]

$\therefore$ Number of solutions $= 5$

64. (c) : (a) $[\vec{a}\,\vec{b}\,\vec{c}] + [\vec{c}\,\vec{a}\,\vec{b}] = 2[\vec{a}\,\vec{b}\,\vec{c}] = 2(\vec{a} \cdot (\vec{b} \times \vec{c}))$

$= 2\vec{a} \cdot \vec{a} = 2|\vec{a}|^2 = 8$

(b) Projection of $\vec{a}$ on $\vec{b} \times \vec{c} = \frac{\vec{a} \cdot (\vec{b} \times \vec{c})}{|\vec{b} \times \vec{c}|} = \frac{\vec{a} \cdot \vec{a}}{|\vec{a}|} = |\vec{a}| = 2$

(c) If $\vec{a}\ \vec{b}\ \vec{c}$ are mutually perpendicular vectors, then

$|\vec{a} \times \vec{b}| = |\vec{c}| \Rightarrow |\vec{a}||\vec{b}| = |\vec{c}| \Rightarrow |\vec{b}| = \frac{|\vec{c}|}{2}$

Also, $|\vec{b} \times \vec{c}| = |\vec{a}|$

$\Rightarrow |\vec{b}||\vec{c}| = 2 \Rightarrow |\vec{c}| = 2$ and $|\vec{b}| = 1$

$|3\vec{a} + \vec{b} - 2\vec{c}|^2 = (3\vec{a} + \vec{b} - 2\vec{c}) \cdot (3\vec{a} + \vec{b} - 2\vec{c})$

$= 9|\vec{a}|^2 + |\vec{b}|^2 + 4|\vec{c}|^2 = (9 \times 4) + 1 + (4 \times 4)$

$= 36 + 1 + 16 = 53$

(d) $\vec{a} \times ((\vec{b} + \vec{c}) \times (\vec{b} - \vec{c}))$

$= \vec{a} \times ((-\vec{b} \times \vec{c}) + (\vec{c} \times \vec{b})) = -2(\vec{a} \times (\vec{b} \times \vec{c})) = -2(\vec{a} \times \vec{a}) = \vec{0}$

65. (d) : Given, equation is $z^2 + 3\bar{z} = 0$

Let $z = x + iy$, then $\bar{z} = x - iy$

$\therefore \quad z^2 + 3\bar{z} = 0 \Rightarrow (x + iy)^2 + 3(x - iy) = 0$

$\Rightarrow x^2 - y^2 + 2ixy + 3x - 3iy = 0$

$\Rightarrow x^2 - y^2 + 3x + i(2xy - 3y) = 0$

$\Rightarrow x^2 - y^2 + 3x = 0$ and $y(2x - 3) = 0$ ...(i)

When $y = 0$, $x^2 - y^2 + 3x = 0 \Rightarrow x = 0$ or $x = -3$

When $y \neq 0 \Rightarrow x = \frac{3}{2}$; $x^2 - y^2 + 3x = 0 \Rightarrow y = \pm\frac{3\sqrt{3}}{2}$

$\therefore$ Solutions are $(0, 0), (-3, 0), \left(\frac{3}{2}, \frac{3\sqrt{2}}{2}\right)$ and $\left(\frac{3}{2}, \frac{-3\sqrt{2}}{2}\right)$

$\Rightarrow$ Total solutions $= 4 \Rightarrow n = 4$

$$\therefore \sum_{k=0}^{\infty} \frac{1}{4^k} = \frac{1}{1 - \frac{1}{4}} = \frac{4}{3} \qquad \text{(Sum of infinite G.P.)}$$

66. (d) : Let L be the line of intersection of planes $\pi_1 : \vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k}) = 2$ and $\pi_2 : \vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k}) = 2$ and let it cuts xy – plane at point A.

