JEE ADVANCED 2018

20th May

PHYSICS

SECTION 1 (Maximum Marks : 24)

- *This section contains SIX (06) questions.*
- *Each question has FOUR options for correct answer(s). ONE OR MORE THAN ONE of these four option(s) is (are) correct option(s).*
- *For each question, choose the correct option(s) to answer the question.*
- *Answer to each question will be evaluated according to the following marking scheme:*

Full Marks :	*+4*	*If only (all) the correct option(s) is (are) chosen.*
Partial Marks :	*+3*	*If all the four options are correct but ONLY three options are chosen.*
Partial Marks :	*+2*	*If three or more options are correct but ONLY two options are chosen, both of which are correct options.*
Partial Marks :	*+1*	*If two or more options are correct but ONLY one option is chosen and it is a correct option.*
Zero Marks :	*0*	*If none of the options is chosen (i.e. the question is unanswered).*
Negative Marks :	*–2*	*In all other cases.*

- *For Example : If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in -2 marks.*

1. The potential energy of a particle of mass m at a distance r from a fixed point O is given by $V(r) = kr^2/2$, where k is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius R about the point O. If v is the speed of the particle and L is the magnitude of its angular momentum about O, which of the following statements is (are) true?

(a) $v = \sqrt{\frac{k}{2m}}R$

(b) $v = \sqrt{\frac{k}{m}}R$

(c) $L = \sqrt{mk}R^2$

(d) $L = \sqrt{\frac{mk}{2}}R^2$

2. Consider a body of mass 1.0 kg at rest at the origin at time t = 0. A force $\vec{F} = (\alpha t\hat{i} + \beta\hat{j})$ is applied on the body, where α = 1.0 N s^{-1} and β = 1.0 N. The torque acting on the body about the origin at time t = 1.0 s is $\vec{\tau}$. Which of the following statements is (are) true?

(a) $|\vec{\tau}| = \frac{1}{3}$ N m

(b) The torque $\vec{\tau}$ is in the direction of the unit vector $+\hat{k}$

(c) The velocity of the body at t = 1 s is $\vec{v} = \frac{1}{2}(\hat{i} + 2\hat{j})$ m s^{-1}

(d) The magnitude of displacement of the body at t = 1 s is $\frac{1}{6}$ m

3. A uniform capillary tube of inner radius r is dipped vertically into a beaker filled with water. The water rises to a height h in the capillary tube above the water surface in the beaker. The surface tension of water is σ. The angle of contact between water and the wall of the capillary tube is θ. Ignore the mass of water in the meniscus. Which of the following statements is (are) true?

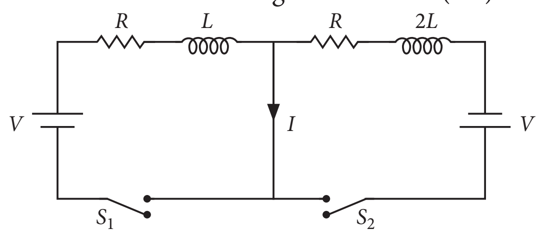
(a) For a given material of the capillary tube, h decreases with increase in r

(b) For a given material of the capillary tube, h is independent of σ

(c) If this experiment is performed in a lift going up with a constant acceleration, then h decreases

(d) h is proportional to contact angle θ

4. In the given figure, the switches S_1 and S_2 are closed simultaneously at t = 0 and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude I_{max} at time $t = \tau$. Which of the following statements is (are) true?



(a) $I_{max} = \frac{V}{2R}$ (b) $I_{max} = \frac{V}{4R}$

(c) $\tau = \frac{L}{R}\ln 2$ (d) $\tau = \frac{2L}{R}\ln 2$

5. Two infinitely long straight wires lie in the xy-plane along the lines $x = \pm R$. The wire located at $x = +R$ carries a constant current I_1 and the wire located at $x = -R$ carries a constant current I_2. A circular loop of radius R is suspended with its centre at $(0, 0, \sqrt{3}R)$ and in a plane parallel to the xy-plane. This loop carries a constant current I in the clockwise direction as seen from above the loop. The current in the wire is taken to be positive if it is in the $+\hat{j}$ direction. Which of the following statements regarding the magnetic field $\vec{B}$ is (are) true?

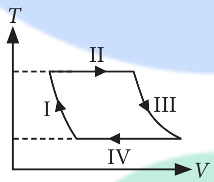
(a) If $I_1 = I_2$, then $\vec{B}$ cannot be equal to zero at the origin (0, 0, 0)

(b) If $I_1 > 0$ and $I_2 < 0$, then $\vec{B}$ can be equal to zero at the origin (0, 0, 0)

(c) If $I_1 < 0$ and $I_2 > 0$, then $\vec{B}$ can be equal to zero at the origin (0, 0, 0)

(d) If $I_1 = I_2$, then the z-component of the magnetic field at the centre of the loop is $\left(-\frac{\mu_0 I}{2R}\right)$

6. One mole of a monatomic ideal gas undergoes a cyclic process as shown in the figure (where V is the volume and T is the temperature). Which of the following statements is (are) true?



(a) Process I is an isochoric process

(b) In process II, gas absorbs heat

(c) In process IV, gas releases heat

(d) Processes I and III are not isobaric

SECTION 2 (Maximum Marks : 24)

- *This section contains EIGHT (08) questions. The answer to each question is a NUMERICAL VALUE.*
- *For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.*
- *Answer to each question will be evaluated according to the following marking scheme:*

  Full Marks : +3 If ONLY the correct numerical value is entered as answer.

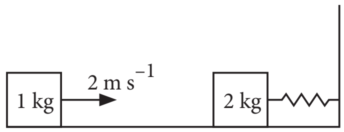
  Zero Marks : 0 In all other cases.

7. Two vectors $\vec{A}$ and $\vec{B}$ are defined as $\vec{A} = a\hat{i}$ and $\vec{B} = a(\cos\omega t\hat{i} + \sin\omega t\hat{j})$, where a is a constant and $\omega = \pi/6$ rad s^{-1}. If $|\vec{A}+\vec{B}| = \sqrt{3}|\vec{A}-\vec{B}|$ at time $t = \tau$ for the first time, the value of τ, in seconds, is ______.

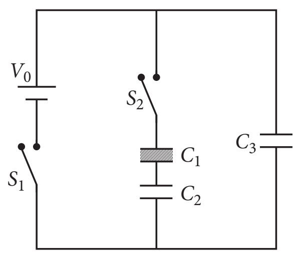
8. Two men are walking along a horizontal straight line in the same direction. The man in front walks at a speed 1.0 m s^{-1} and the man behind walks at a speed 2.0 m s^{-1}. A third man is standing at a height 12 m above the same horizontal line such that all three men are in a vertical plane. The two walking men are blowing identical whistles which emit a sound of frequency 1430 Hz. The speed of sound in air is 330 m s^{-1}. At the instant, when the moving men are 10 m apart, the stationary man is equidistant from them. The frequency of beats in Hz, heard by the stationary man at this instant, is ______.

9. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $(2-\sqrt{3})/\sqrt{10}$ s, then the height of the top of the inclined plane, in metres, is ______. Take $g = 10$ m s^{-2}.

10. A spring-block system is resting on a frictionless floor as shown in the figure. The spring constant is 2.0 N m^{-1} and the mass of the block is 2.0 kg. Ignore the mass of the spring. Initially the spring is in an unstretched condition. Another block of mass 1.0 kg moving with a speed of 2.0 m s^{-1} collides elastically with the first block. The collision is such that the 2.0 kg block does not hit the wall. The distance, in metres, between the two blocks when the spring returns to its unstretched position for the first time after the collision is ______.

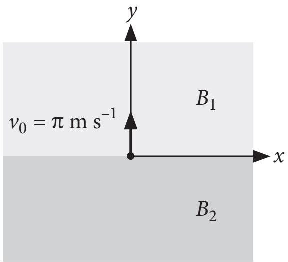


11. Three identical capacitors C_1, C_2 and C_3 have a capacitance of 1.0 μF each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity ε_r. The cell electromotive force (emf) $V_0 = 8$ V. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be 5 μC. The value of ε_r = ______.



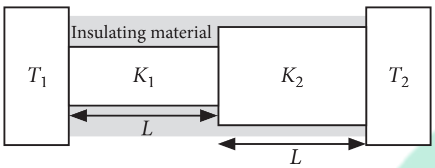
12. In the xy-plane, the region $y > 0$ has a uniform magnetic field $B_1\hat{k}$ and the region $y < 0$ has another uniform magnetic field $B_2\hat{k}$. A positively charged particle is projected from the origin along the positive

y-axis with speed $v_0 = \pi$ m s^{-1} at $t = 0$, as shown in the figure. Neglect gravity in this problem. Let $t = T$ be the time when the particle crosses the x-axis from below for the first time. If $B_2 = 4B_1$, the average speed of the particle, in m s^{-1}, along the x-axis in the time interval T is ______ .



13. Sunlight of intensity 1.3 kW m^{-2} is incident normally on a thin convex lens of focal length 20 cm. Ignore the energy loss of light due to the lens and assume that the lens aperture size is much smaller than its focal length. The average intensity of light, in kW m^{-2}, at a distance 22 cm from the lens on the other side is ______.

14. Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures $T_1 = 300$ K and $T_2 = 100$ K, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2 respectively. If the temperature at the junction of the two cylinders in the steady state is 200 K, then $K_1/K_2 =$ _______.



SECTION 3 (Maximum Marks : 12)

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- *Each question has FOUR options. ONLY ONE of these four options corresponds to the correct answer.*
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PARAGRAPH "X"

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, [E] and [B] stand for dimensions of electric and magnetic fields respectively, while [ε_0] and [μ_0] stand for dimensions of the permittivity and permeability of free space respectively. [L] and [T] are dimensions of length and time respectively. All the quantities are given in SI units.

15. The relation between [E] and [B] is

(a) [E] = [B][L][T] (b) [E] = [B][L]$^{-1}$[T]
(c) [E] = [B][L][T]$^{-1}$ (d) [E] = [B][L]$^{-1}$[T]$^{-1}$

16. The relation between [ε_0] and [μ_0] is

(a) [μ_0] = [ε_0][L]^2[T]$^{-2}$ (b) [μ_0] = [ε_0][L]$^{-2}$[T]^2
(c) [μ_0] = [ε_0]$^{-1}$[L]^2[T]$^{-2}$ (d) [μ_0] = [ε_0]$^{-1}$[L]$^{-2}$[T]^2

PARAGRAPH "A"

If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation $z = x/y$. If the errors in x, y and z are Δx, Δy and Δz, respectively, then

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$$

The series expansion for $\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$, to first power in $\Delta y/y$, is $1 \mp (\Delta y/y)$. The relative errors in independent variables are always added. So the error in z will be

$$\Delta z = z\left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right)$$

The above derivation makes the assumption that $\Delta x/x << 1$, $\Delta y/y << 1$. Therefore, the higher powers of these quantities are neglected.

17. Consider the ratio $r = \dfrac{(1-a)}{(1+a)}$ to be determined by measuring a dimensionless quantity a. If the error in the measurement of a is Δa ($\Delta a/a << 1$), then what is the error Δr in determining r?

(a) $\dfrac{\Delta a}{(1+a)^2}$ (b) $\dfrac{2\Delta a}{(1+a)^2}$
(c) $\dfrac{2\Delta a}{(1-a^2)}$ (d) $\dfrac{2a\Delta a}{(1-a^2)}$

18. In an experiment the initial number of radioactive nuclei is 3000. It is found that 1000±40 nuclei decayed in the first 1.0 s. For $|x| \ll 1$, $\ln(1 + x) = x$ up to first power in x. The error $\Delta\lambda$, in the determination of the decay constant λ, in s^{-1}, is

(a) 0.04 (b) 0.03 (c) 0.02 (d) 0.01

CHEMISTRY

SECTION 1 (Maximum Marks : 24)

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1. The compound(s) which generate(s) N_2 gas upon thermal decomposition below 300 °C is (are)

(a) NH_4NO_3 (b) $(NH_4)_2Cr_2O_7$

(c) $Ba(N_3)_2$ (d) Mg_3N_2

2. The correct statement(s) regarding the binary transition metal carbonyl compounds is (are)

(Atomic numbers: Fe = 26, Ni = 28)

(a) total number of valence shell electrons at metal centre in $Fe(CO)_5$ or $Ni(CO)_4$ is 16

(b) these are predominantly low spin in nature

(c) metal–carbon bond strengthens when the oxidation state of the metal is lowered

(d) the carbonyl C—O bond weakens when the oxidation state of the metal is increased.

3. Based on the compounds of group 15 elements, the correct statement(s) is (are)

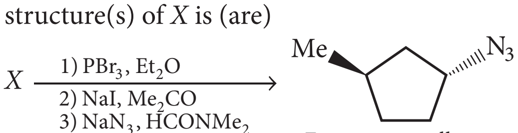
(a) Bi_2O_5 is more basic than N_2O_5

(b) NF_3 is more covalent than BiF_3

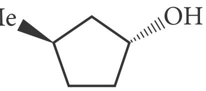
(c) PH_3 boils at lower temperature than NH_3

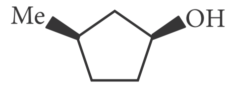
(d) the N—N single bond is stronger than the P—P single bond.

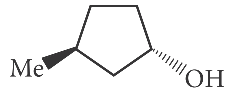
4. In the following reaction sequence, the correct structure(s) of X is (are)

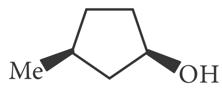


Enantiomerically pure

(a) 

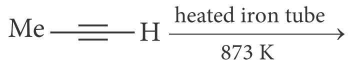
(b) 

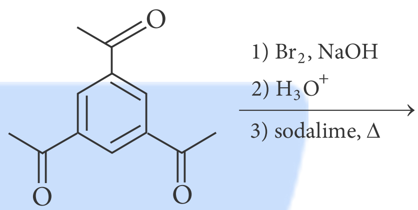
(c) 

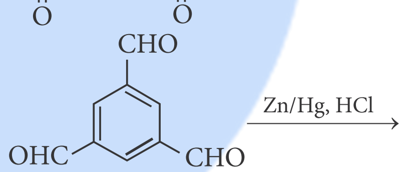
(d) 

5. The reaction(s) leading to the formation of 1,3,5-trimethylbenzene is (are)

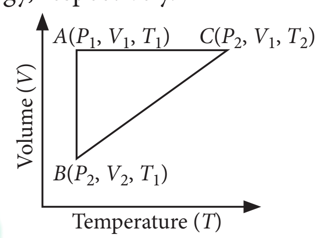
(a) 

(b) 

(c) 

(d) 

6. A reversible cyclic process for an ideal gas is shown below. Here, P, V and T are pressure, volume and temperature, respectively. The thermodynamic parameters q, w, H and U are heat, work, enthalpy and internal energy, respectively.



The correct option(s) is (are)

(a) $q_{AC} = \Delta U_{BC}$ and $w_{AB} = P_2(V_2 - V_1)$

(b) $w_{BC} = P_2(V_2 - V_1)$ and $q_{BC} = \Delta H_{AC}$

(c) $\Delta H_{CA} < \Delta U_{CA}$ and $q_{AC} = \Delta U_{BC}$

(d) $q_{BC} = \Delta H_{AC}$ and $\Delta H_{CA} > \Delta U_{CA}$

SECTION 2 (Maximum Marks : 24)

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- *For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.*
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Zero Marks :	*0*	*In all other cases.*

7. Among the species given below, the total number of diamagnetic species is ____.
H atom, NO_2 monomer, O_2^- (superoxide), dimeric sulphur in vapour phase, Mn_3O_4, $(NH_4)_2[FeCl_4]$, $(NH_4)_2[NiCl_4]$, K_2MnO_4, K_2CrO_4

8. The ammonia prepared by treating ammonium sulphate with calcium hydroxide is completely used by $NiCl_2.6H_2O$ to form a stable coordination compound. Assume that both the reactions are 100% complete. If 1584 g of ammonium sulphate and 952 g of $NiCl_2.6H_2O$ are used in the preparation, the combined weight (in grams) of gypsum and the nickel-ammonia coordination compound thus produced is _____.
(Atomic weights in g mol^{-1}: H = 1, N = 14, O = 16, S = 32, Cl = 35.5, Ca = 40, Ni = 59)

9. Consider an ionic solid *MX* with NaCl structure. Construct a new structure (*Z*) whose unit cell is constructed from the unit cell of *MX* following the sequential instructions given below. Neglect the charge balance.
(i) Remove all the anions (*X*) except the central one
(ii) Replace all the face centered cations (*M*) by anions (*X*)
(iii) Remove all the corner cations (*M*)
(iv) Replace the central anion (*X*) with cation (*M*)

The value of $\left(\frac{\text{number of anions}}{\text{number of cations}}\right)$ in *Z* is ______ .

10. For the electrochemical cell,
$Mg_{(s)} \mid Mg^{2+}(aq, 1\ M) \parallel Cu^{2+}(aq, 1\ M) \mid Cu_{(s)}$
the standard emf of the cell is 2.70 V at 300 K. When the concentration of Mg^{2+} is changed to *x* M, the cell potential changes to 2.67 V at 300 K. The value of *x* is _____ .
(Given, $\frac{F}{R} = 11500\ K\ V^{-1}$, where *F* is the Faraday constant and *R* is the gas constant, ln(10) = 2.30)

11. A closed tank has two compartments *A* and *B*, both filled with oxygen (assumed to be ideal gas). The partition separating the two compartments is fixed and is a perfect heat insulator (Figure 1). If the old partition is replaced by a new partition which can slide and conduct heat but does not allow the gas to leak across (Figure 2), the volume (in m^3) of the compartment *A* after the system attains equilibrium is _____.

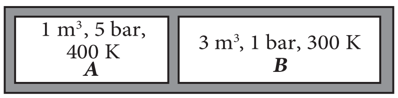


Figure 1

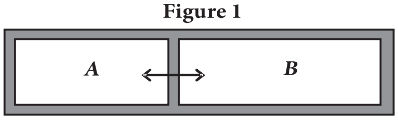
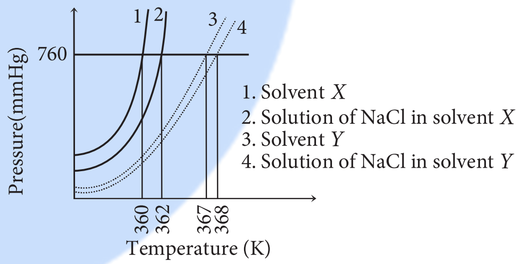


Figure 2

12. Liquids *A* and *B* form ideal solution over the entire range of composition. At temperature *T*, equimolar binary solution of liquids *A* and *B* has vapour pressure 45 Torr. At the same temperature, a new solution of *A* and *B* having mole fractions x_A and x_B, respectively, has vapour pressure of 22.5 Torr. The value of x_A/x_B in the new solution is _____.
(Given that the vapour pressure of pure liquid *A* is 20 Torr at temperature *T*)

13. The solubility of a salt of weak acid (*AB*) at pH 3 is $Y \times 10^{-3}$ mol L^{-1}. The value of *Y* is _____.
(Given that the value of solubility product of AB (K_{sp}) = 2×10^{-10} and the value of ionization constant of HB (K_a) = 1×10^{-8})

14. The plot given below shows *P*—*T* curves (where *P* is the pressure and *T* is the temperature) for two solvents *X* and *Y* and isomolal solutions of NaCl in these solvents. NaCl completely dissociates in both the solvents.



On addition of equal number of moles of a non-volatile solute *S* in equal amount (in kg) of these solvents, the elevation of boiling point of solvent *X* is three times that of solvent *Y*. Solute *S* is known to undergo dimerization in these solvents. If the degree of dimerization is 0.7 in solvent *Y*, the degree of dimerization in solvent *X* is _____.

SECTION 3 (Maximum Marks : 12)

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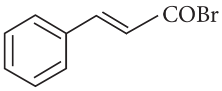
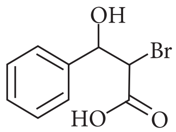
PARAGRAPH "X"

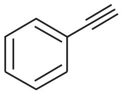
Treatment of benzene with CO/HCl in the presence of anhydrous $AlCl_3/CuCl$ followed by reaction with $Ac_2O/NaOAc$ gives compound *X* as the major product. Compound *X* upon reaction with Br_2/Na_2CO_3, followed by heating at 473 K with moist KOH furnishes *Y* as the major product. Reaction of *X* with $H_2/Pd\text{-}C$, followed by

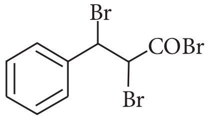
H_3PO_4 treatment gives *Z* as the major product.

15. The compound *Y* is

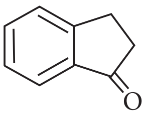
(a) 

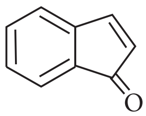
(b) 

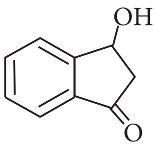
(c) 

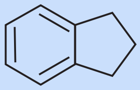
(d) 

16. The compound *Z* is

(a) 

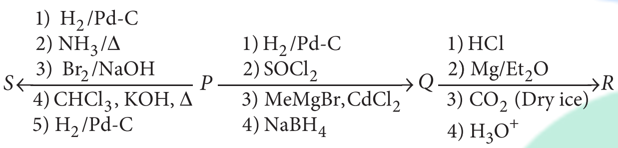
(b) 

(c) 

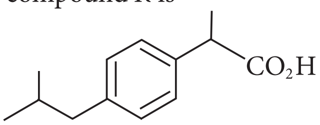
(d) 

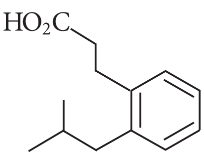
PARAGRAPH "A"

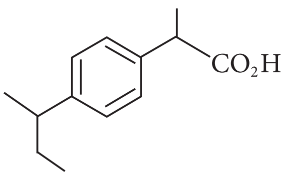
An organic acid *P* ($C_{11}H_{12}O_2$) can easily be oxidized to a dibasic acid which reacts with ethylene glycol to produce a polymer dacron. Upon ozonolysis, *P* gives an aliphatic ketone as one of the products. *P* undergoes the following reaction sequences to furnish *R* via *Q*. The compound *P* also undergoes another set of reactions to produce *S*.

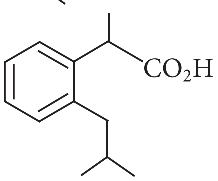


17. The compound *R* is

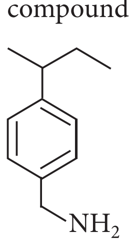
(a) 

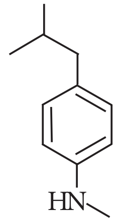
(b) 

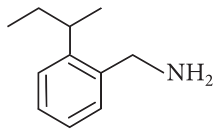
(c) 

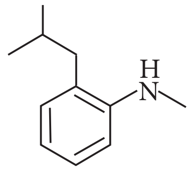
(d) 

18. The compound *S* is

(a) 

(b) 

(c) 

(d) 

MATHEMATICS

SECTION 1 (Maximum Marks : 24)

- *This section contains SIX (06) questions.*
- *Each question has FOUR options for correct answer(s). ONE OR MORE THAN ONE of these four option(s) is (are) correct option(s).*
- *For each question, choose the correct option(s) to answer the question.*
- *Answer to each question will be evaluated according to the following marking scheme:*

Full Marks :	+4	*If only (all) the correct option(s) is (are) chosen.*
Partial Marks :	+3	*If all the four options are correct but ONLY three options are chosen.*
Partial Marks :	+2	*If three or more options are correct but ONLY two options are chosen, both of which are correct options.*
Partial Marks :	+1	*If two or more options are correct but ONLY one option is chosen and it is a correct option.*
Zero Marks :	0	*If none of the options is chosen (i.e. the question is unanswered).*
Negative Marks :	−2	*In all other cases.*

- *For Example: If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option) ,without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in -2 marks.*

1. For a non-zero complex number *z*, let arg(*z*) denote the principal argument with $-\pi < \arg(z) \le \pi$. Then, which of the following statement(s) is (are) FALSE?

(a) $\arg(-1-i) = \frac{\pi}{4}$, where $i = \sqrt{-1}$

(b) The function $f : \mathbb{R} \to (-\pi, \pi]$, defined by $f(t) = \arg(-1+it)$ for all $t \in \mathbb{R}$, is continuous at all points of $\mathbb{R}$, where $i = \sqrt{-1}$

(c) For any two non-zero complex numbers z_1 and z_2, $\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$ is an integer multiple of 2π.

(d) For any three given distinct complex numbers z_1, z_2 and z_3, the locus of the point z satisfying the condition $\arg\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi$, lies on a straight line.

2. In a triangle PQR, let $\angle PQR = 30°$ and the sides PQ and QR have lengths $10\sqrt{3}$ and 10, respectively. Then, which of the following statement(s) is (are) TRUE?
(a) $\angle QPR = 45°$
(b) The area of the triangle PQR is $25\sqrt{3}$ and $\angle QRP = 120°$
(c) The radius of the incircle of the triangle PQR is $10\sqrt{3} - 15$
(d) The area of the circumcircle of the triangle PQR is $100\,\pi$

3. Let P_1: $2x + y - z = 3$ and $P_2 : x + 2y + z = 2$ be two planes. Then, which of the following statement(s) is (are) TRUE?
(a) The line of intersection of P_1 and P_2 has direction ratios 1, 2,–1
(b) The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2
(c) The acute angle between P_1 and P_2 is 60°
(d) If P_3 is the plane passing through the point (4, 2, –2) and perpendicular to the line of intersection of P_1 and P_2, then the distance of the point (2, 1, 1) from the plane P_3 is $\frac{2}{\sqrt{3}}$

4. For every twice differentiable function $f : \mathbb{R} \to [-2, 2]$ with $(f(0))^2 + (f'(0))^2 = 85$, which of the following statement(s) is (are) TRUE?
(a) There exist $r, s \in \mathbb{R}$, where $r < s$, such that f is one-one on the open interval (r, s)
(b) There exists $x_0 \in (-4, 0)$ such that $|f'(x_0)| \le 1$
(c) $\lim_{x\to\infty} f(x) = 1$
(d) There exists $\alpha \in (-4, 4)$ such that $f(\alpha) + f''(\alpha) = 0$ and $f'(\alpha) \ne 0$

5. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two non-constant differentiable functions. If $f'(x) = (e^{(f(x) - g(x))})g'(x)$ for all $x \in \mathbb{R}$, and $f(1) = g(2) = 1$, then which of the following statement(s) is (are) TRUE?
(a) $f(2) < 1 - \log_e 2$
(b) $f(2) > 1 - \log_e 2$
(c) $g(1) > 1 - \log_e 2$
(d) $g(1) < 1 - \log_e 2$

6. Let $f : [0, \infty) \to \mathbb{R}$ be a continuous function such that $f(x) = 1 - 2x + \int_0^x e^{x-t} f(t)\,dt$ for all $x \in [0, \infty)$. Then, which of the following statement(s) is (are) TRUE?
(a) The curve $y = f(x)$ passes through the point (1, 2)
(b) The curve $y = f(x)$ passes through the point (2,–1)
(c) The area of the region $\{(x, y) \in [0,1] \times \mathbb{R} : f(x) \le y \le \sqrt{1-x^2}\}$ is $\frac{\pi-2}{4}$
(d) The area of the region $\{(x, y) \in [0, 1] \times \mathbb{R} : f(x) \le y \le \sqrt{1-x^2}\}$ is $\frac{\pi-1}{4}$

SECTION 2 (Maximum Marks : 24)

- *This section contains EIGHT (08) questions. The answer to each question is a NUMERICAL VALUE.*
- *For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.*
- *Answer to each question will be evaluated according to the following marking scheme :*
  Full Marks : +3 If ONLY the correct numerical value is entered as answer.
  Zero Marks : 0 In all other cases.

7. The value of $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$ is _____ .

8. The number of 5 digit numbers which are divisible by 4, with digits from the set {1, 2, 3, 4, 5} and the repetition of digits is allowed, is ______ .

9. Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11, … , and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, … . Then, the number of elements in the set $X \cup Y$ is ______.

10. The number of real solutions of the equation

$$\sin^{-1}\left(\sum_{i=1}^{\infty} x^{i+1} - x\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^i\right) = \frac{\pi}{2} - \cos^{-1}\left(\sum_{i=1}^{\infty}\left(-\frac{x}{2}\right)^i - \sum_{i=1}^{\infty}(-x)^i\right)$$

lying in the interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$ is ______ .

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\cos^{-1}x$ assume values in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ and $[0, \pi]$, respectively.)

11. For each positive integer n, let

$$y_n = \frac{1}{n}\left((n+1)(n+2)\ldots(n+n)\right)^{\frac{1}{n}}$$

For $x \in \mathbb{R}$, let $[x]$ be the greatest integer less than or equal to x. If $\lim_{n\to\infty} y_n = L$, then the value of $[L]$ is ____ .

12. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a}\cdot\vec{b}=0$. For some $x, y \in \mathbb{R}$, let $\vec{c}=x\vec{a}+y\vec{b}+(\vec{a}\times\vec{b})$. If $|\vec{c}|=2$ and the vector $\vec{c}$ is inclined at the same angle α to both $\vec{a}$ and $\vec{b}$, then the value of $8\cos^2\alpha$ is _____ .

13. Let a, b, c be three non-zero real numbers such that the equation $\sqrt{3}a\cos x+2b\sin x=c, x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, has two distinct real roots α and β with $\alpha+\beta=\frac{\pi}{3}$. Then, the value of $\frac{b}{a}$ is _____ .

14. A farmer F_1 has a land in the shape of a triangle with vertices at $P(0, 0)$, $Q(1, 1)$ and $R(2, 0)$. From this land, a neighbouring farmer F_2 takes away the region which lies between the side PQ and a curve of the form $y = x^n$ $(n > 1)$. If the area of the region taken away by the farmer F_2 is exactly 30% of the area of ΔPQR, then the value of n is ______ .

SECTION 3 (Maximum Marks : 12)

- *This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO (02) questions.*
- *Each question has FOUR options. ONLY ONE of these four options corresponds to the correct answer.*
- *For each question, choose the option corresponding to the correct answer.*
- *Answer to each question will be evaluated according to the following marking scheme:*
  Full Marks : +3 If ONLY the correct option is chosen.
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : −1 In all other cases.

PARAGRAPH "X"

Let S be the circle in the xy-plane defined by the equation $x^2 + y^2 = 4$.

15. Let E_1E_2 and F_1F_2 be the chords of S passing through the point P_0 (1, 1) and parallel to the x-axis and the y-axis, respectively. Let G_1G_2 be the chord of S passing through P_0 and having slope −1. Let the tangents to S at E_1 and E_2 meet at E_3, the tangents to S at F_1 and F_2 meet at F_3, and the tangents to S at G_1 and G_2 meet at G_3. Then, the points E_3, F_3 and G_3 lie on the curve

(a) $x + y = 4$ (b) $(x - 4)^2 + (y - 4)^2 = 16$
(c) $(x - 4)(y - 4) = 4$ (d) $xy = 4$

16. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve

(a) $(x + y)^2 = 3xy$ (b) $x^{2/3} + y^{2/3} = 2^{4/3}$
(c) $x^2 + y^2 = 2xy$ (d) $x^2 + y^2 = x^2y^2$

PARAGRAPH "A"

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, i =1, 2, 3, 4, 5. But, on the examination day, the five students are randomly allotted the five seats.

17. The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1, and **NONE** of the remaining students gets the seat previously allotted to him/her, is

(a) $\frac{3}{40}$ (b) $\frac{1}{8}$ (c) $\frac{7}{40}$ (d) $\frac{1}{5}$

18. For i = 1, 2, 3, 4, let T_i denote the event that the students S_i and S_{i+1} do **NOT** sit adjacent to each other on the day of the examination. Then, the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is

(a) $\frac{1}{15}$ (b) $\frac{1}{10}$ (c) $\frac{7}{60}$ (d) $\frac{1}{5}$

PAPER - II

PHYSICS

SECTION 1 (Maximum Marks : 24)

- *This section contains SIX (06) questions.*
- *Each question has FOUR options for correct answer(s).ONE OR MORE THAN ONE of these four option(s) is (are) correct option(s).*
- *For each question, choose the correct option(s) to answer the question.*
- *Answer to each question will be evaluated according to the following marking scheme:*
  Full Marks : +4 If only (all) the correct option(s) is (are) chosen.
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.
  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : −2 In all other cases.
- *For Example: If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option) ,without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in -2 marks.*

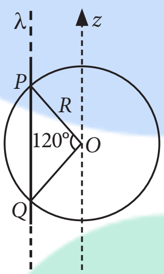
1. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true?

(a) The force applied on the particle is constant
(b) The speed of the particle is proportional to time
(c) The distance of the particle from the origin increases linearly with time
(d) The force is conservative

2. Consider a thin square plate floating on a viscous liquid in a large tank. The height h of the liquid in the tank is much less than the width of the tank. The floating plate is pulled horizontally with a constant velocity u_0. Which of the following statements is (are) true?

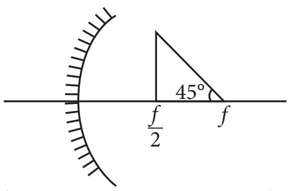
(a) The resistive force of liquid on the plate is inversely proportional to h
(b) The resistive force of liquid on the plate is independent of the area of the plate
(c) The tangential (shear) stress on the floor of the tank increases with u_0
(d) The tangential (shear) stress on the plate varies linearly with the viscosity η of the liquid

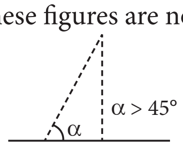
3. An infinitely long thin non-conducting wire is parallel to the z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120° at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is ε_0. Which of the following statements is (are) true?

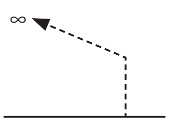


(a) The electric flux through the shell is $\sqrt{3}R\lambda/\varepsilon_0$
(b) The z-component of the electric field is zero at all the points on the surface of the shell
(c) The electric flux through the shell is $\sqrt{2}R\lambda/\varepsilon_0$
(d) The electric field is normal to the surface of the shell at all points

4. A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length f, as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire? (These figures are not to scale.)



(a)


(b)


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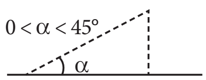


(d)


5. In a radioactive decay chain, $^{232}_{90}\text{Th}$ nucleus decays to $^{212}_{82}\text{Pb}$ nucleus. Let N_α and N_β be the number of α and β^- particles, respectively, emitted in this decay process. Which of the following statements is (are) true?

(a) $N_\alpha = 5$ (b) $N_\alpha = 6$ (c) $N_\beta = 2$ (d) $N_\beta = 4$

6. In an experiment to measure the speed of sound by a resonating air column, a tuning fork of frequency 500 Hz is used. The length of the air column is varied by changing the level of water in the resonance tube. Two successive resonances are heard at air columns of length 50.7 cm and 83.9 cm. Which of the following statements is (are) true?

(a) The speed of sound determined from this experiment is 332 m s^{-1}
(b) The end correction in this experiment is 0.9 cm
(c) The wavelength of the sound wave is 66.4 cm
(d) The resonance at 50.7 cm corresponds to the fundamental harmonic

SECTION 2 (Maximum Marks : 24)

- ***This section contains EIGHT (08) questions. The answer to each question is a NUMERICAL VALUE.***
- ***For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 6.25, 7.00, –0.33, –.30, 30.27, –127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.***
- ***Answer to each question will be evaluated according to the following marking scheme:***

Full Marks : +3 If ONLY the correct numerical value is entered as answer.
Zero Marks : 0 In all other cases.

7. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass m = 0.4 kg is at rest on this surface. An impulse of 1.0 N s is applied to the block at time t = 0 so that it starts moving along the x-axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and τ = 4 s. The displacement of the block, in metres, at $t = \tau$ is ___ . Take $e^{-1} = 0.37$.

8. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is ______ .

9. A particle, of mass 10^{-3} kg and charge 1.0 C, is initially at rest. At time t = 0, the particle comes under the influence of an electric field $\vec{E}(t) = E_0 \sin \omega t\,\hat{i}$, where

$E_0 = 1.0$ N C^{-1} and $\omega = 10^3$ rad s^{-1}. Consider the effect of only the electrical force on the particle. Then the maximum speed, in m s^{-1}, attained by the particle at subsequent times is ______.

10. A moving coil galvanometer has 50 turns and each turn has an area 2×10^{-4} m^2. The magnetic field produced by the magnet inside the galvanometer is 0.02 T. The torsional constant of the suspension wire is 10^{-4} N m rad^{-1}. When a current flows through the galvanometer, a full scale deflection occurs if the coil rotates by 0.2 rad. The resistance of the coil of the galvanometer is 50 Ω. This galvanometer is to be converted into an ammeter capable of measuring current in the range 0 - 1.0 A. For this purpose, a shunt resistance is to be added in parallel to the galvanometer. The value of this shunt resistance, in ohms, is _______.

11. A steel wire of diameter 0.5 mm and Young's modulus 2×10^{11} N m^{-2} carries a load of mass M. The length of the wire with the load is 1.0 m. A vernier scale with 10 divisions is attached to the end of this wire. Next to the steel wire is a reference wire to which a main scale, of least count 1.0 mm, is attached. The 10 divisions of the vernier scale correspond to 9 divisions of the main scale. Initially, the zero of vernier scale coincides with the zero of main scale. If the load on the steel wire is increased by 1.2 kg, the vernier scale division which coincides with a main scale division is _____ . Take $g = 10$ m s^{-2} and $\pi = 3.2$.

12. One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant $R = 8.0$ J $mol^{-1}K^{-1}$, the decrease in its internal energy, in joule, is ______ .

13. In a photoelectric experiment a parallel beam of monochromatic light with power of 200 W is incident on a perfectly absorbing cathode of work function 6.25 eV. The frequency of light is just above the threshold frequency so that the photoelectrons are emitted with negligible kinetic energy. Assume that the photoelectron emission efficiency is 100%. A potential difference of 500 V is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4}$ N due to the impact of the electrons. The value of n is_____. Mass of the electron $m_e = 9 \times 10^{-31}$ kg and 1.0 eV = 1.6×10^{-19} J.

14. Consider a hydrogen-like ionized atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the $n = 2$ to $n = 1$ transition has energy 74.8 eV higher than the photon emitted in the $n = 3$ to $n = 2$ transition. The ionization energy of the hydrogen atom is 13.6 eV. The value of Z is _____ .

SECTION 3 (MAXIMUM MARKS : 12)

- *This section contains FOUR (04) questions.*
- *Each question has TWO (02) matching lists: LIST-I and LIST-II.*
- *FOUR options are given representing matching of elements from LIST-I and LIST-II. ONLY ONE of these four options corresponds to a correct matching.*
- *For each question, choose the option corresponding to the correct matching.*
- *For each question, marks will be awarded according to the following marking scheme:*

Full Marks : *+3* *If ONLY the option corresponding to the correct matching is chosen.*

Zero Marks : *0* *If none of the options is chosen (i.e. the question is unanswered).*

Negative Marks : *-1* *In all other cases.*

15. The electric field E is measured at a point $P(0, 0, d)$ generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d. List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II.

List-I		List-II	
P	E is independent of d	1.	A point charge Q at the origin
Q.	$E \propto \frac{1}{d}$	2.	A small dipole with point charges Q at$(0, 0, l)$ and $-Q$ at $(0, 0, -l)$. Take $2l << d$
R.	$E \propto \frac{1}{d^2}$	3.	An infinite line charge coincident with the x-axis, with uniform linear charge density λ
S.	$E \propto \frac{1}{d^3}$	4.	Two infinite wires carrying uniform linear charge density parallel to the x- axis. The one along ($y = 0$, $z = l$) has a charge density $+\lambda$ and the one along ($y = 0$, $z = -l$) has a charge density $-\lambda$. Take $2l << d$
		5.	Infinite plane charge coincident with the xy-plane with uniform surface charge density

(a) P → 5; Q → 3, 4; R → 1; S → 2

(b) P → 5; Q → 3; R → 1, 4; S → 2

(c) P → 5; Q → 3; R → 1, 2; S → 4

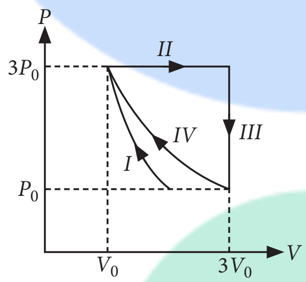
(d) P → 4; Q → 2, 3; R → 1; S → 5

16. A planet of mass M, has two natural satellites with masses m_1 and m_2. The radii of their circular orbits are R_1 and R_2 respectively. Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1; and v_2, L_2, K_2 and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II.

List-I		List-II	
P.	$\frac{v_1}{v_2}$	1.	$\frac{1}{8}$
Q.	$\frac{L_1}{L_2}$	2.	1
R.	$\frac{K_1}{K_2}$	3.	2
S.	$\frac{T_1}{T_2}$	4.	8

(a) P → 4; Q → 2; R → 1; S → 3
(b) P → 3; Q → 2; R → 4; S → 1
(c) P → 2; Q → 3; R → 1; S → 4
(d) P → 2; Q → 3; R → 4; S → 1

17. One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-I with the corresponding statements in List-II.



List-I		List-II	
P.	In process I	1.	Work done by the gas is zero
Q.	In process II	2.	Temperature of the gas remains unchanged
R.	In process III	3.	No heat is exchanged between the gas and its surroundings
S.	In process IV	4.	Work done by the gas is $6P_0V_0$

(a) P → 4; Q → 3; R → 1; S → 2
(b) P → 1; Q → 3; R → 2; S → 4
(c) P → 3; Q → 4; R → 1; S → 2
(d) P → 3; Q → 4; R → 2; S → 1

18. In the List-I below, four different paths of a particle are given as functions of time. In these functions, α and β are positive constants of appropriate dimensions and $\alpha \neq \beta$. In each case, the force acting on the particle is either zero or conservative. In List-II, five physical quantities of the particle are mentioned: $\vec{p}$ is the linear momentum, $\vec{L}$ is the angular momentum about the origin, K is the kinetic energy, U is the potential energy and E is the total energy. Match each path in List-I with those quantities in List-II, which are conserved for that path.

List-I		List-II	
P.	$\vec{r}(t) = \alpha t\hat{i} + \beta t\hat{j}$	1.	$\vec{p}$
Q.	$\vec{r}(t) = \alpha\cos\omega t\hat{i} + \beta\sin\omega t\hat{j}$	2.	$\vec{L}$
R.	$\vec{r}(t) = \alpha(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j})$	3.	K
S.	$\vec{r}(t) = \alpha t\,\hat{i} + \frac{\beta}{2}t^2\,\hat{j}$	4.	U
		5.	E

(a) P → 1, 2, 3, 4, 5; Q → 2, 5; R → 2, 3, 4, 5; S → 5
(b) P → 1, 2, 3, 4, 5; Q → 3, 5; R → 2, 3, 4, 5; S → 2, 5
(c) P → 2, 3, 4; Q → 5; R → 1, 2, 4; S → 2, 5
(d) P → 1, 2, 3, 5; Q → 2, 5; R → 2, 3, 4, 5; S → 2, 5

CHEMISTRY

SECTION 1 (Maximum Marks : 24)

- *This section contains SIX (06) questions.*
- *Each question has FOUR options for correct answer(s). ONE OR MORE THAN ONE of these four option(s) is (are) correct option(s).*
- *For each question, choose the correct option(s) to answer the question.*
- *Answer to each question will be evaluated according to the following marking scheme:*

Full Marks : +4 If only (all) the correct option(s) is (are) chosen.
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : –2 In all other cases.

- *For Example: If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option) ,without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.*

1. The correct option(s) regarding the complex $[Co(en)(NH_3)_3(H_2O)]^{3+}$ ($en = H_2NCH_2CH_2NH_2$) is (are)

(a) it has two geometrical isomers

(b) it will have three geometrical isomers if bidentate '*en*' is replaced by two cyanide ligands

(c) it is paramagnetic

(d) it absorbs light at longer wavelength as compared to $[Co(en)(NH_3)_4]^{3+}$.

2. The correct option(s) to distinguish nitrate salts of Mn^{2+} and Cu^{2+} taken separately is (are)

(a) Mn^{2+} shows the characteristic green colour in the flame test

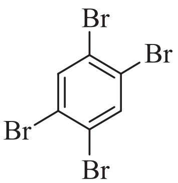
(b) only Cu^{2+} shows the formation of precipitate by passing H_2S in acidic medium

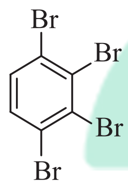
(c) only Mn^{2+} shows the formation of precipitate by passing H_2S in faintly basic medium

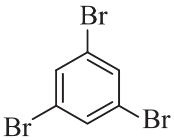
(d) Cu^{2+}/Cu has higher reduction potential than Mn^{2+}/Mn (measured under similar conditions).

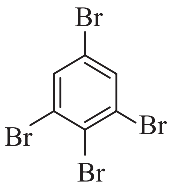
3. Aniline reacts with mixed acid (conc. HNO_3 and conc. H_2SO_4) at 288 K to give P (51 %), Q (47%) and R (2%). The major product(s) of the following reaction sequence is (are)

R —[1) Ac_2O, Pyridine; 2) Br_2, CH_3CO_2H; 3) H_3O^+; 4) $NaNO_2$, HCl/273-278K; 5) EtOH, Δ]→ S —[1) Sn/HCl; 2) Br_2/H_2O (excess); 3) $NaNO_2$, HCl/273-278K; 4) H_3PO_2]→ Major product(s)

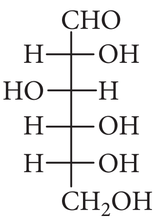
(a) 

(b) 

(c) 

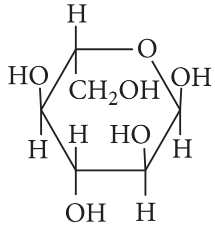
(d) 

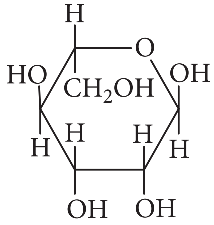
4. The Fischer presentation of *D*-glucose is given below.

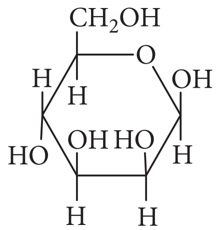


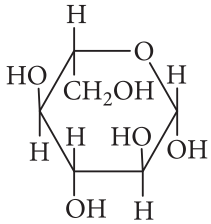
D-glucose

The correct structure(s) of β-*L*-glucopyranose is (are)

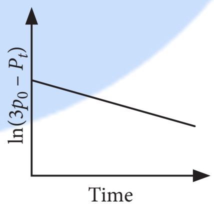
(a) 

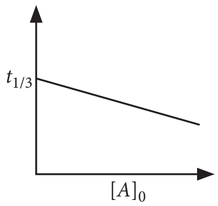
(b) 

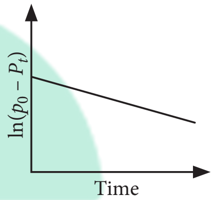
(c) 

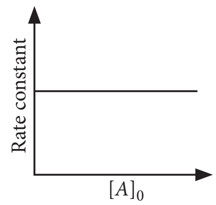
(d) 

5. For a first order reaction $A_{(g)} \rightarrow 2B_{(g)} + C_{(g)}$ at constant volume and 300 K, the total pressure at the beginning ($t = 0$) and at time t are P_0 and P_t, respectively. Initially, only A is present with concentration $[A]_0$, and $t_{1/3}$ is the time required for the partial pressure of A to reach $1/3^{rd}$ of its initial value. The correct option(s) is (are) (Assume that all these gases behave as ideal gases)

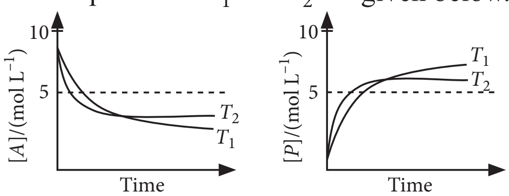
(a) 

(b) 

(c) 

(d) 

6. For a reaction, $A \rightleftharpoons P$, the plots of $[A]$ and $[P]$ with time at temperatures T_1 and T_2 are given below.



If $T_2 > T_1$, the correct statement(s) is (are) (Assume ΔH° and ΔS° are independent of temperature and ratio of ln$K$ at T_1 to ln$K$ at T_2 is greater than T_2/T_1. Here H, S, G and K are enthalpy, entropy, Gibbs energy and equilibrium constant, respectively.)