$\Rightarrow$ $x - y = 2$ and $2x + y = 2$ [Using equations of given plane]

$$\Rightarrow \quad x=\frac{4}{3}, y=\frac{-2}{3}$$

$$\therefore \ A\left(\frac{4}{3},\frac{-2}{3},0\right)$$

Vector parallel to the line of intersection is given by

$$\vec{n}=\vec{n}_1\times\vec{n}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 2\\ 2 & 1 & -1\end{vmatrix}=-\hat{i}+5\hat{j}+3\hat{k}$$

Thus, equation of line of intersection is

$$\frac{x-\frac{4}{3}}{-1}=\frac{y+\frac{2}{3}}{5}=\frac{z-0}{3}=\lambda(\text{say})$$

Let coordinates of foot of perpendicular be

$$B\left(-\lambda+\frac{4}{3},5\lambda-\frac{2}{3},3\lambda\right)$$

$\because$ $C(1, 2, 0)$ lies on the perpendicular line.

$$\therefore \quad \overrightarrow{CB}=\left(-\lambda+\frac{1}{3}\right)\hat{i}+\left(5\lambda-\frac{8}{3}\right)\hat{j}+(3\lambda)\hat{k}$$

Now, $\overrightarrow{CB}\cdot\vec{n}=0$

$$\Rightarrow \quad \lambda-\frac{1}{3}+25\lambda-\frac{40}{3}+9\lambda=0 \Rightarrow \quad 35\lambda=\frac{41}{3}\Rightarrow\lambda=\frac{41}{105}$$

Now, as $\alpha=-\lambda+\frac{4}{3}$, $\beta=5\lambda-\frac{2}{3}$ and $\gamma=3\lambda$

$$\therefore \quad \alpha+\beta+\gamma=7\lambda+\frac{2}{3}=7\left(\frac{41}{105}\right)+\frac{2}{3}=\frac{51}{15}$$

$$\Rightarrow \quad 35(\alpha+\beta+\gamma)=\frac{51}{15}\times 35=119.$$

67. (d) : (a) $(p \Rightarrow {\sim}q) \vee ({\sim}q \Rightarrow p)$

$\equiv ({\sim}p \vee {\sim}q) \vee (q \vee p) \equiv ({\sim}p \vee p) \vee ({\sim}q \vee q) \equiv t \vee t \equiv t$

(b) $(p \Rightarrow q) \vee ({\sim}q \Rightarrow p)$

$\equiv ({\sim}p \vee q) \vee (q \vee p) \equiv ({\sim}p \vee p) \vee q \equiv t \vee q \equiv t$

(c) $(q \Rightarrow p) \vee ({\sim}q \Rightarrow p)$

$\equiv ({\sim}q \vee p) \vee (q \vee p) \equiv ({\sim}q \vee q) \vee p \equiv t \vee p \equiv t$

(d) $({\sim}p \Rightarrow q) \vee ({\sim}q \Rightarrow p) \equiv (p \vee q) \vee (q \vee p)$

$\equiv (p \vee p) \vee (q \vee q) \equiv p \vee q$, which is not a tautology.

68. (b) : $[e^x]^2 + [e^x + 1] - 3 = 0$

$\Rightarrow$ $[e^x]^2 + [e^x] - 2 = 0$

Let $[e^x] = y$, then we have

$y^2 + y - 2 = 0$

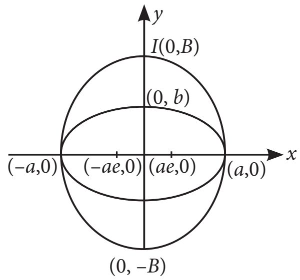
$\Rightarrow$ $(y + 2)(y - 1) = 0$

$\Rightarrow$ $y = -2, 1$

$\therefore$ $[e^x] = 1, -2 \Rightarrow [e^x] = 1 \Rightarrow$ $x \in [0, \log_e 2)$

69. (d) : We have, E_1: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a > b$

Also, it is given that, eccentricities $e_1 = e_2$.



$$\Rightarrow \quad 1-\frac{b^2}{a^2}=1-\frac{a^2}{B^2} \quad (\because B > a)$$

$$\Rightarrow \quad B^2=\frac{a^4}{b^2}$$

$$\Rightarrow B=\frac{a^2}{b} \qquad \text{...(i)}$$

Now, as foci of E_2 are end points of minor axis of E_1, therefore $B{\cdot}e = b$ ...(ii)

From (i) and (ii), we get $e=\frac{b^2}{a^2}$

Since, $e^2=1-\frac{b^2}{a^2}$

$\therefore$ $e^2 = 1 - e$

$$\Rightarrow \quad e^2+e-1=0 \Rightarrow \quad e=\frac{\sqrt{5}-1}{2}.$$

70. (d) : We have, $f(x)=\dfrac{\cos^{-1}\sqrt{x^2-x+1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$.