(a) $\Delta H^\circ < 0, \Delta S^\circ < 0$

(b) $\Delta G^\circ < 0, \Delta H^\circ > 0$

(c) $\Delta G^\circ < 0, \Delta S^\circ < 0$

(d) $\Delta G^\circ < 0, \Delta S^\circ > 0$

SECTION 2 (Maximum Marks : 24)

- ***This section contains EIGHT (08) questions. The answer to each question is a NUMERICAL VALUE.***
- ***For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.***
- ***Answer to each question will be evaluated according to the following marking scheme:***
  Full Marks : +3 If ONLY the correct numerical value is entered as answer.
  Zero Marks : 0 In all other cases.

7. The total number of compounds having at least one bridging oxo group among the molecules given below is ________ .

N_2O_3, N_2O_5, P_4O_6, P_4O_7, $H_4P_2O_5$, $H_5P_3O_{10}$, $H_2S_2O_3$, $H_2S_2O_5$

8. Galena (an ore) is partially oxidized by passing air through it at high temperature. After some time, the passage of air is stopped, but the heating is continued in a closed furnace such that the contents undergo self-reduction. The weight (in kg) of Pb produced per kg of O_2 consumed is _____.
(Atomic weights in g mol^{-1} : O = 16, S = 32, Pb = 207)

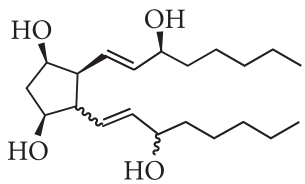
9. To measure the quantity of $MnCl_2$ dissolved in an aqueous solution, it was completely converted to $KMnO_4$ using the reaction,

$MnCl_2 + K_2S_2O_8 + H_2O \rightarrow KMnO_4 + H_2SO_4 + HCl$

(equation not balanced)

Few drops of concentrated HCl were added to this solution and gently warmed. Further, oxalic acid (225 mg) was added in portions till the colour of the permanganate ion disappeared. The quantity of $MnCl_2$ (in mg) present in the initial solution is _____ .
(Atomic weights in g mol^{-1}: Mn = 55, Cl = 35.5)

10. For the given compound X, the total number of optically active stereoisomers is _______ .

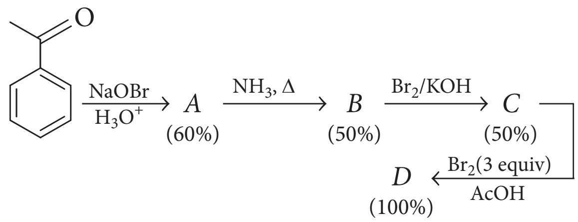


X

This type of bond indicates that the configuration at the specific carbon and the geometry of the double bond is fixed.

This type of bond indicates that the configuration at the specific carbon and the geometry of the double bond is **NOT** fixed.

11. In the following reaction sequence, the amount of D (in g) formed from 10 moles of acetophenone is ________ .
(Atomic weights in g mol^{-1}: H = 1, C = 12, N = 14, O = 16, Br = 80. The yield (%) corresponding to the product in each step is given in the parenthesis.)



12. The surface of copper gets tarnished by the formation of copper oxide. N_2 gas was passed to prevent the oxide formation during heating of copper at 1250 K. However, the N_2 gas contains 1 mole % of water vapour as impurity. The water vapour oxidises copper as per the reaction given below :

$$2Cu_{(s)} + H_2O_{(g)} \rightarrow Cu_2O_{(s)} + H_{2(g)}$$

p_{H_2} is the minimum partial pressure of H_2 (in bar) needed to prevent the oxidation at 1250 K. The value of $\ln(p_{H_2})$ is _______ .
(Given: total pressure = 1 bar, R (universal gas constant) = 8 J K^{-1} mol^{-1}, ln(10) = 2.3, $Cu_{(s)}$ and $Cu_2O_{(s)}$ are mutually immiscible.
At 1250 K: $2Cu_{(s)} + 1/2O_{2(g)} \rightarrow Cu_2O_{(s)}$; $\Delta G° = -78{,}000$ J mol^{-1}
$H_{2(g)} + 1/2O_{2(g)} \rightarrow H_2O_{(g)}$; $\Delta G° = -1{,}78{,}000$ J mol^{-1}; G is the Gibbs energy.)

13. Consider the following reversible reaction,

$$A_{(g)} + B_{(g)} \rightleftharpoons AB_{(g)}$$

The activation energy of the backward reaction exceeds that of the forward reaction by $2RT$ (in J mol^{-1}). If the pre-exponential factor of the forward reaction is 4 times that of the reverse reaction, the absolute value of $\Delta G°$ (in J mol^{-1}) for the reaction at 300 K is ____ .
(Given : ln(2) = 0.7, RT = 2500 J mol^{-1} at 300 K and G is the Gibbs energy)

14. Consider an electrochemical cell :
$A_{(s)} \mid A^{n+}$ (aq, 2 M) $\|$ B^{2n+} (aq, 1 M) $\mid B_{(s)}$. The value of $\Delta H°$ for the cell reaction is twice that of $\Delta G°$ at 300 K. If the emf of the cell is zero, the $\Delta S°$ (in J K^{-1} mol^{-1}) of the cell reaction per mole of B formed at 300 K is _____.
(Given: ln(2) = 0.7, R (universal gas constant) = 8.3 J K^{-1}mol^{-1}. H, S and G are enthalpy, entropy and Gibbs energy, respectively).

SECTION 3 (Maximum Marks : 12)

- ***This section contains FOUR (04) questions.***
- ***Each question has TWO (02) matching lists: LIST-I and LIST-II.***
- ***FOUR options are given representing matching of elements from LIST-I and LIST-II. ONLY ONE of these four options corresponds to a correct matching.***
- ***For each question, choose the option corresponding to the correct matching.***
- ***For each question, marks will be awarded according to the following marking scheme:***
  Full Marks : +3 If ONLY the option corresponding to the correct matching is chosen.

Zero Marks : **0** *If none of the options is chosen (i.e. the question is unanswered).*

Negative Marks : **–1** *In all other cases.*

15. Match each set of hybrid orbitals from List-I with complex(es) given in List-II.

List-I	List-II
P. dsp^2	1. $[FeF_6]^{4-}$
Q. sp^3	2. $[Ti(H_2O)_3Cl_3]$
R. sp^3d^2	3. $[Cr(NH_3)_6]^{3+}$
S. d^2sp^3	4. $[FeCl_4]^{2-}$
	5. $Ni(CO)_4$
	6. $[Ni(CN)_4]^{2-}$

The correct option is

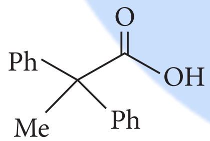
(a) P → 5; Q → 4,6; R → 2, 3; S → 1

(b) P → 5,6; Q → 4; R → 3; S → 1, 2

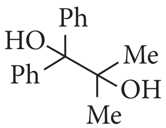
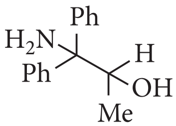
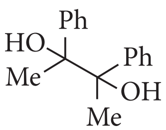
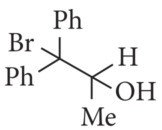
(c) P → 6; Q → 4, 5; R → 1; S → 2,3

(d) P → 4, 6; Q → 5, 6; R → 1, 2; S → 3

16. The desired product *X* can be prepared by reacting the major product of the reactions in List-I with one or more appropriate reagents in List-II. (given, order of migratory aptitude: aryl > alkyl > hydrogen)



X

List-I	List-II
P.  + H_2SO_4	1. I_2, NaOH
Q.  + HNO_2	2. $[Ag(NH_3)_2]OH$
R.  + H_2SO_4	3. Fehling solution
S.  + $AgNO_3$	4. HCHO, NaOH
	5. NaOBr

The correct option is

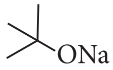
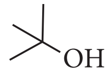
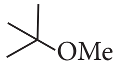
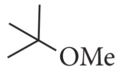
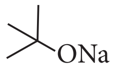
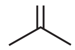
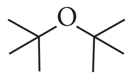
(a) P → 1; Q → 2, 3; R → 1, 4; S → 2, 4

(b) P → 1, 5; Q → 3, 4; R → 4, 5; S → 3

(c) P → 1, 5; Q → 3, 4; R → 5; S → 2, 4

(d) P → 1, 5; Q → 2, 3; R → 1, 5; S → 2, 3

17. List-I contains reactions and List-II contains major products.

List-I	List-II
P.  +  ⟶	1. 
Q.  + HBr ⟶	2. 
R.  + NaOMe ⟶	3. 
S.  + MeBr ⟶	4. 
	5. 

Match each reaction in List-I with one or more products in List-II and choose the correct option.

The correct option is

(a) P → 1, 5; Q → 2; R → 3; S → 4

(b) P → 1, 4; Q → 2; R → 4; S → 3

(c) P → 1, 4; Q → 1, 2; R → 3, 4; S → 4

(d) P → 4, 5; Q → 4; R → 4; S → 3, 4

18. Dilution processes of different aqueous solutions, with water, are given in List-I. The effects of dilution of the solutions on $[H^+]$ are given in List-II.

(Note: Degree of dissociation (α) of weak acid and weak base is ≪1; degree of hydrolysis of salt ≪1; $[H^+]$ represents the concentration of H^+ ions)

List-I	List-II
P. (10 mL of 0.1 M NaOH + 20 mL of 0.1 M acetic acid) diluted to 60 mL	1. The value of $[H^+]$ does not change on dilution
Q. (20 mL of 0.1 M NaOH + 20 mL of 0.1 M acetic acid) diluted to 80 mL	2. The value of $[H^+]$ changes to half of its initial value on dilution
R. (20 mL of 0.1 M HCl + 20 mL of 0.1 M ammonia solution) diluted to 80 mL	3. The value of $[H^+]$ changes to two times of its initial value on dilution
S. 10 mL saturated solution of $Ni(OH)_2$ in equilibrium with excess solid $Ni(OH)_2$ is diluted to 20 mL (solid $Ni(OH)_2$ is still present after dilution).	4. The value of $[H^+]$ changes to $\frac{1}{\sqrt{2}}$ times of its initial value on dilution
	5. The value of $[H^+]$ changes to $\sqrt{2}$ times of its initial value on dilution

Match each process given in List-I with one or more effect(s) in List-II.

The correct option is

(a) $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 3; S \rightarrow 1$

(b) $P \rightarrow 4; Q \rightarrow 3; R \rightarrow 2; S \rightarrow 3$

(c) $P \rightarrow 1; Q \rightarrow 4; R \rightarrow 5; S \rightarrow 3$

(d) $P \rightarrow 1; Q \rightarrow 5; R \rightarrow 4; S \rightarrow 1$

MATHEMATICS

SECTION 1 (MAXIMUM MARKS : 24)

- *This section contains SIX (06) questions.*
- *Each question has FOUR options for correct answer(s). ONE OR MORE THAN ONE of these four option(s) is (are) correct option(s).*
- *For each question, choose the correct option(s) to answer the question.*
- *Answer to each question will be evaluated according to the following marking scheme:*

Full Marks :	*+4*	*If only (all) the correct option(s) is (are) chosen.*
Partial Marks :	*+3*	*If all the four options are correct but ONLY three options are chosen.*
Partial Marks :	*+2*	*If three or more options are correct but ONLY two options are chosen, both of which are correct options.*
Partial Marks :	*+1*	*If two or more options are correct but ONLY one option is chosen and it is a correct option.*
Zero Marks :	*0*	*If none of the options is chosen (i.e. the question is unanswered).*
Negative Marks :	*–2*	*In all other cases.*

- *For Example: If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option) ,without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.*

1. For any positive integer n, define $f_n : (0, \infty) \rightarrow \mathbb{R}$ as

$$f_n(x) = \sum_{j=1}^{n} \tan^{-1}\left(\frac{1}{1+(x+j)(x+j-1)}\right) \text{ for all } x \in (0, \infty)$$

(Here, the inverse trigonometric function $\tan^{-1}x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.) Then, which of the following statement(s) is (are) TRUE?

(a) $\sum_{j=1}^{5} \tan^2(f_j(0)) = 55$

(b) $\sum_{j=1}^{10} (1+f_j'(0))\sec^2(f_j(0)) = 10$

(c) For any fixed positive integer n, $\lim_{x\to\infty} \tan(f_n(x)) = \frac{1}{n}$

(d) For any fixed positive integer n, $\lim_{x\to\infty} \sec^2(f_n(x)) = 1$

2. Let T be the line passing through the points $P(-2, 7)$ and $Q(2, -5)$. Let F_1 be the set of all pairs of circles (S_1, S_2) such that T is tangent to S_1 at P and tangent to S_2 at Q, and also such that S_1 and S_2 touch each other at a point, say, M. Let E_1 be the set representing the locus of M as the pair (S_1, S_2) varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point $R(1, 1)$ be F_2. Let E_2 be the set of the mid-points of the line segments in the set F_2. Then, which of the following statement(s) is (are) TRUE?

(a) The point $(-2, 7)$ lies in E_1

(b) The point $\left(\frac{4}{5}, \frac{7}{5}\right)$ does **NOT** lie in E_2

(c) The point $\left(\frac{1}{2}, 1\right)$ lies in E_2

(d) The point $\left(0, \frac{3}{2}\right)$ does **NOT** lie in E_1

3. Let S be the set of all column matrices $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_1, b_2, b_3 \in \mathbb{R}$ and the system of equations (in real variables)

$$-x + 2y + 5z = b_1$$
$$2x - 4y + 3z = b_2$$
$$x - 2y + 2z = b_3$$

has at least one solution. Then, which of the following system(s) (in real variables) has (have) at least one solution for each $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in S$?

(a) $x + 2y + 3z = b_1$, $4y + 5z = b_2$ and $x + 2y + 6z = b_3$

(b) $x + y + 3z = b_1$, $5x + 2y + 6z = b_2$ and $-2x - y - 3z = b_3$

(c) $-x + 2y - 5z = b_1$, $2x - 4y + 10z = b_2$ and $x - 2y + 5z = b_3$

(d) $x + 2y + 5z = b_1$, $2x + 3z = b_2$ and $x + 4y - 5z = b_3$

4. Consider two straight lines, each of which is tangent to both the circle $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point Q. Consider the ellipse whose center is at the origin $O(0, 0)$ and whose semi-major axis is OQ. If the length of the minor axis of this ellipse is $\sqrt{2}$, then which of the following statement(s) is (are) TRUE?

(a) For the ellipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is 1

(b) For the ellipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is $\frac{1}{2}$

(c) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{4\sqrt{2}}(\pi - 2)$

(d) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{16}(\pi - 2)$

5. Let s, t, r be non-zero complex numbers and L be the set of solutions $z = x + iy$ ($x, y \in \mathbb{R}$, $i = \sqrt{-1}$) of the equation $sz + t\bar{z} + r = 0$, where $\bar{z} = x - iy$. Then, which of the following statement(s) is (are) TRUE?

(a) If L has exactly one element, then $|s| \neq |t|$

(b) If $|s| = |t|$, then L has infinitely many elements

(c) The number of elements in $L \cap \{z : |z - 1 + i| = 5\}$ is at most 2

(d) If L has more than one element, then L has infinitely many elements

6. Let $f : (0, \pi) \to \mathbb{R}$ be a twice differentiable function such that $\lim_{t \to x} \frac{f(x)\sin t - f(t)\sin x}{t - x} = \sin^2 x$ for all $x \in (0, \pi)$. If $f\left(\frac{\pi}{6}\right) = -\frac{\pi}{12}$, then which of the following statement(s) is (are) TRUE?

(a) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4\sqrt{2}}$

(b) $f(x) < \frac{x^4}{6} - x^2$ for all $x \in (0, \pi)$

(c) There exists $\alpha \in (0, \pi)$ such that $f'(\alpha) = 0$

(d) $f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 0$

SECTION 2 (Maximum Marks : 24)

- *This section contains EIGHT (08) questions. The answer to each question is a NUMERICAL VALUE.*
- *For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30) using the mouse and the on- screen virtual numeric keypad in the place designated to enter the answer.*
- *Answer to each question will be evaluated according to the following marking scheme:*
  Full Marks : +3 If ONLY the correct numerical value is entered as answer.
  Zero Marks : 0 In all other cases.

7. The value of the integral $\int_0^{1/2} \frac{1+\sqrt{3}}{((x+1)^2(1-x)^6)^{1/4}} dx$ is ________.

8. Let P be a matrix of order 3×3 such that all the entries in P are from the set $\{-1, 0, 1\}$. Then, the maximum possible value of the determinant of P is ______ .

9. Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta - \alpha)$ is _____ .

10. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f(0) = 0$. If $y = f(x)$ satisfies the differential equation $\frac{dy}{dx} = (2+5y)(5y-2)$, then the value of $\lim_{x \to -\infty} f(x)$ is ______ .

11. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f(0) = 1$ and satisfying the equation $f(x + y) = f(x)f'(y) + f'(x)f(y)$ for all $x, y \in \mathbb{R}$. Then, the value of $\log_e(f(4))$ is _____ .

12. Let P be a point in the first octant, whose image Q in the plane $x + y = 3$ (that is, the line segment PQ is perpendicular to the plane $x + y = 3$ and the mid-point of PQ lies in the plane $x + y = 3$) lies on the z-axis. Let the distance of P from the x-axis be 5. If R is the image of P in the xy-plane, then the length of PR is ______ .

13. Consider the cube in the first octant with sides OP, OQ and OR of length 1, along the x-axis, y-axis and z-axis, respectively, where $O(0,0,0)$ is the origin. Let $S\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ be the centre of the cube and T be the vertex of the cube opposite to the origin O such that S lies on the diagonal OT. If $\vec{p} = \overrightarrow{SP}$, $\vec{q} = \overrightarrow{SQ}$, $\vec{r} = \overrightarrow{SR}$ and $\vec{t} = \overrightarrow{ST}$, then the value of $|(\vec{p} \times \vec{q}) \times (\vec{r} \times \vec{t})|$ is _____ .

14. Let $X = ({}^{10}C_1)^2 + 2({}^{10}C_2)^2 + 3({}^{10}C_3)^2 + \ldots + 10({}^{10}C_{10})^2$, where ${}^{10}C_r$, $r \in \{1, 2, \ldots, 10\}$ denote binomial coefficients. Then, the value of $\frac{1}{1430}X$ is ______ .

SECTION 3 (MAXIMUM MARKS : 12)

- *This section contains FOUR (04) questions.*
- *Each question has TWO (02) matching lists: LIST-I and LIST-II.*
- *FOUR options are given representing matching of elements from LIST-I and LIST-II. ONLY ONE of these four options corresponds to a correct matching.*
- *For each question, choose the option corresponding to the correct matching.*

- *For each question, marks will be awarded according to the following marking scheme:*

Full Marks : +3 *If ONLY the option corresponding to the correct matching is chosen.*

Zero Marks : 0 *If none of the options is chosen (i.e. the question is unanswered).*

Negative Marks : −1 *In all other cases.*

15. Let $E_1 = \left\{x \in \mathbb{R} : x \neq 1 \text{ and } \frac{x}{x-1} > 0\right\}$ and

$$E_2 = \left\{x \in E_1 : \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right) \text{ is a real number}\right\}.$$

(Here, the inverse trigonometric function $\sin^{-1}x$ assumes values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.)

Let $f : E_1 \to \mathbb{R}$ be the function defined by

$$f(x) = \log_e\left(\frac{x}{x-1}\right)$$

and $g : E_2 \to \mathbb{R}$ be the function defined by

$$g(x) = \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right).$$

	List-I		List-II
P.	The range of f is	1.	$\left(-\infty, \frac{1}{1-e}\right] \cup \left[\frac{e}{e-1}, \infty\right)$
Q.	The range of g contains	2.	$(0, 1)$
R.	The domain of f contains	3.	$\left[-\frac{1}{2}, \frac{1}{2}\right]$
S.	The domain of g is	4.	$(-\infty, 0) \cup (1, \infty)$
		5.	$\left(-\infty, \frac{e}{e-1}\right]$
		6.	$(-\infty, 0) \cup \left(\frac{1}{2}, \frac{e}{e-1}\right]$

The correct option is :

(a) $P \to 4; Q \to 2; R \to 1; S \to 1$

(b) $P \to 3; Q \to 3; R \to 6; S \to 5$

(c) $P \to 4; Q \to 2; R \to 1; S \to 6$

(d) $P \to 4; Q \to 3; R \to 6; S \to 5$

16. In a high school, a committee has to be formed from a group of 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls G_1, G_2, G_3, G_4, G_5.

(i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boys and 2 girls.

(ii) Let α_2 be the total number of ways in which the committee can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let α_3 be the total number of ways in which the committee can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let α_4 be the total number of ways in which the committee can be formed such that the committee has 4 members, having at least 2 girls and such that both M_1 and G_1 are **NOT** in the committee together.

	List-I		List-II
P.	The value of α_1 is	1.	136
Q.	The value of α_2 is	2.	189
R.	The value of α_3 is	3.	192
S.	The value of α_4 is	4.	200
		5.	381
		6.	461

The correct option is :

(a) $P \to 4; Q \to 6; R \to 2; S \to 1$

(b) $P \to 1; Q \to 4; R \to 2; S \to 3$

(c) $P \to 4; Q \to 6; R \to 5; S \to 2$

(d) $P \to 4; Q \to 2; R \to 3; S \to 1$

17. Let $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a > b > 0$, be a hyperbola in the xy-plane whose conjugate axis LM subtends an angle of 60° at one of its vertices N. Let the area of the triangle LMN be $4\sqrt{3}$.

	List-I		List-II
P.	The length of the conjugate axis of H is	1.	8
Q.	The eccentricity of H is	2.	$\frac{4}{\sqrt{3}}$
R.	The distance between the foci of H is	3.	$\frac{2}{\sqrt{3}}$
S.	The length of the latus rectum of H is	4.	4

The correct option is:

(a) $P \to 4; Q \to 2; R \to 1; S \to 3$

(b) $P \to 4; Q \to 3; R \to 1; S \to 2$

(c) $P \to 4; Q \to 1; R \to 3; S \to 2$

(d) $P \to 3; Q \to 4; R \to 2; S \to 1$

18. Let $f_1 : \mathbb{R} \to \mathbb{R}$, $f_2 : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$, $f_3 : \left(-1, e^{\frac{\pi}{2}} - 2\right) \to \mathbb{R}$, and $f_4 : \mathbb{R} \to \mathbb{R}$ be functions defined by

(i) $f_1(x)=\sin\left(\sqrt{1-e^{-x^2}}\right)$,

(ii) $f_2(x)=\begin{cases}\dfrac{|\sin x|}{\tan^{-1}x} & \text{if } x\neq 0\\ 1 & \text{if } x=0\end{cases}$, where the inverse trigonometric function $\tan^{-1}x$ assumes values in $\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$,

(iii) $f_3(x) = [\sin(\log_e(x + 2))]$, where, for $t \in \mathbb{R}$, $[t]$ denotes the greatest integer less than or equal to t,

(iv) $f_4(x)=\begin{cases}x^2\sin\left(\dfrac{1}{x}\right) & \text{if } x\neq 0\\ 0 & \text{if } x=0\end{cases}$.

List-I		List-II	
P.	The function f_1 is	1.	**NOT** continuous at $x = 0$
Q.	The function f_2 is	2.	continuous at $x = 0$ and **NOT** differentiable at $x = 0$
R.	The function f_3 is	3.	differentiable at $x = 0$ and its derivative is **NOT** continuous at $x = 0$
S.	The function f_4 is	4.	differentiable at $x = 0$ and its derivative is continuous at $x = 0$

The correct option is:

(a) $P \to 2;\ Q \to 3;\ R \to 1;\ S \to 4$

(b) $P \to 4;\ Q \to 1;\ R \to 2;\ S \to 3$

(c) $P \to 4;\ Q \to 2;\ R \to 1;\ S \to 3$

(d) $P \to 2;\ Q \to 1;\ R \to 4;\ S \to 3$

HINTS & EXPLANATIONS

PHYSICS

1. (b, c) : Potential energy of a particle,

$$V=\frac{kr^2}{2}$$

Force acting on the particle,

$$F=-\frac{dV}{dr}=-\frac{k}{2}(2r)=-kr$$

As particle is moving on a circular path of radius R so F is the centripetal force.

$$F=\frac{mv^2}{R} \text{ or } \frac{mv^2}{R}=kR \quad\text{or}\quad v=\sqrt{\frac{k}{m}}R$$

Now angular momentum of the particle about O,

$$L=mvR=m\sqrt{\frac{k}{m}}R^2=\sqrt{mk}\,R^2$$

2. (a,c) : Here $m = 1$ kg, $\vec{F}=(\alpha t\hat{i}+\beta\hat{j})=(t\hat{i}+\hat{j})$ N

As, $\vec{F}=m\dfrac{d\vec{v}}{dt}=(t\,\hat{i}+\hat{j})$ or $\vec{v}=\displaystyle\int_0^t(t\,\hat{i}+\hat{j})\,dt$

$$\vec{v}=\left[\frac{t^2}{2}\hat{i}+t\,\hat{j}\right]$$

Also, $\vec{r}=\displaystyle\int_0^t\vec{v}\,dt=\int_0^t\left(\frac{t^2}{2}\hat{i}+t\,\hat{j}\right)dt$; $\vec{r}=\left(\dfrac{t^3}{6}\hat{i}+\dfrac{t^2}{2}\hat{j}\right)$

Velocity of the body at $t = 1$ s,

$$\vec{v}=\left(\frac{1}{2}\hat{i}+\hat{j}\right)\text{ m s}^{-1}=\frac{1}{2}(\hat{i}+2\hat{j})\text{ m s}^{-1}$$

Displacement of the body at $t = 1$ s,

$$\vec{s}=\vec{r}(t=1\text{ s})-\vec{r}(t=0\text{ s})=\left(\frac{1}{6}\hat{i}+\frac{1}{2}\hat{j}\right)-0=\frac{1}{6}(\hat{i}+3\hat{j})\text{ m}$$

$$|\vec{s}|=\frac{1}{6}\sqrt{1^2+3^2}=\frac{\sqrt{10}}{6}$$

Torque on the body at $t = 1$ s,

$$\vec{\tau}=\vec{r}\times\vec{F}=\left(\frac{1}{6}\hat{i}+\frac{1}{2}\hat{j}\right)\times(\hat{i}+\hat{j})$$

$$=\frac{1}{6}(\hat{i}\times\hat{j})+\frac{1}{2}(\hat{j}\times\hat{i})=\frac{1}{6}\hat{k}-\frac{1}{2}\hat{k}=-\frac{1}{3}\hat{k}$$

$$|\vec{\tau}|=\frac{1}{3}\text{ N m}$$

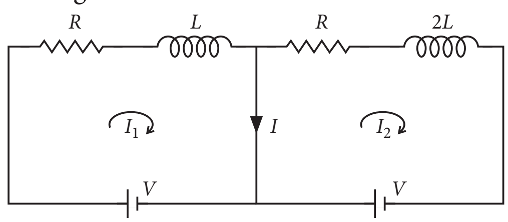
3. (a,c) : Rise of water in capillary tube, $h=\dfrac{2\sigma\cos\theta}{\rho rg}$

For a given material of the capillary tube, $h\propto\dfrac{1}{r}$.

When lift is going up with a constant acceleration a then effective acceleration in the experiment will be $a + g$.

$$\therefore\quad h'=\frac{2\sigma\cos\theta}{\rho r(a+g)} \text{ and } h'<h$$

4. (b,d) : Let I_1 and I_2 be the currents in both loops as shown in figure.



$I = (I_1 - I_2)$

$$I=\frac{V}{R}[1-e^{-\left(\frac{R}{L}\right)t}]-\frac{V}{R}[1-e^{-\left(\frac{R}{2L}\right)t}]$$

$$I=\frac{V}{R}[e^{-\left(\frac{R}{2L}\right)t}-e^{-\left(\frac{R}{L}\right)t}] \quad ...(i)$$

For I_{max}, $\frac{dI}{dt}=0$

$$-\frac{V}{2L}e^{-\left(\frac{R}{2L}\right)t}+\frac{V}{L}e^{-\left(\frac{R}{L}\right)t}=0$$

$$e^{-\left(\frac{R}{L}\right)t}=\frac{1}{2}e^{-\left(\frac{R}{2L}\right)t} \quad \text{or} \quad e^{-\left(\frac{R}{2L}\right)t}=\frac{1}{2}$$

$$\Rightarrow \left(\frac{R}{2L}\right)t=\ln 2$$

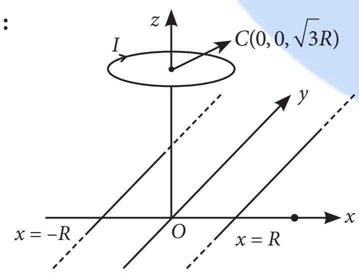
$\Rightarrow \quad t=\frac{2L}{R}\ln 2=\tau \rightarrow$ time when I is maximum.

Using t in equation (i),

$$\Rightarrow \quad I_{max}=\frac{V}{R}[e^{-\frac{R}{2L}\left(\frac{2L}{R}\ln 2\right)}-e^{-\frac{R}{L}\left(\frac{2L}{R}\ln 2\right)}]$$

$$\text{or} \quad I_{max}=\frac{V}{R}\left[\frac{1}{2}-\frac{1}{4}\right]=\frac{V}{4R}$$

5. (a,b,d) :



Magnetic field due to ring at origin

$$=\frac{\mu_0\times I\times R^2}{2\times 8R^3}(-\hat{k})=\frac{\mu_0 I}{16R}(-\hat{k})$$

Magnetic field at origin due to wires

$$\vec{B}_1+\vec{B}_2=\left(\frac{\mu_0 I_1}{2\pi R}-\frac{\mu_0 I_2}{2\pi R}\right)\hat{k}$$

(Assume current I_1 and I_2 are along $+\hat{j}$ direction.)

(a) If $I_1 = I_2$, then $\vec{B}_O=\frac{\mu_0 I}{16R}(-\hat{k})$

It cannot be zero.

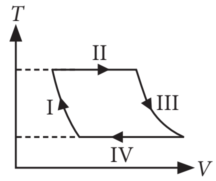
(b) If $I_1 > 0$ and $I_2 < 0$, $\vec{B}$ at origin due to wires will be along $+\hat{k}$ direction and $\vec{B}$ due to ring is along $-\hat{k}$ direction. Hence net magnetic field at point O can be equal to zero.

(c) If $I_1 < 0$, $I_2 > 0$, then net magnetic field at origin will be

$$\vec{B}_O=-\left[\frac{\mu_0(I_1+I_2)}{2\pi R}+\frac{\mu_0\cdot I}{16R}\right]\hat{k}$$. It cannot be zero.

(d) As $I_1 = I_2$, then $\vec{B}_1=-\vec{B}_2$, so magnetic field along z-axis is only due to the ring. Magnetic field at centre of the loop is $\vec{B}=\frac{\mu_0 I}{2R}$ in $-z$ direction.

6. (b,c,d) : (a) Process-I is not isochoric as, V is decreasing.



(b) Process-II is isothermal expansion.

$\Delta U = 0$, $W > 0$,

So, $\Delta Q > 0$

(c) Process-IV is isothermal compression,

$\Delta U = 0$, $W < 0$, $\Delta Q < 0$

(d) Process-I and III are not isobaric because in isobaric process $T \propto V$.

7. (2.00) : $\vec{A}=a\hat{i}$ and $\vec{B}=a(\cos\omega t\,\hat{i}+\sin\omega t\,\hat{j})$

$$|\vec{A}+\vec{B}|=|(a+a\cos\omega t)\hat{i}+a\sin\omega t\,\hat{j}|=2a\cos\frac{\omega t}{2} \quad ...(i)$$

$$|\vec{A}-\vec{B}|=|(a-a\cos\omega t)\hat{i}-\sin\omega t\,\hat{j}|=2a\sin\frac{\omega t}{2} \quad ...(ii)$$

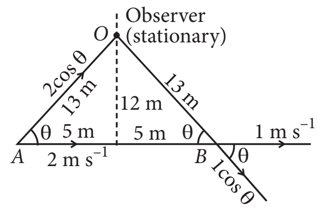
Using equations (i) and (ii) in $|\vec{A}+\vec{B}|=\sqrt{3}|\vec{A}-\vec{B}|$

$$2a\cos\frac{\omega t}{2}=\sqrt{3}\left(2a\sin\frac{\omega t}{2}\right) \Rightarrow \tan\frac{\omega t}{2}=\frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{\omega t}{2}=\frac{\pi}{6} \text{ or } \frac{\pi}{6}\cdot t=\frac{\pi}{3} \quad \left(\because \omega=\frac{\pi}{6}\right)$$

$\Rightarrow \quad t = 2.00$ s

8. (5.00) : Apparent frequency at O due to source at A,



$$\upsilon_A=\upsilon\left(\frac{v}{v-2\cos\theta}\right)$$

$$\upsilon_A=1430\left[\frac{330}{330-2\cos\theta}\right]$$

$$\upsilon_A=1430\left[\frac{1}{1-\frac{2\cos\theta}{330}}\right]$$

$$\approx 1430\left[1+\frac{2\cos\theta}{330}\right] \quad \text{(Using Binomial expansion)}$$

Apparent frequency at O due to source at B,

$$\upsilon_B=\upsilon\left[\frac{v}{v+1\cos\theta}\right]$$

$$\upsilon_B=1430\left[\frac{330}{330+\cos\theta}\right]\approx 1430\left[1-\frac{\cos\theta}{330}\right]$$

(Using bionomial expansion)

Beat frequency $(\upsilon)=\upsilon_A-\upsilon_B=1430\left[\frac{3\cos\theta}{330}\right]$

$\upsilon = 13\cos\theta$

From figure, $\cos\theta=\frac{5}{13}$ $\therefore$ $\upsilon=13\left(\frac{5}{13}\right)=5.00$ Hz

9. **(0.75) :** Time of descend, $t=\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{K^2}{R^2}\right)}$

$$t_{\text{ring}}=\frac{1}{\sin 60^\circ}\sqrt{\frac{2h}{g}(1+1)}=\sqrt{\frac{16h}{3g}}\qquad\left[\because \frac{K^2}{R^2}=1\right]$$

$$t_{\text{disc}}=\frac{1}{\sin 60^\circ}\sqrt{\frac{2h}{g}\left(1+\frac{1}{2}\right)}=\sqrt{\frac{4h}{g}}\qquad\left[\because \frac{K^2}{R^2}=\frac{1}{2}\right]$$

Time difference $(\Delta t)=t_{\text{ring}}-t_{\text{disc}}$

$$\sqrt{\frac{16h}{3g}}-\sqrt{\frac{4h}{g}}=\frac{(2-\sqrt{3})}{\sqrt{10}}$$

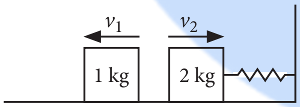
$$\sqrt{h}\left[\frac{4}{\sqrt{3}}-2\right]=2-\sqrt{3},\quad \sqrt{h}=\frac{\sqrt{3}}{2}$$

$$\therefore\quad h=\frac{3}{4}=0.75\text{ m}$$

10. **(2.09) :** Time period of spring-block system

$$T=2\pi\sqrt{\frac{m}{k}}=2\pi\sqrt{\frac{2}{2}}=2\pi\text{ s}$$

Just after the collision speeds of blocks are shown in figure.



Using momentum conservation principle,

$1\times 2+2\times 0=-1\times v_1+2\times v_2$

or, $\;-v_1+2v_2=2$ ...(i)

As collision is elastic so

$$e=\frac{2-0}{v_1+v_2}\Rightarrow 1=\frac{2}{v_1+v_2}$$

$v_1+v_2=2$ ...(ii)

From eqns (i) and (ii),

$$v_1=\frac{2}{3}\text{m s}^{-1},\ v_2=\frac{4}{3}\text{m s}^{-1}$$

Spring returns to its unstretched position for the first time after $\frac{T}{2}=\pi$ s

So, required separation between the blocks

$$=v_1\times\left(\frac{T}{2}\right)=\frac{2}{3}(\pi)=\frac{2}{3}\times 3.14=2.09\text{ m}$$

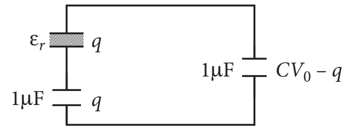
11. **(1.50) :** When the switch S_1 is closed and S_2 is opened, and capacitor C_3 becomes fully charged. Then charge on $C_3=C\times V_0=1\times 8\ \mu\text{C}=8\ \mu\text{C}$

When the switch S_2 is closed and S_1 is opened

Applying loop rule

$$\frac{CV_0-q}{C}-\frac{q}{\varepsilon_r C}-\frac{q}{C}=0$$

$$5-\frac{q}{\varepsilon_r}-q=0$$



$$\varepsilon_r=\frac{q}{5-q}$$

As per question, $CV_0-q=5\ \mu\text{C}$

$\therefore\quad q=8\mu\text{C}-5\ \mu\text{C}=3\ \mu\text{C}$

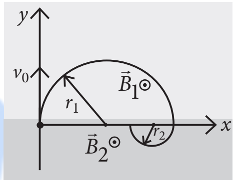
So, $\varepsilon_r=\frac{3}{2}=1.5$

12. **(2.00) :** Average speed along x-axis, $v_x=\frac{d_1+d_2}{t_1+t_2}$

We have, $r_1=\frac{mv_0}{qB_1}$ and $r_2=\frac{mv_0}{qB_2}$

Since $B_1=\frac{B_2}{4}$

$\therefore\quad r_1=4r_2$



Time spent by charged particle in B_1 is $t_1=\frac{\pi m}{qB_1}$

Time spent by charged particle in B_2 is $t_2=\frac{\pi m}{qB_2}$

Total distance along x-axis is

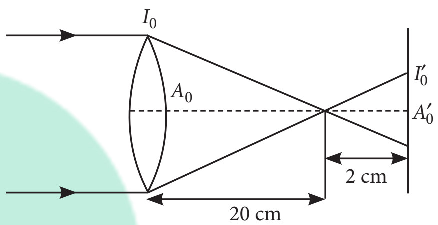
$d_1+d_2=2r_1+2r_2$

$=2(r_1+r_2)=2(5r_2)=10r_2$

$$\text{Average speed}=\frac{10r_2}{5t_2}=2\frac{mv_0}{qB_2}\times\frac{qB_2}{\pi m}$$

$=2\text{ m s}^{-1}\qquad(\because\ v_0=\pi\text{ m s}^{-1})$

13. **(130.00) :**



Let A_0 be the initial area covered by light and A_0' be the final area covered by light at 22 cm.

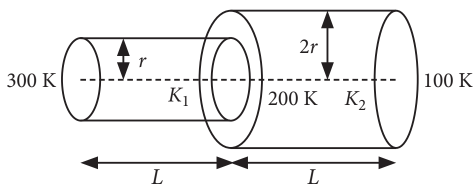
From figure,

$$\frac{A_0'}{A_0}=\left(\frac{2}{20}\right)^2=\frac{1}{100};\ A_0'=\frac{A_0}{100}$$

since, $I_0'A_0'=I_0A_0$

$$\Rightarrow\quad I_0'=\frac{I_0A_0}{\frac{A_0}{100}}=100I_0=130\text{ kW m}^{-2}$$

14. **(4.00) :**



Rate of heat flow is given by $\frac{dQ}{dt} = \frac{1}{R}(T_2 - T_1)$

where $R = \frac{L}{KA}$

Since rate of heat flow is same, we can write

$$\frac{300-200}{R_1} = \frac{200-100}{R_2} \text{ or } R_1 = R_2$$

$$\frac{L_1}{K_1A_1} = \frac{L_2}{K_2A_2};\ \frac{K_1}{K_2} = \frac{A_2}{A_1} = \frac{\pi(2r)^2}{\pi r^2} = 4$$

15. **(c) :** From Maxwell's equations, electric and magnetic fields are related as $E = cB$, where c is the velocity of electromagnetic wave

$[E] = [B][LT^{-1}] = [B][L][T]^{-1}$

16. **(d) :** We have, $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ or $[c^2] = \left[\frac{1}{\mu_0\varepsilon_0}\right]$

$$\Rightarrow [L^2T^{-2}] = \frac{1}{[\mu_0][\varepsilon_0]} \Rightarrow [\mu_0] = [\varepsilon_0]^{-1}[L]^{-2}[T]^2$$

17. **(b) :** $r = \left(\frac{1-a}{1+a}\right) \Rightarrow \frac{\Delta r}{r} = \frac{\Delta(1-a)}{(1-a)} + \frac{\Delta(1+a)}{(1+a)}$

$$\Rightarrow \frac{\Delta r}{r} = \frac{\Delta a}{1-a} + \frac{\Delta a}{1+a} = \frac{\Delta a(1+a+1-a)}{(1-a)(1+a)}$$

$$\therefore \Delta r = \frac{2\Delta a}{(1-a)(1+a)}\frac{(1-a)}{(1+a)} = \frac{2\Delta a}{(1+a)^2}$$

18. **(c) :** $N = N_0e^{-\lambda t} \Rightarrow \ln N = \ln N_0 - \lambda t$

Differentiation with respect to λ

$$\Rightarrow \frac{1}{N}\frac{dN}{d\lambda} = 0 - t$$

Converting to error,

$$\frac{\Delta N}{Nt} = \Delta\lambda;\ \therefore \Delta\lambda = \frac{40}{2000\times 1} = 0.02$$

CHEMISTRY

1. **(b, c) :** Ammonium nitrate decomposes below 300°C to produce N_2O and H_2O.

$$NH_4NO_3 \rightarrow N_2O + 2H_2O$$

On further heating *i.e.*, above 300°C,

$$2NH_4NO_3 \xrightarrow{\Delta} 2N_2 + O_2 + 4H_2O$$

Ammonium dichromate on heating below 300°C decomposes to give N_2 and Cr(III) oxide.

$$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + 4H_2O$$

Barium azide on heating around 180°C decomposes to give N_2 gas and Ba.

$$Ba(N_3)_2 \rightarrow 3N_2 + Ba$$

Magnesium nitride decomposes above 700°C to give Mg and N_2 gas.

$$Mg_3N_2 \rightarrow 3Mg + N_2$$

So, on heating below 300°C only $(NH_4)_2Cr_2O_7$ and $Ba(N_3)_2$ produce N_2 gas.

2. **(b, c) :** (a) Total number of valence shell electrons in central metal atom are

$Fe(CO)_5$; $8 + 2 \times 5 = 18$; $Ni(CO)_4$; $10 + 2 \times 4 = 18$

(b) Due to the presence of strong field ligand (CO) both complexes are low spin in nature.

(c) In lower oxidation state, number of electrons in d-subshell are higher, so due to π- backbonding, electrons from filled t_{2g} of metal are transferred to vacant π^* of CO which strengthens the M—C bond in complexes.

(d) In higher oxidation number, metal may have less number of electrons in d-orbitals, which decreases the extent of synergic bonding. So, in this case M—C bond is weaker while C—O bond is stronger.

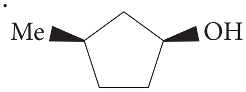
3. **(a, b, c) :** (a) Basic character of oxide increases as we move down the group. Therefore, Bi_2O_5 is more basic than N_2O_5.

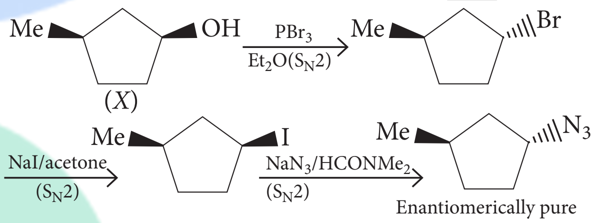
(b) Covalent nature depends on the electronegativity difference between the bonded atoms. Therefore, NF_3 is more covalent than BiF_3.

(c) Due to H-bonding, boiling point of NH_3 is more than PH_3.

(d) Due to small size of N-atom, *l.p-l.p.* repulsion will be more in N—N single bond than in P—P single bond. Therefore, N—N single bond is weaker than P—P single bond.

4. **(b) :** The correct structure of X is :

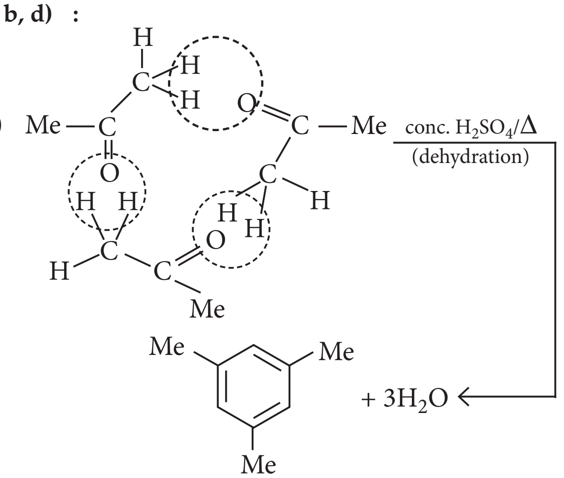




Enantiomerically pure product after several substitution reactions, is only possible when each reaction is stereospecific in nature which confirms the pathway used is S_N2 in nature.

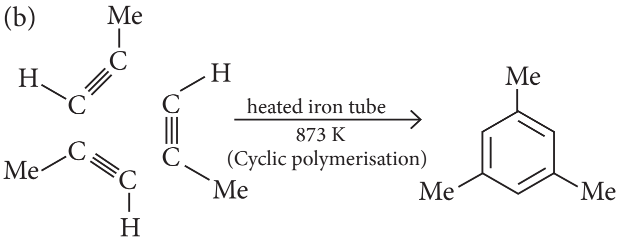
5. **(a, b, d) :**

(a)

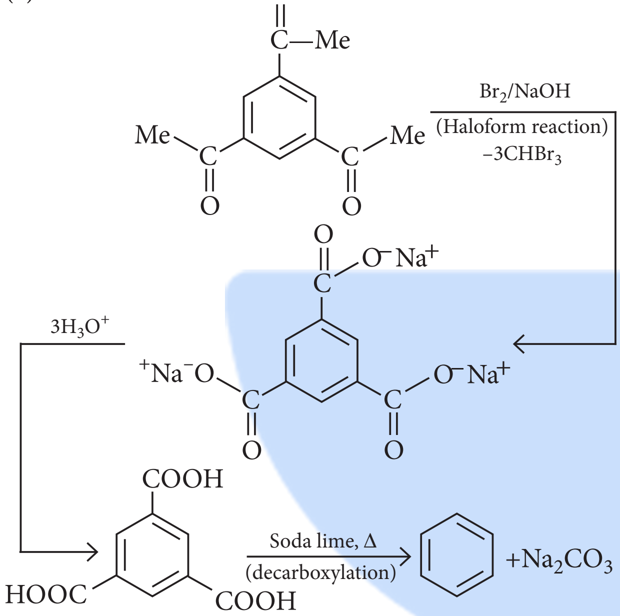


1, 3, 5-Trimethylbenzene

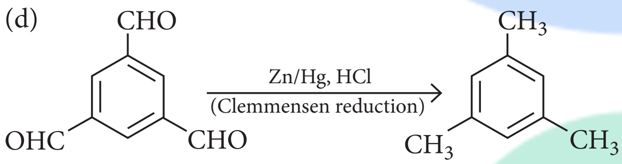
(b)



(c)



(d)



6. **(b, c) :** $A - C$ (Isochoric process) $\Rightarrow w_{AC} = 0$ and $\Delta U_{AC} = q_{AC}$

$B - C$ (Isobaric process) $\Rightarrow \Delta U_{BC} = q_{BC} + w_{BC}$

$w_{BC} = -P_2(V_1 - V_2) = P_2(V_2 - V_1)$

$\Rightarrow \quad q_{BC} = \Delta H_{BC}$

$\because \quad (\Delta T)_{A-C} = (\Delta T)_{B-C}$

$\therefore \quad \Delta U_{BC} = \Delta U_{AC} = q_{AC}$

$\Delta H_{BC} = \Delta H_{AC} = q_{BC} \qquad (\because \; T_2 > T_1)$

ΔH_{CA} and ΔU_{CA} are negative.