Clearly, $f(x)$ will be well-defined if $1 \ge x^2 - x + 1 \ge 0$ and $0<\frac{2x-1}{2}\le 1$

$\Rightarrow$ $x(x - 1) \le 0$ and $1 < 2x \le 3$

$$\Rightarrow \quad x\in[0,1]\cap x\in\left(\frac{1}{2},\frac{3}{2}\right]$$

$$\Rightarrow \quad x\in[0,1]\cap\left(\frac{1}{2},\frac{3}{2}\right] \quad\Rightarrow\quad x\in\left(\frac{1}{2},1\right]$$

$$\Rightarrow \quad \alpha=\frac{1}{2} \text{ and } \beta=1 \quad \therefore \ \alpha+\beta=\frac{3}{2}.$$

71. (d) : We have, $f(x)=\begin{cases}\dfrac{x^3}{(1-\cos 2x)^2}\log_e\left(\dfrac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right) & ,x\ne 0\\ \alpha & ,x=0\end{cases}$

Now, as f is continuous at $x = 0$

$$\therefore \quad \lim_{x\to 0} f(x)=\alpha$$

Consider, $\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{x^3}{(1-\cos 2x)^2}\log\left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right)$

$$=\lim_{x\to 0}\frac{x^3\times x}{(1-1+2\sin^2 x)^2}\,\frac{\log(1+2xe^{-2x})-2\log(1-xe^{-x})}{x}$$

$$=\lim_{x\to 0}\frac{x^4}{4\sin^4 x}\cdot\frac{\log(1+2xe^{-2x})-2\log(1-xe^{-x})}{x}$$

$$=\lim_{x\to 0}\left[\frac{1}{4}\left(\frac{2(xe^{-2x}(-2)+e^{-2x})}{(1+2xe^{-2x})}-\frac{2(xe^{-x}-e^{-x})}{(1-xe^{-x})}\right)\right]=\frac{1}{4}\times 4$$

$\therefore$ $\alpha = 1$.

72. (a) : We have, $\displaystyle\int_0^{100\pi}\frac{\sin^2 x}{e^{\left(\frac{x}{\pi}-\left[\frac{x}{\pi}\right]\right)}}dx=\frac{\alpha\pi^3}{1+4\pi^2},\alpha\in R$

Clearly, L.H.S. $=\displaystyle\int_0^{100\pi}\frac{\sin^2 x}{e^{\left\{\frac{x}{\pi}\right\}}}dx\ =100\int_0^{\pi}\frac{\sin^2 x}{e^{\frac{x}{\pi}}}dx$

$\Rightarrow\ 50\displaystyle\int_0^{\pi}e^{-\frac{x}{\pi}}[1-\cos 2x]\,dx$

$\Rightarrow\ 50\left[e^{\frac{-x}{\pi}}\cdot(-\pi)\right]_0^{\pi}-50\displaystyle\int_0^{\pi}e^{-\frac{x}{\pi}}\cos 2x\,dx$

$$=50\times(-\pi)(e^{-1}-1)-\frac{50\times\left[e^{-\frac{x}{\pi}}\left(-\frac{1}{\pi}\times\cos 2x+2\sin 2x\right)\right]_0^{\pi}}{\left(\frac{1}{\pi^2}+4\right)}$$

$$=-50\pi(e^{-1}-1)-\frac{50\pi^2}{(1+4\pi^2)}\left[e^{-1}\left(-\frac{1}{\pi}\right)+\frac{1}{\pi}\right]$$

$$=\frac{200\pi^3(1-e^{-1})}{1+4\pi^2}.$$

So, $\alpha = 200\ (1-e^{-1})$

73. (d) : Given, line $L: 2x + y = k$, $k > 0$ is a tangent to the hyperbola $x^2 - y^2 = 3$ as well as to the parabola $y^2 = \alpha x$.