$\Delta H_{CA} = \Delta U_{CA} + \underset{(-ve)}{V\Delta P}$

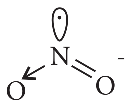
$\therefore \quad \Delta H_{CA} < \Delta U_{CA}$

$A - B$ (Isothermal process)

$\Delta U_{AB} = \Delta H_{AB} = 0$

$w_{AB} = -nRT_1 \ln \dfrac{V_2}{V_1}$

7. **(1) :** H atom : $\underset{1s^1}{\boxed{\uparrow}}$ - Paramagnetic

NO_2 monomer :  - Paramagnetic

(Due to presence of one unshared electron)

O_2^- (Superoxide) : $\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2, \pi 2p_y^2, \pi^* 2p_x^2, \pi^* 2p_y^1$

One unpaired electron is present in either $\pi^* 2p_x$ or $\pi^* 2p_y$, hence, it is paramagnetic in nature.

Dimeric sulphur in vapour phase : It is similar as O_2 in vapour phase. Hence, it is paramagnetic in nature.

Mn_3O_4 : It is combined form of MnO and Mn_2O_3.

Mn^{2+} has 5 unpaired electrons and Mn^{3+} has 4 unpaired electrons. Hence, it is paramagnetic in nature.

$(NH_4)_2[FeCl_4]$ or $[Fe^{2+}Cl_4]^{2-}$ ion

$[FeCl_4]^{2-}$: It is tetrahedral, sp^3-hybridized with e^3, t_2^3 configuration, hence, it is paramagnetic in nature.

$(NH_4)_2[NiCl_4]$ or $[Ni^{2+}Cl_4]^{2-}$ ion

$[NiCl_4]^{2-}$: It is tetrahedral, sp^3-hybridized with e^4, t_2^4 configuration, hence, it is paramagnetic in nature.

K_2MnO_4

Mn^{6+} : $[Ar]3d^1$

Hence, it is paramagnetic in nature.

K_2CrO_4 : Cr^{6+} has zero unpaired electron, hence, it is diamagnetic in nature.

8. **(2992) :**

$(NH_4)_2SO_4 + Ca(OH)_2 \longrightarrow \underset{\substack{\text{Gypsum} \\ 12 \text{ mol}}}{CaSO_4 \cdot 2H_2O} + 2NH_3$

$n = \dfrac{1584}{132} = 12$ mol

$NiCl_2 \cdot 6H_2O + 6NH_3 \longrightarrow \underset{4 \text{ mol}}{[Ni(NH_3)_6]Cl_2} + 6H_2O$

$n = \dfrac{952}{238} = 4$ mol

Combined weight of Gypsum and nickel - ammonia coordination compound

$= 12 \times M_{CaSO_4 \cdot 2H_2O} + 4M_{[Ni(NH_3)_6]Cl_2}$

$= (12 \times 172) + (4 \times 232) = 2992$ g

9. **(3.00):** MX has NaCl type structure.

From given instructions, it is clear that in MX ionic solid :

Cation M^+ - occupies face centres and corners (fcc lattice).

Anion X^- - occupies all octahedral voids (body centre + edge centres)

(i) No. of anions left = 1

(ii) No. of anions added = 3

No. of cations left = 1

(iii) No. of cations left = 0

(iv) No. of cations added = 1

No. of anions left = 3

Final no. of cations in the unit cell of $Z = 1$.

Final no. of anions in the unit cell of $Z = 3$.

The value of $\left(\dfrac{\text{number of anions}}{\text{number of cations}}\right) = \dfrac{3}{1} = 3.00$

10. **(10.00) :** For the given cell,

Anodic reaction : $Mg_{(s)} \longrightarrow Mg^{2+} + 2e^-$

Cathodic reaction : $Cu^{2+} + 2e^- \longrightarrow Cu_{(s)}$

and balanced reaction is

$Mg_{(s)} + Cu^{2+}(1\text{ M}) \rightarrow Cu_{(s)} + Mg^{2+}(x\text{ M})$

$$E_{cell} = E^{o}_{cell} - \frac{RT}{nF}\ln\frac{x}{1} \Rightarrow E_{cell} = E^{o}_{cell} - \frac{T}{\frac{nF}{R}}\ln x$$

$$\Rightarrow 2.67 = 2.70 - \frac{300}{2\times 11500}\ln x$$

$$\Rightarrow 0.03 = \frac{300}{2\times 11500}\ln x$$

$$\Rightarrow 2.3 = \ln x \;\therefore\; x = 10$$

11. (2.22) : From figure 1,

$$n_A = \frac{5\times 1}{R\times 400} = \frac{5}{400\,R}\;;\; n_B = \frac{1\times 3}{R\times 300} = \frac{3}{300\,R} = \frac{1}{100\,R}$$

Figure 2 - After the system attains equilibrium,

$P_A = P_B$ and $T_A = T_B = T$

$$\therefore \frac{n_A RT}{V_A} = \frac{n_B RT}{V_B}$$

$$\Rightarrow \frac{5}{400\,RV_A} = \frac{1}{100\,RV_B} \Rightarrow \frac{V_A}{V_B} = \frac{5}{4} \Rightarrow V_B = \frac{4}{5}V_A$$

$$\because V_A + V_B = 4\text{ m}^3 \Rightarrow V_A + \frac{4}{5}V_A = 4$$

$$\Rightarrow V_A = \frac{20}{9} = 2.22\text{ m}^3$$

12. (19.00) : $p^{o}_{A} = 20$ Torr

For equimolar binary solution : $x_A = x_B = \frac{1}{2}$

$$\therefore \frac{p^{o}_{A} + p^{o}_{B}}{2} = 45 \Rightarrow p^{o}_{B} = 70\text{ Torr}$$

If mole fractions are x_A and x_B then according to Dalton's law of partial pressures,

$$p^{o}_{B} + (p^{o}_{A} - p^{o}_{B})x_A = 22.5$$

$$\Rightarrow 70 + (20 - 70)x_A = 22.5$$

$$\Rightarrow x_A = \frac{47.5}{50} \text{ and } x_B = \frac{2.5}{50}$$

$$\frac{x_A}{x_B} = \frac{47.5}{2.5} = 19.00$$

13. (4.47) : $AB_{(s)} \rightleftharpoons \underset{s}{A^{+}_{(aq)}} + \underset{s-x}{B^{-}_{(aq)}}$

$$\underset{s-x}{B^{-}} + \underset{10^{-3}}{H^{+}} \rightleftharpoons \underset{x}{HB} \quad (\text{given, pH} = 3)$$

K_a of HB $= 1\times 10^{-8}$

$$K_a = \frac{[H^{+}][B^{-}]}{[HB]} = 10^{-8} = \frac{10^{-3}\times(s-x)}{x}$$

$$\frac{s-x}{x} = 10^{-5} \Rightarrow s - x = x\times 10^{-5}$$

$$K_{sp} = [A^{+}]\,[B^{-}] \Rightarrow 2\times 10^{-10} = s(s-x)$$

$$\Rightarrow sx = 2\times 10^{-5} \text{ and } s^2 - sx = 2\times 10^{-10}$$

$$s^2 = 2\times 10^{-10} + 2\times 10^{-5}$$

$$s^2 = 2\times 10^{-5}$$

$$\therefore s = 4.47\times 10^{-3}$$

14. (0.05) : When NaCl as solute is used

For solvent X; $2 = 2K_b m$

For solvent Y; $1 = 2\times K'_b m$

$$\therefore \frac{K_b}{K'_b} = 2$$

When solute S is used then molality in both the solvents is equal.

For solvent X; $i = 1 - \frac{\alpha}{2}$, $\Delta T_b = \left(1 - \frac{\alpha}{2}\right)K_b m$

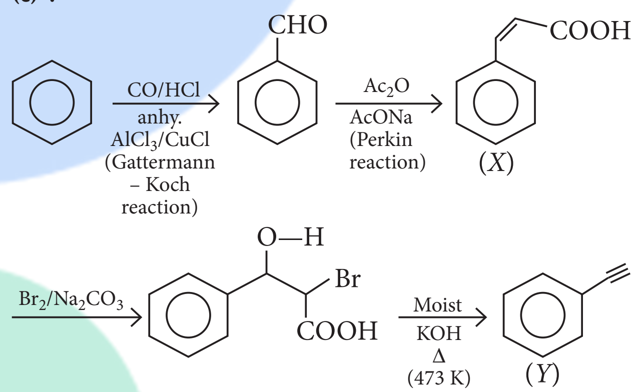
For solvent Y; $i = 1 - \frac{0.7}{2} = 0.65$, $\Delta T'_b = (0.65)\,K'_b m$

$$3 = \frac{\Delta T_b}{\Delta T'_b} = \frac{\left(1 - \frac{\alpha}{2}\right)\times 2}{0.65}$$

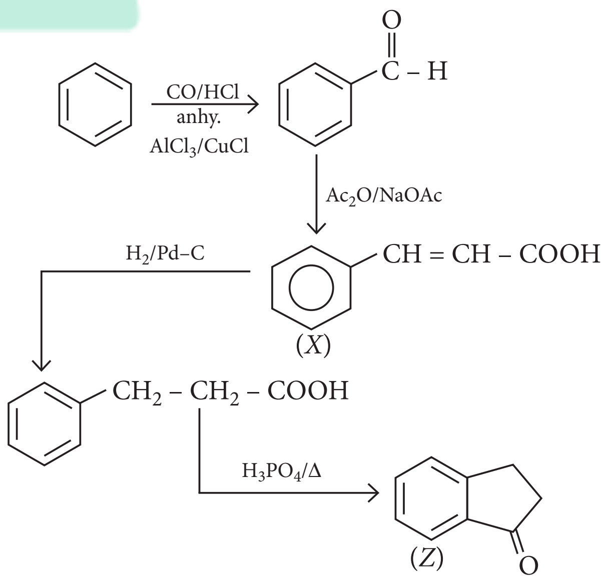
$$1 - \frac{\alpha}{2} = \frac{3}{2}\times 0.65 \Rightarrow \frac{\alpha}{2} = 1 - \frac{3}{2}\times 0.65$$

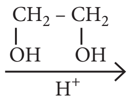
$$\therefore \alpha = 0.05$$

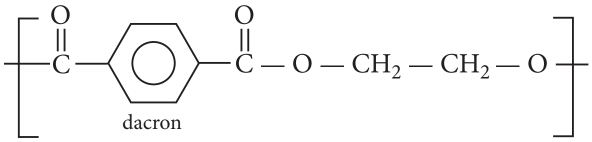
15. (c) :



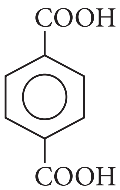
16. (a) :



17. (a) : $(C_{11}H_{12}O_2) \xrightarrow{\text{oxidation}}$ dibasic acid 

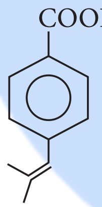


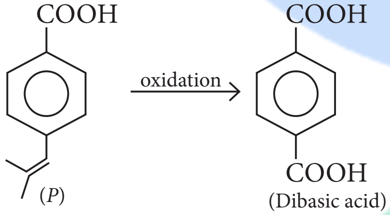
Dibasic acid must be terephthalic acid *i.e.,*

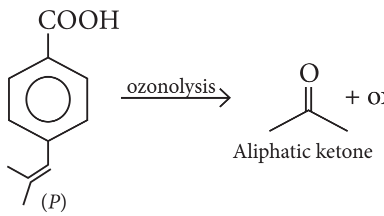


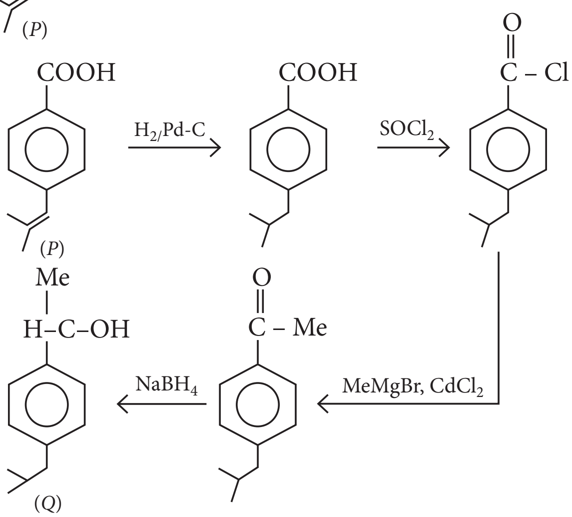
To give dacron, compound *P* must have benzene based structure.

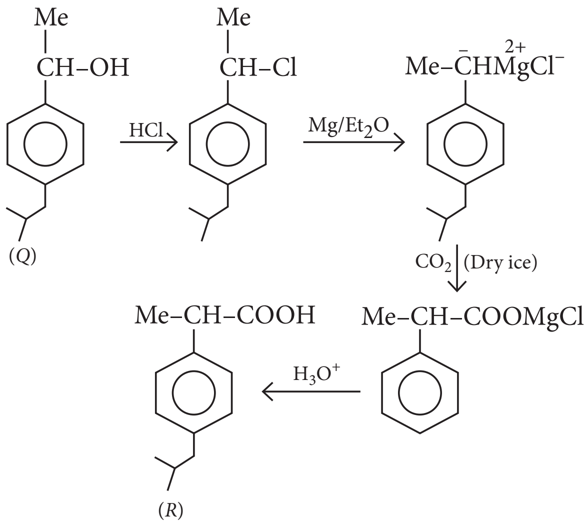
$C_{11}H_{12}O_2 \xrightarrow{\text{Ozonolysis}}$ ketone + oxidized products of benzene

Possible structure of *P* is  $(C_{11}H_{12}O_2)$.

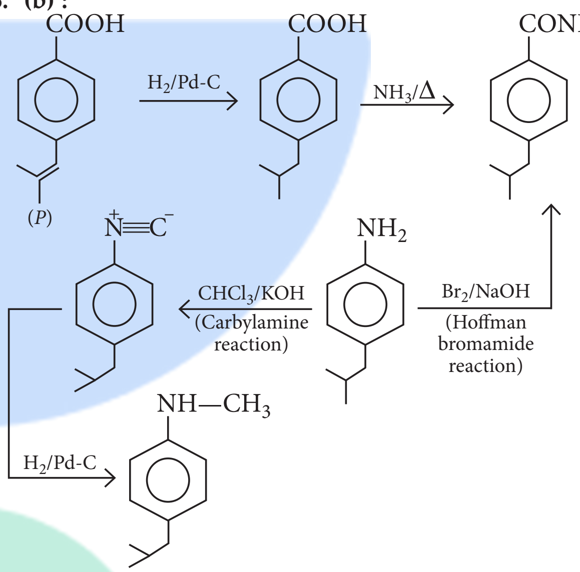








18. (b) :



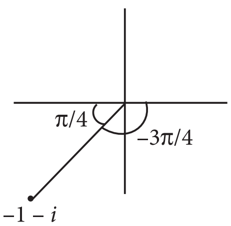
MATHEMATICS

1. (a, b, d) : $\arg(-1-i) = -3\pi/4$

We have, $f(t) = \arg|-1+it| = \begin{cases} \pi - \tan^{-1} t, & t \geq 0 \\ -\pi + \tan^{-1} t, & t < 0 \end{cases}$

$\lim_{t\to 0^+} f(t) = \pi, \quad \lim_{t\to 0^-} f(t) = -\pi$

Hence *f* is not continuous at 0.

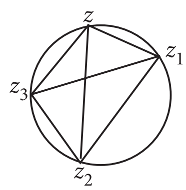


$\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$

$= (\arg z_1 - \arg z_2) + 2n\pi - \arg z_1 + \arg z_2$

$= 2n\pi, n \in Z$

Now, $\arg\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi$



$\therefore \quad \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$ is real.

$\Rightarrow \quad z, z_1, z_2, z_3$ are concyclic.

2. **(b, c, d) :** Using cosine rule,

$$\cos 30^\circ = \frac{\sqrt{3}}{2} = \frac{(10\sqrt{3})^2 + 10^2 - PR^2}{2\cdot 10\cdot 10\sqrt{3}}$$

$$\Rightarrow \frac{\sqrt{3}}{2}\cdot 2\times 10\sqrt{3}\times 10 = 400 - PR^2$$

$\Rightarrow$ $300 = 400 - PR^2$ $\Rightarrow$ $PR^2 = 100$ $\therefore$ $PR = 10$

So, $QR = PR$ $\Rightarrow$ $\angle QPR = 30^\circ$

$$ar\,(\Delta PQR) = \frac{1}{2}\cdot 10\sqrt{3}\cdot 10\sin 30^\circ = \frac{1}{2}\cdot 10\cdot 10\sqrt{3}\cdot\frac{1}{2} = 25\sqrt{3}$$

Also, $\angle QRP = 180^\circ - 2\times 30^\circ = 120^\circ$

$$\text{Now, } r = \frac{\Delta}{s} = \frac{25\sqrt{3}}{\frac{10+10+10\sqrt{3}}{2}} = \frac{25\sqrt{3}}{10+5\sqrt{3}} = \frac{25\sqrt{3}}{5(2+\sqrt{3})}$$

$$= 5\sqrt{3}(2-\sqrt{3}) = 10\sqrt{3} - 15$$

$$\text{Also, } R = \frac{10}{2\sin 30^\circ} = 10$$

Area $= \pi R^2 = 100\pi$

3. **(c, d) :** Let DR's of line of intersection of P_1 and P_2 be (a, b, c).

Then, $2a + b - c = 0$, $a + 2b + c = 0$

$$\Rightarrow \frac{a}{1+2} = \frac{b}{-1-2} = \frac{c}{4-1} \Rightarrow \frac{a}{3} = \frac{b}{-3} = \frac{c}{3}$$

So, the DR's are $(1, -1, 1)$.

The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ can be rewritten as

$$\frac{x-\frac{4}{3}}{3} = \frac{y-\frac{1}{3}}{-3} = \frac{z}{3}$$

So, the line is parallel to the line of intersection of plane P_1 and P_2.

The angle between P_1 and P_2

$$= \cos^{-1}\left(\frac{2\cdot 1 + 1\cdot 2 - 1\cdot 1}{\sqrt{6}\sqrt{6}}\right) = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

The plane P_3 is given by

$1(x-4) - 1(y-2) + 1(z+2) = 0$

i.e., $x - y + z = 0$

Distance of $(2, 1, 1)$ from the plane $P_3 = \frac{2-1+1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

4. **(a, b, d) :** As f is continuous and it is not constant, we must have an interval (r, s) on which f is one-one. Applying L.M.V.T. on $(-4, 0)$, we have

$$\frac{f(0) - f(-4)}{4} = f'(c), \quad \text{where } c \in (-4, 0)$$

$$\Rightarrow |f'(c)| \le \left|\frac{2+2}{4}\right| = 1$$

$\lim_{x\to\infty} f(x) = 1$ is incorrect as a counter example.

$f(x) = \sin(\sqrt{85}x)$ satisfies the given conditions but $\lim_{x\to\infty}\sin(\sqrt{85}x)$ doesn't exist.

Consider $A(x) = f^2(x) + f'^2(x)$

$A(0) = 85$

$\Rightarrow$ $c \in (-4, 0)$ such that $|f'(x)|^2 \le 1$

$A(c) = f^2(c) + (f'(c))^2$

$\le 4 + 1 = 5$ then $c \in (-4, 0)$

Similarly, let d be such that $A(d) \le 5$ then $d \in (0, 4)$

$A(0) = 85$

$\therefore$ $A(x)$ must have maxima in (c, d), say at β

$\Rightarrow$ $A'(\beta) = 0$ and $A(\beta) \ge 85$

Now, $2f'(\beta)\,[f(\beta) + f''(\beta)] = 0$

$f'(\beta) = 0$ is not possible, hence $f(\beta) + f''(\beta) = 0$.

5. **(b, c) :** We have $f'(x) = e^{(f(x)-g(x))}\,g'(x)$

$\Rightarrow$ $e^{-f(x)}f'(x) = e^{-g(x)}g'(x)$

Integrating we get, $-e^{-f(x)} = -e^{-g(x)} + \lambda$

i.e., $e^{-g(x)} - e^{-f(x)} = \lambda$

Putting $x = 1$ and $x = 2$ respectively, we get

$e^{-g(1)} - e^{-f(1)} = \lambda$ and $e^{-g(2)} - e^{-f(2)} = \lambda$

We have $e^{-g(1)} - e^{-f(1)} = e^{-g(2)} - e^{-f(2)}$

$\Rightarrow$ $e^{-g(1)} - e^{-1} = e^{-1} - e^{-f(2)} \Rightarrow e^{-g(1)} + e^{-f(2)} = 2/e$

$$\Rightarrow e^{-f(2)} < \frac{2}{e} \Rightarrow -f(2) < \log_e 2 - 1$$

$\therefore$ $f(2) > 1 - \log_e 2$

Similar things holds for $g(1)$.

6. **(b, c) :** $f(x) = 1 - 2x + \int_0^x e^{x-t} f(t)\,dt$ ...(i)

$$\Rightarrow e^{-x}f(x) = e^{-x}(1-2x) + \int_0^x e^{-t} f(t)\,dt$$

Differentiating w.r.t. x, we get

$$e^{-x}f'(x) - e^{-x}f(x) = e^{-x}(-2) - e^{-x}(1-2x) + e^{-x}f(x)$$

This simplifies to $f'(x) - 2f(x) = 2x - 3$ (Linear D.E.)

$\therefore$ I.F. $= e^{-2x}$

$\therefore$ The required solution is $f(x)e^{-2x} = \int (2x-3)e^{-2x}dx$

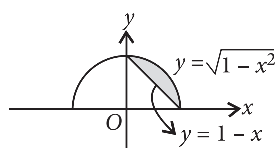
$$= (2x-3)\cdot\frac{e^{-2x}}{-2} + 2\int\frac{e^{-2x}}{2}dx + \lambda$$

$$= \frac{-(2x-3)}{2}e^{-2x} - \frac{1}{2}e^{-2x} + \lambda$$

$\therefore$ $y = f(x) = (1-x) + \lambda e^{2x}$

Put $x = 0$ in (i) $\Rightarrow$ $y = 1$ $\Rightarrow$ $1 = 1 + \lambda$ $\therefore$ $\lambda = 0$

$\therefore$ $y = 1 - x$



$\therefore$ Required area $= \frac{1}{4}\cdot\pi.(1)^2 - \frac{1(1)(1)}{2} = \frac{\pi}{4} - \frac{1}{2}$

7. (8) : $((\log_2 9)^2)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$

$= (\log_2 9)^{\frac{2}{\log_2(\log_2 9)}} \cdot (\sqrt{7})^{\log_7 4}$

$= (\log_2 9)^{2\log_{\log_2 9}(2)} \cdot 7^{\frac{1}{2}\log_7 4} = 4\cdot 2 = 8$

8. (625) : The last two digits can be 12, 24, 32, 44, 52 for the number to be divisible by 4.

$\therefore$ The number of ways $= 5\cdot5\cdot5\cdot5 = 625$

9. (3748) : Here $X = \{1, 6, 11, ..., 10086\}$

and $Y = \{9, 16, 23, ..., 14128\}$

The intersection of X and Y is an A.P. with 16 as first term and 35 as common difference.

The series becomes 16, 51, 86, ...

Now, k^{th} term $= 16 + (k-1)\,35 \le 10086$

i.e. $k \le \frac{10105}{35}$ $\therefore k \le 288$ (as k is to be an integer)

Hence, $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$= 2018 + 2018 - 288 = 3748$

10. (2) : Let $f(x) = \sum_{i=1}^{\infty} x^{i+1} - x\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^i$

$= (x^2 + x^3 + ... \text{ to } \infty) - x\left(\frac{x}{2} + \left(\frac{x}{2}\right)^2 + ... \text{ to } \infty\right)$

$= \frac{x^2}{1-x} - \frac{x\cdot\frac{x}{2}}{1-\frac{x}{2}} = \frac{x^2}{1-x} - \frac{x^2}{2-x}$

Again, let $g(x) = \sum_{i=1}^{\infty}\left(-\frac{x}{2}\right)^i - \sum_{i=1}^{\infty}(-x)^i$

$= \left[\left(\frac{-x}{2}\right) + \left(\frac{-x}{2}\right)^2 + ... \text{ to } \infty\right] - \left[(-x) + (-x)^2 + ... \text{ to } \infty\right]$

$= \frac{-\frac{x}{2}}{1+\frac{x}{2}} - \frac{-x}{1+x} = \frac{x}{1+x} - \frac{x}{2+x}$

$\therefore$ The given equation becomes

$\sin^{-1}f(x) + \cos^{-1}g(x) = \pi/2$

So, we must have $f(x) = g(x)$

$\Rightarrow \frac{x^2}{1-x} - \frac{x^2}{2-x} = \frac{x}{1+x} - \frac{x}{2+x}$

$\Rightarrow \frac{x^2}{(1-x)(2-x)} = \frac{x}{(2+x)(1+x)}$

$\Rightarrow$ $x = 0$ and $x(2+x)(1+x) = (1-x)(2-x)$

Let $h(x) = x(2+x)(1+x) - (1-x)(2-x)$

$= x^3 + 2x^2 + 5x - 2$

Now, $h\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3 + 2\left(-\frac{1}{2}\right)^2 + 5\left(-\frac{1}{2}\right) - 2$

$= -\frac{1}{8} + \frac{1}{2} - \frac{5}{2} - 2 = -\frac{33}{8} < 0$

$h\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 2\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) - 2$

$= \frac{1}{8} + \frac{1}{2} + \frac{5}{2} - 2 = \frac{9}{8} > 0$

$\therefore$ There exist a root between $-\frac{1}{2}$ and $\frac{1}{2}$.