Consider, $2x + y = k\ \Rightarrow\ y = -2x + k$

$\Rightarrow\ k^2 = 3(-2)^2 - 3$

$[\because$ If $y = mx + c$ is a tangent to the hyperbola $x^2 - y^2 = a^2$, then $c^2 = a^2m^2 - a^2]$

$\Rightarrow k^2 = 9 \Rightarrow k = \pm 3 \Rightarrow k = 3\ (\because\ k > 0)$

$\therefore\ y = -2x + 3$

We know that if $y = mx + c$ is a tangent to $y^2 = 4ax$, then

$c=\dfrac{a}{m}$

$\therefore\ 3=\dfrac{\alpha}{4(-2)}\qquad\left[\because\ \text{Here, } 4a=\alpha\Rightarrow\ a=\dfrac{\alpha}{4}\right]$

$\Rightarrow\ \alpha = -24.$

74. (c) : Given, circle S is $36x^2 + 36y^2 - 108x + 120y + C = 0$

$\Rightarrow\ x^2+y^2=\dfrac{108x}{36}-\dfrac{120y}{36}-\dfrac{C}{36}$

$\Rightarrow\ x^2+y^2=3x-\dfrac{10}{3}y-\dfrac{C}{36}$

or $\ x^2+y^2-3x+\dfrac{10}{3}y+\dfrac{C}{36}=0$

Given that circle does not touch the co-ordinate axes.

$\Rightarrow\ g^2 - c < 0$ and $f^2 - c < 0$

$\Rightarrow\ \left(-\dfrac{3}{2}\right)^2-\dfrac{C}{36}<0$ and $\left(\dfrac{10}{6}\right)^2-\dfrac{C}{36}<0$

$\left[\because\ \text{Here } g=\dfrac{-3}{2},\ f=\dfrac{10}{6} \text{ and } c=\dfrac{C}{36}\right]$

$\Rightarrow\ \dfrac{9}{4}-\dfrac{C}{36}<0$ and $\dfrac{100}{36}-\dfrac{C}{36}<0$

$\Rightarrow\ \dfrac{9}{4}<\dfrac{C}{36}$ and $\dfrac{100}{36}<\dfrac{C}{36}$

$\Rightarrow\ C>\dfrac{9\times 36}{4}$ and $C>\dfrac{36\times 100}{36}$

$\Rightarrow\ C > 81\quad$ ...(i) and $C > 100\quad$ ...(ii)

Now, as $g^2 + f^2 - C > 0$

$\therefore\ \left(-\dfrac{3}{2}\right)^2+\left(\dfrac{5}{3}\right)^2-\dfrac{C}{36}>0$

$\Rightarrow\ \dfrac{C}{36}<\dfrac{9}{4}+\dfrac{25}{9}\ \Rightarrow C<181\quad$ ...(iii)

Intersection point of $2x - y = 5$ and $x - 2y = 4$ is $(2, -1)$ and $(2, -1)$ lies inside the circle. So, $S\ (2, -1) < 0$

$\therefore\ 36(2)^2 + 36(-1)^2 - 108(2) + 120(-1) + C < 0$

$\Rightarrow\ C < 156\quad$ ...(iv)

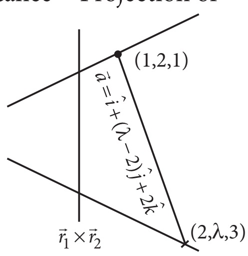
$\therefore$ By (ii), (iii) and (iv), we have

$100 < C < 156.$

75. (a) : Let $L_1:\dfrac{(x-1)}{2}=\dfrac{(y-2)}{1}=\dfrac{(z-1)}{3};\ \vec r_1=2\hat i+\hat j+3\hat k$

$L_2:\dfrac{(x-2)}{1}=\dfrac{(y-\lambda)}{2}=\dfrac{(z-3)}{4};\ \vec r_2=\hat i+2\hat j+4\hat k$