Also, $h'(x) = 3x^2 + 4x + 5 > 0$

$\Rightarrow$ $h(x)$ has exactly one real root in $\left(-\frac{1}{2}, \frac{1}{2}\right)$.

11. (1) : We have

$\log y_n = \frac{1}{n}\left\{\log\left(1+\frac{1}{n}\right) + \log\left(1+\frac{2}{n}\right) + ... + \log\left(1+\frac{n}{n}\right)\right\}$

$\therefore \lim_{n\to\infty}\log y_n = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\left(1+\frac{r}{n}\right)$

$= \int_0^1 \log(1+x)dx = \int_1^2 \log x\,dx$

$= [x\log x - x]_1^2 = 2\log 2 - 1 = \log\frac{4}{e}$

$\therefore$ $L = 4/e$ Thus, $[L] = 1$.

12. (3) : Here $\vec{c} = x\vec{a} + y\vec{b} + \vec{a}\times\vec{b}$

Also, $\vec{a}\cdot\vec{c} = \vec{b}\cdot\vec{c} = 2\cos\alpha$

We have, $\vec{c}\cdot\vec{a} = x\vec{a}\cdot\vec{a} + 0 + 0$

$\therefore$ $x = \vec{c}\cdot\vec{a} = 2\cos\alpha$ Similarly, $y = 2\cos\alpha$

Now, $c^2 = x^2 + y^2 + 1$

We have $4 = 2(4\cos^2\alpha) + 1 \Rightarrow 3 = 8\cos^2\alpha$

13. (0.5) : As α and β are the roots of the given equation.

$\therefore$ $\sqrt{3}\,a\cos\alpha + 2b\sin\alpha = c$ ...(i)

and $\sqrt{3}\,a\cos\beta + 2b\sin\beta = c$ ...(ii)

On subtracting (ii) from (i), we get

$\sqrt{3}(\cos\alpha - \cos\beta) + \frac{2b}{a}(\sin\alpha - \sin\beta) = 0$

$\Rightarrow \sqrt{3}\left(-2\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}\right) + \frac{2b}{a}2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2} = 0$

$\Rightarrow -\sqrt{3}\sin\frac{\pi}{6} + \frac{2b}{a}\cdot\cos\frac{\pi}{6} = 0$

$\Rightarrow -\sqrt{3}\cdot\frac{1}{2} + \frac{2b}{a}\cdot\frac{\sqrt{3}}{2} = 0 \Rightarrow -1 + \frac{2b}{a} = 0 \quad \therefore \frac{b}{a} = \frac{1}{2} = 0.5$

Alternative Solution :

We have $\sqrt{3}a\cos t + 2b\sin t = c$

$\Rightarrow \sqrt{3}a\dfrac{1-t^2}{1+t^2} + 2b\dfrac{2t}{1+t^2} = c \qquad \left[\text{Put } t = \tan\dfrac{x}{2}\right]$

$\Rightarrow \sqrt{3}a(1-t^2) + 4bt = c(1+t^2)$

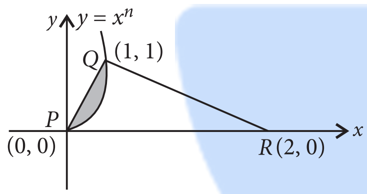
i.e., $(c+\sqrt{3}a)t^2 - 4bt + c - \sqrt{3}a = 0$

As $\tan\dfrac{\alpha+\beta}{2} = \tan\dfrac{\pi}{6} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \dfrac{t_1+t_2}{1-t_1t_2} = \dfrac{1}{\sqrt{3}} \qquad \left[\text{where } t_1 = \tan\dfrac{\alpha}{2} \text{ and } t_2 = \tan\dfrac{\beta}{2}\right]$

$\Rightarrow \dfrac{4b}{c+\sqrt{3}a - c + \sqrt{3}a} = \dfrac{1}{\sqrt{3}} \Rightarrow \dfrac{b}{a} = \dfrac{1}{2} = 0.5$

14. **(4) :**



$\int_0^1 (x - x^n)dx = \dfrac{3}{10}\cdot\left(\dfrac{1}{2}\times 2\times 1\right)$

$\Rightarrow \left[\dfrac{x^2}{2} - \dfrac{x^{n+1}}{n+1}\right]_0^1 = \dfrac{3}{10} \Rightarrow \dfrac{1}{2} - \dfrac{1}{n+1} = \dfrac{3}{10}$

$\Rightarrow \dfrac{1}{2} - \dfrac{3}{10} = \dfrac{1}{n+1} \Rightarrow \dfrac{2}{10} = \dfrac{1}{n+1}$

$\Rightarrow n+1 = \dfrac{10}{2} = 5 \quad \therefore\ n = 4$

15. **(a) :** Point E_1 is $(-\sqrt{3}, 1)$ and E_2 is $(\sqrt{3}, 1)$

Similarly, coordinates of F_1 and F_2 are $(1, \sqrt{3})$ and $(1, -\sqrt{3})$ respectively.

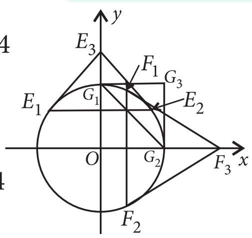
The tangent at E_1 is $-\sqrt{3}x$ is $y = 4$

and tangent at E_2 is $\sqrt{3}x + y = 4$

So, E_3 is (0, 4)

Now, tangent at F_1 is $x + \sqrt{3}y = 4$

and tangent at F_2 is $x - \sqrt{3}y = 4$



So, F_3 is (4, 0)

Also, G_3 happens to be (2, 2).

So, (0, 4), (4, 0) and (2, 2) clearly lies on $x + y = 4$.

16. **(d) :** Let P be $(2\cos\theta, 2\sin\theta)$

Equation of tangent at P is $x\cos\theta + y\sin\theta = 2$

So, N is $\left(\dfrac{2}{\cos\theta}, 0\right)$ and M is $\left(0, \dfrac{2}{\sin\theta}\right)$

Let mid-point of MN be (x, y).

$\therefore\ x = \dfrac{1}{\cos\theta}$ and $y = \dfrac{1}{\sin\theta}$

$\therefore$ The locus is $\dfrac{1}{x^2} + \dfrac{1}{y^2} = 1$

i.e., $x^2 + y^2 = x^2y^2$

17. **(a) :** The required probability is

$$\dfrac{4!\left(1 - \dfrac{1}{1!} + \dfrac{1}{2!} - \dfrac{1}{3!} + \dfrac{1}{4!}\right)}{5!} = \dfrac{9}{120} = \dfrac{3}{40}$$

18. **(c) :** Total cases = 5!

Lets count the number of favourable ways.

2 4 1 × ×
2 5 3 1 4 } 3 ways × 2 = 6

1 3 5 2 4
1 4 2 5 3 } 2 ways × 2 = 4

3 1 5 2 4
3 1 4 2 5 } 2 ways × 2 = 4

$\therefore$ Total favourable ways = 14

Alternative Solution :

(By P.I.E.)

$|T_1 \cap T_2 \cap T_3 \cap T_4| = |T| - |T'_1 \cup T'_2 \cup T'_3 \cup T'_4|$

$= 5! - \{{}^4C_1 \cdot 4! \cdot 2! - ({}^3C_1 \cdot 3! \cdot 2! + {}^3C_1 \cdot 3! \cdot 2! \cdot 2!) + ({}^2C_1 \cdot 2! \cdot 2! + {}^4C_1 \cdot 2! \cdot 2!) - 2\}$

$= 5! - \{4 \cdot 48 - (3 \cdot 6 \cdot 2 + 3 \cdot 6 \cdot 2 \cdot 2) + (2 \cdot 2 \cdot 2 + 4 \cdot 2 \cdot 2) - 2\}$

$= 120 - (192 - 108 + 24 - 2) = 120 - 106 = 14$

Hence, the required probability $= \dfrac{14}{120} = \dfrac{7}{60}$

PAPER - II

PHYSICS

1. **(a,b,d) :** As $\dfrac{dK}{dt} = \dfrac{d}{dt}\left(\dfrac{1}{2}mv^2\right) = \dfrac{1}{2}m\left(2v\dfrac{dv}{dt}\right) = mv\dfrac{dv}{dt}$

As per question, $mv\dfrac{dv}{dt} = \gamma t$

$\therefore \int_0^v v\,dv = \dfrac{\gamma}{m}\int_0^t t\,dt \Rightarrow \dfrac{v^2}{2} = \dfrac{\gamma}{m}\dfrac{t^2}{2};\ v = \sqrt{\dfrac{\gamma}{m}}\,t$

or $\quad v \propto t \qquad$ ...(i)

Hence, the speed of the particle is proportional to time.

From equation (i),

$\frac{dv}{dt} = a = \text{constant} \quad \therefore \quad F = ma = \text{constant}$

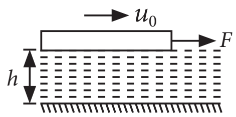
Also, $v = \frac{dx}{dt} \propto t$ so, $x \propto t^2$

Since work done by the force F is

$W = \int_{x_i}^{x_f} F dx = F(x_f - x_i)$ $\qquad$ ($\because$ F is constant)

As work done by this force depends only on initial and final position. So, the force is conservative in nature.

2. **(a,c,d) :** Viscous force on the square plate moving on viscous liquid is



$F = -\eta A \frac{dv}{dy}$

According to the question,

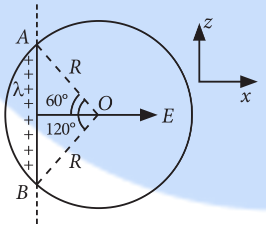
$dv = u_0$ and $dy = h$

$\therefore \quad F = -\eta A \frac{u_0}{h}$ or $F \propto \frac{1}{h}$ and $F \propto A$

Tangential stress $(S) = \frac{F}{A} = -\eta \frac{u_0}{h}$

$\therefore \quad S \propto \eta$ and $S \propto u_0$

3. **(a, b) :** Electric field due to a long straight wire carrying uniform linear charge density is perpendicular to the wire. As shown in figure electric filed will be in x-direction.



$\therefore \quad E_z = 0$

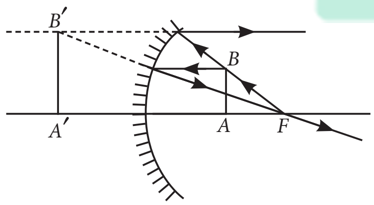
Length of wire inside the spherical shell is

$AB = 2R \sin 60° = \sqrt{3}R$

As per Gauss's law, electric flux through the shell,

$\oint \vec{E}.d\vec{S} = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda\sqrt{3}R}{\varepsilon_0}$

4. **(d) :** Ray diagram is not to scale.



Distance of point A from the mirror is $f/2$.

Using mirror formula,

$$\frac{1}{v} + \frac{1}{-f/2} = \frac{1}{-f} \Rightarrow \frac{1}{v} = \frac{2}{f} - \frac{1}{f} = \frac{1}{f}$$

$\therefore \quad v = f$

Image $A'B'$ of line AB would be perpendicular to the principle axis. Image of F will be formed at infinity.

Also light ray from infinity or towards infinity seems parallel to the principle axis of the mirror. So correct choice is (d).

5. **(a, c) :** According to question,

$^{232}_{90}\text{Th} \longrightarrow ^{212}_{82}\text{Pb} + x\,^4_2\text{He} + y\,_{-1}e^0$

Change in mass number is 20.

$\therefore \quad 4x = 20 \Rightarrow x = 20/4 = 5$; $\quad \therefore \quad N_\alpha = 5$

In order to balance the charge on both sides,

$90 = 82 + 2x + y$

$90 = 82 + 2 \times 5 + y \Rightarrow y = -2$; $\quad \therefore \quad N_\beta = 2$

Hence, 5 α and 2 β^-particles are emitted from $^{232}_{90}\text{Th}$.

6. **(a,c) :** Let n and n+1 be the two consecutive harmonics corresponding to the length 50.7 cm and 83.9 cm respectively.

$\therefore \quad (83.9 - 50.7) = \lambda/2 \Rightarrow \lambda = 66.4$ cm

Length corresponding to the fundamental mode is $\lambda/4 = 16.6$ cm.

As length of the resonating air column is an odd multiple of $\lambda/4$.

$\therefore$ At 3rd harmonic, $L = 3 \times \lambda/4 = 3 \times 16.6 = 49.8$ cm

This means the resonance at 50.7 cm is the 3rd harmonic.

End correction e is given by

$e + 50.7 = \frac{3\lambda}{4}$; $e = 49.8 - 50.7 = -0.9$ cm

Velocity of sound is $v = \upsilon \times \lambda = 500 \times 66.4$ cm s^{-1}
$= 332$ m s^{-1}

7. **(6.30) :** Given: $v = v_0 e^{-t/\tau}$ $\qquad$ ...(i)

At initial condition, $v_0 = J/m = 2.5$ m s^{-1}

where J is impulse given to the block.

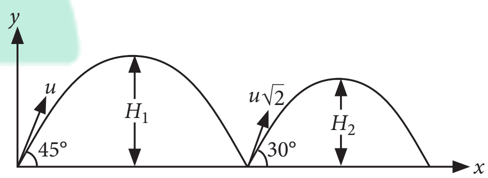
From eqn. (i), $\frac{dx}{dt} = v_0 e^{-t/\tau} \Rightarrow \int_0^x dx = v_0 \int_0^\tau e^{-t/\tau} dt$

$\therefore \quad x = v_0 \left[\frac{e^{-t/\tau}}{-1/\tau}\right]_0^\tau = -v_0\tau(e^{-1} - 1)$

At $t = \tau = 4$ s

$x = -2.5 \times 4\,(e^{-1} - 1) = -10\,(0.37 - 1) = 6.30$ m

8. **(30) :**



Height attained by the ball during projectile is

$H_1 = \frac{u^2 \sin^2 45°}{2g}$ or $\frac{u^2}{4g} = 120$ $\qquad$ ...(i)

Let v be the velocity of the ball after the bounce.

As per question,

$\frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{1}{2}mu^2\right) \Rightarrow v = \frac{u}{\sqrt{2}}$

Maximum height attained by the ball after the bounce is

$$H_2 = \frac{v^2 \sin^2 30°}{2g} = \frac{\left(\frac{u}{\sqrt{2}}\right)^2 \sin^2 30°}{2g}$$

$$=\frac{u^2}{16g}=\frac{1}{4}\left(\frac{u^2}{4g}\right)=\frac{120}{4} \quad \text{(using eqn. (i))}$$

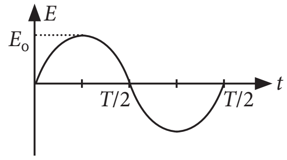
$\therefore \quad H_2 = 30$ m

9. (2) : Given: $\vec{E}=\vec{E}_0 \sin\omega t\,\hat{i},\ m = 10^{-3}$ kg; $q = 1$ C

Force on a charged particle placed in an external electric field is given by

$F = qE = qE_0 \sin\omega t$

$$\therefore \ a=\frac{F}{m}=\frac{q}{m}E_0 \sin\omega t$$



$$\frac{dv}{dt}=\frac{q}{m}E_0 \sin\omega t$$

$$\text{or} \quad \int_0^v dv=\frac{qE_0}{m}\int_0^{\pi/\omega}\sin\omega t\ dt$$

(Particle is initial accelerating for time interval $t = 0$ to $t = T/2 = \pi/\omega$.)

$$v-0=\frac{qE_0}{m\omega}[-\cos\omega t]_0^{\pi/\omega}$$

$$v=\frac{qE_0}{m\omega}[-\cos\pi-(-\cos 0)=\frac{qE_0}{m\omega}\times 2$$

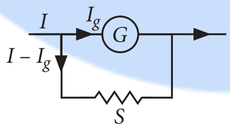
$$\therefore \quad v=\frac{1\times 1}{10^3\times 10^{-3}}\times 2 = 2 \text{ m s}^{-1}$$

10. (5.56) : Given, $N = 50$, $A = 2 \times 10^{-4}$ m^2

Torsional constant,

$C = 10^{-4}$ N m rad^{-1}

$B = 0.02$ T, $\phi = 0.2$ rad



As per question,

$NI_g AB = C\theta$, where I_g is the current flowing through galvanometer

$$\therefore \quad I_g=\frac{C\theta}{NAB}=\frac{10^{-4}\times 0.2}{50\times 2\times 10^{-4}\times 0.02}=0.1 \text{ A}$$

To convert galvanometer to ammeter, we put a shunt resistance S in parallel to the galvanometer.

$I_g \times G = (I - I_g)S$ (using Kirchhoff's law)

$\therefore \quad 0.1 \times 50 = (1 - 0.1) \times S$

(To measure current in the range of 0-1.0 A)

$$5 = 0.9\,S \quad \Rightarrow \quad S=\frac{50}{9}\,\Omega = 5.56\,\Omega$$

So, the shunt resistance is $= 5.56\ \Omega$.

11. (3) : $d = 0.5$ mm, $Y = 2 \times 10^{11}$, $l = 1$ m

$$\Delta l=\frac{Fl}{Ay}=\frac{mgl}{\frac{\pi d^2}{4}y}=\frac{1.2\times 10\times 1}{\frac{\pi}{4}\times(5\times 10^{-4})^2\times 2\times 10^{11}}$$

$$\Delta l=\frac{1.2\times 10}{\frac{3.2}{4}\times 25\times 10^{-8}\times 2\times 10^{11}}$$

$$=\frac{12}{0.8\times 25\times 2\times 10^3}=\frac{12}{40\times 10^3}=0.3 \text{ mm}$$

Since least count of vernier callipers is 0.1 mm so 3$^{\text{rd}}$ division of vernier scale will coincicle with the main scale division.

12. (900) : For an adiabatic process,

PV^{γ} = constant or $TV^{\gamma-1}$ = constant

$$\therefore \quad \frac{T_1}{T_2}=\left(\frac{V_2}{V_1}\right)^{\gamma-1} \quad \text{or} \quad \frac{T_1}{T_2}=\left(\frac{8V_1}{V_1}\right)^{\gamma-1}$$

$(\because V_2 = 8V_1$ (given))

$$\Rightarrow \quad T_2=\frac{T_1}{8^{\gamma-1}} \quad \text{...(i)}$$

For a monatomic gas,

$$C_V=\frac{3}{2}R \text{ and } C_P=\frac{5}{2}R \quad \therefore \quad \gamma=\frac{C_P}{C_V}=\frac{5}{3}$$

So, from eqn (i), $T_2=\dfrac{T_1}{8^{(5/3-1)}}=\dfrac{T_1}{4}$

Change in internal energy of the gas,

$\Delta U = nC_V(T_2 - T_1)$

$$=1\times\frac{3}{2}R\times\left(\frac{T_1}{4}-T_1\right)=\frac{3}{2}R\times\left(\frac{-3T_1}{4}\right)$$

$$=\frac{3}{2}\times 8\times\left(\frac{-3}{4}\right)\times 100=-900 \text{ J}$$

So, decrease in internal energy of the gas is 900 J.

13. (24) : Since frequency of incident light is just above threshold frequency so the kinetic energy of photoelectrons will be equivalent to zero.

As photoelectron emission efficiency is 100% then number of photoelectrons emitted per second is,

$$N=\frac{P}{\phi_0}=\frac{200}{6.25\times 1.6\times 10^{-19}}$$

There is a accelerating voltage of 500 V.

$\therefore$ K.E. of each electron $= qV = 1.6 \times 10^{-19} \times 500$ J

Momentum of each electron, $P=\sqrt{2m \text{ K.E.}}$

Net force on anode, F = rate of change of momentum

$$=N\times\sqrt{2m \text{ K.E.}}$$

$$=\frac{200}{6.25\times 1.6\times 10^{-19}}\times\sqrt{2\times 9\times 10^{-31}\times 1.6\times 10^{-19}\times 500}$$

$= 24 \times 10^{-4}$ N $= n \times 10^{-4}$ N

So $n = 24$

14. (3) : Energy of electron in the n^{th} orbit of hydrogen like atom having atomic number Z is given by

$$E_n=-13.6\frac{Z^2}{n^2}$$

As per given transitions in questions,

$$13.6\times Z^2\left(1-\frac{1}{4}\right)=74.8+13.6\times Z^2\left(\frac{1}{4}-\frac{1}{9}\right)$$

$$13.6\times Z^2\left(\frac{3}{4}\right)=74.8+13.6\times Z^2\left(\frac{5}{36}\right)$$

$$13.6\times Z^2\left(\frac{11}{18}\right)=74.8;\ Z^2=9\ \Rightarrow\ Z=3$$

15. (b) : For a point charge Q,

$$E=\frac{1}{4\pi\varepsilon_0}\frac{Q}{d^2}\ \Rightarrow\ E\propto\frac{1}{d^2}$$

For an electron dipole,

$$E=\frac{kp}{d^3}\sqrt{1+3\cos^2\theta}\ \Rightarrow\ E\propto\frac{1}{d^3}$$

For a line charge, $E=\frac{\lambda}{2\pi\varepsilon_0 d}\ \Rightarrow\ E\propto\frac{1}{d}$

For two infinite wires carrying uniform linear charge density parallel to x-axis electric field is given by

$$E=\frac{\lambda}{2\pi\varepsilon_0(d-l)}-\frac{\lambda}{2\pi\varepsilon_0(d+l)}$$

$$=\frac{\lambda}{2\pi\varepsilon_0}\left(\frac{1}{d-l}-\frac{1}{d+l}\right)=\frac{\lambda}{2\pi\varepsilon_0}\frac{2l}{d^2(1-l^2/d^2)}$$

$$\Rightarrow\ E\propto\frac{1}{d^2}\qquad(\because l^2<<<d^2)$$

For infinite plane sheet,

$$E=\frac{\sigma}{2\varepsilon_0}\ \Rightarrow\ E \text{ is independent of } d$$

16. (b) : Orbital speed, $v=\sqrt{\frac{GM}{R}}\Rightarrow v\propto\frac{1}{\sqrt{R}}$

$$\therefore\ \frac{v_1}{v_2}=\sqrt{\frac{R_2}{R_1}}=\sqrt{4}=2$$

Angular momentum, $L = mvR$

$$\therefore\ \frac{L_1}{L_2}=\frac{m_1}{m_2}\times\frac{v_1}{v_2}\times\frac{R_1}{R_2}=\frac{2}{1}\times\frac{2}{1}\times\frac{1}{4}=1$$

K.E. of satellite, $K=\frac{GMm}{2R}$

$$\therefore\ \frac{K_1}{K_2}=\frac{m_1}{m_2}\times\frac{R_2}{R_1}=2\times4=8$$

From Kepler's second law, $T^2\propto R^3\Rightarrow T\propto R^{3/2}$

$$\therefore\ \frac{T_1}{T_2}=\left(\frac{R_1}{R_2}\right)^{3/2}=\left(\frac{1}{4}\right)^{3/2}=\frac{1}{8}$$

17. (c) : Process-I is adiabatic (ΔQ = 0), so no heat is exchanged between the gas and its surroundings.

Process II is isobaric process ($\Delta P = 0$), work done by the gas = area under the curve = (3 P_0) × (2 V_0) = 6 P_0V_0

Process-III is isochoric process ($\Delta V = 0$), work done by the gas = $P\Delta V = P\times 0 = 0$

Process-IV is isothermal process (ΔT = 0), so temperatures of the gas remains constant.

18. (a) : For $\vec r(t)=\alpha t\hat i+\beta t\hat j$

$$\vec v=\frac{d\vec r(t)}{dt}=\alpha\hat i+\beta\hat j;\ \vec a=\frac{d\vec v}{dt}=0$$

$\vec p=m\vec v$ (remains constant)

$K=\frac{1}{2}mv^2$ (remains constant)

$$\vec F=-\left[\frac{\partial U}{\partial x}\hat i+\frac{\partial U}{\partial y}\hat j\right]=0\ \Rightarrow\ U=\text{constant}$$

$E = K + U$ = constant

$$\frac{d\vec L}{dt}=\vec\tau=\vec r\times\vec F=0$$

$\vec L$ = constant

For $\vec r(t)=\alpha\cos(\omega t)\hat i+\beta\sin(\omega t)\hat j$

$$\vec v=\frac{d\vec r}{dt}=-\alpha\omega\sin(\omega t)\hat i+\beta\omega\cos(\omega t)\hat j$$

$$\vec a=\frac{d\vec v}{dt}=-\alpha\omega^2\cos(\omega t)\hat i-\beta\omega^2\sin(\omega t)\hat j$$

$$=-\omega^2[\alpha\cos(\omega t)\hat i+\beta\sin(\omega t)\hat j]$$

$$\vec a=-\omega^2\vec r$$

$\vec\tau=\vec r\times\vec F=0$ ($\because \vec r$ and $\vec F$ are parallel)

So, $\vec L$ = constant

$$U=-\int\vec F\cdot d\vec r=+\int_0^r m\omega^2.rdr=m\omega^2\left[\frac{r^2}{2}\right]$$

$U\propto r^2$

$$r=\sqrt{\alpha^2\cos^2(\omega t)+\beta^2\sin^2(\omega t)}$$

r is a function of time (t).

U depends on r hence it will change with time.

Total energy remains constant because force is central.

For $\vec r(t)=\alpha(\cos\omega t\hat i+\sin(\omega t)\hat j)$

$$\vec v(t)=\frac{d\vec r}{dt}=\alpha(-\omega\sin\omega t\hat i+\omega\cos\omega t\hat j)$$

$|\vec v|=\alpha\omega$ (Speed remains constant)

$$\vec a(t)=\frac{d\vec v(t)}{dt}=\alpha[-\omega^2\cos(\omega t)\hat i-\omega^2\sin(\omega t)\hat j]$$

$$=-\alpha\omega^2[\cos(\omega t)\hat i+\sin(\omega t)\hat j]$$

$$\vec a(t)=-\omega^2\vec r$$

$\vec\tau=\vec r\times\vec F=0$ $\qquad\therefore\ \vec L$ = constant

$|\vec r|=\alpha$ (remains constant)

Force is central in nature and distance from fixed point is constant.

Potential energy remains constant.

Kinetic energy is also constant. ($\because$ Speed is constant)

For $\vec r=\alpha t\hat i+\frac{\beta}{2}t^2\hat j$

$\vec v=\frac{d\vec r}{dt}=\alpha\hat i+\beta t\hat j$ (speed of particle depends on t)

$$\vec{a} = \frac{d\vec{v}}{dt} = \beta\hat{j} \quad \text{(constant)}$$

$$\vec{F} = m\vec{a} \quad \text{(constant)}$$

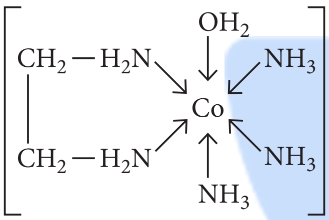
$$U = -\int \vec{F}\cdot d\vec{r} = -m\int_0^t \beta\hat{j}\cdot(\alpha\hat{i} + \beta t\hat{j})dt$$

$$U = \frac{-m\beta^2t^2}{2}\,;\; K = \frac{1}{2}mv^2 = \frac{1}{2}m(\alpha^2 + \beta^2t^2)$$

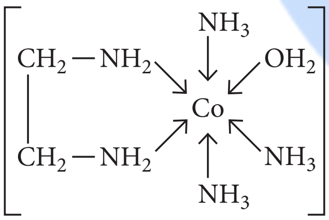
$$E = K + U = \frac{1}{2}m\alpha^2 \quad \text{(remains constant)}$$

CHEMISTRY

1. **(a, b, d) :** (a) $[Co(en)(NH_3)_3(H_2O)]^{3+}$

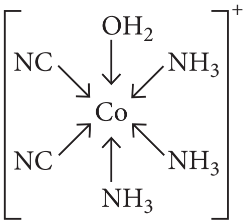


Facial

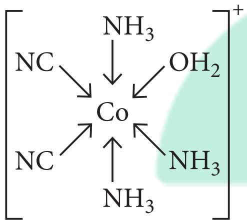


Meridional

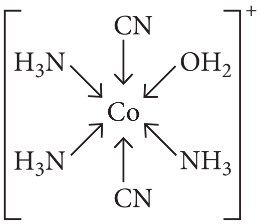
(b) $[Co(CN)_2(NH_3)_3(H_2O)]^{+}$



Facial



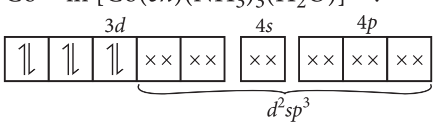
Meridional



Trans

(c) Co^{3+} : $[Ar]3d^6$ in presence of *en* and NH_3 it forms low spin complex.

Co^{3+} in $[Co(en)(NH_3)_3(H_2O)]^{3+}$:



Due to absence of unpaired electron, this complex is diamagentic in nature.

(d) $[Co(en)(NH_3)_4]^{3+}$ has larger energy gap between t_{2g} and e_g than $[Co(en)(NH_3)_3(H_2O)]^{3+}$ as NH_3 is stronger ligand than H_2O. So, $[Co(en)(NH_3)_3(H_2O)]^{3+}$ absorbs longer wavelength than $[Co(en)(NH_3)_4]^{3+}$.

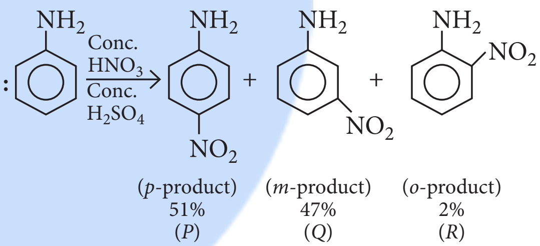
2. **(b, d) :** (a) Manganese show pale purple colour in flame test.

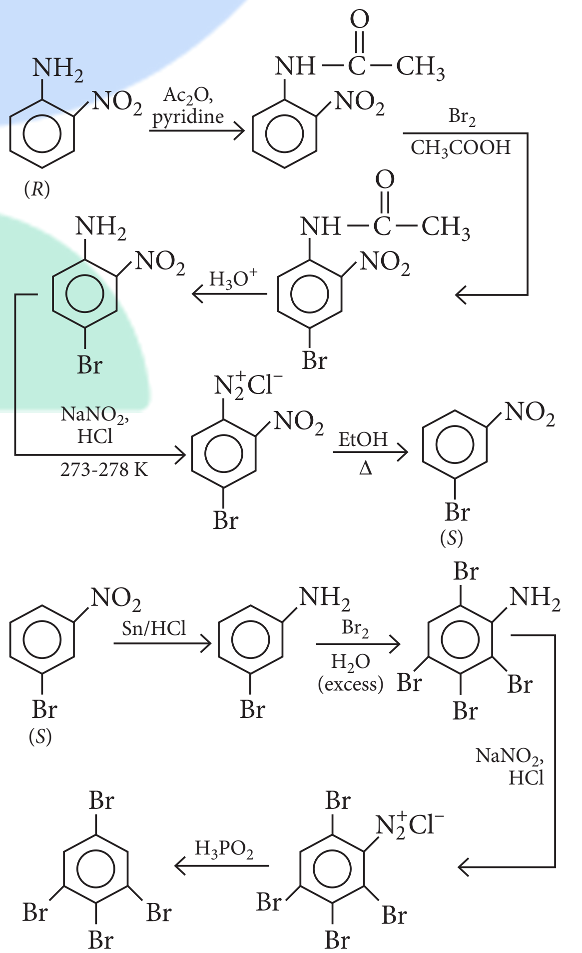
(b) $Cu^{2+} \xrightarrow[HCl]{H_2S} CuS\downarrow$ Black ppt.

(c) Both Cu^{2+} and Mn^{2+} form precipitate with H_2S in basic medium.

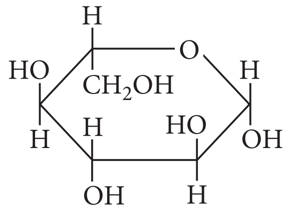
(d) $E^\circ_{Cu^{2+}/Cu} = +0.34$ V, $E^\circ_{Mn^{2+}/Mn} = -1.18$ V

3. **(d) :**





4. **(d) :** Structure of β-L-glucopyranose is



5. **(a, d) :** $A_{(g)} \xrightarrow{\text{First order}} 2B_{(g)} + C_{(g)}$; V = constant, T = 300 K

$t = 0$	P_0	0	0
$t = t_{1/3}$	$\left(P_0 - \frac{2P_0}{3}\right) = \frac{P_0}{3}$	$\frac{4P_0}{3}$	$\frac{2P_0}{3}$
$t = t$	$P_0 - x$	$2x$	x

So, $P_t = P_0 - x + 2x + x = P_0 + 2x$

or $2x = P_t - P_0$

$$t = \frac{1}{k}\ln\frac{P_0}{(P_0 - x)}$$

or $$t = \frac{1}{k}\ln\frac{P_0}{P_0 - \frac{(P_t - P_0)}{2}} = \frac{1}{k}\ln\frac{2P_0}{2P_0 - P_t + P_0}$$

or $$kt = \ln\frac{2P_0}{3P_0 - P_t},\ kt = \ln 2P_0 - \ln(3P_0 - P_t)$$

or $\ln(3P_0 - P_t) = -kt + \ln 2P_0$

Comparing the above equation with general straight line equation we get, slope = $-k$, intercept = $\ln 2P_0$

So, (a) is correct option.

Now, $$t_{1/3} = \frac{1}{k}\ln\frac{P_0}{(P_0/3)} = \frac{1}{k}\ln 3$$

$\Rightarrow$ It is independent of initial concentration.

So, (b) is wrong option.

For first order reaction, rate constant is independent of initial concentration.

So, graph (d) is correct.

6. **(a, c) :** $$\frac{\ln K_1}{\ln K_2} > \frac{T_2}{T_1}$$

On increasing temperature, concentration of product decreases and hence, K decreases.

Since, reaction is exothermic, therefore, $\Delta H° < 0$

From the graph,

$[P]_{eq} > 5$, $[A]_{eq} < 5$

$$K_{eq} = \frac{[P]}{[A]} > 1$$

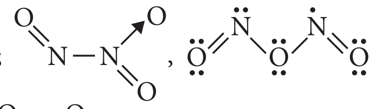
$\Delta G° = -RT\ln K_{eq} \Rightarrow \Delta G° < 0$

$$\frac{\ln K_1}{\ln K_2} = \frac{\frac{-\Delta H°}{T_1 R} + \frac{\Delta S°}{R}}{\frac{-\Delta H°}{T_2 R} + \frac{\Delta S°}{R}} > \frac{T_2}{T_1}$$

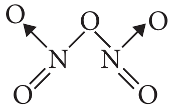
$$\frac{(-\Delta H° + T_1\Delta S°)}{(-\Delta H° + T_2\Delta S°)}\frac{T_2}{T_1} > \frac{T_2}{T_1}$$

$-\Delta H° + T_1\Delta S° > -\Delta H° + T_2\Delta S° \Rightarrow \Delta S° < 0$

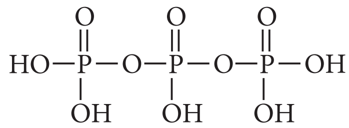
7. **(5 or 6) :** N_2O_3;



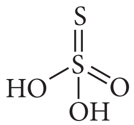
N_2O_5;



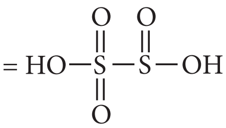
$H_5P_3O_{10}$;



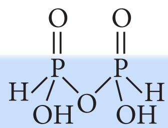
$H_2S_2O_3$;



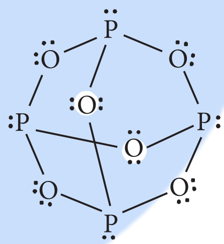
, $H_2S_2O_5$ =



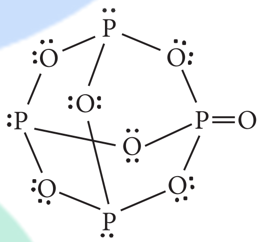
$H_4P_2O_5$;



P_4O_6;



P_4O_7;



8. **(6.47) :** $2PbS + 3O_2 \longrightarrow 2PbO + 2SO_2$

$2PbO + PbS \longrightarrow 3Pb + SO_2$

3 moles of O_2 produce 3 moles of lead.

96 kg of oxygen produce 621 kg of lead.

1 kg of oxygen produce $\frac{621}{96}$ = 6.468 = 6.47 kg

9. **(126) :** From principle of atom conservation,

$2MnCl_2 + 5K_2S_2O_8 + 8H_2O \longrightarrow 2KMnO_4 + 4K_2SO_4 + 6H_2SO_4 + 4HCl$ ...(i)

mmoles of $MnCl_2$ = mmoles of $KMnO_4$ = x(let)

$2KMnO_4 + 5H_2C_2O_4 + 3H_2SO_4 \longrightarrow K_2SO_4 + 2MnSO_4 + 8H_2O + 10CO_2$ ...(ii)

meq of $KMnO_4$ = meq of oxalic acid

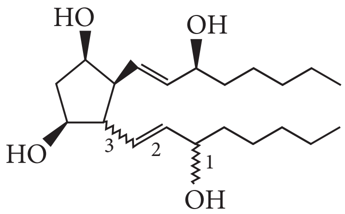
$$x \times 5 = \left(\frac{225}{90}\right) \times 2 \Rightarrow x = 1$$

($\therefore$ mass of oxalic acid added = 225 mg)

$\therefore$ mmoles of $MnCl_2$ = 1

mg of $MnCl_2$ = (55 + 71) = 126 mg

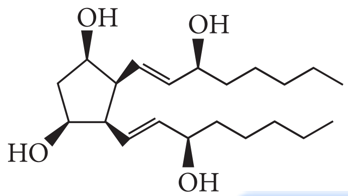
10. (7) :



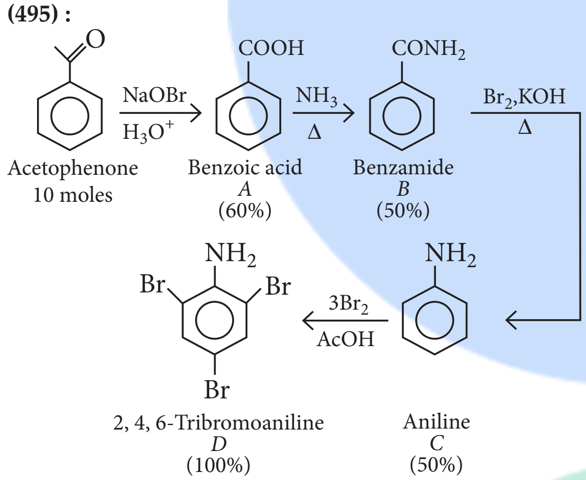
Stereochemistry around these three centres can vary.

$\therefore$ Total isomers = $2^3 = 8$

Out of these eight possible isomers, one isomer will be optically inactive.



11. (495) :



Yield of D in moles $= 10 \times \frac{60}{100} \times \frac{50}{100} \times \frac{50}{100}$

$= 1.5$ moles

Amount of D = Number of moles × Molecular weight

$= 1.5 \times 330 = 495$

12. (–14.6) : $2Cu_{(s)} + \frac{1}{2}O_{2(g)} \longrightarrow Cu_2O_{(s)}$,

$\Delta G° = -78$ kJ/mole ...(i)

$H_{2(g)} + \frac{1}{2}O_{2(g)} \longrightarrow H_2O_{(g)}$, $\Delta G° = -178$ kJ/mole ...(ii)

Subtracting eqn. (ii) from (i)

$2Cu_{(s)} + H_2O_{(g)} \longrightarrow Cu_2O_{(s)} + H_{2(g)}, \Delta G° = 100$ kJ/mole

$\Delta G = \Delta G° + RT \ln K \geq 0$

$$\Rightarrow \quad 10^5 + 8 \times 1250 \ln\left(\frac{p_{H_2}}{p_{H_2O}}\right) \geq 0$$

$$10^4 \ln\left(\frac{p_{H_2}}{p_{H_2O}}\right) + 10^5 \geq 0$$

$\ln p_{H_2} - \ln p_{H_2O} \geq -10$

Now, $\ln P_{H_2O} = X_{H_2O} \times P_{Total} = 0.01 \times 1 = 10^{-2}$

$\therefore \ln P_{H_2} + 2\ln 10 \geq -10$

$\ln p_{H_2} + 4.6 \geq -10 \Rightarrow \ln p_{H_2} \geq -14.60$

13. (8500) : $A_{(g)} + B_{(g)} \rightleftharpoons AB_{(g)}$

Given, $E_{a_b} - E_{a_f} = 2RT$ and $\frac{A_f}{A_b} = 4 \Rightarrow K_{eq} = \frac{K_f}{K_b}$

Also, $K_f = A_f e^{-E_{a_f}/RT}$ ...(i)

$K_b = A_b\, e^{-E_{a_b}/RT}$ ...(ii)

Now, $\frac{K_f}{K_b} = \frac{A_f}{A_b} e^{(E_{a_b} - E_{a_f})/RT}$

or $K_{eq} = 4e^{2RT/RT}$; $K_{eq} = 4e^2$

$\Delta G° = -RT \ln K_{eq} = -RT \ln(4e^2) = -RT\,(2 + \ln 4)$

$= -2500\,(2 + 2 \times 0.7) = -8500$ J mol^{-1}

$\therefore$ Absolute value of $\Delta G°$ is 8500.

14. (–11.62) : $A_{(s)} \longrightarrow A^{n+} + ne^-] \times 2$ ...(i)

$B^{2n+} + 2ne^- \longrightarrow B_{(s)}$

$2A_{(s)} + B^{2n+} \longrightarrow 2A^{n+} + B_{(s)}$

Given, $\Delta H° = 2\Delta G°$, $E_{cell} = 0$

As, $\Delta G° = \Delta H° - T\Delta S°$ $\therefore$ $\Delta G° = 2\Delta G° - T\Delta S°$

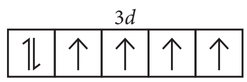
or, $\Delta G° = T\Delta S°$ or $\Delta S° = \frac{\Delta G°}{T}$

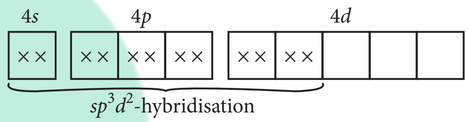
Also, $\Delta S° = \frac{-RT \ln K}{T} = -R\ln\frac{[A^{n+}]^2}{[B^{2n+}]} = -8.3 \times \ln\frac{2^2}{1}$

$\therefore$ $\Delta S° = -\,11.62$ J K^{-1} mol^{-1}

15. (c) : 1. $[FeF_6]^{4-}$, Oxidation state of Fe = + 2

E.C. of Fe^{2+} in $[FeF_6]^{4-}$:

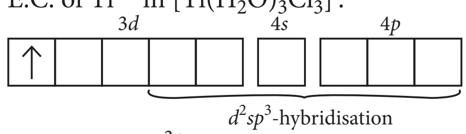




It forms high spin complex because F^- is weak field ligand.

2. $[Ti(H_2O)_3Cl_3]$, oxidation state of Ti = +3

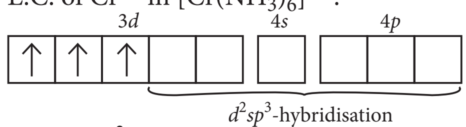
E.C. of Ti^{3+} in $[Ti(H_2O)_3Cl_3]$:



3. $[Cr(NH_3)_6]^{3+}$, oxidation state of Cr = +3

Due to presence of strong field ligand, inner orbital complex is formed.

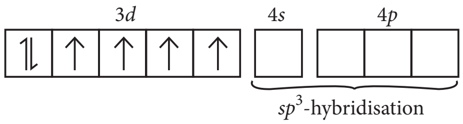
E.C. of Cr^{3+} in $[Cr(NH_3)_6]^{3+}$:



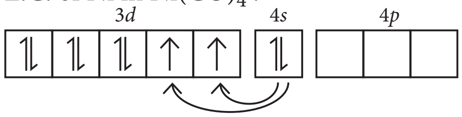
4. $[FeCl_4]^{2-}$, oxidation state of Fe = +2

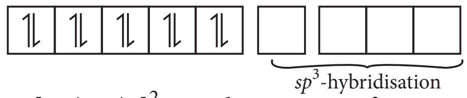
It forms high spin complex as Cl^- is weak field ligand.

E.C. of Fe^{2+} in $[FeCl_4]^{2-}$:

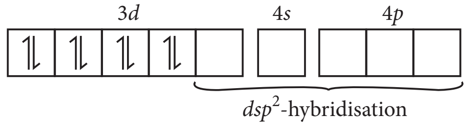


5. $Ni(CO)_4$, oxidation state of Ni = 0
It forms low spin complex as CO is strong field ligand.
E.C. of Ni in $Ni(CO)_4$:

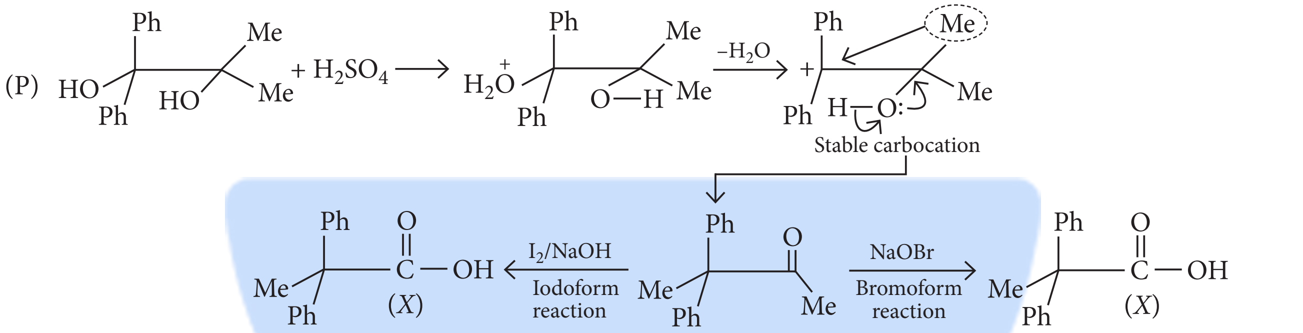




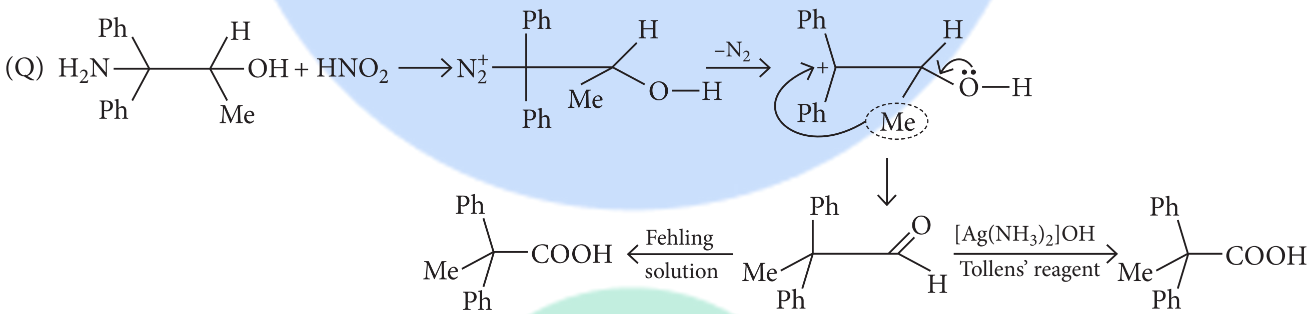
6. $[Ni(CN)_4]^{2-}$, oxidation state of Ni = +2
It forms low spin complex as CN is strong field ligand.
E.C. of Ni^{2+} in $[Ni(CN)_4]^{2-}$:



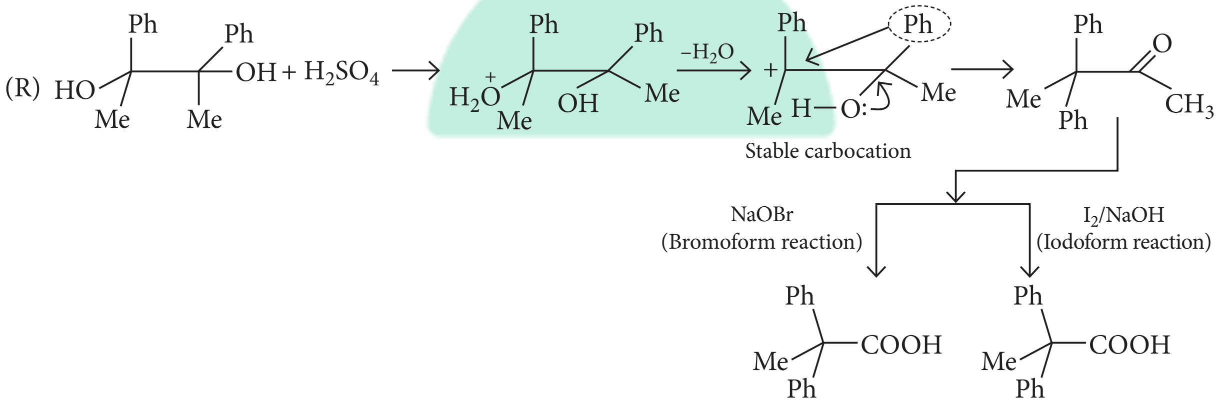
16. (d) :



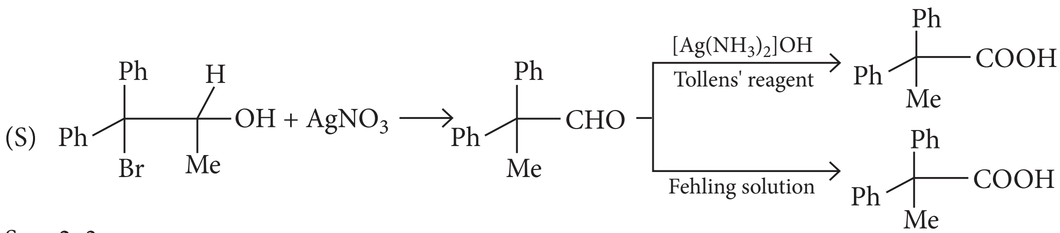
P → 1, 5



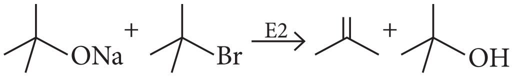
Q → 2, 3



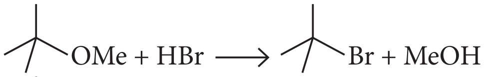
R → 1, 5



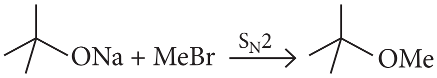
S → 2, 3

17. **(b) :** (P) 

With 3° halide and strong base, elimination predominates.