Now, shortest distance = Projection of $\vec a$ on $\vec r_1\times\vec r_2$



$$=\frac{|a\cdot(\vec r_1\times\vec r_2)|}{|\vec r_1\times\vec r_2|}$$

Now, $|\vec a\cdot(\vec r_1\times\vec r_2)|=\begin{vmatrix}1 & \lambda-2 & 2\\ 2 & 1 & 3\\ 1 & 2 & 4\end{vmatrix}=|14-5\lambda|$

and $|\vec r_1\times\vec r_2|=\sqrt{38}$

$\Rightarrow\ \dfrac{1}{\sqrt{38}}=\dfrac{|14-5\lambda|}{\sqrt{38}}$

$\Rightarrow\ |14 - 5\lambda| = 1$

$\Rightarrow\ 14 - 5\lambda = 1$ or $14 - 5\lambda = -1$

$\therefore\ \lambda=\dfrac{13}{5}$ or $\lambda = 3$

$\therefore$ Integral value of $\lambda = 3$.

76. (d) : Given, vector $\vec{a}$ is coplanar with vector $\vec{b}=2\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}+\hat{k}$. Also, we have $\vec{a}$ is perpendicular to $\vec{d}=3\hat{i}+2\hat{j}+6\hat{k}$ and $|\vec{a}|=\sqrt{10}$.

$\therefore \quad \vec{a}=\lambda\vec{b}+\mu\vec{c} = \lambda(2\hat{i}+\hat{j}+\hat{k})+\mu(\hat{i}-\hat{j}+\hat{k})$

$= \hat{i}(2\lambda+\mu)+\hat{j}(\lambda-\mu)+\hat{k}(\lambda+\mu)$

and $\vec{a}\cdot\vec{d}=0$

$\Rightarrow \quad 3(2\lambda+\mu)+2(\lambda-\mu)+6(\lambda+\mu)=0$

$\Rightarrow \quad 14\lambda+7\mu=0 \Rightarrow \mu=-2\lambda$

$\Rightarrow \quad \vec{a}=(0)\hat{i}+3\lambda\hat{j}+(-\lambda)\hat{k} \Rightarrow |\vec{a}|=\sqrt{10}\,|\lambda|=\sqrt{10}$

$\Rightarrow \quad |\lambda|=1 \Rightarrow \lambda=1$ or -1

Now, as $[\vec{a}\;\vec{b}\;\vec{c}]=0$

$\therefore \; [\vec{a}\;\vec{b}\;\vec{c}]+[\vec{a}\;\vec{b}\;\vec{d}]+[\vec{a}\;\vec{c}\;\vec{d}]=[\vec{a}\;\;\vec{b}+\vec{c}\;\;\vec{d}]$

$$=\begin{vmatrix}0 & -3\lambda & \lambda\\ 3 & 0 & 2\\ 3 & 2 & 6\end{vmatrix}$$

$\Rightarrow \quad -3\lambda(12)-\lambda(6)=-42\lambda=42$ or -42.

Thus, the possible value is -42.

77. (d) : We have, $\operatorname{cosec}^2 x\,dy+2dx=(1+y\cos 2x)\operatorname{cosec}^2 x\,dx$

$\Rightarrow \quad \operatorname{cosec}^2 x\,dy=(1+y\cos 2x)\operatorname{cosec}^2 x\,dx-2dx$

$\Rightarrow \quad \operatorname{cosec}^2 x\,dy=\{(1+y\cos 2x)\operatorname{cosec}^2 x-2\}dx$

$$\Rightarrow \quad \frac{dy}{dx}+2\sin^2 x=1+y\cos 2x$$

$$\Rightarrow \quad \frac{dy}{dx}+(-\cos 2x)y=\cos 2x$$

I.F. $= e^{\int -\cos 2x\,dx}=e^{-\frac{\sin 2x}{2}}$

Now, solution of differential equation is

$$y\left(e^{-\frac{\sin 2x}{2}}\right)=\int(\cos 2x)\left(e^{-\frac{\sin 2x}{2}}\right)dx+c$$

$$\Rightarrow \quad ye^{\frac{-\sin 2x}{2}}=-e^{-\frac{\sin 2x}{2}}+c$$

On applying given conditions, we get $c=e^{-\frac{1}{2}}$

$$\therefore \quad y\left(e^{-\frac{\sin 2x}{2}}\right)=-e^{-\frac{\sin 2x}{2}}+e^{-\frac{1}{2}}$$