(Q) 

(R) 

(S) 

P → 1, 4; Q → 2; R → 4; S → 3

18. **(d) :** (P) $[CH_3COOH]_{old} = \frac{20\times 0.1 - 10\times 0.1}{30} = \frac{1}{30}$

$[CH_3COO^-]_{new} = \frac{1}{30}$

Buffer with [Salt] = [Acid]

pH does not change on dilution (P) → (1)

(Q) $[CH_3COO^-]_{old} = \frac{20\times 0.1}{40} = \frac{2}{40}$

$[CH_3COO^-]_{new} = \frac{2}{80}$

$$\underset{c}{CH_3COO^-} + H_2O \longrightarrow \underset{x}{CH_3COOH} + \underset{x}{OH^-}$$

$$K_h = \frac{x^2}{c} = \frac{[OH^-]^2_{old}}{2/40} = \frac{[OH^-]^2_{new}}{2/80}$$

or, $[OH^-]^2_{new} = \frac{[OH^-]^2_{old}}{2}$ or, $[OH^-]_{new} = \frac{[OH^-]_{old}}{\sqrt{2}}$

$\therefore$ $[H^+]_{new} = \sqrt{2}[H^+]_{old}$

(Q) → (5)

(R) $[NH_4^+]_{old} = \frac{20\times 0.1}{40} = \frac{2}{40}$, $[NH_4^+]_{new} = \frac{2}{80}$

$$\underset{c}{NH_4^+} + H_2O \rightleftharpoons \underset{y}{NH_4OH} + \underset{y}{H^+}$$

$$K_h = \frac{y^2}{c} = \frac{[H^+]^2_{old}}{2/40} = \frac{[H^+_{new}]^2}{2/80}$$

or $[H^+_{new}]^2 = \frac{[H^+_{old}]^2}{2} \Rightarrow [H^+_{new}] = \frac{[H^+]_{old}}{\sqrt{2}}$

(R) → (4)

(S) For a saturated solution, $Ni(OH)_2 \rightleftharpoons \underset{s}{Ni^{2+}} + \underset{2s}{2OH^-}$

$K_{sp} = s \times (2s)^2 = 4s^3$

$s = [OH^-] = \sqrt[3]{\frac{K_{sp}}{4}}$

Irrespective of volume of solution, $[H^+]$ remains constant.

(S) → (1)

MATHEMATICS

1. **(d) :** We have, $f_n(x) = \sum_{j=1}^{n} \tan^{-1}\left(\frac{1}{1+(x+j)(x+j-1)}\right)$

$= \sum_{j=1}^{n} [\tan^{-1}(x+j) - \tan^{-1}(x+j-1)]$

$= \tan^{-1}(x+n) - \tan^{-1}(x)$

$\Rightarrow$ $\tan(f_n(x)) = \tan(\tan^{-1}(x+n) - \tan^{-1}x)$

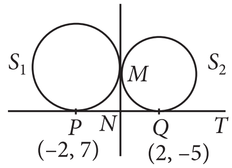
$= \frac{x+n-x}{1+x(x+n)} = \frac{n}{1+x^2+nx}$

$\lim_{x\to\infty} \tan(f_n(x)) = \lim_{x\to\infty} \frac{n}{x^2+nx+1} = 0$

$\lim_{x\to\infty} \sec^2(f_n(x)) = \lim_{x\to\infty} 1 + \left(\frac{n}{1+x^2+nx}\right)^2 = 1$

Note that $f_n(x)$ and $f'_n(x)$ are not defined.

2. **(b, d) :** MN is the common tangent which is the radical axis of the pair of circles.



As $NM = NP = NQ$ we have locus of M happens to be a circle with PQ as diameter. Then it is

$(x - 2)(x + 2) + (y - 7)(y + 5) = 0$

$\Rightarrow$ $x^2 + y^2 - 2y - 39 = 0$

Again, locus of midpoint is a circle with (1, 1) and (0, 1) as ends of diameter, so it is $x(x - 1) + (y - 1)^2 = 0$

i.e. $x^2 + y^2 - x - 2y + 1 = 0$

The equation of line through (–2, 7) and (1, 1) is

$(y - 1) = -2(x - 1)$ *i.e.* $2x + y - 3 = 0$

The foot of perpendicular from (0, 1) on line $2x + y - 3 = 0$ is given by

$$\frac{x-0}{2} = \frac{y-1}{1} = -\frac{2\cdot 0+1-3}{2^2+1^2} \quad i.e. \left(\frac{4}{5}, \frac{7}{5}\right)$$

Note that this is midpoint of chord PR of circle $x^2 + y^2 - 2y - 39 = 0$

As P doesn't lies in the locus of M, PQ can't be a chord of locus of M. So E_2 doesn't contain the point $\left(\frac{4}{5}, \frac{7}{5}\right)$.

3. **(a, d) :** The condition for at least one solution is $D = 0, D_1 = D_2 = D_3 = 0$

For the choice (a), $D \neq 0$, so we have unique solution.

For the choice (b), $D = 0$ but the plane P_3 is not a linear combination of planes P_1 and P_2, hence rejected.

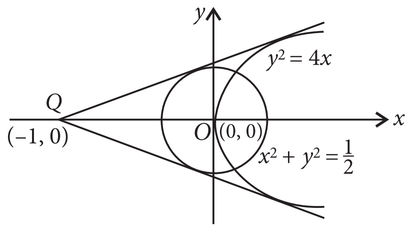
For the choice (c), planes are parallel, but there exists infinitely many points for which the planes become coincident. Hence (c) is false.

The choice (d) satisfies that $D \neq 0$.

4. **(a, c) :** The equation of common tangent be $y = px + 1/p$.

We have, $\frac{|1/p|}{\sqrt{1+p^2}} = \frac{1}{\sqrt{2}} \Rightarrow p^4 + p^2 - 2 = 0$

$\Rightarrow$ $(p^2 + 2)(p^2 - 1) = 0 \Rightarrow p^2 = 1\ (\because p^2 \geq 0)$



$\therefore$ The equations of common tangents are
$x + y + 1 = 0$ and $x - y + 1 = 0$

So, the point Q is (–1, 0).

The equation of ellipse is $\dfrac{x^2}{1} + \dfrac{y^2}{1/2} = 1$

The eccentricity of the ellipse = $\sqrt{1 - \dfrac{1}{2}} = \dfrac{1}{\sqrt{2}}$

Length of latus rectum = $\dfrac{2b^2}{a} = \dfrac{2 \cdot \frac{1}{2}}{1} = 1$

The required area = $2 \displaystyle\int_{1/\sqrt{2}}^{1} \frac{1}{\sqrt{2}} \sqrt{1 - x^2}\, dx$

$$= \sqrt{2}\left[\left(\frac{x}{2}\right)\sqrt{1 - x^2} + \frac{1}{2}\sin^{-1} x\right]_{1/\sqrt{2}}^{1} = \frac{\pi - 2}{4\sqrt{2}}$$

5. **(a, c, d) :** We have, $sz + t\bar{z} + r = 0$
Taking conjugate on both sides, we get $\bar{s}\bar{z} + \bar{t}z + \bar{r} = 0$
Now we have a system of equations in z and $\bar{z}$, given by
$sz + t\bar{z} + r = 0$ and $\bar{t}z + \bar{s}\bar{z} + \bar{r} = 0$
If $s\bar{s} - t\bar{t}$ i.e. $|s|^2 - |t|^2 \neq 0$, then the equation has a unique solution.
If $|s| = |t|$ and $\bar{r}t = r\bar{s}$, then we have infinite solutions.
If $|s| = |t|$ but $\bar{r}t \neq r\bar{s}$, then we have no solution.
So, if $|s| = |t|$, then we are not able to know how many elements L has.
As z is a circle, and L may be line or a point or null set and the intersection can have at most two elements.
If L has more than one element, then L has infinitely many elements as L is a line.

6. **(b, c, d) :** $\displaystyle\lim_{t \to x} \frac{f(x)\sin t - f(t)\sin x}{t - x} = \sin^2 x$

Using L'hospital's rule, we have

$$\lim_{t \to x} \frac{f(x)\cos t - f'(t)\sin x}{1} = \sin^2 x$$

We have, $f(x)\cos x - f'(x)\sin x = \sin^2 x$

$\Rightarrow \dfrac{-f'(x)\sin x + f(x)\cos x}{\sin^2 x} = 1$

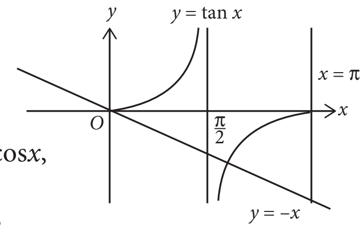
$\Rightarrow d\left(\dfrac{f(x)}{\sin x}\right) = -1 \Rightarrow \dfrac{f(x)}{\sin x} = -x + c$

Now, $f\left(\dfrac{\pi}{6}\right) = -\dfrac{\pi}{12}$ gives $c = 0$. Thus $f(x) = -x\sin x$

(a) $f\left(\dfrac{\pi}{4}\right) = -\dfrac{\pi}{4} \cdot \dfrac{1}{\sqrt{2}} = -\dfrac{\pi}{4\sqrt{2}}$

(b) $\sin x > x - x^3/6$ gives $-x\sin x < -x^2 + x^4/6$

(c) $f'(x) = 0$
$\Rightarrow \tan x = -x$
$\Rightarrow \alpha \in (0, \pi)$
such that $f'(\alpha) = 0$



(d) $f''(x) = x\sin x - 2\cos x$,
$\Rightarrow f''(\pi/2) = \pi/2$
and $f(\pi/2) = -\pi/2$
Thus $f''(\pi/2) + f(\pi/2) = 0$

7. **(2) :** Let $I = \displaystyle\int_0^{1/2} \frac{(1+\sqrt{3})}{[(1+x)^2(1-x)^6]^{1/4}}\, dx$

$$= \int_0^{1/2} \frac{(1+\sqrt{3})dx}{(1+x)^{1/2}(1-x)^{3/2}} = \int_0^{1/2} \frac{(1+\sqrt{3})dx}{(1+x)^2\left(\frac{1-x}{1+x}\right)^{3/2}}$$

Put $\left(\dfrac{1-x}{1+x}\right) = t$ so that $-\dfrac{2dx}{(1+x)^2} = dt$

$$\therefore\ I = (1+\sqrt{3})\int_1^{1/3} \frac{dt}{-2t^{3/2}} = -\left(\frac{1+\sqrt{3}}{2}\right)\left(-\frac{2}{\sqrt{t}}\right)\Bigg|_1^{1/3} = 2$$

8. **(4) :** Let $P = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$\therefore\ \det P = (a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2)$
$- (a_3b_2c_1 + a_2b_1c_3 + a_1b_3c_2)$

As each term is at most 1, then $\det(P) \leq 6$
But it can't be realised.
Suppose $a_1b_2c_3 = a_2b_3c_1 = a_3b_1c_2 = 1$
and $a_3b_2c_1 = a_2b_1c_3 = a_1b_3c_2 = -1$
But $\det(P)$ can not be 6.
Now, for the value to become 5, one term will be zero, but it is not possible. So it is ruled out.
So, 4 is the next possible value which is realised by the

matrix $\begin{vmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$ or $\begin{vmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 1 \end{vmatrix}$.

9. **(119) :** The number of one-one functions
$= {}^7C_5 \cdot 5! = 21 \cdot 120 = 2520 = \alpha$
For the number of onto functions, we do division into groups as follows : 1 1 1 1 3 or 1 1 1 2 2
The number of onto functions is the sum in the two possible scenario, given by

$$\beta = \frac{7!}{3!\ 4!} \times 5! + \frac{7!}{2!2!2!3!} \times 5! = 4 \cdot {}^7C_3 \times 5!$$

$$\therefore\ \frac{\beta - \alpha}{5!} = 4 \cdot {}^7C_3 - {}^7C_5 = 4 \times 35 - 21 = 119$$

10. (0.4) : $\frac{dy}{dx}=(5y+2)(5y-2) \Rightarrow \frac{dy}{25y^2-4}=dx$

On integrating, we get $\frac{1}{25}\cdot\frac{5}{2\cdot 2}\log\left|\frac{y-2/5}{y+2/5}\right|=x+\lambda$

Then, $\log\left(\frac{5y-2}{5y+2}\right)=20(x+\lambda)$.

Again, $f(0) = 0$, so $\lambda = 0$. Then, $\left(\frac{5y-2}{5y+2}\right)=e^{20x}$

Now, $\lim_{x\to-\infty}\left|\frac{5f(x)-2}{5f(x)+2}\right| = \lim_{x\to-\infty} e^{20x}=0$

$\therefore \lim_{x\to-\infty} f(x)=\frac{2}{5}=0.4.$

11. (2) : $f(x+y)=f(x)f'(y)+f'(x)f(y)$ ...(i)

Putting $x = 0$, $y = 0$ in (i), we get

$f(0)=f(0)f'(0)+f'(0)f(0) \Rightarrow f'(0)=1/2$

Again put $x = x$ and $y = 0$ in (i), we get

$f(x)=f(x)f'(0)+f'(x)f(0) \Rightarrow f(x)=\frac{1}{2}f(x)+f'(x)$

$\Rightarrow f'(x)=\frac{1}{2}f(x) \Rightarrow \frac{f'(x)}{f(x)}=\frac{1}{2}$

Thus, $f(x)=e^{(1/2)x+\lambda}$

As $f(0) = 1$, so we have $\lambda = 0 \Rightarrow f(x)=e^{(1/2)x}$

Thus, $\log f(4)=\log e^{4/2}=2.$

12. (8) : Let P be (x_0, y_0, z_0), $Q(x, y, z)$

We have $\frac{x-x_0}{1}=\frac{y-y_0}{1}=\frac{z-z_0}{0}=-\frac{2(x_0+y_0-3)}{2}$

So $x = 3 - y_0$, $y = 3 - x_0$, $z = z_0$

The point lies on z-axis.

$\therefore y_0 = 3$ and $x_0 = 3$

Now the point $(3, 3, z_0)$ is at a distance of 5 from x-axis.

$\therefore 9+z_0^2=25 \Rightarrow z_0^2=16 \Rightarrow z_0=4$

Thus, P is (3, 3, 4) and R is (3, 3, –4). $\therefore PR = 8$

13. (0.5) : After coordinating the point, we have

$S\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$; $T(1, 1, 1)$

$\vec{p}=\frac{\hat{i}-\hat{j}-\hat{k}}{2}, \vec{q}=\frac{-\hat{i}+\hat{j}-\hat{k}}{2}, \vec{r}=\frac{-\hat{i}-\hat{j}+\hat{k}}{2}, \vec{t}=\frac{\hat{i}+\hat{j}+\hat{k}}{2}$

$$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & -1\\ -1 & 1 & -1\end{vmatrix}\times\frac{1}{4}=\frac{1}{4}(2\hat{i}+2\hat{j})=\frac{1}{2}(\hat{i}+\hat{j})$$

$$\vec{r}\times\vec{t}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & -1 & 1\\ 1 & 1 & 1\end{vmatrix}\times\frac{1}{4}=\frac{1}{2}(-\hat{i}+\hat{j})$$

Again, $$(\vec{p}\times\vec{q})\times(\vec{r}\times\vec{t})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 0\\ -1 & 1 & 0\end{vmatrix}\times\frac{1}{4}=\frac{\hat{k}}{2}$$

$\therefore |(\vec{p}\times\vec{q})\times(\vec{r}\times\vec{t})|=\frac{1}{2}=0.5$

14. (646) : $X=\sum_{r=1}^{10}({}^{10}C_r)^2=\sum_{r=1}^{10} r\cdot{}^{10}C_r\cdot{}^{10}C_r$

$=10\cdot\sum_{r=1}^{10}{}^{9}C_{r-1}\cdot{}^{10}C_{10-r}=10\cdot{}^{19}C_9$

$\therefore \frac{X}{1430}=\frac{10\cdot{}^{19}C_9}{1430}=\frac{{}^{19}C_9}{143}=646$

15. (a) : For E_1 the solution is given by

$\frac{x}{x-1}>0$, which gives $x\in(-\infty,0)\cup(1,\infty)$

E_2 is given by $-1\le\log\frac{x}{x-1}\le 1$

$\Rightarrow \frac{1}{e}\le\frac{x}{x-1}\le e \Rightarrow \frac{1}{e}\le 1+\frac{1}{x-1}\le e$

$\Rightarrow \frac{1}{e}-1\le\frac{1}{x-1}\le e-1$

$\Rightarrow x-1\in\left(-\infty,\frac{e}{1-e}\right]\cup\left[\frac{1}{e-1},\infty\right)$

$\Rightarrow x\in\left(-\infty,\frac{1}{1-e}\right]\cup\left[\frac{e}{e-1},\infty\right)$

Also, $\frac{x}{x-1}\in(0,\infty)-\{1\}\ \forall\, x\in E_1$

Then $\log\left(\frac{x}{x-1}\right)\in((-\infty,0)-\{0\})$

$\therefore \sin^{-1}\left\{\log\left(\frac{x}{x-1}\right)\right\}\in\left\{\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}\right\}$

So, we have P → 4, Q → 2, R → 1, S → 1

16. (c) : $\alpha_1={}^6C_3\cdot{}^5C_2=200$

$\alpha_2={}^6C_1\cdot{}^5C_1+{}^6C_2\cdot{}^5C_2+{}^6C_3\cdot{}^5C_3+{}^6C_4\cdot{}^5C_4+{}^6C_5\cdot{}^5C_5$

$=461$

$\alpha_3={}^{11}C_5-{}^6C_5-{}^6C_4\cdot{}^5C_1={}^{11}C_5-6-75$

$={}^{11}C_5-81=381$

$\alpha_4\to$
- G_1 include ${}^4C_1\cdot{}^5C_2+{}^4C_2\,{}^5C_1+{}^4C_3=74$
- M_1 include ${}^4C_2\cdot{}^5C_1+{}^4C_3=34$
- G_1 and M_1, both not there ${}^4C_4+{}^4C_3\cdot{}^5C_1+{}^4C_2\cdot{}^5C_2=81$

$\therefore$ The total number of ways $(\alpha_4)=74+34+81=189$

17. (b) : Given, the area of $\Delta LMN=4\sqrt{3}$ sq. units

$\Rightarrow \frac{1}{2}(2b)(\sqrt{3}b)=4\sqrt{3} \Rightarrow b^2=4 \therefore b=2$

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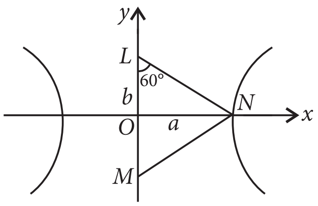

As $\frac{a}{b} = \tan 60° \Rightarrow a = 2\sqrt{3}$

Also, $b^2 = a^2(e^2 - 1) \Rightarrow 4 = 12(e^2 - 1) \Rightarrow e^2 = 1 + \frac{1}{3} = \frac{4}{3}$

$\Rightarrow e = \frac{2}{\sqrt{3}}$

Now, $2ae = 2 \times 2\sqrt{3} \times \frac{2}{\sqrt{3}} = 8$

$\therefore$ Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \cdot 4}{2\sqrt{3}} = \frac{4}{\sqrt{3}}$

18. (d) : (i) $f_1'(0) = \lim_{h \to 0} \frac{\sin\sqrt{1 - e^{-h^2}}}{h}$

$$= \lim_{h \to 0} \frac{\sin\sqrt{1-e^{-h^2}}}{\sqrt{1-e^{-h^2}}} \cdot \sqrt{\frac{1-e^{-h^2}}{h^2}} \cdot \frac{|h|}{h}$$

As $\lim_{h \to 0} \frac{|h|}{h}$ doesn't exist, hence $f_1'(0)$ doesn't exist.

So, P $\to$ 2

(ii) $\lim_{x \to 0} f_2(x) = \lim_{x \to 0} \frac{|\sin x|}{\tan^{-1} x}$

$$= \lim_{x \to 0} \frac{|\sin x|}{|x|} \cdot \frac{x}{\tan^{-1} x} \cdot \frac{|x|}{x} = 1 \cdot 1 \cdot \lim_{x \to 0} \frac{|x|}{x}$$

As $\lim_{x \to 0} \frac{|x|}{x}$ doesn't exist, hence $\lim_{x \to 0} f_2(x)$ doesn't exist.

So, Q $\to$ 1

(iii) For $1 < x + 2 < e^{\pi/2}$, we have $0 < \log(x + 2) < \pi/2$

$\Rightarrow \quad 0 < \sin(\log(x + 2)) < 1$

$\therefore \quad [\sin(\log(x + 2))] = 0$

Thus, $f_3(x) = 0$. Hence R $\to$ 4

(iv) $f_4(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

$$\lim_{x \to 0} f_4(x) = \lim_{x \to 0}\left(x^2 \sin\frac{1}{x}\right) = 0$$

$$f_4'(x) = \lim_{h \to 0} \frac{h^2 \sin\frac{1}{h}}{h} = \lim_{h \to 0} h \sin\frac{1}{h} = 0$$

$\therefore \quad f_4'(x) = -\cos\frac{1}{x} + 2x \sin\frac{1}{x}$

Now, $\lim_{x \to 0} f_4'(x)$ doesn't exist. Hence S $\to$ 3

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