Now, $y(0)=-1+e^{-1/2} \Rightarrow (y(0)+1)^2=e^{-1}$

78. (a) : We have, $f(x)=\begin{cases}-\frac{4}{3}x^3+2x^2+3x, & x>0\\ 3xe^x, & x\le 0\end{cases}$

$$\Rightarrow \quad f'(x)=\begin{cases}-\frac{4}{3}(3x^2)+4x+3, & x>0\\ 3[xe^x+e^x], & x<0\end{cases}$$

$$\Rightarrow \quad f'(x)=\begin{cases}-4x^2+4x+3, & x>0\\ 3xe^x+3e^x, & x<0\end{cases}$$

$\because \quad f'(x)>0$ in the interval $\left(-1,\frac{3}{2}\right)$.

$\therefore \quad f$ is increasing function in the interval $\left(-1,\frac{3}{2}\right)$.

79. (a) : The given system of equations is

$x+y+z=6,\ 3x+5y+5z=26,\ x+2y+\lambda z=\mu$

Coefficient matrix corresponding to above system of linear equations is

$$A=\begin{bmatrix}1 & 1 & 1\\ 3 & 5 & 5\\ 1 & 2 & \lambda\end{bmatrix}$$

For no solution, $|A|=0$ and $(\text{adj}A)B\neq O$.

$$\therefore \quad |A|=0 \Rightarrow \begin{vmatrix}1 & 1 & 1\\ 3 & 5 & 5\\ 1 & 2 & \lambda\end{vmatrix}=0$$

$\Rightarrow \quad 1(5\lambda-10)-1(3\lambda-5)+1(6-5)=0$

$\Rightarrow \quad 2\lambda-4=0 \Rightarrow \lambda=2$

Now, $(\text{adj } A)B\neq O \Rightarrow \mu\neq 10$

80. (a) : We have, $S_{10}=530$ and $S_5=140$

$\Rightarrow \quad S_{10}=\frac{10}{2}[2a+9d]=530$

$\Rightarrow \quad 5[2a+9d]=530$

$\Rightarrow \quad 2a+9d=106$ ...(i)

and $S_5=\frac{5}{2}[2a+4d]=140 \Rightarrow 2a+4d=56$ ...(ii)

$\therefore \quad$ By (i) and (ii), we get $a=8$ and $d=10$

$\therefore \quad S_{20}-S_6=\frac{20}{2}[2a+19d]-\frac{6}{2}[2a+5d]$

$=10[2a+19d]-3[2a+5d]$

$=20a+190d-6a-15d=14a+175d$

Putting $a=8$ and $d=10$, we get

$S_{20}-S_6=14\times 8+175\times 10=1862$

81. (1251) : Clearly, $2040=2^3\times 3\times 5\times 17$

$\therefore \quad n$ should not be multiple of 2, 3, 5 and 17

Sum of all such $n=(1+3+5+\ldots\ldots+99)-(3+9+15+21+\ldots.+99)-(5+25+35+55+65+85+95)-(17)=1251$

$\therefore \quad$ The sum of all the elements in the set $\{n\in\{1,2,\ldots\ldots 100\}\ |\text{H.C.F of } n \text{ and } 2040 \text{ is } 1\}=1251$

82. (4) : We have a function $f: R\to R$ defined as

$$f(x)=\begin{cases}3\left(1-\frac{|x|}{2}\right), & \text{if } |x|\le 2\\ 0, & \text{if } |x|>2\end{cases}$$

$$f(x+2)=\begin{cases}3\left(\frac{4+x}{2}\right), & \text{if } -4\le x\le -2\\ \frac{-3x}{2}, & \text{if } -2<x\le 0\\ 0, & \text{if } x\in(-\infty,-4)\cup(0,\infty)\end{cases}$$

$$-f(x-2)=\begin{cases}\dfrac{-3x}{2}, & \text{if } 0\le x\le 2\\ \dfrac{3(x-4)}{2}, & \text{if } 2<x\le 4\\ 0, & \text{if } x\in(-\infty,0)\cup(4,\infty)\end{cases}$$

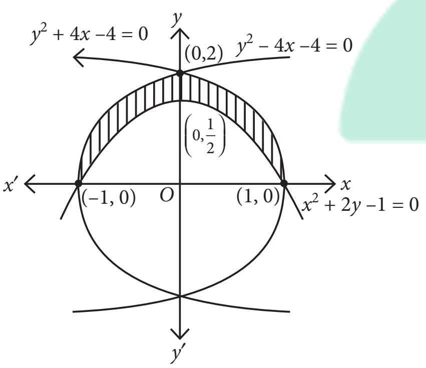
Now, $g(x) = f(x+2) - f(x-2)$

$$\therefore\quad g(x)=\begin{cases}0+0, & \text{if } x\in(-\infty,-4]\\ \dfrac{3(4+x)}{2}, & \text{if } -4<x\le -2\\ \dfrac{-3x}{2}, & \text{if } -2<x\le 0\\ \dfrac{-3x}{2}, & \text{if } 0<x\le 2\\ \dfrac{3(x-4)}{2}, & \text{if } 2<x\le 4\\ 0, & \text{if } x\in(4,\infty)\end{cases}$$

$$\text{and } g'(x)=\begin{cases}0, & \text{if } x\in(-\infty,-4]\\ \dfrac{3}{2}, & \text{if } -4<x\le -2\\ \dfrac{-3}{2}, & \text{if } -2<x\le 0\\ \dfrac{-3}{2}, & \text{if } 0<x\le 2\\ \dfrac{3}{2}, & \text{if } 2<x\le 4\\ 0, & \text{if } x\in(4,\infty)\end{cases}$$

$\therefore\quad n = 0$ and $m = 4 \Rightarrow n + m = 4$

83. (2) : Region bounded by the given curves is the shaded region, shown below.



$$\therefore\quad \text{Required area } = 2\left[\int_0^2\left(\frac{4-y^2}{4}\right)dy-\int_0^1\left(\frac{1-x^2}{2}\right)dx\right]$$

$$=2\left[\frac{4}{3}-\frac{1}{3}\right] = 2 \text{ sq.units.}$$

84. (8) : Given, constant term in the binomial expansion of

$$\left(2x^r+\frac{1}{x^2}\right)^{10} = 180$$

Let the $(k+1)^{\text{th}}$ term is $T_{k+1} = {}^{10}C_k(2x^r)^{10-k}\left(\dfrac{1}{x^2}\right)^k$

$= {}^{10}C_k(2)^{(10-k)}\,.\,x^{r(10-k)}.(x^{-2})^k = {}^{10}C_k(2)^{(10-k)}x^{10r-rk-2k}$

Now, for constant term $10r - rk - 2k = 0$

$\Rightarrow r = \dfrac{2k}{10-k}$ and ${}^{10}C_k(2)^{10-k} = 180$

$\Rightarrow\; k = 8 \;\;\therefore\;\; r = \dfrac{2\times 8}{10-8} = 8.$

85. (720) : Let $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$.

To find the number of bijective functions $f : A \to A$ such that $f(1) + f(2) + f(3) = 3$, the only possibility for this sum is $0 + 1 + 2 = 3$.

$\Rightarrow$ Elements 1, 2, 3 in the domain can be mapped with 0, 1, 2 only.

Therefore, number of bijective functions $= 3! \times 5! = 6 \times 120 = 720$.

86. (4) : Given D.E. is $\left[(x+2)e^{\left(\frac{y+1}{x+2}\right)}+(y+1)\right]dx = (x+2)dy$, $y(1) = 1$.

Let $y + 1 = Y \Rightarrow dy = dY$ and $x + 2 = X \Rightarrow dx = dX$

$\Rightarrow \left(Xe^{\frac{Y}{X}}+Y\right)dX = X\,dY \;\Rightarrow X\,dY - Y\,dX = Xe^{\frac{Y}{X}}dX$

$\Rightarrow d\left(\dfrac{Y}{X}\right)e^{\frac{-Y}{X}} = \dfrac{dX}{X} \;\Rightarrow\; -e^{-\frac{Y}{X}} = \log|X| + C$

At $y(1) = 1$, $X = 3$, $Y = 2$

$\therefore\quad Y(3) = 2 \;\Rightarrow\; -e^{-\frac{2}{3}} = \log|3| + C$

$\Rightarrow\; e^{-\frac{Y}{X}} = e^{-\frac{2}{3}} + \log 3 - \log|X| > 0$

$\Rightarrow\; \log|X| < e^{-\frac{2}{3}} + \log 3 = \lambda$ (say)

$\Rightarrow\; |x+2| < e^{\lambda}$

$\Rightarrow\; -e^{\lambda} < x + 2 < e^{\lambda} \Rightarrow -e^{\lambda} - 2 < x < e^{\lambda} - 2$

$\Rightarrow\; \alpha + \beta = -4 \;\Rightarrow\; |\alpha + \beta| = 4$

87. (96) : We have digits 0, 2, 4, 6, 8.

A	B	C	D	E

Number of possibility for $A = 4$

Number of possibility for $B = 4$

Number of possibility for $C = 3$

Number of possibility for $D = 2$

Number of possibility for $E = 1$

$\therefore$ Number of numbers greater than 10,000 formed $= 4 \times 4 \times 3 \times 2 \times 1 = 96$.

88. (96) : Let $f(n)=\left(\frac{10}{11}\right)^n+\left(\frac{9}{11}\right)^n$, then

$$f'(n)=\left(\frac{10}{11}\right)^n\log\left(\frac{10}{11}\right)+\left(\frac{9}{11}\right)^n\log\left(\frac{9}{11}\right)<0$$

$\left(\because \frac{d}{dx}(a^x)=a^x\log_e a\right)$

$\Rightarrow$ $f(n)$ is decreasing function.

Putting $n = 1, 2, 3, 4$, ...,we see that

$$f(1)=\frac{10}{11}+\frac{9}{11}=\frac{19}{11}>1$$

$$f(2)=\left(\frac{10}{11}\right)^2+\left(\frac{9}{11}\right)^2=\frac{100}{121}+\frac{81}{121}=\frac{181}{121}>1$$

Similarly, $f(3)$ and $f(4) > 0$. But $f(5)=\left(\frac{10}{11}\right)^5+\left(\frac{9}{11}\right)^5<1$

Similarly, for $n = 6, 7, 8,$ .... 100, $f(n) < 1$.

$\therefore$ Required number of elements = 96.

89. (4) :

Class	Frequency	x_i	f_ix_i
0-6	a	3	$3a$
6-12	b	9	$9b$
12-18	12	15	180
18-24	9	21	189
24-30	5	27	135
	$N = 26 + a + b$		$3a + 9b + 504$

$\therefore$ Mean $=\frac{3a+9b+504}{a+b+26}=\frac{309}{22}$ (Given)

$\Rightarrow$ $66a + 198b + 11088 = 309a + 309b + 8034$

$\Rightarrow$ $81a + 37b = 1018$ ...(i)

Clearly, median $= 12 + \frac{\frac{a+b+26}{2}-(a+b)}{12}\times 6 = 14$ (Given)

$$\Rightarrow \frac{13}{2}-\left(\frac{a+b}{4}\right)=2$$

$\Rightarrow$ $a + b = 18$ ...(ii)

By (i) and (ii), we get $a = 8$, $b = 10$

$\therefore$ $(a - b)^2 = (8 - 10)^2 = 4$

90. (3125) : Here, $A=\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}$.

Let $B=\begin{bmatrix}p&q&r\\s&t&u\\v&w&x\end{bmatrix}$ be 3×3 matrix such that $AB = BA$

$$\Rightarrow \begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}\begin{bmatrix}p&q&r\\s&t&u\\v&w&x\end{bmatrix}=\begin{bmatrix}p&q&r\\s&t&u\\v&w&x\end{bmatrix}\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}s&t&u\\p&q&r\\v&w&x\end{bmatrix}=\begin{bmatrix}q&p&r\\t&s&u\\w&v&x\end{bmatrix}$$

$\Rightarrow s = q, t = p, u = r, v = w$

So, we have to select 5 elements for B.

$\because$ Each element of B have five options 1, 2, 3, 4, 5.

$\therefore$ Total number of ways $= 5^5 = 3125$.

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